Title: Applied QBism and its Potential

Speakers: John DeBrota

Series: Quantum Foundations

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Abstract: The Quantum Bayesian, or QBist, interpretation regards the quantum formalism to be a tool that a single agent may adopt to help manage their expectations for the consequences of their actions. In other words, quantum theory is an addition to decision theory, and its shape, we hope, can teach us something about the nature of reality. Beyond simple consistency, an interpretation is judged by its capacity to point the way forward. In the first half of the talk, I will highlight several ways in which my collaborators and I have applied QBist intuitions to pose and solve technical questions regarding the informational structure and conceptual function of quantum theory. At the root of many of these developments is the notion of a reference measurement, the key to a probabilistic representation of quantum theory. In this setting, we can explore the boundary of the quantum reasoning structure from a uniquely QBist angle. Working with such representations grants a new perspective and inspires questions which wouldn't have occurred otherwise; as examples, we will meet downstream results concerning quantum channels, discrete quasiprobability representations, and a variant of the information-disturbance tradeoff. Most recently, I have pursued ways in which QBism could be applied to the construction of new tools and strategies for existing problems in quantum information and computation. In the second half of the talk, we will encounter the first of these, an agent-based modeling proposal where multiple, suitably interacting, QBist decision-makers might collectively work out the solution to a task of interest in the right circumstances. I will describe some initial explorations of modeling agent belief dynamics in two contexts: first, an expectation sampling interaction with an eye to agential agreement, and, second, a setting where agents are players of quantum games. In the future, we imagine it is possible that a sufficiently mature development of the agent-based program we have begun could suggest new approaches to quantum algorithm design.

Zoom Link: https://pitp.zoom.us/j/95668668835?pwd=MUJtRGMxbEFzSEdVVmZ3TkR3dVVVZz09

Applied QBism and its Potential

John B. DeBrota

Perimeter Institute Seminar

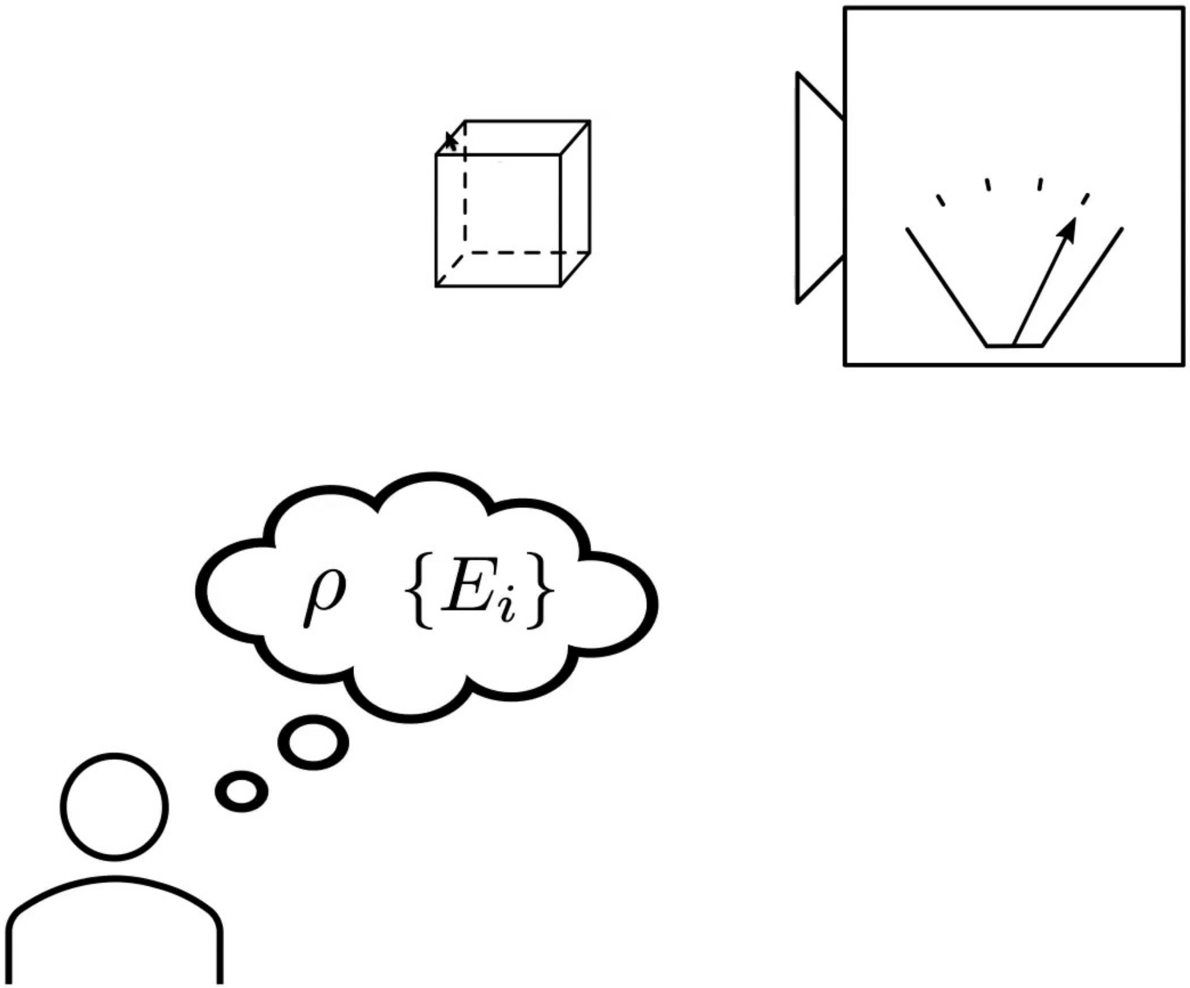
February 1, 2022

QBism at a glance

- Interpretation
  - Epistemic/doxastic
  - Quantum formalism is empirical addition to Bayesian decision theory
- Ontological project
  - What does shape of quantum theory tell us about character of the world?

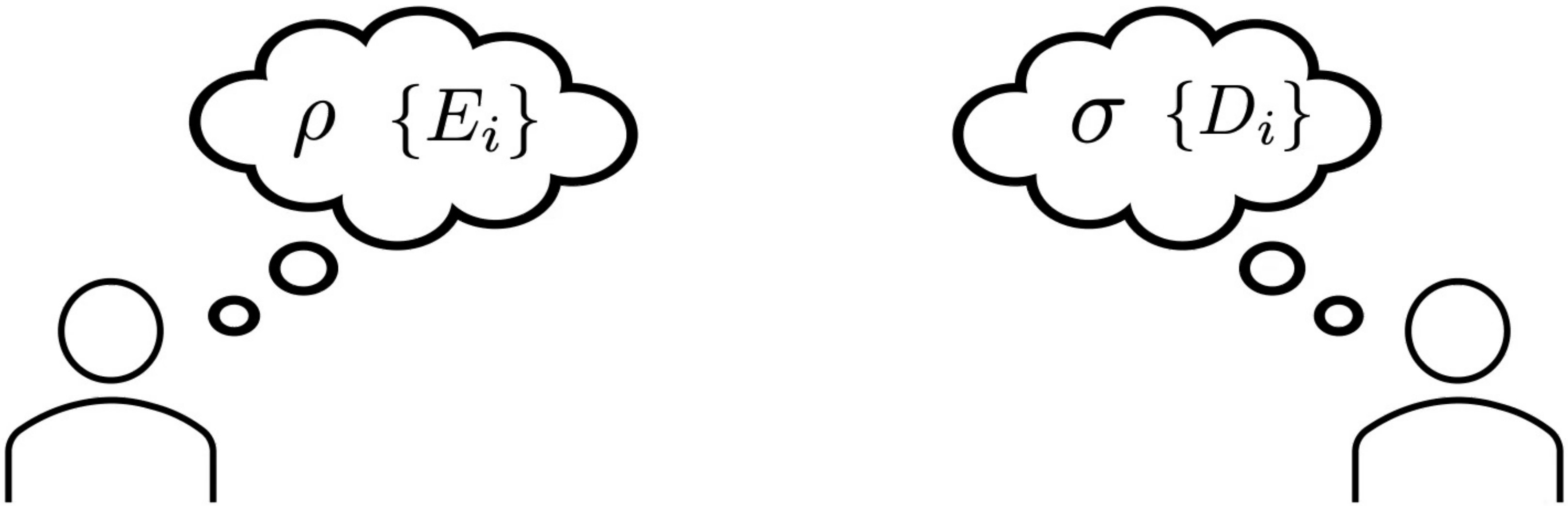
Tenets of QBism

- States, channels, and measurements are personal judgments of an agent
- A measurement device is an extension of an agent
- Measurement outcomes are personal
- The Born rule is a normative and interpersonal


$\rho \quad \{E_i\}$

- Although states and measurements are personal, the *relation* to probabilities is shared:

$$P(E_i) = \operatorname{tr} \rho E_i \qquad\qquad P(D_i) = \operatorname{tr} \sigma D_i$$



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$$P(E_i) = \mathrm{tr}\,\rho E_i \qquad\qquad P(D_i) = \mathrm{tr}\,\sigma D_i$$

- Born rule is reminiscent of probabilistic consistency, but seems to come from the world rather than pure self-consistency
- Asserting P(A)=0.5 *and* P(¬A)=0.6 inconsistent because P(A)+P(¬A)≠1

Why quantum theory?

- Goal is to learn something about the character of reality

Technical work on the road to QBism

- Quantum de Finetti theorems for “unknown” states[1] and processes[2]
- Classical no-cloning and the no-broadcasting theorem[3]
- Conditions for compatibility of quantum state assignments[4]
- Information-disturbance tradeoffs[5] and eavesdropping[6]
- Decoherence and the reflection principle[7]
- Symmetric informationally complete quantum measurements[8] (SICs)!

1. C. M. Caves, C. A. Fuchs and R. Schack, “Unknown quantum states: the quantum de Finetti representation,” *Journal of Mathematical Physics* **43** (2002), 4537–59.
2. C. A. Fuchs, R. Schack and P. F. Scudo, “De Finetti representation theorem for quantum-process tomography,” *Physical Review A* **69** (2004), 062305.
3. H. Barnum, C. M. Caves, C. A. Fuchs, R. Jozsa and B. Schumacher, “Noncommuting mixed states cannot be broadcast,” *Physical Review Letters* **76** (1996), 2818.
4. C. M. Caves, C. A. Fuchs and R. Schack, “Conditions for compatibility of quantum-state assignments,” *Physical Review A* **66** (2002), 062111.
5. C. A. Fuchs and A. Peres, “Quantum state disturbance vs. information gain: Uncertainty relations for quantum information,” *Physical Review A* **53** (1996), 2038.
6. C. A. Fuchs, N. Gisin, R. B. Griffiths, C.-S. Niu and A. Peres, “Optimal eavesdropping in quantum cryptography. I,” arXiv:quant-ph/9701039 (1997).
7. C. A. Fuchs and R. Schack, “Bayesian conditioning, the reflection principle, and quantum decoherence,” *Probability in Physics* (2012), 233–47.
8. J.M. Renes, R. Blume-Kohout, A.J. Scott and C.M. Caves, “Symmetric Informationally Complete Quantum Measurements,” *J. Math. Phys.* **45**, 2171 (2004).

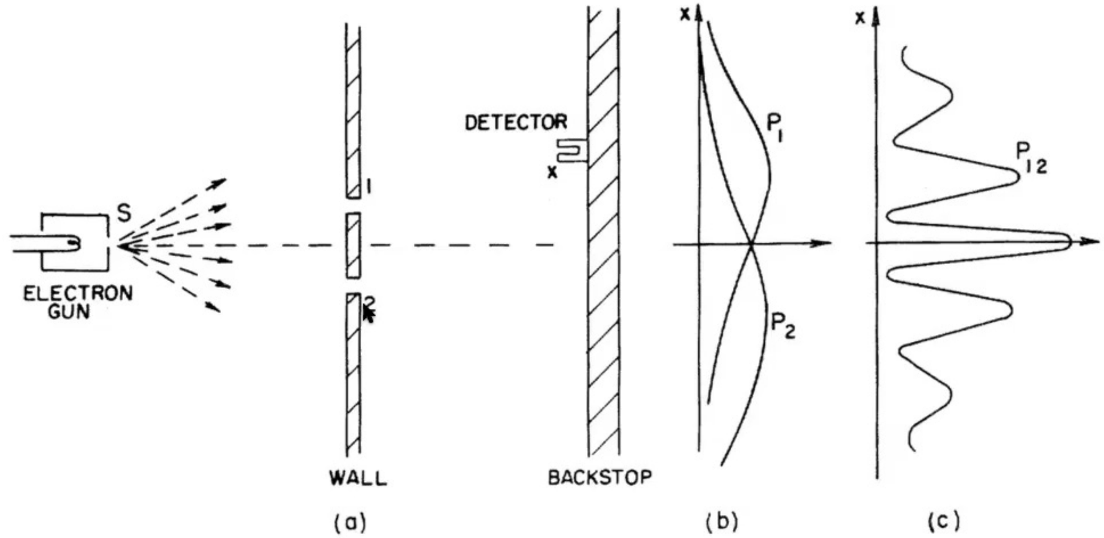
The way forward

- If QT is an addition to decision theory, the way forward could be obscured by the standard Hilbert space representation
  - There, probabilities are the result of manipulations of terms which lack an obvious, direct, decision theoretic interpretation

The way forward

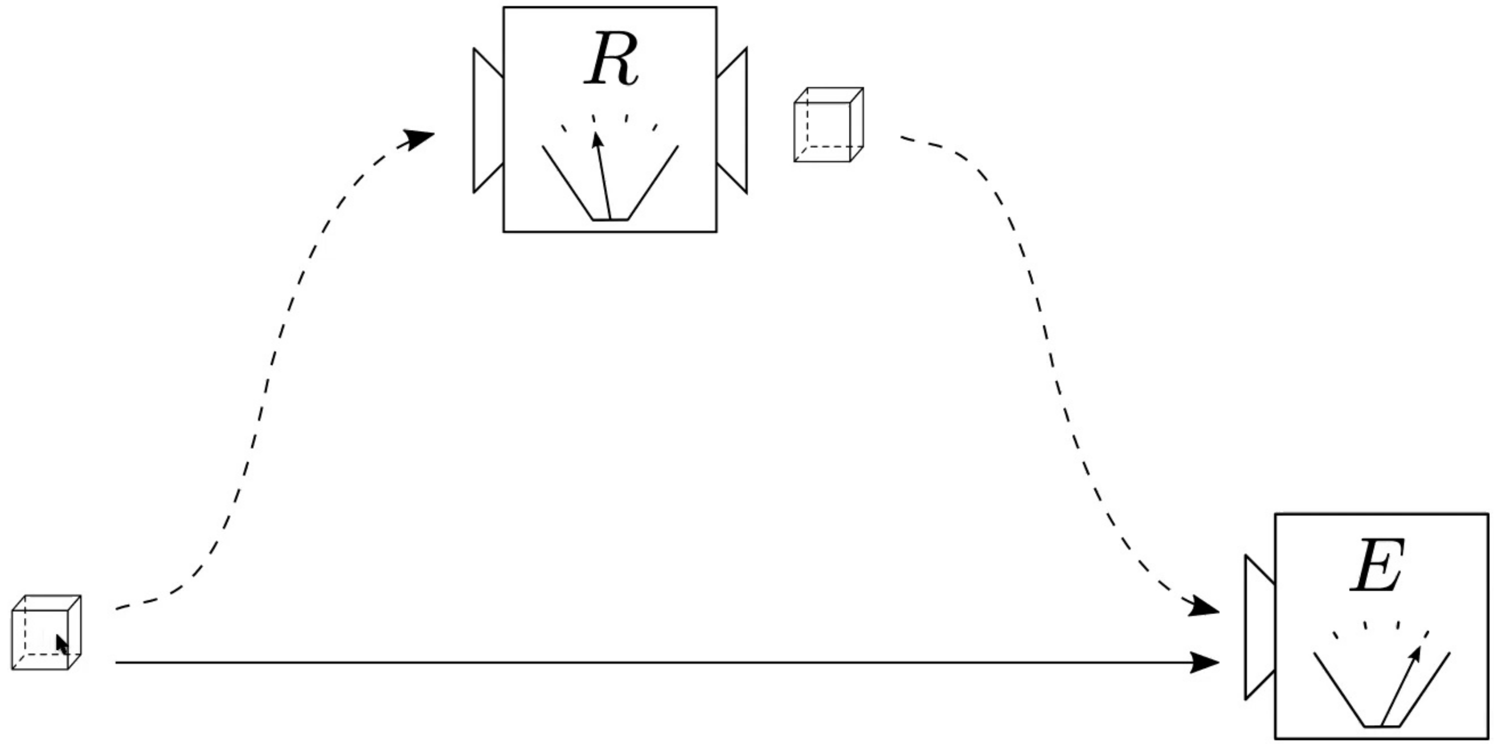
- If QT is an addition to decision theory, the way forward could be obscured by the standard Hilbert space representation
  - There, probabilities are the result of manipulations of terms which lack an obvious, direct, decision theoretic interpretation
- Can we understand quantum theory directly in terms of the probabilities an agent uses in their decision making?
- A fully probabilistic representation may help us see where consistent decision-making ends and physical postulate begins

“The only mystery”



“The result P_{12} obtained with both holes is clearly not the sum of P_1 and P_2 […] [This is] a phenomenon which is impossible, absolutely impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the only mystery.”

Abstraction of double slit experiment



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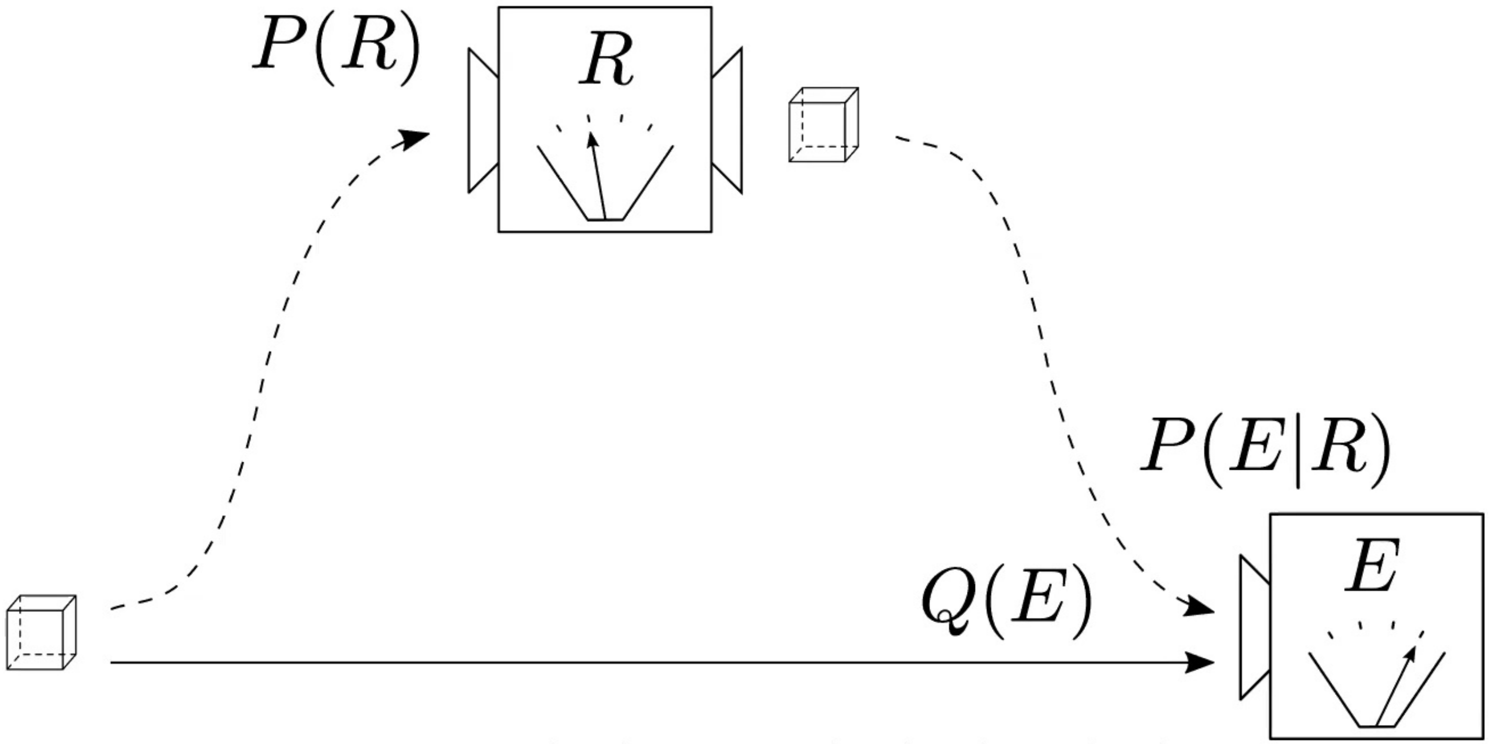


Classical relation: $Q(E) = P(E|R)P(R)$

Reference measurements

- We can still have a probabilistic representation if we can arrange for top path to fix our expectations for the bottom path
- For this, we need $P(R)$ to tell us everything relevant about the system for us to produce probabilities for any other measurement
  - Such a measurement is called a *reference measurement*
- A classical reference measurement simply *reveals* underlying physical coordinates…but these do not exist in quantum mechanics

Informational completeness

- To produce probabilities for any other measurement, $P(R)$ must contain the information of ρ
- Since the Born rule is an inner product, $P(R_i) = \mathrm{tr}\,\rho R_i$, we need the effects of the POVM to span $\mathcal{L}(\mathbb{C}^d)$
- $\dim(\mathcal{L}(\mathbb{C}^d)) = d^2$, so we need at least this many effects
- One can construct a minimal informationally complete POVM (MIC) for any *d*
  - This is a quantum reference measurement and we have $\rho \leftrightarrow P(R)$
  - By similar reasoning, a measurement is a set of probabilities conditional on a reference measurement outcome $E \leftrightarrow P(E|R)$

MIC examples

d=2
N=4

$$|\psi_1\rangle = \begin{pmatrix} \sqrt{\frac{1}{6}(3+\sqrt{3})} \\ \sqrt{\frac{1}{6}(3-\sqrt{3})}e^{i\pi/4} \end{pmatrix} \quad |\psi_2\rangle = \begin{pmatrix} \sqrt{\frac{1}{6}(3-\sqrt{3})} \\ \sqrt{\frac{1}{6}(3+\sqrt{3})}e^{-i\pi/4} \end{pmatrix} \quad |\psi_3\rangle = \begin{pmatrix} \sqrt{\frac{1}{6}(3-\sqrt{3})} \\ \sqrt{\frac{1}{6}(3+\sqrt{3})}e^{3i\pi/4} \end{pmatrix} \quad |\psi_4\rangle = \begin{pmatrix} \sqrt{\frac{1}{6}(3+\sqrt{3})} \\ \sqrt{\frac{1}{6}(3-\sqrt{3})}e^{-3i\pi/4} \end{pmatrix}$$

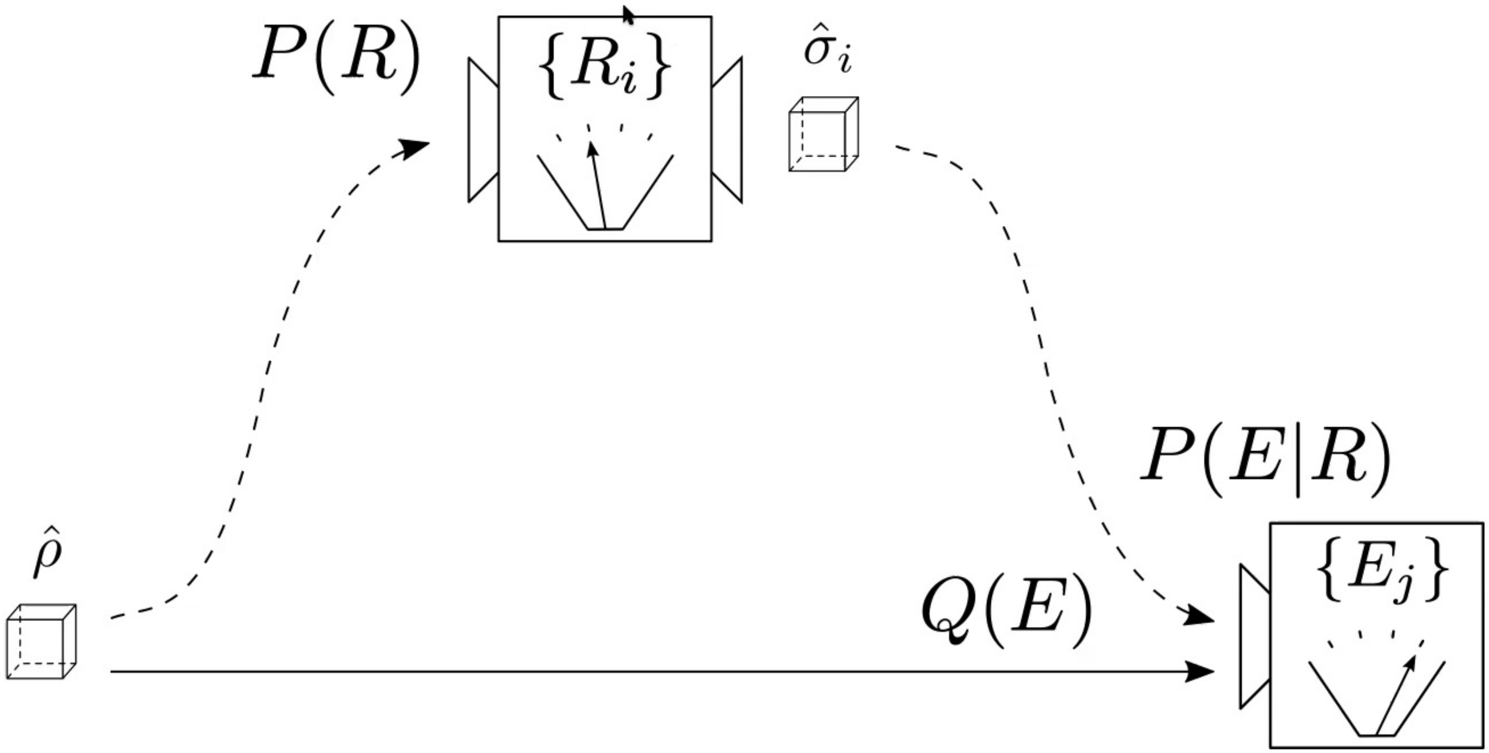
$$R_i = \frac{1}{2}\,|\psi_i\rangle\langle\psi_i|$$

d=3
N=9

$$|\psi_i\rangle = \Big\{(1,0,0), (0,1,0), \frac{1}{\sqrt{2}}(1,0,1), \frac{1}{2}(0,1,1), \frac{1}{\sqrt{2}}(1,0,-i), \frac{1}{\sqrt{2}}(0,1,-i), \frac{1}{\sqrt{3}}(1,-i,i), \\ \frac{1}{\sqrt{40}}(5,-1+2i,-3+i), \frac{1}{\sqrt{24}}(1,3+2i,-3+i)\Big\}.$$

$$R_i = \frac{1}{3}\,|\psi_i\rangle\langle\psi_i|$$

Let R be a MIC and E an arbitrary POVM



A simple manipulation then casts the Born rule in the form

$$Q(E) = P(E|R)\Phi P(R)$$

Where we define the “Born matrix” through its inverse:

$$[\Phi^{-1}]_{ij} = \mathrm{tr}\,\hat{R}_i\hat{\sigma}_j$$

This allows us to directly study the distinction between quantum and classical as different additions to bare probability theory. It is all captured in the fact that

$$\Phi \neq I$$

How nonclassical is quantum?

- If, for some reference process, $\Phi = I$, then quantum mechanics would just be a stochastic theory
- But $\Phi \neq I$. It is *quasistochastic*
  - Generalization allowing negative numbers
  - $\Phi P(R)$ must contain negativity for some ρ
- How "close" can Born rule get to the classical rule?
  - Lower bound $\|I - \Phi\|$

Symmetric Informationally Complete POVMs

For any reference process, $\|I - \Phi\| \geq \|I - \Phi_{\mathrm{SIC}}\|$ with equality if and only if the MIC is a *symmetric* MIC (SIC), where

$\Phi_{\mathrm{SIC}} = (d+1)I - \frac{1}{d}J$ and the Born rule becomes

$$Q(E_j) = \sum_{i=1}^{d^2} \left[(d+1)P(R_i) - \frac{1}{d}\right] P(E_j|R_i)$$

9. JBD, C. A. Fuchs, B. C. Stacey, "Symmetric Informationally Complete Measurements Identify the Irreducible Difference between Classical and Quantum Systems", *Phys. Rev. Research* **2**, 013074 (2020).

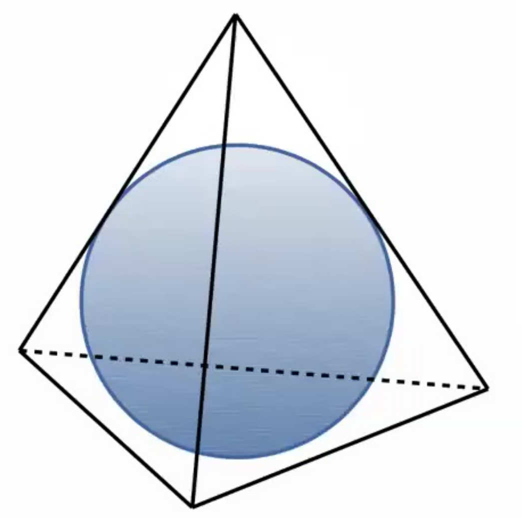
What is a SIC?

- How many equiangular lines fit in $\mathbb{C}^d$?
- It seems $N_{max} = d^2$
- Explicit examples are known for d=2-193 and in several higher dimensions, but a general proof remains elusive
- Given such a set, a SIC-POVM is the quantum reference measurement

$$|\psi_i\rangle \to R_i = \frac{1}{d}\,|\psi_i\rangle\langle\psi_i|$$

SIC optimality

- Probability region corresponding to quantum state space is maximized[9] when the MIC is a SIC



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Do they exist?

- More promising than brute force searches: SICs sit at the intersection of many otherwise distinct topics
  - Design theory, frame theory, Lie and Jordan algebras, graph theory, combinatorics, discrete geometry
  - Most recently, algebraic number theory[12]—Hilbert's 12th unsolved problem!

12. M. Appleby, S. Flammia, G. McConnell, and J. Yard, "SICs and Algebraic Number Theory," *Found Phys* **47**, 1042 (2017).

Could a reconstruction *start* with SICs?

- Why the quantum? $\longleftrightarrow$ SIC existence[13]
  - Ideal reference measurement (think Carnot engine, ideal gas, light clock)
  - Certainty is possible for some measurements, but *reference* distributions inevitably overlap

$$L \leq \langle p, s \rangle \leq U$$

- Born rule from Bayesian coherence (and only a little more)[14]
  - Equiangular reference measurement
  - Measurement outcomes are noncontextual
  - "Everything not forbidden must be allowed"
  - Minimum number of outcomes for a reference is d^2

13. B. C. Stacey, "Quantum Theory as Symmetry Broken by Vitality," arXiv:1907.02432.
14. JBD, C. A. Fuchs, J. L. Pienaar, and B. C. Stacey, "Born's rule as a quantum extension of Bayesian coherence," *Physical Review A*, **104**(2):022207, 2021.

Mapping the MIC landscape

- Studying the full MIC landscape may also help us build intuition about the shape of the reasoning structure[15]
    - Focus on Gram matrices which are useful to characterize bases and frames
    - Unlike “observables” a MIC can never be orthogonal; departure from orthogonality is an indication of nonclassicality
- There are low dimension mysteries to unpack:
    - How many orthogonal pairs are possible in a MIC?
    - Interesting features in *randomly* generated low dimension MIC Gram matrix spectra

- *Open question*: Given a computational task, is there an optimal/approximate reference measurement which improves efficiency?

15. JBD, C. A. Fuchs, and B. C. Stacey. “The varieties of minimal tomographically complete measurements,” *Int. J. Quantum Inf.,* 2040005, 2020.

What about update rules?

- Post-measurement states in general may depend on ρ
  - For entanglement breaking, only "memory" of ρ is in reference probability
  - We could consider MIC measurements with respect to another interaction mode
- In the usual context of measuring an observable, one projects into the eigenspace corresponding to the observable
  - This is a special case of Lüders rule:

$$\rho \to \sigma_i = \frac{\sqrt{E_i}\rho\sqrt{E_i}}{\operatorname{tr} \rho E_i}$$

SICs and depolarization

- The SIC LMC reduces to $\mathcal{E}_{\mathrm{SIC}}(\rho) = \frac{I+\rho}{d+1}$
- This is a depolarizing channel with $\alpha = \frac{1}{d+1}$

$$\mathcal{E}_\alpha(\rho) = \alpha\rho + \frac{1-\alpha}{d} I, \quad \frac{-1}{d^2-1} \leq \alpha \leq 1$$

- Which LMCs are depolarizing? It turns out[16] a depolarizing LMC must satisfy $\frac{1}{d+1} \leq \alpha \leq 1$, and the lower bound is saturated *only* by a SIC
- Using majorization and Schur concavity, we also found SIC LMCs maximize the average post-channel entropy for pure state inputs and the entropy exchange for the maximally mixed input

16. JBD and B. C. Stacey, "Lüders channels and the existence of symmetric-informationally-complete measurements," *Physical Review A* **100**, 062327 (2019).

Discrete quasiprobability representations

- Recall the probabilistic Born rule $Q(E) = P(E|R)\Phi P(R)$
- As Φ is quasistochastic, $Q(E) = P(E|R)(\Phi P(R))$ is a quasiprobability representation
  - One where all the negativity is in the state of the system. Contrast this with discrete Wigner functions, where negativity is shared
- So, we thought[17], why not $Q(E) = \left(P(E|R)\Phi^{1/2}\right)\left(\Phi^{1/2}P(R)\right)?$
- This square root, it turns out, corresponds to an orthogonalization of a rescaling of the MIC
  - Discrete Wigner functions may be thought of as orthogonalized MICs

17. JBD and B. C. Stacey, "Discrete Wigner functions from informationally complete quantum measurements," *Physical Review A*, **102**(3):032221, 2020.

An inverse problem

- Orthogonalization is many-to-one
- What are the sensible MIC inverses of the phase point operators of a discrete Wigner function?
  - How are phase space properties reflected in such MICs?
  - When are there desirable MICs (e.g. rank-1, group covariant, equiangular) in an equivalence class?

17. JBD and B. C. Stacey, "Discrete Wigner functions from informationally complete quantum measurements," *Physical Review A*, **102**(3):032221, 2020

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  - When are there desirable MICs (e.g. rank-1, group covariant, equiangular) in an equivalence class?
- One strategy might be look for the “closest” MIC by sum of operator distances
  - By this metric, a SIC is the closest an unbiased MIC can get to a Wigner function

17. JBD and B. C. Stacey, “Discrete Wigner functions from informationally complete quantum measurements,” *Physical Review A*, **102**(3):032221, 2020

Disturbance, information, and orthogonality

- Given a measurement, the Lüders interaction mode is minimally disturbing: maximizes input-output fidelity averaged over pure inputs[18]
- Restricting to Lüders updating, we[19] quantify a measurement's intrinsic disturbance with an upper bound of the difference between an input state and the expected post-measurement state: $D(\mathcal{E}) = \|\mathcal{E} - \mathcal{I}\|^2$
- This expression decomposes into parts we can interpret independently:
  - Measurement strength *R*: Expected gain in an invariant information quantity based on the Wigner—Yanase skew information
$$I(\rho, X) = -\frac{1}{2}\,\mathrm{tr}[\sqrt{\rho}, X]^2$$
  - Orthogonality *O*: A measure of a POVM's deviation from an orthonormal set

18. H. N. Barnum, Quantum information theory, Ph.D. thesis, University of New Mexico (1998).
19. Y. Liu and JBD, "Relating measurement disturbance, information and orthogonality," *Physical Review A*, **104**:052216, 2021.

Disturbance, information, and orthogonality

$$D = 2d(R+1) - d^2 - n + O$$

- Projective measurements are orthogonal, $O = 0$, and maximize $R = d - 1$, but since $n = d$, $D_{\mathrm{vN}} = d^2 - d$
- Increasing *n* incurs *O* cost. This is minimized by equiangularity: $D_{n\mathrm{EA}} = \frac{n(d-1)^2}{n-1}$ which decreases until $n = d^2$ at a SIC
- Classical ideal measurement is perfectly informative, orthogonal, and perfectly nondisturbing; other measurements are coarse grainings
- Maximizing measurement strength while minimizing disturbance and orthogonality suggests SICs are the quantum analogs of classical ideal measurements

19. Y. Liu and JBD, "Relating measurement disturbance, information and orthogonality," *Physical Review A*, **104**:052216, 2021.

We've seen how QBism can be applied to pose new foundational and technical questions regarding the informational structure and conceptual function of quantum theory

- Some more projects currently underway along these lines:
  - Real Hilbert space QM in dimension 4
  - Learning the Born Rule from an optical experiment
  - VQE with reference measurements

Can QBism additionally motivate the development of new tools with which to address existing problems?

What to make of agents?

- Recall it is assumed in QBism that there is someone who uses the theory; QBism is natively agent-based
  - While it is possible for two agents to have same beliefs/states, it is never ensured without additional assumptions
  - Same with measurement outcomes
- This is a feature, not a bug
  - For Bayesians, it is more natural to examine the situations where agreement or consistency between agents might occur than to assume it

Wigner's friend

- Friend makes measurement on system, obtains outcome. Wigner outside lab, treats lab, friend, and system as continuously evolving quantum system. For Wigner, no outcome of Friend's measurement. Who is right?
- Problem if outcomes or states are agent-independent facts of the world
- But for QBism, differing accounts *between* agents regarding the occurrence of an outcome is no more unusual than the fact that different people have different experiences
- Extensions of WF scenario drove home how different a QBist treatment of multiple agents is from the norm[20]

20. JBD, C.A. Fuchs, and R. Schack, "Respecting One's Fellow: QBism's Analysis of Wigner's Friend," *Foundations of Physics*, 2020.

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We should lean into the differences

- I want to see if QBism has something new to say about computation and algorithm design
- If the treatment of multiple agents is where QBism diverges most, then we should simulate multiple users of quantum theory and see where it takes us

Agent-based physics

- An agent is an actor, a decision-maker
- Agent-based model is when agents are a basic object
  - May be varyingly complex, “smart”, informed, as desired

Agent-based physics

- An agent is an actor, a decision-maker
- Agent-based model is when agents are a basic object
  - May be varyingly complex, “smart”, informed, as desired
- Simulate interactions, look for emergent properties
  - Economics, biology, ecology, social sciences…typically not physics
- What does it mean to be an agent capable of quantum communication, cryptography, games, QIP protocols, etc?

Bayesian decision-makers

- Although so far, we've been concerned with probabilities alone, the point of probabilities are to inform decisions
- In Bayesian decision theory, probabilities operationally arise out of "coherent preference" in decision making
  - Quantified degrees of belief are *probabilities*
  - Preference among consequences can be given values with a *utility function*
  - Most preferred actions *maximize expected utility*

Inference

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  - Most preferred actions *maximize expected utility*
- So, a QBist decision maker chooses among actions (measurements) based on a utility function over outcomes

Inference

- Bayes rule tells us how beliefs in hypotheses are affected by data

$$P(H|D) \propto P(D|H)P(H)$$

- If the data are outcomes of measurements, the hypotheses would be reference probabilities (e.g., quantum states) and P(D|H) is the agent's physical postulate for a chosen measurement

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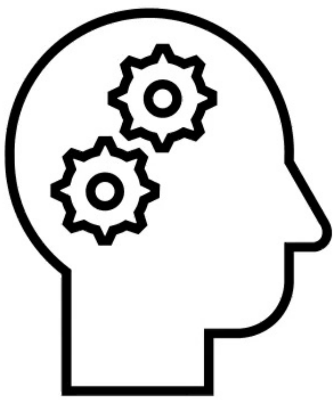
$$P(\rho|j) = \frac{p(j|\rho)P(\rho)}{\int P(\rho)p(j|\rho)d\rho}$$

- In this way, quantum and classical agents can revise and refine their beliefs about systems they receive
  - This will allow us to study agents' belief dynamics in simulations

Quantum state tomography

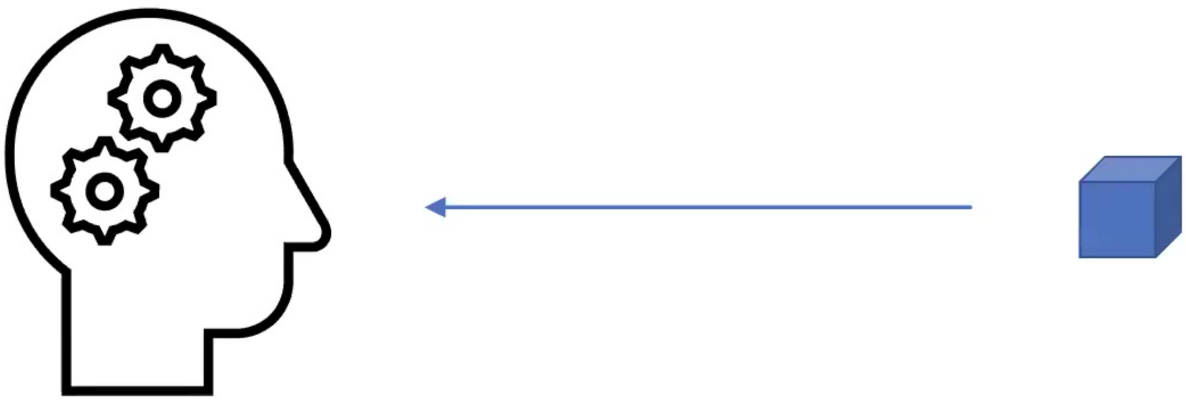
- Rather than just “flipping the coin”, agent can take any of an IC set of measurements (e.g., Pauli X, Y, and Z for a qubit)



$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

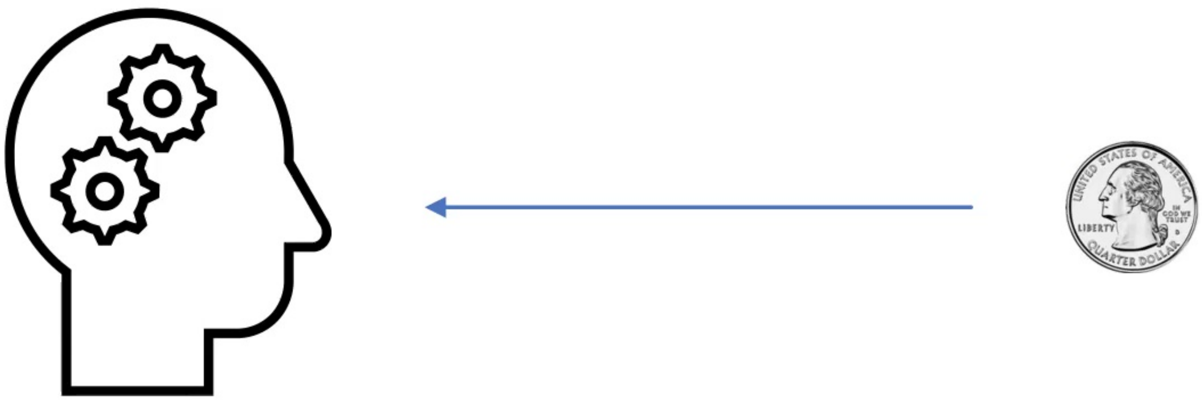
State tomography

- Classical and quantum agents alike can estimate states produced by a source



Estimating the bias of coins

- Source prepares a sequence of biased coins, agent flips each in succession and updates prior with Bayes rule based on outcomes



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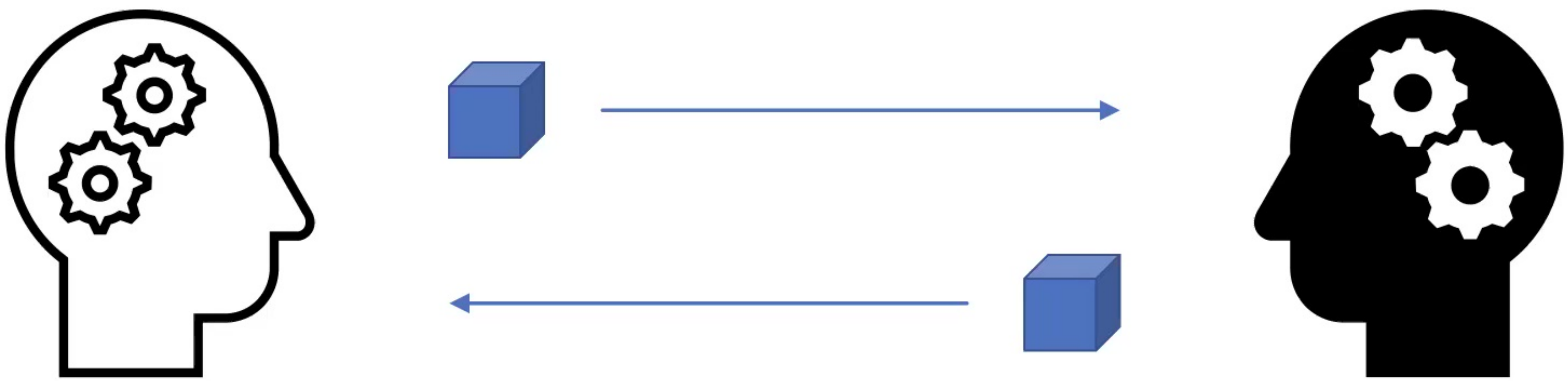
$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

How can two agents interact?

- We need properties of each agent to affect outcome for the other
- Recall, for QBism, the physics is in the structure of each agent's reasoning
  - How we, the simulators, model an interaction is not constrained by physics
- On the other hand, for agents to behave in interesting ways, we need to think up clever ways for them to interact
- We invented an interaction[21] to illustrate the framework with two motivations in mind:
  - Each agent might be thought of as a source for the other
  - Sometimes agreement can be reached through interaction

21. JBD and P.J. Love, "Quantum and Classical Bayesian Agents," arXiv:2106.09057.

Broadcasting and receiving systems/signals

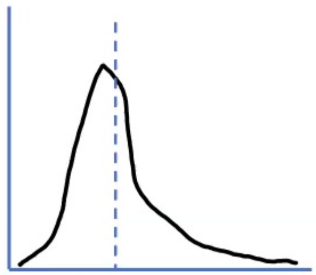


In each step, each agent sends a system to the other and acts on the system they receive

Expectation sampling

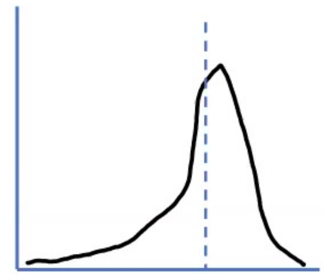
Outcomes for each agent are sampled from other's expectation

$P_A(\rho)$



$\langle \rho_A \rangle$

$P_B(\rho)$

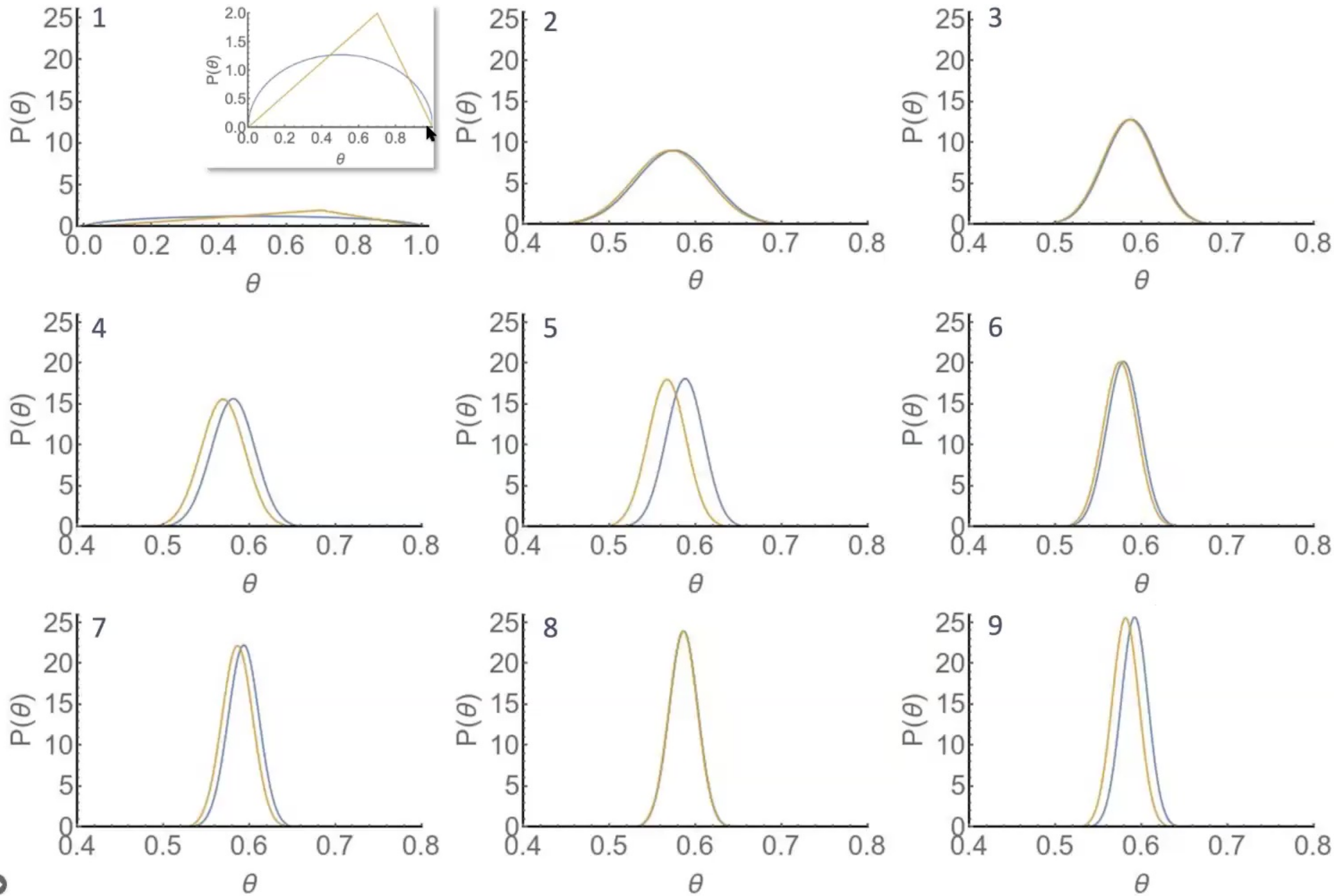


$\langle \rho_B \rangle$

- Alice chooses a measurement $\{E_i\}$
- Gets outcome distributed by $\langle \rho_B \rangle$

- Bob chooses a measurement $\{D_i\}$
- Gets outcome distributed by $\langle \rho_A \rangle$

Then each updates their prior with Bayes rule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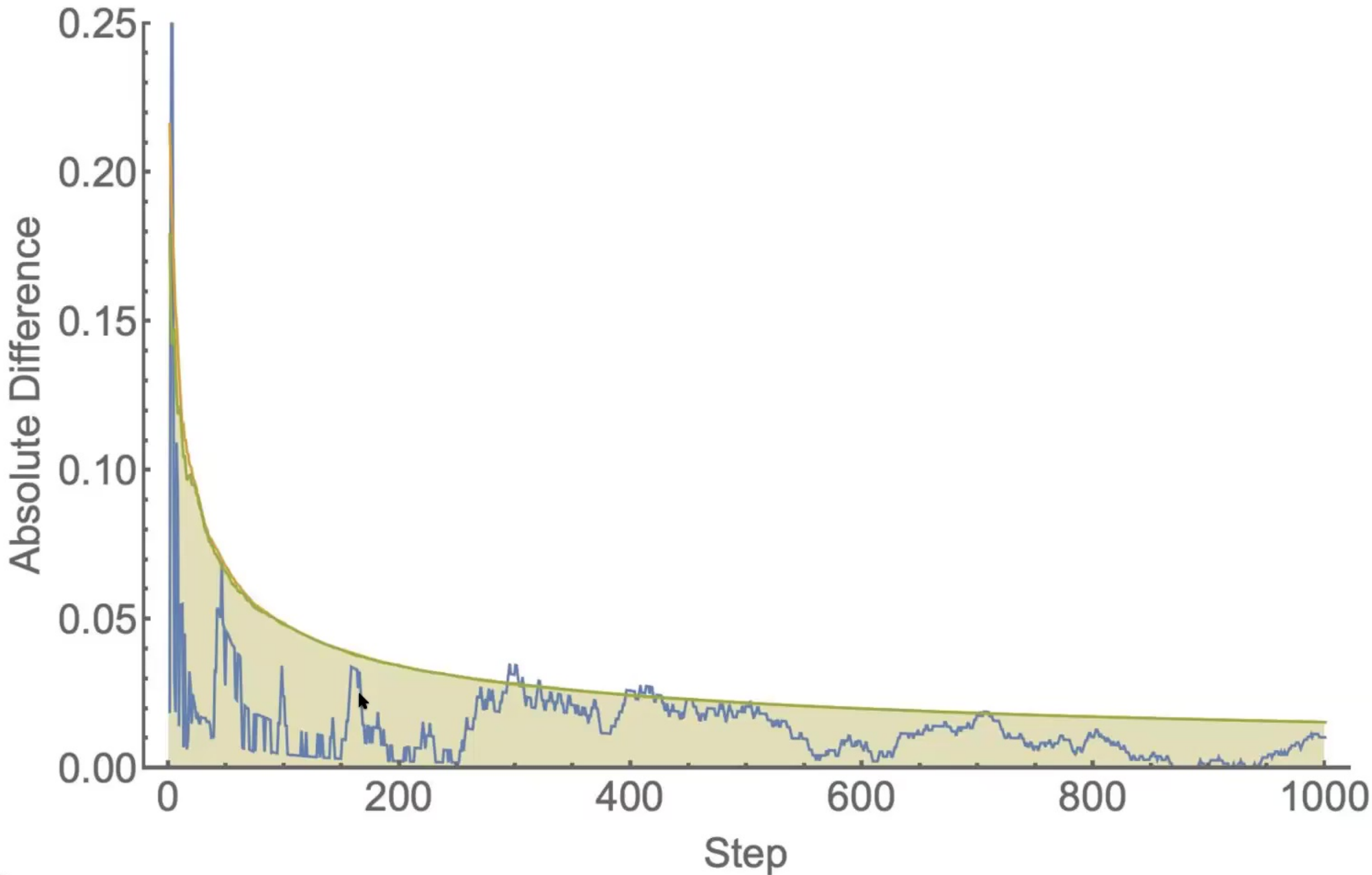
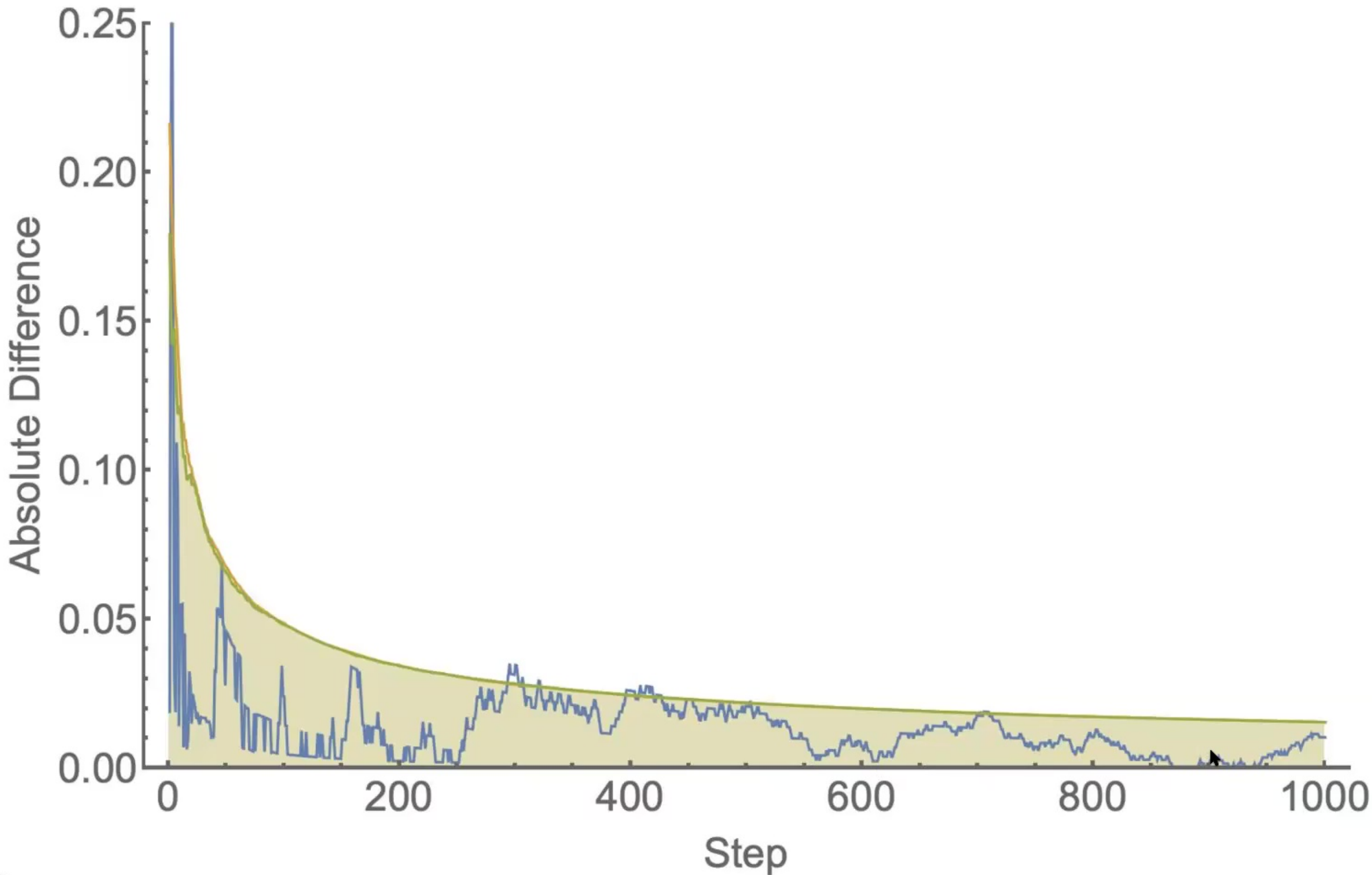
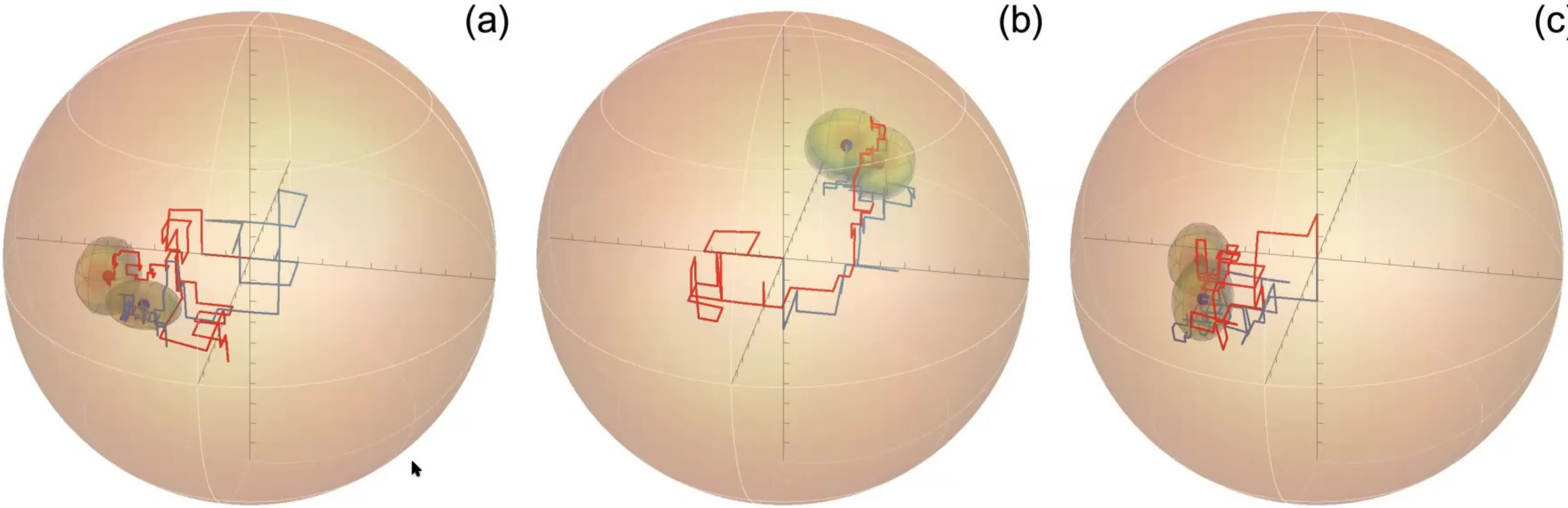
6
7
8
9
P(θ)
θ
0 5 10 15 20 25
0.0 0.2 0.4 0.6 0.8 1.0
0.0 0.5 1.0 1.5 2.0
0.4 0.5 0.6 0.7 0.8


0.25
0.20
0.15
0.10
0.05
0.00
Absolute Difference
0
200
400
600
800
1000
Step

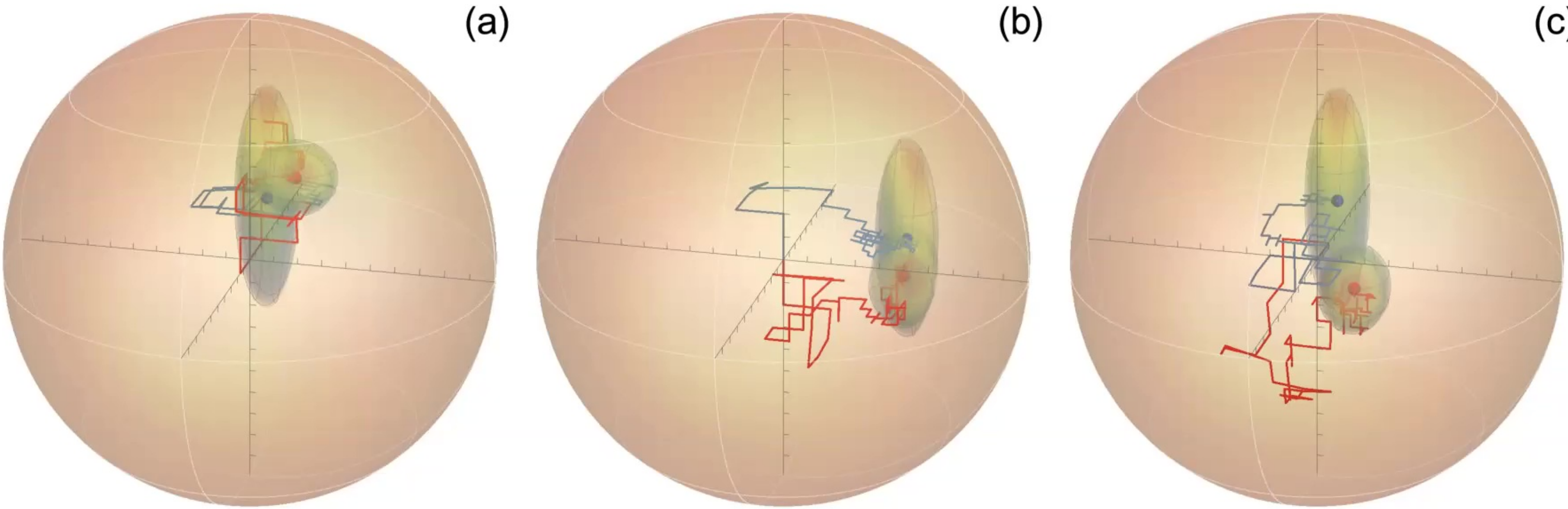

0.25
0.20
0.15
0.10
0.05
0.00
Absolute Difference
0
200
400
600
800
1000
Step

We can do the same with qubits

- Now instead of coins with a particular bias, each prepares a quantum system with a particular state based on current prior
- Suppose again that each agent can choose among the Pauli measurements for each interaction
- As the measurement is a choice, each decides by choosing the Pauli which maximizes their expected utility
  - They tiebreak by choosing uniformly at random from the maximizing options



- Final posterior densities for 3 simulations of 100 updates of 2 agents interacting by expectation sampling
  - Uniform priors over Bloch ball
  - Uniform utilities: Each randomly choose one of 3 Pauli measurements each step
- Red and blue dots are means of final posteriors. Red and blue lines are mean trajectory
  - Final distributions are similar, suggests agreement
  - Where they end up depends on choices and outcomes



- Same situation, but this time Bob has a nonuniform utility function
  - u({X+,X-,Y+,Y-,Z+Z-})={1,1,1,1,0.98,1.02}
- After a few interactions, the Z measurement is no longer a rational choice for him
  - His posteriors end up smeared out vertically
- Seems to approach agreement in *x* and *y* directions

Various quantum-classical interactions

- With minor modifications to expectation sampling, we can have several kinds of interactions between quantum and classical agents
  - May help address how physical postulate affects belief dynamics
  - Investigate when/how/why quantum outperforms classical reasoning
- Classical because of experimental limitation
  - Quantum subtheory, e.g., single axis of Bloch ball as counterpart to qubit
  - Interaction between agents with different sized reference measurements

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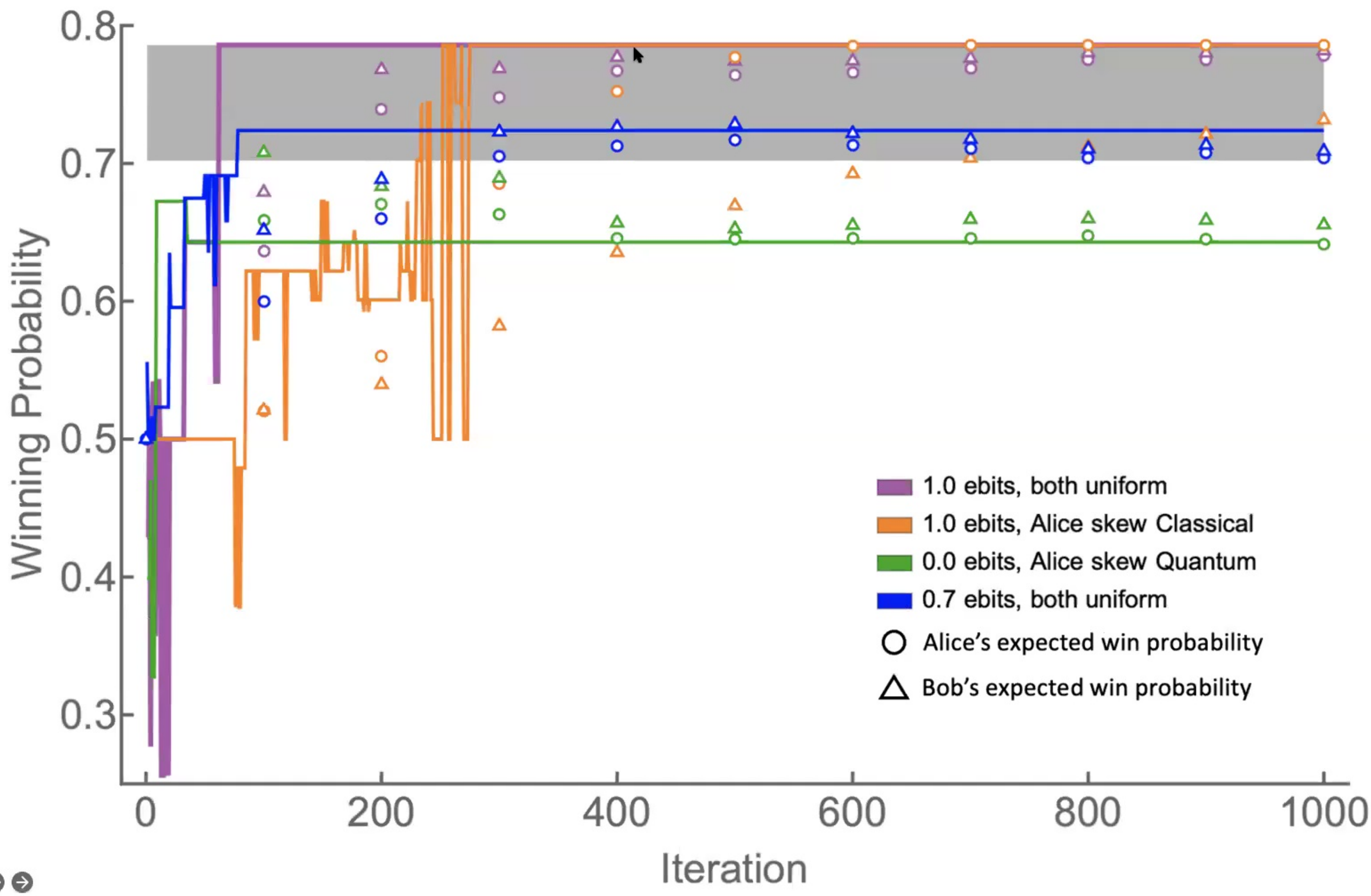
  - Quantum subtheory, e.g., single axis of Bloch ball as counterpart to qubit
  - Interaction between agents with different sized reference measurements
- Classical because of disbelief in quantum theory
  - Same measurement apparatus calibrations, that is, $P(E|R)$, but $\Phi = I$
  - Classical agent limited by their choice of theory rather than experimental capabilities

Agents playing games

- Interaction is now derived from the game's rules
- How do QBist agents fare in game settings?
  - As with many things, the answer depends on their priors
- So far, we have considered the CHSH game and the prisoners' dilemma
- In some circumstances, initially ignorant players can learn to leverage quantum resources and take optimal or nearly optimal actions

CHSH game

- Alice and Bob are given bits *x* and *y* and return bits *a* and *b.* They win if $x \wedge y = a \oplus b$
- Always returning a=b=0 or a=b=1 wins with probability 3/4 and is superior to any probabilistic strategy based on shared random variable
- If they initially share a maximally entangled state and condition replies on measurement outcomes, the optimal winning probability is ≈ 10% higher
- With priors for the action of their partner and the amount of shared entanglement, rational agents can play iterations of this game
  - Being rational may not be optimal for a given round
  - But after many iterations they tend to play well, beating out the classical optimum by detecting shared entanglement, if present

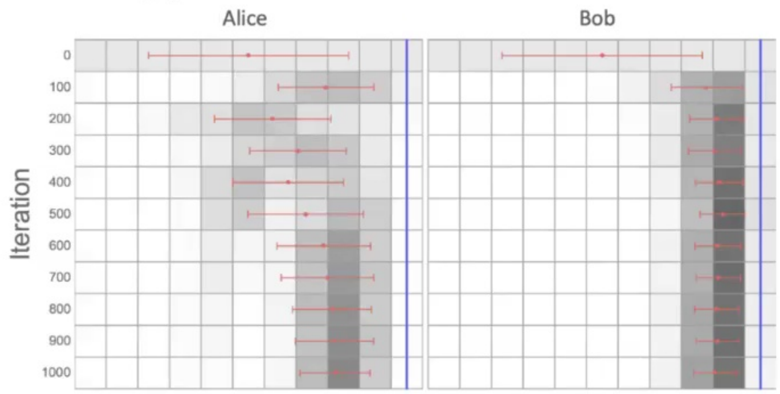


Winning Probability
0.8
0.7
0.6
0.5
0.4
0.3
0
200
400
600
800
1000
Iteration
1.0 ebits, both uniform
1.0 ebits, Alice skew Classical
0.0 ebits, Alice skew Quantum
0.7 ebits, both uniform
Alice's expected win probability
Bob's expected win probability

Prisoners' dilemma

A/B	C	D
C	3,3	0,5
D	5,0	1,1

- Alice and Bob are prisoners, each presented a choice. They choose bit value *a* and *b* to send to warden and are rewarded by payoff matrix
- Mutual defection is a dominant Nash equilibrium when each only has two choices, although mutual cooperation is Pareto optimal
- If they share a maximally entangled state, apply one of a family of unitaries on their half, and allow their choice to be conditioned on a measurement by the warden, a new Pareto optimal Nash equilibrium appears
- Rational agents can similarly play a discretized version, updating their beliefs about their opponent's action and the game entanglement with each iteration



Future plans with agents

- More games → enacting a simple known algorithm?
  - If so, based on what they needed to know to do it, perhaps they can find new algorithms as well
- Agents on graphs/networks indicating who can interact
  - Graph rewriting → social network
  - Emergent metric structure → belief geometry
- Physics emerging from agency
  - Updating physical postulate? Postulate from collective beliefs?
  - The pressures which lead agents to adopt QM would strongly indicate the kind of postulates we seek and could point to what's next

Thank you!

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