Title: TBA

Speakers: Daniel Ranard

Series: Perimeter Institute Quantum Discussions

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Abstract: Abstract: TBD

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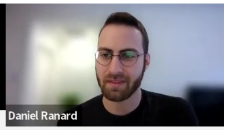


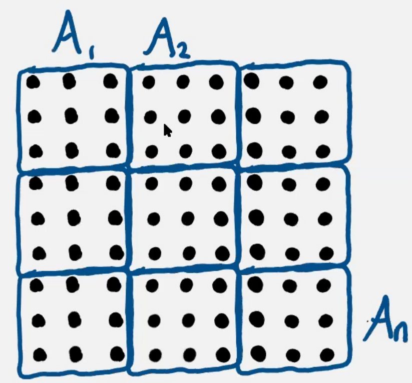
Coarse-grained entropy, microstates, and the quantum marginal problem

Daniel Ranard (MIT)

Perimeter Institute, February 2022

Outline

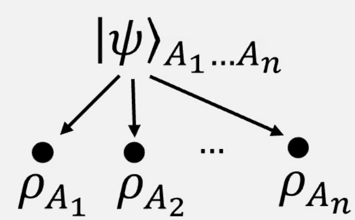
- Review:
  - Coarse-grained entropy
  - Quantum marginal problem
- Main results
  - $\log M < S_{CG}(\rho)$ (Osborne 2008)
  - $S_{CG}(\rho) \tilde{\approx} \log M$ (Today)
- Proof sketches
- Open questions

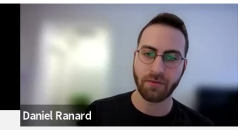


$$S_{CG}(\rho) = \sum S(\rho_{A_i})$$

$$S_{CG}(\rho) \approx \log M$$

M = # of solutions $|\psi\rangle$ to marginal problem





Entropy: classical vs. quantum

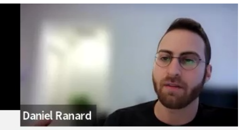
Classically, entropy satisfies the 2nd Law of Thermo.

What's the analog in quantum many–body systems?

Take closed quantum system with state $\rho(t)$

$\rightarrow S\big(\rho(t)\big) = \text{constant}$

Global von Neumann entropy not a great candidate for the 2nd Law!

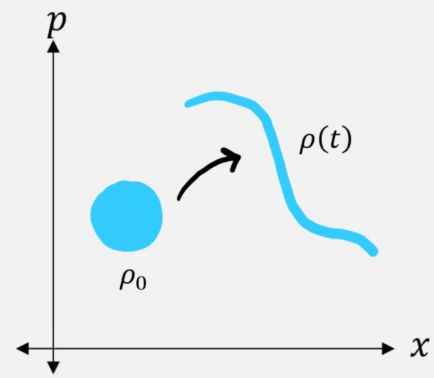


Entropy: classical vs. quantum

Analogy from classical physics:

Classical “state” $\rho(x,p)$ =
probability distribution over phase space
$S(\rho(t)) = -\int dx\, dp\, \rho(x,p,t) \log \rho\,(x,p,t)$

That entropy doesn’t change either!



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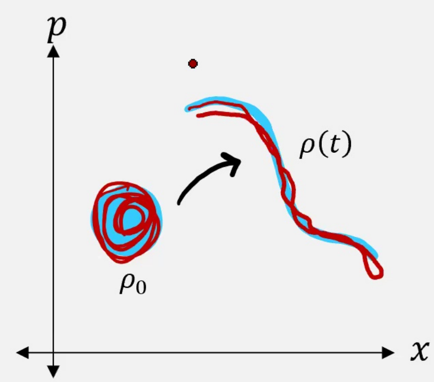

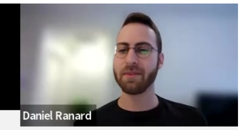

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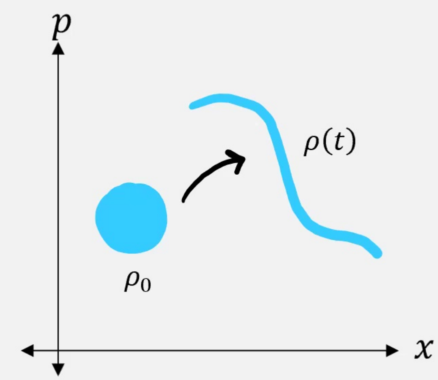

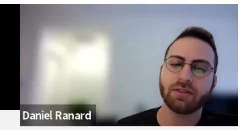

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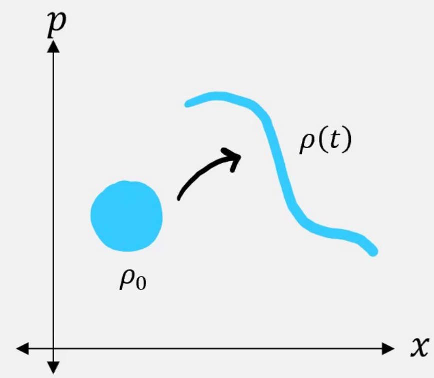


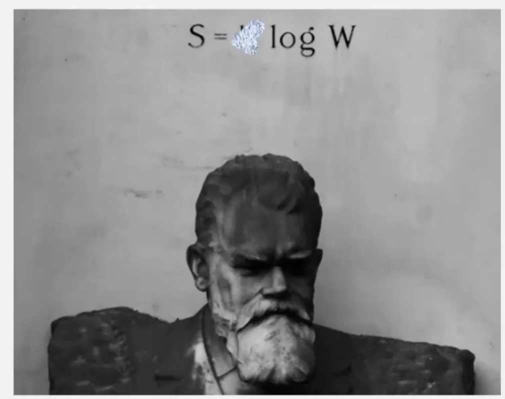
That entropy doesn't change either! Instead….

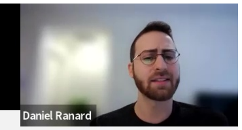
Coarse-grained (Boltzmann) entropy

For microstate α,

$S(\alpha) = \log \Omega$

$\Omega = \#$ microstates with same coarse-grained properties as α
e.g. total energy

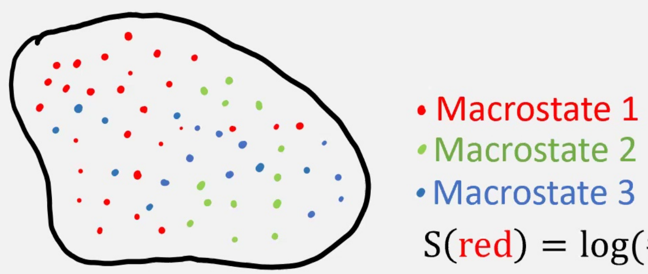




Coarse-grained entropy in QM?

Lessons from classical case:

- Useful notion of entropy involves coarse-graining
- Entropy associated to microstate counts how many *other* microstates have the same coarse-grained properties



$S(\text{red}) = \log(\#\ \text{red})$

Space of microstates

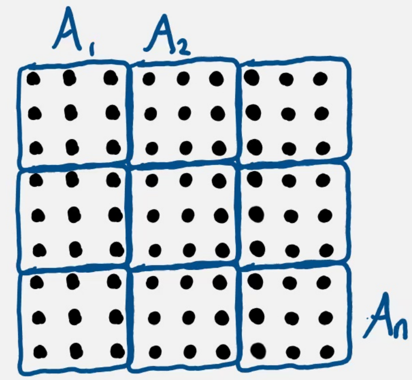
Define analogous "coarse-grained entropy" S_{CG} in quantum?

Lots of options. We'll study a very simple one!

Quantum coarse-grained entropy S_{CG}

Partition the lattice into regions A_i. Define **coarse-grained entropy**:

$$S_{CG}(\rho) = \sum_i S(\rho_{A_i})$$



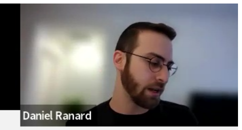
Properties:

- Depends on partition, but often insensitive to exact choice.
- If ρ is pure product state, $S_{CG}(\rho) = 0$.
- If ρ is thermal state $\rho = e^{-\beta H}/Z$ for local Hamiltonian,

$$S_{CG}(\rho) \approx \sum_i S\left(\frac{e^{-\beta H_{A_i}}}{Z_i}\right) \approx S\left(\frac{e^{-\beta H}}{Z}\right) + boundary\ terms \approx \text{thermal entropy.}$$

- If ρ is a pure state that looks thermal on regions A_i, same as above.
- Under natural time-evolution, S_{CG} tends to increase.

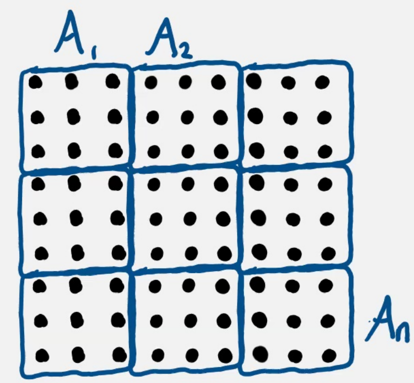
But what does this entropy *count*?



Coarse-grained entropy as microstate counting

Given ρ,

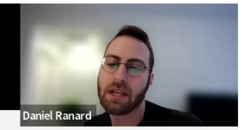
ask about states $|\phi\rangle$ that match ρ on all regions A_i , i.e.
$\mathrm{Tr}_{\overline{A_i}} |\phi\rangle\langle\phi| \approx \rho_{A_i}$



Such states $|\phi\rangle$ like "microstates," with the same local properties as ρ.

If you can only observe ρ locally, then for all you know the system could really be in state $|\phi\rangle$.

We show: The quantity $S_{CG}(\rho) = \sum_i S(\rho_{A_i})$ *counts microstates.*



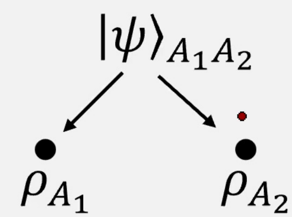
Quantum marginal problem

"Marginal" = "Reduced density matrix" = "RDM".

Quantum marginal problem:
Given list of marginals ρ_{A_i}, does there exist a global state σ consistent with them?

Any σ with $\sigma_{A_i} = \rho_{A_i}$ is a "solution" to that marginal problem.

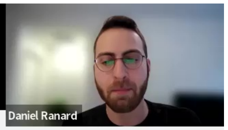
Example:
Given ρ_{A_1} and ρ_{A_2} on two qubits A_1 and A_2, does there exist a pure state $|\psi\rangle$ with those marginals?
(**Answer:** Yes iff ρ_{A_1} and ρ_{A_2} have same spectrum.)



General marginal problem: **hard!** (QMA-complete.)

But we're interested in approximate solutions for large systems: **easier.**

We want to **count how many solutions**: what's the largest number of pure, orthogonal states we can find, such that each state has marginals approximately given by the ρ_{A_i} ?



Prior work

Coarse-grained entropy $S_{CG}(\rho) = \sum_i S(\rho_{A_i})$ and related quantities discussed by many.

- Quantum thermodynamics: Gell-Mann + Hartle
- Holography: Susskind, Kelly + Wall, Englehardt + Wall

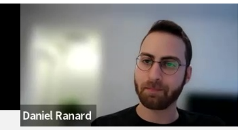
Tobias Osborne noted the upper bound

$$\log M \ \leq S_{CG}(\rho)$$

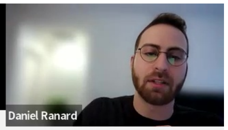
where M is the max number of pure, orthogonal solutions to marginal problem. We'll be showing (approximate) equality!

Brandao + Dalzell show the existence of *one* approximate solution to the marginal problem, given arbitrary marginals on overlapping local regions. (With an MPS!)

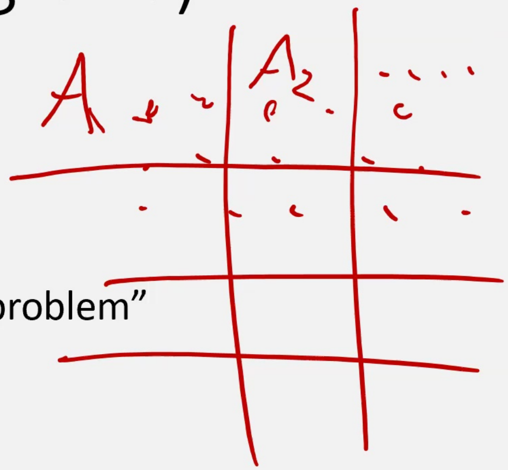
We'll be interested in showing *many* solutions.



Main results and intuition



Main result (for case of disjoint regions)



Partition the system into disjoint regions A_i.
Define *coarse-grained entropy:*

$$S_{CG}(\rho) = \sum_{i=1}^{n} S(\rho_{A_i})$$

Let $M =$ "number of orthogonal, pure, approximate solutions to marginal problem"

$= $ size of the largest set of orthogonal states $\{|\phi_\alpha\rangle\}_{\alpha=1}^{M}$ such that $\|\mathrm{Tr}_{\overline{A_i}} |\phi_\alpha\rangle\langle\phi_\alpha| - \rho_{A_i}\|_1 \leq \epsilon$ for each A_i , $|\phi_\alpha\rangle$.

Then

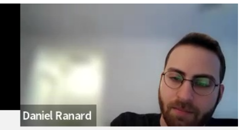
$$S_{CG} \approx \log M$$

In particular:

$$S_{CG} - \sqrt{\mathrm{n} \log \epsilon^{-1}} \log d \leq \log M \leq S_{CG} + \epsilon\, n \log d$$

$$d = \max_i \dim(A_i)$$

Note generally $S_{CG} \propto n$ is extensive.
$S_{CG} \approx \log M$ up to "sub-extensive corrections" if e.g. $\epsilon \sim \frac{1}{n^2}$.



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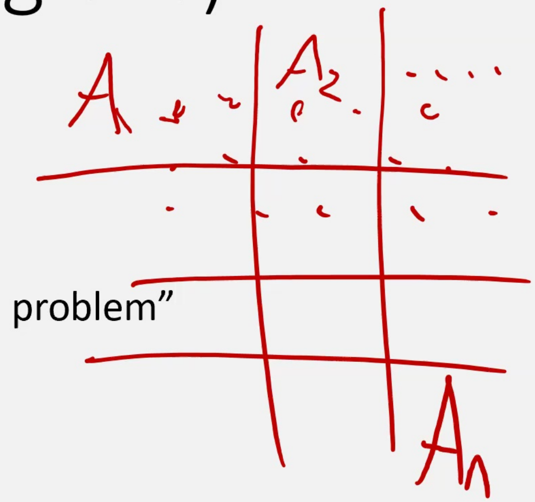

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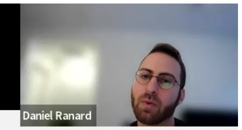
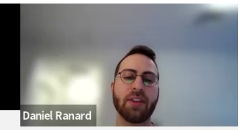


Intuition for $S_{CG} \approx \log M$

If each $\rho_{A_i} = |\psi_{A_i}\rangle\langle\psi_{A_i}|$ is pure,
the only solution is $|\psi\rangle = |\psi_{A_1}\rangle \dots |\psi_{A_n}\rangle$.
$S_{CG} = 0, \quad M = 1.$

If each ρ_{A_i} maximally mixed, Haar-random $|\psi\rangle$ gives approximate solution. So "most" pure states work.
$S_{CG} = n \log d, \quad M \approx \dim H = d^n.$

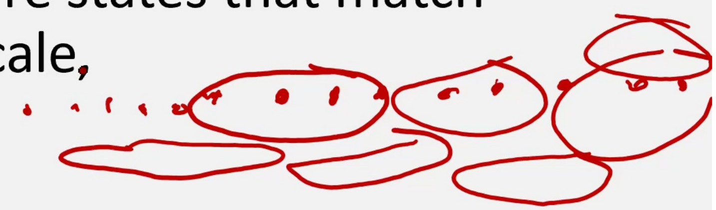
$d = \dim A_i$

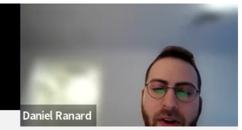


Generalization to overlapping marginals

With caveats:

$S_{CG}(\rho)$ also counts the number of orthogonal pure states that match the marginals ρ_A on *all* regions of some length scale, i.e. including overlapping regions.





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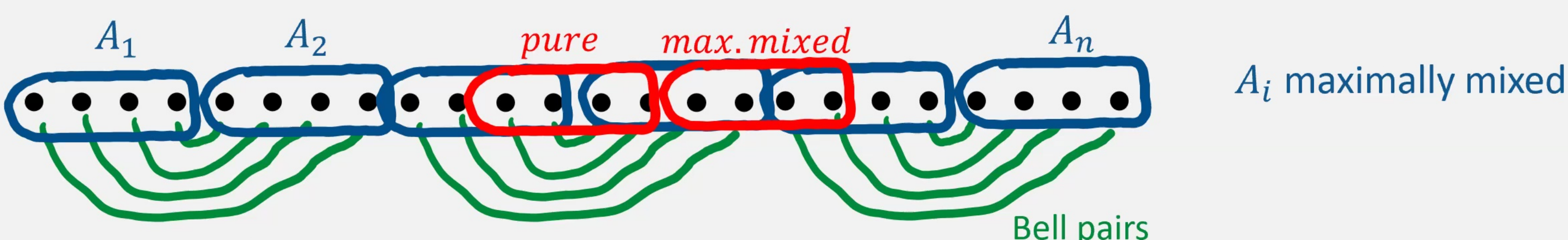
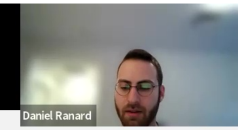
Example of issues:



S_{CG} for blue and red partitions very different.
We'll define a version of S_{CG} that's minimized over all partitions of fixed length scale.
Then we'll count pure states that match the marginals for all regions (independent of partition) below some **smaller** scale.



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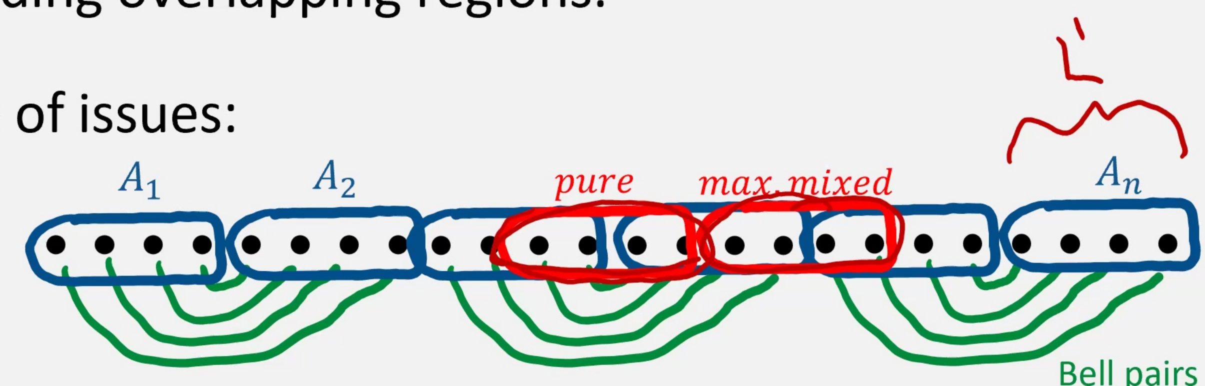

A_i maximally mixed

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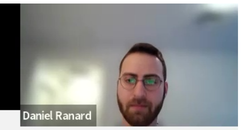


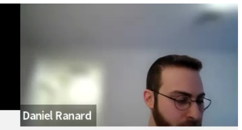
Upper bounding number of marginal solutions

Statement: [specialized to case $\epsilon = 0$]

Partition the system into disjoint regions A_i. Assume there exist orthogonal states $\{|\phi_\alpha\rangle\}_{\alpha=1}^{M}$ such that

$$\mathrm{Tr}_{\overline{A_i}} |\phi_\alpha\rangle\langle\phi_\alpha| = \rho_{A_i}$$

for each $|\phi_\alpha\rangle$ and A_i. Then $\log M \leq S_{CG}(\rho) \equiv \sum_i S(\rho_{A_i})$.



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for each $|\phi_\alpha\rangle$ and A_i. Then $\log M \leq S_{CG}(\rho) \equiv \sum_i S(\rho_{A_i})$.

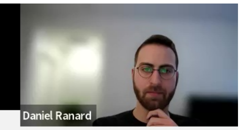
Proof (Osborne):

Take the average: $\sigma = \frac{1}{M}\sum_\alpha |\phi_\alpha\rangle\langle\phi_\alpha|$. Note $\sigma_{A_i} = \rho_{A_i}$.

Then

$$\log \mathrm{M} = \mathrm{S}(\sigma) \underset{subadditivity}{\leq} \sum_i S(\sigma_{A_i}) = \sum_i S(\rho_{A_i}) .$$

Easy! 🎉



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subadditivity

Easy!

Lower bounding number of marginal solutions

Statement:

Partition the system into disjoint regions A_i. Let $S_{CG}(\rho) \equiv \sum_i S(\rho_{A_i})$.

Then there exist M orthogonal pure states $\{|\phi_\alpha\rangle\}_{\alpha=1}^{M}$ such that

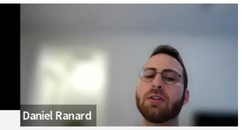
$$\mathrm{Tr}_{\overline{A_i}} |\phi_\alpha\rangle\langle\phi_\alpha| \approx \rho_{A_i}$$

for some M with

$$\log M > S_{CG}(\rho) - o\left(\sqrt{\mathrm{n} \log \epsilon^{-1}} \log d\right). \qquad d = \max_i \dim(A_i)$$

Extra desideratum:
Would be nice for the $|\phi_\alpha\rangle$ to be low complexity, as possible "microstates."

Daniel Ranard



Lower bounding number of marginal solutions

Statement: $\log M > S_{CG}(\rho) - o\left(\sqrt{\mathrm{n}\log \epsilon^{-1}}\log d\right)$

Proof sketch:

Specialize to case $A_1, \ldots, A_n$ are each single qubits, with same $\rho_{A_i} = p_1|0\rangle\langle 0| + p_2|1\rangle\langle 1|$.

Want to construct $M \approx e^{S_{CG}}$ states $|\phi\rangle$ with marginals $\approx \rho_{A_i}$.
$S_{CG} = nS(\rho_A) = nS(\{p_1, p_2\})$.

Let $T = \{$product states $|011011\ldots\rangle$ with $\approx p_1$ zeros and $\approx p_2$ ones$\}$

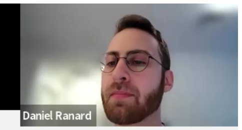
Number of such states is $|T| \approx 2^{nS(\{p_1,p_2\})}$.

Randomly choose k states from T, denoted $\{|\psi_i\rangle\}_{i=1}^{k}$. Take $k \propto \log n$.
The state $\frac{1}{k}\sum_{i=1}^{k}|\psi_i\rangle\langle\psi_i|$ has marginals close to ρ_{A_i}, with "sampling error" like $1/\sqrt{k}$.
Note $|\phi_1\rangle \equiv \frac{1}{\sqrt{k}}\sum_{i=1}^{k}|\psi_i\rangle$ has same marginals as the $\frac{1}{k}\sum_{i=1}^{k}|\psi_i\rangle\langle\psi_i|$, because the terms are unlikely to interfere.

$\rightarrow$ We already found one approximate solution $|\phi_1\rangle$.

Now repeat: Pick k more random states from states from T (avoiding the ones we already picked). Repeat to obtain another solution $|\phi_2\rangle$.

Can repeat process for $M \approx \exp(S_{CG}(\rho) - o(n))$ iterations without significantly depleting T.



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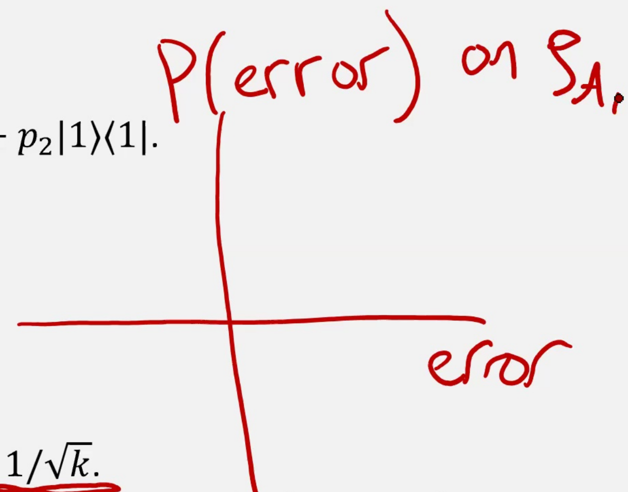

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Let $T =$ {product states $|011011 \ldots\rangle$ with $\approx p_1$ zeros and $\approx p_2$ ones}

Number of such states is $|T| \approx 2^{nS(\{p_1,p_2\})}$.

Randomly choose k states from T, denoted $\{|\psi_i\rangle\}_{i=1}^{k}$. Take $k \propto \log n$.
The state $\frac{1}{k}\sum_{i=1}^{k}|\psi_i\rangle\langle\psi_i|$ has marginals close to ρ_{A_i}, with "sampling error" like $1/\sqrt{k}$.
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$\rightarrow$ We already found one approximate solution $|\phi_1\rangle$.

Now repeat: Pick k more random states from states from T (avoiding the ones we already picked). Repeat to obtain another solution $|\phi_2\rangle$.

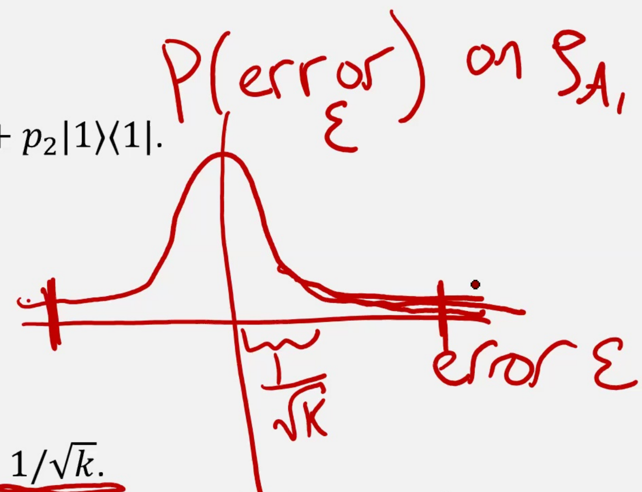
Can repeat process for $M \approx \exp(S_{CG}(\rho) - o(n))$ iterations without significantly depleting T.

Lower bounding number of marginal solutions

Statement: $\log M > S_{CG}(\rho) - o\left(\sqrt{\text{n} \log \epsilon^{-1}} \log d\right)$

Proof sketch:

Specialize to case $A_1, \ldots, A_n$ are each single qubits, with same $\rho_{A_i} = p_1|0\rangle\langle 0| + p_2|1\rangle\langle 1|$.



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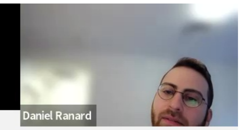

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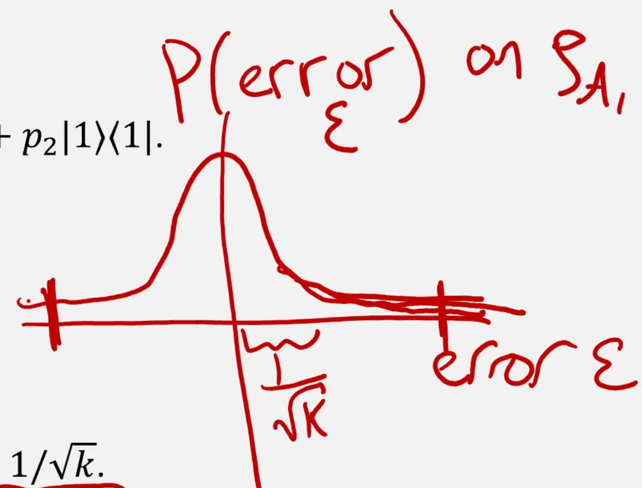

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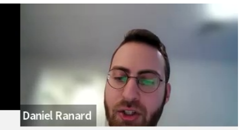


Lower bounding number of marginal solutions

Statement for "overlapping" case:

Choose two length scales, $L' \gg L$. Define $S_{CG}^{(L')}(\rho) = \min_{partitions} \sum_{i=1}^{n} S(\rho_{A_i})$ with minimum over partitions into **disjoint** regions A_i of size $|A_i| = L'$

Then $\exists\ \mathrm{M_L}$ orthogonal pure states that match ρ (to error L/L') on all regions of size $\leq L$ (including overlapping regions), with

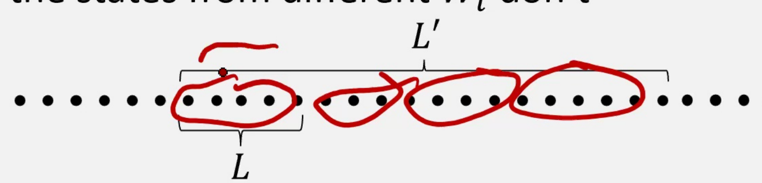
$$\log \mathrm{M_L} > S_{CG}^{(L')} - o\left(n\frac{L'}{L}\right)$$

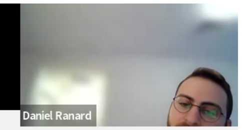
Proof sketch:

Fix a partition into regions A_i of size L'. Already showed how to find $\approx \exp(S_{CG}^{(L')})$ pure states with marginals ρ_i. Call this set of pure states W_1. Then shift regions A_i by one site and repeat: call this set of states W_2. Repeat to $W_{L'}$.

Choose one state from each W_i and take uniform superposition. New state has correct marginals on every region of size $\leq L$, to error L'/L. (How do we ensure the states from different W_i don't interfere? Trick adapted from Brandao + Dalzell.)

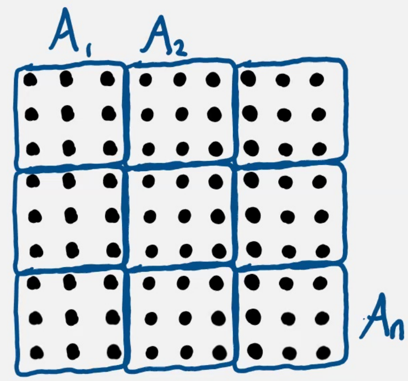
Then repeat, choosing a different state from each W_i.





Summary

We took a common proxy for thermodynamic entropy in quantum many-body systems, the coarse-grained entropy:



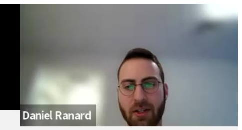
$$S_{CG}(\rho) = \sum_i S(\rho_{A_i})$$

What "microstates" does this quantity count?

We showed

$$S_{CG}(\rho) \approx \log M$$

counts the number M of orthogonal pure states with the same marginals as ρ.

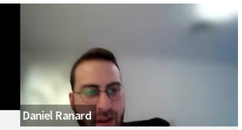


Concrete open questions

- Can we count **exact** solutions to the marginal problem?
  For non-overlapping marginals, I suspect there are $\exp(S_{CG}(\rho))$ *exact* solutions. Sketchy proof, with numerical support.

- Can we construct **O(1)-complexity** solutions to the marginal problem?
  Brandao + Dalzell already showed how to construct a *single* approximate solution of O(1)-complexity. Can you construct $\exp(S_{CG}(\rho))$ such solutions?

- Can we extend our bounds to the alternative "max-entropy" definition of S_{CG} (e.g. as used in holography)?



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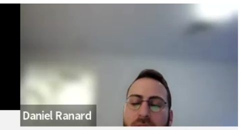
- Connection to quantum PCP conjecture?

- What are dynamics of $S_{CG}(\rho(t))$? Microstate interpretation suggests it should increase?
  - Point of tension: recent work (Cotler, Jones, DR) suggests S_{CG} fluctuates in time much *less* in quantum case than classical.

- What's structure of space of consistent marginals? To specify all k-body RDMs on an n-body system only takes $\approx k\, n \log n$ bits, rather than naïve k^n. What's the right "data structure" to encode the set of k-body RDMs?



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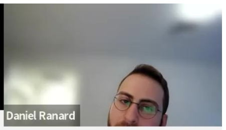
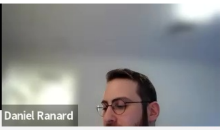


Acknowledgments

Thanks to Michael Walter for discussions on the quantum marginal problem, and Geoff Penington for discussions on holographic coarse-grained entropies.



Thank you!