Title: TBA

Speakers: Chris Akers

Series: Perimeter Institute Quantum Discussions

Date: February 02, 2022 - 11:00 AM

URL: https://pirsa.org/22020043

Abstract: Abstract: TBD

Zoom Link: https://pitp.zoom.us/j/93553787795?pwd=U1RURnkzeHNsL1B0cE91S1JMVW1rZz09

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Jul 29, 2021 at 2:42 PM

Quantum minimal surfaces from quantum codes

[Chris Akers]

Quantum gravity suggests that spacetime emerges from something more fundamental.

Our best example of this is AdS/CFT. [Maldacena 97]

D+1 dimensional gravity theory in Anti-de Sitter space is equivalent to D dimensional non-gravitational conformal field theory

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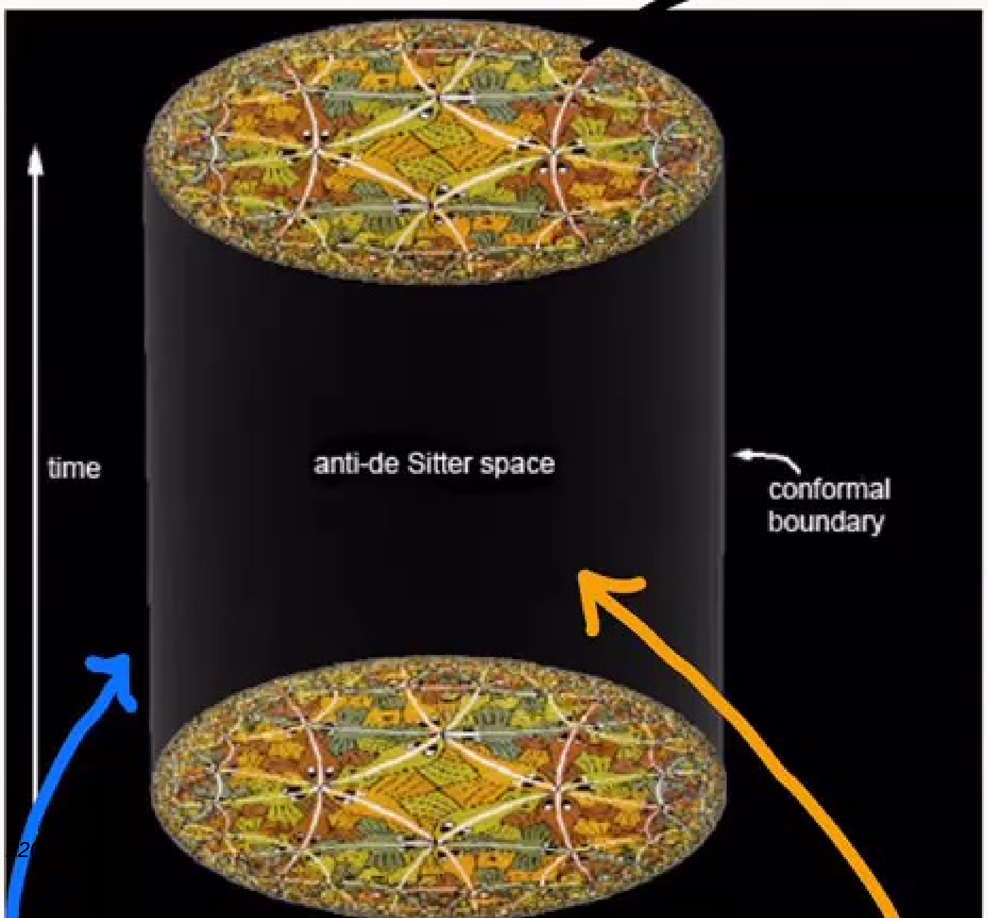
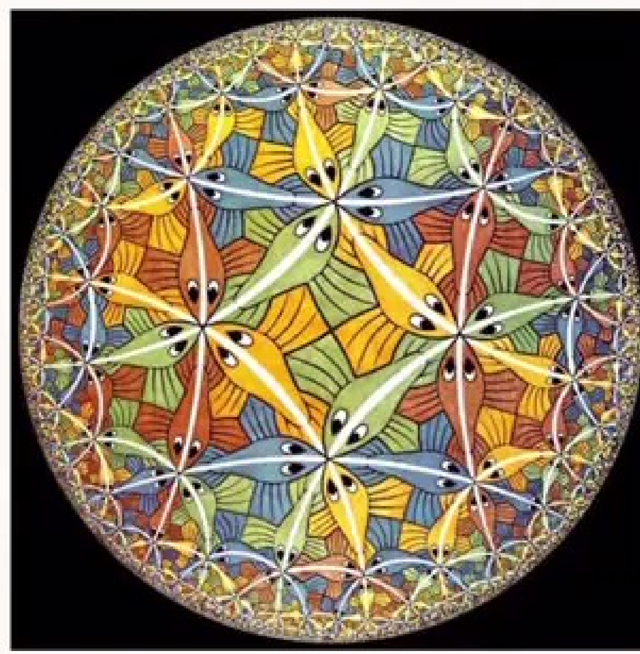
D+1 dimensional gravity theory in Anti-de Sitter space is equivalent to D dimensional non-gravitational conformal field theory.





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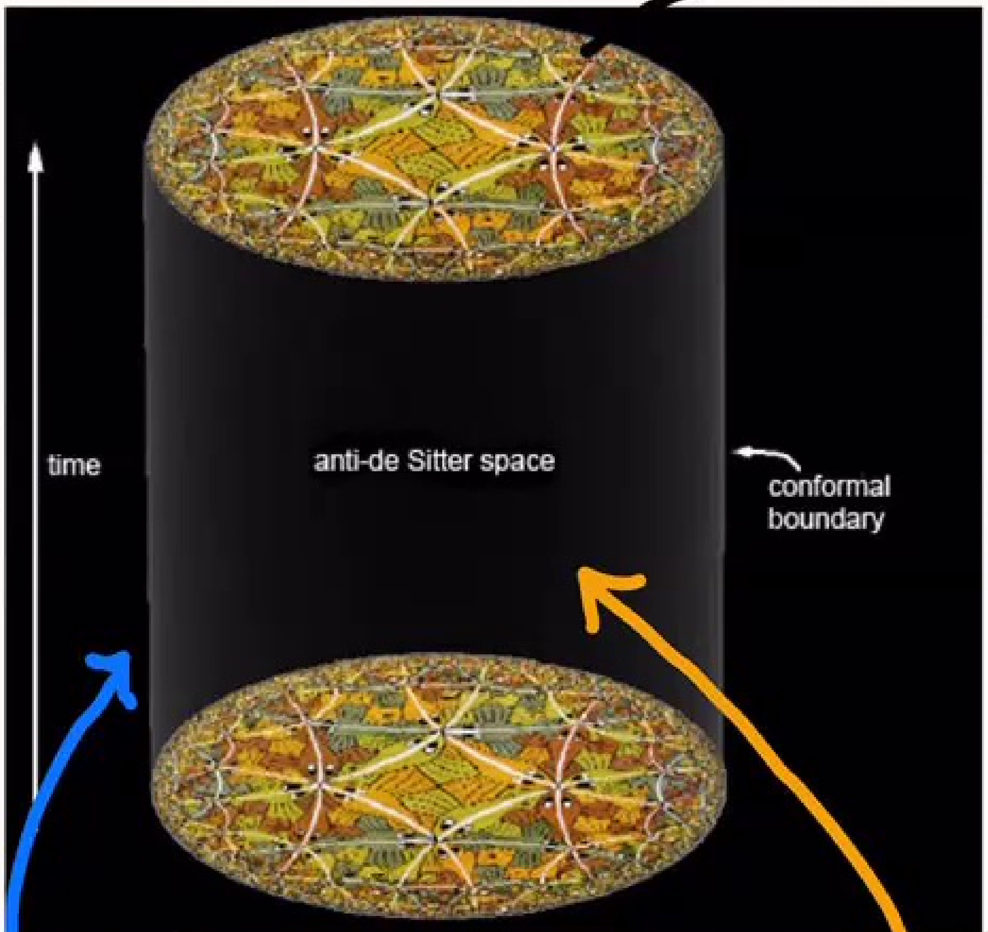

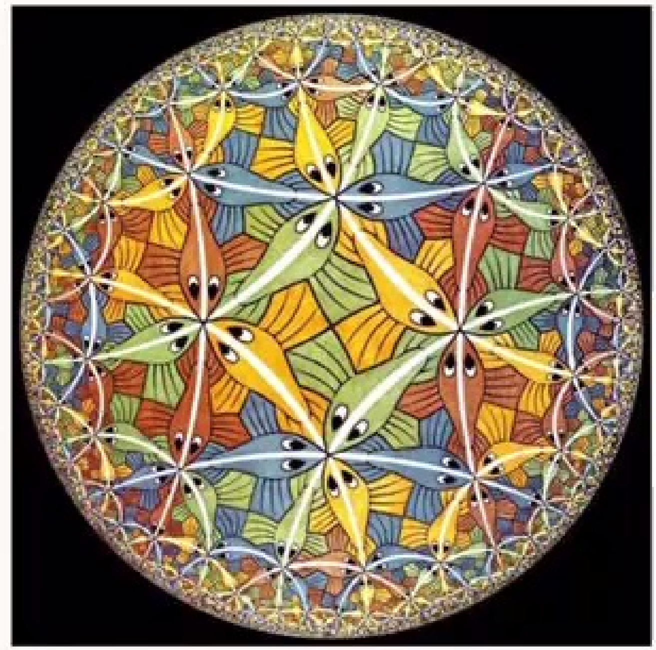

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Gravity theory

Non-gravity theory

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Gravity theory

Non-gravity theory

The question of spacetime emergence:
How can there be a duality that doesn't preserve the locality structure?

Our best clue is the famous "quantum extremal surface" formula. It is believed to be a part of the AdS/CFT dictionary.

[Ryu-Takayanagi 06, Hubeny-Rangamani-Takayanagi 07, Engelhardt-Wall 14, CA-Engelhardt-Penington-Usatyuk 19]

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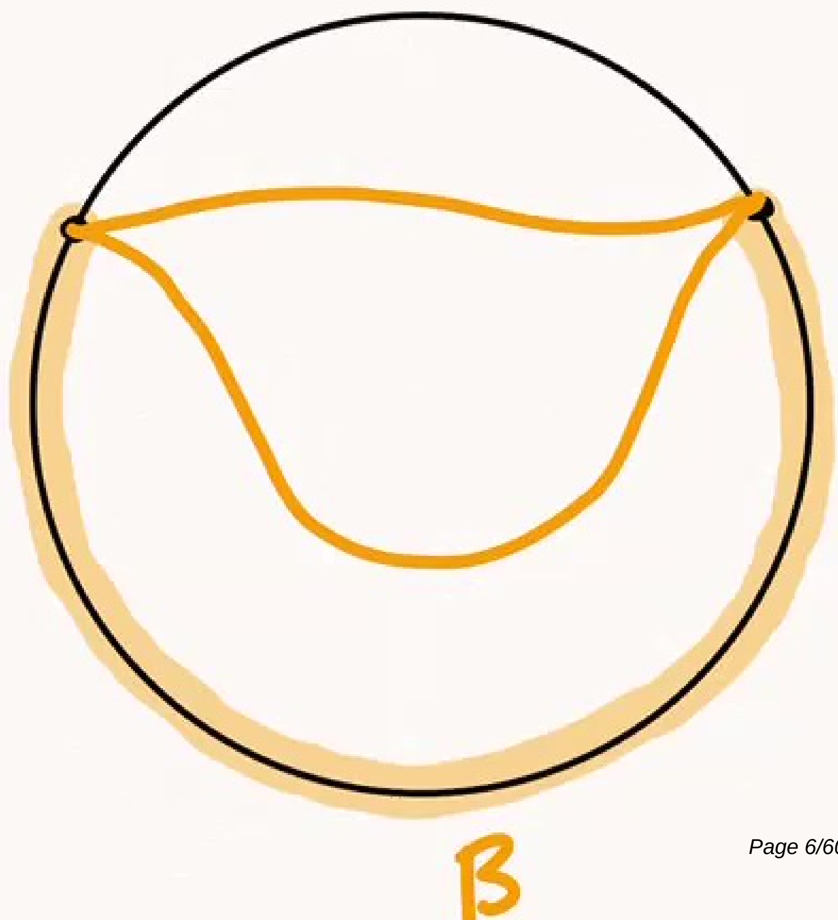
Our best clue is the famous "quantum extremal surface" formula. It is believed to be a part of the AdS/CFT dictionary.

[Ryu-Takayanagi 06, Hubeny-Rangamani-Takayanagi 07, Engelhardt-Wall 14, **CA**-Engelhardt-Penington-Usatyuk 19]

$$S(B) = \min_{\gamma}\left(\frac{A_\gamma}{4G} + S_\gamma\right)$$



surface" formula. It is believed to be a part of the AdS/CFT dictionary.

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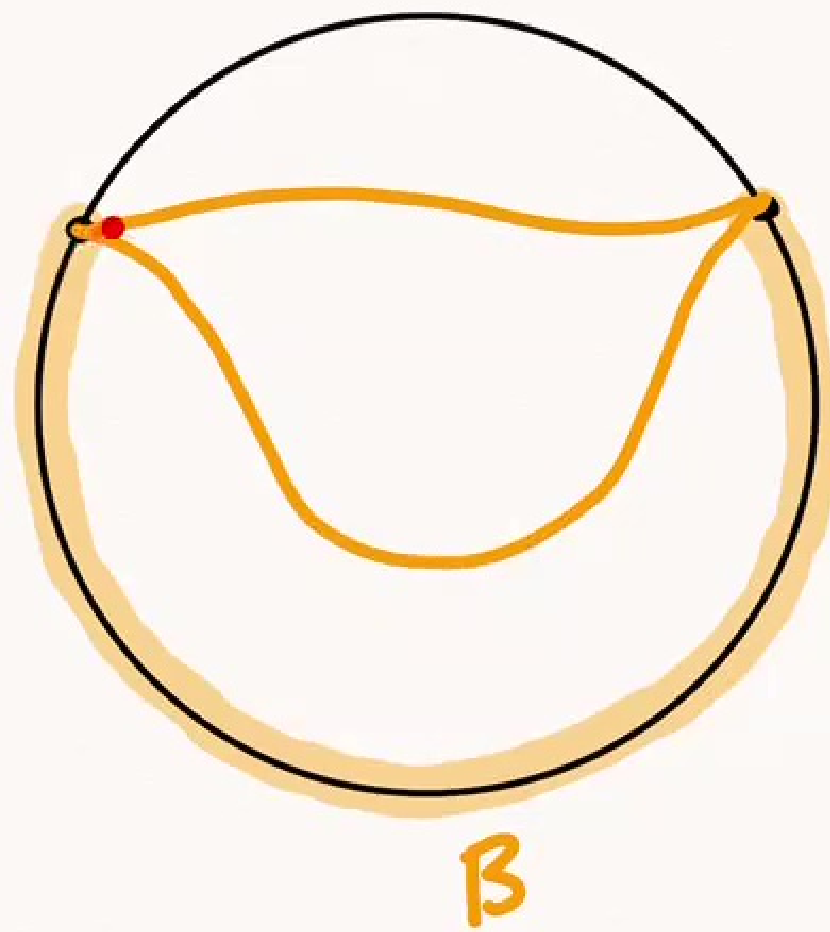


The quantum minimal surface (QMS) formula clues how the extra dimension emerges by suggesting that AdS/CFT should be understood as a certain kind of quantum code.

Isometry $V : \mathcal{H}_{AdS} \rightarrow \mathcal{H}_{CFT}$

The idea is that the logical Hilbert space of a code can have a different locality structure than the physical Hilbert space.

suggesting that AdS/CFT should be understood as a certain kind of quantum code.

Isometry $V: \mathcal{H}_{AdS} \longrightarrow \mathcal{H}_{CFT}$

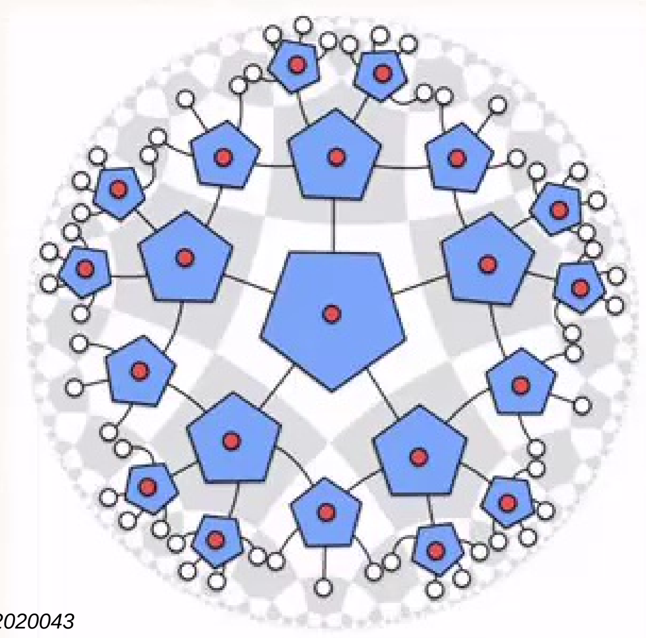
The idea is that the logical Hilbert space of a code can have a different locality structure than the physical Hilbert space.

The upshot: it is important to understand the QMS formula, its connection to quantum codes, and from that learn about spacetime emergence. The main point of this talk is to tell you a theorem we proved, solidifying these connections. [CA-Penington 21]

First, let me remind you of some examples and special cases in which people have already understood the connection.

Example 1: Tensor networks.

[Swingle 09, Pastawski-Yoshida-Harlow-Preskill 15, Hayden-Nezami-Qi-Thomas-Walter-Yang 16]



- Many small isometries combined to form one large one

- Emergence of spacetime:

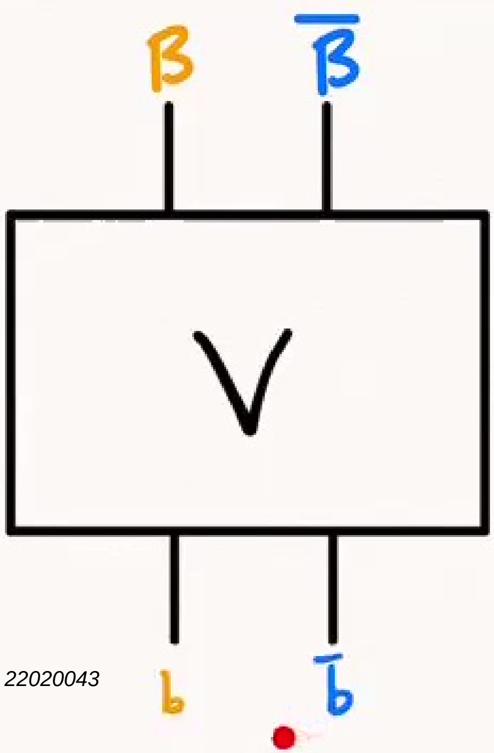
  $\mathcal{H}_{phys}$ circle

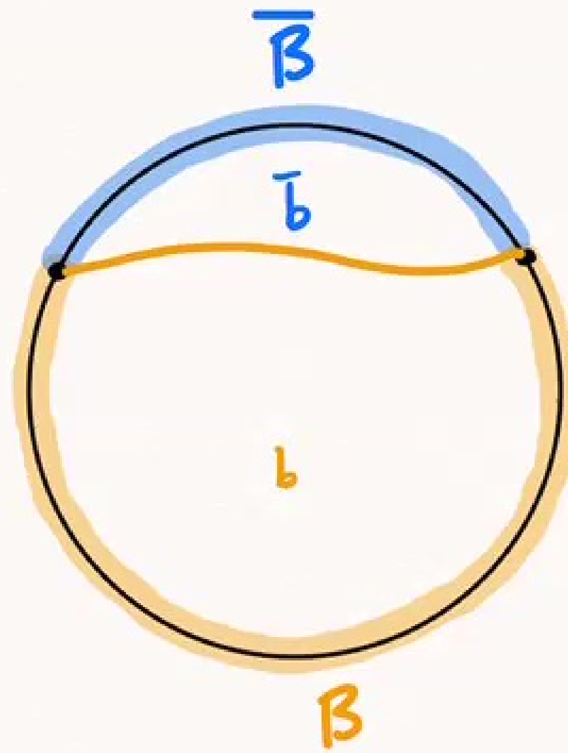
  $\mathcal{H}_{logical}$ disk

- $S(B) = A + S(b)$

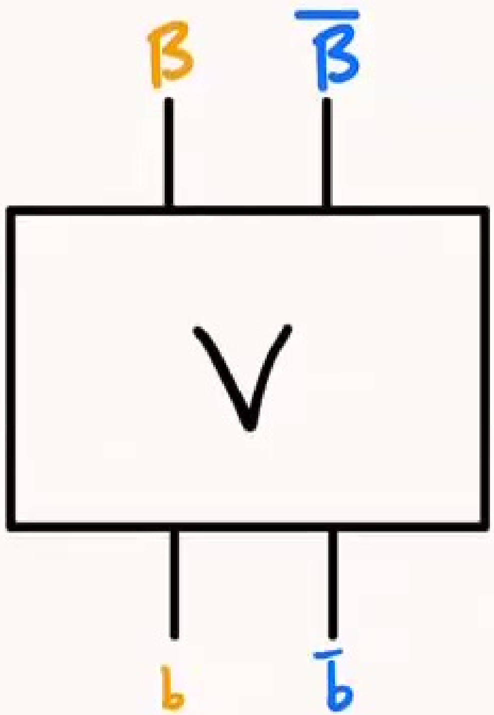
- $S(B) = A + S(b)$

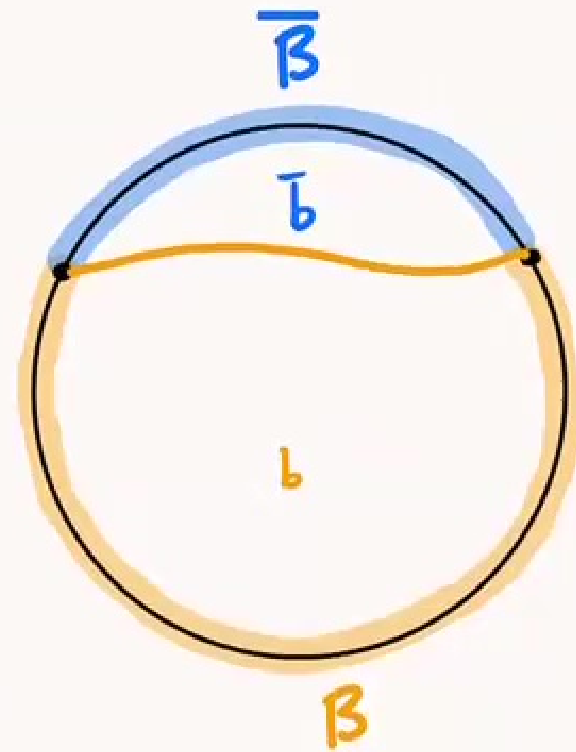
Example 2: "Quantum error correcting codes with complementary recovery." [Harlow 16]





complementary recovery." [Harlow 16]





$\exists R$ s.t. $\forall |\psi\rangle \in \mathcal{H}_{AdS}$, $R(\psi_B) = \psi_b$

$$B \updownarrow \bar{B} \qquad b \updownarrow \bar{b}$$

It follows that

$b \qquad b \qquad B$

$$\exists R \quad \text{s.t.} \quad \forall |\psi\rangle \in \mathcal{H}_{AdS}, \quad R(\psi_{B}) = \psi_{b}$$

$$\updownarrow \qquad \updownarrow$$

$$\bar{B} \qquad \bar{b}$$

It follows that

$$S(B)_{\forall|\psi\rangle} = A + S(b)_{|\psi\rangle}$$

Harlow *defined* area "A" as the quantity that showed up in the formula. This "A" quantified particular entanglement in the code.

That was the state-of-the-art of our code understanding of the QMS formula.

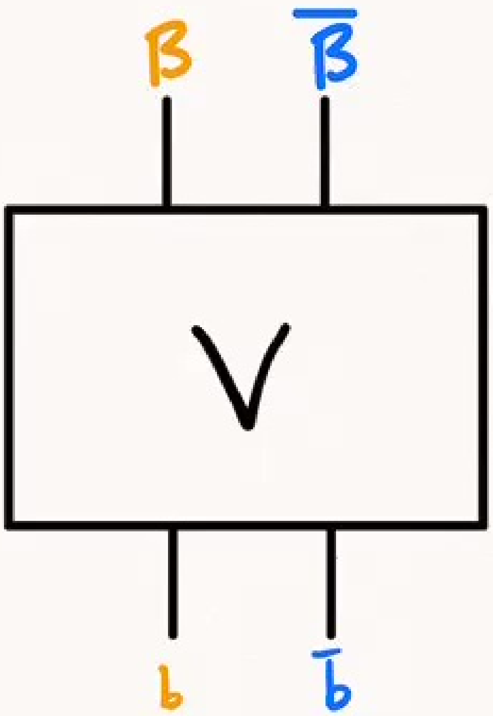
What's left to do?

These works addressed special cases. The QMS in AdS/CFT is more sophisticated than that in tensor networks and more generally quantum error correcting codes with complementary recovery.

In tensor networks the "Area" is too simple. Tensor networks are best understood as representing special "fixed-area states" in AdS/CFT. [CA-Rath 18, Dong-Harlow-Marolf 18]

Harlow's formalism was special for a different

Recall he defined the QMS to delineate the recoverable factor, and the recoverable factor does not depend on the logical state:



$$\exists R \text{ s.t. } \forall |\psi\rangle \in \mathcal{H}_{AdS},\quad R(\psi_B) = \psi_b$$

$$\begin{array}{ccc} \psi_B & & \psi_b \\ \updownarrow & & \updownarrow \\ \bar{B} & & \bar{b} \end{array}$$

$$\Updownarrow$$

$$S(B)_{V|\psi\rangle} = A + S(b)_{|\psi\rangle}$$

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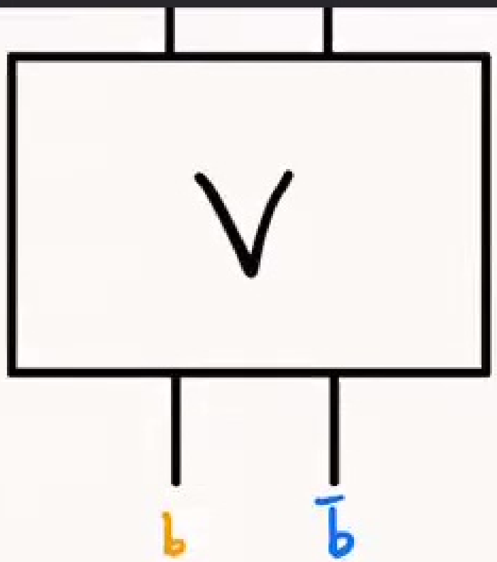

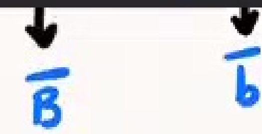

$$S(B)_{V|\psi\rangle} = A + S(b)_{|\psi\rangle}$$

Which means *the quantum minimal surface position is independent of the logical state.*

But in general setups in AdS/CFT, the QMS *does* depend on the state. E.g., if you change the state by adding some entropy somewhere, that can move the QMS.

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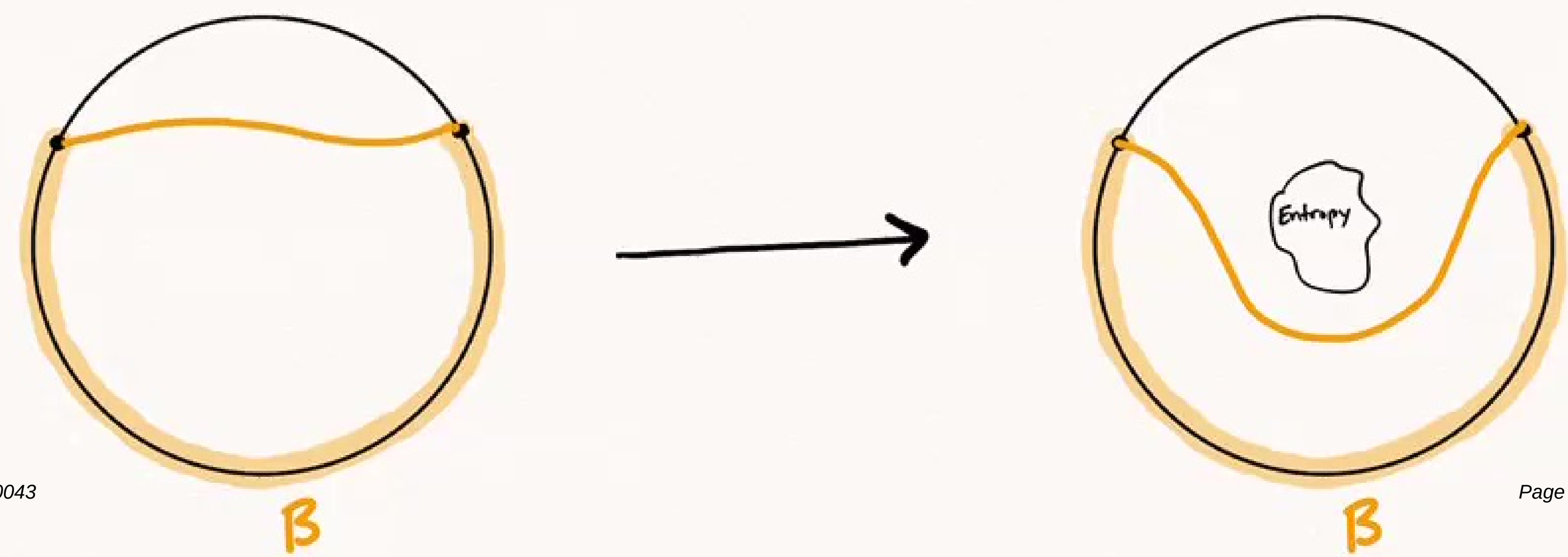
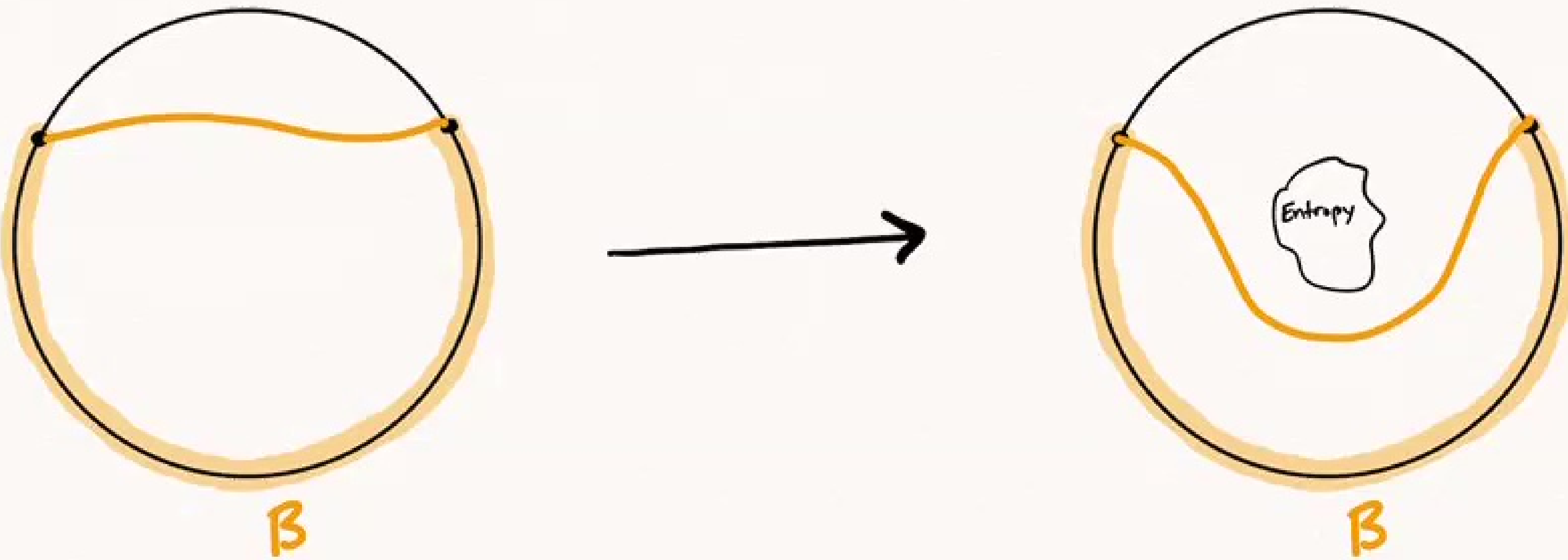
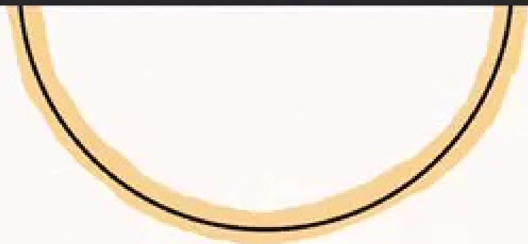
$$S(B) = \min_{\gamma}\left(\frac{A_\gamma}{4G} + S_\gamma\right)$$





So QMS can't be defined by the logical factor recoverable in this "state-independent" way.

Raises the question: if this type of code doesn't define the QMS in the full generality of AdS/CFT, is there another type of code that *does*?



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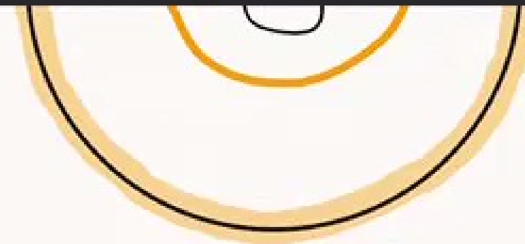

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<u>THEOREM</u>:
Given a code

$$V: \mathcal{H}_{b_1} \otimes \mathcal{H}_{b_2} \otimes \dots \otimes \mathcal{H}_{b_n} \longrightarrow \mathcal{H}_B \otimes \mathcal{H}_{\bar{B}}$$

$$|\psi\rangle \in \mathcal{H}_{b_1} \otimes \mathcal{H}_{b_2} \otimes \dots \otimes \mathcal{H}_{b_n} \otimes \mathcal{H}_R \otimes \mathcal{H}_{\bar{R}}$$

the following two statements are equivalent.

(1): Complementary state-specific reconstruction

For all U_b^{Product} and $\hat{U}_{\bar{b}}^{\text{Product}}$,

$$U_{BR}\,\hat{U}_{\bar{B}\bar{R}}\,V|\psi\rangle = V\,U_b^{\text{Product}}\,\hat{U}_{\bar{b}}^{\text{Product}}|\psi\rangle$$

(2): Holographic entropy formula

For all U^{Product}

$$S(BR)_{VU^{\text{Product}}|\psi\rangle} = A_B(b) + S(bR)_{U^{\text{Product}}|\psi\rangle}$$

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For all U_b and $U_{\bar{b}}$,

$$U_{BR}\,\hat{U}_{\bar{B}\bar{R}}\,V|\Psi\rangle = V\,U_b^{\text{Product}}\,\hat{U}_{\bar{b}}^{\text{Product}}|\Psi\rangle$$

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Moreover, both imply

(3): Minimality

For all b'

$$A_B(b) + S(bR) \leq A_B(b') + S(b'R)$$

Some notes about this theorem—

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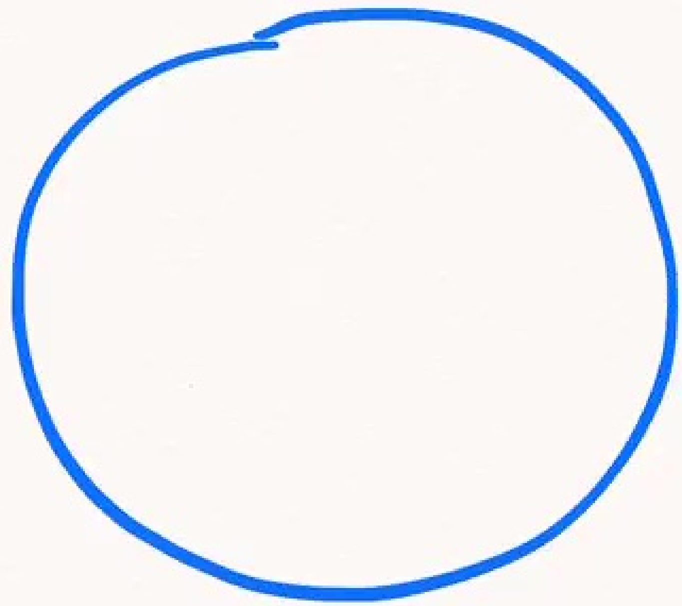
Some notes about this theorem—
-Won't have time to prove, but I hope to give you intuition for it as I explain what it says.
-Will explain state-specific reconstruction & A(b).

<u>State-specific reconstruction of product unitaries</u>

We're going to motivate it by thinking about what properties an encoding must have if it's going to be equivalent to the QMS formula.

$$|\Psi\rangle = V\, U^{Product}_{b}\, \hat{U}^{Product}_{\bar{b}} |\Psi\rangle$$

phic entropy formula



$$U^{Product}$$

$$= A_{B}(b) + S(bR)_{U^{Product}|\Psi\rangle}$$

$$U^{Product}|\Psi\rangle$$

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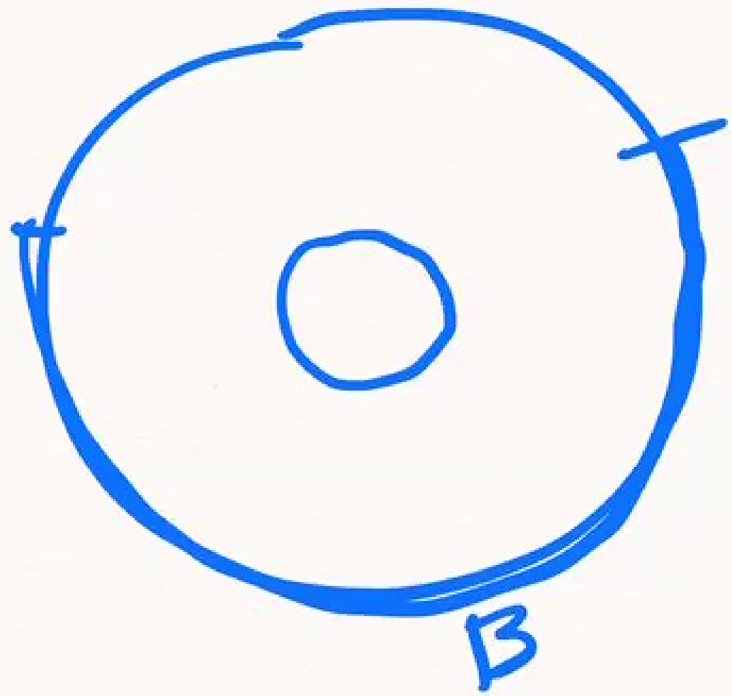
$$|\Psi\rangle = V\, U^{\text{Product}}_{b}\, \hat{U}^{\text{Product}}_{\bar{b}}\, |\Psi\rangle$$

$$|\mathcal{H}| \sim e^{A/4G}$$

phic entropy formula



$$U^{\text{Product}}$$

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$$U^{\text{Product}}\, |\Psi\rangle$$

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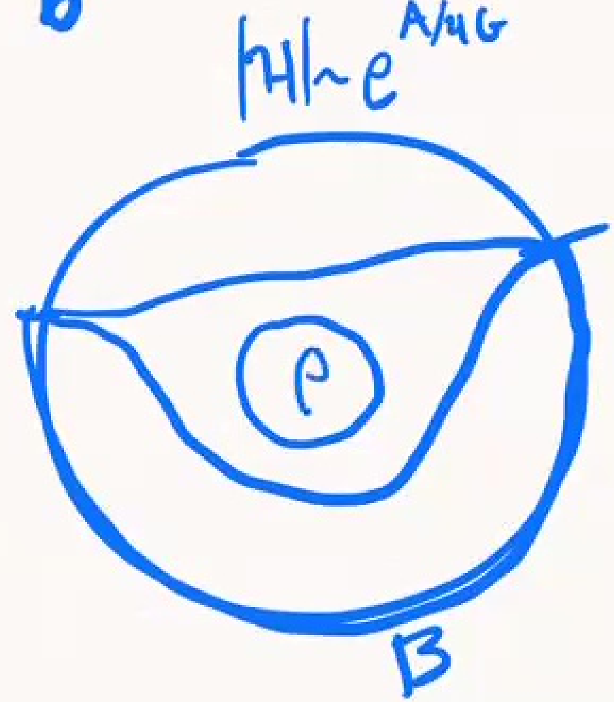

Holographic entropy formula

For all U^{Product}

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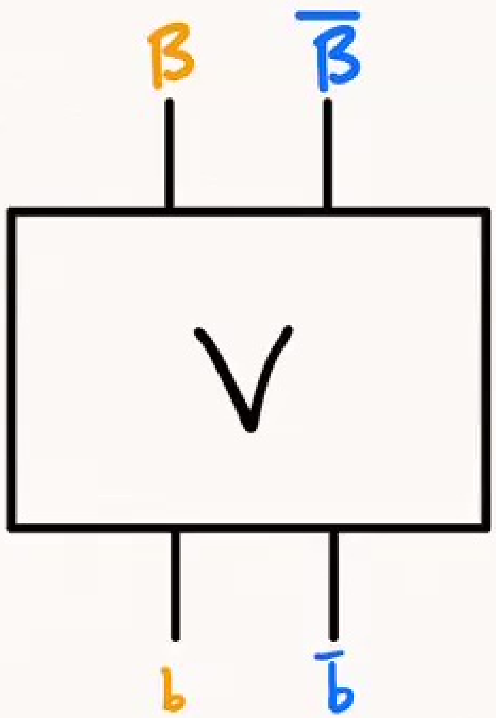
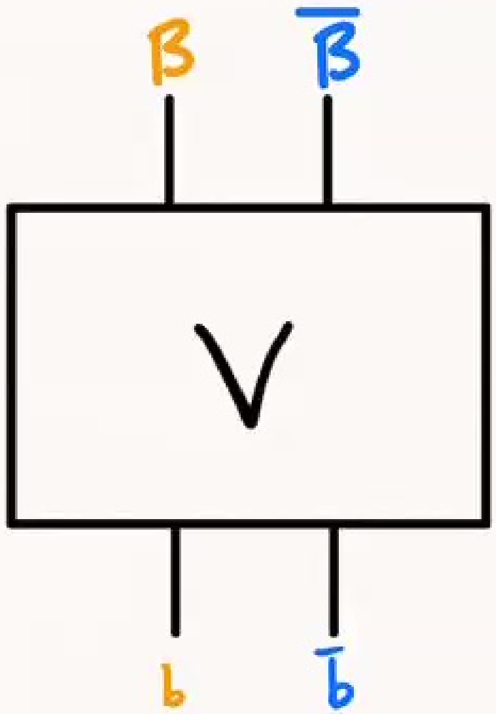
We're going to motivate it by thinking about what properties an encoding must have if it's going to be equivalent to the QMS formula.

Recall usual quantum error correction:



$$\mathcal{N}(\rho_{b\bar{b}}) = \mathrm{tr}_{\bar{B}}\left[V \rho_{b\bar{b}} V^{\dagger}\right]$$

$$\exists R \text{ s.t. } R \circ \mathcal{N}(\rho_{b\bar{b}}) = \rho_b \otimes \mathbb{1}_{\bar{b}}$$



$$\mathcal{N}(\rho_{b\bar{b}}) = \mathrm{tr}_{\bar{B}}\left[V \rho_{b\bar{b}} V^{\dagger}\right]$$

$$\exists R \text{ s.t. } R \circ \mathcal{N}(\rho_{b\bar{b}}) = \rho_b \otimes \mathbb{1}_{\bar{b}}$$

Equivalent:

$$\forall U_b,\ \exists U_B \text{ s.t. } \forall |\psi\rangle \in \mathcal{H}_b \otimes \mathcal{H}_{\bar{b}} \otimes \mathcal{H}_R$$

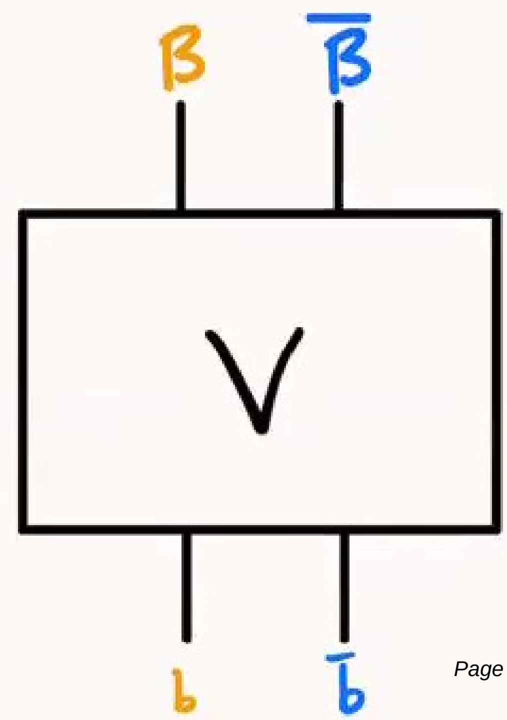
$$U_B V |\psi\rangle = V U_b |\psi\rangle$$

regardless of the logical state. The problem was that therefore the logical factor that B can recover in this way can't define a *state-dependent* QMS.

The recoverability that defines the QMS must depend on the state. First naive guess:

$$U_B V|\psi\rangle = V U_b |\psi\rangle$$

Allowed to depend on both U_b and $|\psi\rangle$.



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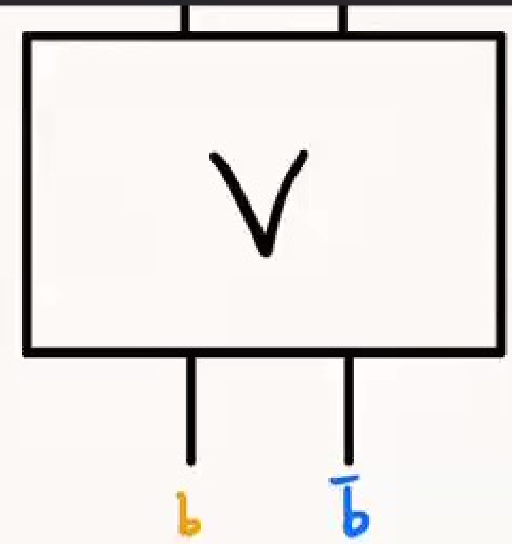

No good. This won't define the QMS. One way to see this is that we don't want to talk about recovering any operators that can change the QMS. (In that case, which QMS would be defined?)

General unitary can change the QMS even when acting only b:

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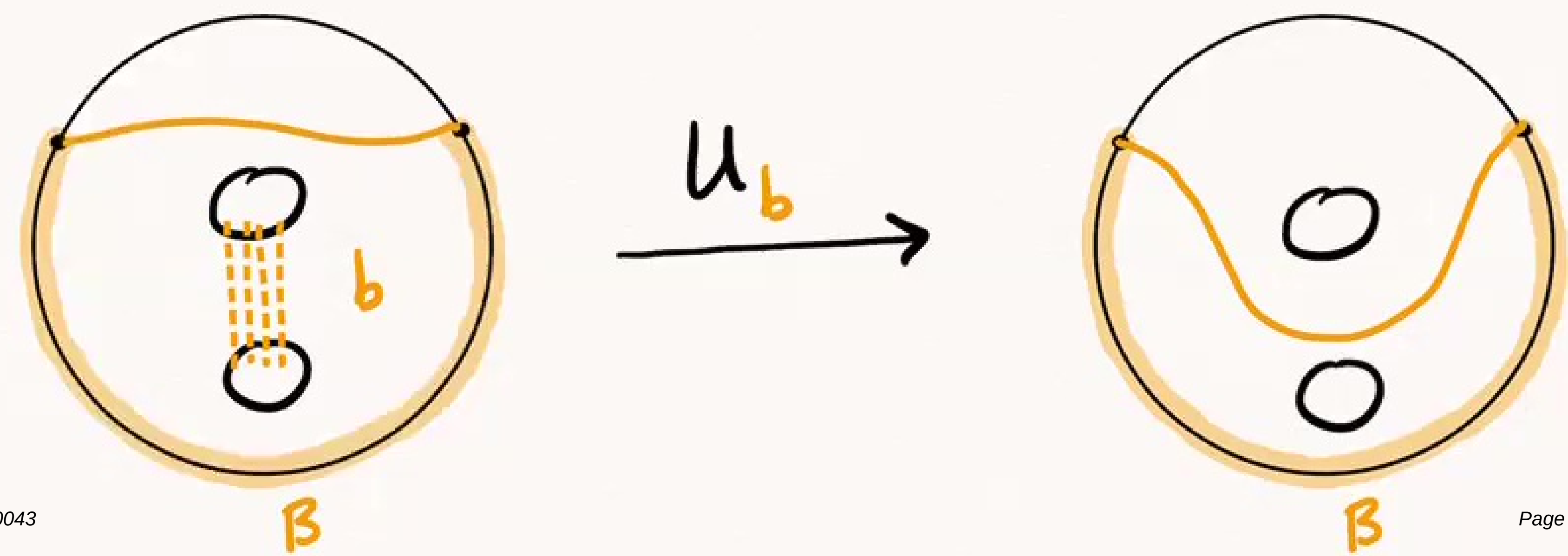
(In that case, which QMS would be defined?)

General unitary can change the QMS even when acting only b:

$$S(B) = \min_{\gamma} \left(\frac{A_\gamma}{4G} + S_\gamma \right)$$



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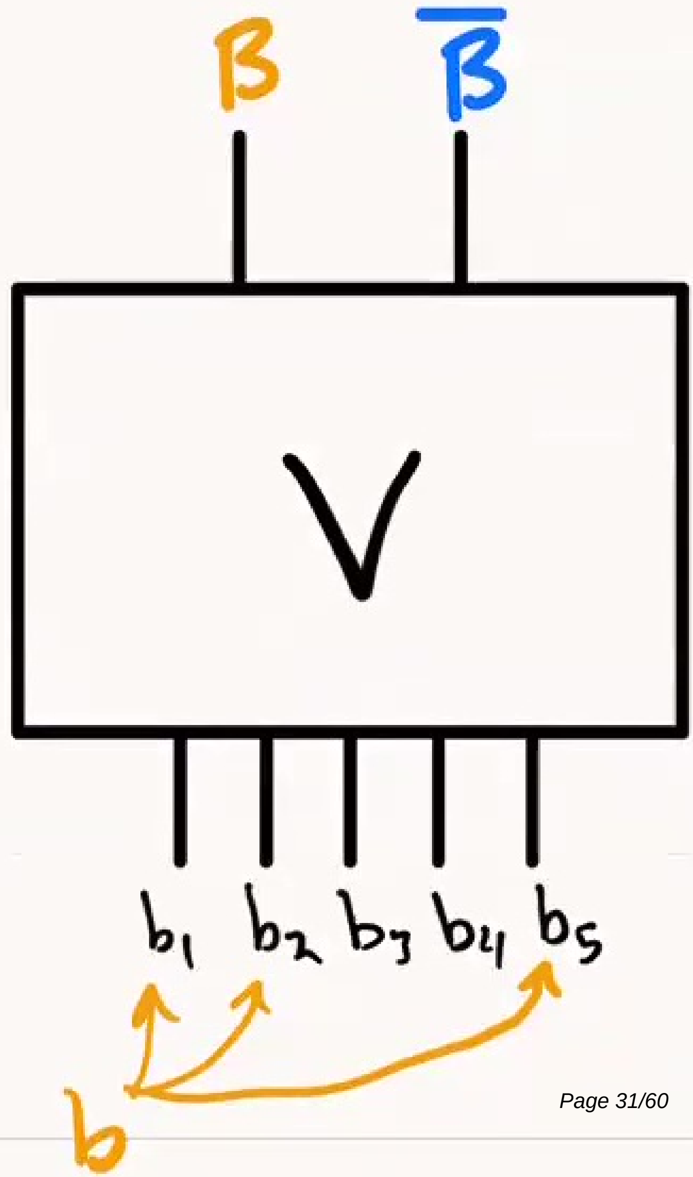
structure are *product unitaries*, which factorize across the bulk inputs.

$$U^{\text{Product}} = U_{b_1} \otimes \dots \otimes U_{b_5}$$

$$U_b^{\text{Product}} = U_{b_1} \otimes U_{b_2} \otimes U_{b_5}$$



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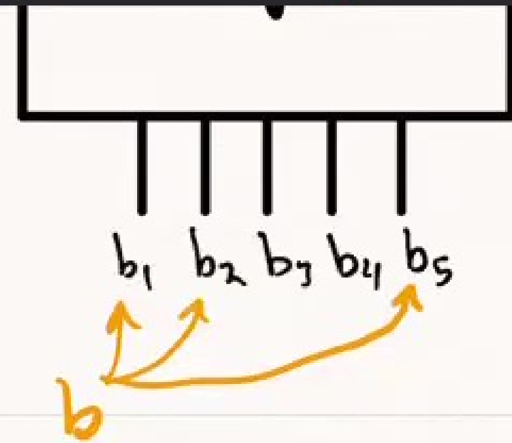
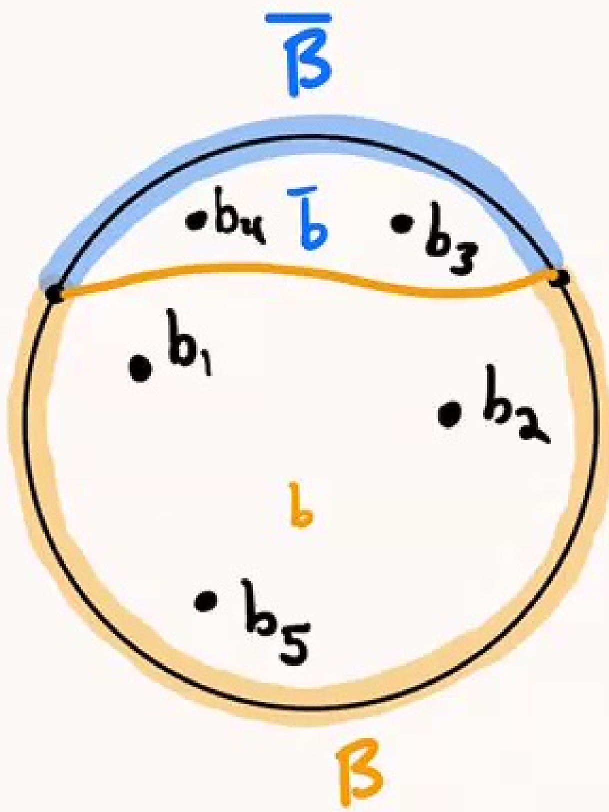
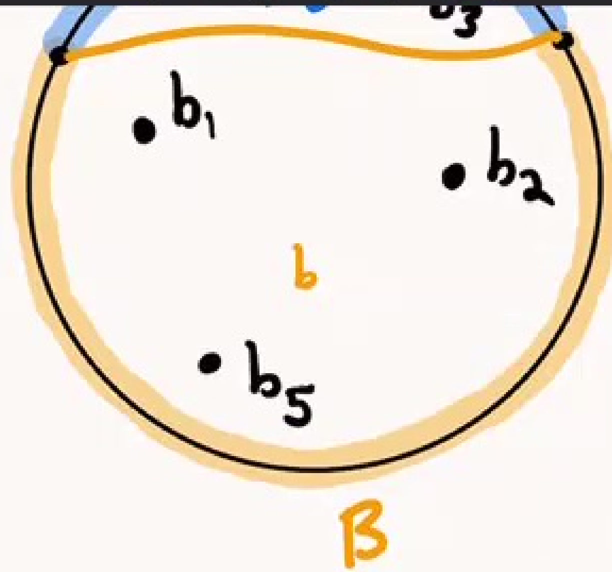
$$U_b^{\text{product}} = U_{b_1} \otimes U_{b_2} \otimes U_{b_5}$$







With this, can define the relevant reconstruction—*state-specific product unitary* reconstruction:

$$U_B V |\psi\rangle = V U_b^{\text{Product}} |\psi\rangle$$

Depends on U_b^{Product} and $|\psi\rangle$

$$U_{B} V |\psi\rangle = V\, U_{b}^{\text{Product}} |\psi\rangle$$

Depends on U_{b}^{Product}

and $|\psi\rangle$

In our theorem, this was the first of the two equivalent statements. Specifically, the

complementary recovery version of this:
(1):

$$\hat{U}_{\bar{B}\bar{R}} U_{BR} V |\psi\rangle = V\, \hat{U}_{\bar{b}}^{\text{Product}} U_{b}^{\text{Product}} |\psi\rangle$$

Now I'll explain the second statement of the

$$\hat{U}_{\bar{B}\bar{R}} U_{BR} V |\psi\rangle = V \hat{U}^{\text{Product}}_{\bar{b}} U^{\text{Product}}_{b} |\psi\rangle$$

Now I'll explain the second statement of the theorem, which is equivalent to complementary state-specific reconstruction. It said:
(2):

For all U^{Product}

$$S(B)_{VU^{\text{Product}}|\psi\rangle} = A_B(b) + S(b)_{U^{\text{Product}}|\psi\rangle}$$

First, note the "For all U" clause is new, compared to the usual statement in AdS/CFT. Why is this important for us? If we didn't require this, it would be too weak to imply any recovery statement — we have counterexamples. Also, it's true for QMS in gravity

For all $U^{Product}$

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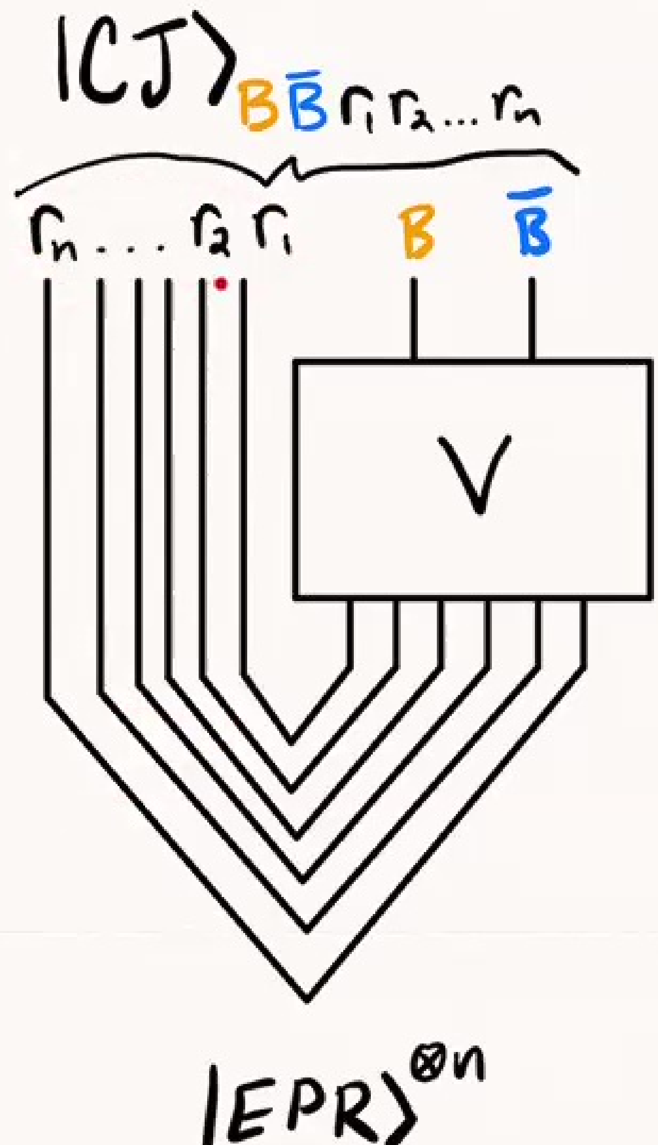
Now, what is this area term A(b)?

Area

This A(b) is a function we define for general `logical regions' of general quantum codes. (Geometry from entanglement!) Generalizes Harlow's definition. Was necessary because QMS can depend on the state. Need an area for all of

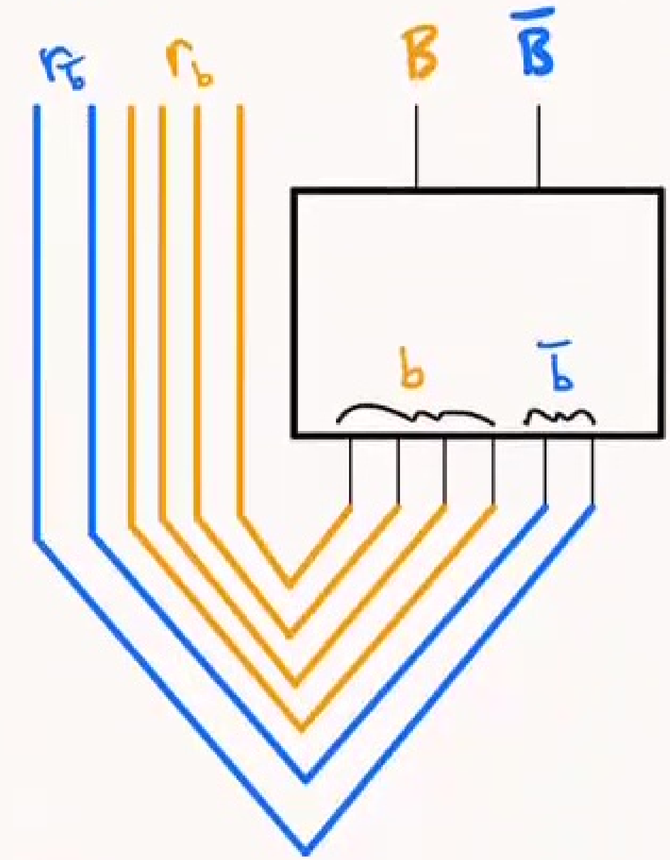
Every code V comes with a special *Choi-Jamiolkowski state*:



$$|EPR\rangle^{\otimes n}$$

Area is a von Neumann entropy in this state:

$$A_B(b) := S(B\, r_b)_{|CJ\rangle}$$



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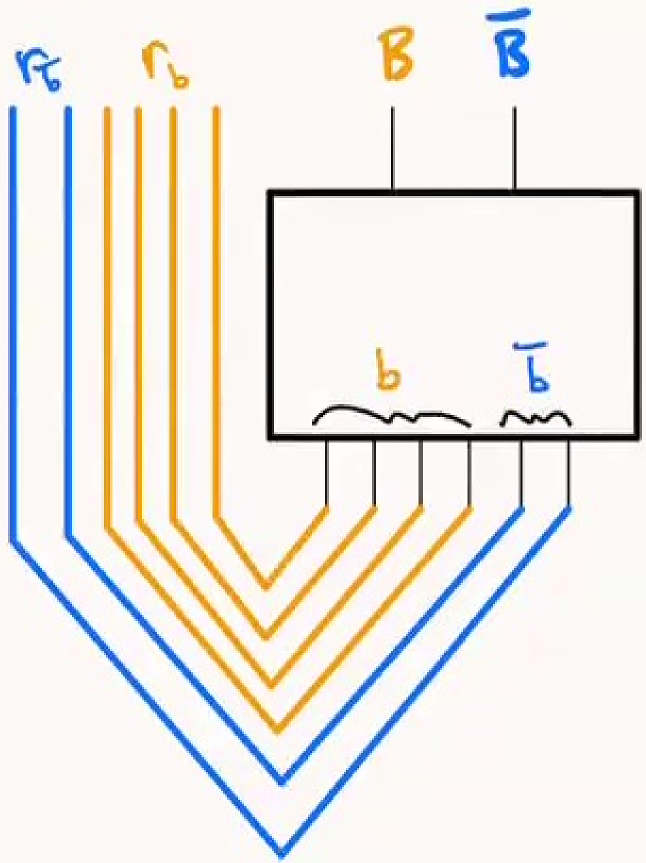

Why *this* area? It is the *functionally unique* definition that leads to a QMS formula.

definition that leads to a QMS formula.

THEOREM:

Given a code

$$V : \mathcal{H}_{b_1} \otimes \mathcal{H}_{b_2} \otimes \dots \otimes \mathcal{H}_{b_n} \longrightarrow \mathcal{H}_B \otimes \mathcal{H}_{\bar{B}}$$

and a state

$$|\psi\rangle \in \mathcal{H}_{b_1} \otimes \mathcal{H}_{b_2} \otimes \dots \otimes \mathcal{H}_{b_n} \otimes \mathcal{H}_R \otimes \mathcal{H}_{\bar{R}}$$

the following two statements are equivalent.

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(2): Holographic entropy formula

For all U^{Product}

$$S(BR)_{V U^{\text{Product}} |\psi\rangle} = A_B(b) + S(bR)_{U^{\text{Product}} |\psi\rangle}$$

Moreover, both imply

(3): Minimality

For all b'

$$A_B(b) + S(bR) \leq A_B(b') + S(b'R)$$

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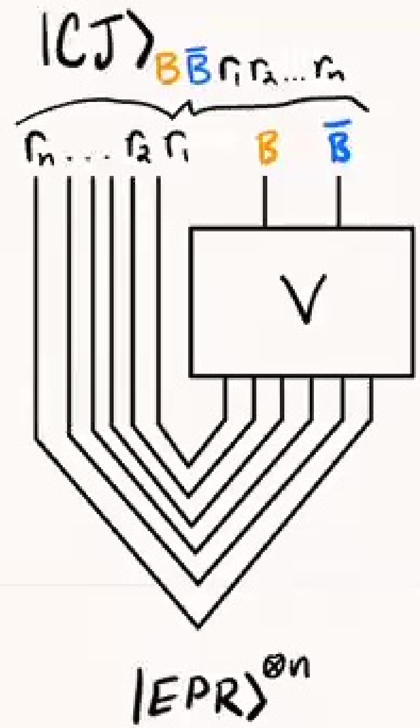
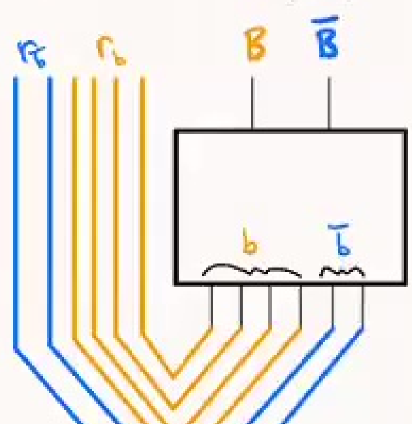
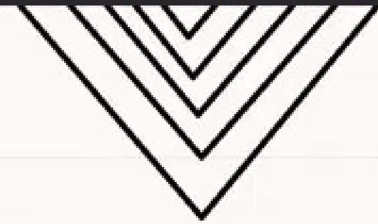
Jamiołkowski state:



Area is a von Neumann entropy in this state:

$$A_B(b) = S(B\, r_b)_{|CJ\rangle}$$





$$|EPR\rangle^{\otimes n}$$

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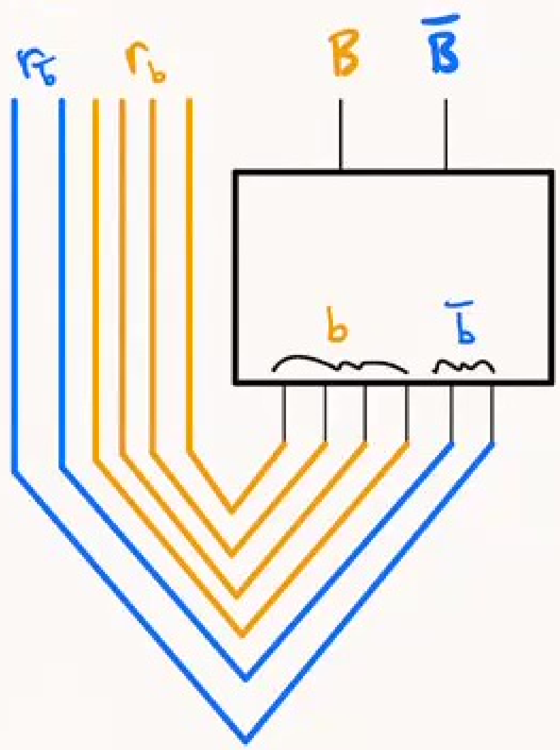

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$$|\psi\rangle \in \mathcal{H}_{b_1} \otimes \mathcal{H}_{b_2} \otimes \dots \otimes \mathcal{H}_{b_n} \otimes \mathcal{H}_R \otimes \mathcal{H}_{\bar{R}}$$

the following two statements are equivalent.

(1): Complementary state-specific reconstruction

For all U_b^{Product} and $\hat{U}_{\bar{b}}^{\text{Product}}$,

$$U_{BR}\,\hat{U}_{\bar{B}\bar{R}}\,V|\psi\rangle = V\,U_b^{\text{Product}}\,\hat{U}_{\bar{b}}^{\text{Product}}|\psi\rangle$$

(2): Holographic entropy formula

For all U^{Product}

$$S(BR)_{VU^{\text{Product}}|\psi\rangle} = A_B(b) + S(bR)_{U^{\text{Product}}|\psi\rangle}$$

Moreover, both imply

(3): Minimality

For all b'

$$A_B(b) + S(bR) \leq A_B(b') + S(b'R)$$

Black hole information paradox

Now that we understand the coding interpretation of QMS, next step is to apply this to black holes.
[CA-Penington 21, CA-Engelhardt-Harlow-Penington-Vardhan in progress]

Primary new subtlety is black holes have to be described by *non-isometric codes*.

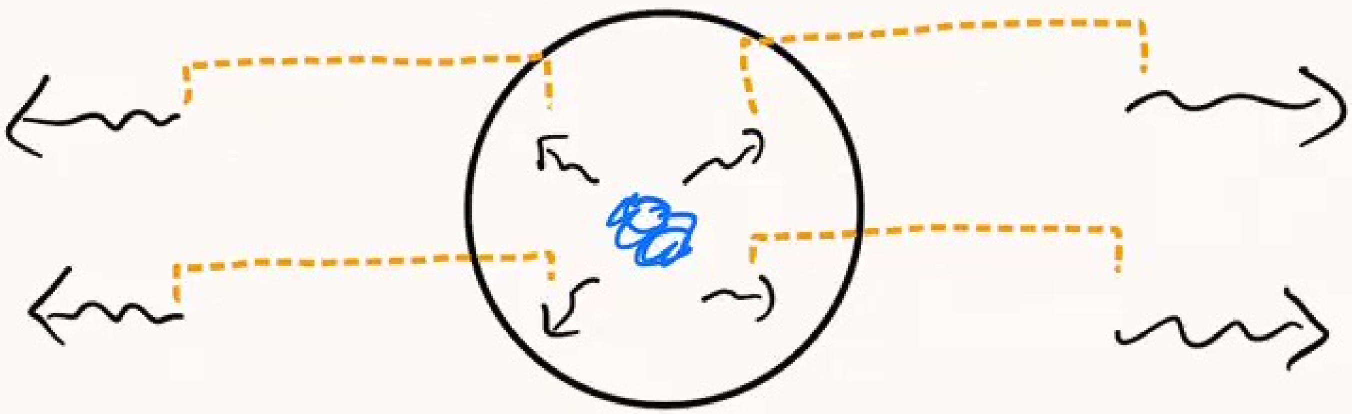
In 2019 there was excitement over a large advance in understanding the information paradox.
[Penington 19, Almheiri-Engelhardt-Marolf-Maxfield 19]

described by *non-isometric codes*.

In 2019 there was excitement over a large advance in understanding the information paradox.
[Penington 19, Almheiri-Engelhardt-Marolf-Maxfield 19]

What they did is alleviate the tension between

1) Hawking's picture of black hole evaporation, which famously predicted information loss.



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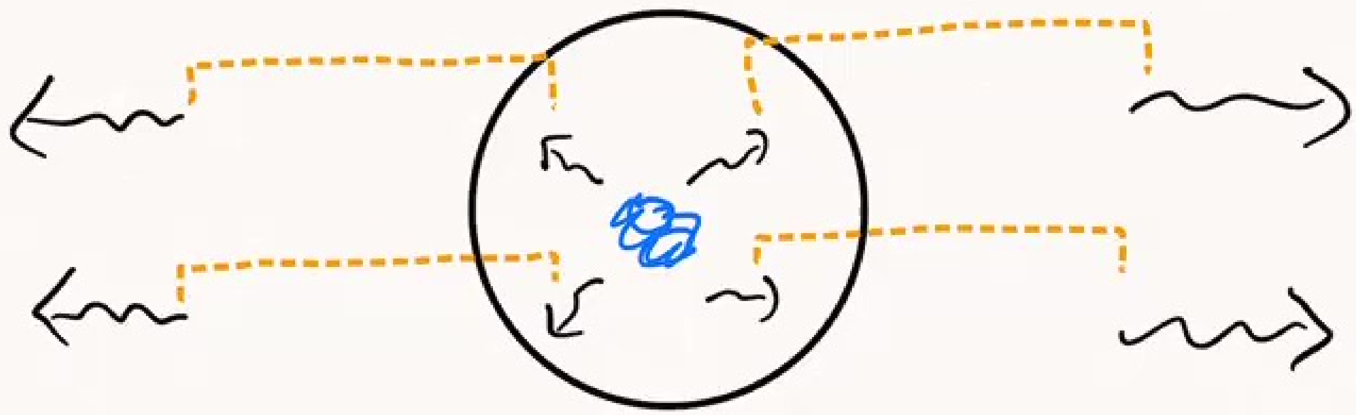
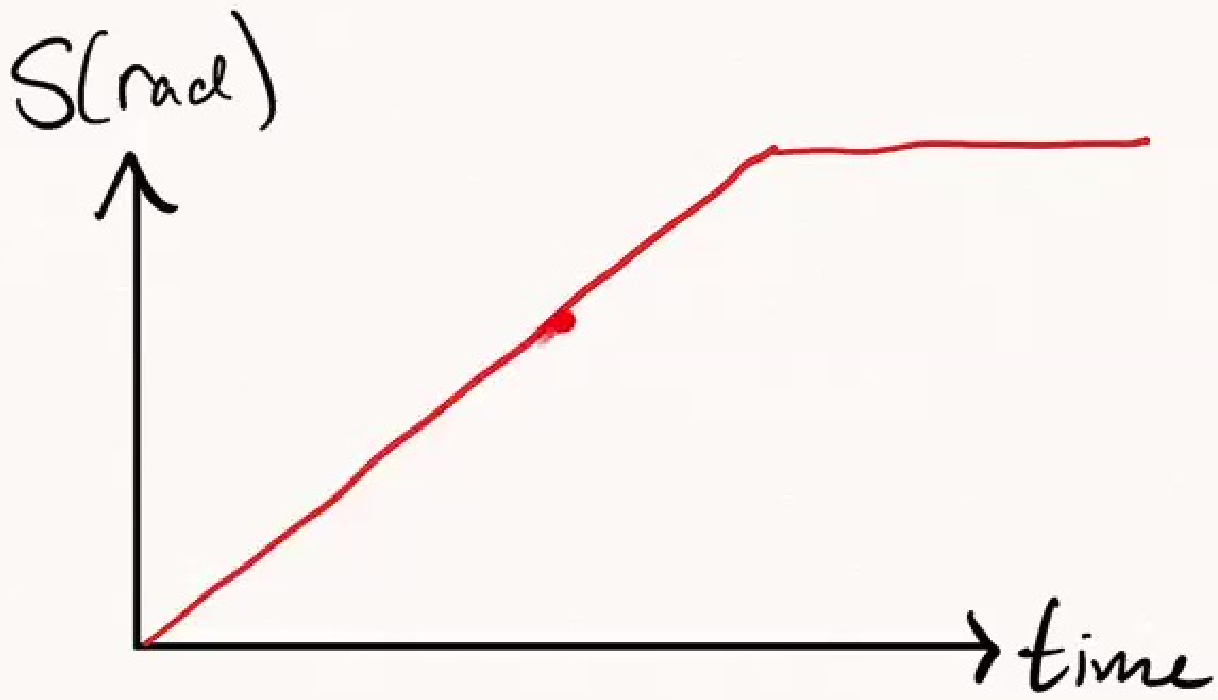
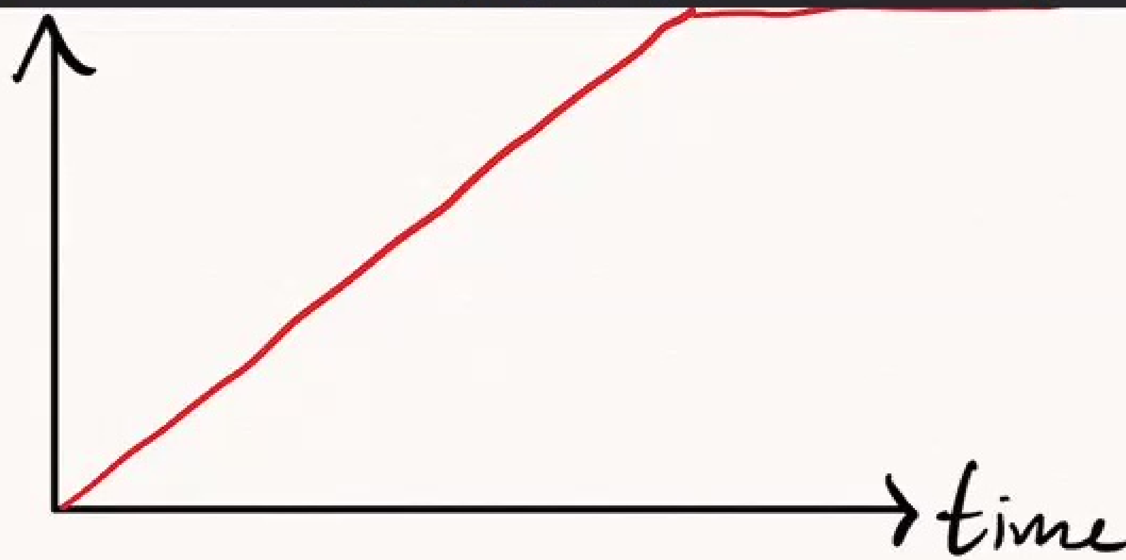
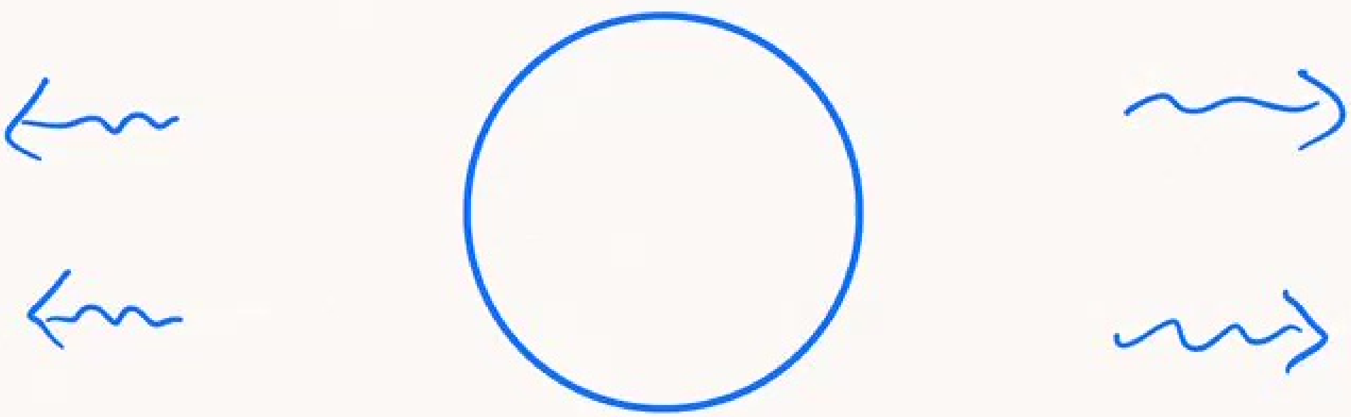


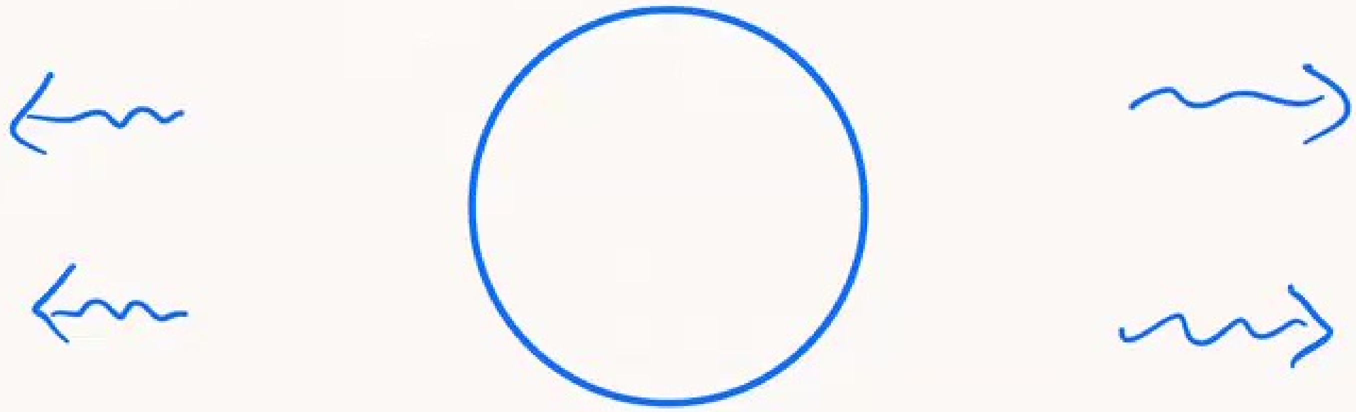


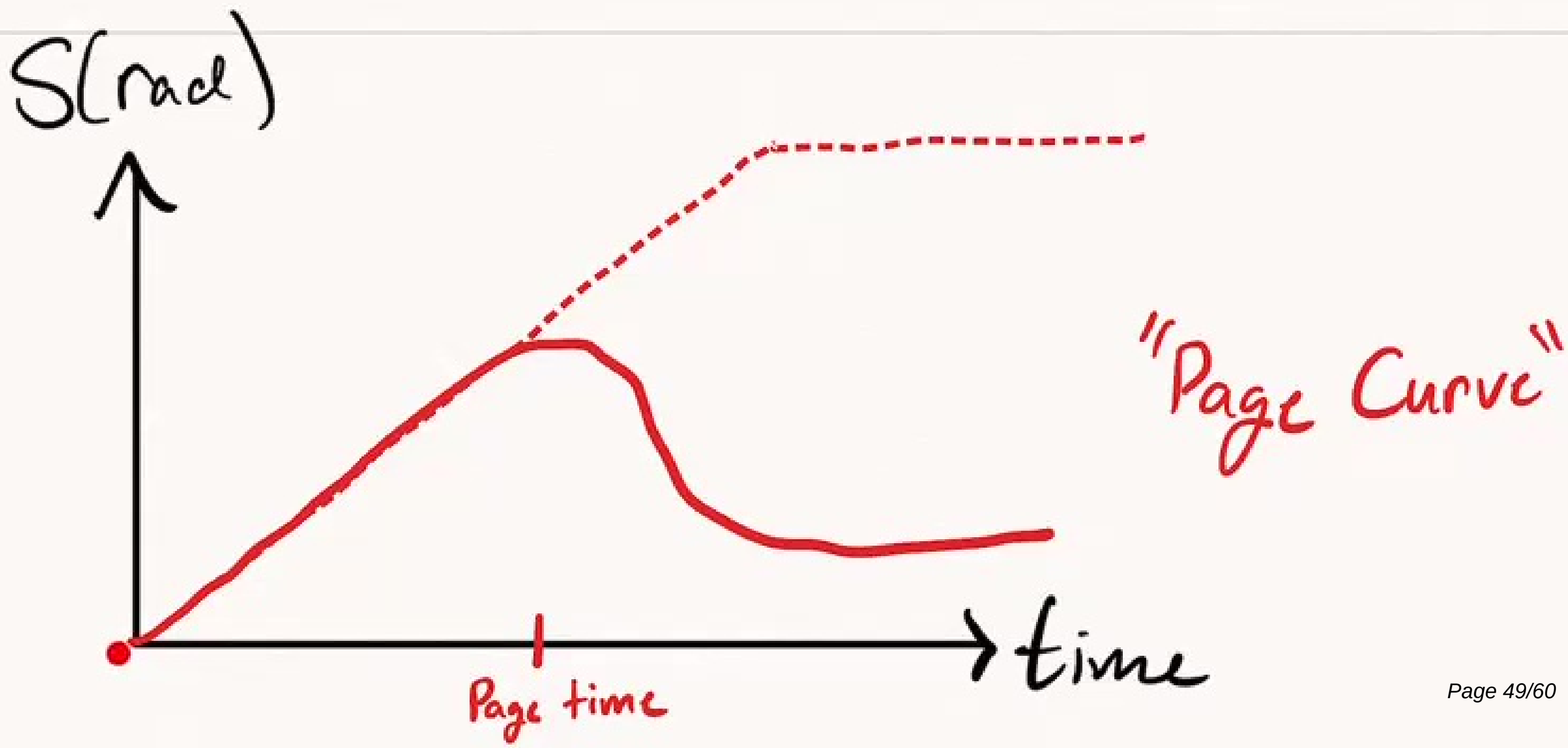


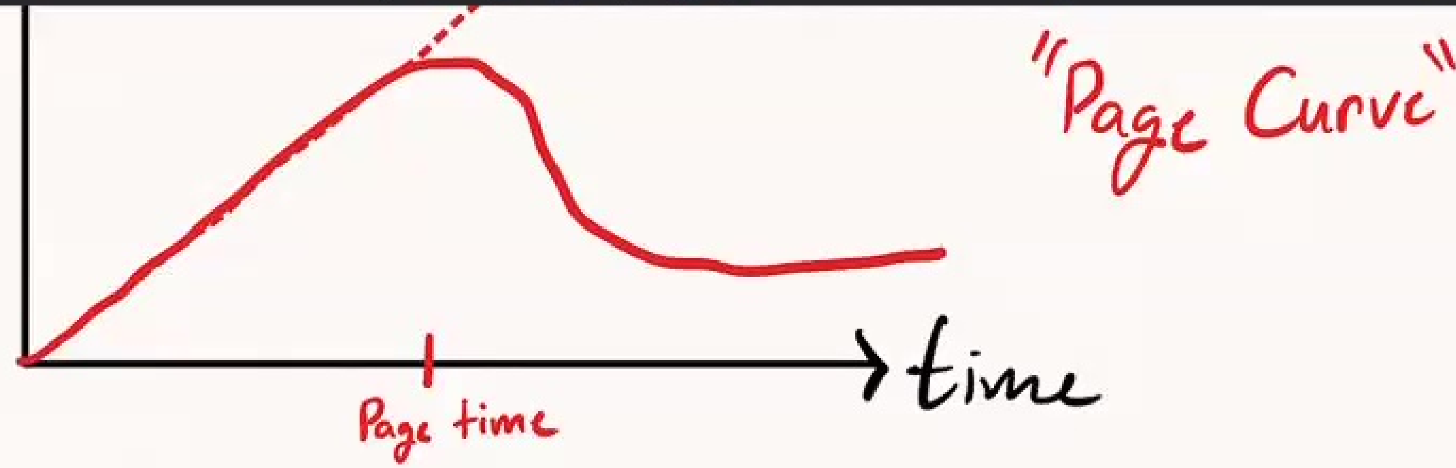
2) Unitarity of quantum mechanics. Someone who stays outside the black hole will still have access to all information.



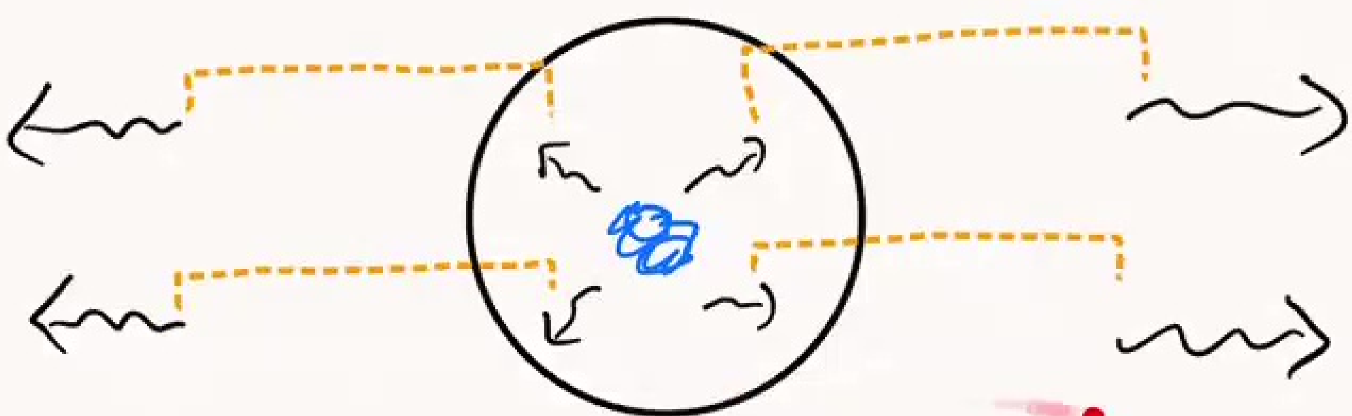
all information.

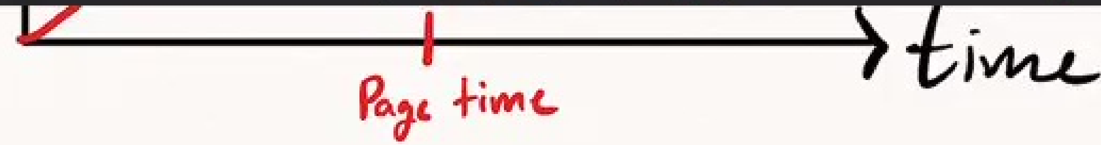






They computed the Page curve using the QMS formula, applied in the Hawking picture.





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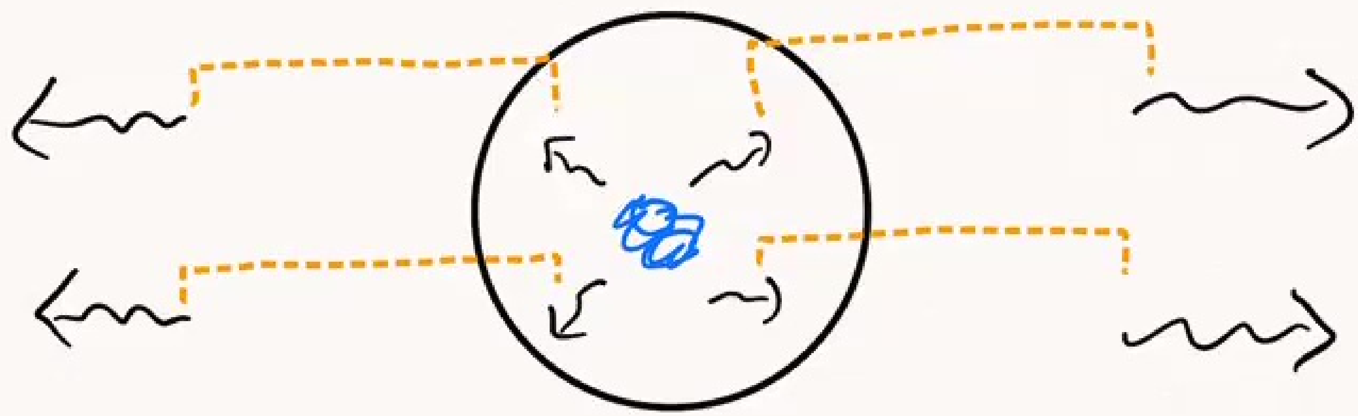

$$S(\mathrm{rad})_{\mathrm{Unitary}} = \min_{b} \left(\frac{A(b)}{4G} + S(\mathrm{rad} \cup b)_{\mathrm{Hawking}} \right)$$

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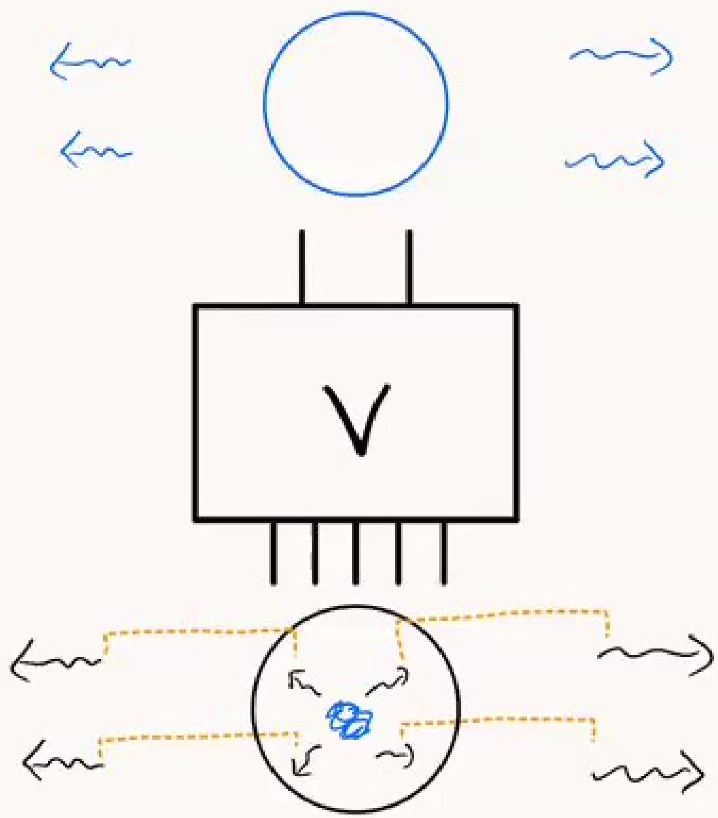
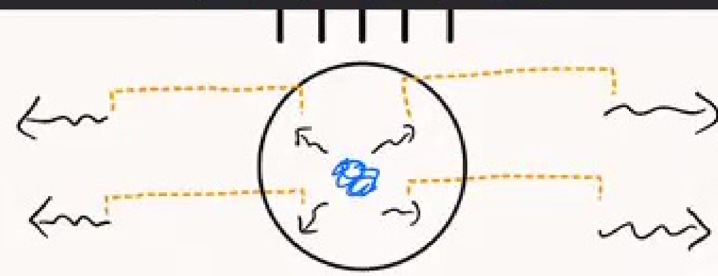
Our new coding interpretation of QMS suggests that we should therefore understand the Hawking picture as a logical description and the Unitary picture as the physical description:





However, the Hawking description can have *more* degrees of freedom than the Unitary description!

Hawking Picture

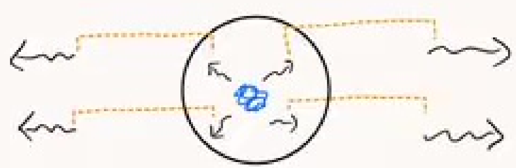
N radiation qubits

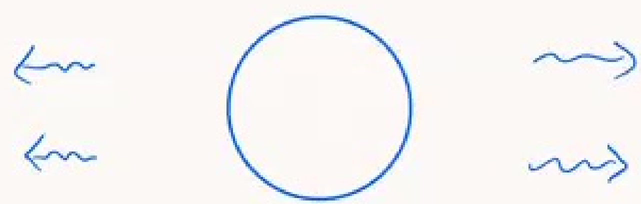
S interior qubits

Unitary Picture

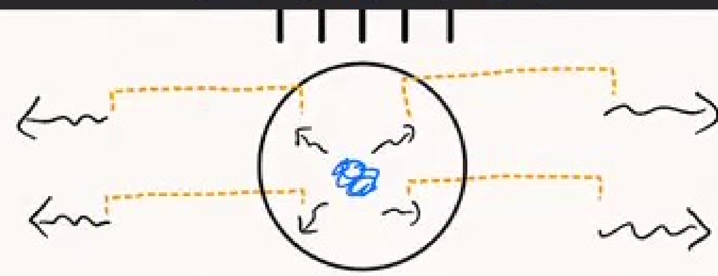
N radiation qubits

$A/4G$ black hole qubits





After Page time, S > A / 4G. In these cases V must be non-isometric.



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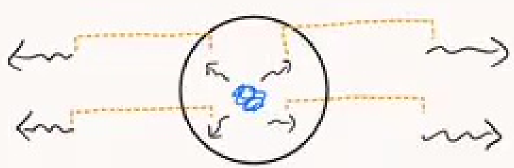
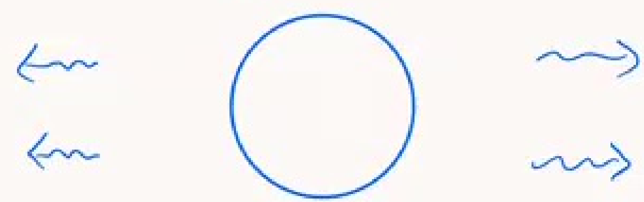
Hawking Picture	Unitary Picture
N radiation qubits	N radiation qubits
S interior qubits	A/4G black hole qubits





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Unitary Picture

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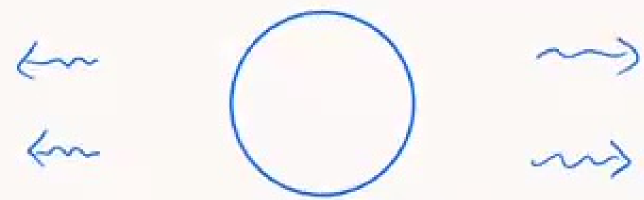

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While heretical in usual quantum error correction, non-isometric codes do exist that exhibit our weaker sort of encoding.

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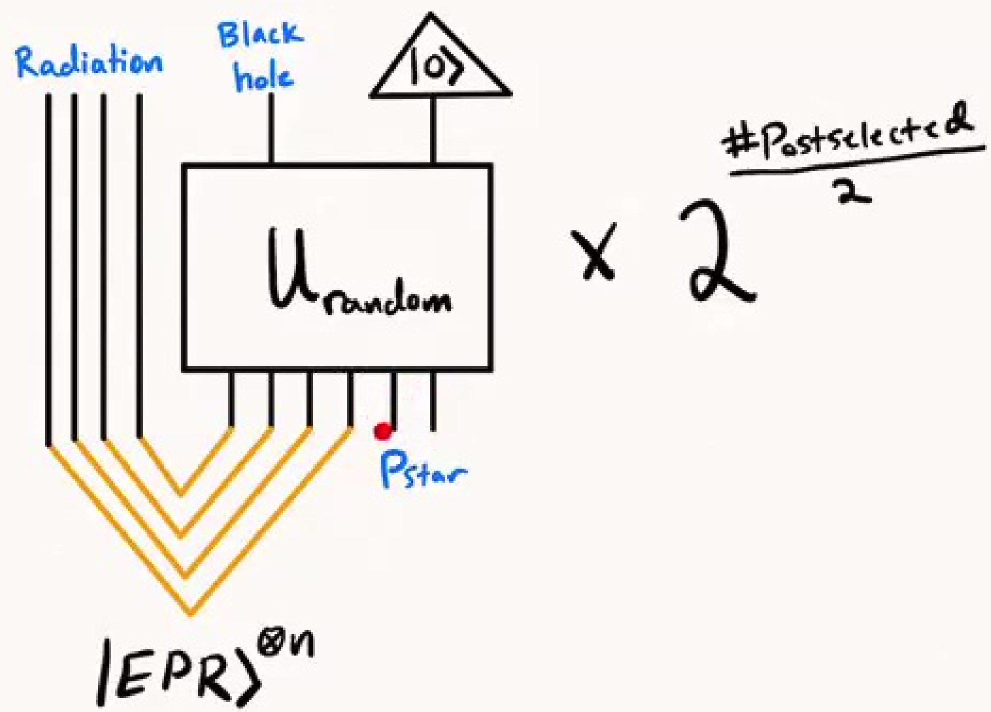
A model we are thinking about:

[CA-Penington 21, CA-Engelhardt-Harlow-Penington-Vardhan in progress]



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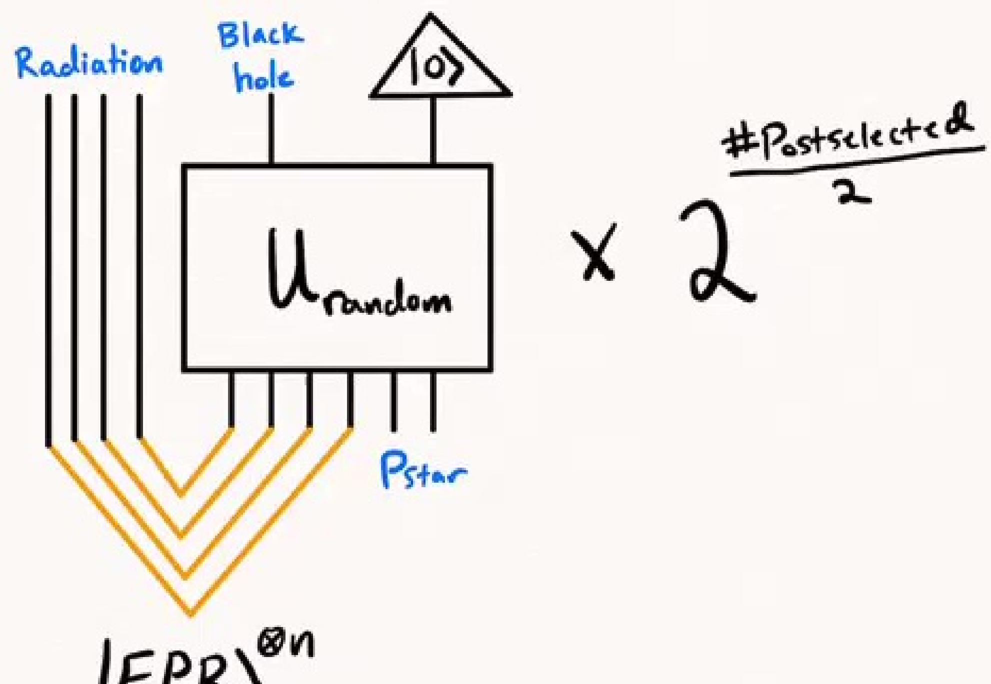

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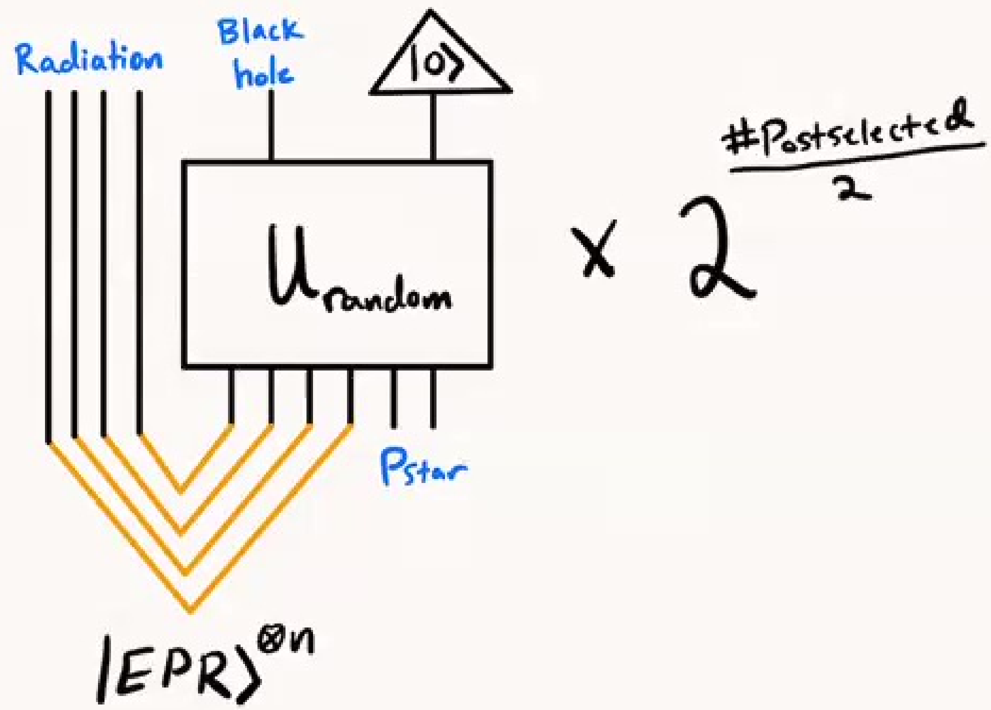

Radiation in output encodes all input qubits in this state-specific way. Therefore the QMS formula computes its entropy, exactly as in black holes.

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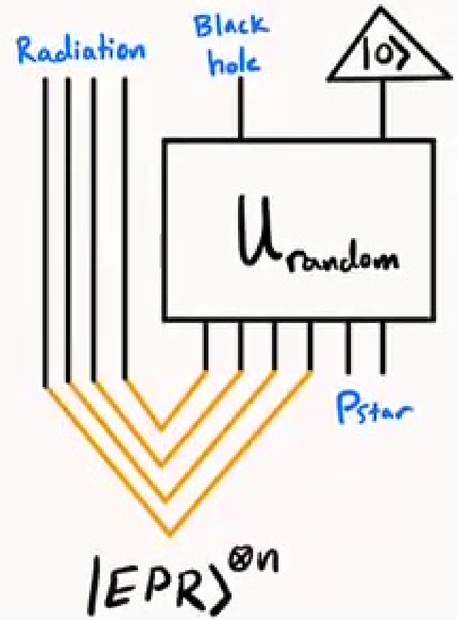

$$\times\, 2^{\frac{\#\text{Postselected}}{2}}$$

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Also suggests we should understand semiclassical gravity as an effective description that to understand requires talking about quantum codes.

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understand requires talking about quantum codes.

Here are some things I wanted to talk about but didn't have time:

1) Everything here has been about asymptotic quantities like von Neumann entropy. In full generality, better to work with one-shot quantities like the min-/max-entropy. There are situations where that makes a big difference. [CA-Leichenauer-Levine 19, CA-Penington 2020, CA-Levine-Penington-Wildenhain in progress]

2) The Page curve for the "reflected entropy." The reflected entropy is a new quantum information theoretic quantity that's become important in holography. It diagnoses when two entangled quantum systems have a geometrically connected AdS dual. We have understood the details of a formula similar to QMS, but for reflected entropy. [CA-Rath 19, CA-Faulkner-Lin-Rath 21/22]