Title: QFT III 2021/2022

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Collection: QFT III 2021/2022

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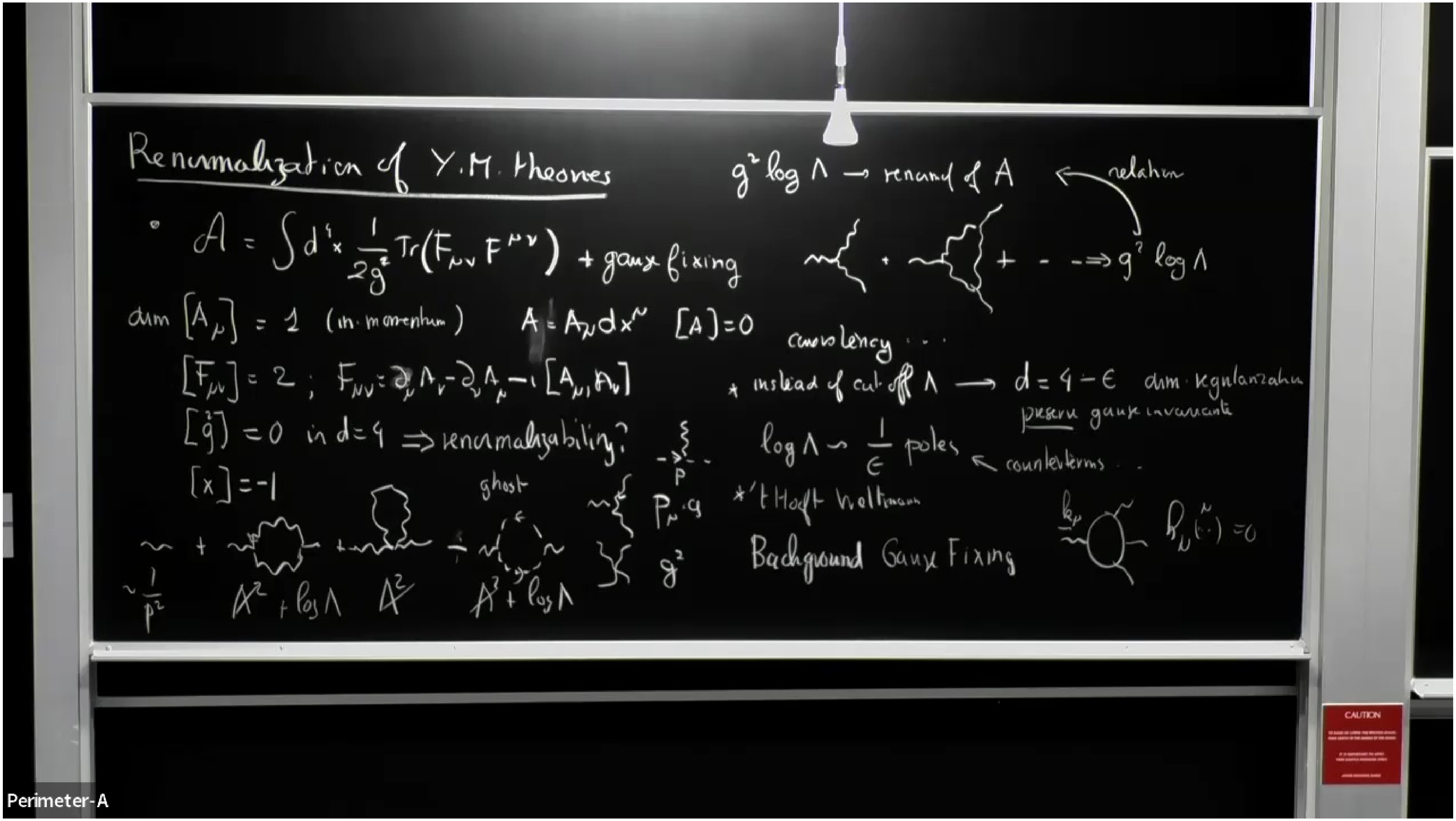
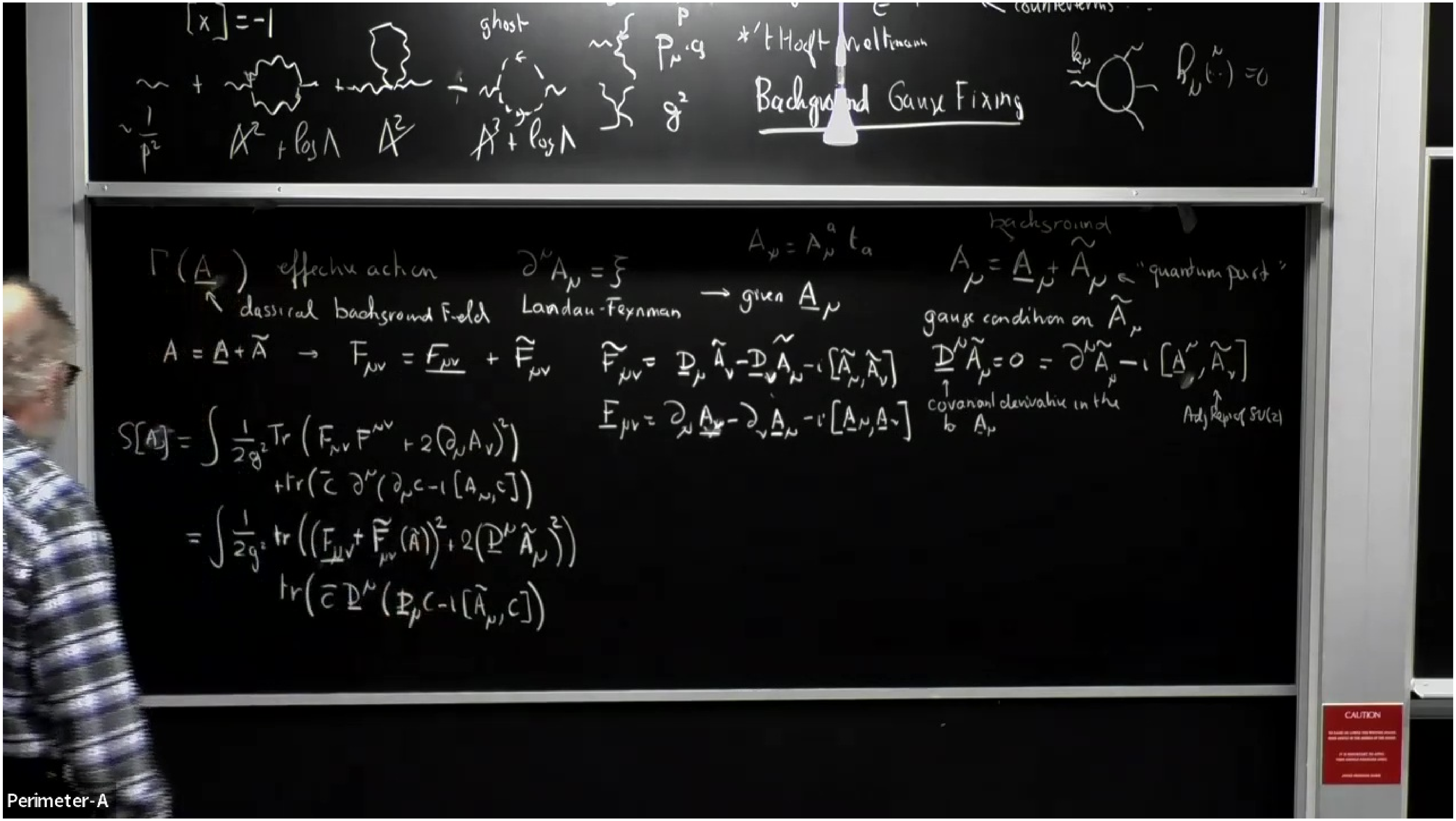
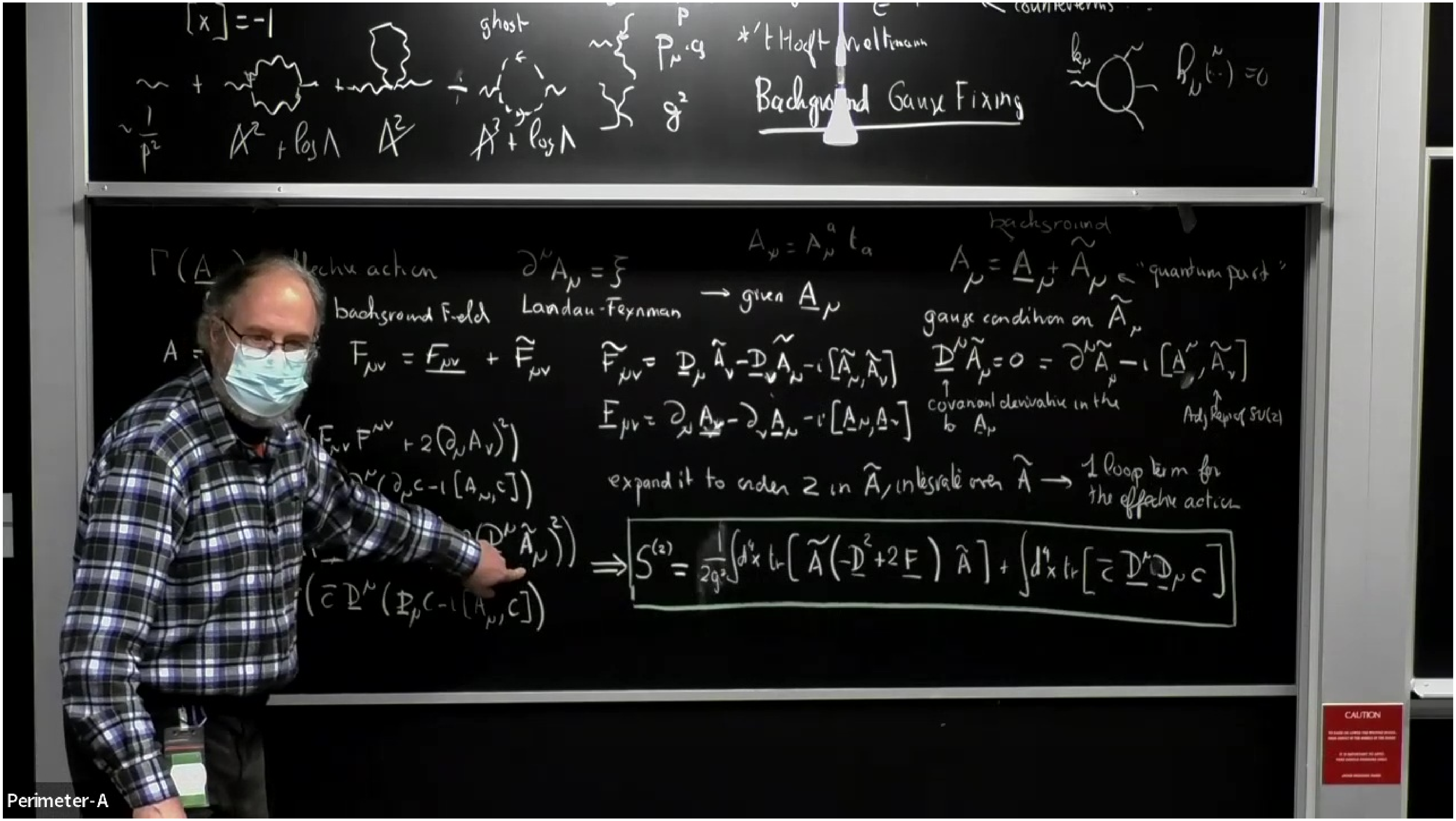
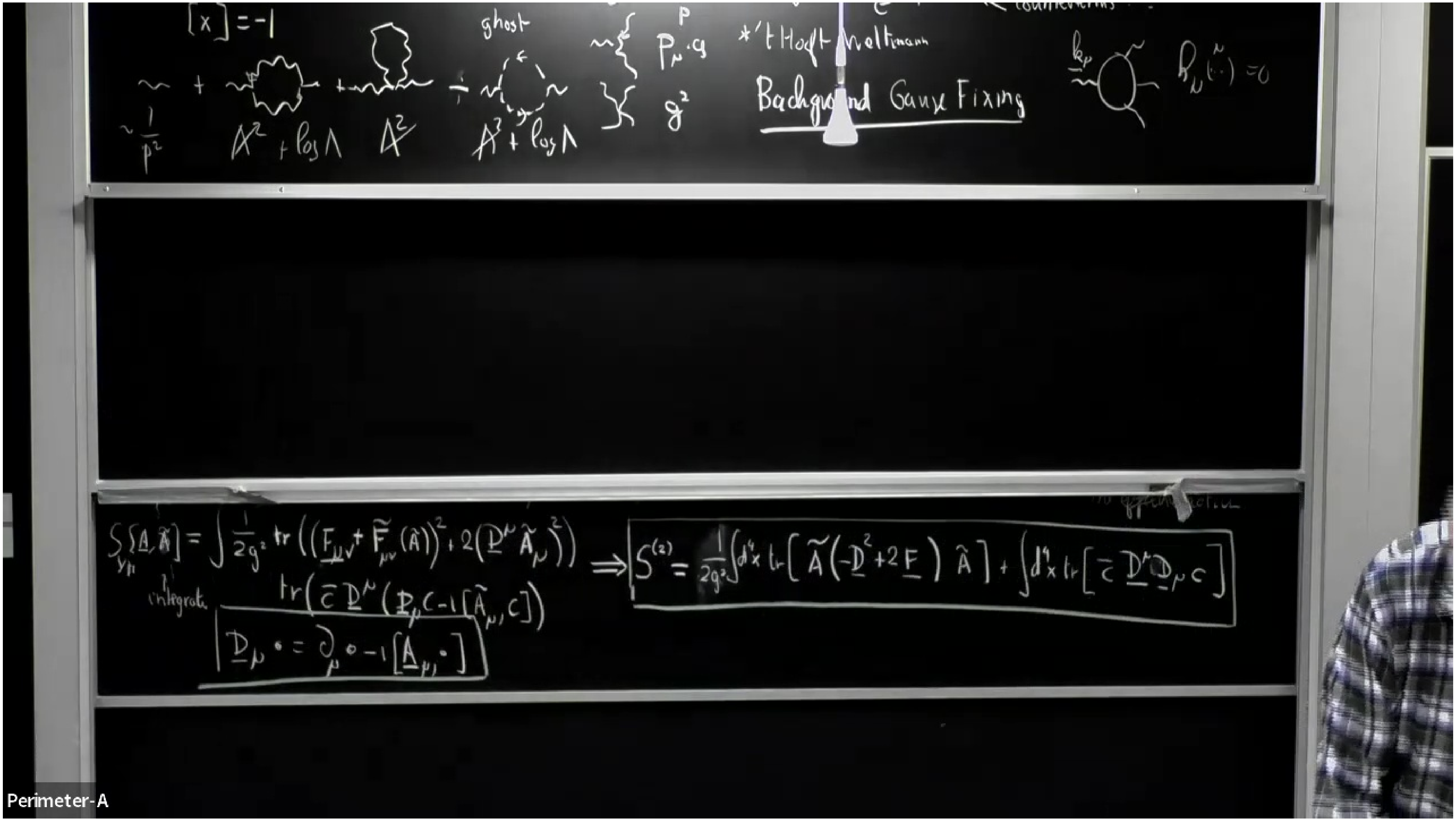
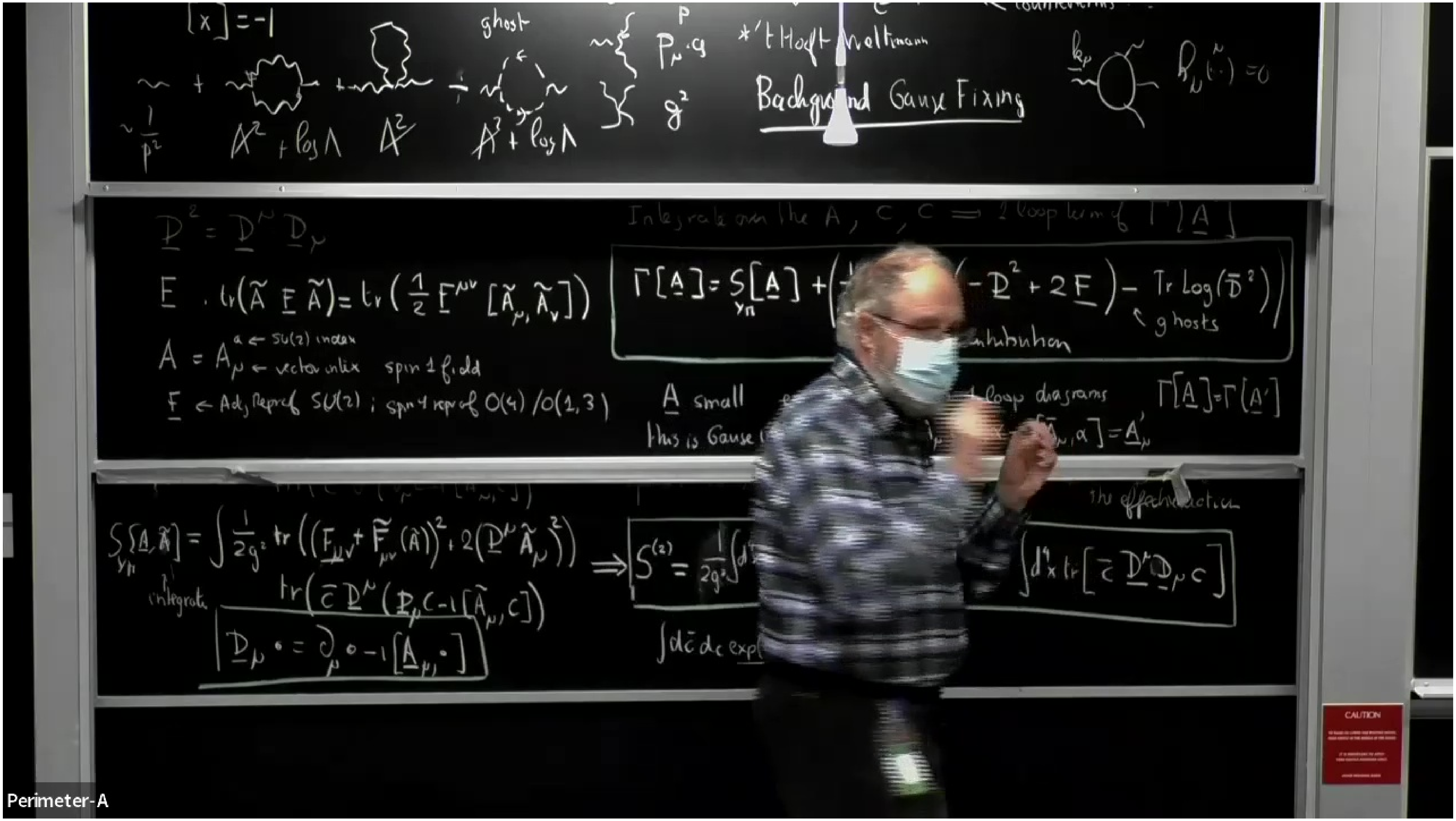
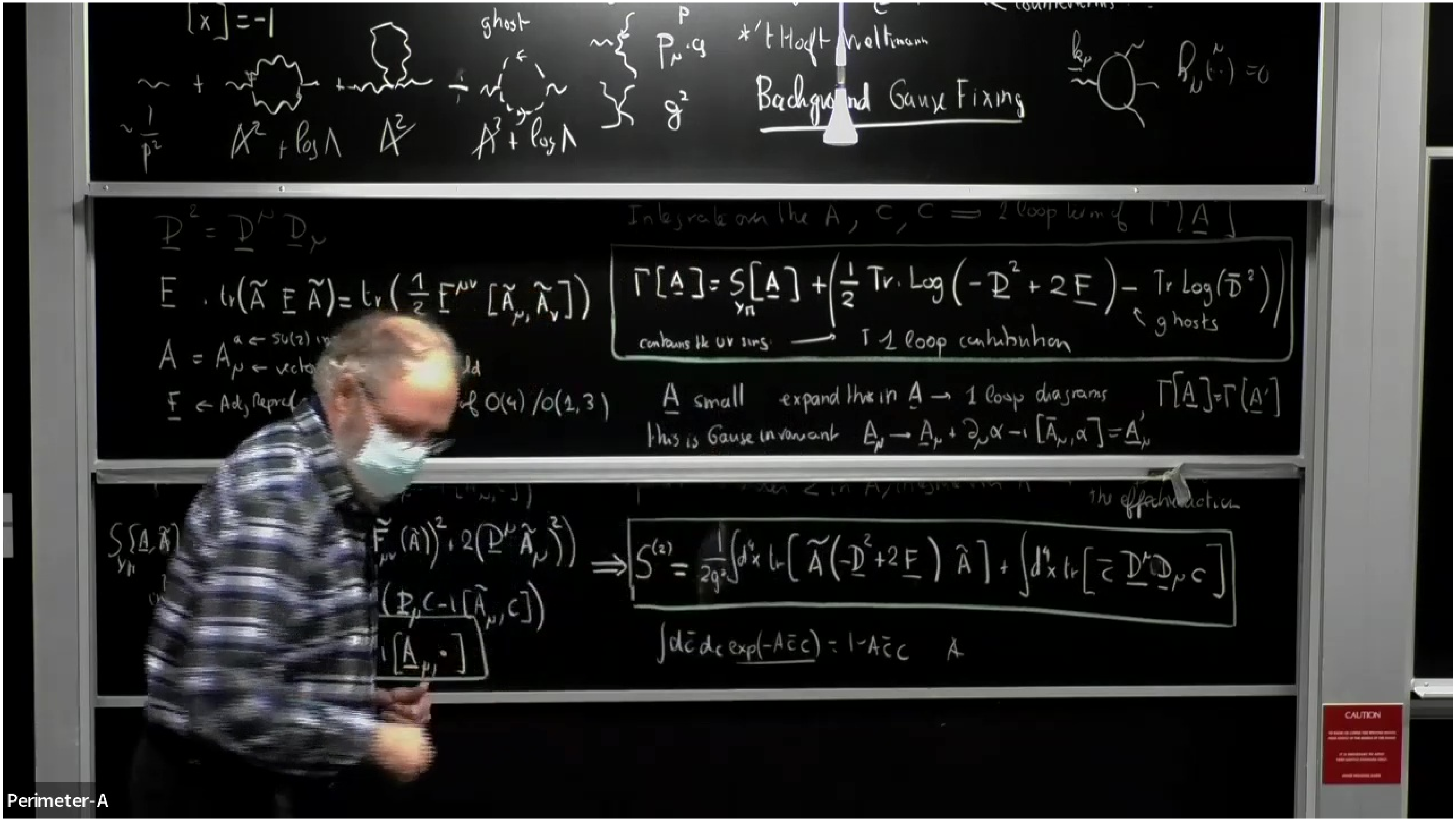

Renormalization of Y.M. theories
$g^2 \log\Lambda \to$ renorm of A
relation
• $A = \int d^4x \frac{1}{2g^2} Tr(F_{\mu\nu}F^{\mu\nu})$ + gauge fixing
$+ \dots \Rightarrow g^2 \log\Lambda$
dim $[A_\mu] = 1$ (in momentum)
$A = A_\mu dx^\mu$ $[A]=0$
covariancy ...
$[F_{\mu\nu}] = 2$; $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$
* instead of cut-off $\Lambda \longrightarrow d = 4-\epsilon$ dim. regularization
preserve gauge invariance
$[g] = 0$ in $d=4 \Rightarrow$ renormalizability?
$\log\Lambda \rightsquigarrow \frac{1}{\epsilon}$ poles
counterterms ...
$[x] = -1$
ghost
$p_\mu \cdot q$
*'t Hooft Veltman
k_μ
$k_\nu(\dots) = 0$
g^2
Background Gauge Fixing
$\sim \frac{1}{p^2}$
$\Lambda^2 + \log\Lambda$
Λ^2
$\Lambda^2 + \log\Lambda$
Perimeter-A
CAUTION

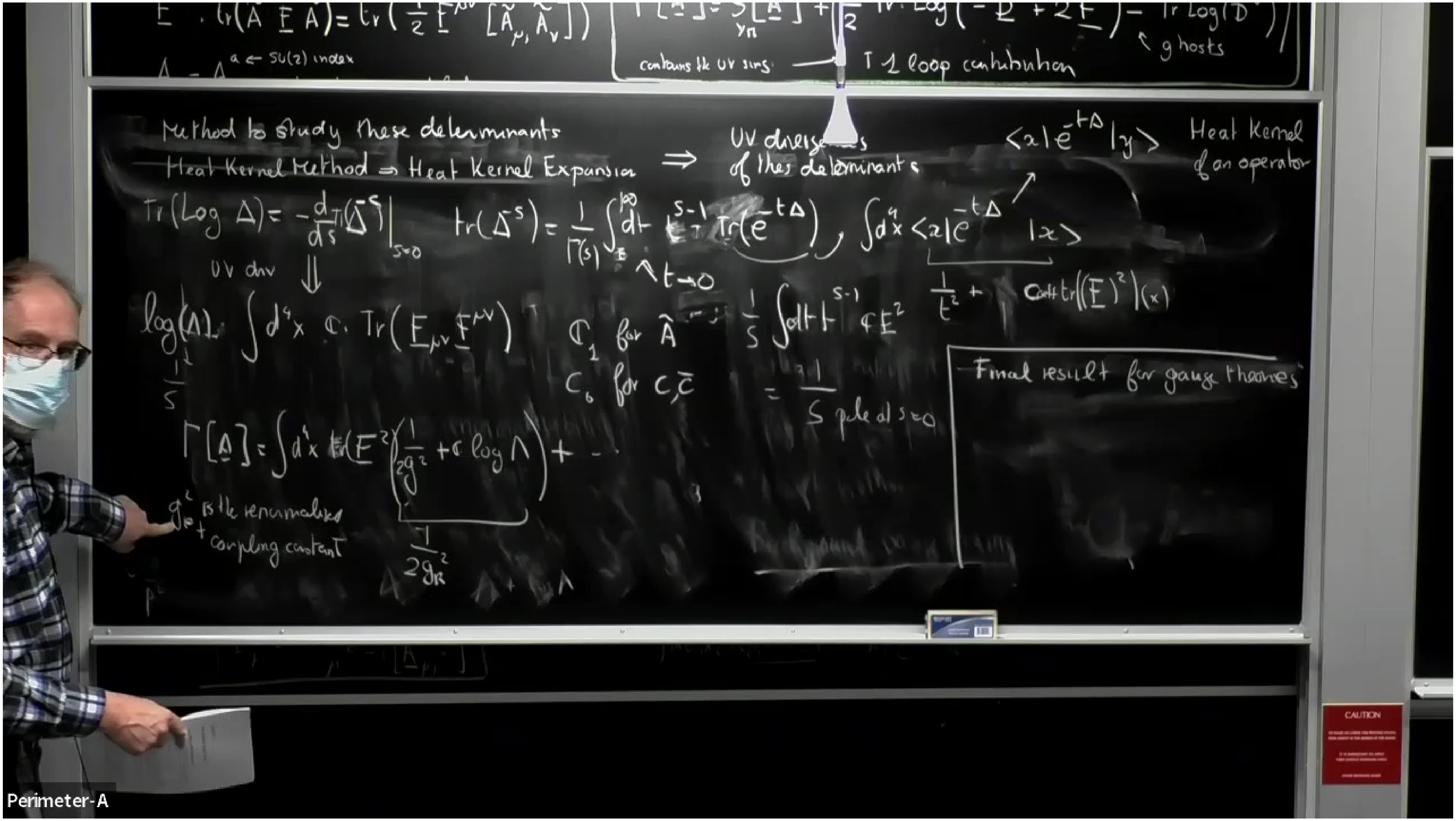

[x] = -1
ghost
Pμ·q
g²
*'t Hooft Veltmann
Background Gauge Fixing
1/p²
Λ² + log Λ
Λ²
Λ³ + log Λ
counterterms
Γ(A) effective action
classical background field
∂^μ A_μ = ξ
Landau-Feynman
→ given A_μ
A_μ = A_μ^a t_a
background
A_μ = A_μ + Ã_μ ← "quantum part"
gauge condition on Ã_μ
A = A + Ã → F_μν = F_μν + F̃_μν
F̃_μν = D_μ Ã_ν − D_ν Ã_μ − i[Ã_μ, Ã_ν]
F_μν = ∂_μ A_ν − ∂_ν A_μ − i[A_μ, A_ν]
D^μ Ã_μ = 0 = ∂^μ Ã_μ − i[A^μ, Ã_ν]
covariant derivative in the adj rep of SU(2) b A_μ
Adj Rep of SU(2)
S[A] = ∫ 1/(2g²) Tr(F_μν F^μν + 2(∂_ν A_ν)²)
+ tr(c̄ ∂^μ(∂_μ c − i[A_μ, c]))
= ∫ 1/(2g²) tr((F_μν + F̃_μν(Ã))² + 2(D^μ Ã_μ)²)
tr(c̄ D^μ(D_μ c − i[Ã_μ, c]))
CAUTION
Perimeter-A

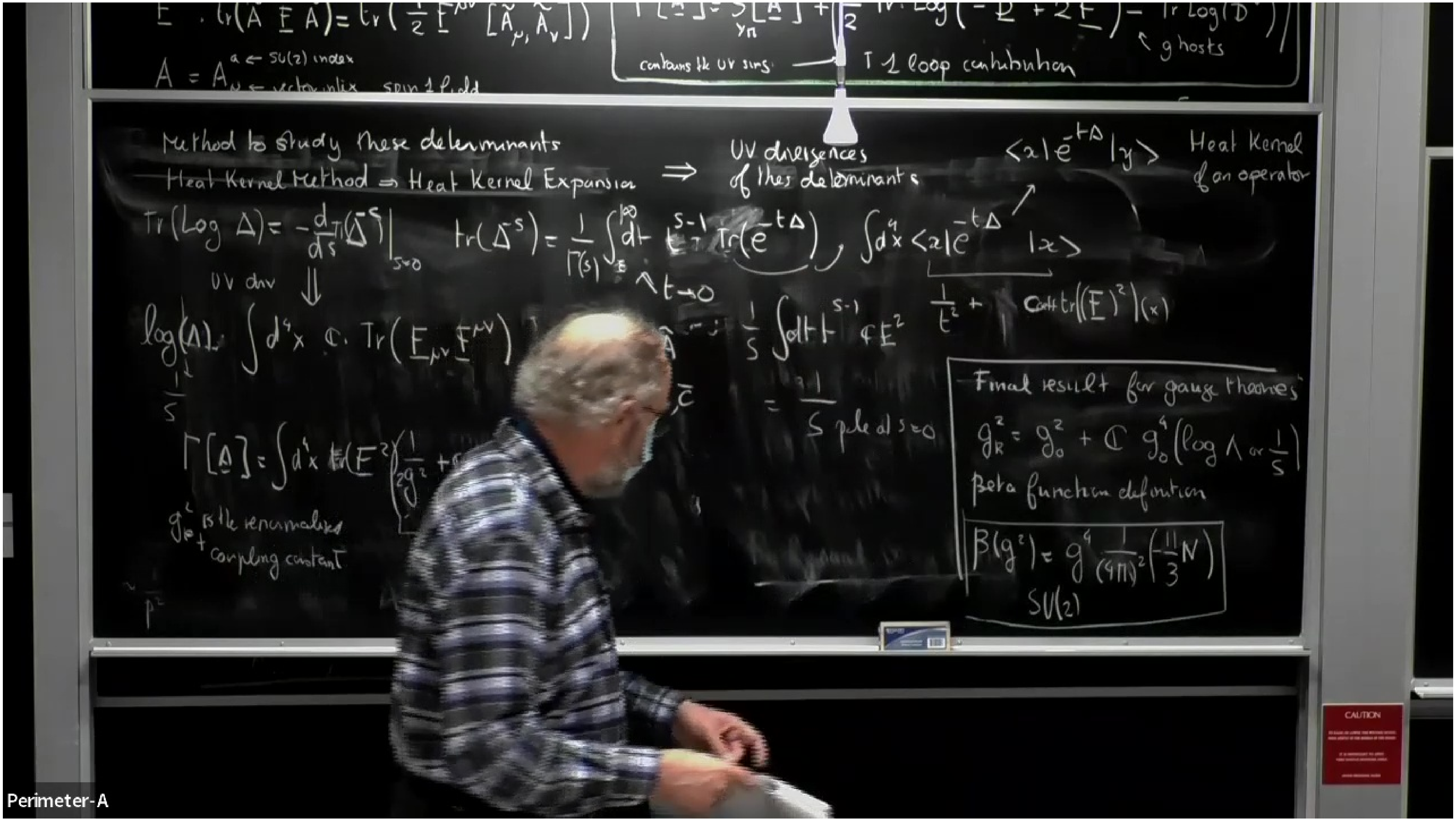

[x] = -1
ghost
P_μ·q
*'t Hooft Veltman
g^2
Background Gauge Fixing
counterterms
Λ^2 + log Λ
Λ^2
Λ^3 + log Λ
~ 1/p^2
k_μ
k_ν(...) = 0
A_μ = A_μ^a t_a
background
A_μ = A_μ + Ã_μ ← "quantum part"
effective action
∂^μ A_μ = ξ
background Field
Landau-Feynman → given A_μ
gauge condition on Ã_μ
F_μν = F_μν + F̃_μν
F̃_μν = D_μ Ã_ν − D_ν Ã_μ − i[Ã_μ, Ã_ν]
F_μν = ∂_μ A_ν − ∂_ν A_μ − i[A_μ, A_ν]
D^μ Ã_μ = 0 = ∂^μ Ã_μ − i[A^μ, Ã_ν]
covariant derivative in the Adj Rep of SU(2)
expand it to order 2 in Ã, integrate over Ã → 1 loop term for the effective action
⇒ S^(2) = 1/2g^2 ∫d^4x tr[Ã(−D^2 + 2F) Ã] + ∫d^4x tr[c̄ D^μ D_μ c]
Perimeter-A

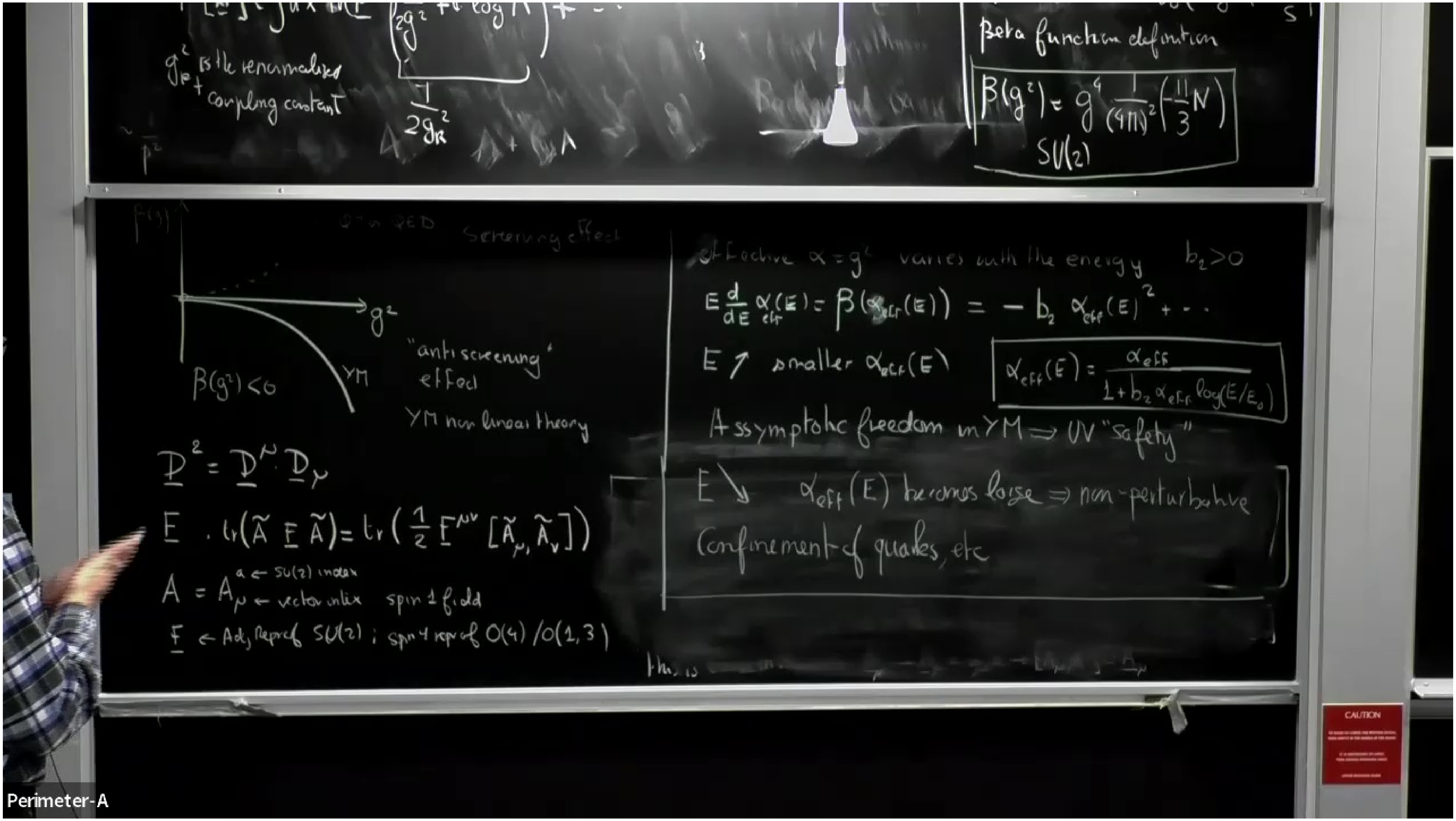

[x] = -1
ghost
$P_\mu \cdot q$
*'t Hooft Veltmann
Background Gauge Fixing
$A^2 + \log\Lambda$
A^2
$A^2 + \log\Lambda$
g^2
k_μ
integrate
$S[\underline{A}, \tilde{A}] = \int \frac{1}{2g^2} tr\left(\left(\underline{F}_{\mu\nu} + \tilde{F}_{\mu\nu}(\tilde{A})\right)^2 + 2\left(\underline{D}^\mu \tilde{A}_\mu\right)^2\right)$
$tr\left(\bar{c}\,\underline{D}^\mu\left(\underline{D}_\mu c - i[\tilde{A}_\mu, c]\right)\right)$
$\underline{D}_\mu \bullet = \partial_\mu \bullet - i[\underline{A}_\mu, \bullet]$
$\Rightarrow S^{(2)} = \frac{1}{2g^2}\int d^4x\, tr\left[\tilde{A}\left(-\underline{D}^2 + 2\underline{F}\right)\tilde{A}\right] + \int d^4x\, tr\left[\bar{c}\,\underline{D}^\mu \underline{D}_\mu c\right]$
Perimeter-A


ghost
*'t Hooft Veltmann
Background Gauge Fixing
ghosts
SU(2) index
vector index spin 1 field
A small
integrate
the effective action
CAUTION
Perimeter-A


Perimeter-A
ghost
*'t Hooft Veltmann
Background Gauge Fixing
Integrate over the A, c, c̄ ⟹ 1 loop term of Γ[A]
Γ[A] = S_YM[A] + (1/2 Tr Log(−D² + 2F) − Tr Log(D̄²))
ghosts
contains the UV sing. ⟶ Γ 1 loop contribution
A small expand this in A → 1 loop diagrams
Γ[A] = Γ(A′)
This is Gauge invariant A_μ → A_μ + ∂_μα − i[A_μ, α] = A′_μ
the effective action
S^(2) = 1/(2g²) ∫d⁴x tr[Ã(−D² + 2F) Â] + ∫d⁴x tr[c̄ D^μ D_μ c]
∫dc̄ dc exp(−Ac̄c) = 1 − Ac̄c
CAUTION


a ← SU(2) index
contains the UV sing.
1 loop contribution
ghosts
Method to study these determinants
Heat Kernel Method ⇒ Heat Kernel Expansion
⇒ UV divergences of the determinants
Heat Kernel of an operator
UV div
s=0
t→0
for
pole at s=0
Final result for gauge theories
is the renormalized coupling constant
Perimeter-A
CAUTION


contains the UV sings
1 loop contribution
ghosts
Method to study these determinants
Heat Kernel Method ⇒ Heat Kernel Expansion
⇒ UV divergences of these determinants
Heat Kernel of an operator
UV div
Final result for gauge theories
Beta function definition
SU(2)
g_R is the renormalized coupling constant
S pole at s=0
CAUTION
Perimeter-A


g_R^2 is the renormalized coupling constant
-1/2g_R^2
Beta function definition
β(g²) = g⁴ 1/(4π)² (−11/3 N)
SU(2)
β(g)
Screening effect
g²
β(g²) < 0
YM
"anti screening" effect
YM non linear theory
D² = D^μ D_μ
E . tr(Ã E Ã) = tr(1/2 E^μν [Ã_μ, Ã_ν])
A = A_μ^a   a ← SU(2) index   μ ← vector index   spin 1 field
E ← Adj Rep of SU(2) ; spin 4 rep of O(4) / O(1,3)
effective α = g² varies with the energy   b₂ > 0
E d/dE α_eff(E) = β(α_eff(E)) = − b₂ α_eff(E)² + . . .
E ↗ smaller α_eff(E)
α_eff(E) = α_eff / (1 + b₂ α_eff log(E/E₀))
Assymptotic freedom in YM ⇒ UV "safety"
E ↘ α_eff(E) becomes large ⇒ non-perturbative
Confinement of quarks, etc
Perimeter-A
CAUTION