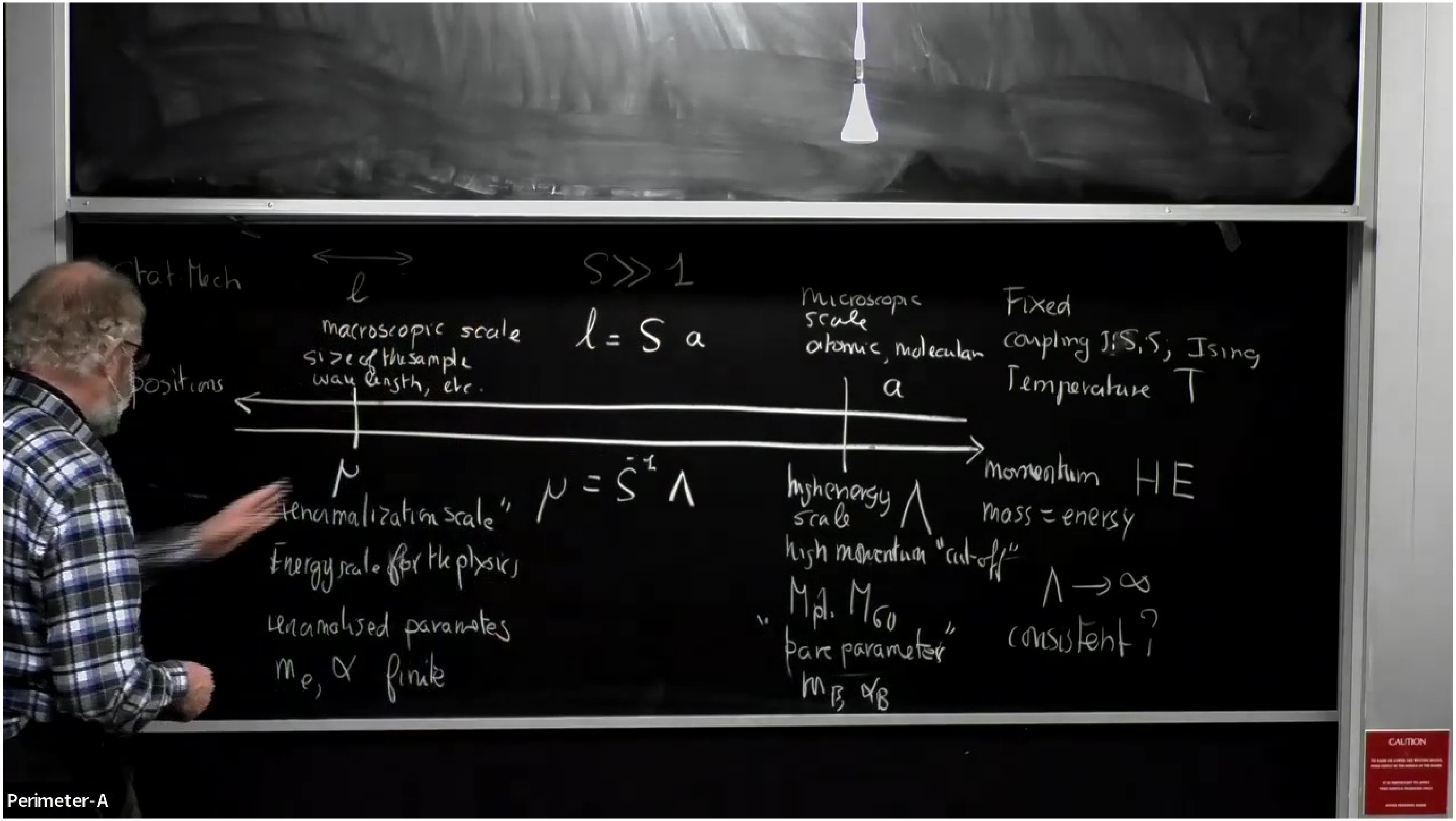
Title: QFT III 2021/2022

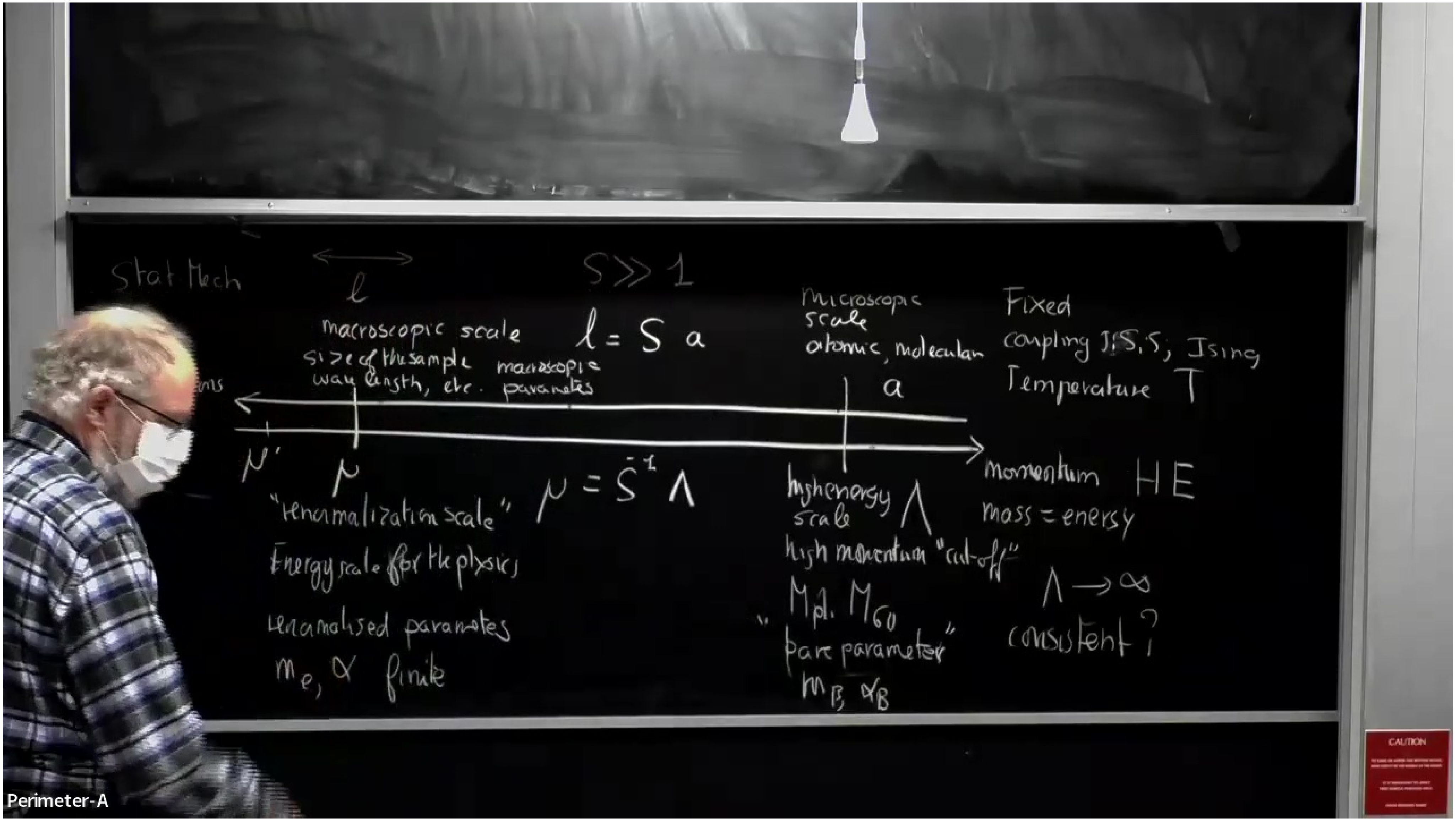
Speakers: Francois David

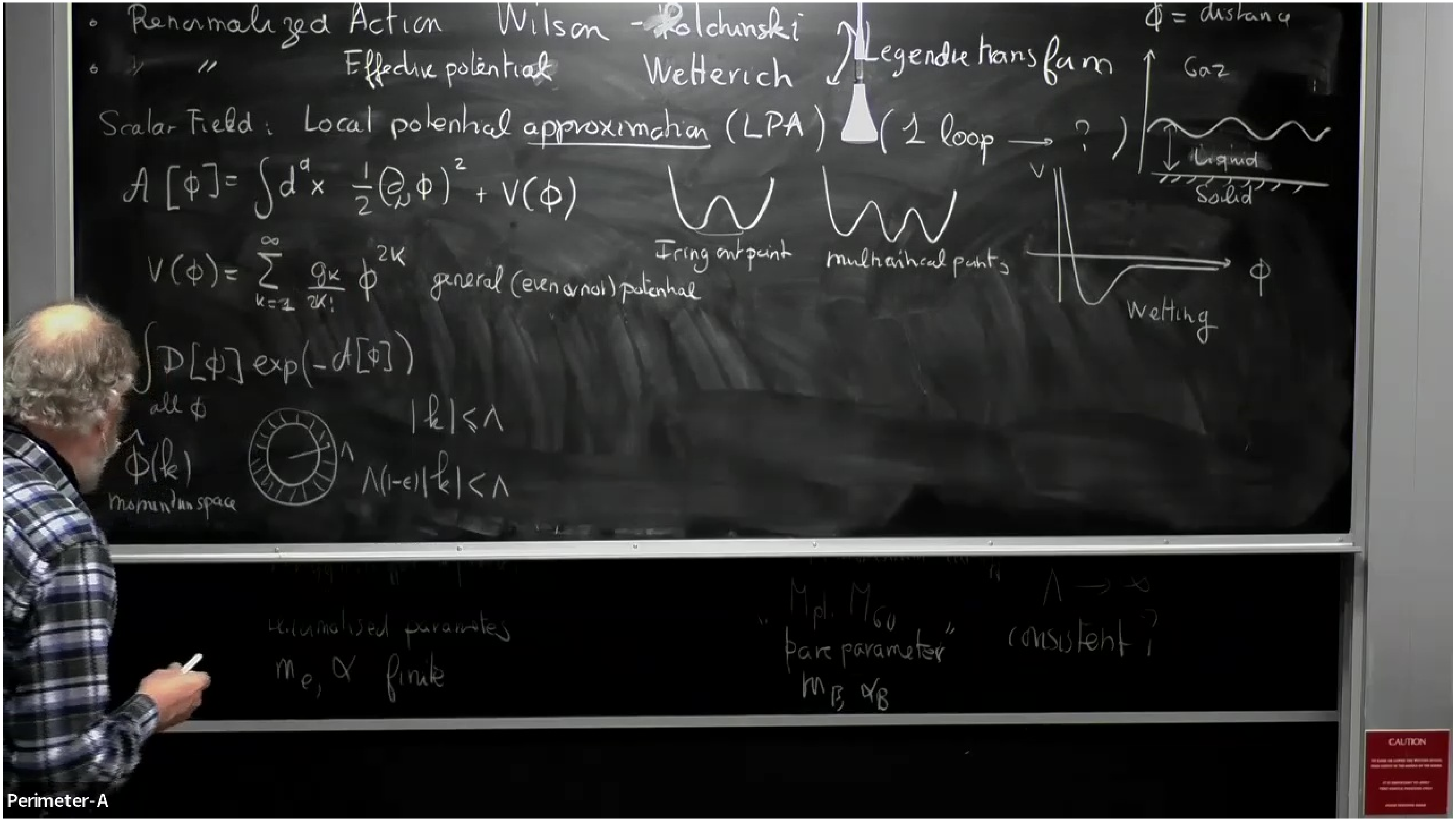
Collection: QFT III 2021/2022

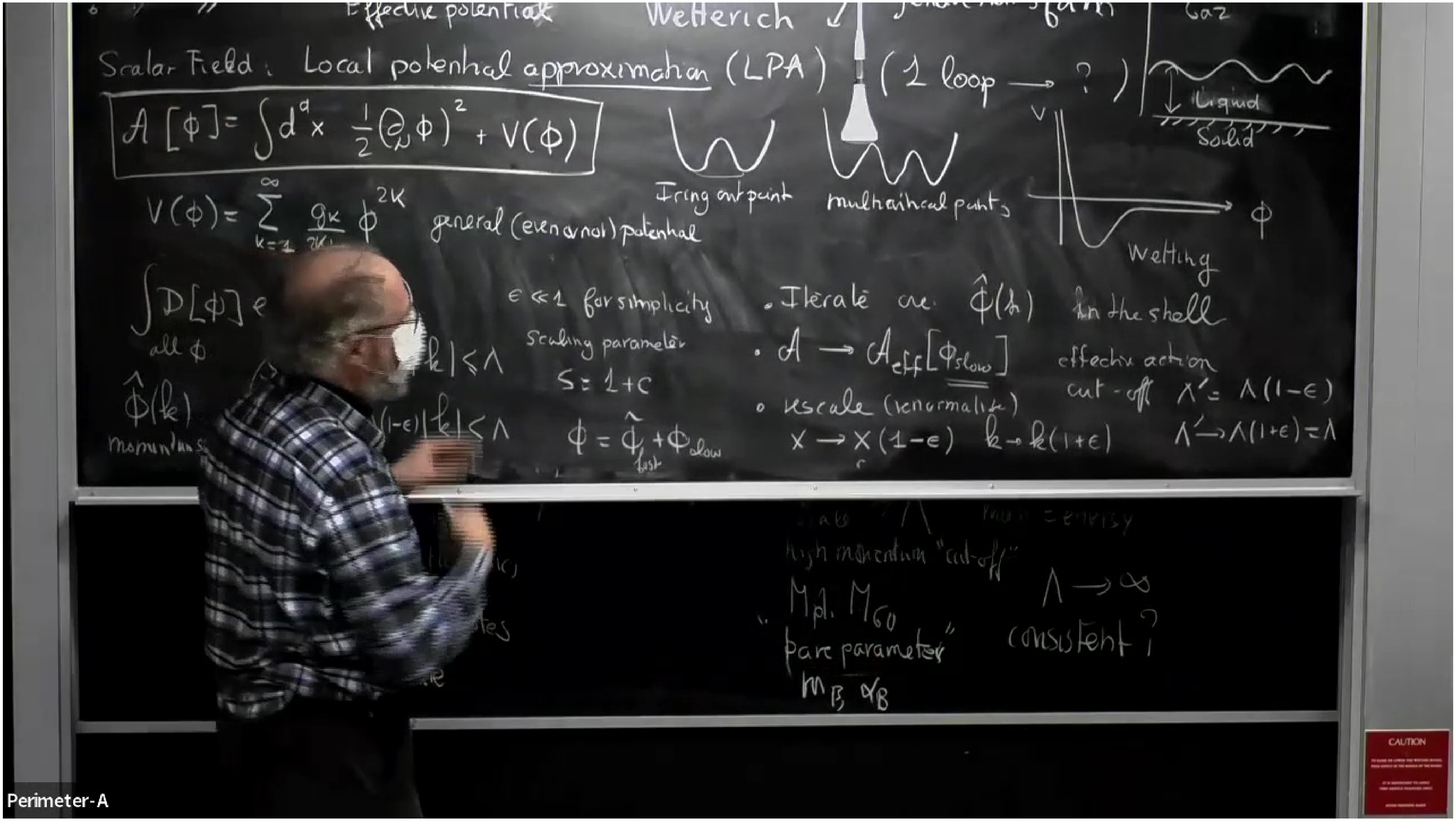
Date: February 07, 2022 - 10:15 AM

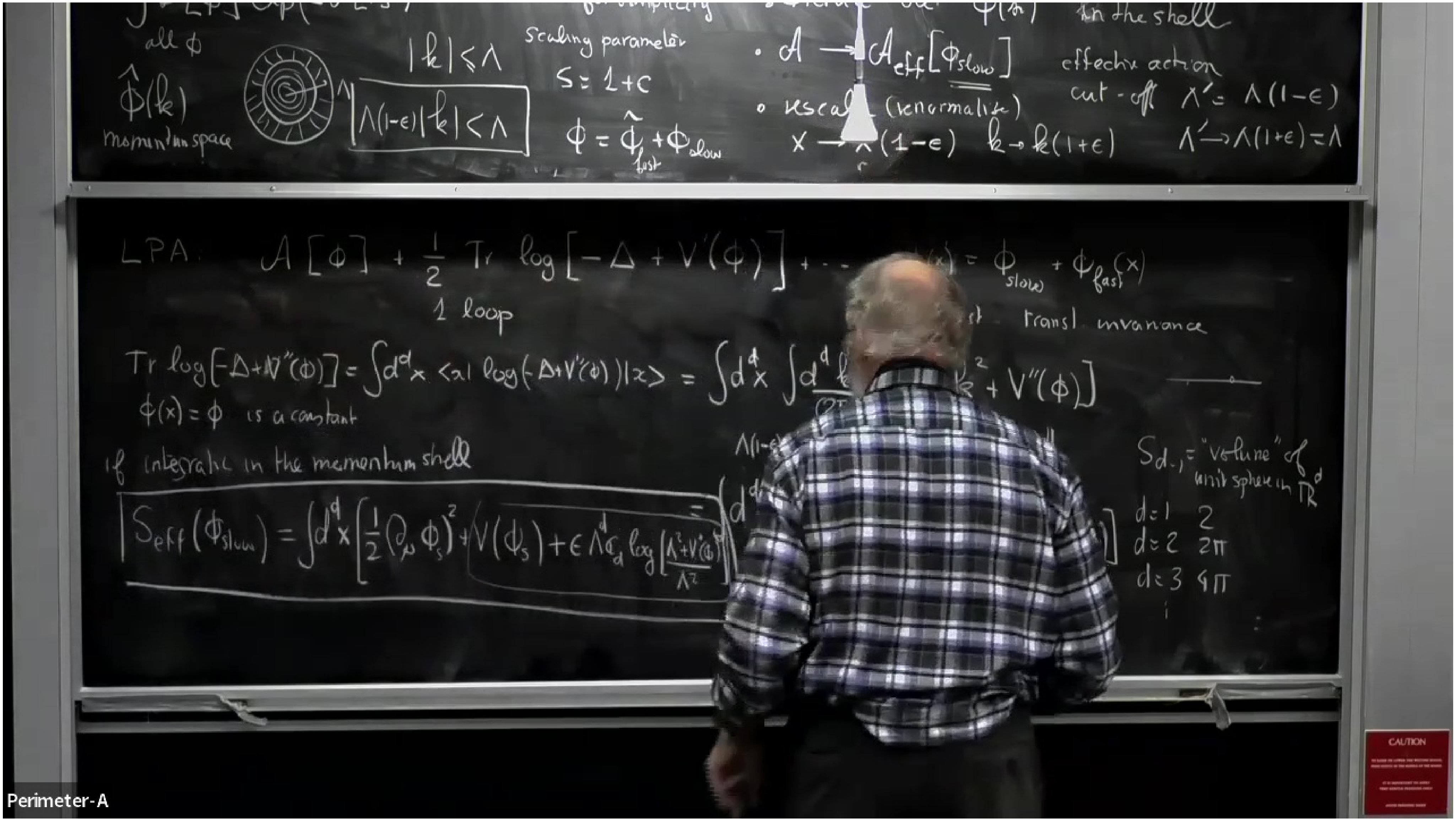
URL: https://pirsa.org/22020015

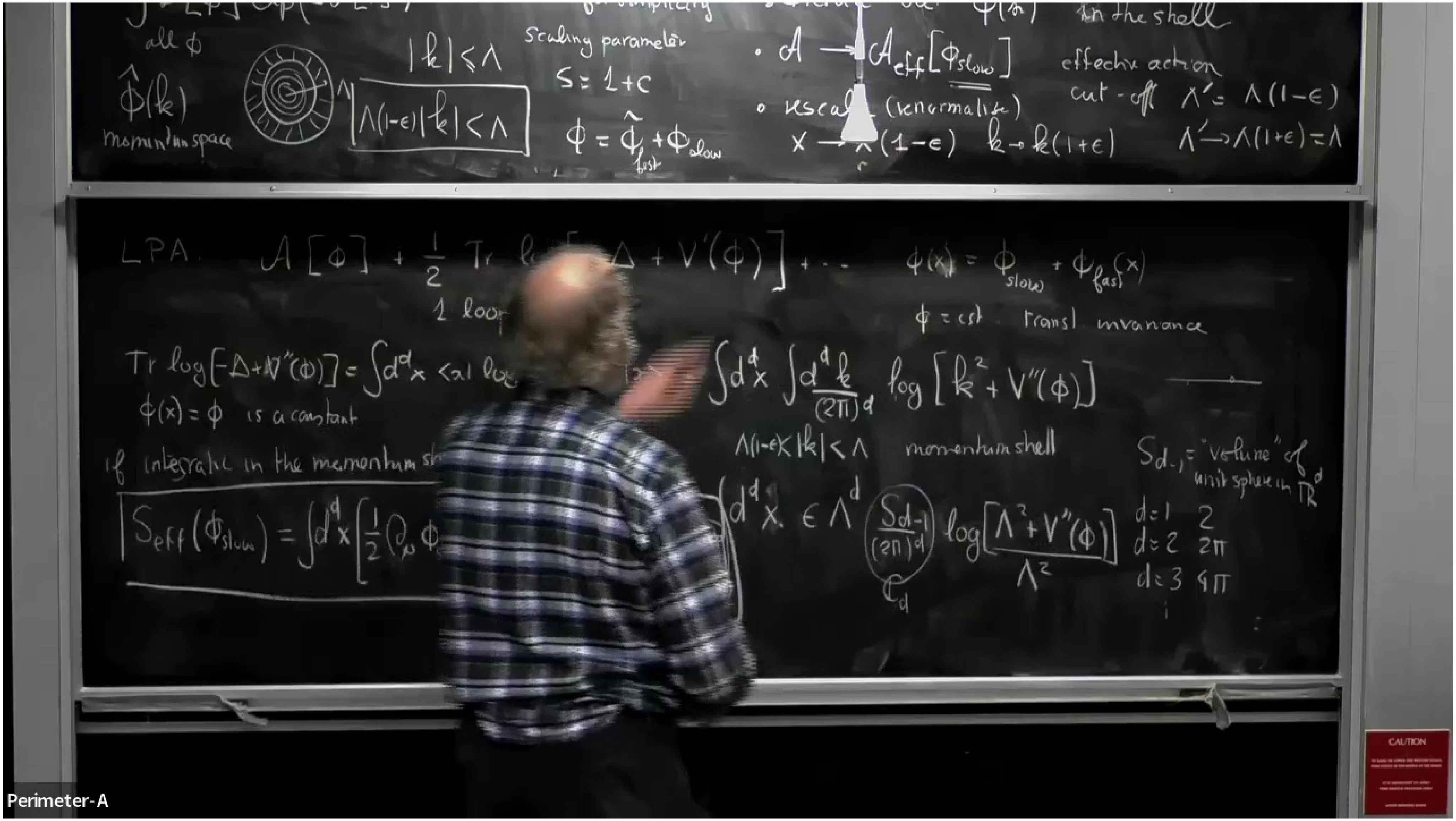

Stat Mech
l
S >> 1
macroscopic scale
size of the sample
wave length, etc.
l = S a
microscopic scale
atomic, molecular
Fixed
coupling J S S, Ising
Temperature T
positions
a
μ
μ = S⁻¹ Λ
momentum HE
mass = energy
"renormalization scale"
Energy scale for the physics
renormalised parameters
mₑ, α finite
high energy scale Λ
high momentum "cut-off"
Λ → ∞
consistent ?
"bare parameter"
m_B, α_B
Perimeter-A
CAUTION

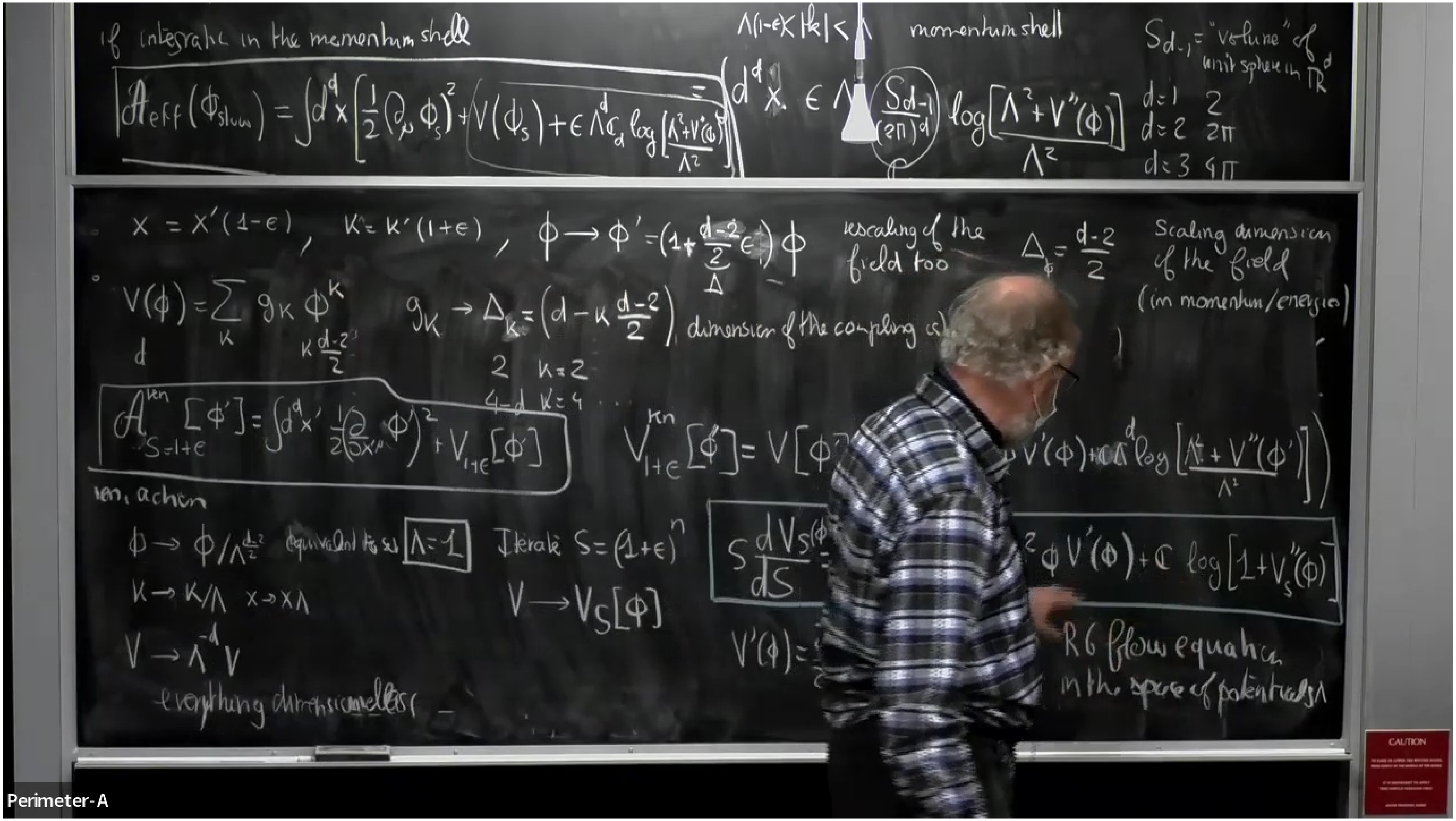

Stat Mech
ℓ
S ≫ 1
macroscopic scale
ℓ = S a
size of the sample
wave length, etc.
macroscopic parameters
microscopic scale
atomic, molecular
a
Fixed
coupling J S_i S_j Ising
Temperature T
μ'
μ
"renormalization scale"
μ = S⁻¹ Λ
high energy scale Λ
momentum HE
mass = energy
Energy scale for the physics
high momentum "cut-off"
Λ → ∞
renormalised parameters
M_pl, M_GU
"bare parameters"
consistent?
m_e, α finite
m_B, α_B
Perimeter-A
CAUTION

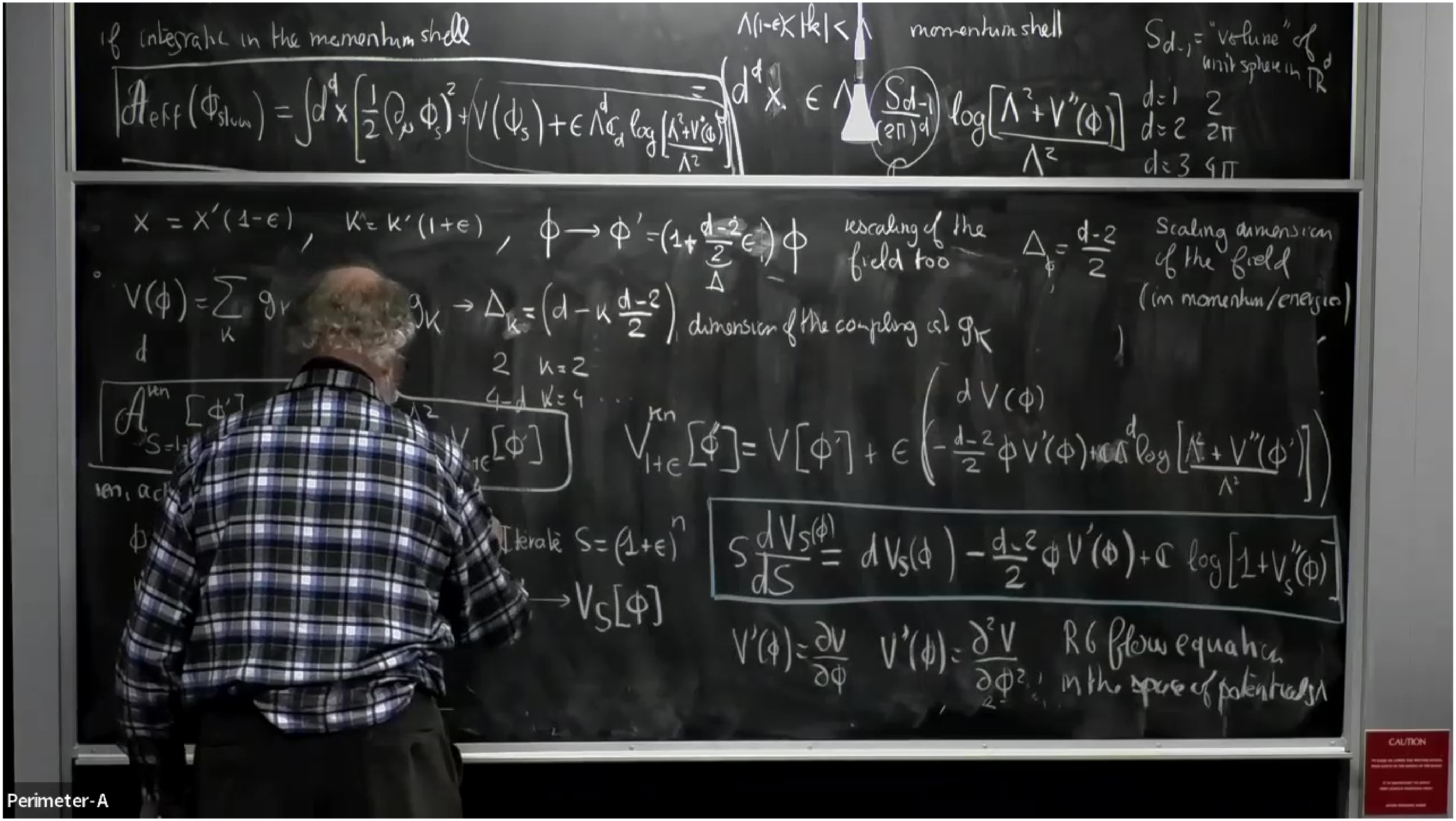

Renormalized Action Wilson - Polchinski
" " Effective potential Wetterich
Legendre transform
Scalar Field : Local potential approximation (LPA) (1 loop → ?)
A [ϕ] = ∫ d^d x 1/2 (∂_ν ϕ)^2 + V(ϕ)
V(ϕ) = Σ_{k=1}^{∞} g_k/2k! ϕ^{2k} general (even or not) potential
Ising crit point
multicritical points
ϕ = distance
Gaz
Liquid
Solid
V
ϕ
Wetting
∫_{all ϕ} D[ϕ] exp(-A[ϕ])
|k| ≤ Λ
ϕ̂(k)
momentum space
Λ
Λ(1-ε) |k| < Λ
renormalised parameters
m_e, α finite
" M_pl, M_GU "
bare parameter
m_B, α_B
Λ → ∞
consistent ?
Perimeter-A
CAUTION


Effective potential
Wetterich
Gaz
Scalar Field : Local potential approximation (LPA)
(1 loop → ?)
Liquid
Solid
A [φ] = ∫ d^d x ½(∂_μ φ)² + V(φ)
V
Ising crit point
multicritical points
φ
V(φ) = Σ_{k=1}^∞ g_k/2k! φ^{2k} general (even or not) potential
Wetting
ε ≪ 1 for simplicity
Scaling parameter
s = 1 + ε
φ = φ̂_fast + φ_slow
. Iterate over φ̂(k) in the shell
. A → A_eff[φ_slow] effective action
. rescale (renormalize)
cut-off Λ' = Λ(1−ε)
x → x(1−ε)
k → k(1+ε)
Λ' → Λ(1+ε) = Λ
∫D[φ] e
all φ
φ̂(k)
momentum
high momentum "cut-off"
M_pl, M_GU
"bare parameter"
m_B, α_B
Λ → ∞
consistent ?
Perimeter-A
CAUTION


all ϕ
|k| ≤ Λ
Scaling parameter
s = 1 + ϵ
in the shell
effective action
cut-off Λ' = Λ(1 − ϵ)
(renormalise)
x → x(1 − ϵ) k → k(1 + ϵ)
Λ' → Λ(1 + ϵ) = Λ
Λ(1 − ϵ) |k| < Λ
momentum space
ϕ = ϕ̂fast + ϕslow
LPA
1 loop
ϕslow + ϕfast(x)
transl. invariance
ϕ(x) = ϕ is a constant
if integrate in the momentum shell
Sd-1 = "volume" of unit sphere in ℝ^d
d = 1 2
d = 2 2π
d = 3 4π
Perimeter-A
CAUTION


all ϕ
|k| ≤ Λ
momentum space
Λ(1-ε) |k| < Λ
Scaling parameter
s = 1+ε
ϕ = ϕ̂ fast + ϕ slow
• A → A eff [ϕ slow]
effective action
(renormalize)
cut-off Λ' = Λ(1-ε)
x → x(1-ε) k → k(1+ε)
Λ' → Λ(1+ε) = Λ
in the shell
LPA A[ϕ] + 1/2 Tr log [-Δ + V''(ϕ)] + ...
1 loop
ϕ(x) = ϕ slow + ϕ fast(x)
ϕ = cst transl invariance
Tr log[-Δ+V''(ϕ)] = ∫ d^d x
ϕ(x) = ϕ is a constant
∫ d^d x ∫ d^d k/(2π)^d log[k² + V''(ϕ)]
Λ(1-ε) < |k| < Λ momentum shell
if integrate in the momentum sh
S eff (ϕ slow) = ∫ d^d x [1/2 ∂μ ϕ
∫ d^d x ε Λ^d S d-1/(2π)^d log[(Λ² + V''(ϕ))/Λ²]
S d-1 = "volume" of unit sphere in R^d
d=1 2
d=2 2π
d=3 4π
Perimeter-A
CAUTION


if integrate in the momentum shell
Λ(1-ε) < |k| < Λ momentum shell
S_{d-1} = "volume" of unit sphere in $\mathbb{R}^d$
d=1 2
d=2 2π
d=3 4π
$A_{eff}(\phi_{slow}) = \int d^d x \left[\frac{1}{2}(\partial_\mu \phi_s)^2 + V(\phi_s) + \epsilon \Lambda^d c_d \log\left[\frac{\Lambda^2 + V''(\phi)}{\Lambda^2}\right]\right]$
$\int d^d x \, \epsilon \Lambda \frac{S_{d-1}}{(2\pi)^d} \log\left[\frac{\Lambda^2 + V''(\phi)}{\Lambda^2}\right]$
$x = x'(1-\epsilon)$, $k = k'(1+\epsilon)$, $\phi \to \phi' = (1 + \frac{d-2}{2}\epsilon)\phi$
rescaling of the field too
$\Delta_\phi = \frac{d-2}{2}$
Scaling dimension of the field (in momentum/energies)
$V(\phi) = \sum_k g_k \phi^k$
$g_k \to \Delta_k = \left(d - k\frac{d-2}{2}\right)$ dimension of the coupling
2 k=2
4-d k=4
$A^{ren}_{s=1+\epsilon}[\phi'] = \int d^d x' \frac{1}{2}\left(\frac{\partial}{\partial x'^\mu}\phi'\right)^2 + V_{1+\epsilon}[\phi']$
$V^{ren}_{1+\epsilon}[\phi'] = V[\phi$
$V'(\phi) + c \Lambda^d \log\left[\frac{\Lambda^2 + V''(\phi')}{\Lambda^2}\right]$
ren. action
$\phi \to \phi / \Lambda^{\frac{d-2}{2}}$ equivalent to set $\Lambda = 1$
$k \to k/\Lambda$ $x \to x\Lambda$
$V \to \Lambda^{-d} V$
everything dimensionless
Iterate $S = (1+\epsilon)^n$
$V \to V_S[\phi]$
$S \frac{dV_S(\phi}{dS}$
$\phi V'(\phi) + c \log\left[1 + V_S''(\phi)\right]$
RG flow equation in the space of potentials
$V'(\phi) =$
Perimeter-A


if integrate in the momentum shell
Λ(1-ε) < |k| < Λ momentum shell
S_{d-1} = "volume" of unit sphere in R^d
d=1 2
d=2 2π
d=3 4π
rescaling of the field too
Scaling dimension of the field
(in momentum/energies)
dimension of the coupling cst g_K
Iterate S=(1+ε)^n
RG flow equation in the space of potentials
Perimeter-A
CAUTION

$k \to k/\Lambda \quad x \to x\Lambda$

$V \to V_S[\phi]$

$$\boxed{s\frac{\partial V_s}{ds} = d\,V_s(\phi) - \frac{d-2}{2}\phi V'(\phi) + C\log\left[1+V_s''(\phi)\right]}$$

$V \to \Lambda^{-d} V$

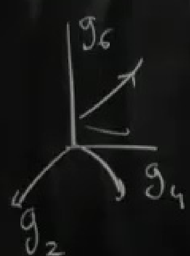
everything dimensionless

$V'(\phi) = \frac{\partial V}{\partial \phi} \quad V''(\phi) = \frac{\partial^2 V}{\partial \phi^2}$ RG flow equation in the space of potentials

Use this to study features of RG flows and continuum limit

$\boxed{\text{we neglect } \partial\phi\,\partial\phi\phi \; \phi}$

$$V(\phi) = \sum_k \frac{g_k}{k!}\phi^k \qquad V'(\phi) = \sum_k \frac{g^k}{(k-1)!}\phi^{k-1} \qquad V''(\phi) = \sum_k g^k/(k-2)!\;\phi^{k-2}$$



$\phi^2 + \phi^4 + \phi^6$

flow in the space of couplings

truncation & RG flows structure

next lecture

expand in ϕ but keep only ϕ^k $\quad k \le k_{MAX}$

$k_{max} \to \infty$

$$\underset{\text{dimensionless}}{D[\phi]} = \prod_x \left[\frac{d\phi(x)}{\Lambda^{\frac{d-2}{2}}}\right] = \prod_x \underset{\text{dimensionless}}{d\phi(x)}$$

Perimeter-A