Title: QFT III 2021/2022

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Collection: QFT III 2021/2022

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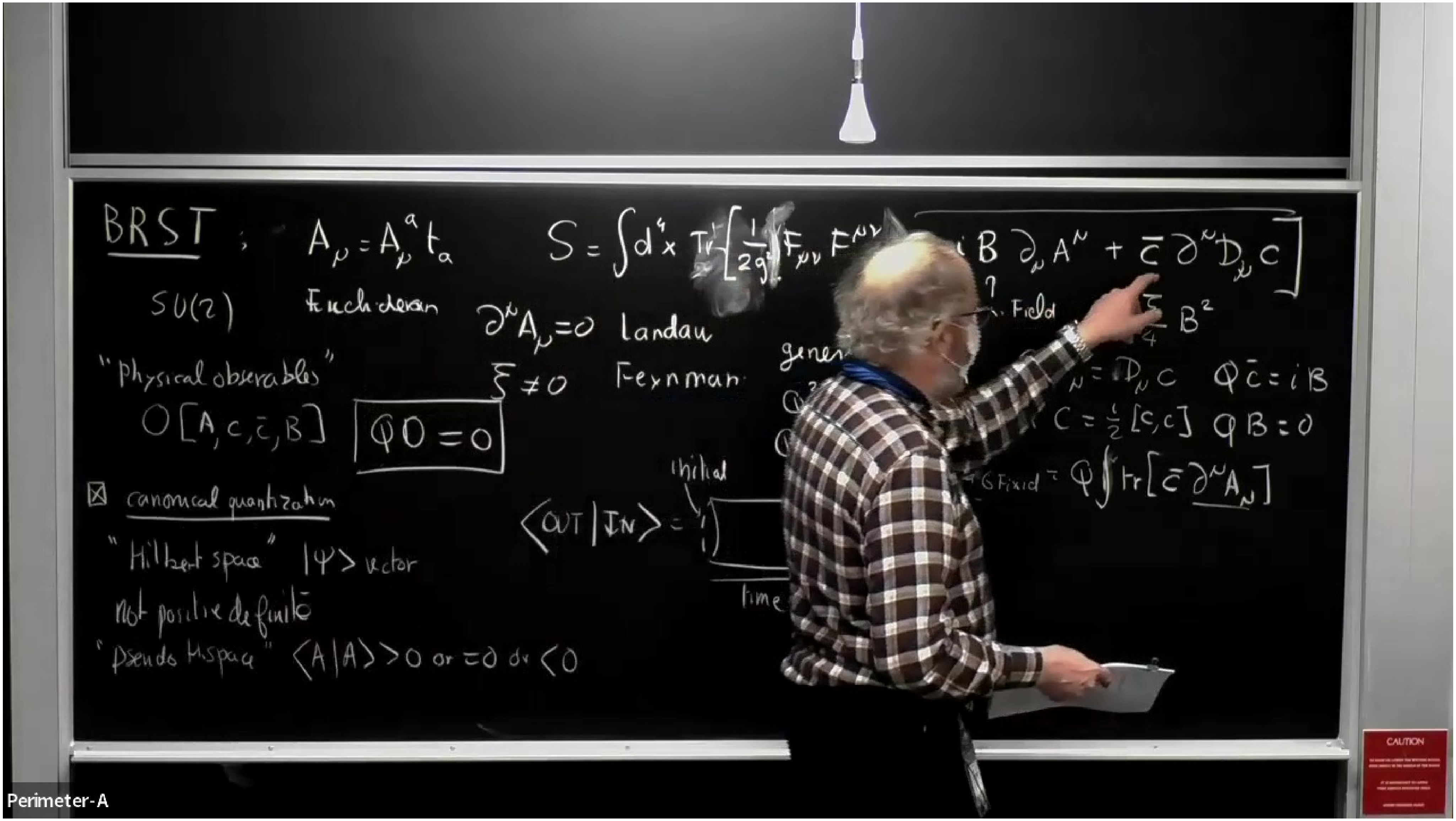
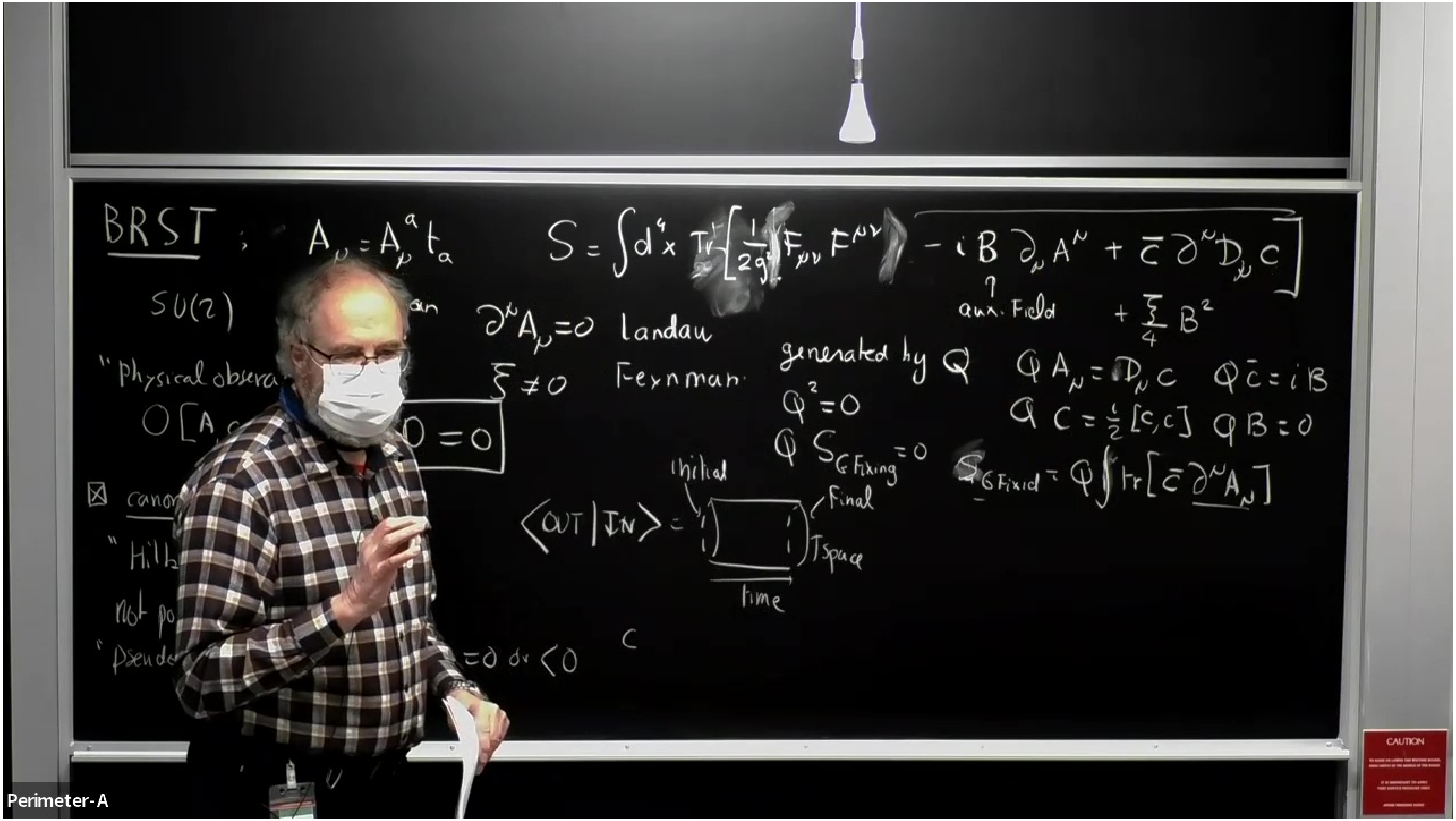
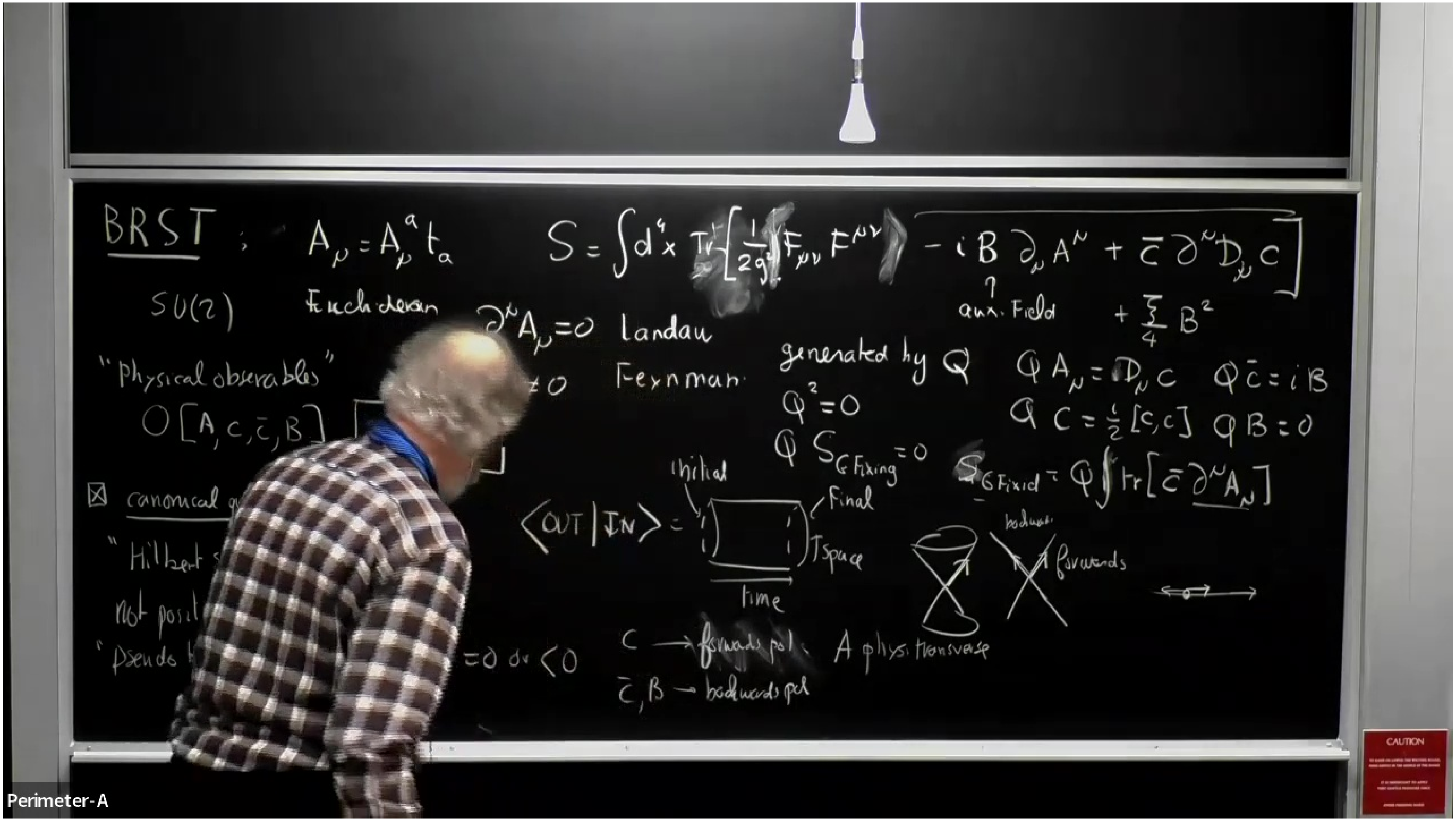
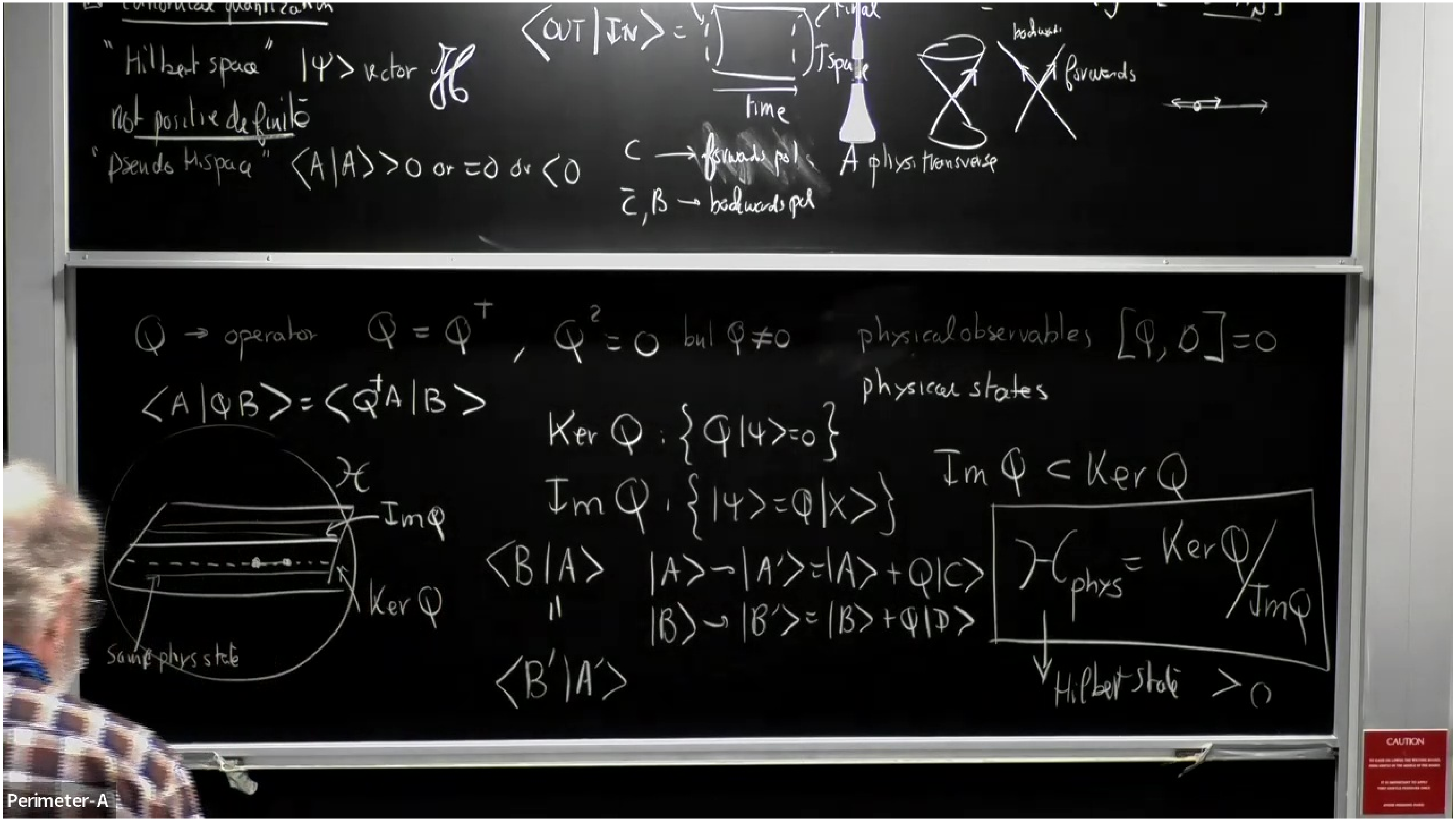
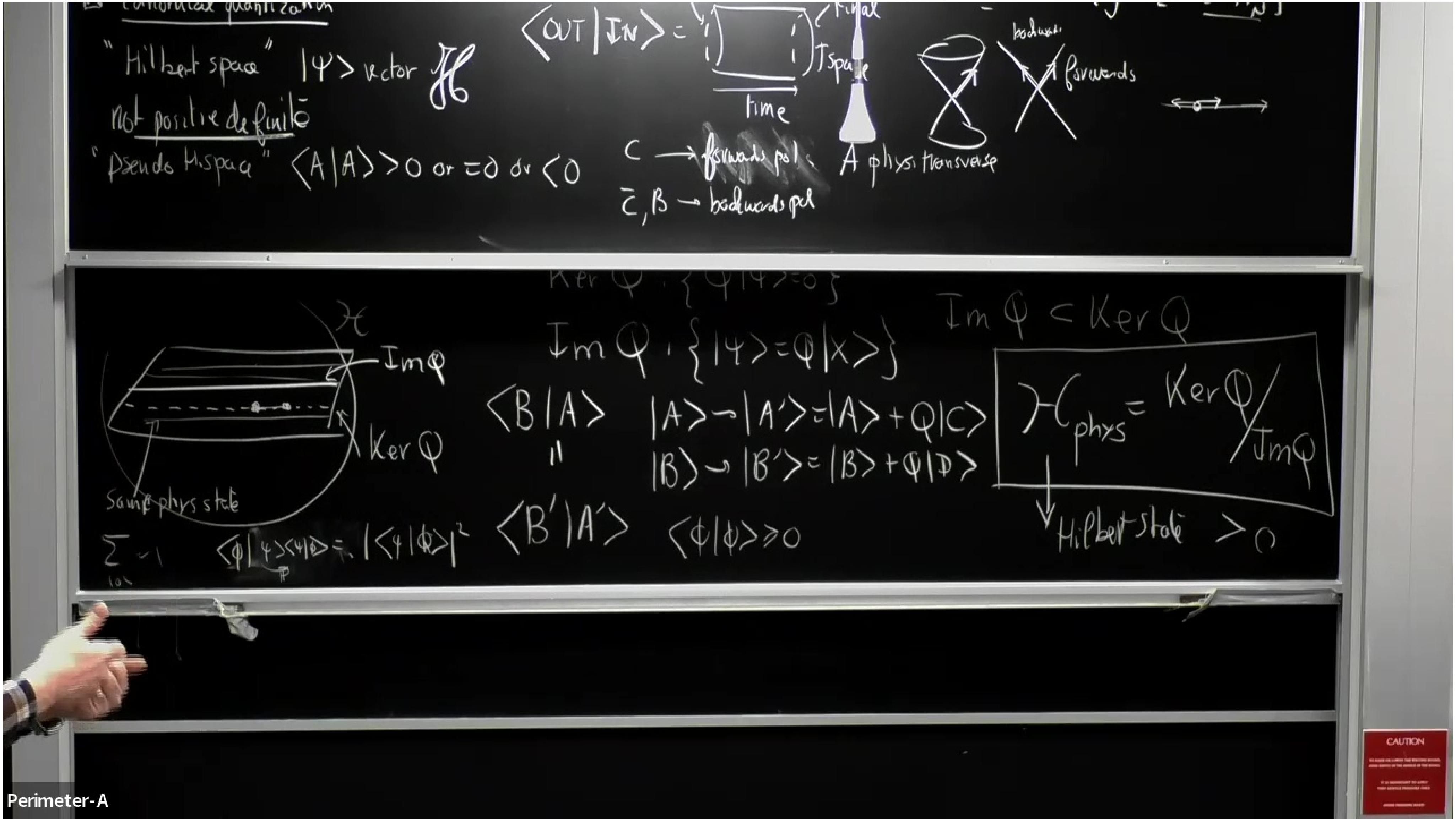
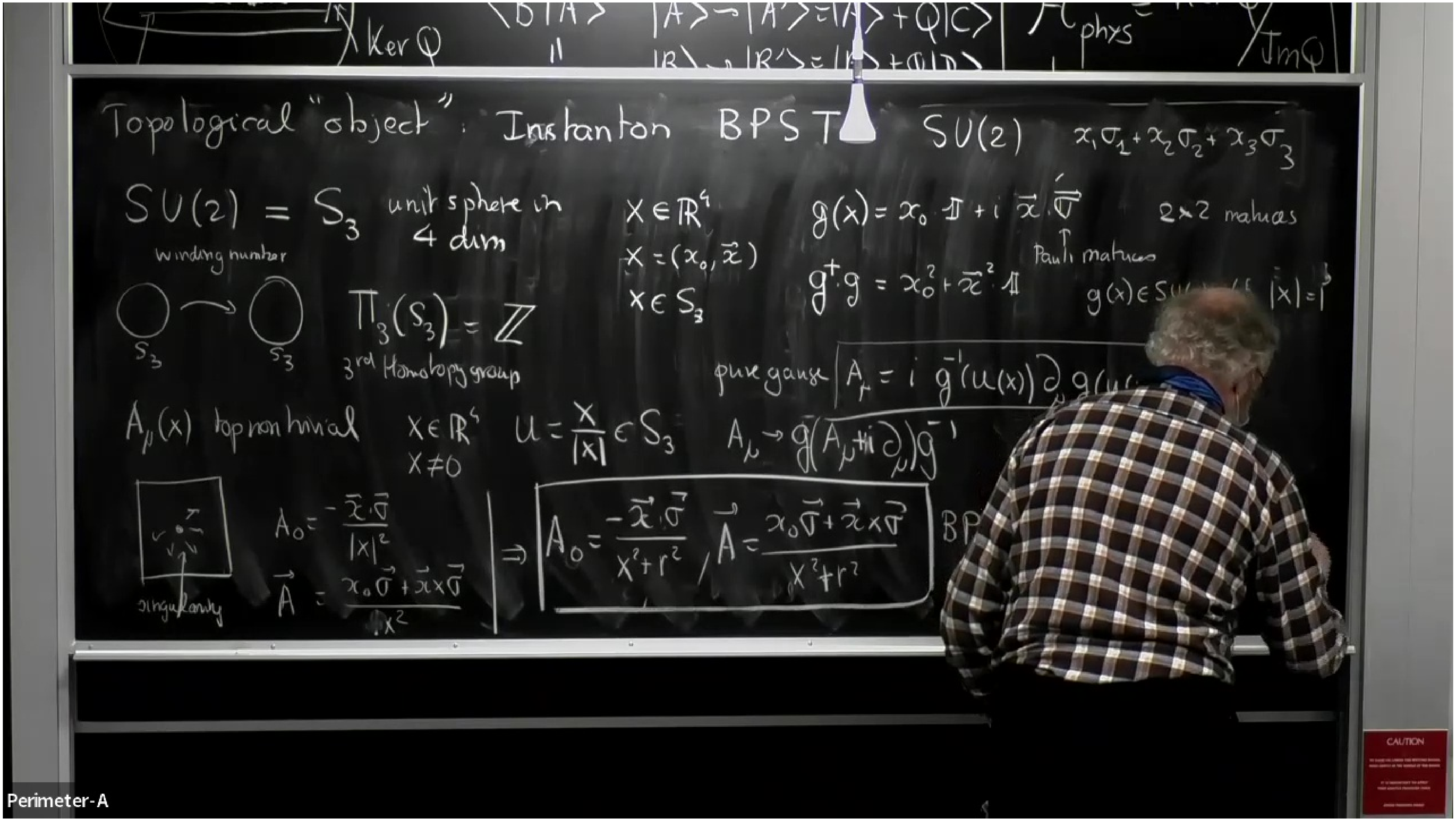

BRST
SU(2)
Euclidean
Landau
Feynman
"Physical observables"
canonical quantization
"Hilbert space" |Ψ> vector
not positive definite
"pseudo H.space" <A|A> > 0 or = 0 or < 0
initial
time
Perimeter-A
CAUTION

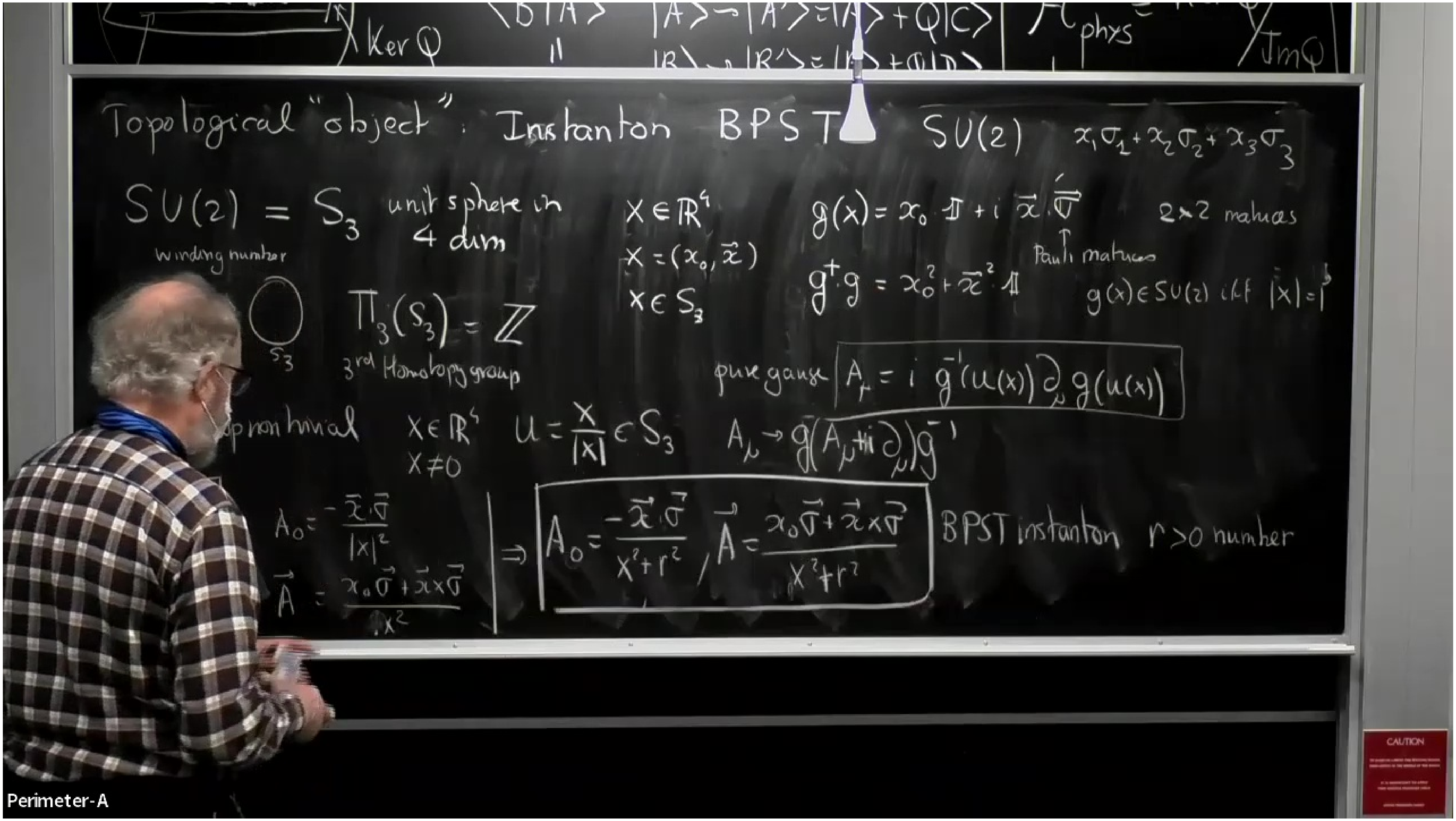

BRST
SU(2)
landau
Feynman
generated by Q
aux. Field
initial
Final
space
time
Perimeter-A
CAUTION

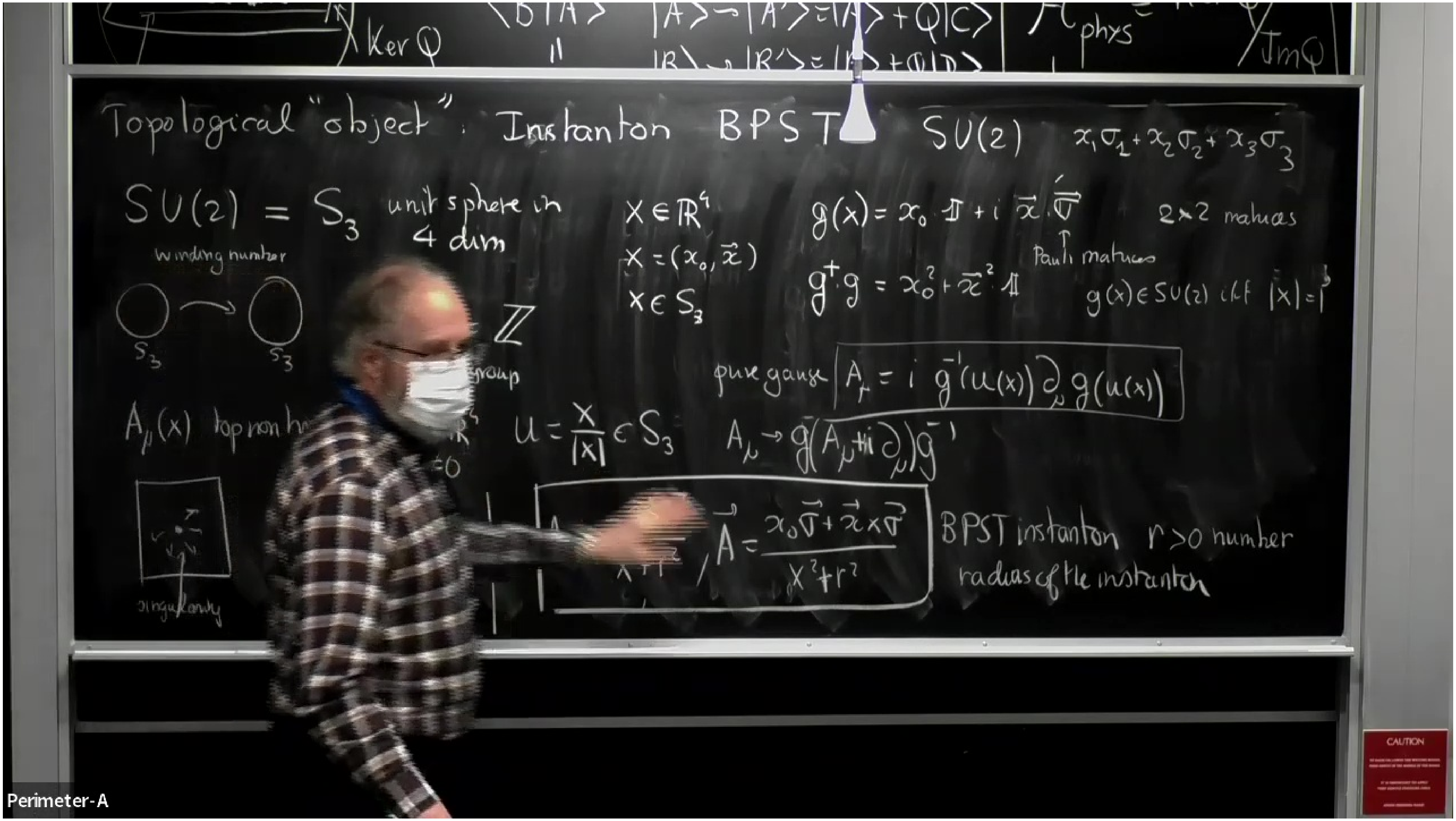

BRST
$A_\mu = A_\mu^a t_a$
$S = \int d^4x \, \mathrm{Tr}\left[\frac{1}{2g^2} F_{\mu\nu} F^{\mu\nu} - iB \partial_\mu A^\mu + \bar{c} \partial^\mu D_\mu c\right]$
aux. Field
$+ \frac{\xi}{4} B^2$
SU(2)
$\partial^\mu A_\mu = 0$ Landau
Feynman
"Physical observables"
$O[A, c, \bar{c}, B]$
generated by Q
$Q A_\mu = D_\mu c$
$Q \bar{c} = iB$
$Q^2 = 0$
$Q c = \frac{i}{2}[c, c]$
$Q B = 0$
$Q S_{G\,Fixing} = 0$
$S_{G\,Fixed} = Q \int \mathrm{Tr}[\bar{c} \partial^\mu A_\mu]$
canonical
Initial
Final
$\langle OUT | IN \rangle =$
space
time
backwards
forwards
"Hilbert
not posit
"pseudo
$= 0$ or < 0
$c \to$ forwards pol.
A phys. transverse
$\bar{c}, B \to$ backwards pol
Perimeter-A
CAUTION

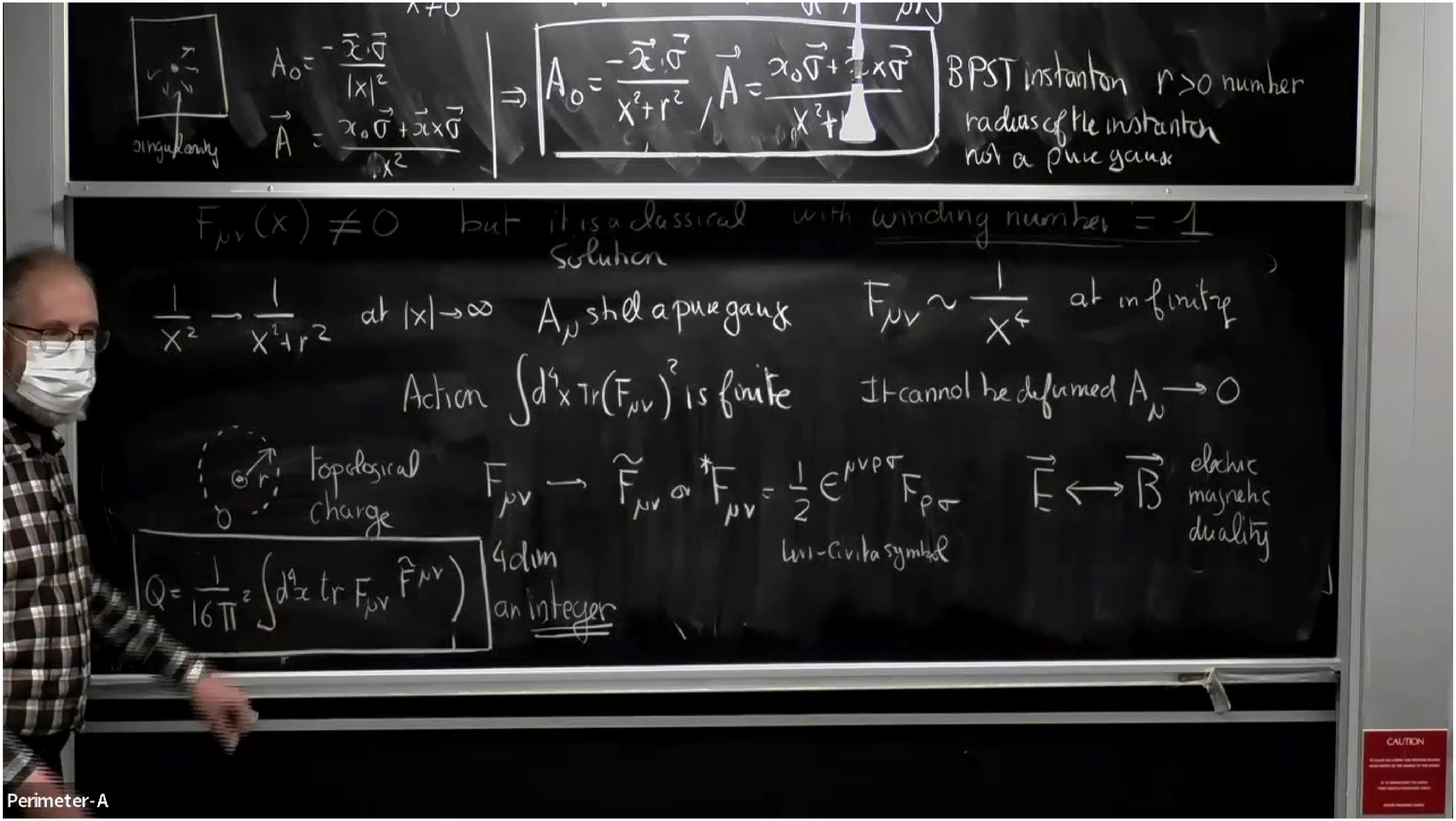

"Hilbert space" |Ψ> vector ℋ
not positive definite
"pseudo H space" <A|A> > 0 or = 0 or < 0
<OUT|IN> =
time
Tspace
final
c → forwards pol
c̄, B → backwards pol
A phys transverse
backwards
forwards
Q → operator Q = Q†, Q² = 0 but Q ≠ 0
physical observables [Q, O] = 0
physical states
<A|QB> = <Q†A|B>
Ker Q : {Q|ψ> = 0}
Im Q : {|ψ> = Q|χ>}
Im Q ⊂ Ker Q
ℋ_phys = Ker Q / Im Q
Hilbert state > 0
<B|A>
|A> → |A'> = |A> + Q|C>
|B> → |B'> = |B> + Q|D>
<B'|A'>
ℋ
Im Q
Ker Q
same phys state
Perimeter-A
CAUTION

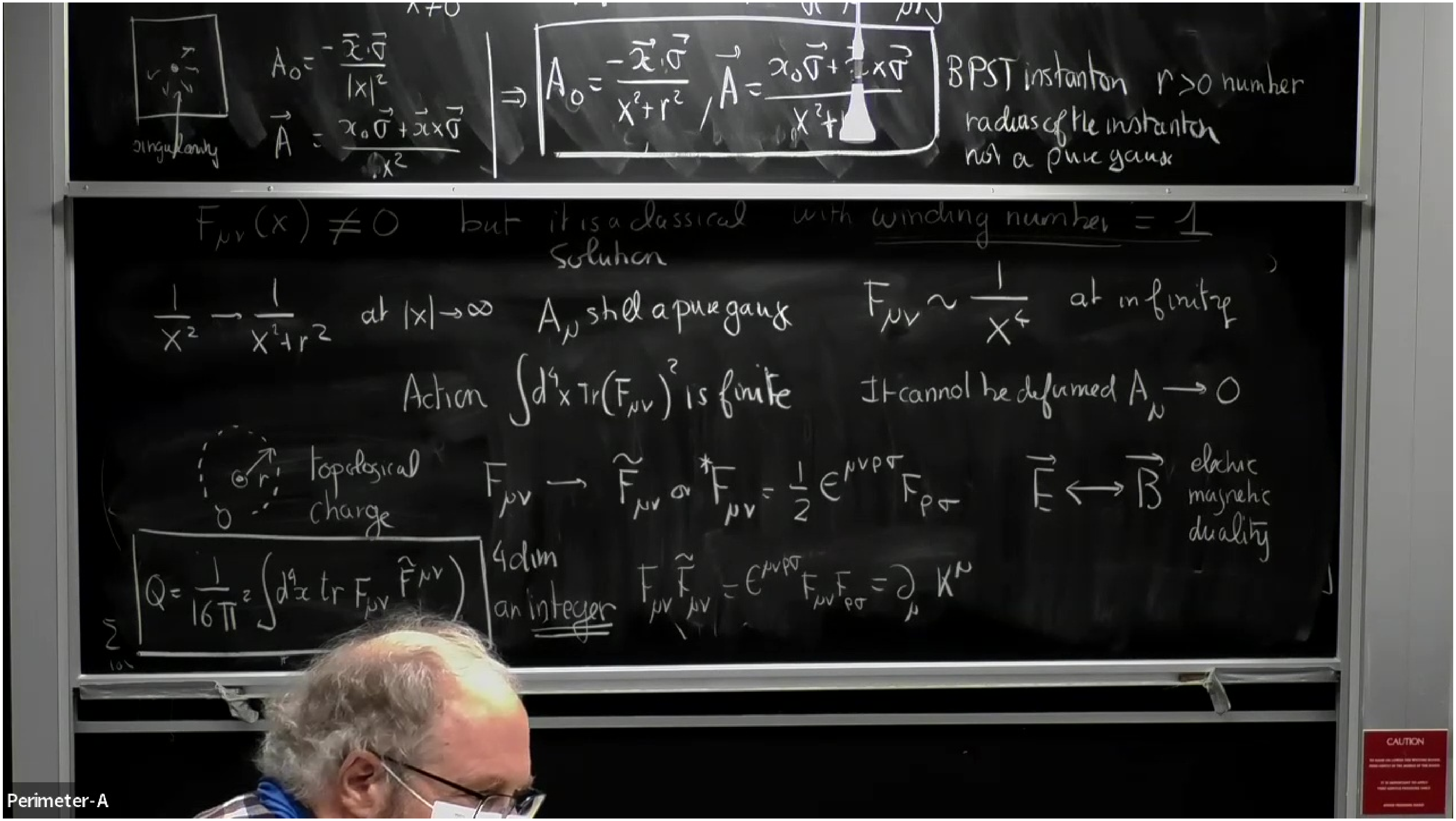

"Hilbert space" |Ψ> vector
not positive definite
"pseudo H.space" <A|A> > 0 or = 0 or < 0
<OUT|IN> =
final
time
backwards
forwards
c → forwards pol
A phys: transverse
c̄, B → backwards pol
Im φ ⊂ Ker Q
Im Q : {|Ψ> = Q|χ>}
Im φ
Ker Q
same phys state
<B|A>
|A> → |A'> = |A> + Q|C>
|B> → |B'> = |B> + Q|D>
<B'|A'>
<φ|φ> ≥ 0
H_phys = Ker Q / Im Q
Hilbert state > 0
Perimeter-A
CAUTION

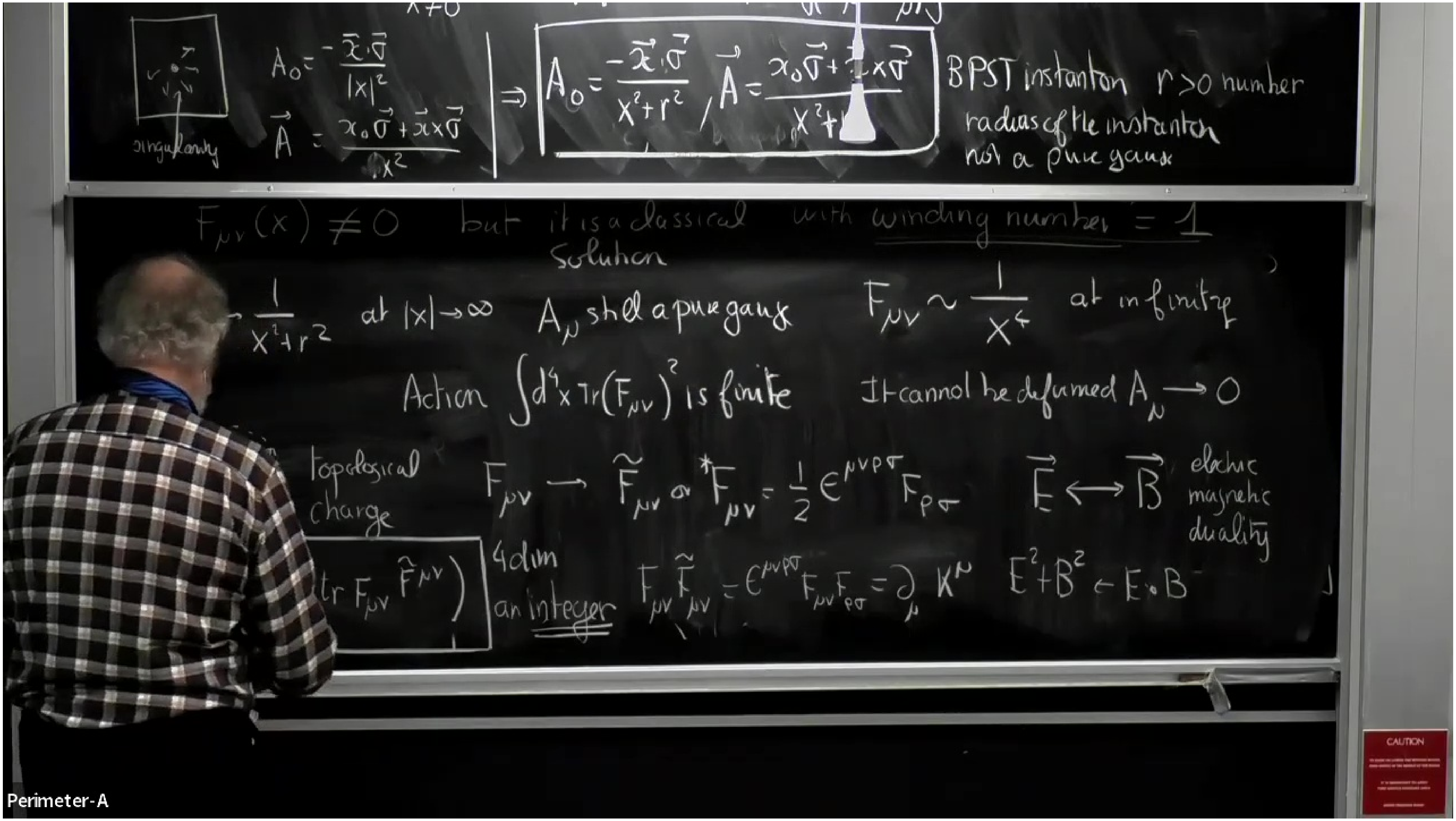

Topological "object": Instanton BPST
SU(2) $x_1\sigma_1 + x_2\sigma_2 + x_3\sigma_3$
SU(2) = S_3 unit sphere in 4 dim
$X \in \mathbb{R}^4$
$X = (x_0, \vec{x})$
$X \in S_3$
$g(x) = x_0 \mathbb{1} + i\, \vec{x}\cdot\vec{\sigma}$
2×2 matrices
Pauli matrices
$g^\dagger g = x_0^2 + \vec{x}^2\, \mathbb{1}$
winding number
$\Pi_3(S_3) = \mathbb{Z}$
3rd Homotopy group
pure gauge
$A_\mu(x)$ top non trivial
$X \in \mathbb{R}^4$
$X \neq 0$
$U = \frac{X}{|X|} \in S_3$
$A_\mu \to g(A_\mu + i\partial_\mu)g^{-1}$
$A_0 = \frac{-\vec{x}\cdot\vec{\sigma}}{|X|^2}$
$\vec{A} = \frac{x_0\vec{\sigma} + \vec{x}\times\vec{\sigma}}{X^2}$
singularity
$A_0 = \frac{-\vec{x}\cdot\vec{\sigma}}{X^2 + r^2}$, $\vec{A} = \frac{x_0\vec{\sigma} + \vec{x}\times\vec{\sigma}}{X^2 + r^2}$
Perimeter-A
CAUTION

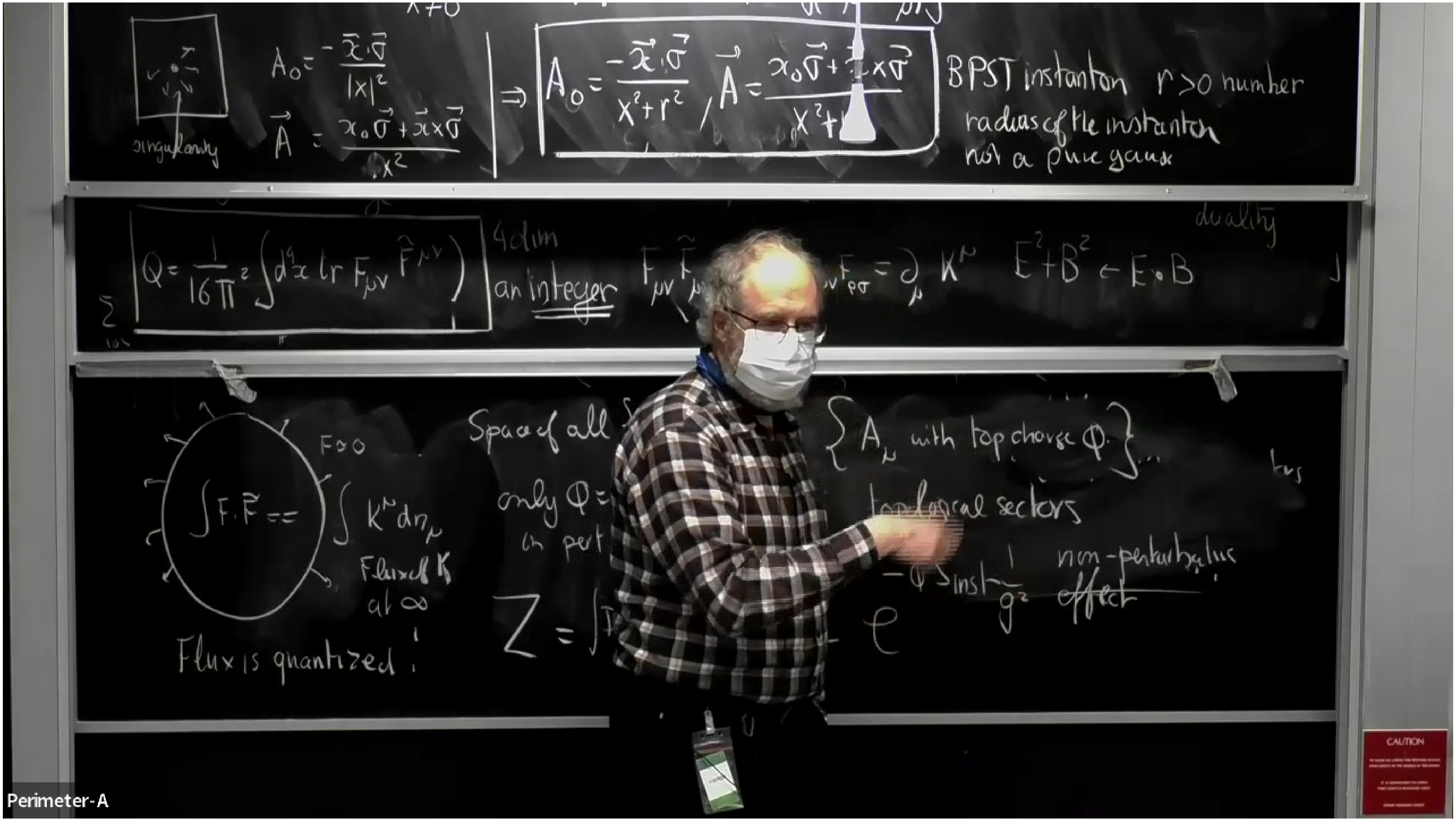

Ker Q
phys
Im Q
Topological "object": Instanton BPST
SU(2) $x_1\sigma_1 + x_2\sigma_2 + x_3\sigma_3$
SU(2) = S_3 unit sphere in 4 dim
winding number
$X \in \mathbb{R}^4$
$X = (x_0, \vec{x})$
$X \in S_3$
$g(x) = x_0 \mathbb{1} + i\, \vec{x} \cdot \vec{\sigma}$
2×2 matrices
Pauli matrices
$g^\dagger g = x_0^2 + \vec{x}^2\, \mathbb{1}$
g(x) ∈ SU(2) iff |x| = 1
S_3
$\pi_3(S_3) = \mathbb{Z}$
3rd Homotopy group
pure gauge $A_\mu = i\, g^{-1}(u(x))\, \partial_\mu g(u(x))$
non trivial
$X \in \mathbb{R}^4$, $X \neq 0$
$u = \frac{X}{|X|} \in S_3$
$A_\mu \to g(A_\mu + i\partial_\mu) g^{-1}$
$A_0 = \frac{-\vec{x}\cdot\vec{\sigma}}{|x|^2}$
$\vec{A} = \frac{x_0\vec{\sigma} + \vec{x}\times\vec{\sigma}}{x^2}$
$\Rightarrow A_0 = \frac{-\vec{x}\cdot\vec{\sigma}}{x^2+r^2}$, $\vec{A} = \frac{x_0\vec{\sigma} + \vec{x}\times\vec{\sigma}}{x^2+r^2}$
BPST instanton r > 0 number
Perimeter-A
CAUTION


Ker Q
Topological "object": Instanton BPST
SU(2)
$x_1\sigma_1 + x_2\sigma_2 + x_3\sigma_3$
SU(2) = S_3 unit sphere in 4 dim
winding number
S_3 S_3
$\mathbb{Z}$
group
$X \in \mathbb{R}^4$
$X = (x_0, \vec{x})$
$X \in S_3$
$g(x) = x_0 \mathbb{1} + i\, \vec{x}\cdot\vec{\sigma}$
2×2 matrices
Pauli matrices
$g^+ g = x_0^2 + \vec{x}^2\, \mathbb{1}$
$g(x) \in SU(2)$ iff $|X| = 1$
pure gauge $A_\mu = i\, g^{-1}(u(x))\, \partial_\mu\, g(u(x))$
$A_\mu(x)$ top non
$u = \frac{X}{|X|} \in S_3$
$A_\mu \to g(A_\mu + i\partial_\mu)g^{-1}$
$\vec{A} = \frac{x_0\vec{\sigma} + \vec{x}\times\vec{\sigma}}{X^2 + r^2}$
BPST instanton r > 0 number
radius of the instanton
singularity
Perimeter-A
CAUTION

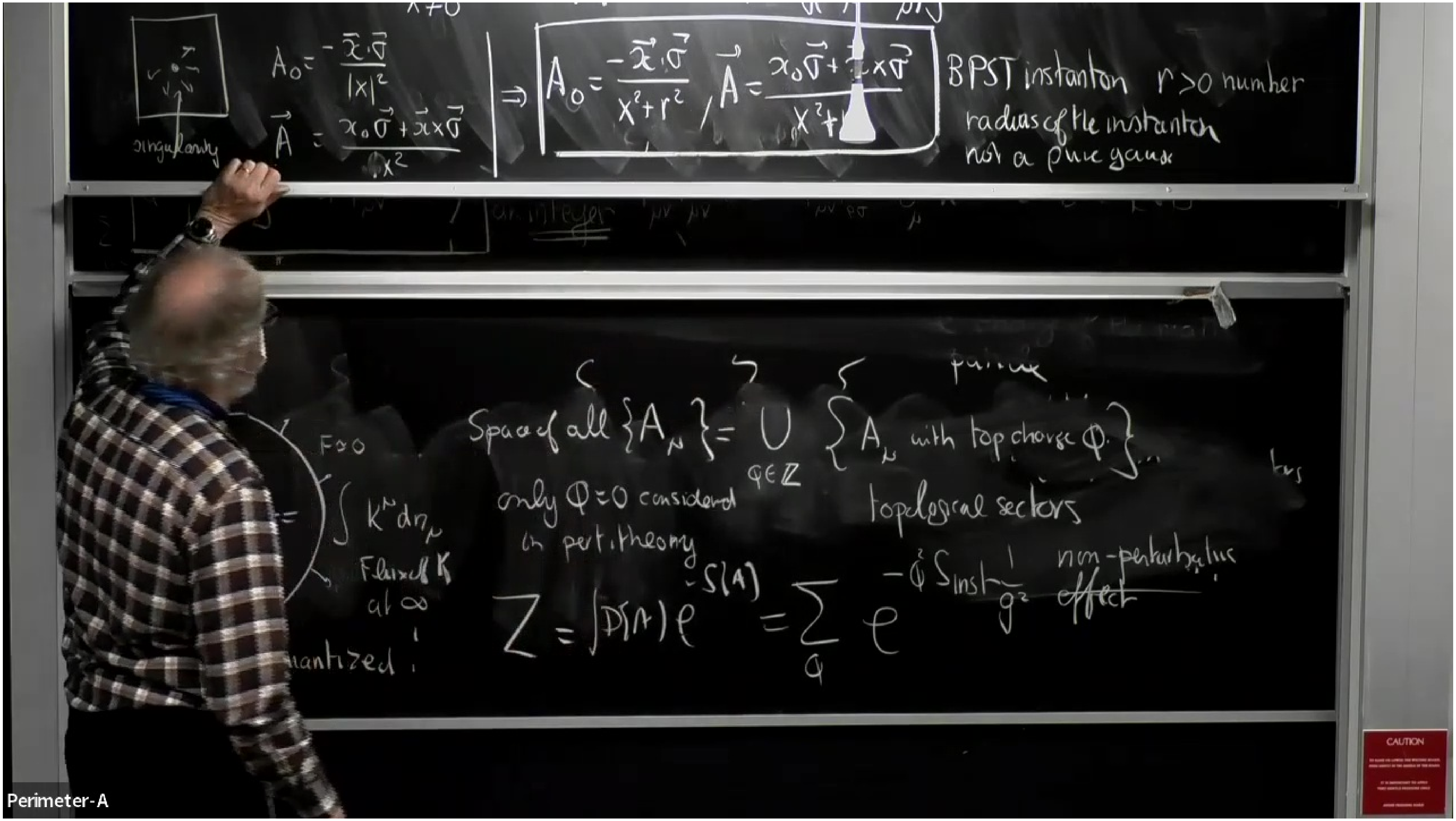

singularity
$A_0 = \frac{-\vec{x}\cdot\vec{\sigma}}{|x|^2}$
$\vec{A} = \frac{x_0\vec{\sigma} + \vec{x}\times\vec{\sigma}}{x^2}$
$\Rightarrow A_0 = \frac{-\vec{x}\cdot\vec{\sigma}}{x^2+r^2}, \vec{A} = \frac{x_0\vec{\sigma}+\vec{x}\times\vec{\sigma}}{x^2+r^2}$
BPST instanton r>0 number
radius of the instanton
not a pure gauge
$F_{\mu\nu}(x) \neq 0$ but it is a classical solution with winding number = 1
$\frac{1}{x^2} \to \frac{1}{x^2+r^2}$ at $|x| \to \infty$ A_μ still a pure gauge
$F_{\mu\nu} \sim \frac{1}{x^4}$ at infinity
Action $\int d^4x \, \mathrm{Tr}(F_{\mu\nu})^2$ is finite
It cannot be deformed $A_\mu \to 0$
topological charge
$F_{\mu\nu} \to \tilde{F}_{\mu\nu}$ or ${}^*F_{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$
Levi-Civita symbol
$\vec{E} \longleftrightarrow \vec{B}$ electric magnetic duality
$Q = \frac{1}{16\pi^2}\int d^4x \, \mathrm{tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu}$
4dim
an integer
Perimeter-A
CAUTION


singularity
$A_0 = -\frac{\vec{x}\cdot\vec{\sigma}}{|x|^2}$
$\vec{A} = \frac{x_0\vec{\sigma} + \vec{x}\times\vec{\sigma}}{x^2}$
$\Rightarrow A_0 = \frac{-\vec{x}\cdot\vec{\sigma}}{x^2+r^2}, \vec{A} = \frac{x_0\vec{\sigma}+\vec{x}\times\vec{\sigma}}{x^2+r^2}$
BPST instanton r>0 number
radius of the instanton
not a pure gauge
$F_{\mu\nu}(x) \neq 0$ but it is a classical solution with winding number = 1
$\frac{1}{x^2} \to \frac{1}{x^2+r^2}$ at $|x|\to\infty$ A_μ should a pure gauge
$F_{\mu\nu} \sim \frac{1}{x^4}$ at infinity
Action $\int d^4x\, \mathrm{Tr}(F_{\mu\nu})^2$ is finite
It cannot be deformed $A_\mu \to 0$
topological charge
$F_{\mu\nu} \to \tilde{F}_{\mu\nu}$ or ${}^*F_{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$
$\vec{E} \longleftrightarrow \vec{B}$ electric magnetic duality
$Q = \frac{1}{16\pi^2}\int d^4x\, \mathrm{tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu}$
4dim
an integer
$F_{\mu\nu}\tilde{F}_{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} = \partial_\mu K^\mu$
Perimeter-A
CAUTION


singularity
A0 = - x·σ / |x|²
A = x0 σ + x × σ / x²
A0 = -x·σ / x²+r² , A = x0 σ + x × σ / x²+r²
BPST instanton r>0 number
radius of the instanton
not a pure gauge
Fμν(x) ≠ 0 but it is a classical solution with winding number = 1
1 / x²+r² at |x|→∞ Aμ should a pure gauge Fμν ~ 1 / x⁴ at infinity
Action ∫d⁴x Tr(Fμν)² is finite It cannot be deformed Aμ → 0
topological charge
Fμν → F̃μν or *Fμν = ½ εμνρσ Fρσ
E ⟷ B electric magnetic duality
4 dim an integer
tr Fμν F̃μν
Fμν F̃μν = εμνρσ Fμν Fρσ = ∂μ Kμ E² + B² ← E·B
Perimeter-A
CAUTION



BPST instanton r>0 number
radius of the instanton
not a pure gauge
singularity
4dim
an integer
duality
Space of all
with top charge
sectors
non-perturbative effect
Flux of K at ∞
Flux is quantized!
Perimeter-A
CAUTION


singularity
BPST instanton r > 0 number
radius of the instanton
not a pure gauge
F ≈ 0
Flux of K
at ∞
quantized
Space of all {A_μ} = ∪ {A_μ with top charge φ}
φ ∈ Z
only φ = 0 considered
in pert. theory
topological sectors
non-perturbative
effect
Perimeter-A
CAUTION