Title: QFT III 2021/2022

Speakers: Francois David

Collection: QFT III 2021/2022

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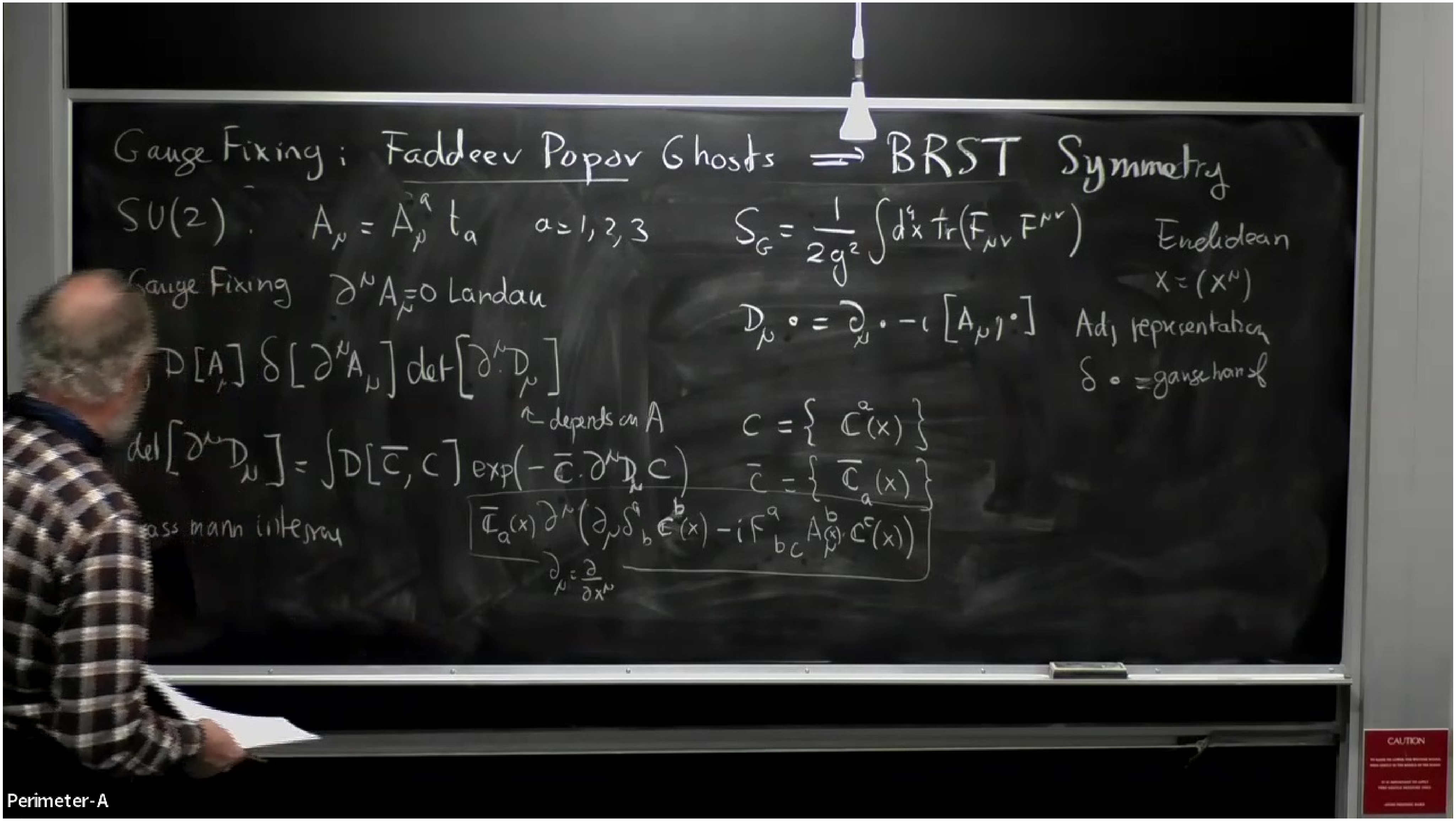
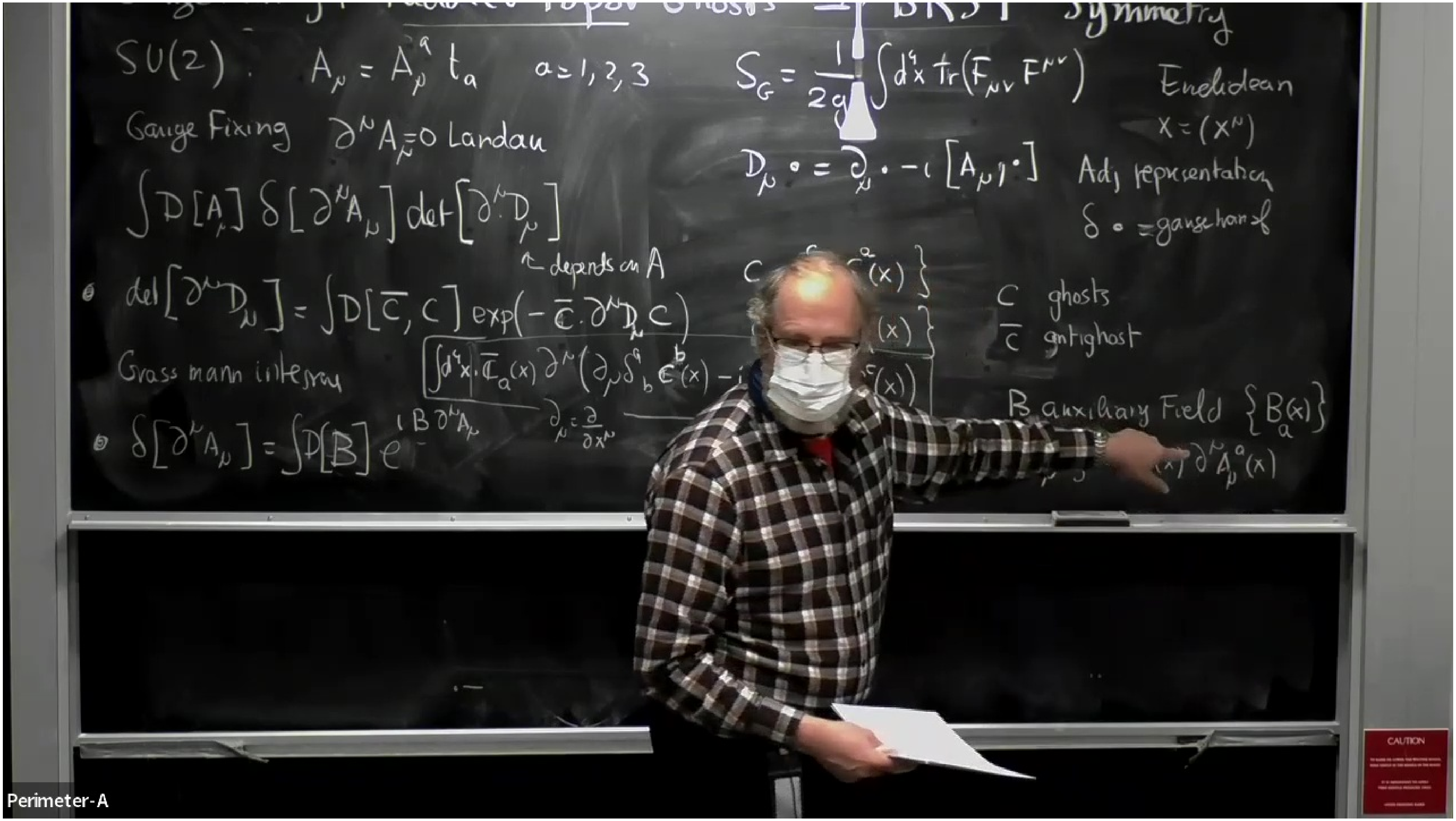
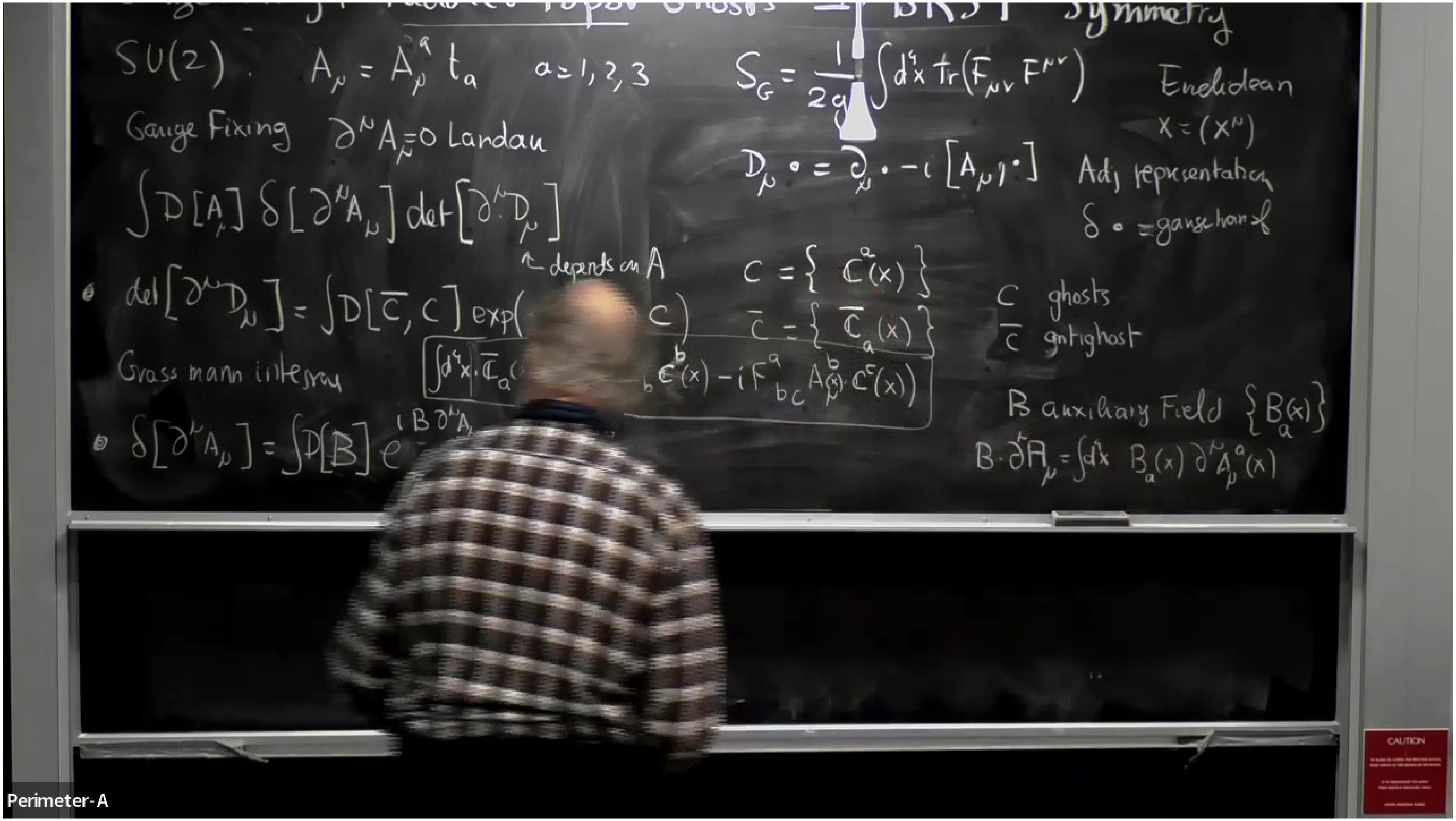


Gauge Fixing: Faddeev Popov Ghosts $\Rightarrow$ BRST Symmetry
$SU(2)$: $A_\mu = A^a_\mu t_a$ $a=1,2,3$
$S_G = \frac{1}{2g^2}\int d^4x\, tr(F_{\mu\nu}F^{\mu\nu})$
Euclidean
$x = (x^\mu)$
Gauge Fixing $\partial^\mu A_\mu = 0$ Landau
$D_\mu \circ = \partial_\mu \circ - i[A_\mu, \circ]$
Adj representation
$\delta \circ =$ gauge transf
$\int D[A]\, \delta[\partial^\mu A_\mu]\, \det[\partial^\mu D_\mu]$
depends on A
$C = \{ C^a(x) \}$
$\det[\partial^\mu D_\mu] = \int D[\bar{C}, C] \exp(-\bar{C}\cdot\partial^\mu D_\mu C)$
$\bar{C} = \{ \bar{C}_a(x) \}$
ass mann integral
$\bar{C}_a(x)\partial^\mu(\partial_\mu \delta^a_b C^b(x) - iF^a_{bc} A^b_\mu(x) C^c(x))$
$\partial_\mu = \frac{\partial}{\partial x^\mu}$
Perimeter-A


Perimeter-A
SU(2): A_μ = A_μ^a t_a   a=1,2,3
S_G = 1/2g ∫d^4x tr(F_μν F^μν)
Euclidean
x = (x^μ)
Gauge Fixing ∂^μ A_μ = 0 Landau
D_μ • = ∂_μ • − i [A_μ, •]
Adj representation
δ • = gauge transf
∫D[A_μ] δ[∂^μ A_μ] det[∂^μ D_μ]
← depends on A
det[∂^μ D_μ] = ∫D[C̄, C] exp(−C̄ · ∂^μ D_μ C)
C ghosts
C̄ antighost
Grassmann integral
B auxiliary Field {B_a(x)}
δ[∂^μ A_μ] = ∫D[B] e^{iB∂^μ A_μ}
∂_μ = ∂/∂x^μ
CAUTION


SU(2): A_μ = A_μ^a t_a a=1,2,3
S_G = 1/2g ∫d^4x tr(F_μν F^μν)
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C = { C^a(x) }
C̄ = { C̄_a(x) }
C ghosts
C̄ antighost
Grassmann integral
B auxiliary Field {B_a(x)}
δ[∂^μ A_μ] = ∫D[B] e^{iB∂^μA_μ}
B·∂^μA_μ = ∫d^4x B_a(x) ∂^μ A_μ^a(x)
Perimeter-A
CAUTION

$$\det[\partial^\mu D_\mu] = \int D[\bar{c}, c]\, \exp(-\bar{c}\cdot\partial^\mu D_\mu c) \qquad c = \{c_a(x)\} \qquad \bar{c} = \{\bar{c}_a(x)\}$$

Grassmann integral $\int d^4x\, \bar{c}_a(x)\,\partial^\mu\left(\partial_\mu \delta^a_b\, c^b(x) - i f^a_{\ bc} A^b_\mu c^c(x)\right)$

$$\delta[\partial^\mu A_\mu] = \int D[B]\, e^{iB\partial^\mu A_\mu} \qquad \partial_\mu = \frac{\partial}{\partial x^\mu}$$

c ghosts, $\bar{c}$ antighost } fermions (Grassmann)

boson (real)

B auxiliary Field $\{B_a(x)\}$

$$B\cdot\partial^\mu A_\mu = \int d^4x\, B_a(x)\,\partial^\mu A^a_\mu(x)$$

$$\int D[A]\, D[\bar{c}, c]\, D[B]\, \exp\left(-S_G[A] - S_{G\,Fixing}[c, \bar{c}, B]\right) \qquad B^2 = \int d^4x\, B_a(x)^2$$

Landau Gauge

$$\boxed{S_{GF} = \bar{c}\,\partial^\mu D_\mu c - i B\cdot\partial^\mu A_\mu + \frac{\xi}{2} B^2}$$

All the A's, but Diagrammatic is OK

Feynman Gauge | $\xi \neq 0$ Integrate over B · $S_{GF} = \bar{c}\,\partial^\mu D_\mu c + \underbrace{\frac{1}{2\xi}(\partial^\mu A_\mu)^2 \to \text{adds to } (\partial_\nu A_\mu - \partial_\mu A_\nu)^2}_{}$ quadratic term

≥ 0 definite form $A_\mu K_{\mu\nu} A_\nu \Rightarrow$ it can be inverted

no zero modes problem

now long. polarization $\epsilon_\mu k^\mu = 0$ propagate

Perimeter-A

$$\det[\partial^\mu D_\nu] = \int D[\bar{c}, c]\, \exp(-\bar{c}\cdot\partial^\mu D_\mu c) \qquad \bar{c} = \{\bar{c}_a(x)\}$$

Grassmann integral $\quad \boxed{\int d^4x\, \bar{c}_a(x)\,\partial^\mu\left(\partial_\mu \delta^a_b\, c^b(x) - i f^a_{bc} A^b_\mu(x)\, c^c(x)\right)} \qquad \partial_\mu = \frac{\partial}{\partial x^\mu}$

$$\delta[\partial^\mu A_\mu] = \int D[B]\, e^{iB\partial^\mu A_\mu}$$

c ghosts, $\bar{c}$ antighost } fermions (Grassmann)

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B auxiliary Field $\{B_a(x)\}$

$$B\cdot\partial^\mu A_\mu = \int d^4x\, B_a(x)\,\partial^\mu A^a_\mu(x)$$

$$\int D[A]\, D[\bar{c}, c]\, D[B]\, \exp\left(-S_G[A] - S_{G\,Fixing}[\bar{c}, c, B]\right)$$

Landau Gauge

$$\boxed{S_{GF} = \bar{c}\,\partial^\mu D_\mu c - i B\cdot\partial^\mu A_\mu + \frac{\xi}{2} B^2}$$

All the A's, but Diagrammatics OK

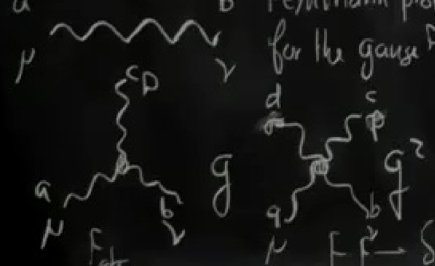
Feynman Gauge $\quad \xi \neq 0 \quad$ Integrate over B $\cdot$

$$S_{GF} = \bar{c}\,\partial^\mu D_\mu c + \underbrace{\frac{1}{2\xi}(\partial^\mu A_\mu)^2 \longrightarrow \text{adds to } (\partial_\mu A_\nu - \partial_\nu A_\mu)^2}_{\geq 0 \text{ definite form } A_\mu K_{\mu\nu} A_\nu \Rightarrow \text{it can be inverted}} \text{ quadratic term}$$

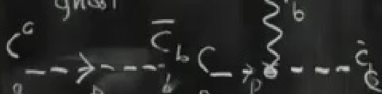
no zero modes problem

now long. polarization $\epsilon_\mu k^\mu = 0$ propagate

Feynmann propagator for the gauge Field



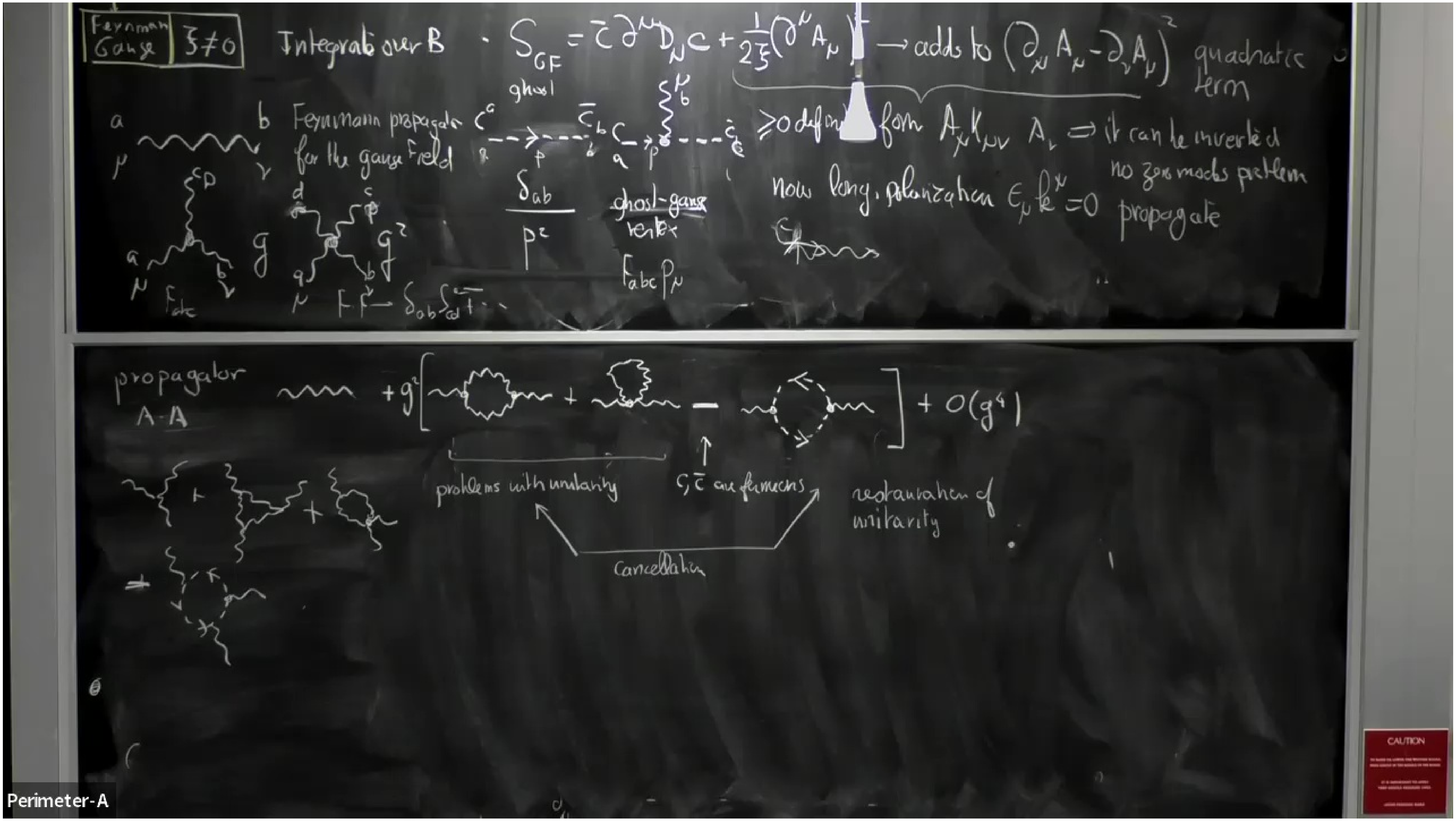
ghost

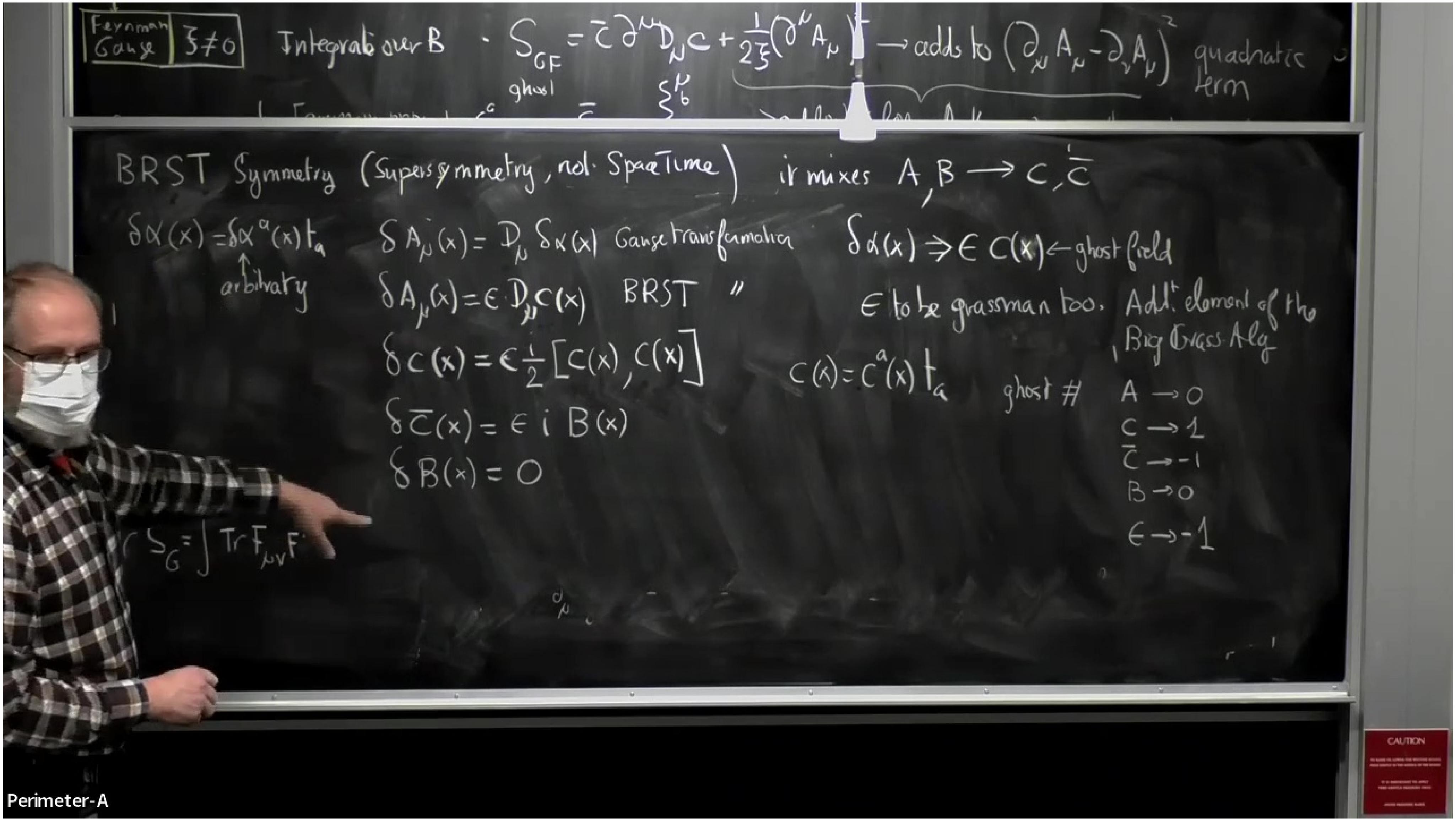


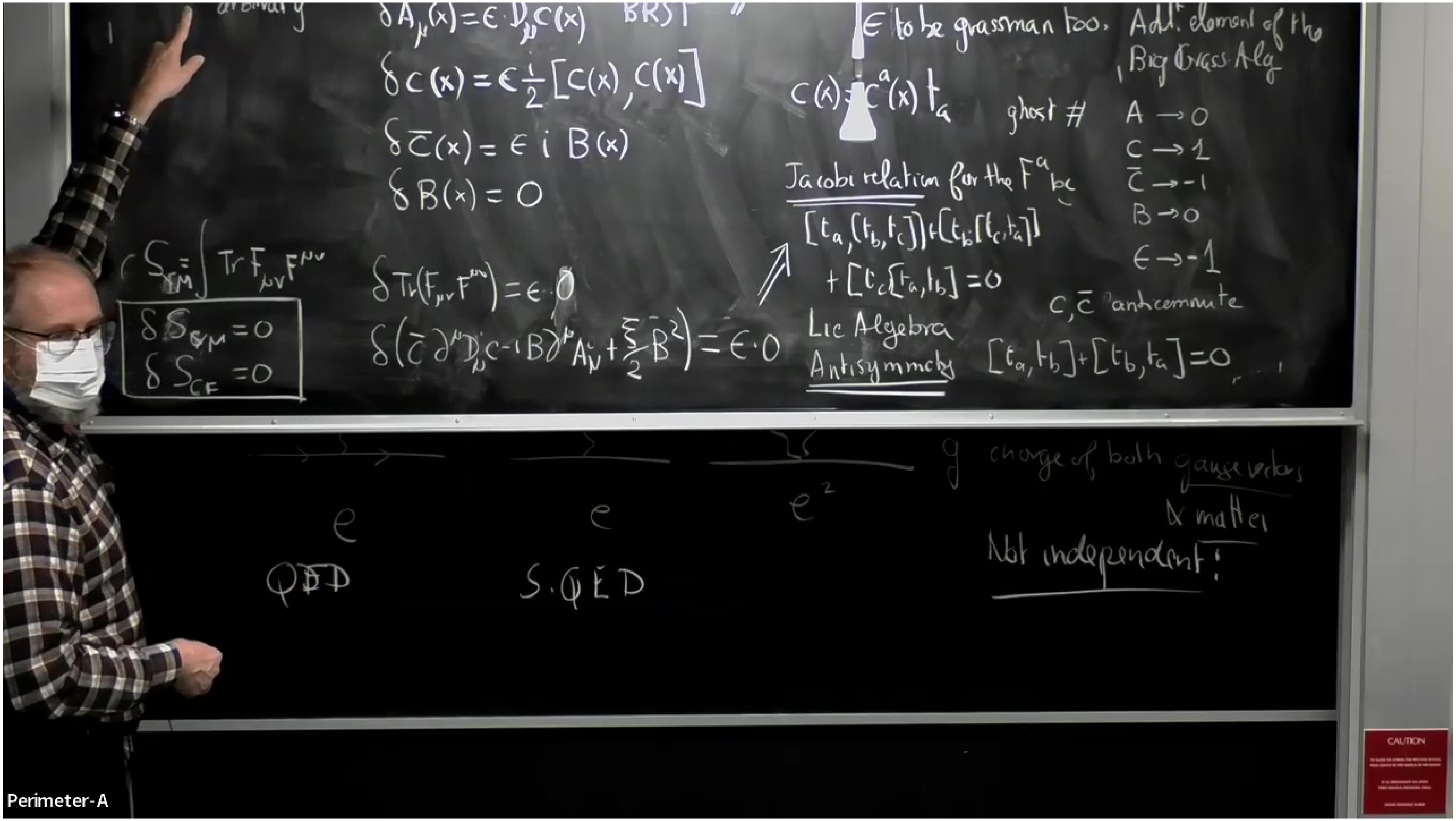
$$\frac{\delta_{ab}}{p^2}$$

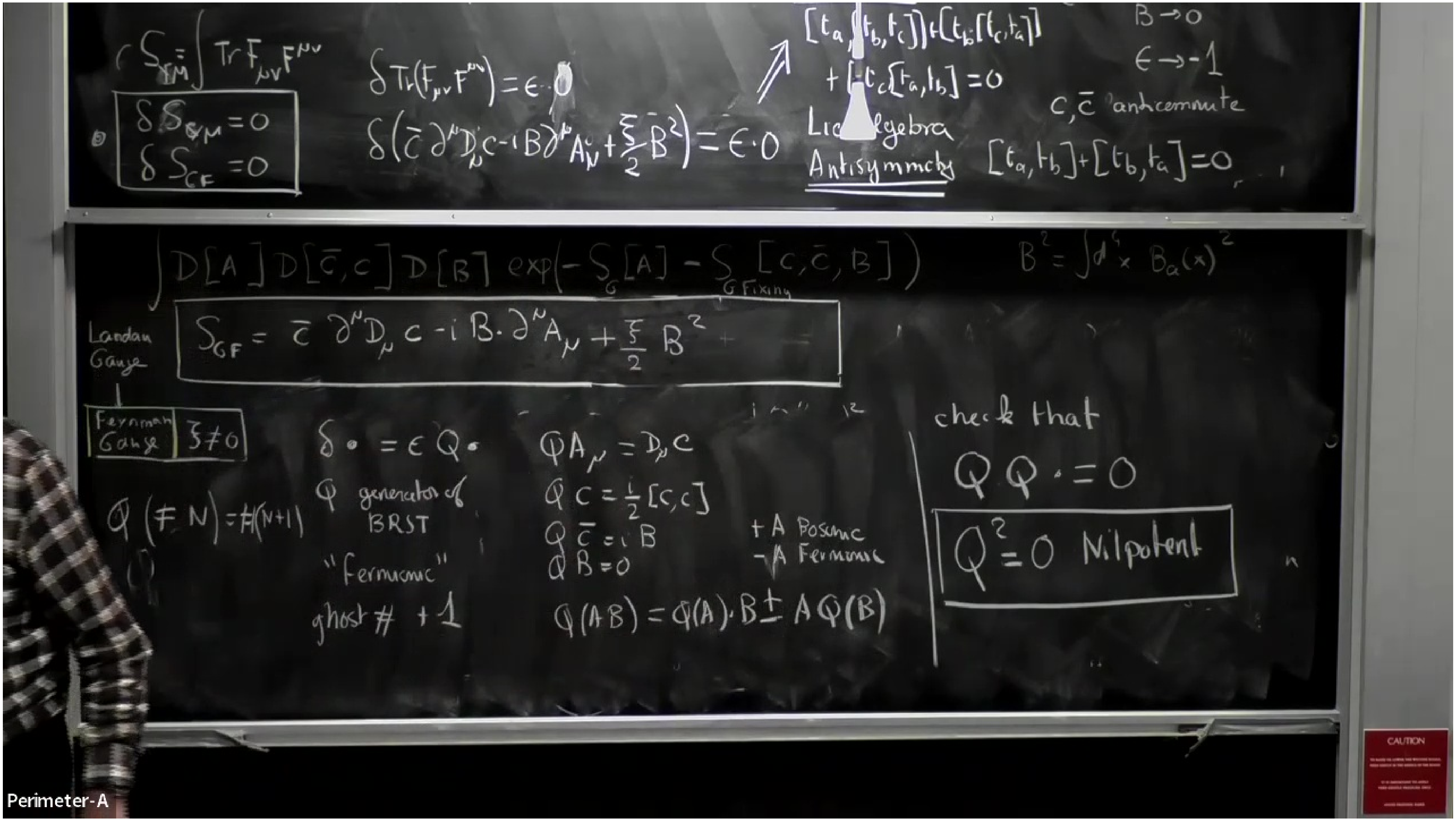
ghost-gauge vertex

$f_{abc}\, p_\mu$


Feynman Gauge ξ≠0
Integrate over B
S_GF = c̄ ∂^μ D_μ c + 1/2ξ (∂^μ A_μ)² → adds to (∂_ν A_μ − ∂_ν A_ν)² quadratic term
ghost
Feynmann propagator for the gauge field
δ_ab / p²
ghost-gauge vertex
f_abc p_μ
≥0 definite form A_μ K_μν A_ν ⇒ it can be inverted
no zero modes problem
now long. polarization ε_μ k^μ = 0 propagate
propagator A-A
+ O(g⁴)
problems with unitarity
c, c̄ are fermions
restauration of unitarity
cancellation
Perimeter-A
CAUTION


Feynman Gauge
ξ≠0
Integrate over B
S_GF = c̄ ∂^μ D_μ c + 1/(2ξ) (∂^μ A_μ)² → adds to (∂_ν A_μ − ∂_ν A_μ)² quadratic term
ghost
BRST Symmetry (Supersymmetry, not SpaceTime) it mixes A, B → c, c̄
δα(x) = δα^a(x) t_a
arbitrary
δA_μ(x) = D_μ δα(x) Gauge transformation
δA_μ(x) = ε D_μ c(x) BRST "
δc(x) = ε 1/2 [c(x), c(x)]
δc̄(x) = ε i B(x)
δB(x) = 0
δα(x) ⇒ ε c(x) ← ghost field
ε to be grassman too. Add. element of the Big Grass Alg
c(x) = c^a(x) t_a
ghost #
A → 0
c → 1
c̄ → −1
B → 0
ε → −1
S_G = ∫ Tr F_μν F
Perimeter-A
CAUTION


δA_μ(x) = ε·D_μ C(x)   BRST
δC(x) = ε ½ [C(x), C(x)]
δC̄(x) = ε i B(x)
δB(x) = 0
S_YM = ∫ Tr F_μν F^μν
δ S_YM = 0
δ S_GF = 0
δ Tr(F_μν F^μν) = ε · 0
δ(C̄ ∂^μ D_μ C − i B ∂^μ A_μ + ξ/2 B²) = ε · 0
ε to be grassman too.
Add. element of the Big Grass. Alg.
C(x) = C^a(x) t_a
ghost #
A → 0
C → 1
C̄ → −1
B → 0
ε → −1
C, C̄ anticommute
Jacobi relation for the F^a_bc
[t_a,[t_b,t_c]] + [t_b,[t_c,t_a]] + [t_c,[t_a,t_b]] = 0
Lie Algebra
Antisymmetry
[t_a,t_b] + [t_b,t_a] = 0
e
e
e²
QED
S.QED
g charge of both gauge vectors & matter
Not independent!
Perimeter-A
CAUTION


S_YM = ∫ Tr F_μν F^μν
δ Tr(F_μν F^μν) = ε · 0
δ S_YM = 0
δ S_GF = 0
δ(c̄ ∂^μ D_μ c - i B ∂^μ A_μ + ξ/2 B²) = ε · 0
[t_a, [t_b, t_c]] + [t_b, [t_c, t_a]] + [t_c, [t_a, t_b]] = 0
Lie algebra
Antisymmetry
B → 0
ε → -1
c, c̄ anticommute
[t_a, t_b] + [t_b, t_a] = 0
∫ D[A] D[c̄, c] D[B] exp(- S_G[A] - S_G Fixing[c, c̄, B])
B² = ∫ d⁴x B_a(x)²
Landau Gauge
S_GF = c̄ ∂^μ D_μ c - i B · ∂^μ A_μ + ξ/2 B²
Feynman Gauge
ξ ≠ 0
δ • = ε Q •
Q generator of BRST
"fermionic"
ghost # +1
Q A_μ = D_μ c
Q c = 1/2 [c, c]
Q c̄ = i B
Q B = 0
+ A Bosonic
- A Fermionic
Q(A B) = Q(A) · B ± A Q(B)
Q (# N) = #(N+1)
check that
Q Q • = 0
Q² = 0 Nilpotent
Perimeter-A
CAUTION