Title: PSI Lecture - Condensed Matter - Lecture 11

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Collection: PSI Lecture - Condensed Matter

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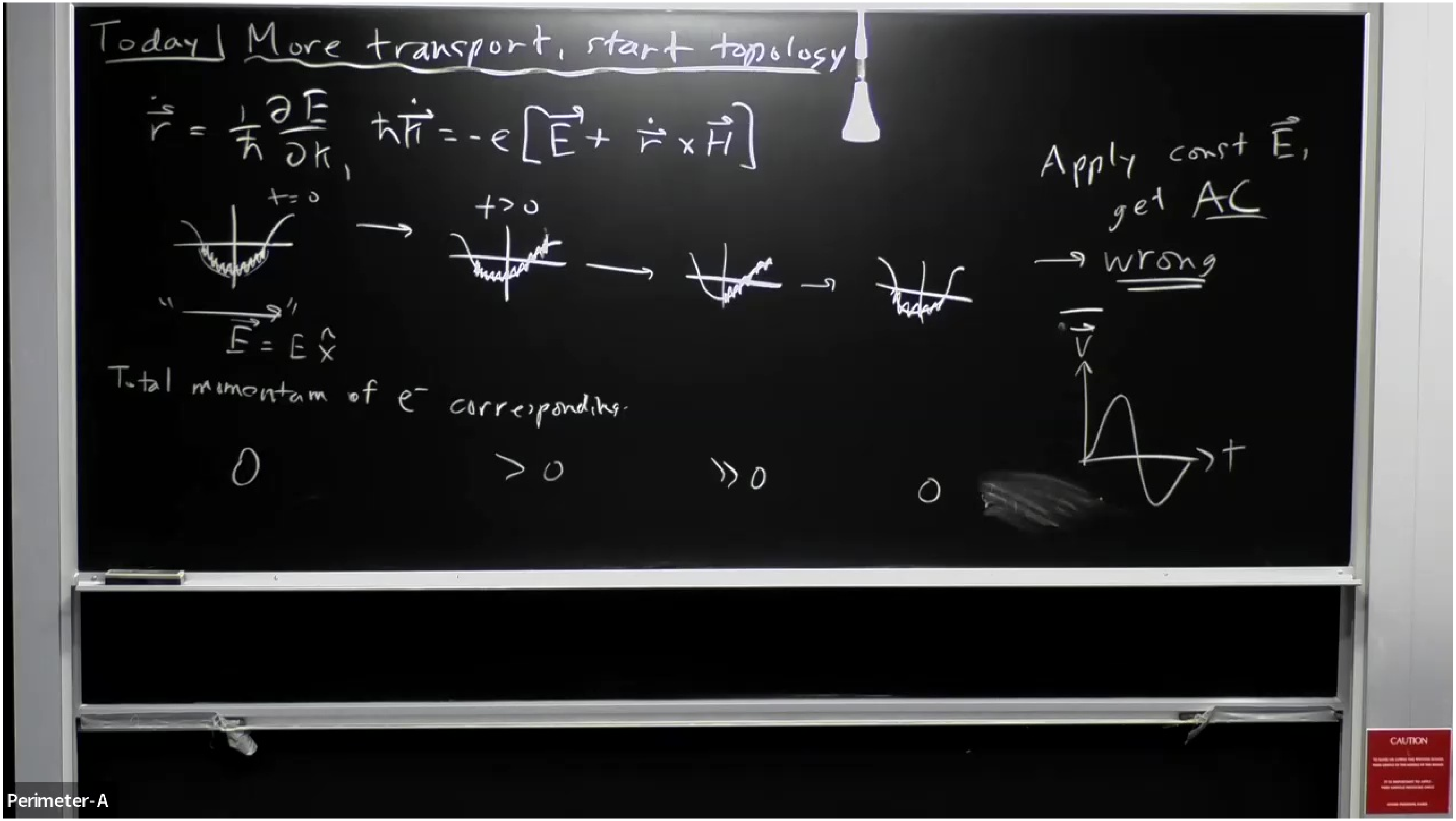
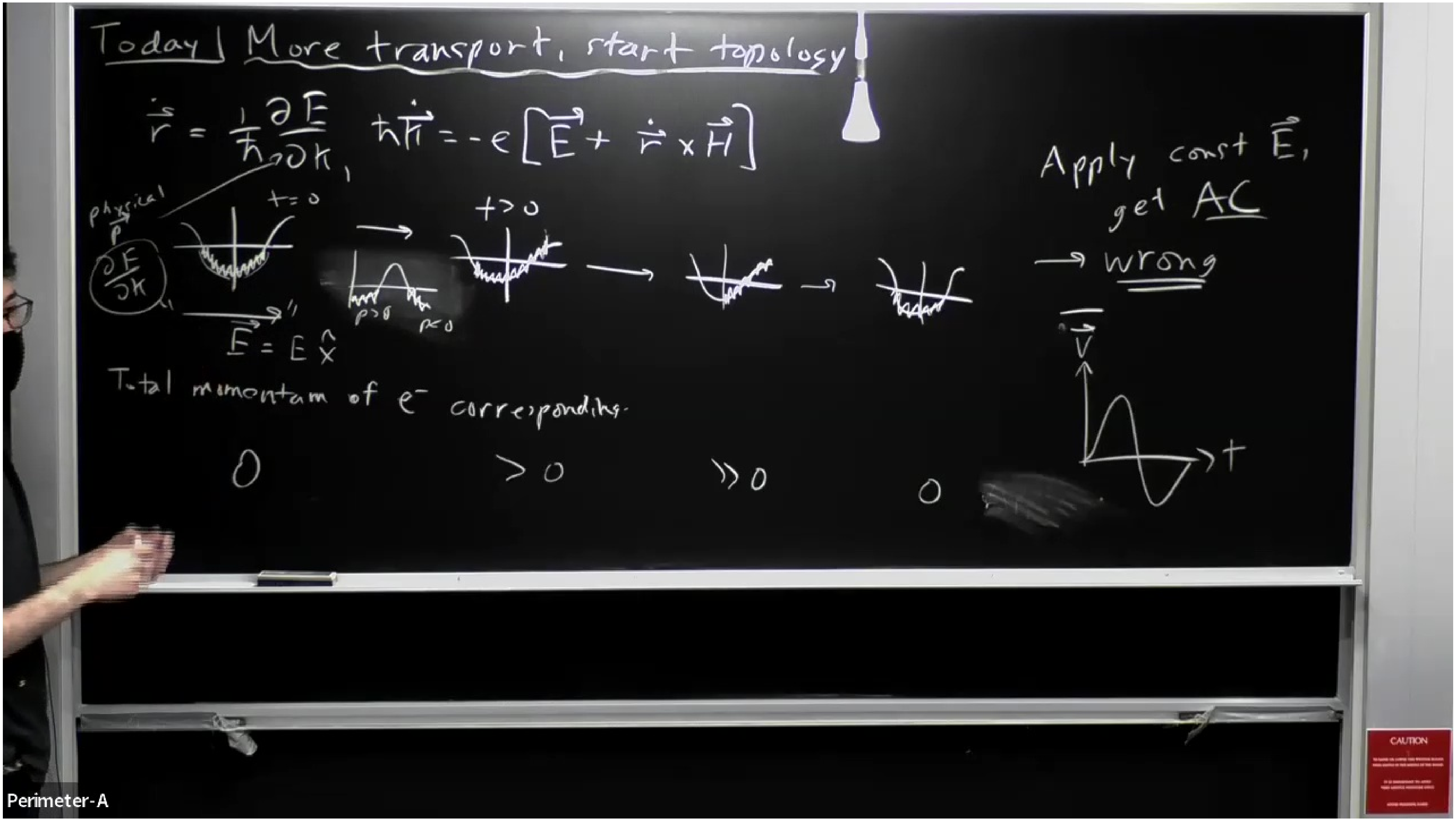
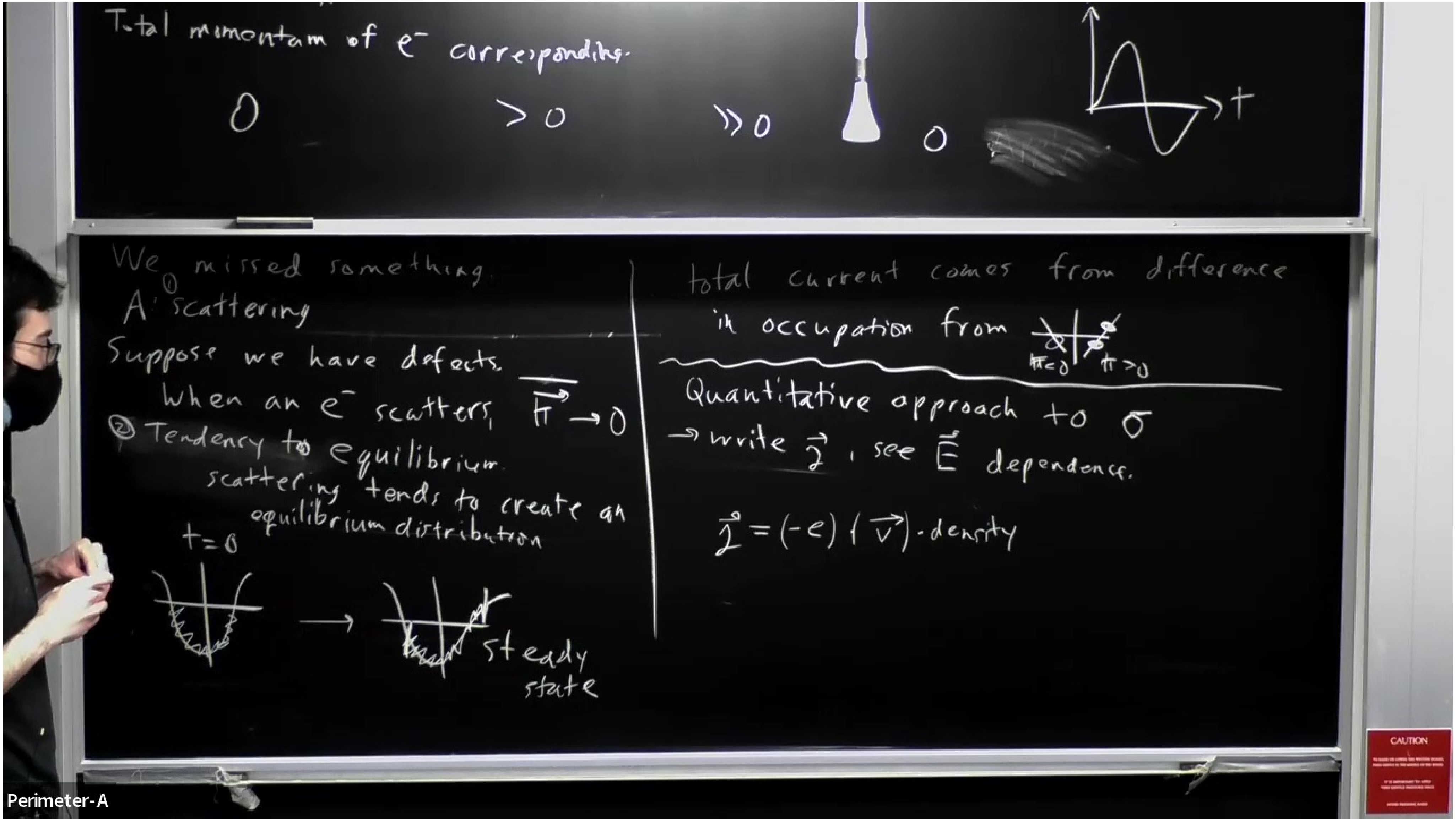
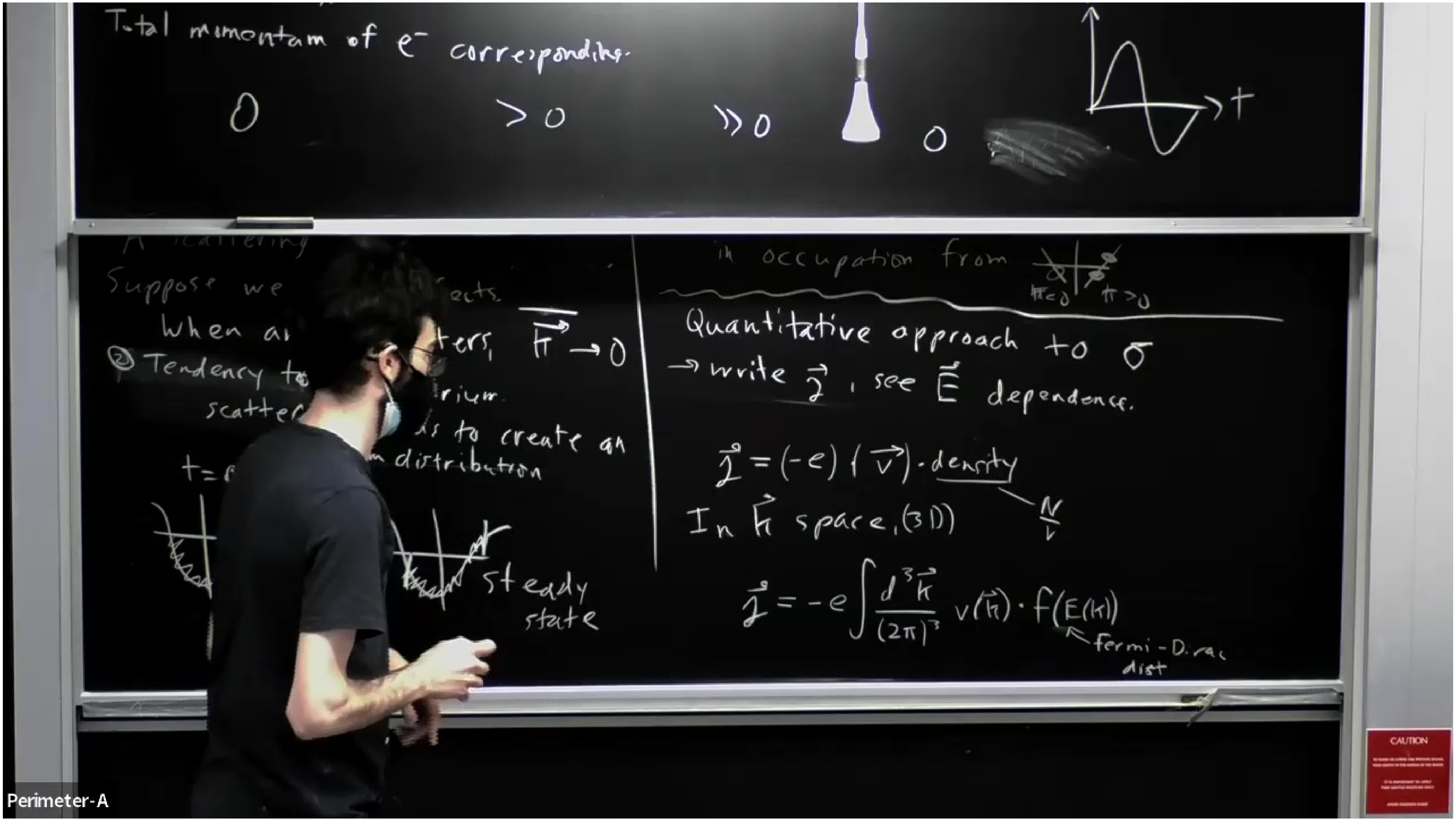
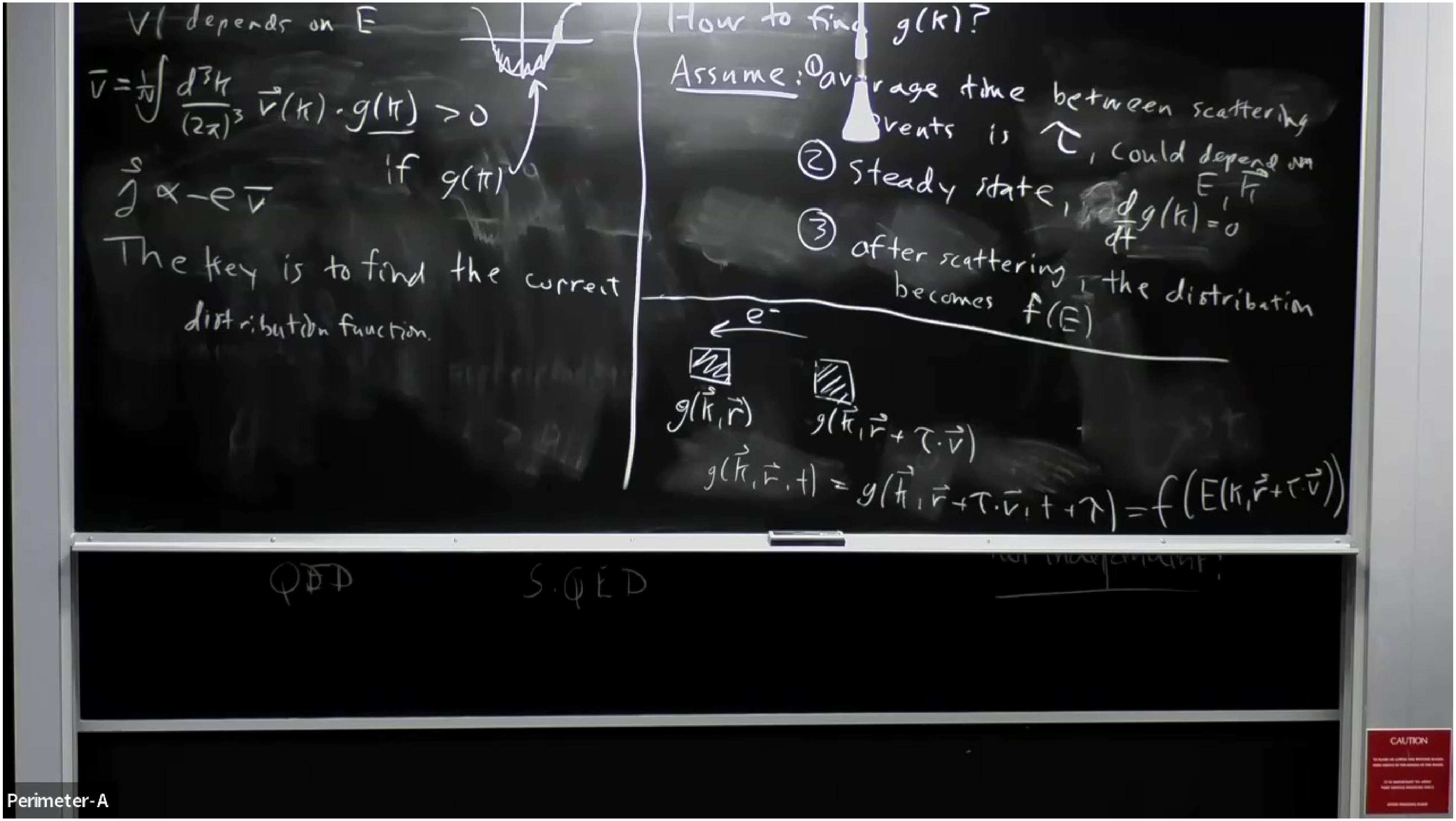

Today | More transport, start topology
$\dot{\vec{r}} = \frac{1}{\hbar}\frac{\partial E}{\partial k}$, $\hbar\dot{\vec{k}} = -e[\vec{E} + \dot{\vec{r}} \times \vec{H}]$
t=0
t>0
$\vec{E} = E\hat{x}$
Total momentum of e^- corresponding
0
> 0
>> 0
0
Apply const $\vec{E}$,
get AC
→ wrong
$\bar{\vec{v}}$
t
Perimeter-A
CAUTION


Today | More transport, start topology
$\dot{r} = \frac{1}{\hbar}\frac{\partial E}{\partial k}$, $\hbar\dot{\vec{k}} = -e[\vec{E} + \dot{\vec{r}} \times \vec{H}]$
physical p
$\frac{\partial E}{\partial k}$
t=0
t>0
p>0
p<0
$\vec{E} = E\hat{x}$
Total momentum of e⁻ corresponding
0
>0
>>0
0
Apply const $\vec{E}$,
get AC
→ wrong
$\bar{\vec{v}}$
t
Perimeter-A
CAUTION


Total momentum of e⁻ corresponding
0
> 0
>> 0
0
t
We missed something
① A: scattering
Suppose we have defects.
When an e⁻ scatters, Π → 0
② Tendency to equilibrium
scattering tends to create an equilibrium distribution
t = 0
steady state
total current comes from difference
in occupation from
Π < 0
Π > 0
Quantitative approach to σ
→ write j, see E dependence.
j = (−e)(v)·density
Perimeter-A
CAUTION


Total momentum of e⁻ corresponding
0
> 0
>> 0
0
t
Suppose we
When a
ters, k → 0
② Tendency to
rium
scatte
s to create an
distribution
steady
state
in occupation from
k<0
k>0
Quantitative approach to σ
→ write j, see E dependence.
j = (-e) (v) · density
N/V
In k space, (3D)
j = -e ∫ d³k/(2π)³ v(k) · f(E(k))
fermi-Dirac dist
Perimeter-A
CAUTION



V depends on E
$\bar{v} = \frac{1}{N}\int \frac{d^3k}{(2\pi)^3}\, \vec{v}(k)\cdot g(k) > 0$
if $g(k)$
$\vec{j} \propto -e\bar{v}$
The key is to find the correct distribution function.
How to find $g(k)$?
Assume: ① average time between scattering events is τ, could depend on $E, \vec{k}$
② Steady state, $\frac{d}{dt} g(k) = 0$
③ after scattering, the distribution becomes $f(E)$
e^-
$g(\vec{k},\vec{r})$
$g(\vec{k},\vec{r}+\tau\cdot\vec{v})$
$g(\vec{k},\vec{r},t) = g(\vec{k},\vec{r}+\tau\cdot\vec{v},t+\tau) = f(E(k,\vec{r}+\tau\cdot\vec{v}))$
QED
S.QED
Perimeter-A
CAUTION

$$g(\vec{k},\vec{r}) \qquad g(\vec{k},\vec{r}+\tau\cdot\vec{v})$$

$$g(\vec{k},\vec{r},t) = g(\vec{k},\vec{r}+\tau\cdot\vec{v},t+\tau) = f\big(E(\vec{k},\vec{r}+\tau\cdot\vec{v})\big)$$

$$\frac{1}{1+e^{\beta(E-\mu)}} \qquad E(\vec{k}) \to eE\cdot\underbrace{\vec{r}}_{\tau\cdot\vec{v}}$$

Assume $\vec{E}$ is small, because σ is a linear response.

→ Taylor expand

$$f(E)\Big|_{\vec{E}=0} - e f'(E)\Big|_{\vec{E}=0}\cdot\tau\vec{v}\cdot\vec{E} = g(\vec{k})$$

$$\sigma_{ij} \propto \int d^3k\, \frac{\partial^2 E(\vec{k})}{\partial k_i \partial k_j}\cdot f(E(\vec{k}))$$

$$\vec{j} = -2e\int \frac{d^3k}{(2\pi)^3}\vec{v}(k)\, f(E(\vec{k})) \to 0, \text{ equilibrium}$$

$$+2e^2\int\frac{d^3k}{(2\pi)^3}\vec{v}(\vec{k})\Big(\tau(E,\vec{k})\big(-f'(E)\big)\,\vec{v}(k)\cdot\vec{E}\Big)$$

$$\boxed{\sigma = 2e^2\int\frac{d^3k}{(2\pi)^3}\tau\,\vec{v}(\vec{k})\,\vec{v}(\vec{k})^T\left(-\frac{\partial f}{\partial E}\right)}$$

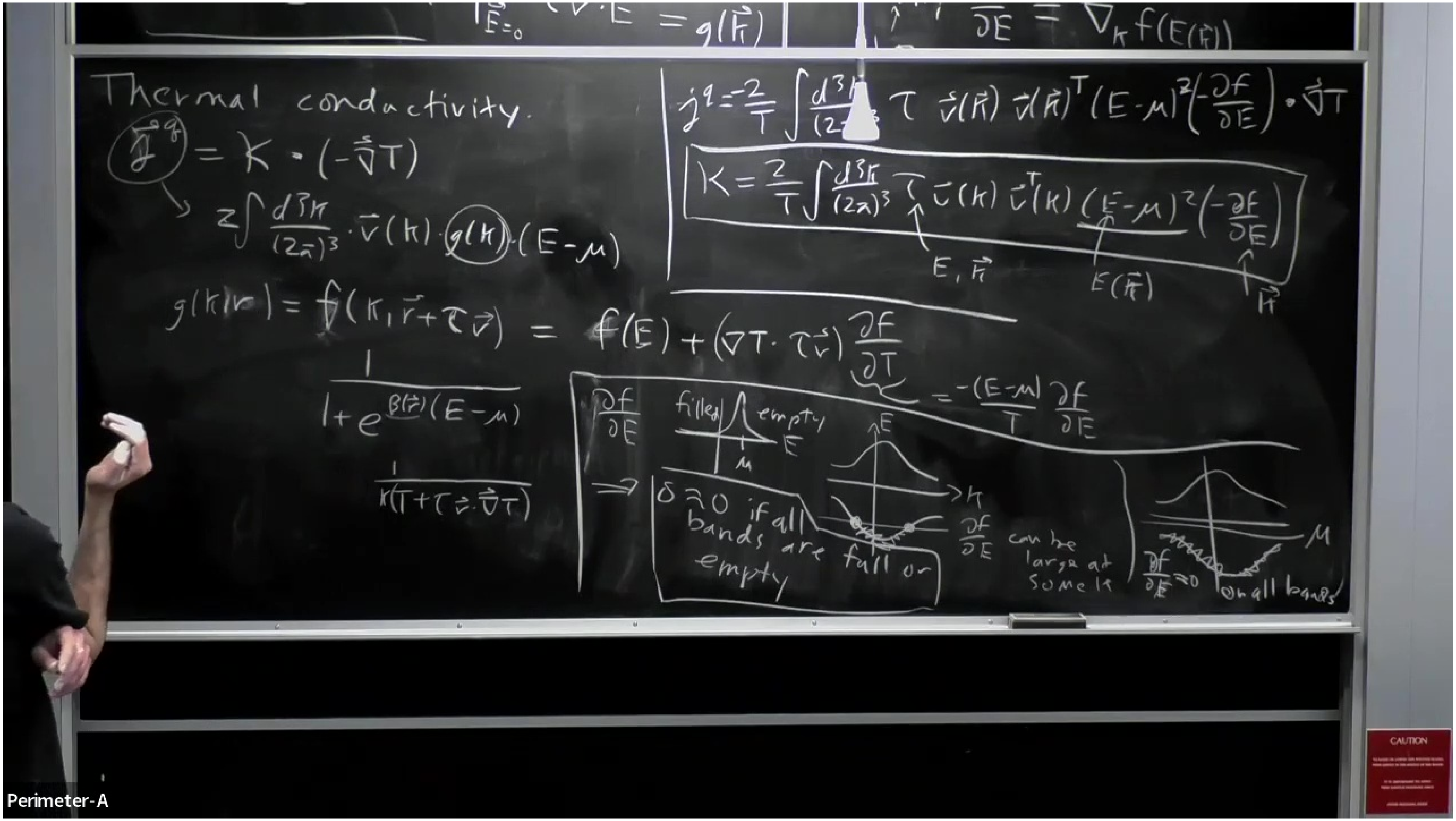
$$\underbrace{\vec{v}(k)}_{\frac{1}{\hbar}\nabla_k E}\cdot\frac{\partial f}{\partial E} = \nabla_k f(E(\vec{k}))$$

integrate by parts

$$\int d^3k\,(\nabla_k E)(\nabla_k f) \xrightarrow{ij\ \text{comp.}} \int d^3k\,\frac{\partial^2 E}{\partial k_i\partial k_j} f(E)$$

Perimeter-A

CAUTION


Thermal conductivity.
$\vec{J}^q = K \cdot (-\vec{\nabla} T)$
$2\int \frac{d^3k}{(2\pi)^3} \cdot \vec{v}(k) \cdot g(k) \cdot (E-\mu)$
$g(k|r) = f(k, \vec{r} + \tau\vec{v}) = f(E) + (\nabla T \cdot \tau\vec{v}) \frac{\partial f}{\partial T}$
$\frac{1}{1+e^{\beta(\vec{r})(E-\mu)}}$
$= \frac{-(E-\mu)}{T}\frac{\partial f}{\partial E}$
$j^q = \frac{-2}{T}\int \frac{d^3k}{(2\pi)^3} \tau\, \vec{v}(\vec{k})\, \vec{v}(\vec{k})^T (E-\mu)^2 \left(-\frac{\partial f}{\partial E}\right) \cdot \vec{\nabla} T$
$K = \frac{2}{T}\int \frac{d^3k}{(2\pi)^3} \tau\, \vec{v}(k)\, \vec{v}^T(k)\, (E-\mu)^2 \left(-\frac{\partial f}{\partial E}\right)$
E, k
E(k)
k
$\frac{\partial f}{\partial E}$
filled
empty
E
μ
$\delta \approx 0$ if all bands are full or empty
k
$\frac{\partial f}{\partial E}$ can be large at some k
μ
$\frac{\partial f}{\partial E} \approx 0$ on all bands
Perimeter-A
CAUTION