Title: Higgs inflation, unitarity, and emergence of scalaron

Speakers: Yohei Ema

Series: Particle Physics

Date: January 25, 2022 - 1:00 PM

URL: https://pirsa.org/22010093

Abstract: Higgs inflation introduces a large non-minimal coupling between the Ricci scalar and Higgs that makes the cut-off scale well below the Planck scale. In the first part of the talk, we show that unitarity is indeed violated by a resonant production of longitudinal gauge bosons after inflation, calling for UV completion of Higgs inflation. We then show that unitarity is restored after summing over vacuum polarization-type diagrams that are leading-order in the large-N limit. Scattering amplitude develops a pole after the resummation, which we identify as the scalar component of the metric, or the scalaron. This phenomenon can be understood in the language of the non-linear sigma model (NLSM), with the scalaron identified as the sigma-meson that linearizes the NLSM.

Zoom Link: https://pitp.zoom.us/j/98015814051?pwd=YWdrWTJzbzJkdnFXRUxPWFJnNXVNQT09

Higgs inflation

- Higgs inflation (HI): Standard Model Higgs = inflaton.
- Require non-minimal coupling to gravity ξ :

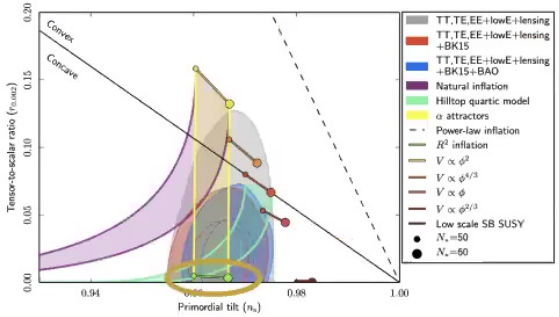
$$\mathscr{L} = \frac{M_P^2}{2}\left(1 + \frac{\xi\phi_i^2}{M_P^2}\right)R + \frac{1}{2}\left(\partial\phi_i\right)^2 - \frac{\lambda}{4}\phi_i^4,$$

where ϕ_i : Higgs with $i = 1,\ldots,N_s(=4)$, R : Ricci scalar.

- Consistent with CMB observation for $\xi^2 \simeq 2\times 10^9\lambda$.

[Bezrukov, Shaposhnikov 07; Planck 2018]

➡ Assume $\xi \gg 1$ in this talk (true unless λ: tiny).



Higgs inflation, unitarity, and emergence of scalaron

Yohei Ema

Minnesota U.

Seminar @ Perimeter Institute 01.25.2022

Based mainly on 1907.00993, 2002.11739 and 2102.12501

with R. Jinno, K. Nakayama, K. Mukaida and J. van de Vis.

See also 1609.05209, 1701.07665 and 2008.01096.

Higgs inflation

- Higgs inflation (HI): Standard Model Higgs = inflaton.
- Require non-minimal coupling to gravity ξ :

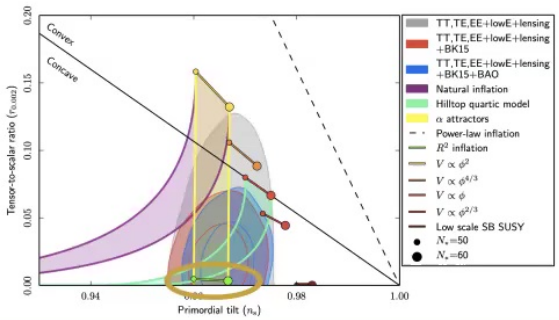
$$\mathscr{L} = \frac{M_P^2}{2}\left(1 + \frac{\xi\phi_i^2}{M_P^2}\right)R + \frac{1}{2}\left(\partial\phi_i\right)^2 - \frac{\lambda}{4}\phi_i^4,$$

where ϕ_i : Higgs with $i = 1,\ldots,N_s(=4)$, R : Ricci scalar.

- Consistent with CMB observation for $\xi^2 \simeq 2\times 10^9\lambda$.

[Bezrukov, Shaposhnikov 07; Planck 2018]

➡ Assume $\xi \gg 1$ in this talk (true unless λ: tiny).



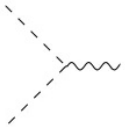
Tree-level unitarity violation

- Large ξ induces tree-level unitarity violation.

[Burgess+ 09,10; Barbon+ 09; Hertzberg 10; ...]

  - Scalar-scalar-graviton vertex:

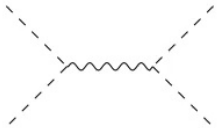
$$\xi R\phi_i^2 \ni \xi\phi_i^2\partial^2 h \sim$$



.

  - 4-point scattering amplitude $\phi_i\phi_i \to \phi_j\phi_j$.

$$A_{\text{tree}}^{(ii\to jj)} =$$



$$= \frac{1}{M_P^2}\left[\frac{(1+6\xi)^2}{6}s - \left(\frac{s}{6} - \frac{tu}{s}\right)\right].$$

➡ Tree-level unitarity violation at $\sqrt{s} \sim \dfrac{M_P}{\xi} \ll M_P$ around $\phi_i = 0$.

- Could be fine during inflation, with finite VEV. [Bezrukov+ 10]
- Problematic after inflation, during (p)reheating epoch. [YE, Jinno, Mukaida, Nakayama 16]

➡ Think more about unitarity of HI.

Outline

1. Introduction
2. Unitarity violation during preheating
3. Unitarity and scalaron in large N
4. Summary

more cosmology-ish

more QFT-ish

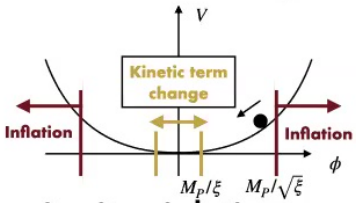
Spiky oscillation after inflation

- Higgs fields have a non-tirivial target space in Einstein frame:

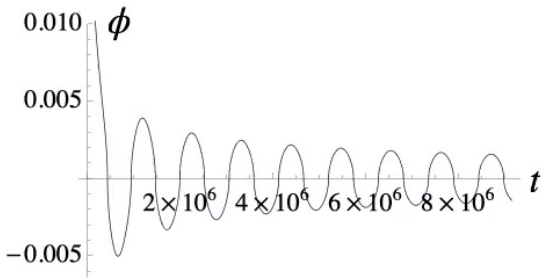
$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + \frac{1}{2}h_{ab}g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi)\right]$$

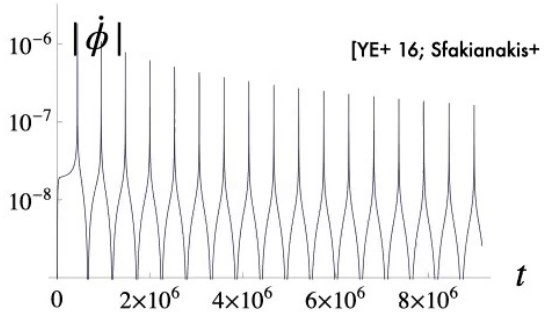
with $h_{ab} = \frac{1}{\Omega^4}\begin{pmatrix}\Omega^2 + \frac{6\xi^2\phi^2}{M_P^2} & \frac{6\xi^2\phi\chi}{M_P^2} \\ \frac{6\xi^2\phi\chi}{M_P^2} & \Omega^2 + \frac{6\xi^2\chi^2}{M_P^2}\end{pmatrix}$ with χ : NG mode(s) and $\Omega^2 = 1 + \xi\frac{\phi^2+\chi^2}{M_P^2}$ for HI.

- Kinetic term drastically changes for $|\phi| \lesssim M_P/\xi$,



a "spiky" feature for $|\phi| \lesssim M_P/\xi$, causing unitarity violation.





[YE+ 16; Sfakianakis+ 18; Nguyen+ 19]

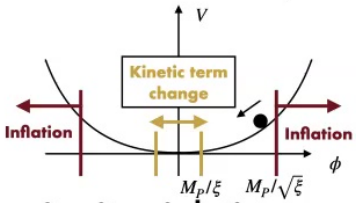
Spiky oscillation after inflation

- Higgs fields have a non-tirivial target space in Einstein frame:

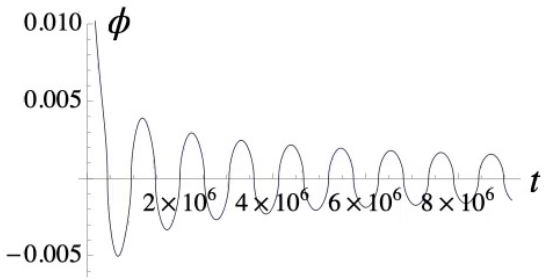
$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + \frac{1}{2}h_{ab}g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi)\right]$$

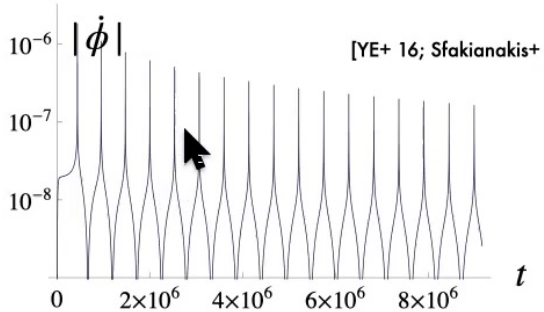
with $h_{ab} = \frac{1}{\Omega^4}\begin{pmatrix} \Omega^2 + \boxed{\frac{6\xi^2\phi^2}{M_P^2}} & \frac{6\xi^2\phi\chi}{M_P^2} \\ \frac{6\xi^2\phi\chi}{M_P^2} & \Omega^2 + \frac{6\xi^2\chi^2}{M_P^2} \end{pmatrix}$ with χ : NG mode(s) and $\Omega^2 = 1 + \xi\frac{\phi^2+\chi^2}{M_P^2}$ for HI.

- Kinetic term drastically changes for $|\phi| \lesssim M_P/\xi$,



➡ a "spiky" feature for $|\phi| \lesssim M_P/\xi$, causing unitarity violation.





[YE+ 16; Sfakianakis+ 18; Nguyen+ 19]

Target space and unitarity

An easy-to-use condition of unitarity violation from target space

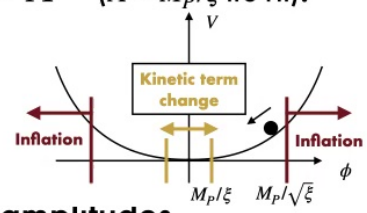
[YE, Jinno, Nakayama, van de Vis 21]

- NG boson has mass from target space curvature → feels spikes:

$$m_\chi^2 = \nabla^\chi V_\chi - \dot{\phi}^2 R^\chi{}_{\phi\phi\chi}\,, \qquad \text{e.g.} \left(1+\frac{\chi^2}{\Lambda^2}\right)(\partial\phi)^2 \to m_\chi^2 = -\frac{\dot{\phi}^2}{\Lambda^2}.$$

- Inflaton motion changes for $|\phi| \lesssim \Lambda$ with curvature $R[h] \sim \Lambda^{-2}$ ($\Lambda \sim M_P/\xi$ fro HI).

  ➡ typical momentum scale: $k_{\rm spike} \sim (\Lambda/\dot{\phi}_{\rm origin})^{-1}$.



- Cut-off also $\sim \Lambda$ since the curvature affects e.g. scattering amplitudes.

- With energy cons. $\dot{\phi}^2_{\rm origin} \sim V_{\rm inf}$, unitarity violation $k_{\rm spike} \gtrsim \Lambda$ translates to

$V_{\rm inf} \gtrsim \Lambda^4$: simply compare inflation energy scale and cut-off.

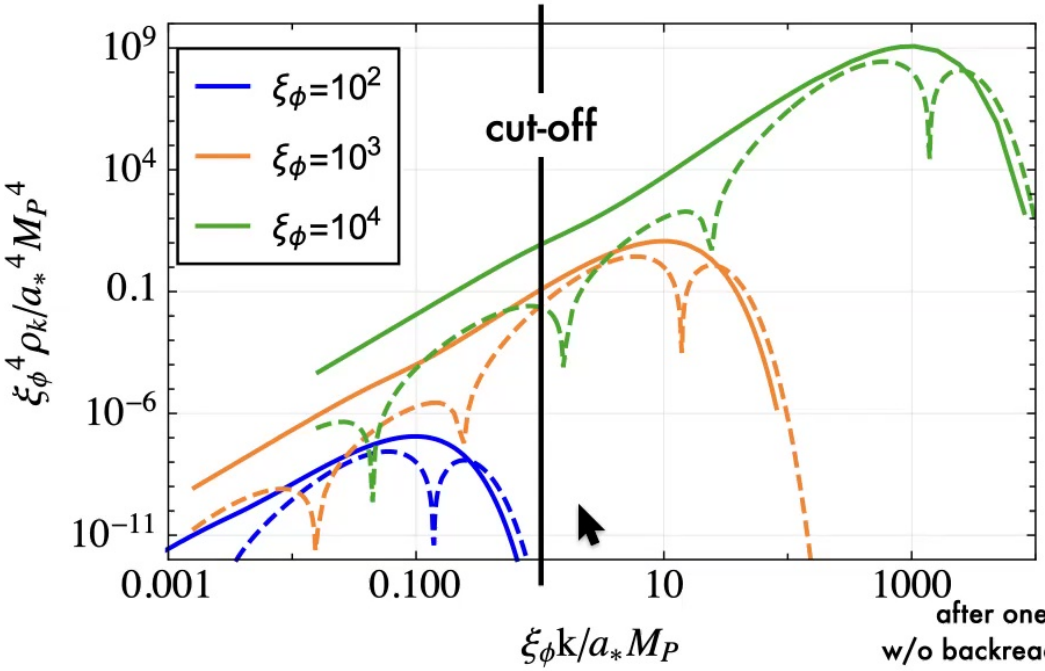
e.g. $V_{\rm inf}/\Lambda^4 \sim \lambda\xi^2 \sim 10^{-9}\xi^4$ for HI → unitarity violation for $\xi \gtrsim 10^2$.

- Applicable to other inflation models (can see e.g. running kinetic inflation violates unitarity).

Numerical result

Numerical result confirms the previous estimation.

[YE, Jinno, Nakayama, van de Vis 21]



after one oscillation,
w/o backreaction to inflaton.

$\rho_\chi \sim k_{\rm spike}^4 \sim V_{\rm inf}^2/\Lambda^4 > V_{\rm inf}$ for $V_{\rm inf} > \Lambda^4$ → this production is fatal.

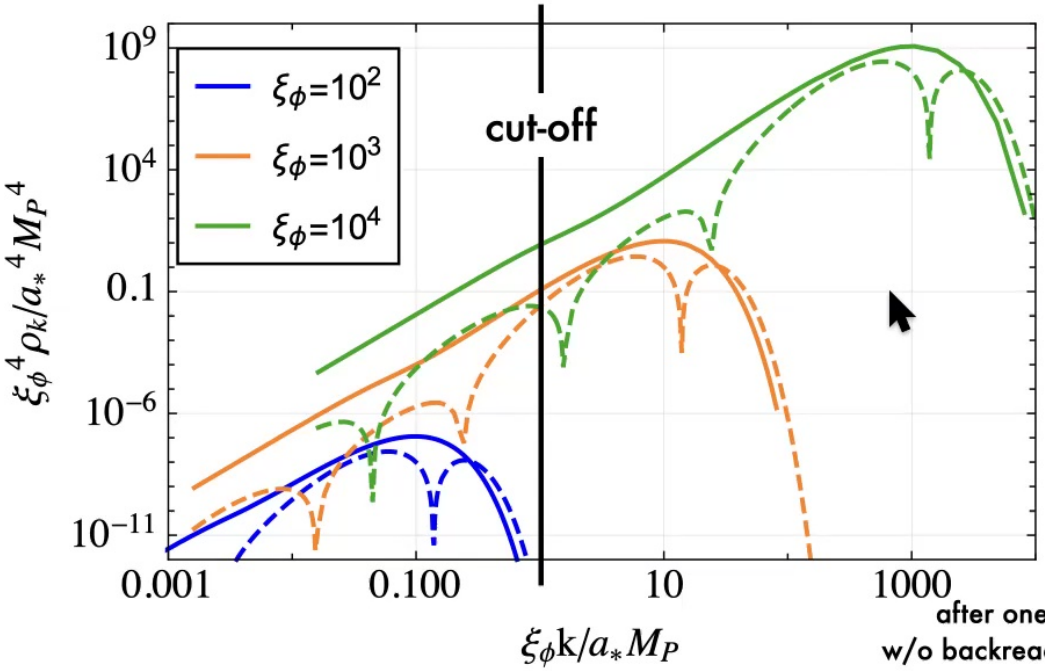
Outline

1. Introduction
2. Unitarity violation during preheating
3. Unitarity and scalaron in large N
4. Summary

Numerical result

Numerical result confirms the previous estimation.

[YE, Jinno, Nakayama, van de Vis 21]



after one oscillation,
w/o backreaction to inflaton.

$\rho_\chi \sim k_{\rm spike}^4 \sim V_{\rm inf}^2/\Lambda^4 > V_{\rm inf}$ for $V_{\rm inf} > \Lambda^4$ → this production is fatal.

Main question

- Tree-level unitarity of Higgs inflation is violated at $\sim M_P/\xi$.

BUT

- Renormalization group equations:

* Scalaron mass: $m_s^2 = \dfrac{M_P^2}{12\alpha}$.

$$\beta_\alpha \equiv \frac{d\alpha}{d\ln\mu} = -\frac{N_s}{1152\pi^2}\left(1+6\xi\right)^2, \quad \mathscr{L}_{\text{c.t.}} = \alpha R^2 .$$

➡ $\alpha = \mathcal{O}(\xi^2) \gg 1$ inevitably induced, implying light scalaron.

- Scalaron lifts the cut-off scale to M_P.

[YE 17; Gorbunov, Tkareva 18]

➡ **quantum correction to the rescue?**

Study quantum correction in a controllable way → **large N_S limit.**

Large N_s analysis

- Take $N_s \to \infty$ with $N_s \xi^2 s / M_P^2$ fixed.
- Leading: vacuum polarization diagrams ("dressed amplitude")

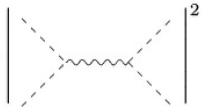
$A^{(ii\to jj)}_{\text{dressed}} \equiv$ 

- Study forward scattering of flavor-singlet state: $|1\rangle \equiv \frac{1}{\sqrt{N_s}} \sum |\phi_i \phi_i\rangle$.

$\Rightarrow$ $A^{(1\to 1)} = N_s A^{(ii\to jj)}_{\text{dressed}}$ at the leading order.

- Partial wave expansion: $A^{(1\to 1)} = 32\pi \sum a^{(l)} P_l(\cos\theta)$.

$$\Rightarrow \quad a^{(0)} = a^{(0)}_{\text{tree}} \left[1 + a^{(0)}_{1-\text{loop}} / a^{(0)}_{\text{tree}} + \left(a^{(0)}_{1-\text{loop}} / a^{(0)}_{\text{tree}} \right)^2 + \cdots \right] = \frac{a^{(0)}_{\text{tree}}}{1 - a^{(0)}_{1-\text{loop}} / a^{(0)}_{\text{tree}}} .$$

- Im [] = |  |^2 $\Rightarrow$ $\text{Im}\left[a^{(0)}_{1-\text{loop}}\right] = \left|a^{(0)}_{\text{tree}}\right|^2$.

$\Rightarrow$ $\boxed{\text{Im}\left[a^{(0)}\right] = \left|a^{(0)}\right|^2 \text{: unitarity satisfied.}}$

phrased "self-healing mechanicsm" in [Aydemir+12; Calmet, Casadio 13]

New degree of freedom

Question: how to understand this "physically"?

- Dressed amplitude develops a pole.

$$A_{\rm dressed} \sim \frac{A_{\rm tree}}{1 - A_{1-{\rm loop}}/A_{\rm tree}}, \quad A_{1-{\rm loop}}/A_{\rm tree} \propto s \text{ (non-renormalizable).}$$

$$A_{\rm dressed} \sim \frac{1}{1 - s/m^2}.$$

A new DOF emerges due to resummation, unitarizing the theory.

- Similar phenomena observed for other models within large N.

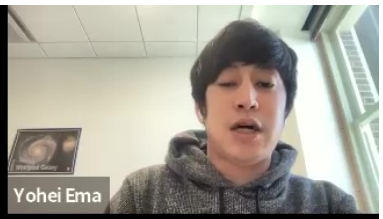
e.g. 4-fermi → pions, [Nambu, Jona-Lasinio 61]

$\mathrm{O}(N)$ NLSM → σ-mesons, [Bardeen+ 76; Brezin+ 76]

CP^{N-1} NLSM → ρ-meson, [D'Adda+ 78,79; Bando+ 85,88; ...]

EWChPT→ Higgs [Dobado+ 90, 00]

- Identified as **scalaron** in the case of Higgs inflation. [YE 19]



Emergence of scalaron

[YE 19]

- Dressed amplitude:

$$a^{(0)} = -\frac{N_s\left(1+6\xi\right)^2 s}{2304\pi\alpha}\left[s\left(1-\frac{i\pi}{\ln(s/\Lambda_\alpha^2)}\right)-\frac{M_P^2}{12\alpha}\right]^{-1}$$

where $\alpha = -\frac{N_s\left(1+6\xi\right)^2}{2304\pi^2}\ln\left(\frac{s}{\Lambda_\alpha^2}\right)$: coefficient of counter term,

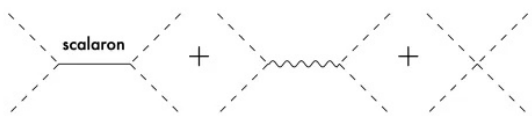
Λ_α: parameter choice of the theory, or "dimensional transmutation" (the same as $\Lambda_{\rm QCD}$).

- Consider a theory with scalaron: $\mathscr{L} = \bar{\alpha}R^2 + \xi R\phi_i^2/2$.

$$\mathscr{L} = -\frac{1}{144\bar{\alpha}}\left[\frac{3M_P^2}{2}-\left(\sigma+\frac{\sqrt{6}M_P}{2}\right)-\frac{6\xi+1}{2}\phi_i^2\right]^2$$ in conformal frame with σ: scalaron.

- Compute 4-point amplitude $\phi_i\phi_i \to \phi_j\phi_j$ with the R^2 term.

$\bar{A}_{\rm tree} =$



.

- We obtain $A_{\rm dressed} = \bar{A}_{\rm tree}$ by identifying $\alpha = \bar{\alpha}$ → new DOF = scalaron.

Correspondence

[YE, Mukaida, van de Vis 20]

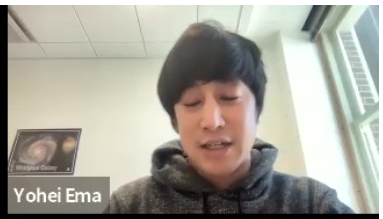
O(N) NLSM	Higgs inflation
pions π_i	Higgs fields ϕ_i, conformal mode of metric Φ
target space: $\pi_i^2 + h^2 = v^2, \quad (\pi_i, h) \in \mathbb{R}^{(N+1)}$	target space: $\frac{6\xi+1}{2}\phi_i^2 + \left(h + \frac{\Phi}{2}\right)^2 = \frac{\Phi^2}{4}, \quad (\Phi, \phi_i, h) \in \mathbb{R}^{(1,N+1)}$
sigma meson σ	scalaron σ

$$\mathscr{L} = \frac{M_P^2}{2}\left(1 + \frac{\xi\phi_i^2}{M_P^2}\right)R + \alpha R^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 - \frac{\lambda}{4}\phi_i^4 .$$

Auxiliary field introduction + Weyl transformation

$$\mathscr{L} = \underline{\frac{M_P^2}{2}\left(1 - \frac{\sigma^2}{6M_P^2} - \frac{\phi_i^2}{6M_P^2}\right)R + \frac{1}{2}\left[\left(\partial\phi_i\right)^2 + (\partial\sigma)^2\right]} - \frac{\lambda}{4}\phi_i^4 - \frac{1}{144\alpha}\left[\frac{3M_P^2}{2} - \left(\sigma + \sqrt{\frac{3}{2}}M_P\right)^2 - \frac{6\xi+1}{2}\phi_i^2\right]^2 .$$

Flat kinetic term in the conformal frame!

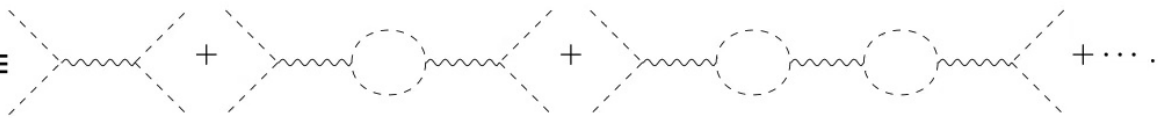


Outline

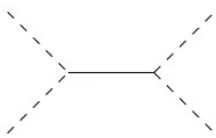
1. Introduction
2. Unitarity violation during preheating
3. Unitarity and scalaron in large N
4. Summary

Summary

- Higgs inflation violates unitarity after inflation, during preheating.
- Higgs inflation "self-heals" unitarity at large N.

$A_{\text{dressed}} \equiv$ 

- Interpreted as emergence of scalaron.

$A_{\text{dressed}} \sim$  $= -\frac{(1+6\xi)^2}{6}\frac{m_s^2}{M_P^2}\frac{s}{s-m_s^2}, \quad m_s^2 = \frac{M_P^2}{12\alpha_1}.$

1. Higgs inflation is actually a "Higgs-R^2" model (at least in large N).
2. (P)reheating can be studied without unitarity issue.

Scalaron smears spiky features, see [He+ 18; Bezrukov+ 19; ...].