Title: Quantum networks theory

Speakers: Pablo Arrighi

Series: Quantum Foundations

Date: January 28, 2022 - 2:00 PM

URL: https://pirsa.org/22010090

Abstract: The formalism of quantum theory over discrete systems is extended in two significant ways. First, tensors and traceouts are generalized, so that systems can be partitioned according to almost arbitrary logical predicates. Second, quantum evolutions are generalized to act over network configurations, in such a way that nodes be allowed to merge, split and reconnect coherently in a superposition. The hereby presented mathematical framework is anchored on solid grounds through numerous lemmas. Indeed, one might have feared that the familiar interrelations between the notions of unitarity, complete positivity, trace-preservation, non-signalling causality, locality and localizability that are standard in quantum theory be jeopardized as the partitioning of systems becomes both logical and dynamical. Such interrelations in fact carry through, albeit two new notions become instrumental: consistency and comprehension.

Joint work with Amélia Durbec and Matt Wilson

Reference: https://arxiv.org/abs/2110.10587

Zoom Link: https://pitp.zoom.us/j/97185954578?pwd=OC9mUzl4L3V4WDZzVEZoekpOS24wQT09

Laboratoire Méthodes Formelles

Quantum networks theory

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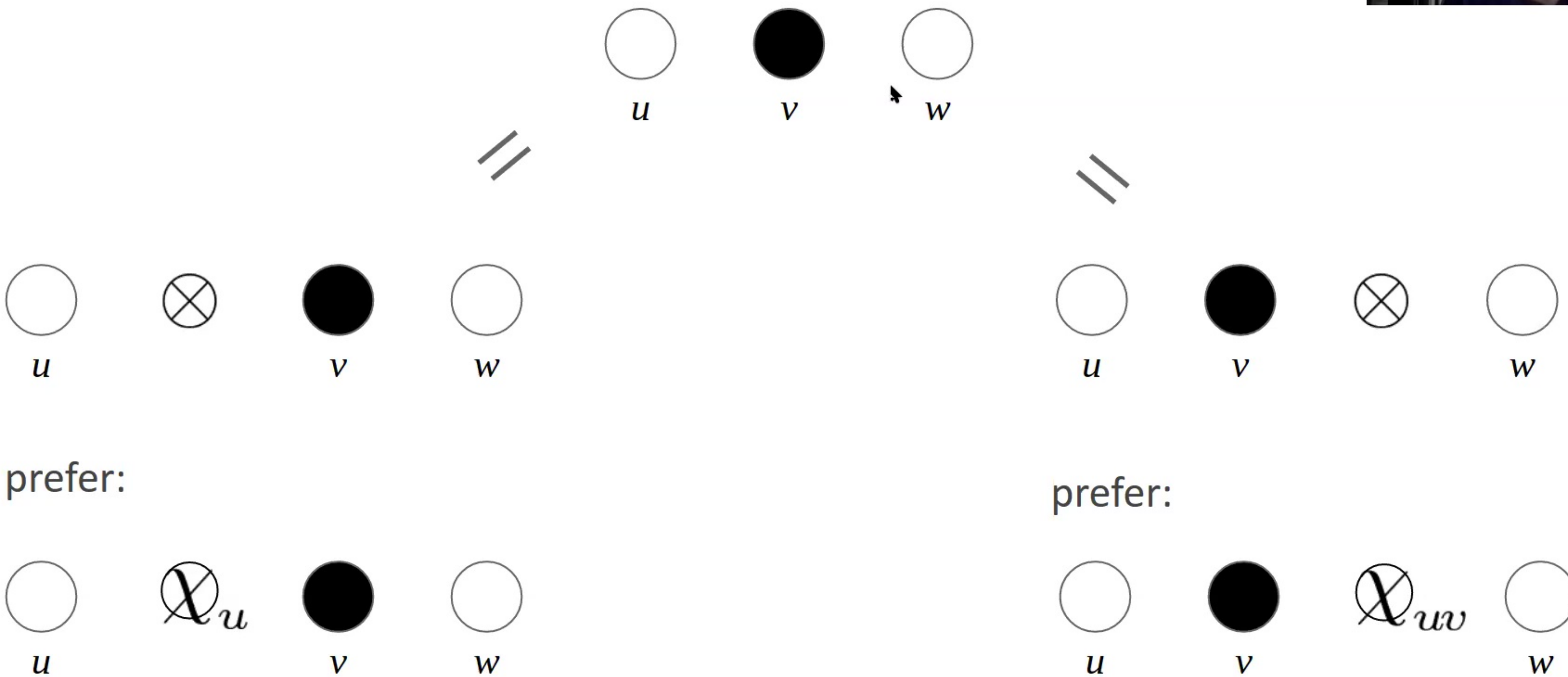
UNIVERSITY OF OXFORD

Foreword » **Tensors**

…just making notations



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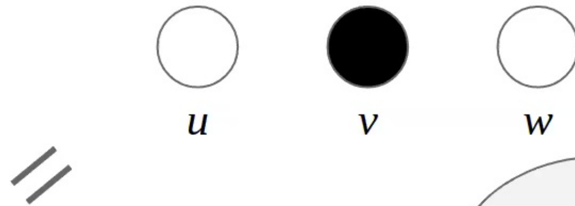
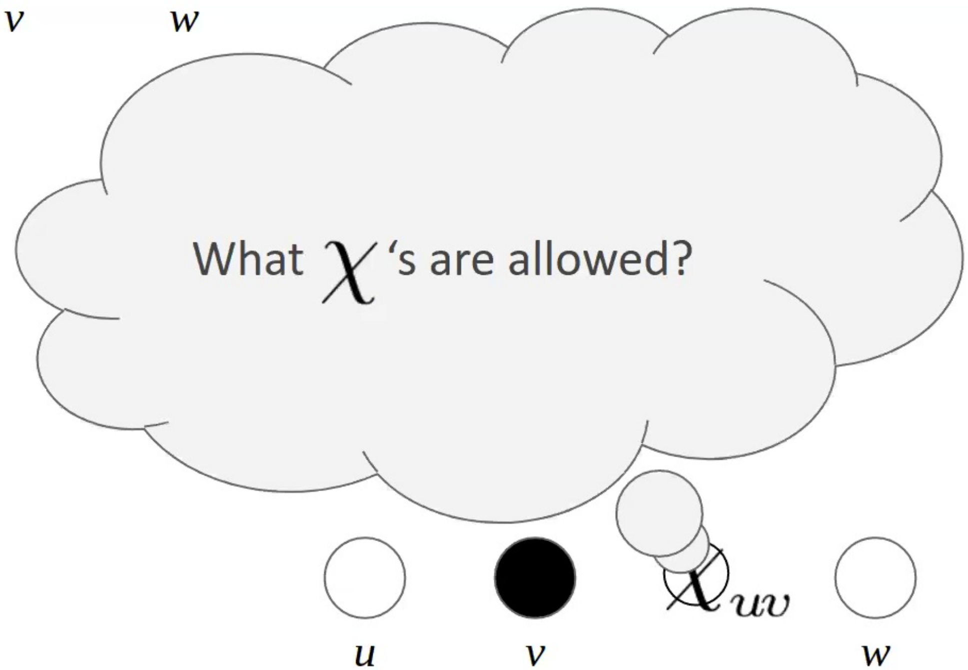
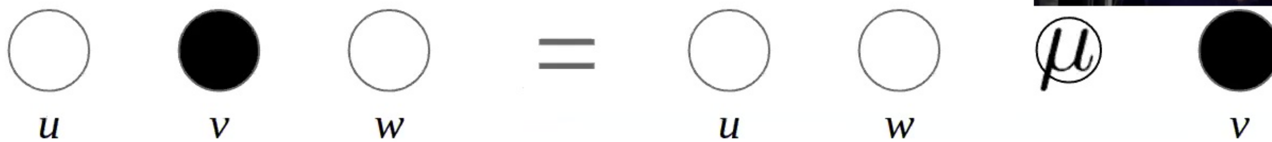


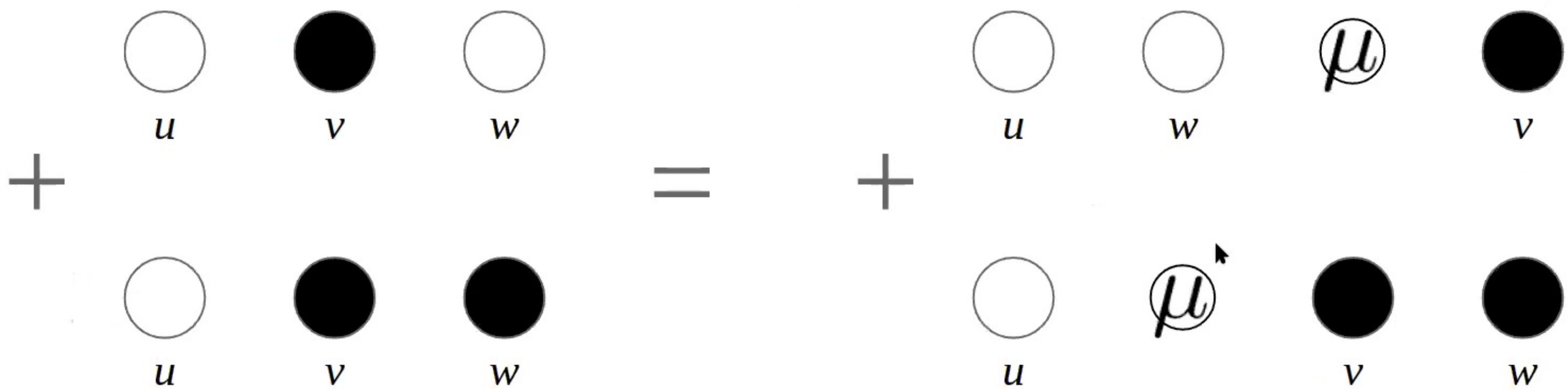
prefer:



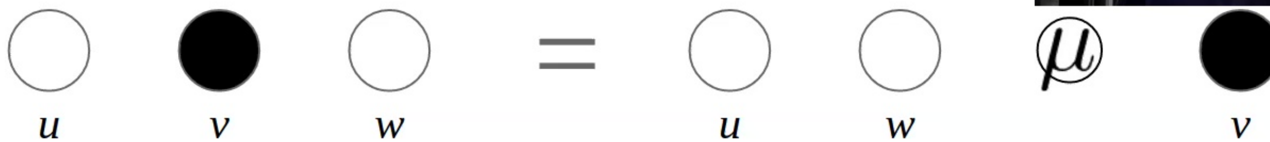
Foreword » **Tensors**



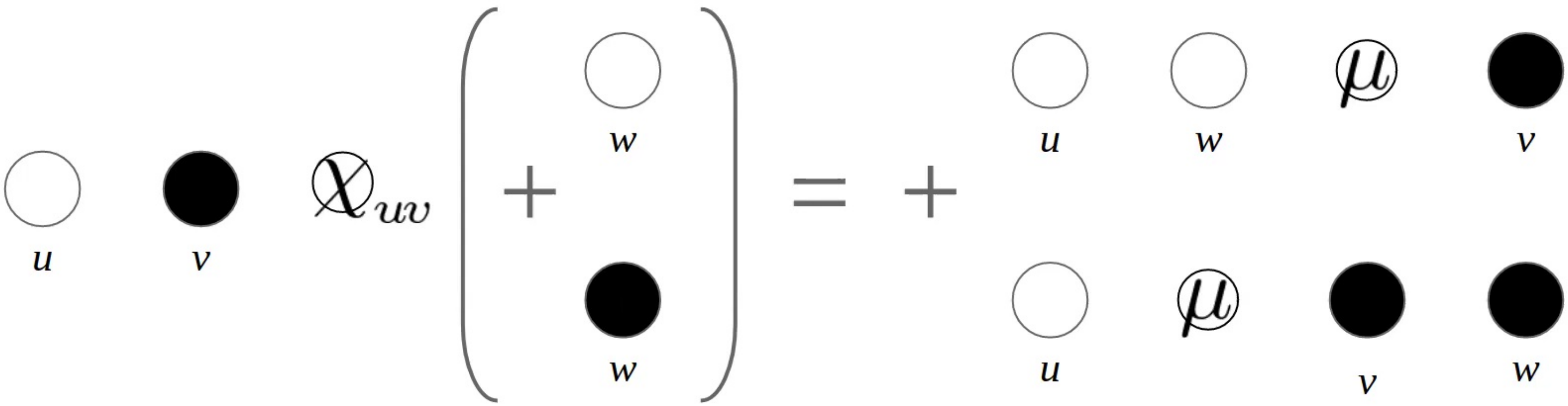
Rk. Because discriminating criterion is quantized, so is the split into subsystems:



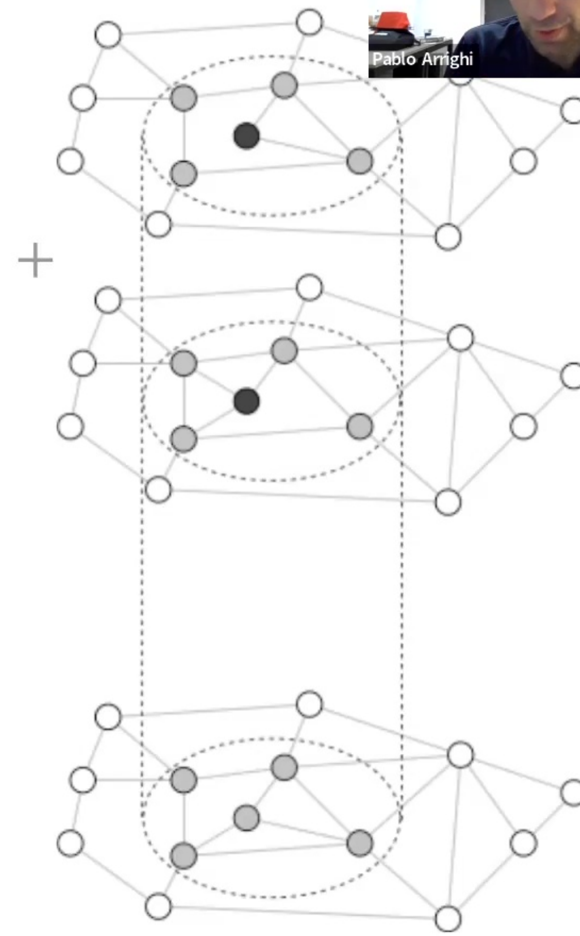
Foreword » **Tensors**

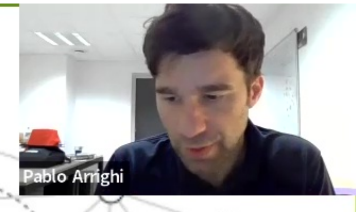


Rk. Entanglement is relative to the choice of χ :



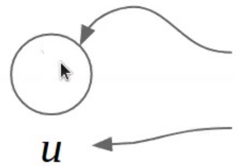
- Generalised tensors & traces
- Why they work.
- Applications?





Gen. tensors & traces » **Hilbert space**

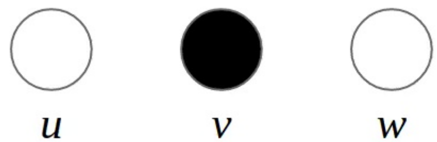
Def. System.



internal state in Σ, some (often finite) set.

unique name in $\mathcal{V}$, a countably infinite set.

Def. Graph.

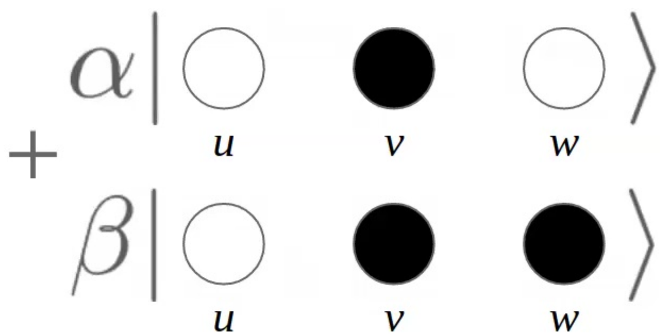


finite but unbounded set of systems.

(edges introduced later in the talk.)

$G \in \mathcal{G}.$

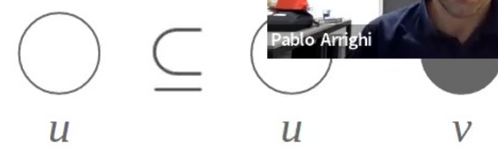
Def. State.

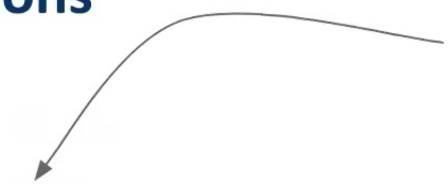


element of the Hilbert space whose o.n.b. is the set of graphs.

$|\psi\rangle \in \mathcal{H}.$

Gen. tensors & traces » **Restrictions**





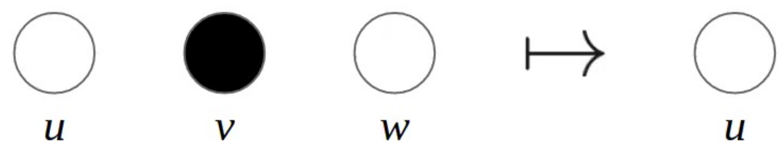
A function $\chi : \mathcal{G} \rightarrow \mathcal{G}$

$$\chi : G \mapsto G_\chi \subseteq G$$

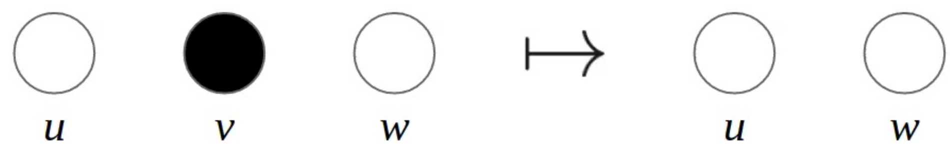
such that $G_\chi \subseteq H \subseteq G \Rightarrow G_\chi = H_\chi$.

Ex.

χ_u :



μ :



Gen. tensors & traces » **Restrictions**

A function

$$\chi : \mathcal{G} \rightarrow \mathcal{G}$$

$$\chi : G \mapsto G_\chi \subseteq G$$

such that $G_\chi \subseteq H \subseteq G \;\Rightarrow\; G_\chi = H_\chi .$

Props.

$G_{\overline{\chi}} = G \setminus G_\chi$

$G_{\chi \cup \zeta} = G_\chi \cup G_\zeta$

- $\chi\chi = \chi, \quad \chi\overline{\chi} = \varnothing$
- If χ and ζ are restrictions, so is $\chi \cup \zeta$.
- *(If χ is a restriction so is χ^r that encompassing r-neighbours in the graphs.)*

Gen. tensors & traces » **Tensors**

A bilinear operator $\otimes : \mathcal{H} \times \mathcal{H} \to \mathcal{H}$

any restriction

internalized

$$|L\rangle \otimes |R\rangle = \begin{cases} |G\rangle & \text{if } L = G_\chi \text{ and } R = G_{\overline{\chi}} \\ 0 & \text{otherwise} \end{cases}$$

Ex.

$$\Big|\underset{u}{\bigcirc}\Big\rangle \otimes_u \Big|\underset{v}{\bullet}\Big\rangle = \Big|\underset{u}{\bigcirc} \quad \underset{v}{\bullet}\Big\rangle$$

$$\Big|\underset{v}{\bullet}\Big\rangle \otimes_u \Big|\underset{u}{\bigcirc}\Big\rangle = 0$$

$$\Big|\underset{u}{\bigcirc}\Big\rangle \otimes_u \Big|\underset{u}{\bigcirc}\Big\rangle = 0$$

Gen. tensors & traces » **Tensors**

Def. $|G\rangle\langle H| \otimes_\chi |G'\rangle\langle H'| := (|G\rangle \otimes_\chi |G'\rangle)(\langle H| \otimes_\chi \langle H'|)$

Def. A is χ-consistent iff $\langle H|A|G_\chi\rangle \neq 0 \Rightarrow |H\rangle \otimes_\chi |G_\chi\rangle \neq 0$
and same holds for $A^\dagger$.

Prop. If A, A' are χ-consistent and B, B' are $\overline{\chi}$-consistent then

$$(A \otimes_\chi B)(A' \otimes_\chi B') = (AA' \otimes_\chi BB')$$

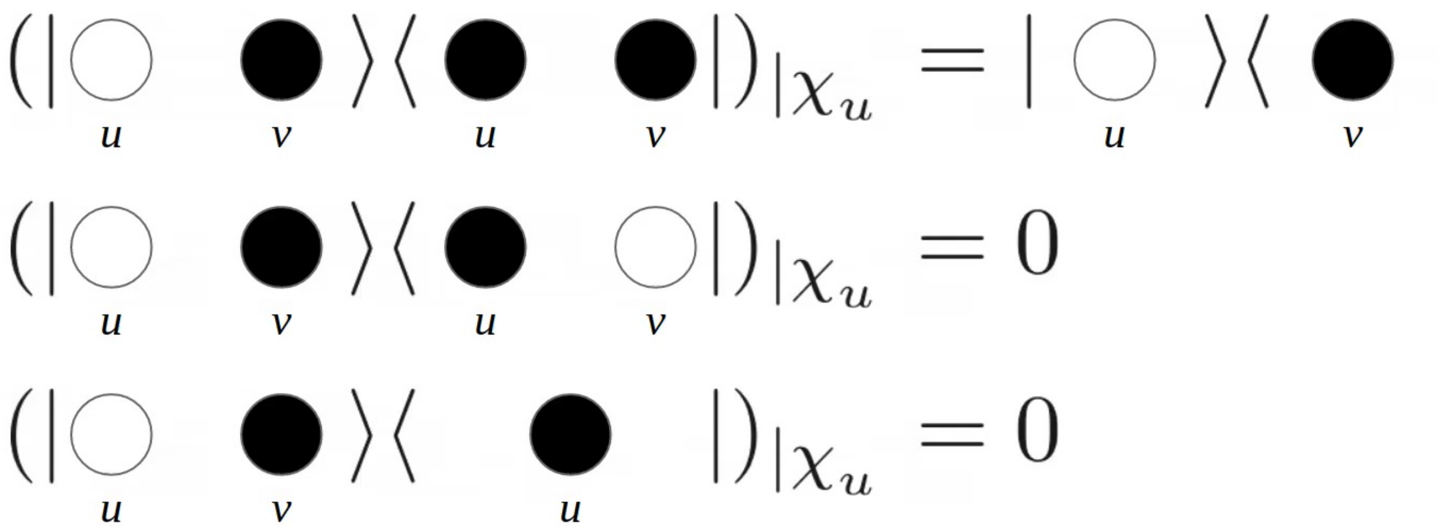
Gen. tensors & traces » **Traceouts**

A linear operator $|_{\chi} : \mathcal{T}(\mathcal{H}) \to \mathcal{T}(\mathcal{H})$

Trace-class operators. Internalized.

$$(|G\rangle\langle H|)_{|\chi} := |G_{\chi}\rangle\langle H_{\chi}|\langle H_{\overline{\chi}}|G_{\overline{\chi}}\rangle$$

Ex.



Gen. tensors & traces » **Traceouts**

A linear operator $|_\chi : \mathcal{T}(\mathcal{H}) \to \mathcal{T}(\mathcal{H})$

$$(|G\rangle\langle H|)_{|\chi} := |G_\chi\rangle\langle H_\chi|\langle H_{\overline{\chi}}|G_{\overline{\chi}}\rangle$$

Prop. $|_\chi$ is a TPCP.

Def. Comprehension.

$$\zeta \sqsubseteq \chi \text{ iff } G_{\chi\zeta} = G_\zeta \text{ and } \langle H_{\overline{\zeta}}|G_{\overline{\zeta}}\rangle = \langle H_{\chi\overline{\zeta}}|G_{\chi\overline{\zeta}}\rangle\langle H_{\overline{\chi}}|G_{\overline{\chi}}\rangle$$

Prop. $\zeta \sqsubseteq \chi \;\Rightarrow\; (\rho_{|\chi})_{|\zeta} = \rho_{|\zeta}$

- Generalised tensors & traces
- Why they work.
- Applications?

Why they work. » **Locality**

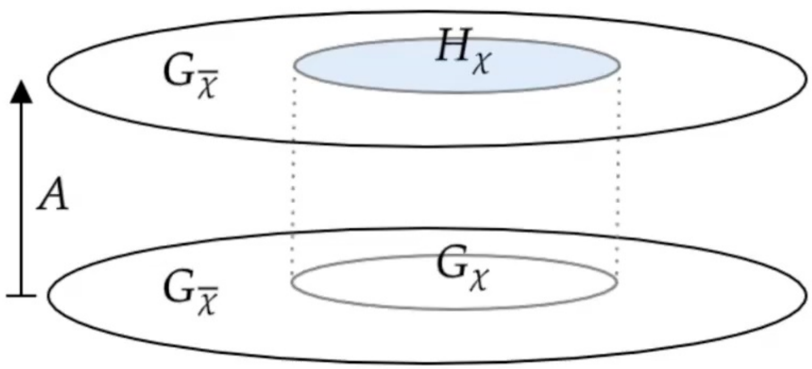
Th.

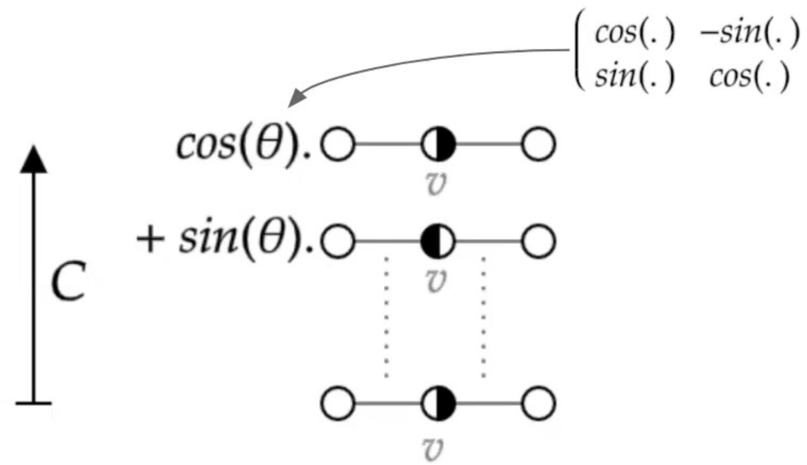
$$\langle H|A|G\rangle = \langle H_\chi|A|G_\chi\rangle\langle H_{\overline{\chi}}|G_{\overline{\chi}}\rangle$$

$$\Leftrightarrow \quad A = A \otimes_\chi I$$

$$\Leftrightarrow \quad \mathrm{Tr}(A\rho) = \mathrm{Tr}(A\rho_{|\chi})$$

$\Bigg\}$ A χ-local



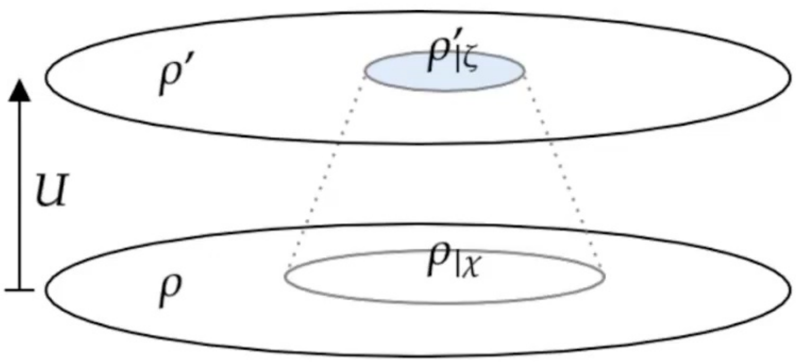


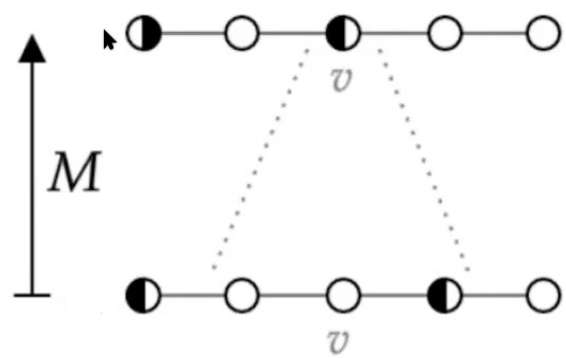
Why they work. » **Causality**

Th. $$(U\rho U^\dagger)_{|\zeta} = (U\rho_{|\chi}U^\dagger)_{|\zeta}$$

$\Leftrightarrow$ U decomposes into χ-local gates.

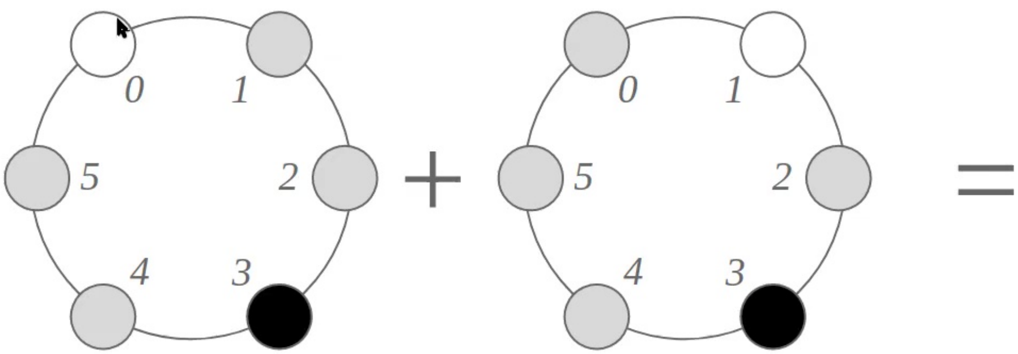
$\Leftrightarrow$ A ζ-local $\Rightarrow$ $UAU^\dagger$ χ-local



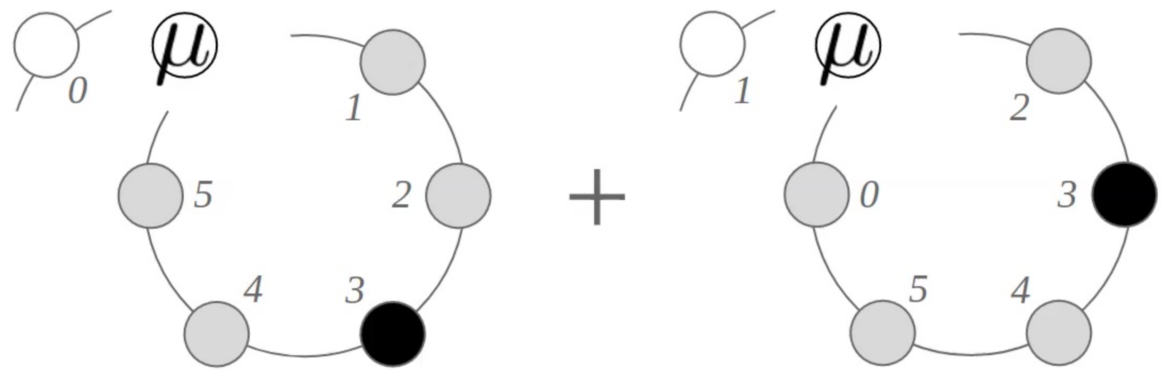


- Generalised tensors & traces
- Why they work.
- Applications?

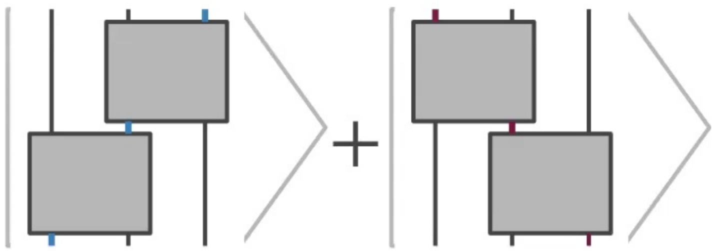
Applications? » **Quantum Reference Frames…**

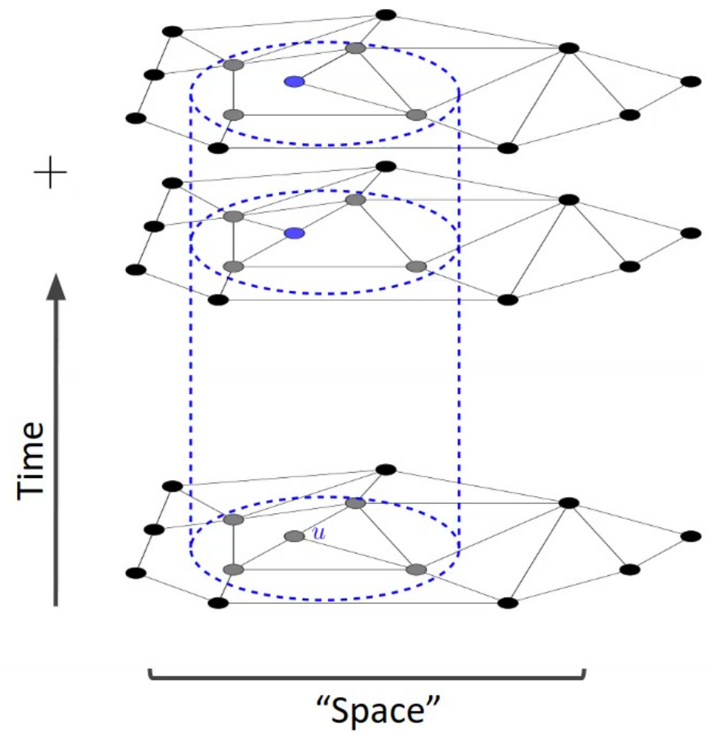


…or simply taking the view of a superposed observer, rather.



Applications? » **Indefinite causal orders & quantum gravity…**





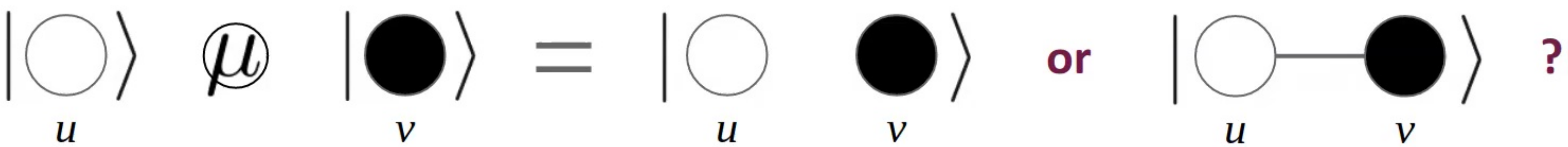
has interaction graph :

… both feature fully quantum networks:

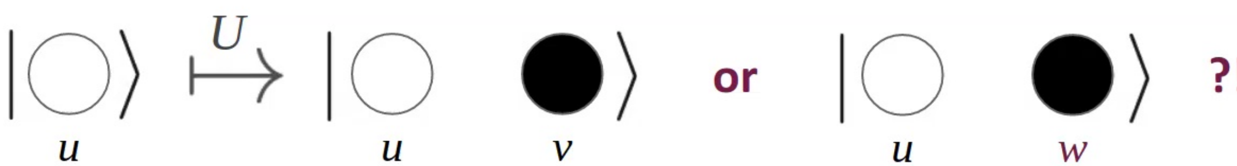
- » They need formalization.
- » Unitarity, locality, causality of evolutions over them are poorly understood.

Applications? » **Fully quantum networks**

Difficulty 1. Splitting through edges.

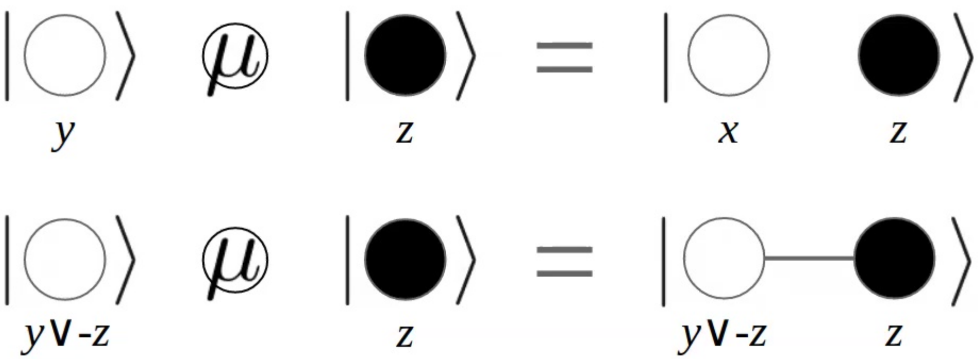


Difficulty 2. Unitarity versus node creation/destruction.



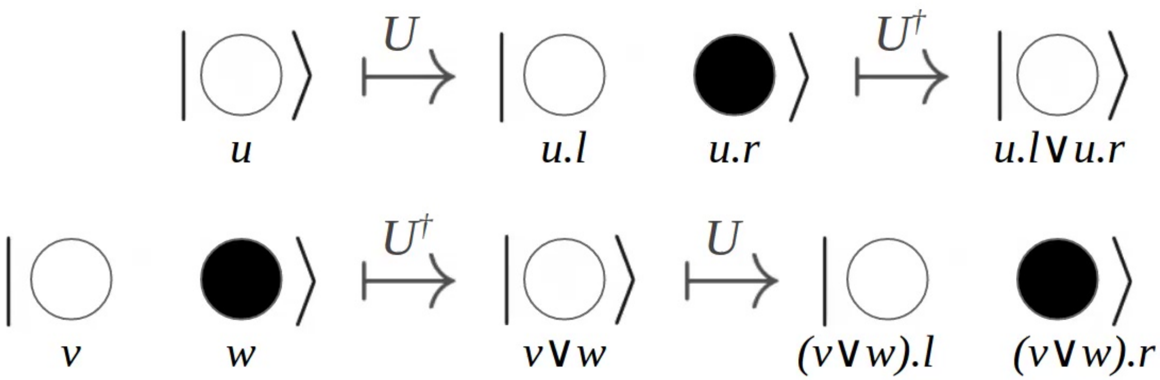
Applications? » Fully quantum networks » **Names hold geome**

~~**Difficulty 1.** Splitting through edges.~~



Applications? » Fully quantum networks » **Name algebra**

~~**Difficulty 2.** Unitarity versus node creation/destruction.~~



Thus, $u.l \vee u.r = u \qquad (v \vee w).l = v \qquad (v \vee w).r = w \quad .$

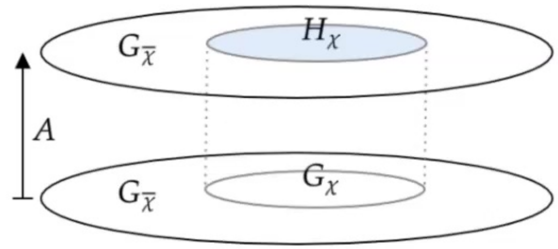
Once these naming conventions adopted… all previous results carries through!

Applications » Fully quantum networks » **Locality**

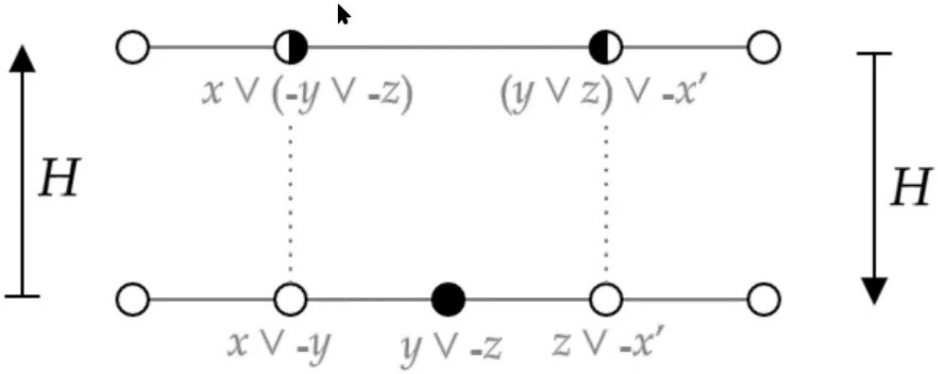
Th. $\langle H|A|G\rangle = \langle H_\chi|A|G_\chi\rangle\langle H_{\overline{\chi}}|G_{\overline{\chi}}\rangle$

$\Leftrightarrow \quad A = A \otimes_\chi I$

$\Leftrightarrow \quad \mathrm{Tr}(A\rho) = \mathrm{Tr}(A\rho_{|\chi})$



Ex.



Applications » Fully quantum networks » **Locality**

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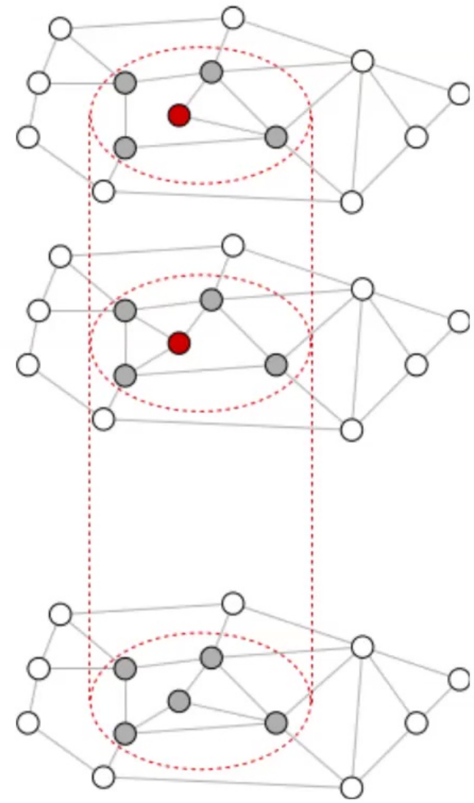

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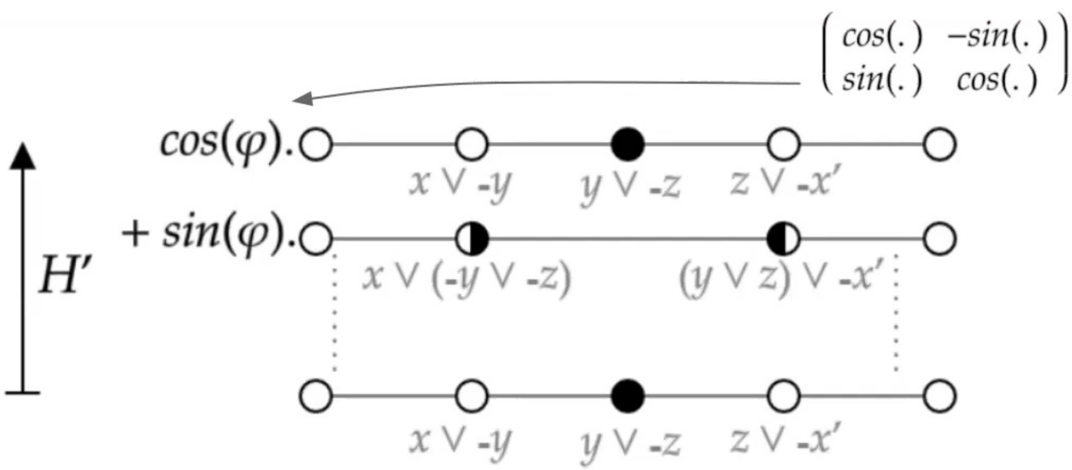

Applications » Fully quantum networks » **Causality**

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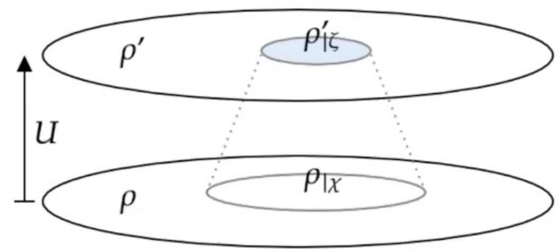

Typically here $\zeta = \chi_u$
And $\chi = \zeta^r = \chi_u^r$.

Prop. Over name-preserving states, $\zeta \sqsubseteq \zeta^r$.

Ex. Combining moves, coins upon walkers, and inflation rule at colliding walkers.

$$U = H'C'M$$

- Generalised tensors & traces
- Why they work.
- Applications?

Summary »

arXiv:211

Generalized tensors & traces »

A more precise notation. Parametrized by any restriction. Internalized.
Entanglement relative to the split.

Why they work. »

Locality, causality in the Shrödinger, Operational and Heisenberg pictures.
Safeguarding their logical interrelations.

Applications »

Taking the viewpoint of a superposed observer.
For the sake of indefinite causal orders or quantum gravity: formalizing fully quantum networks; studying unitary, local and causal evolutions upon them.