Title: The symmetries of the isolated horizons

Speakers: Gaston Giribet

Series: Quantum Gravity

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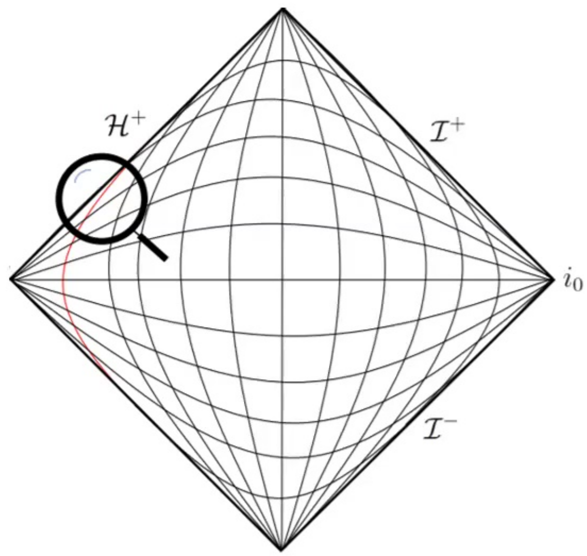
Abstract: In this talk, I will revisit the calculation of infinite-dimensional symmetries that emerge in the vicinity of isolated horizons. In particular, I will focus my attention on extremal black holes, for which the isometry algebra that preserves a sensible set of asymptotic boundary conditions at the horizon strictly includes the BMS algebra. The conserved charges that correspond to this BMS sector, however, reduce to those of superrotation, generating only two copies of Witt algebra. For more general horizon isometries, in contrast, the charge algebra does include both Witt and supertranslations, being similar to BMS but s.str. differing from it. I will also show how this is extended to the case of black holes in the Einstein-Yang-Mills case, where a loop algebra associated to the gauge group is found to emerge at the horizon.

The symmetries of the isolated horizons

Gaston Giribet
University of Buenos Aires



QG Seminar – Perimeter Institute
Online, Thursday 27th January 2022

Based on:

L. Donnay, G. Giribet, H. A. González and M. Pino, Phys. Rev. Lett. **116**, no. 9, 091101 (2016).

L. Donnay, G. Giribet, H. A. González and M. Pino, JHEP **09**, 100 (2016).

L. Donnay, G. Giribet, H. A. González and A. Puhm, Phys. Rev. **D98**, no.12, 124016 (2018).

Motivation:

S. W. Hawking, [arXiv:hep-th/1509.01147].

S. W. Hawking, M. J. Perry and A. Strominger, JHEP **05**, 161 (2017).

S. W. Hawking, M. J. Perry and A. Strominger, Phys. Rev. Lett. **116**, no. 23, 231301 (2016).

Based on collaborations with:

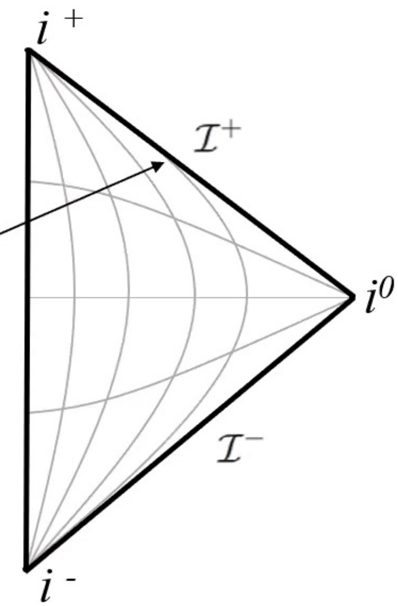
Laura Donnay, Hernán González, Miguel Pino, Andrea Puhm, Luciano Montecchio, Sasha Brenner, Julio Oliva, Andrés Anabalón.

And related work done by:

Glenn Barnich, Daniel Grumiller, Sasha Haco, Pierre-Henry Lambert, Charles Marteau, Sabrina Pasterski, Robert Penna, Ricardo Troncoso, Shahin Sheikh Jabbari, Andrew Strominger, Herman Verlinde, Céline Zwikel, and many others.

BMS symmetry

Infinite-dimensional symmetry at null intinity



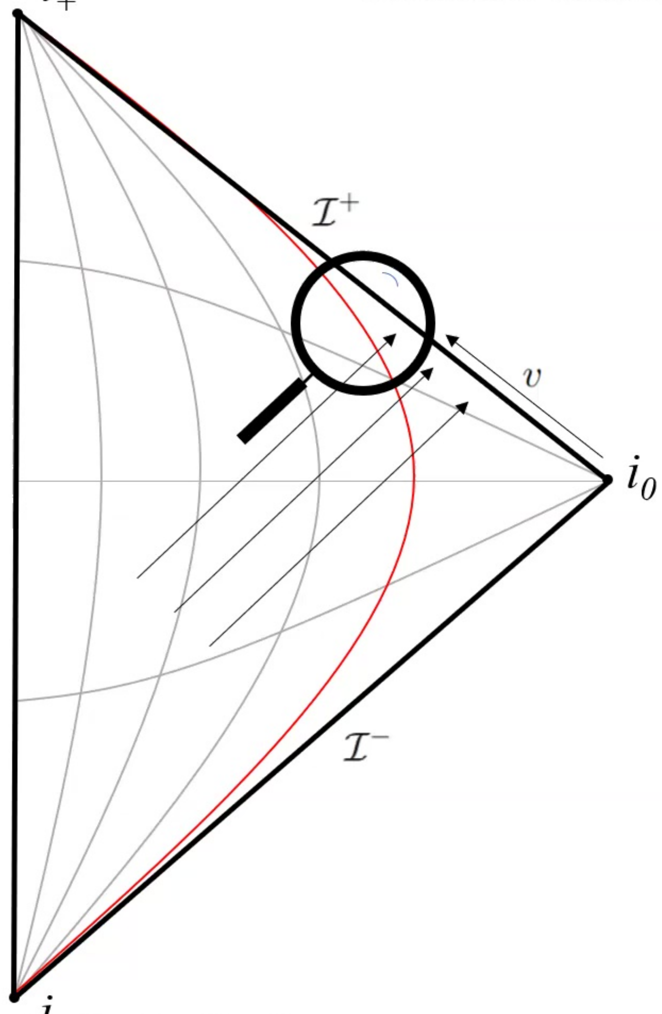
Classics:

. Bondi, van de Burg & Metzner – Sachs, 1960's: Gravitational waves in GR in asymptotically flat spacetime

Recent:

. Infinite-dimensional (superrotation) extension of BMS [Barnich & Compère, 2006]
. BMS/CFT correspondence [Barnich & Troessaert, 2010]
. BMS and memory gravitational effect [Strominger & Zhiboedov, 2014]
. BMS, S-matrix and soft theorems [He, Lysov, Mitra & Strominger, 2014]
. Supertranslation at the black hole horizon [Hawking, Perry & Strominger, 2016]

Infinite-dimensional symmetry at null intinity



$$g_{vv} = (-1 + \mathcal{O}(1/r))e^{2\beta} + g_{AB}U^A U^B \qquad A, B = \theta, \phi$$
$$g_{vr} = -e^{2\beta} \qquad \beta \simeq \mathcal{O}(1/r^2)$$
$$g_{vA} = -g_{AB}U^B \qquad U^A \simeq \mathcal{O}(1/r^2)$$
$$g_{AB} = r^2\gamma_{AB} + \mathcal{O}(r) \qquad \det(g_{AB}) = \det(\gamma_{AB})$$

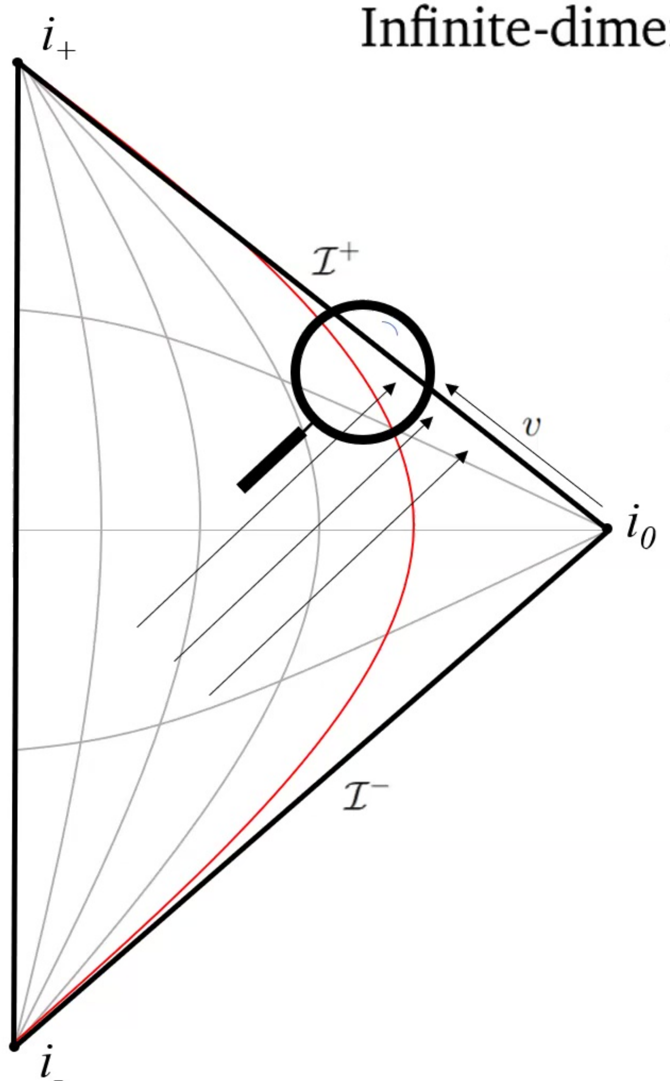
- Gravitational waves in asymptotically flat spacetimes [Bondi, van de Burg & Metzner 1962, Sachs 1962]

$$\delta_\xi g_{\mu\nu} = \mathcal{L}_\xi g_{\mu\nu}$$

$$\xi = P(z, \bar{z})\, \partial_v$$

$$\infty$$

Infinite-dimensional symmetry at null intinity



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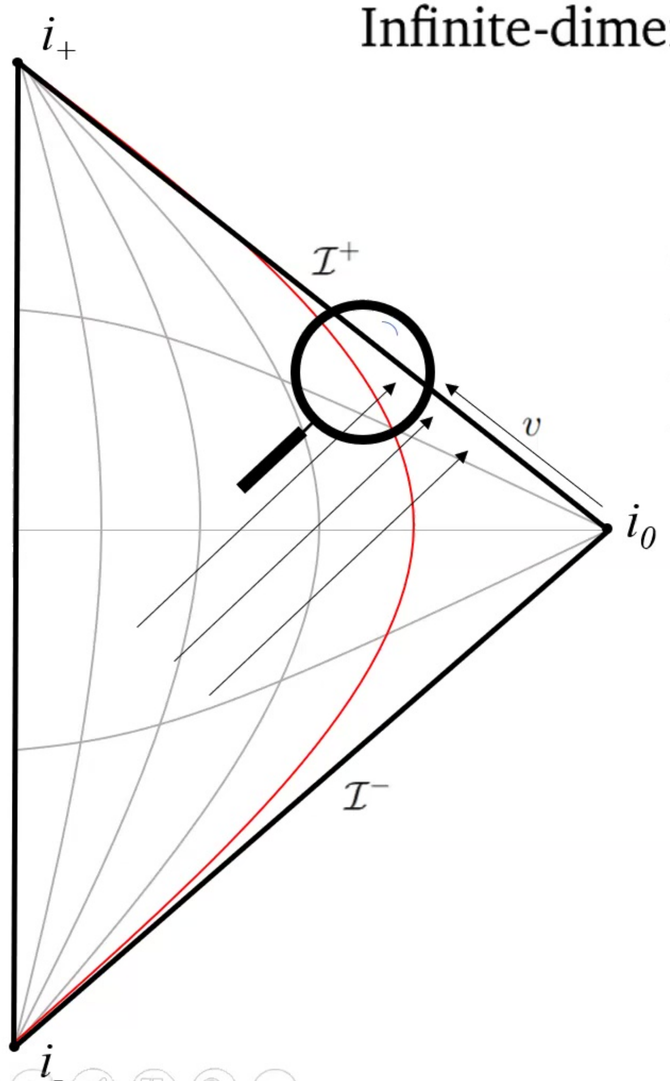
- Gravitational waves in asymptotically flat spacetimes [Bondi, van de Burg & Metzner 1962, Sachs 1962]
- Classical central extension for asymptotic symmetries at null infinity [Barnich & Compère, 2006]
- Conformal symmetries of gravity from asymptotic methods [Lambert, 2014]
- Supertranslations call for superrotations [Barnich & Troessaert, 2011]
- Aspects of BMS / CFT correspondence [Barnich & Troessaert, 2010]

Infinite-dimensional symmetry at null intinity



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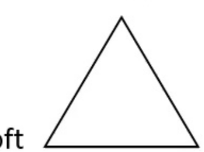
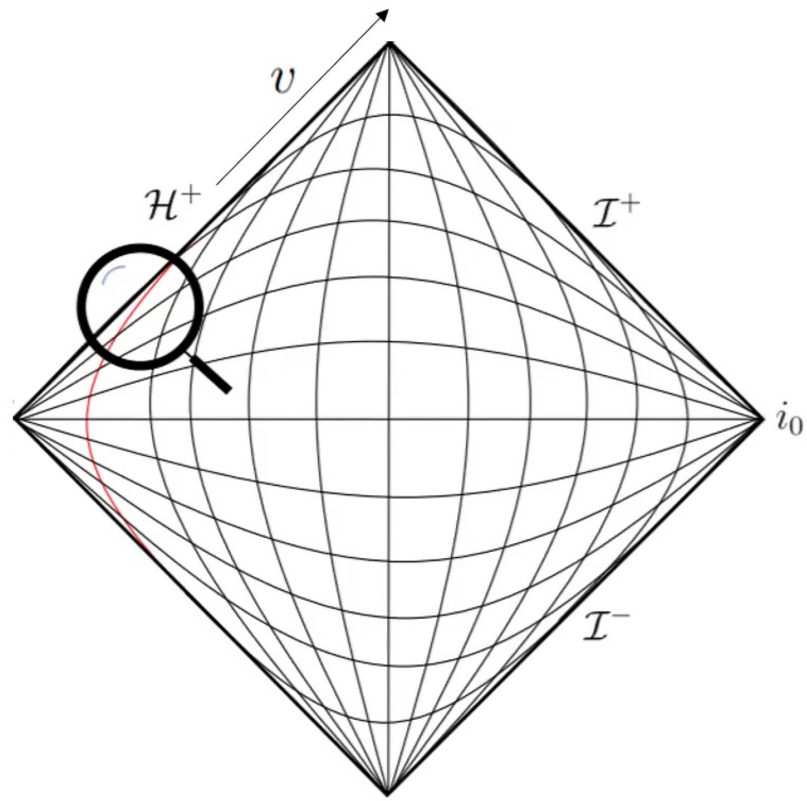
- A. Strominger, JHEP **07**, 152 (2014).
- T. He, V. Lysov, P. Mitra and A. Strominger, JHEP **05**, 151 (2015).
- A. Strominger and A. Zhiboedov, JHEP **01**, 086 (2016).
- A. Strominger, [arXiv:1703.05448].



The symmetries of the horizon



Cfr. $\mathrm{AdS}_3/\mathrm{CFT}_2$

The isometries of AdS_3 are *part* of the conformal symmetry of the 1+1 dimensional field theory. It would be interesting to understand what is the gravitational counterpart of the full conformal symmetry group in 1+1 dimensions. [Maldacena, 1997]

The asymptotic symmetry group of three-dimensional gravity with a negative cosmological constant is the *local* conformal group in 1+1 dimensions with a nontrivial central extension, which turns out to be the Virasoro central charge. [Brown & Henneaux, 1986]

$$g_{tt} \simeq \frac{r^2}{\ell^2} + \mathcal{O}(1) \quad g_{tt} \simeq \frac{\ell^2}{r^2} + \mathcal{O}(1/r^4) \quad g_{\phi\phi} \simeq r^2 + \mathcal{O}(1) \quad g_{t\phi} \simeq \mathcal{O}(1)$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}\delta_{m+n,0}n(n^2-1)$$
$$[\bar{L}_m, \bar{L}_n] = (m-n)\bar{L}_{m+n} + \frac{c}{12}\delta_{m+n,0}n(n^2-1)$$
$$c = \frac{3\ell}{2G}$$

$$so(2,2) \simeq sl(2,\mathbb{R}) \oplus sl(2,\mathbb{R}) \in Vir \oplus Vir$$

The infinite symmetries of black hole horizons

Let us start by considering the near horizon geometry of stationary black holes. Close to the event horizon, we can always consider the spacetime metric in the form

$$ds^2 = -2\kappa\,\rho\,dv^2 + 2\,d\rho dv + 2N_A\,\rho\,dz^A dv + \Omega_{AB}\,dz^A dz^B + ...$$

where the ellipsis stand for subleading terms. z^A with $A = 1,2$ represents coordinates on constant-v slices of the horizon, $v \in \mathbb{R}$ being the advanced time (null) coordinate. $\rho \in \mathbb{R}_{\geq 0}$ measures the distance from the horizon, which is located at $\rho = 0$. Functions N_A and Ω_{AB} depend on the two coordinates z^A, and in principle they might depend on time as well . The specific form of the subleading terms is given by

$$g_{vv} = -2\kappa\,\rho + \mathcal{O}(\rho^2)\,, \quad g_{vA} = N_A(v, z^B)\,\rho + \mathcal{O}(\rho^2)\,, \quad g_{AB} = \Omega_{AB}(v, z^C) + \mathcal{O}(\rho)\,,$$

where $\mathcal{O}(\rho^n)$ stand for functions on the coordinates whose dependence with ρ damps off at least as fast as $\sim \rho^n$ when ρ tends to zero.

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The infinite symmetries of black hole horizons

Now, let us study the diffeomorphisms that preserve such a near horizon form of the metric. To do that, we compute the Lie derivative $\delta_\xi g_{\mu\nu} = \mathcal{L}_\xi g_{\mu\nu}$ with respect to a vector field $\xi = \xi^\mu \partial_\mu$, and demand it to preserve the boundary conditions at the horizon

$$
\begin{aligned}
\xi^v &= T + \mathcal{O}(\rho), \\
\xi^\rho &= -\partial_v T\,\rho + \frac{1}{2}\Omega^{AB} N_A\,\partial_B T\,\rho^2 + \mathcal{O}(\rho^3), \\
\xi^A &= L^A + \Omega^{AB}\,\partial_B T\,\rho + \mathcal{O}(\rho^2),
\end{aligned}
$$

where T is a function on v and z^A, and L^A is a function of z^A. The expressions can be considerably simplified if we consider the conformal gauge, which implies that L^A are conformal Killing vectors on the 2-sphere: $\partial_{\bar{z}} L^z = \partial_z L^{\bar{z}} = 0$, with z, $\bar{z}$ being holomorphic and anti-holomorphic coordinates on the spacelike sections of the horizon. Generalization to the full Diff(S^2) symmetry is also possible

The infinite symmetries of black hole horizons

Variations generate a Lie algebra realized by the application

$$[\delta_{\xi_1}, \delta_{\xi_2}] g_{\mu\nu} = \delta_{\hat{\xi}} g_{\mu\nu}.$$

with

$$\hat{\xi} = [\xi_1, \xi_2] + \delta_{\xi 2}\xi_1 - \delta_{\xi 1}\xi_2 \, ,$$

which suffices to take into account the dependence of ξ on the metric functions. In this way, we find the following algebra of diffeomorphisms

$$\hat{T} = T_1 \partial_v T_2 + L_1^A \partial_A T_2 - T_2 \partial_v T_1 - L_2^A \partial_A T_1 \, ,$$
$$\hat{L}^A = L_1^B \partial_B L_2^A - L_2^B \partial_B L_1^A \, .$$

Symmetry algebra

Let us represent the asymptotic Killing vector as $\chi = \chi(T, L^{z}, L^{\bar{z}})$. By defining the Fourier modes, $T_{(m,n)} = \chi(z^m \bar{z}^n, 0, 0, 0)$, $Y_n = \chi(0, 0, -z^{n+1}, 0)$, $\bar{Y}_n = \chi(0, 0, 0, -\bar{z}^{n+1})$ we find

$$
\begin{aligned}
[Y_m, Y_n] &= (m-n) Y_{m+n}, \\
[\bar{Y}_m, \bar{Y}_n] &= (m-n) \bar{Y}_{m+n}, \\
[Y_k, T_{(m,n)}] &= -m T_{(m+k,n)}, \\
[\bar{Y}_k, T_{(m,n)}] &= -n T_{(m,n+k)},
\end{aligned}
$$

where $\quad T(z,\bar{z}) = \sum_{n,m\in\mathbb{Z}} T_{(m,n)}\, z^n\, \bar{z}^m \ , \quad L^{z}(z) = \sum_{n\in\mathbb{Z}} Y_n\, z^n \ , \quad L^{\bar{z}}(\bar{z}) = \sum_{n\in\mathbb{Z}} \bar{Y}_n\, \bar{z}^n$

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\end{aligned}\right\} \;(\text{Witt} \oplus \text{Witt}) + \hat{u}(1)_0$$

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where $T(z, \bar{z}) = \sum_{n,m \in \mathbb{Z}} T_{(m,n)} z^n \bar{z}^m$, $\quad L^z(z) = \sum_{n \in \mathbb{Z}} Y_n z^n$, $\quad L^{\bar{z}}(\bar{z}) = \sum_{n \in \mathbb{Z}} \bar{Y}_n \bar{z}^n$

Noether charges

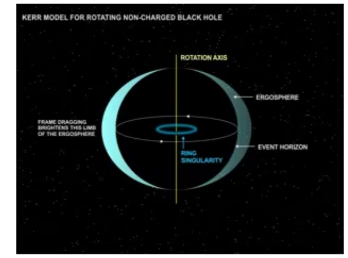
These Noether charges can be computed

$$Q[T, L^A] = \frac{1}{16\pi G} \int d^2z \sqrt{\det \Omega_{AB}} \left(2T\kappa - L^A N_A\right) + Q_0$$

Integrability: $\Omega_{AB} = \Omega\gamma_{AB}, \quad \gamma_{AB} dx^A dx^B = \frac{4}{(1+z\bar{z})^2} dz d\bar{z}, \quad \Omega^{-1}\partial_v \Omega = 2A(\Omega)$

Charge algebra: $\{Q_{\xi_1}[\Phi], Q_{\xi_2}[\Phi]\} = \delta_{\xi_2} Q_{\xi_1}[\Phi] = -\delta_{\xi_1} Q_{\xi_2}[\Phi]$

Stationary black holes



The Kerr metric written in the Eddington-Finkelstein coordinates is given by

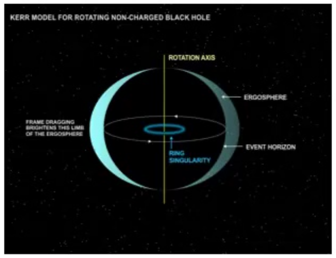
$$ds^2 = \left(\frac{\Delta - \Xi}{\Sigma} - 1\right) dv^2 + 2\ dv\ dr - \frac{2a(\Xi - \Delta)\sin^2\theta}{\Sigma} dv\ d\varphi - $$
$$- 2a \sin^2\theta\ dr\ d\phi + \Sigma d\theta^2 + \frac{(\Xi^2 - a^2 \Delta \sin^2\theta)\sin^2\theta}{\Sigma} d\varphi^2,$$

where the functions Δ, Ξ, and Σ are given by

$$\Delta(r) = r^2 - 2GMr + a^2\ , \quad \Xi(r) = r^2 + a^2\ , \quad \Sigma(r) = r^2 + a^2\cos^2\theta,$$

where M is the mass and a is the angular momentum per unit of mass. The outer horizon of the Kerr black hole is located at $r_+ = GM + \sqrt{G^2M^2 - a^2}$.

Stationary black holes



Close to the event horizon,

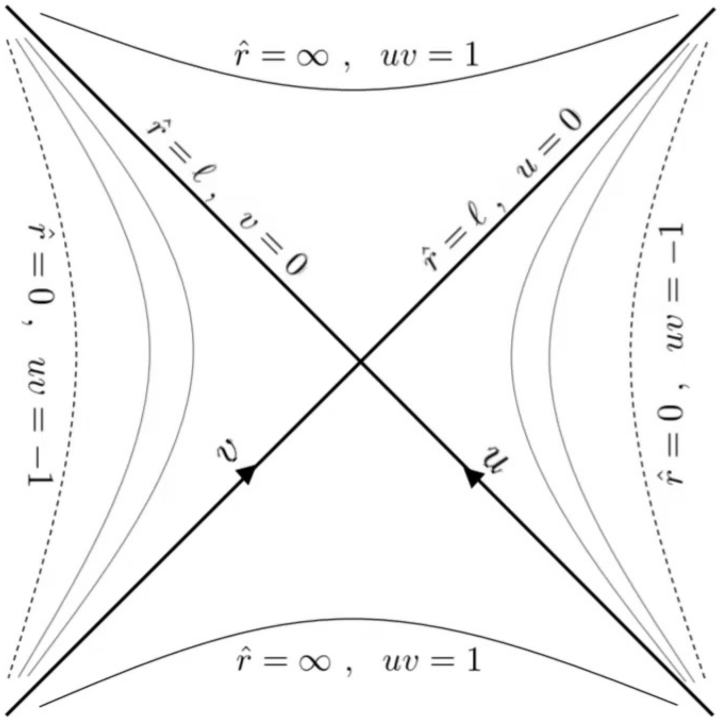
$$ds^2 = -2\kappa\,\rho\,dv^2 + 2\,d\rho dv + 2N_A\,\rho\,dz^A dv + \Omega_{AB}\,dz^A dz^B + ...$$

where $\kappa = -\dfrac{\Delta'(r_+)}{2\Xi(r_+)}, \quad \Omega_{\theta\theta} = \Sigma(r_+), \quad \Omega_{\theta\varphi} = 0, \quad \Omega_{\varphi\varphi} = \dfrac{\Xi^2(r_+)\sin^2\theta}{\Sigma(r_+)},$

$$N_\theta = \frac{2a^2\sin\theta\cos\theta}{\Sigma(r_+)}, \quad N_\varphi = -\left(\frac{a\Delta'(r_+)\sin^2\theta}{\Sigma(r_+)} + \frac{2ar_+\Xi(r_+)\sin^2\theta}{\Sigma^2(r_+)}\right),$$

$$\boxed{Q[1,0] = \frac{\kappa}{2\pi}\,\frac{\Delta\mathcal{A}}{4G} \quad , \quad Q[0,1] = Ma}$$

Cosmological horizons



de Sitter space

$$Q[1,0] = \frac{\kappa}{2\pi}\frac{\mathcal{A}}{4G} = TS$$

$$\mathcal{A} = H^{-2}\int d^2z\,\sqrt{\det\gamma_{AB}}$$

$$T = \frac{\kappa}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\Lambda}{3}}$$

- ✓ Stationary black holes and de Sitter space
- ✓ The Schwarzschild-Taub-NUT/Bolt-de Sitter cosmological horizon
- ✓ Charged, stationary black holes
- ✓ Horizon charges for magnetized black holes
- ✓ Gravitating monopoles in external magnetic field
- ✓ Rindler horizons
- ✓ The near horizon symmetries of the C-metric
- ✓ Accelerated black holes in AdS
- ✓ Black hole binary system
- ✓ Non-Abelian horizons

Higher-derivative corrections

The effective theory

$$I = \frac{1}{16\pi G} \int d^D x \sqrt{|g|} \left(R - 2\Lambda + \alpha_1 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \alpha_2 R_{\mu\nu} R^{\mu\nu} + \alpha_3 R^2 + \ldots \right)$$

With a near horizon expansion of the form

$$g_{AB} = \Omega_{AB}(z^C) \, + \, \lambda_{AB}(v, z^C)\, \rho + \, \theta_{AB}(v, z^C)\, \rho^2 + \ldots$$

The Noether charges take the form

$$Q \, = \, Q(\Omega_{AB}, \lambda_{AB})$$

This capures the higher-curvature corrections to the area law

$$Q[T=1] = \frac{\kappa}{2\pi} \frac{\mathcal{A}}{4G} + \, \mathcal{O}(\alpha_i / G)$$

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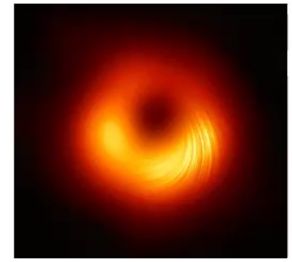
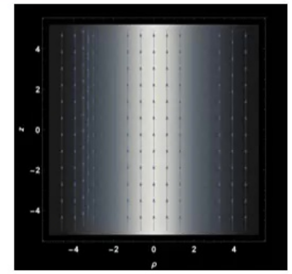
Magnetized horizons



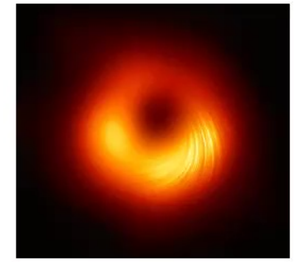


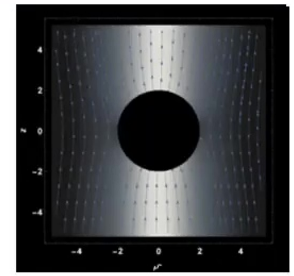
$$ds^2 = |\Lambda(r,\theta)|^2\Big(-f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\theta^2\Big) + |\Lambda(r,\theta)|^{-2} r^2 \sin^2\theta\,(d\phi - \omega(r,\theta)dt)^2$$

$$\begin{aligned}
\Lambda(r,\theta) &= 1 + \frac{1}{4}B_0^2(r^2\sin^2\theta + q^2\cos^2\theta) - iB_0 q\cos\theta\,,\\
f(r) &= 1 - \frac{2M}{r} + \frac{q^2}{r^2}\\
\omega(r,\theta) &= B_0 q\Big[-\Big(\frac{2}{r} - \frac{2}{r_+}\Big) + \frac{B_0^2}{2}\Big(r - r_+ + r f(r)\cos^2\theta\Big)\Big] + \omega_0\,,
\end{aligned}$$

$$\begin{aligned}
A_\phi(r,\theta) &= \frac{1}{B_0}\Big[1 + \Big(\frac{\mathrm{Re}(\Lambda(r,\theta))}{|\Lambda(r,\theta)|}\Big)\Big(\frac{\mathrm{Re}(\Lambda(r,\theta)) - 2}{|\Lambda(r,\theta)|}\Big)\Big],\\
A_t(r,\theta) &= \frac{2q}{r} + \frac{3\omega(r,\theta)}{2B_0} - A_\phi(r,\theta)\omega(r,\theta),
\end{aligned}$$

Magnetized horizons





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$$\Lambda(r,\theta) = 1 + \frac{1}{4}B_0^2(r^2\sin^2\theta + q^2\cos^2\theta) - iB_0 q\cos\theta\,,$$

$$f(r) = 1 - \frac{2M}{r} + \frac{q^2}{r^2}$$

$$\omega(r,\theta) = B_0 q\Big[-\Big(\frac{2}{r} - \frac{2}{r_+}\Big) + \frac{B_0^2}{2}\Big(r - r_+ + r f(r)\cos^2\theta\Big)\Big] + \omega_0\,,$$

$$A_\phi(r,\theta) = \frac{1}{B_0}\Big[1 + \Big(\frac{\mathrm{Re}(\Lambda(r,\theta))}{|\Lambda(r,\theta)|}\Big)\Big(\frac{\mathrm{Re}(\Lambda(r,\theta)) - 2}{|\Lambda(r,\theta)|}\Big)\Big],$$

$$A_t(r,\theta) = \frac{2q}{r} + \frac{3\omega(r,\theta)}{2B_0} - A_\phi(r,\theta)\omega(r,\theta),$$

Magnetized horizons

$$ds^2 = |\Lambda(r,\theta)|^2\Big(-f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\theta^2\Big) + |\Lambda(r,\theta)|^{-2} r^2 \sin^2\theta\,(d\phi - \omega(r,\theta)dt)^2$$

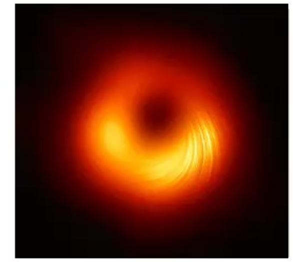
$$\Lambda(r,\theta) = 1 + \frac{1}{4}B_0^2(r^2\sin^2\theta + q^2\cos^2\theta) - iB_0 q\cos\theta\,,$$

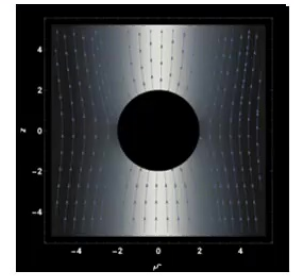
$$f(r) = 1 - \frac{2M}{r} + \frac{q^2}{r^2}$$

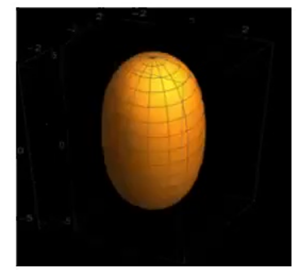
$$\omega(r,\theta) = B_0 q\Big[-\Big(\frac{2}{r} - \frac{2}{r_+}\Big) + \frac{B_0^2}{2}\Big(r - r_+ + r f(r)\cos^2\theta\Big)\Big] + \omega_0\,,$$

$$A_\phi(r,\theta) = \frac{1}{B_0}\Big[1 + \Big(\frac{\mathrm{Re}(\Lambda(r,\theta))}{|\Lambda(r,\theta)|}\Big)\Big(\frac{\mathrm{Re}(\Lambda(r,\theta)) - 2}{|\Lambda(r,\theta)|}\Big)\Big],$$

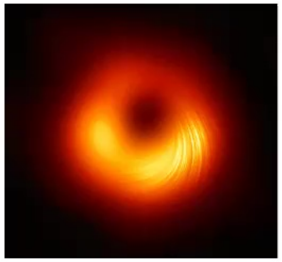
$$A_t(r,\theta) = \frac{2q}{r} + \frac{3\omega(r,\theta)}{2B_0} - A_\phi(r,\theta)\omega(r,\theta),$$

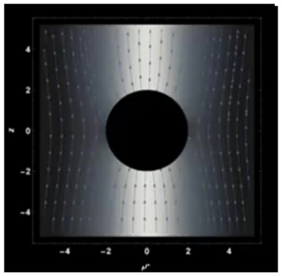


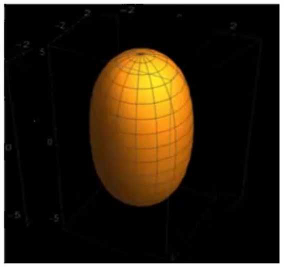




Magnetized horizons







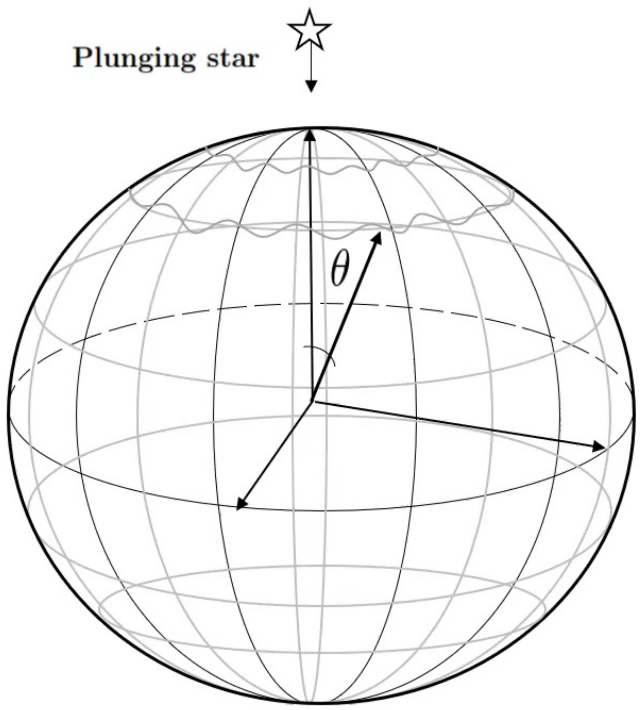
$$ds^2 = |\Lambda(r,\theta)|^2\Big(-f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\theta^2\Big) + |\Lambda(r,\theta)|^{-2} r^2 \sin^2\theta\,(d\phi - \omega(r,\theta)dt)^2$$

$$Q[T=1] = \frac{1}{2}\Big(1 + \frac{3}{2}B_0^2 q^2 + \frac{1}{16}B_0^4 q^4\Big)\sqrt{M^2 - q^2} \quad = \frac{\kappa}{2\pi}\,\frac{\mathcal{A}}{4}$$

$$Q[U=1] = q\Big(1 - \frac{B_0^2 q^2}{4}\Big)$$

$$Q[\,L^A = 1] = \; -q^3 B_0\Big(1 + \frac{1}{4}q^3 B_0\Big)$$

Black hole memory effect



$$ds^2 = r_+^2(d\theta^2 + \sin^2\theta d\phi^2) \simeq r_+^2(d\theta^2 + \theta^2 d\phi^2)$$

$$\Downarrow$$

$$ds^2 = r_+^2(d\theta^2(1-\epsilon(\theta)) + \theta^2(1+\epsilon(\theta))d\phi^2) = r_+^2(d\tilde{\theta}^2 + \tilde{\theta}^2 d\phi^2)$$

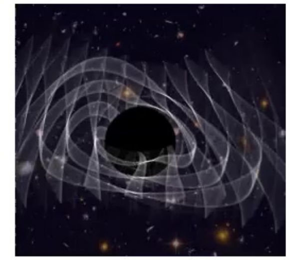
$$\boxed{\theta \to \tilde{\theta} = \theta - \frac{1}{2}\theta\epsilon(\theta)}$$

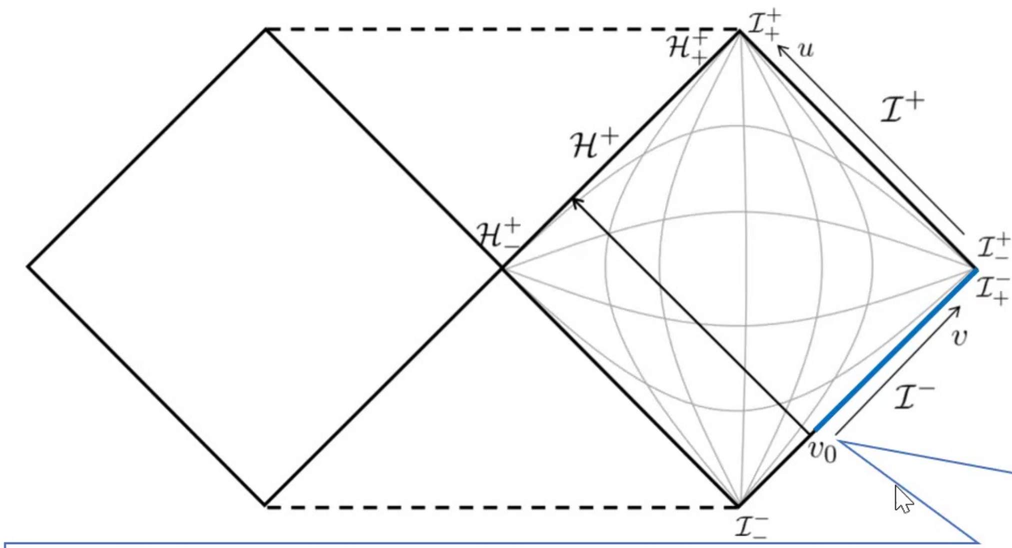
It's a superrotation.

Gravitational perturbations of the horizon
[Suen, Price & Redmount 1988]

[Thorne, Price & Macdonald 1986]

Black hole memory effect

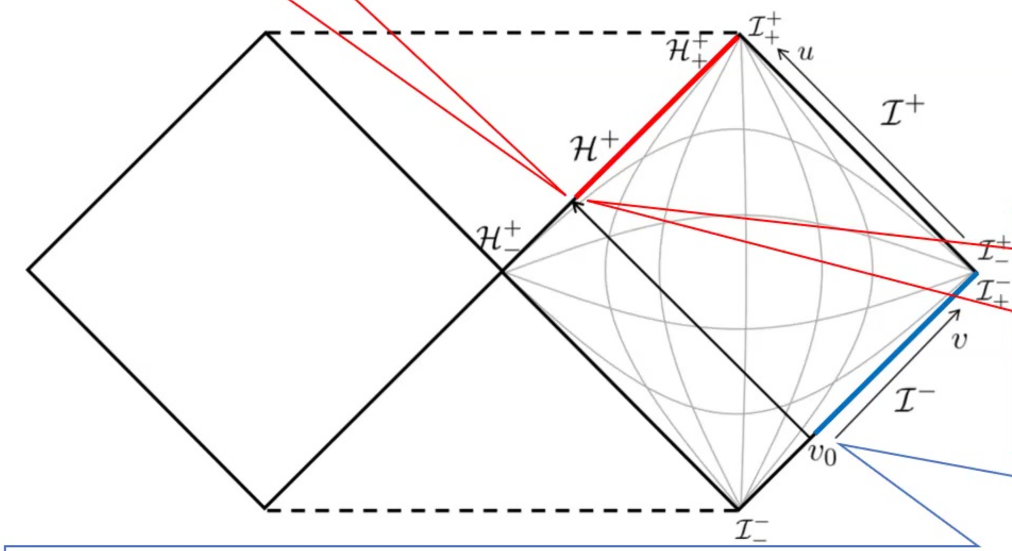


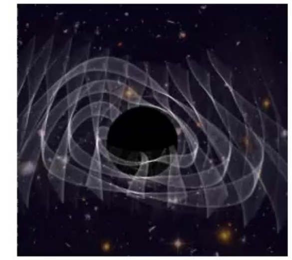


$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \left(\frac{2M}{r} - 1 + \frac{M}{r^2}D^2 f\right)dv^2 + 2dvdr - D_A\left(2f - \frac{4M}{r}f + D^2 f\right)dvdz^A$$
$$+ \left(r^2\gamma_{AB} + 2rD_AD_Bf - r\gamma_{AB}D^2 f\right)dz^A dz^B \,.$$

Black hole memory effect

$$L_A = \frac{1}{r_+} D_A f$$





$$ds^2 = -\frac{1}{r_+}\rho dv^2 + 2dvd\rho - \frac{2}{r_+}\rho D_A f dz^A dv + (r_+^2\gamma_{AB} + 2r_+ D_A D_B f) dz^A dz^B + \dots$$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \left(\frac{2M}{r} - 1 + \frac{M}{r^2}D^2 f\right)dv^2 + 2dvdr - D_A\left(2f - \frac{4M}{r}f + D^2 f\right)dvdz^A + \left(r^2\gamma_{AB} + 2rD_A D_B f - r\gamma_{AB}D^2 f\right)dz^A dz^B \,.$$

Non-Abelian horizons

$$I = \frac{1}{16\pi G}\int_{\mathcal{M}} d^4x\sqrt{|g|}\,R \,-\, \frac{1}{4}\int_{\mathcal{M}} d^4x\sqrt{|g|}\,\mathrm{Tr}F^2$$

We are interested in Einstein gravity coupled to Yang-Mills theory, and therefore we have to introduce, in addition to the metric, the non-Abelian gauge field A^a_μ, which defines the gauge connection 1-form

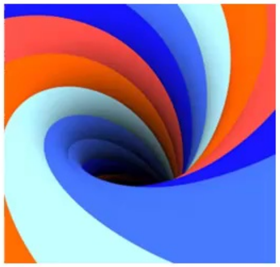
$$A = A^a_\mu\, T_a\, dx^\mu\ ,$$

with T_a being the generators of a Lie algebra g (with $a = 1, 2, ...,$ dim(g)), which satisfy the Lie product

$$[T_b, T_c] = i f^a_{bc} T_a$$

with f^a_{bc} being the structure constants. Let G be a compact, semisimple Lie group generated by g, which enters in the definition of the gauge theory through the standard building blocks: we have the covariant derivative $D_\mu = \partial_\mu - i\alpha A^a_\mu T_a$ and the field strength $F^a = F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \alpha f^a_{bc} A^b_\mu A^c_\nu$

Non-Abelian horizons



$$I = \frac{1}{16\pi G}\int_{\mathcal{M}} d^4x\sqrt{|g|}\,R - \frac{1}{4}\int_{\mathcal{M}} d^4x\sqrt{|g|}\,\mathrm{Tr}F^2$$

$$[Y_m, Y_n] = (m-n)Y_{m+n}, \qquad [\bar{Y}_m, \bar{Y}_n] = (m-n)\bar{Y}_{m+n},$$

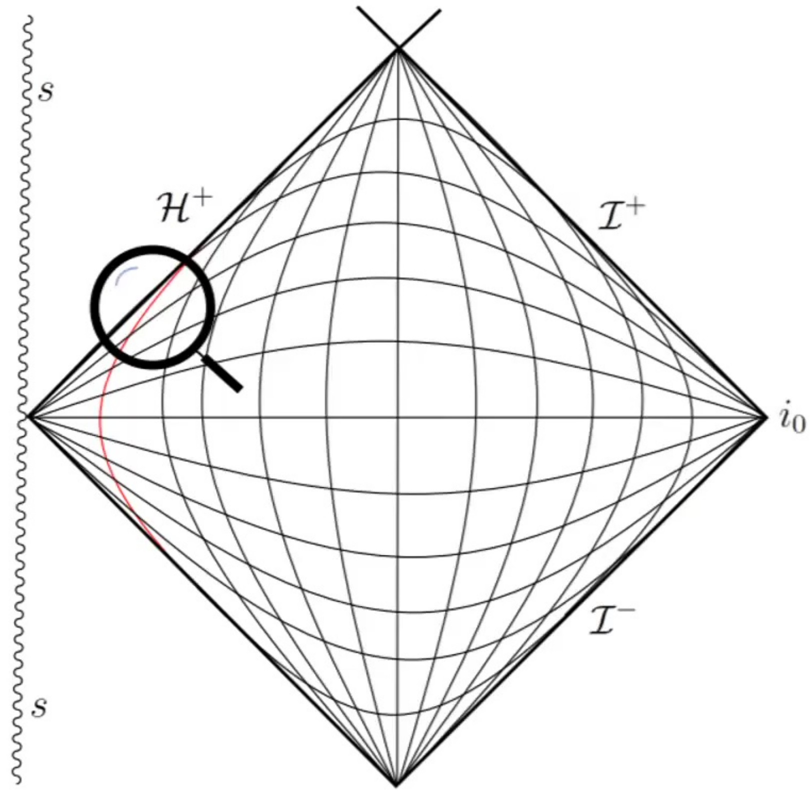
$$[Y_k, T_{(m,n)}] = -mT_{(m+k,n)}, \qquad [\bar{Y}_k, T_{(m,n)}] = -nT_{(m,n+k)},$$

$$[Y_k, U^a_{(m,n)}] = -mU^a_{(m+k,n)}, \qquad [\bar{Y}_k, U^a_{(m,n)}] = -nU^a_{(m,n+k)},$$

$$\boxed{[U^a_{(k,l)}, U^b_{(m,n)}] = f^{ab}{}_c\, U^c_{(k+m,l+n)}}$$

$$(\,\mathrm{Witt} \oplus \mathrm{Witt}\,) + \hat{u}(1)_0 \boxed{+\, \hat{\mathfrak{g}}_0}$$

The extremal horizons



The Penrose diagram of an extremal, asymptotically flat black hole. $\mathcal{I}^+$ ($\mathcal{I}^-$) is the null future (past) infinity, $\mathcal{H}^+$ is the future horizon, i_0 is spatial infinity, and s stands for the timelike singularity. The red line delimits the near horizon region.

BMS isometries of extremal horizons

These Noether charges can be computed and take the relatively

$$Q[J, L^A] = \frac{1}{16\pi G} \int d^2z \sqrt{\det \Omega_{AB}} \left(2J - L^A N_A\right).$$

The zero modes of these charges reproduce the Wald entropy (for $J = 2\pi$) and the angular momentum (for $L^z - L^{\bar{z}} = 1$)

BMS isometries of extremal horizons

What we want to argue here is that the set of transformations strictly includes the BMS algebra. To see this, consider the particular transformations obeying

$$J = \frac{1}{2} D_A L^A \,.$$

This particular subset of isometries yields the algebra

$$\left.\begin{aligned} \hat{L}^A &= L_1^B \partial_B L_2^A - L_2^B \partial_B L_1^A \\ \hat{P} &= \frac{1}{2} P_1 D_A L_2^A + L_1^A D_A P_2 - \frac{1}{2} P_2 D_A L_1^A - L_2^A D_A P_1 \,, \end{aligned}\right\} \text{BMS}$$

which is actually the BMS algebra.

The Noether charges: $Q[L^A] = \dfrac{-1}{16\pi G} \displaystyle\int d^2 z \sqrt{\det \Omega_{AB}}\; L^A N_A \quad \Big\}\ \text{Witt} \oplus \text{Witt}$

Summary

Close to their event horizons, black holes exhibit infinite-dimensional symmetries.

These symmetries correspond to asymptotic isometries that preserve a sensible set of boundary conditions at the horizon.

These symmetries have Noether charges associated to them, which turn out to be integrable, finite, and conserved.

The charges form an ∞-dimensional algebra, containing a Virasoro–Kac-Moody system along with supertranslations.

These charges carry physical information about the black hole, like Wald formula.

This can be realized for charged, spinning Taub-NUT/Bolt (A)dS black holes with acceleration and in presence of external fields in Einstein-Yang-Mills theory.