

Title: Gravitational-wave Source Inference with Data-driven Models

Speakers: Bruce Edelman

Series: Strong Gravity

Date: January 20, 2022 - 1:00 PM

URL: <https://pirsa.org/22010086>

Abstract: With the release of the third gravitational wave transient catalog (GWTC-3), the LIGO and Virgo detectors have reported nearly 100 gravitational waves from colliding black holes and neutron stars. Among these detections there have been numerous surprises, such as the heavy GW190521, the confidently asymmetric GW190412, and the exceptionally small secondary of GW190814. In addition to analyses of each individual sources' properties, such as their masses and spins, one can also summarize the collective properties of the colliding objects as population probability distributions over these parameters. As catalog sizes continue to grow, it enables both finer grained investigations into the population properties of merging compact objects, and robustly testing GR in the strong gravity regime. In this talk I will present data driven statistical models to look for deviations to underlying theoretical expectations, both for individual gravitational waveform models and population models describing the astrophysical distributions of merging compact binaries. I will present the results of an analysis using this novel data-driven model on the 11 compact binary mergers in GWTC-1, then move towards hierarchical models, inferring the binary black hole mass distribution with similar data-driven methods. I will conclude with showing new results from the LVK population analyses of GWTC-3 and motivate the need towards developing more data-driven statistical models for the incoming swath of observations expected in the fourth observing run that, as we have seen, will likely continue to further challenge theoretical expectations.

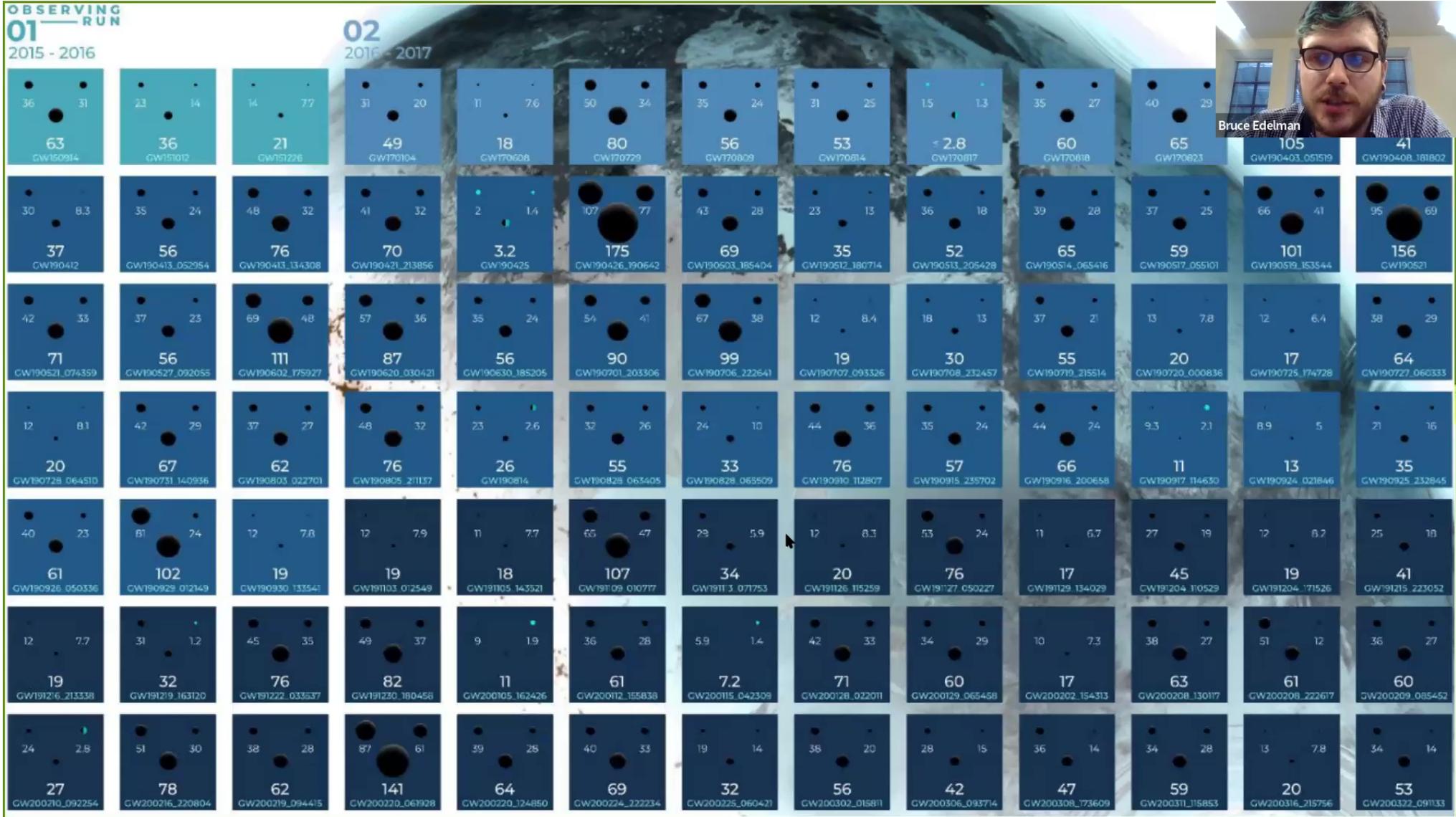


Gravitational-Wave Source Inference with Data-Driven Models



Bruce Edelman, University of Oregon
PI Strong Gravity Seminar 01/20/22

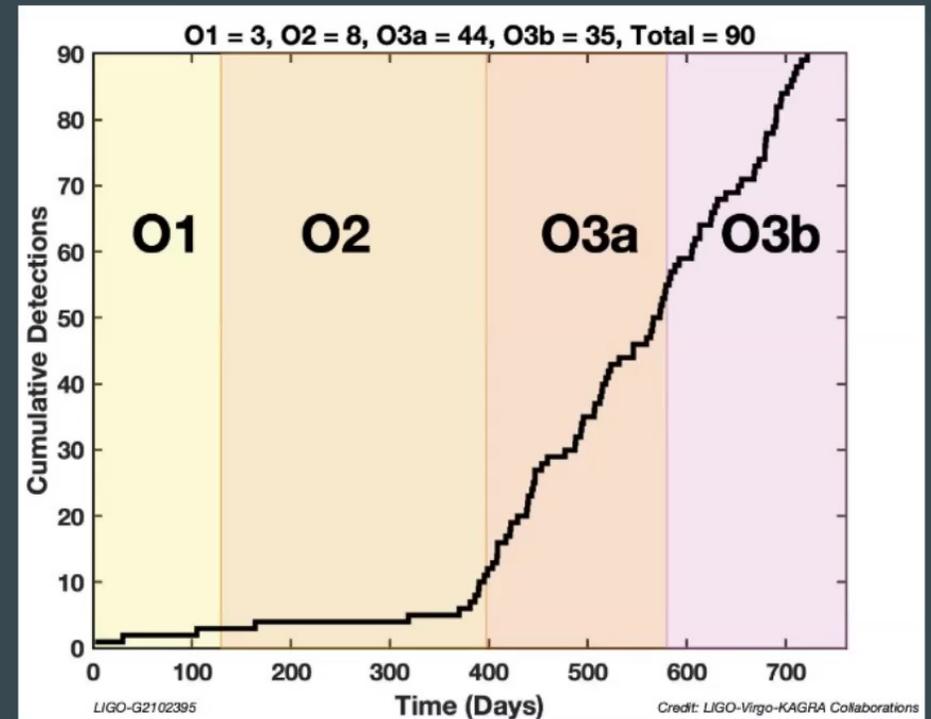




Recent Catalog of Observations – GWTC-3 Papers



- Catalog [[2111.03606](#)]
- Population Analysis [[2111.03634](#)]
- Testing GR [[2112.06861](#)]
- Cosmology [[2111.03604](#)]



Overview

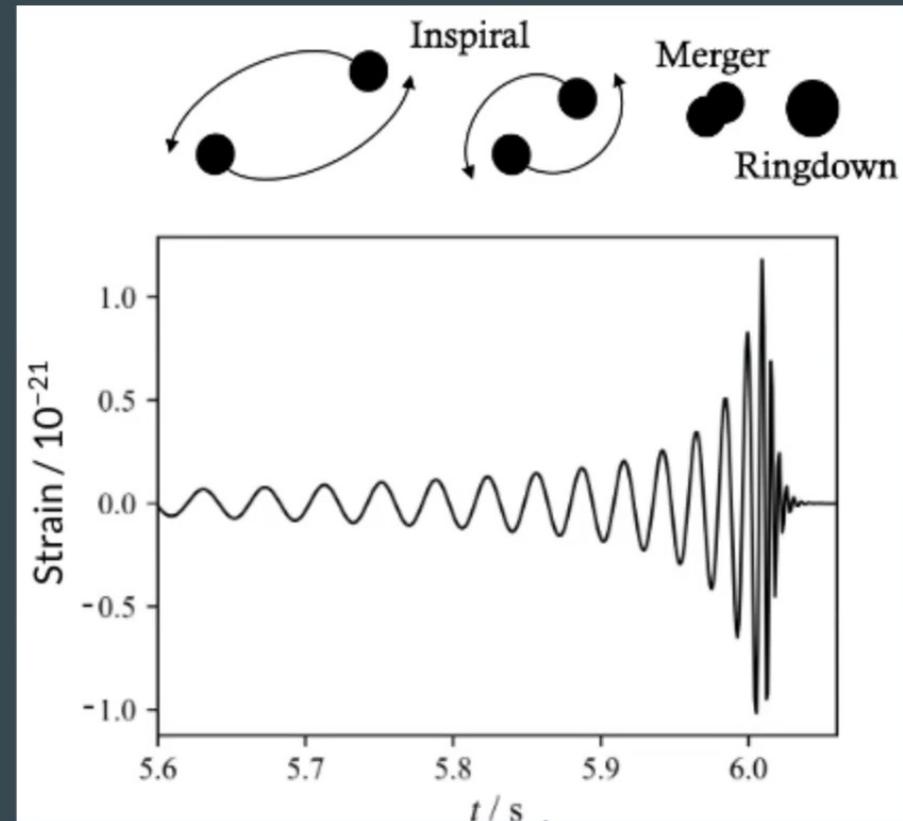


- **Single Source Inference and Parameter Estimation**
 - Coherent Waveform Deviation Model ([Edelman et. al 2021](#))
- **Hierarchical Inference of a Population of Sources**
 - Pair-Instability Mass Gap Model ([Edelman, Doctor, Farr 2021](#))
 - Semi-Parametric Modeling of the BBH Mass Distribution ([Edelman et al 2022](#))
- **Population Updates from GWTC-3**



Parameters Describing a BBH Merger

- Intrinsic (8 parameters)
 - Mass of each BH: $m_1 \geq m_2$ (2)
 - Spin vector of each BH (6)
- Extrinsic (7 parameters)
 - Luminosity Distance D_L
 - Sky localisation (right ascension, declination), Inclination angle, Arrival Time, Phase, Polarization angles

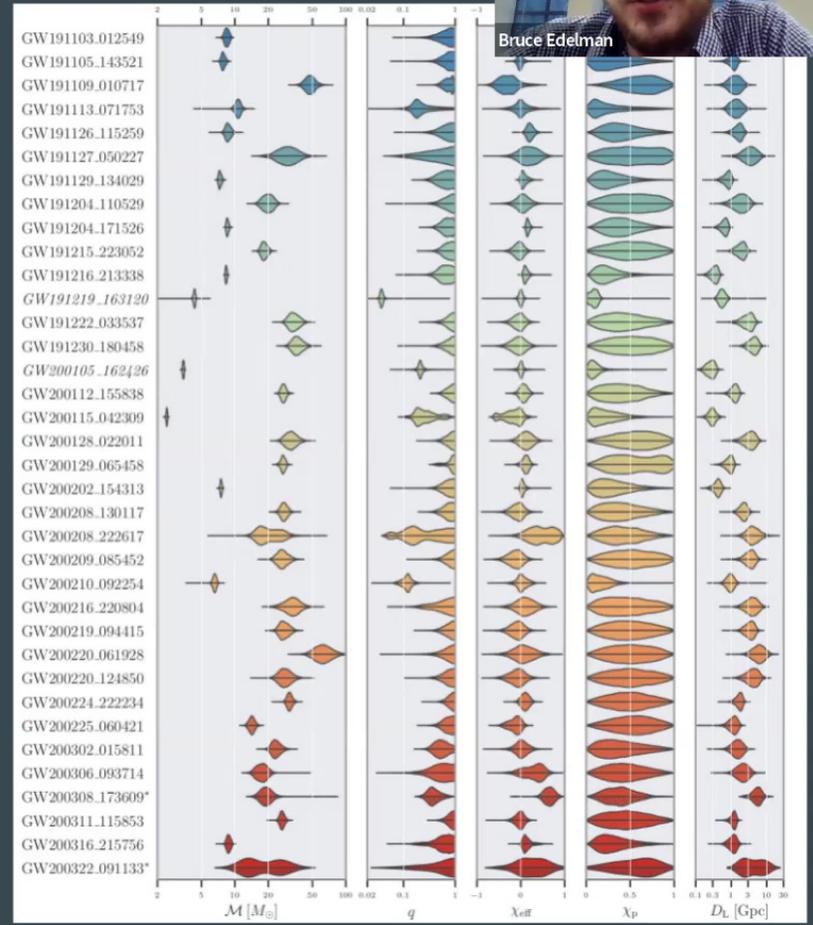


Parameter Estimation – Bayes' Theorem

- Noisy Data Limits our Inferences

$$p(\theta|d) = \frac{\mathcal{L}(d|\theta)p(\theta)}{\mathcal{Z}(d)}$$

- Goal: Given a stream of strain data, d , what is the probability distribution over parameters, θ



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Parameter Estimation – Bayes' Theorem



$$p(\theta|d) = \frac{\mathcal{L}(d|\theta)p(\theta)}{\mathcal{Z}(d)}$$

Likelihood

Prior

Posterior

Evidence

- Uninformative (Agnostic) Priors



Parameter Estimation – Bayes' Theorem

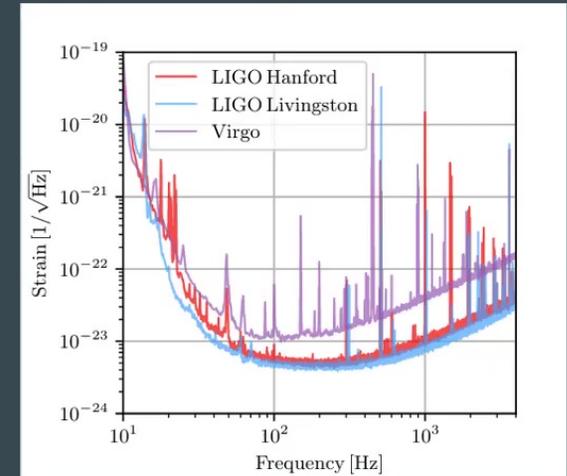
$$p(\theta|d) = \frac{\mathcal{L}(d|\theta)p(\theta)}{\mathcal{Z}(d)}$$

Likelihood Prior

Posterior Evidence

$$\mathcal{L}(d|\theta) \propto \exp\left(-\frac{1}{2} \sum_k \frac{|d_k - h_k(\theta)|^2}{\sigma_k^2}\right)$$

- Uninformative (Agnostic) Priors
- Under Assumption of Uncorrelated Gaussian Stationary Noise, we use the Whittle Likelihood



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Likelihood and GW Waveform model

- The waveform model h , is different for each detector according to antenna pattern

$$\mathcal{L}(d|\theta) \propto \exp \left(-\frac{1}{2} \sum_k \frac{|d_k - h_k(\theta)|^2}{\sigma_k^2} \right)$$

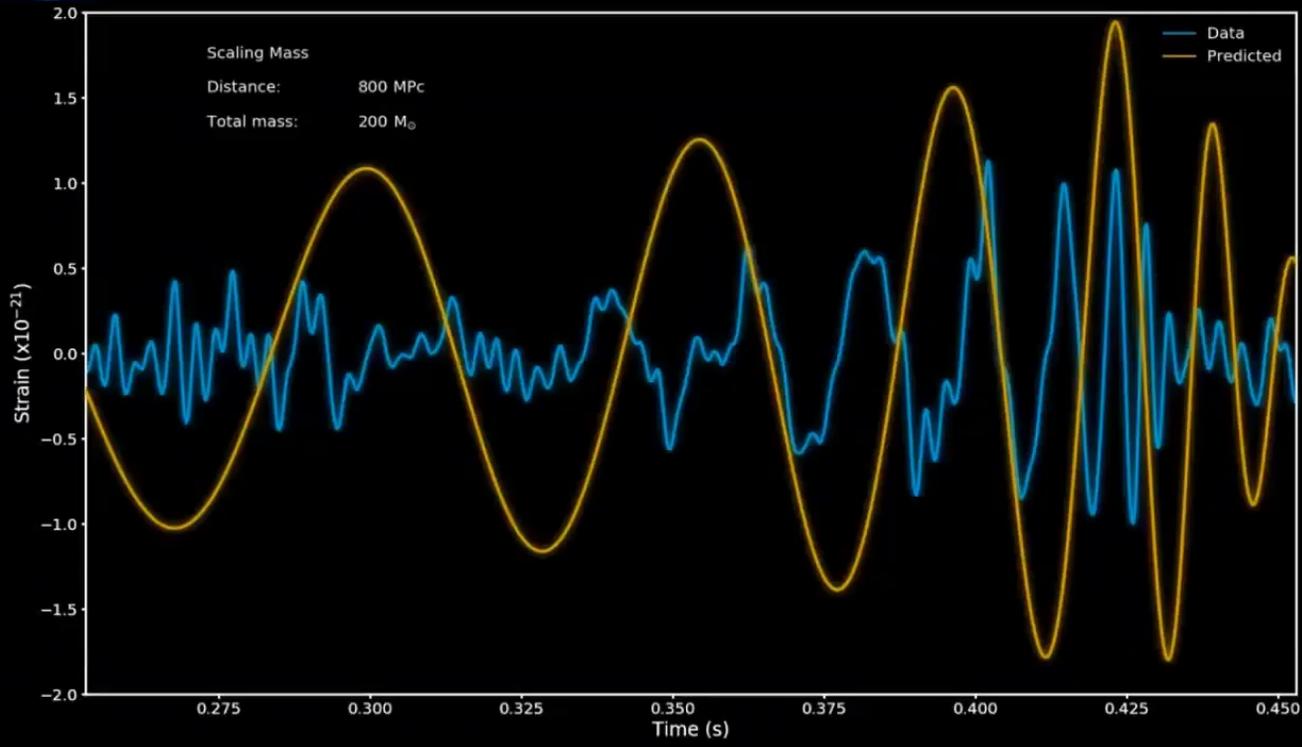
$$d_k(f) = h_{\text{obs},k}(f) + n_k(f)$$

$$h_{\text{obs},k}(f) = h_+(f)F_{+,k}(f) + h_{\times}(f)F_{\times,k}(f)$$

“Observed” signal in detector k

“Intrinsic” or “Coherent” signal across the network

LSC



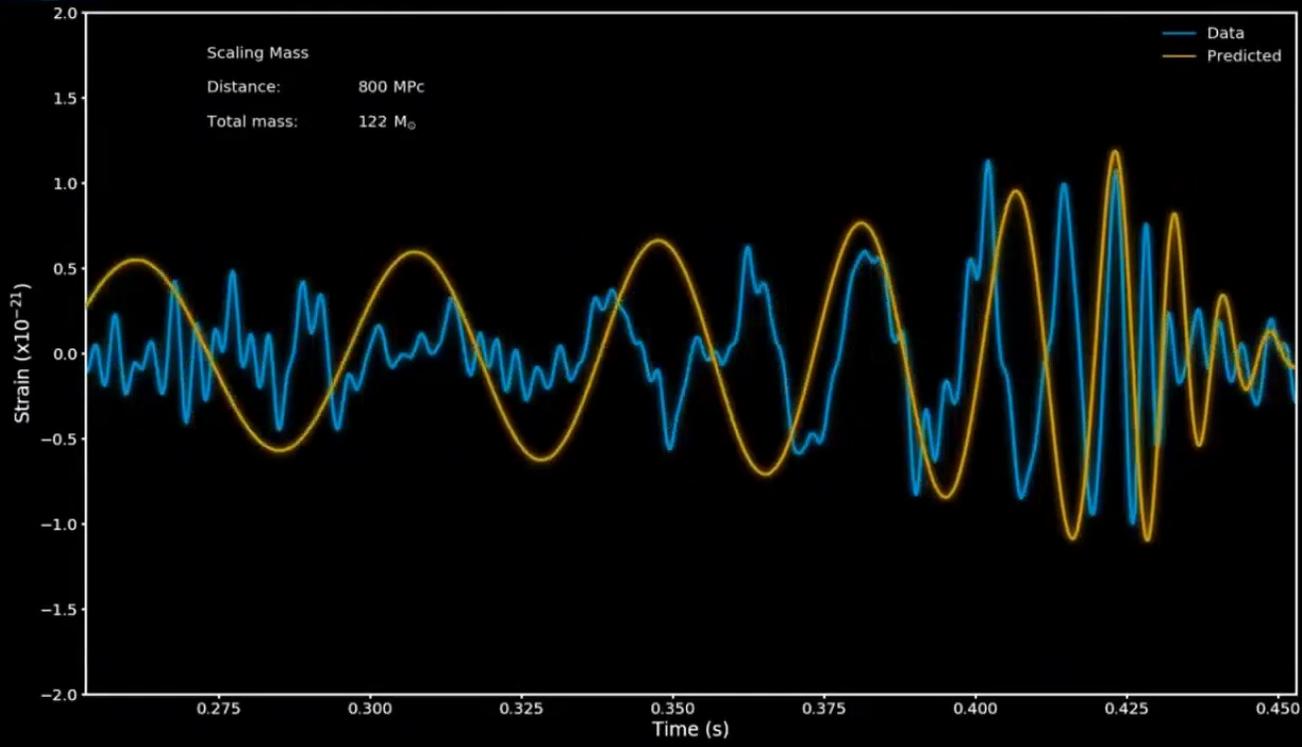
Data & Best-fit Waveform: LIGO Open Science Center (losc.ligo.org); Prediction & Animation: C.North/M.Hannam (Cardiff University)



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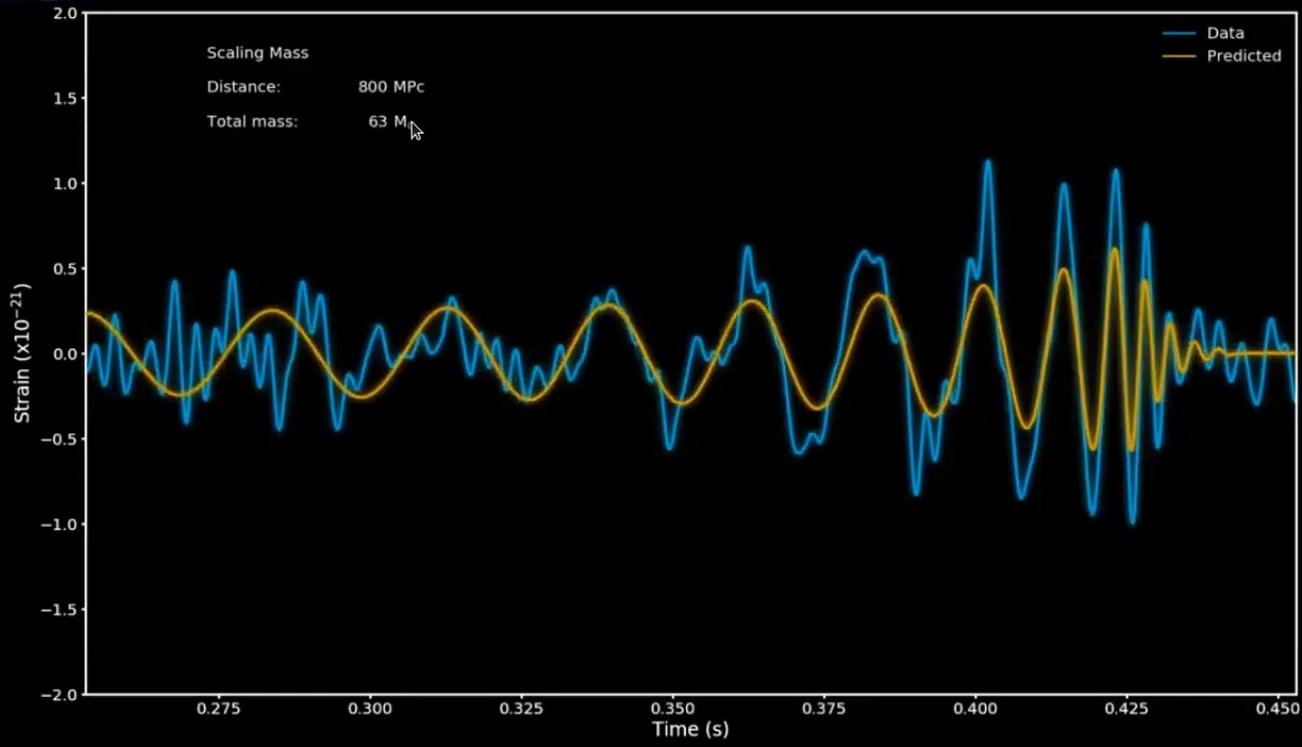
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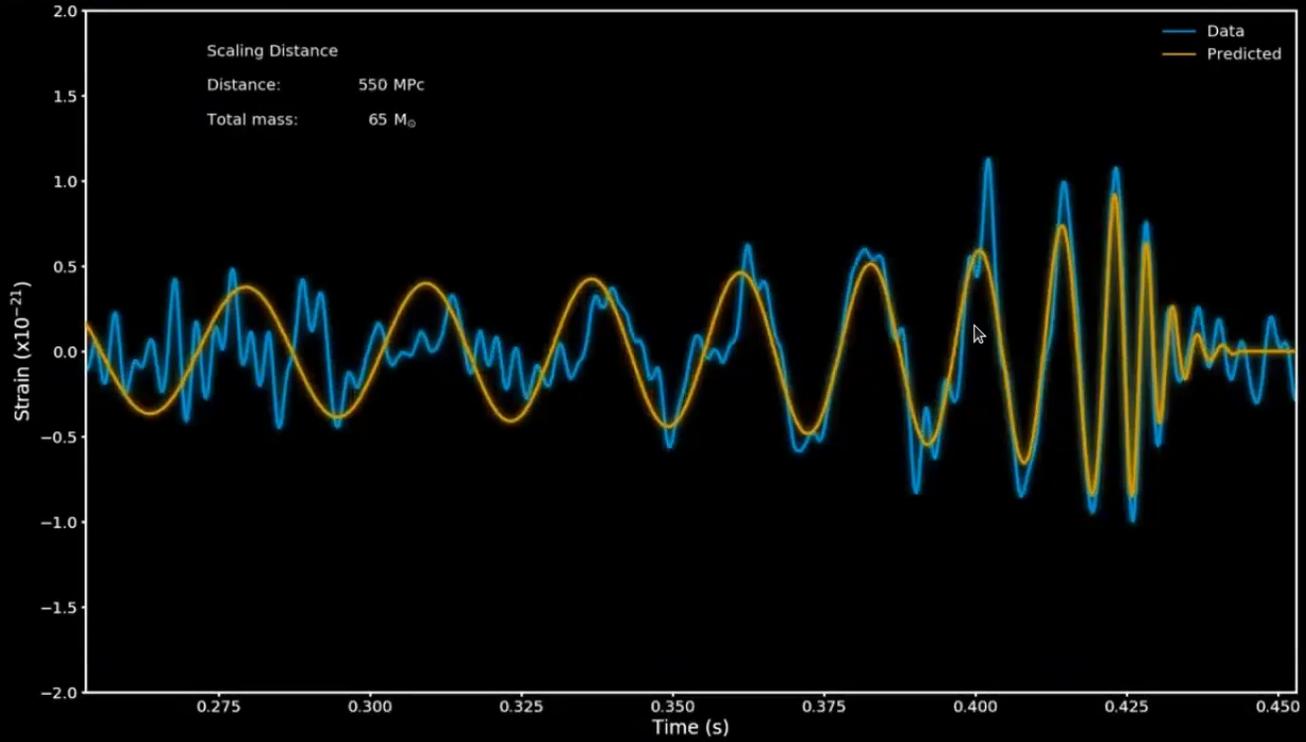
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Bruce Edelman

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LSC

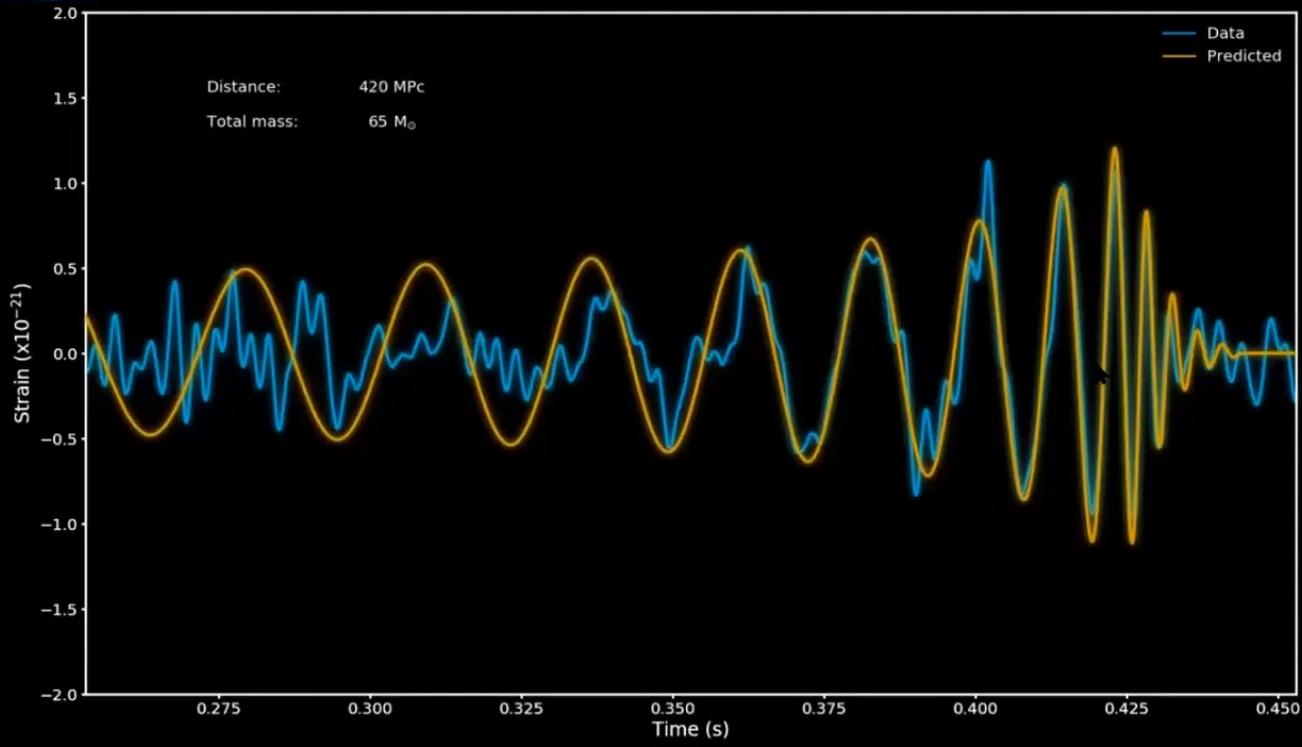


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LSC



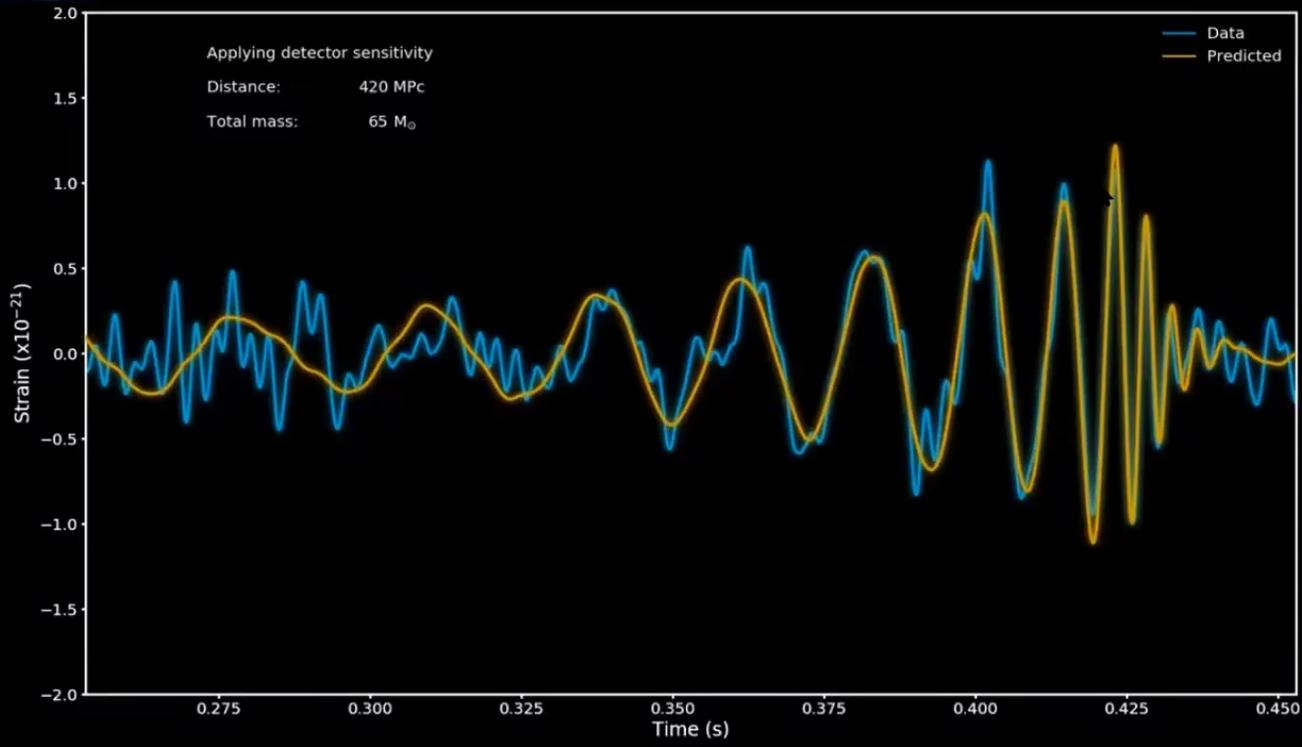
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Bruce Edelman
PRIFYSGOL
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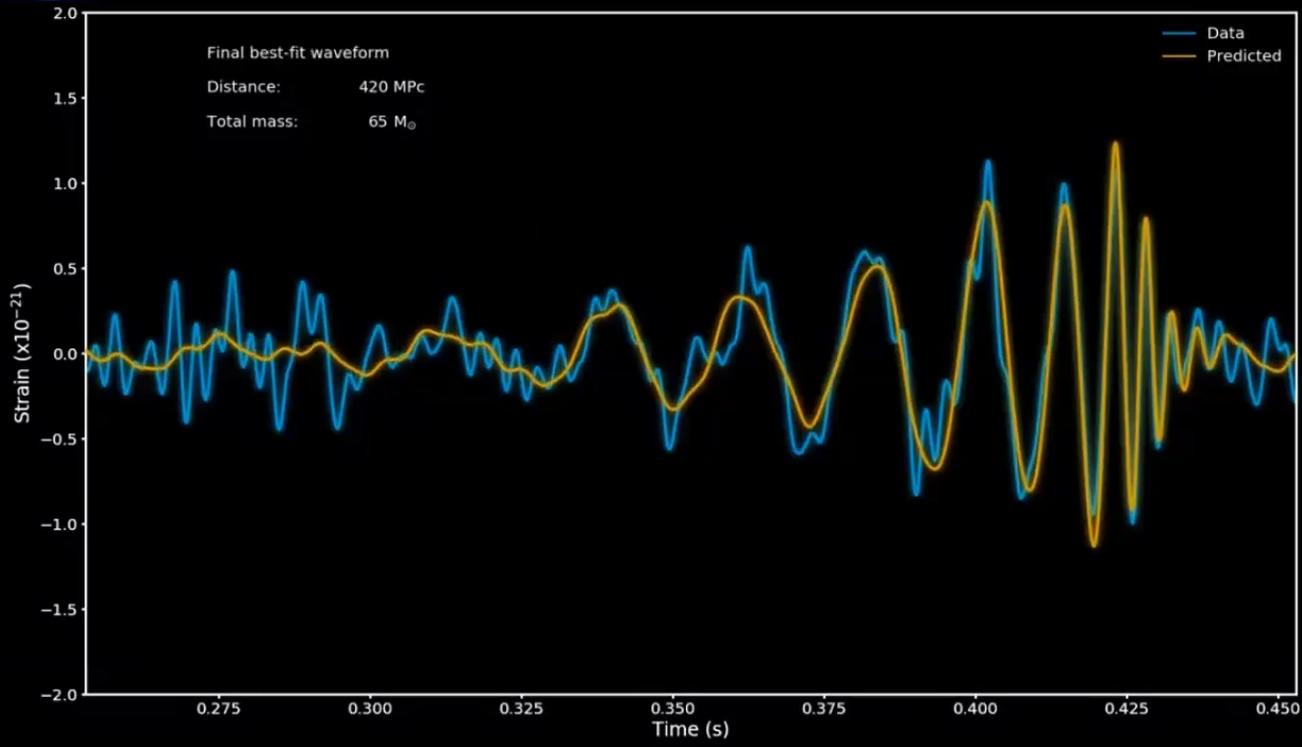
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CC Settings Full Screen



LSC



Data & Best-fit Waveform: LIGO Open Science Center (losc.ligo.org); Prediction & Animation: C.North/M.Hannam (Cardiff University)

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Coherent Waveform Deviations

- How do we know our models are correct? Or “good enough”?
- We parametrize deviations to the COHERENT waveform model
- This could be due to:
 - Waveform Model systematic errors
 - Deviations from GR
 - Presence of physics not included in the waveform model

$$h_{+, \times}(f) = h_{\text{model}, +, \times}(f) [1 + \delta A(f)] e^{i\delta\phi(f)}$$

- What Properties do we want in Amp/Phase deviations?
 - Smoothly varying in frequency
 - Flexible enough to fit generic forms without bias
- Let the Data sort it out! (non-parametric models)

Signal Spline Deviation Model



- Use cubic spline functions to model deviations in amplitude/phase
- Construct set of n knot points initialized uniformly across log-frequency space (f_low -> Nyquist)

$$\delta A(f) = \mathcal{I}_3(f; \{f_i, \delta A_i\})$$
$$\delta \phi(f) = \mathcal{I}_3(f; \{f_i, \delta \phi_i\})$$

Signal Spline Deviation Model



- Use cubic spline functions to model deviations in amplitude/phase
- Construct set of n knot points initialized uniformly across log-frequency space (f_{low} \rightarrow Nyquist)
- Add $3n$ parameters to model
 - n knot locations in freq space
 - n knot heights for amp deviation
 - n knot heights for phase deviation

$$\delta A(f) = \mathcal{I}_3(f; \{f_i, \delta A_i\})$$
$$\delta \phi(f) = \mathcal{I}_3(f; \{f_i, \delta \phi_i\})$$

$$p(\delta \phi_i) = \mathcal{N}(0, \sigma_\phi)$$

$$p(\delta A_i) = \mathcal{N}(0, \sigma_A)$$

$$p(f_i) = \mathcal{U}(f_{\text{low}}, f_{\text{high}})$$

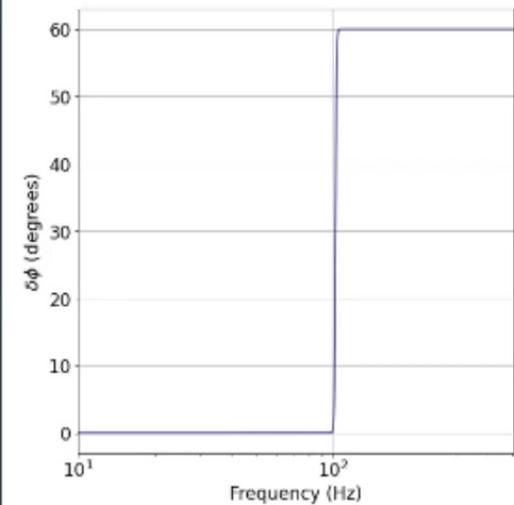
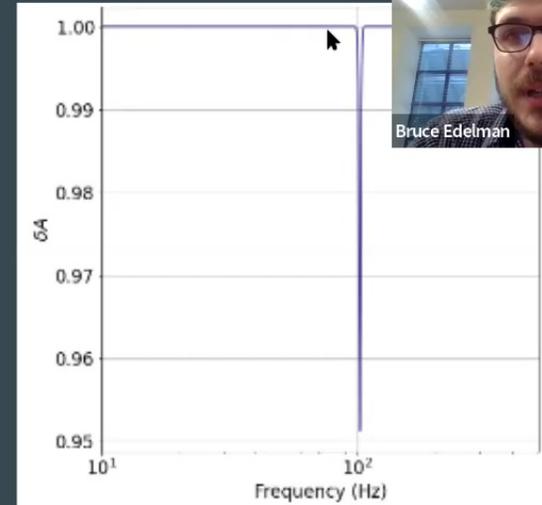
Toy Model of Simulated Deviations

- We construct a toy model deviation based on an extreme spontaneous scalarization
- Rapid jump in phase with temporary amplitude reduction

$$\delta A(f) = \exp\left(\frac{1}{2}dA\left[\tanh\left(\frac{f - f_z}{df}\right)^2 - \tanh\left(\frac{f_{\text{ref}} - f_z}{df}\right)^2\right]\right)$$

$$\delta\phi(f) = \frac{1}{2}d\phi\left[\tanh\left(\frac{f - f_z}{df}\right) - \tanh\left(\frac{f_{\text{ref}} - f_z}{df}\right)\right]$$

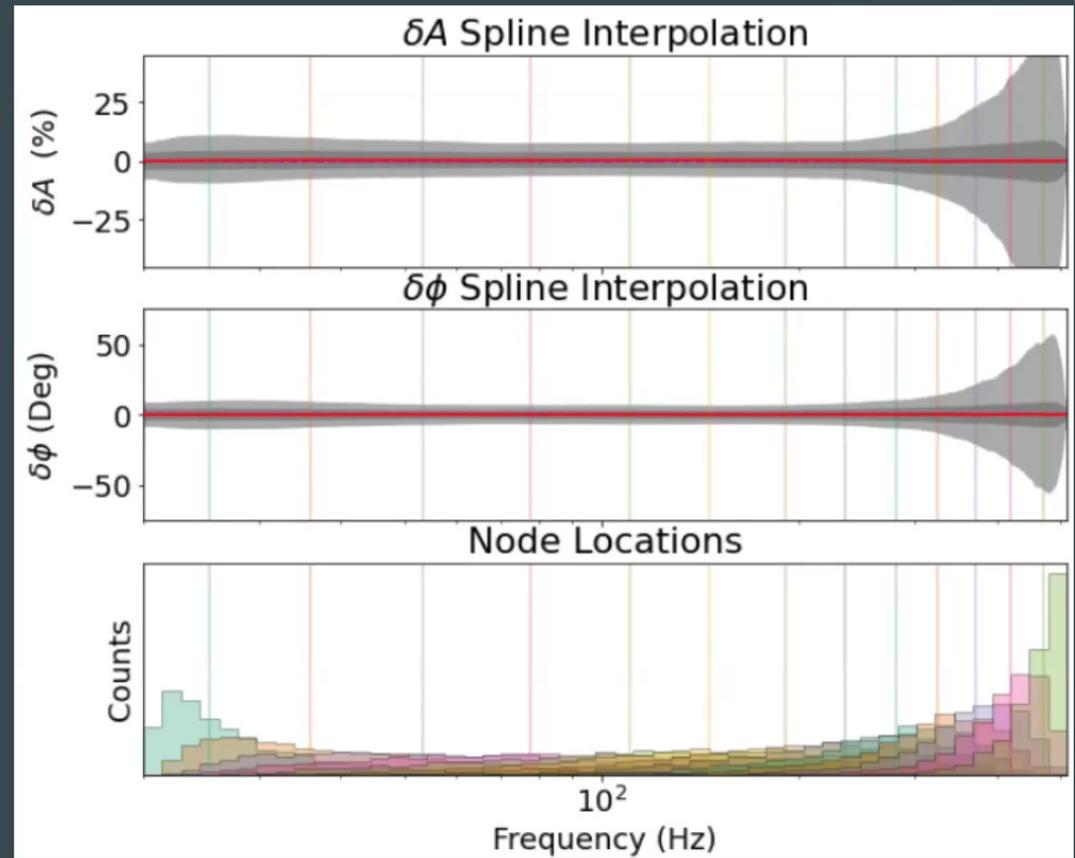
[Edelman et. al 2021](#)



Simulated Deviation Study

- Simulate a GW150914-like signal with IMRPhenomD model with no deviations and fit our deviation model

m_1	$36 M_\odot$
m_2	$29 M_\odot$
D_L	450 Mpc
ϕ	2.76 rad
α	1.37 rad
δ	-1.26 rad
$S_{1,z}$	0.0
$S_{2,z}$	0.0

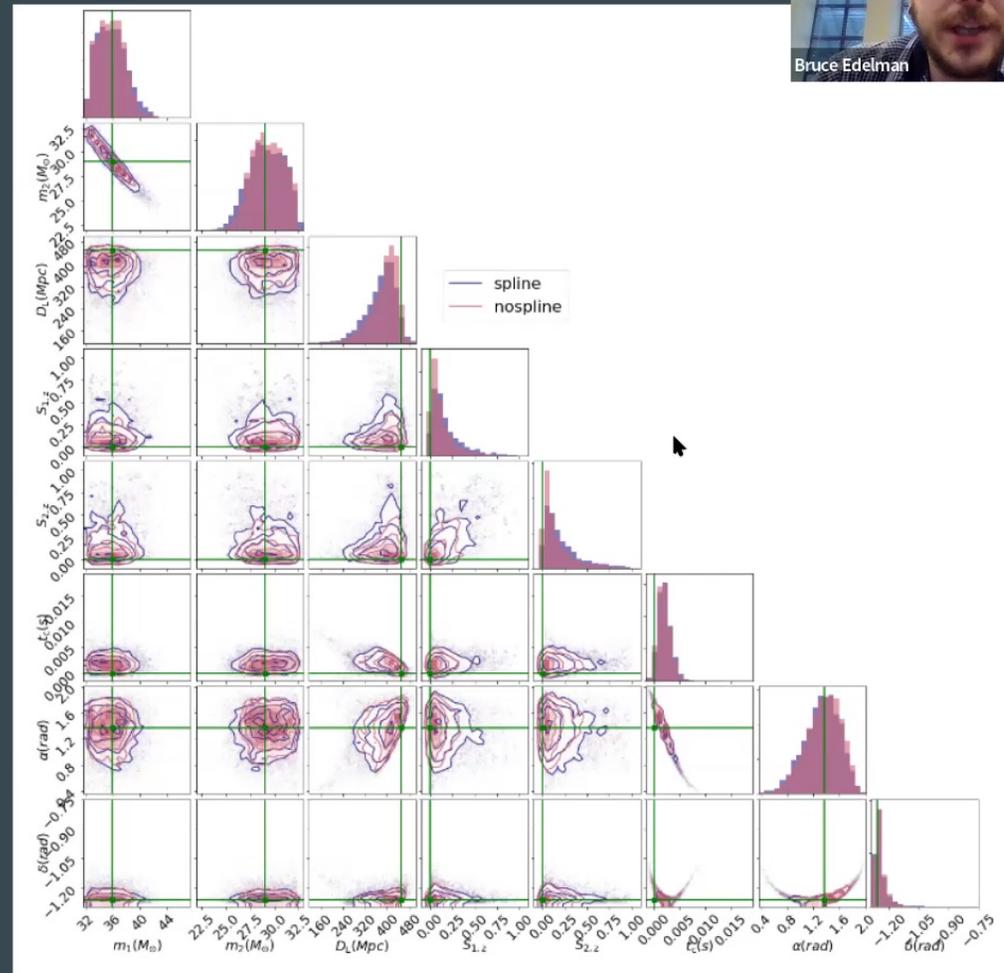


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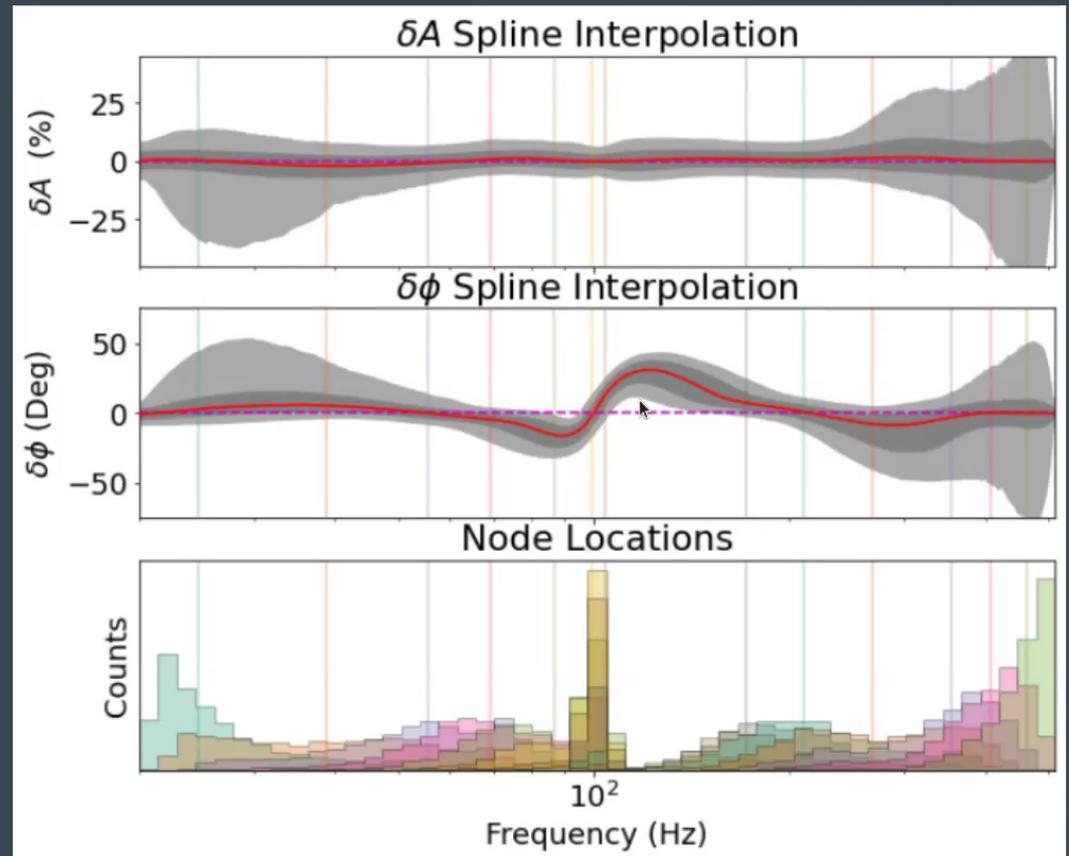
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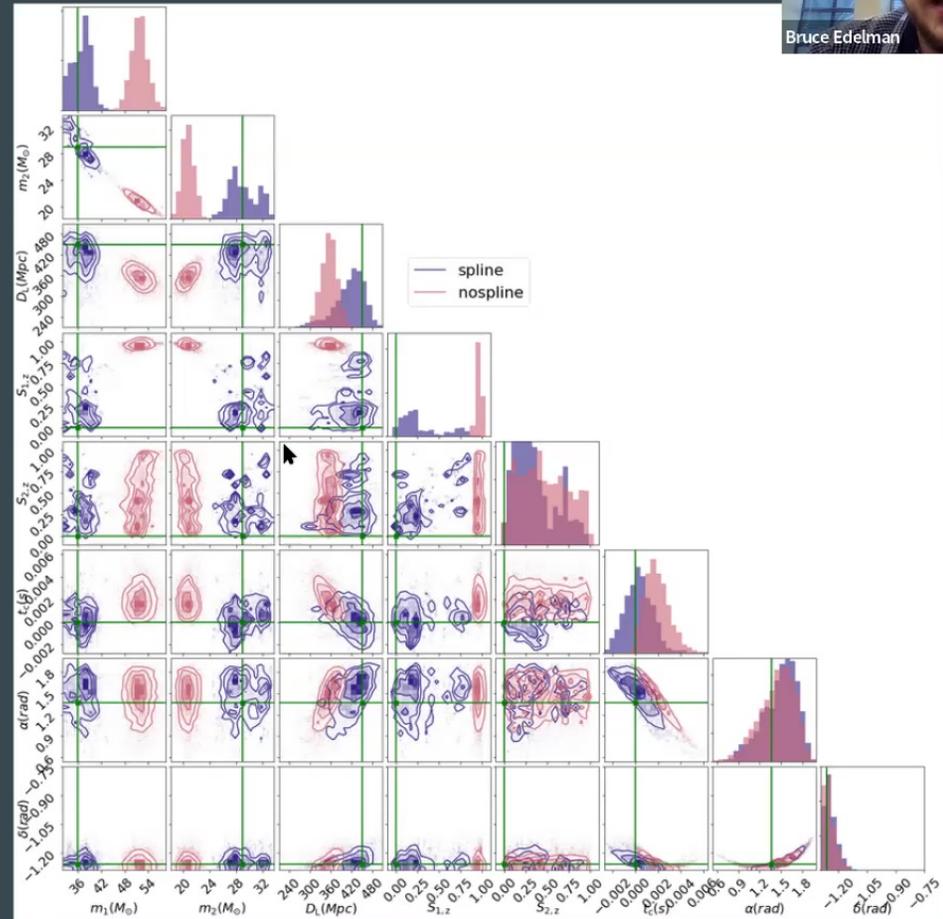


[Edelman et. al 2021](#)

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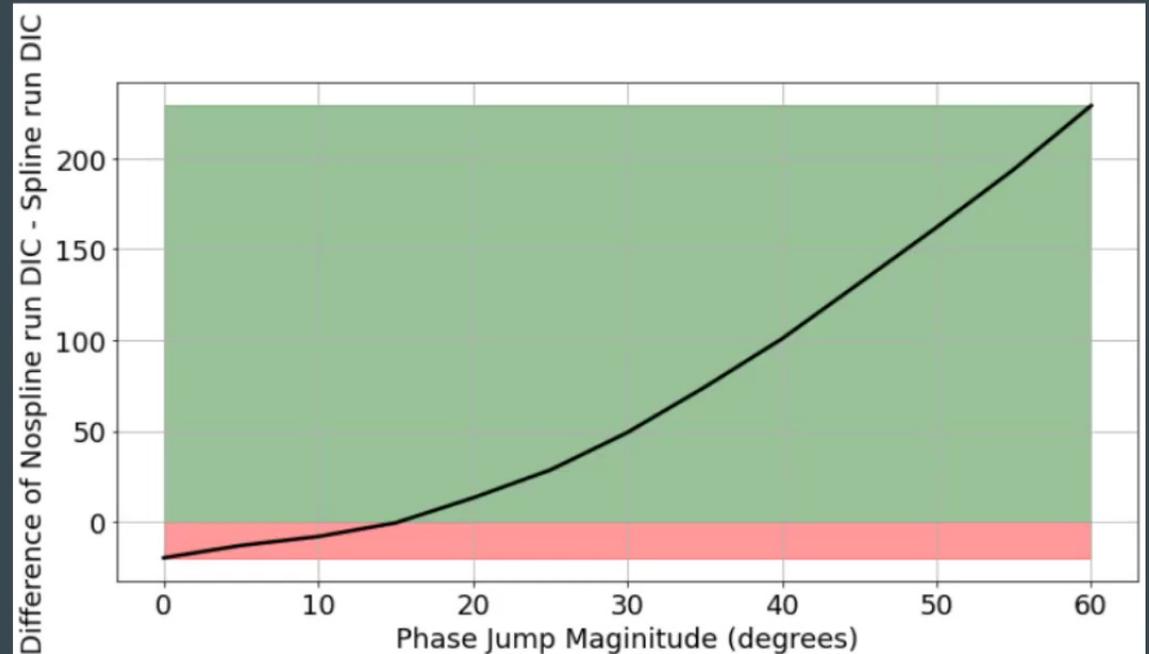
Bruce Edelman

[Edelman et. al 2021](#)

Simulated Deviation Study



- Do same procedure with varying phase jump from 0 -> 60 degrees (increments of 5)
- Use DIC metric to compare using the signal deviation model vs. not



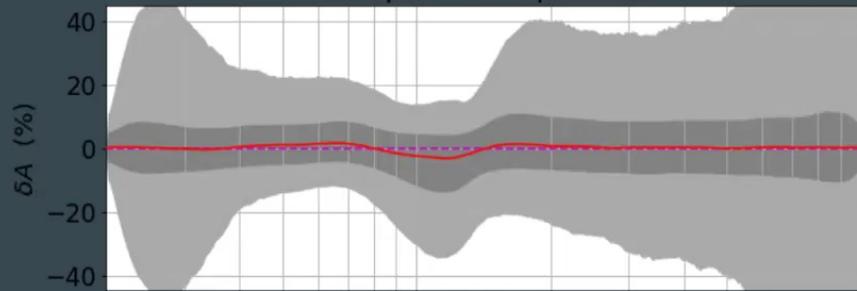
[Edelman et. al 2021](#)

Looking for Deviations in the 11 CBCs of GWTC-1 (2-3)

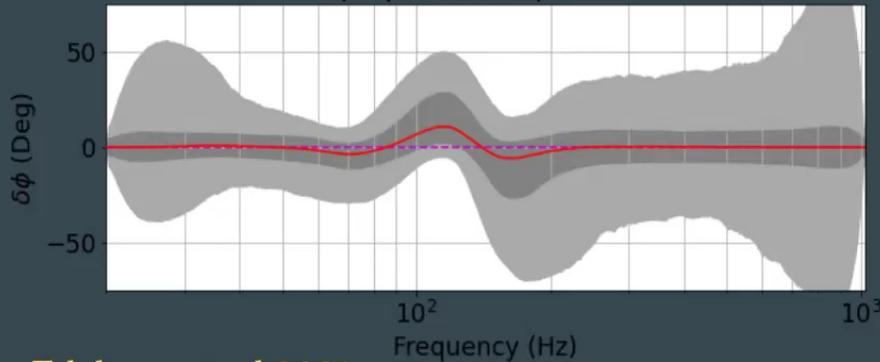


GW170729

δA Spline Interpolation

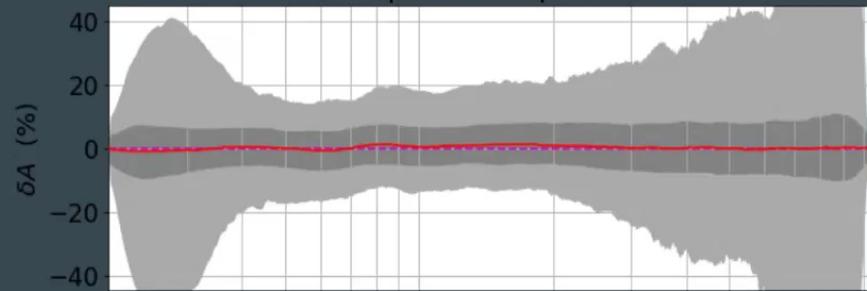


$\delta\phi$ Spline Interpolation

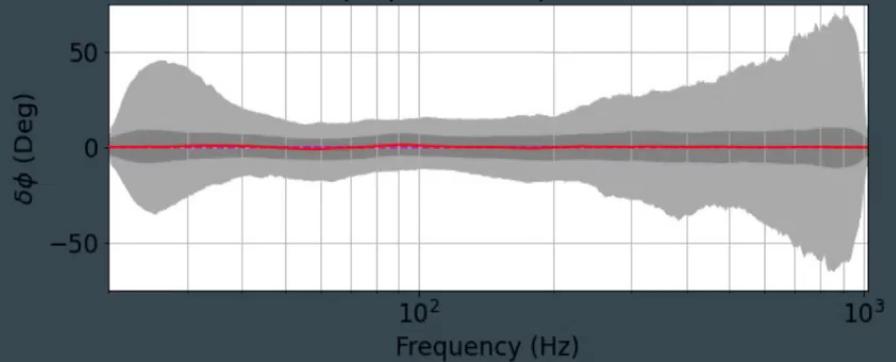


GW190823

δA Spline Interpolation



$\delta\phi$ Spline Interpolation

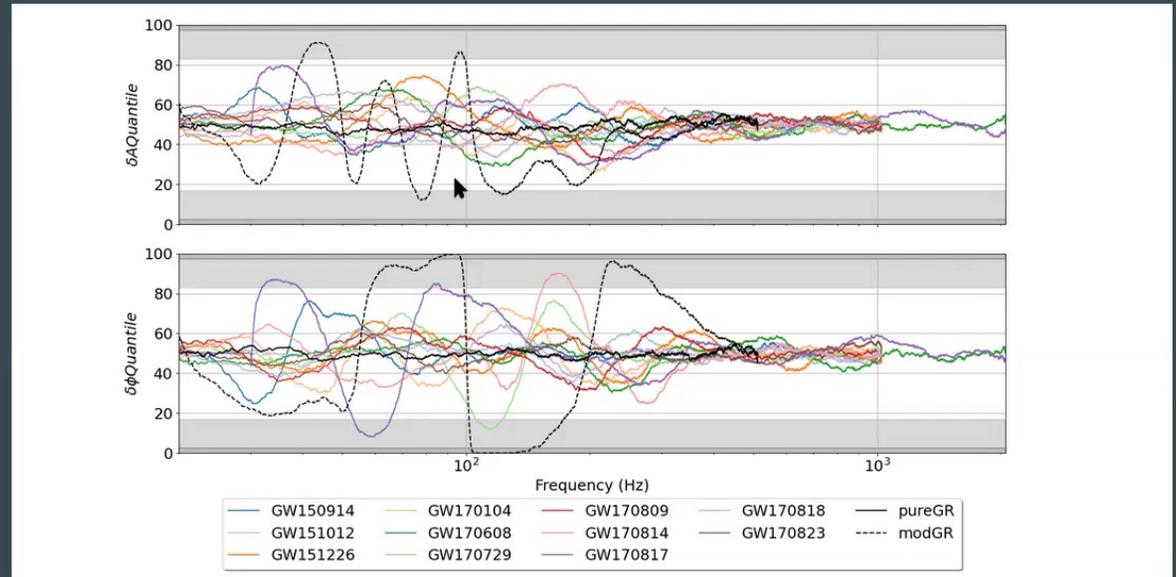
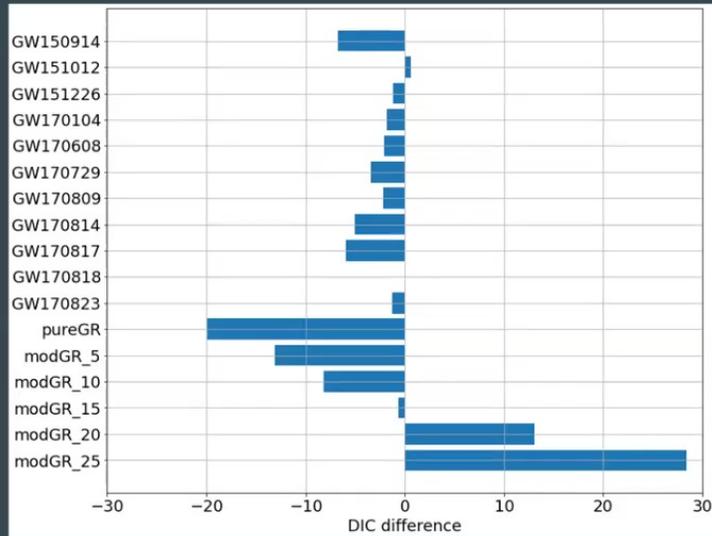


[Edelman et. al 2021](#)



Looking for Deviations in the 11 CBCs of GWTC-1 (2-3)

- no significant deviation from IMR family of waveform models



- Model comparisons show no preference or slight disfavoring of spline mode in all GWTC-1 events

[Edelman et. al 2021](#)

Overview

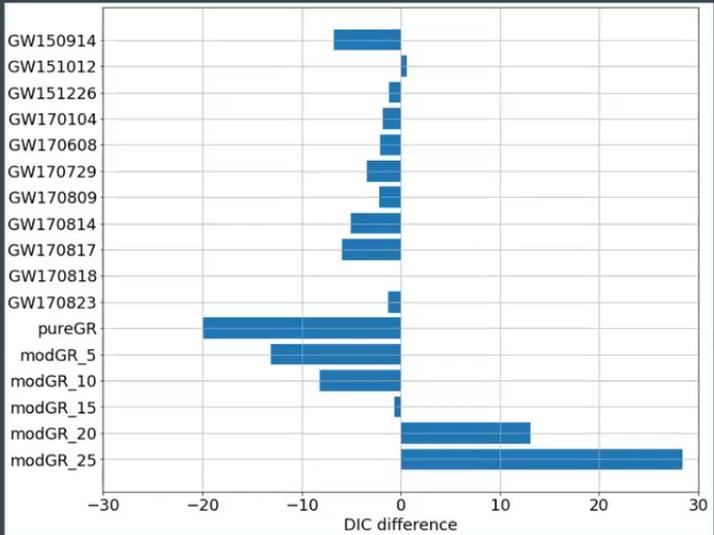
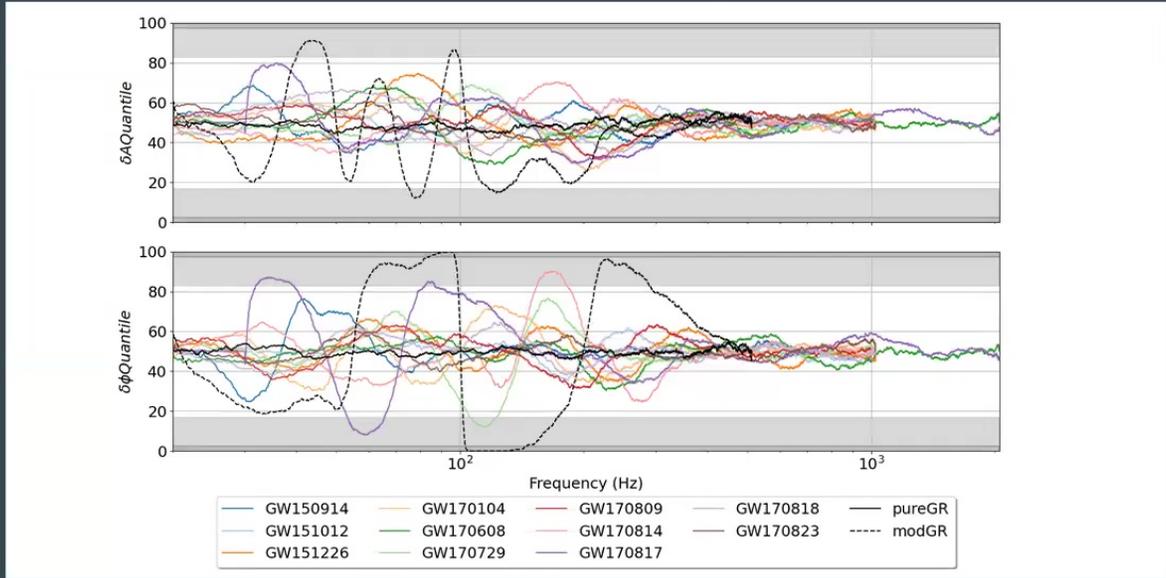


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[Edelman et. al 2021](#)

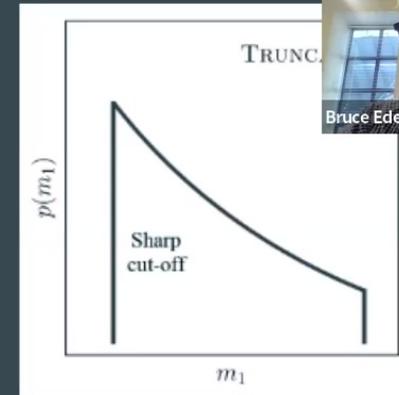
Hierarchical Bayesian Inference

- How many BBH mergers are there in the Universe?
- How often do they merge?
- How are their event properties distributed?

$$p(\theta|d) = \frac{\mathcal{L}(d|\theta)p(\theta)}{\mathcal{Z}(d)}$$

$$p(\Lambda|d) = \frac{p(\Lambda) \int d\theta \mathcal{L}(d|\theta)p(\theta|\Lambda)}{\mathcal{Z}(d)}$$

$$p(\Lambda|\{d_i\}) = \frac{p(\Lambda) \prod_i [\int d\theta_i \mathcal{L}(d_i|\theta_i)p(\theta_i|\Lambda)]}{\mathcal{Z}(\{d_i\})}$$



$$p(\theta|\Lambda)$$

Hyper-Prior



GW Selection Effects



- GW Detection efficiency depends on parameters
 - Distance (Malmquist Bias)
 - Mass (Heavier mass BBHs merge at lower frequencies)
- Number of observed BBHs should be dependent on region of parameter space (Inhomogeneous Poisson Process)
- Use Importance sampling to compute monte carlo Integrals

$$p(\Lambda, N | \{d_i\}) \propto \left[\prod_{i=1}^{N_{\text{obs}}} \int d\theta_i \mathcal{L}(d_i | \theta_i) p(\theta_i | \Lambda) \right] N^{N_{\text{obs}}} e^{-N p_{\text{det}}(\Lambda)} p(\Lambda) p(N)$$

$$p_{\text{det}}(\Lambda) = \int_{\{d_i\} > \text{threshold}} d\{d_i\} d\theta \mathcal{L}(\{d_i\} | \theta) p(\theta | \Lambda)$$

<https://iopscience.iop.org/article/10.3847/1538-4357/ac3667>

GW Selection Effects

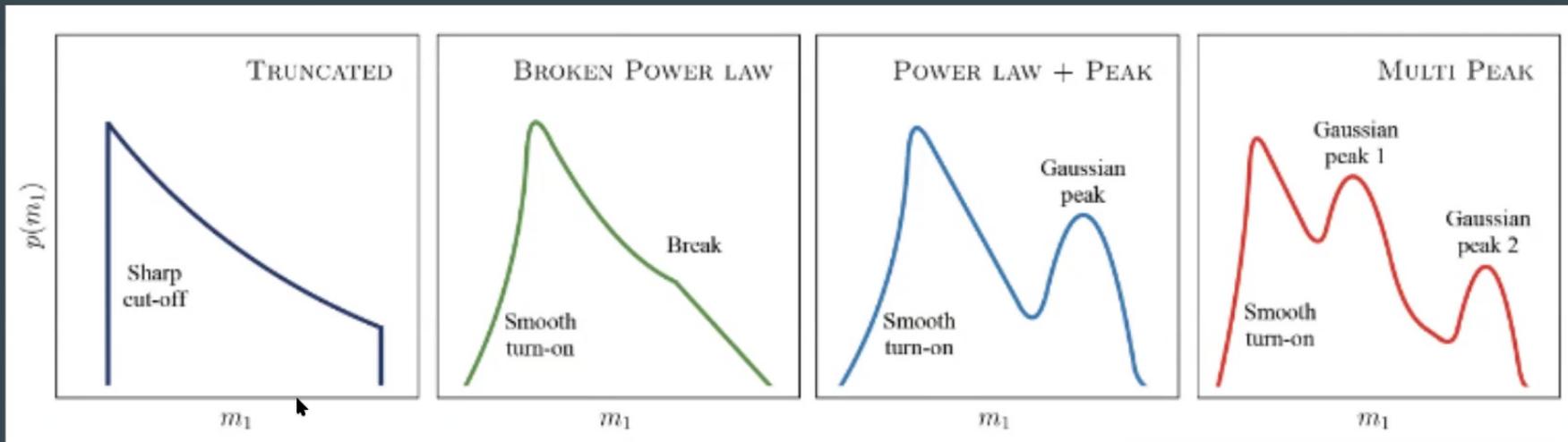


- Use catalog Posterior samples to access the Likelihood
- Use a set of Software “Injections” ran through search pipelines to estimate p_{det}

$$p(\Lambda, N | \{d_i\}) \propto p(\Lambda)p(N) \prod_i \left[\frac{1}{K_i} \sum_j \frac{p(\theta_i^j | \Lambda)}{p_{PE}(\theta_i^k)} \right] N^{N_{\text{obs}}} e^{-N p_{\text{det}}(\Lambda)}$$

$$p_{\text{det}}(\Lambda) = \frac{1}{N_{\text{draw}}} \sum_i^{N_{\text{found}}} \frac{p(\theta_i | \Lambda)}{p_{\text{draw}}(\theta_i)}$$

BBH Mass Population Models

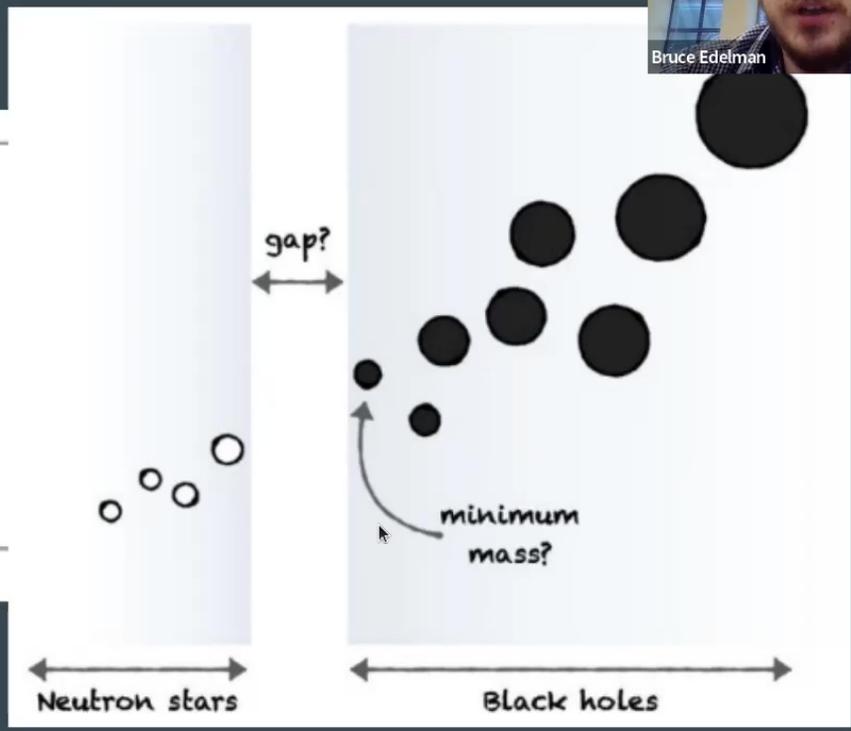
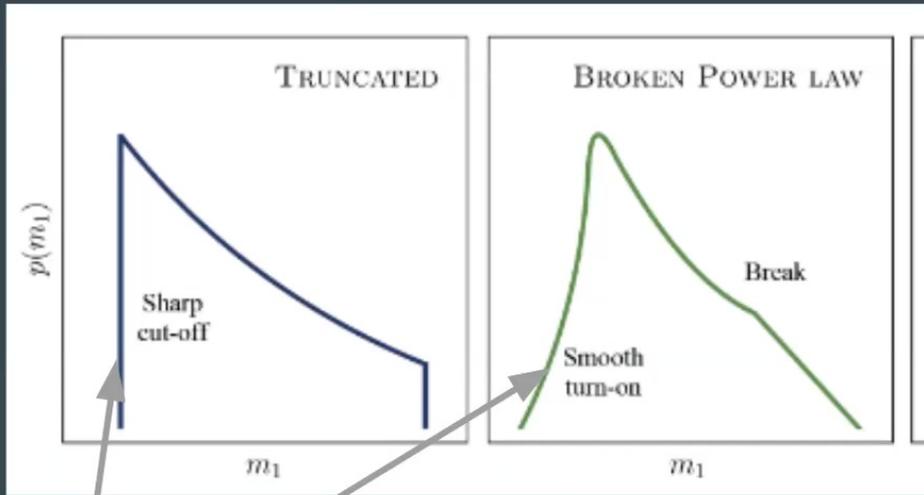


[R. Abbott et al 2021 ApJL 913 L7](#)

Illustration: Shanika



BBH Mass Population Models



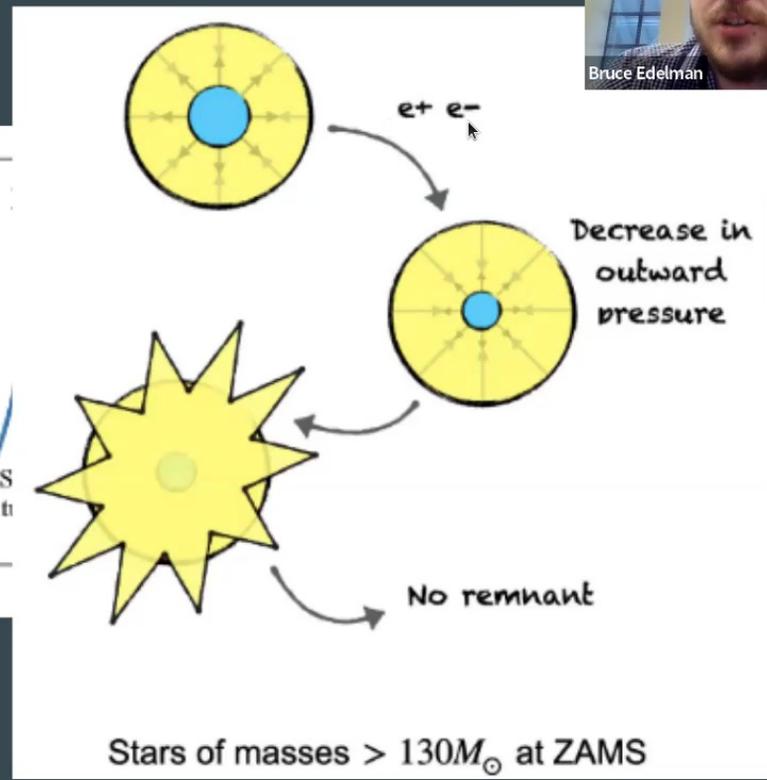
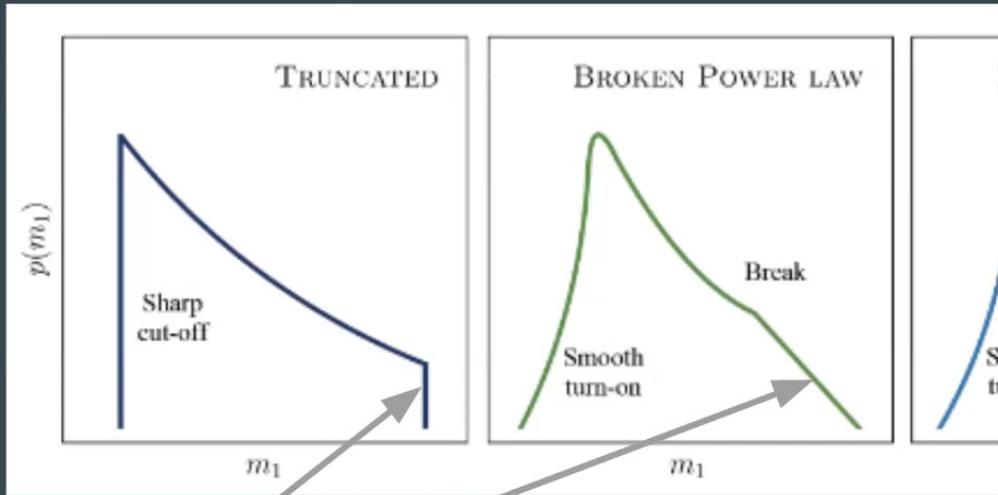
Low-Mass End: Lower mass gap? What's the minimum BH mass? Is the NS-BH gap populated?

[R. Abbott et al 2021 ApJL 913 L7](#)

Illustration: Shanika



BBH Mass Population Models

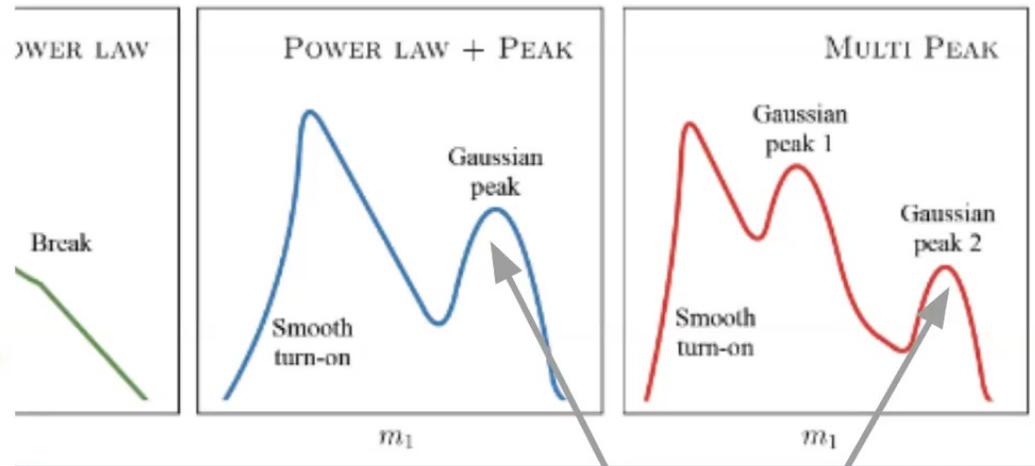
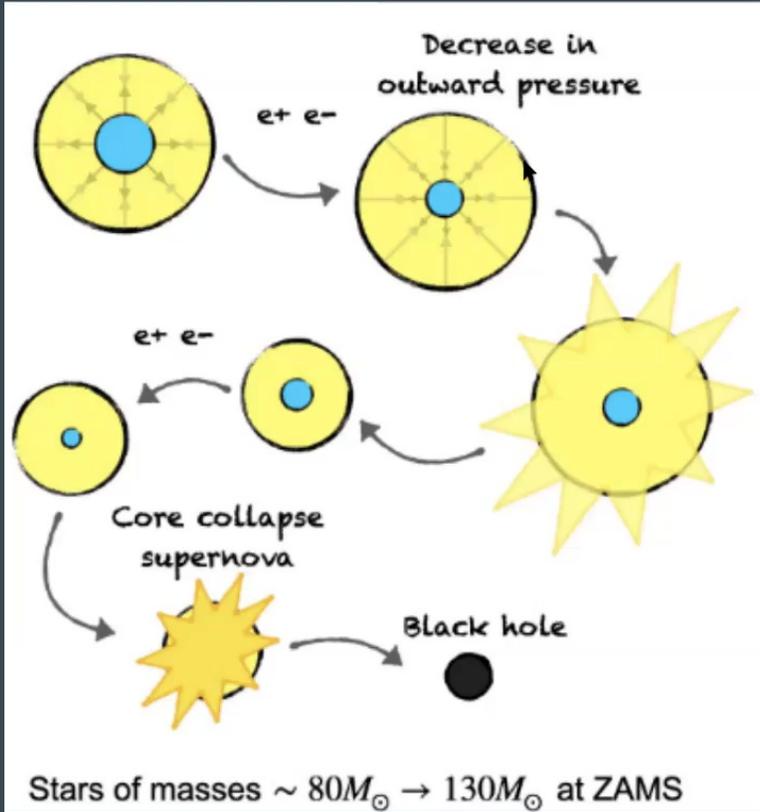


High Mass end: Sharp Cutoff or slow taper? PISN mass gap?

[R. Abbott et al 2021 ApJL 913 L7](#)



BBH Mass Population Models

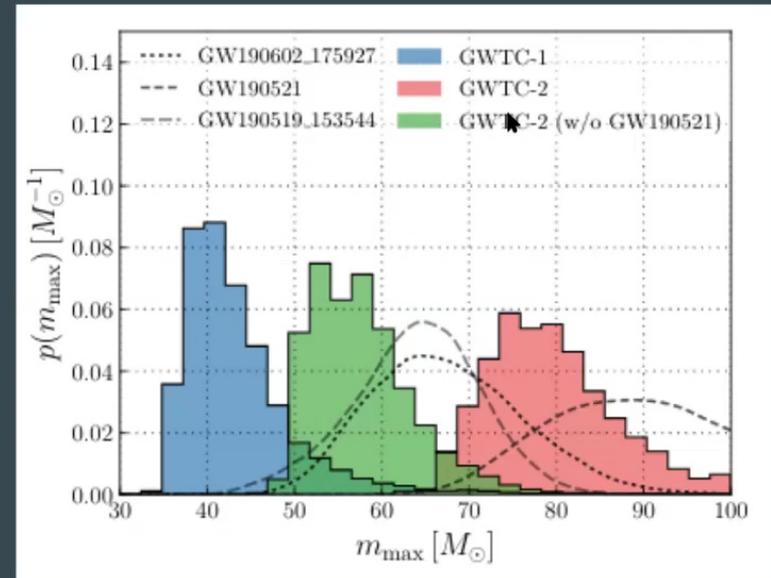


Is there a "pileup" of BBHs from PPISNe? Second peak from hierarchical mergers?

R. Abbott et al 2021 ApJL 913 L7

Astrophysical Lessons from GWTC-2

- Truncated Powerlaw is no longer consistent with the data
- We are seeing more heavy BBHs

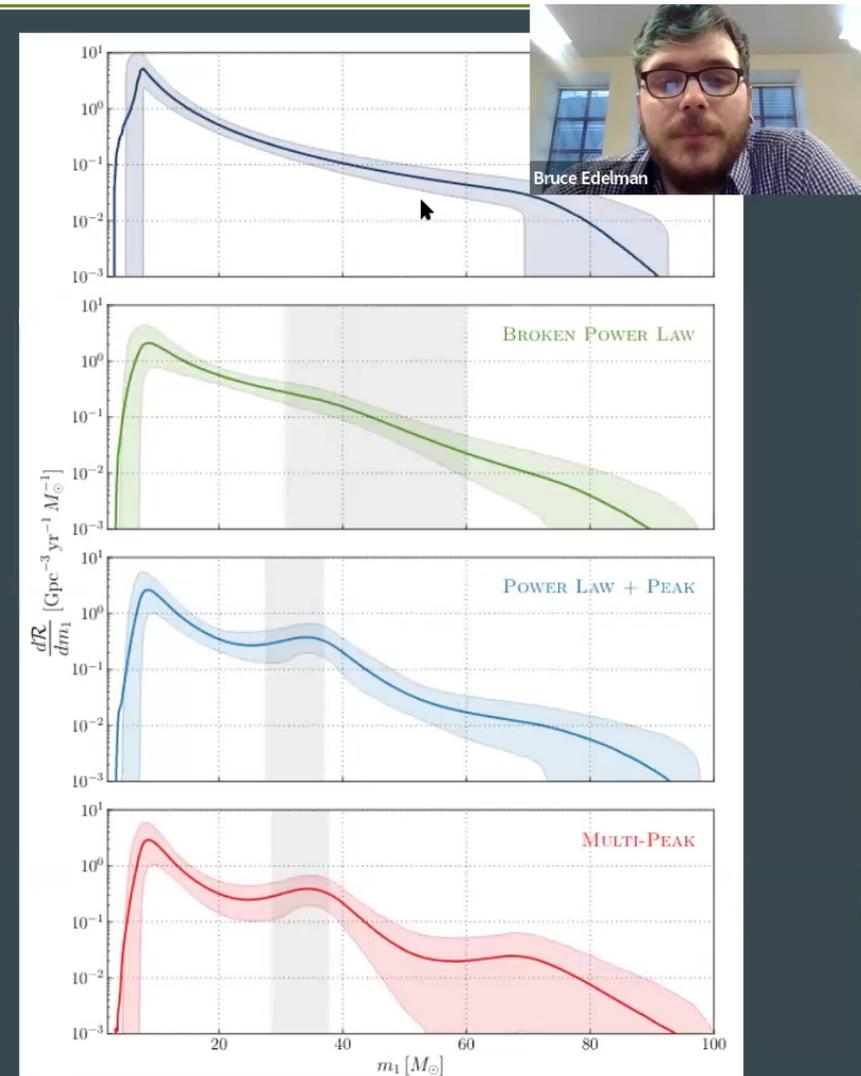


R. Abbott et al 2021 ApJL 913 L7

Astrophysical Lessons from GWTC-2

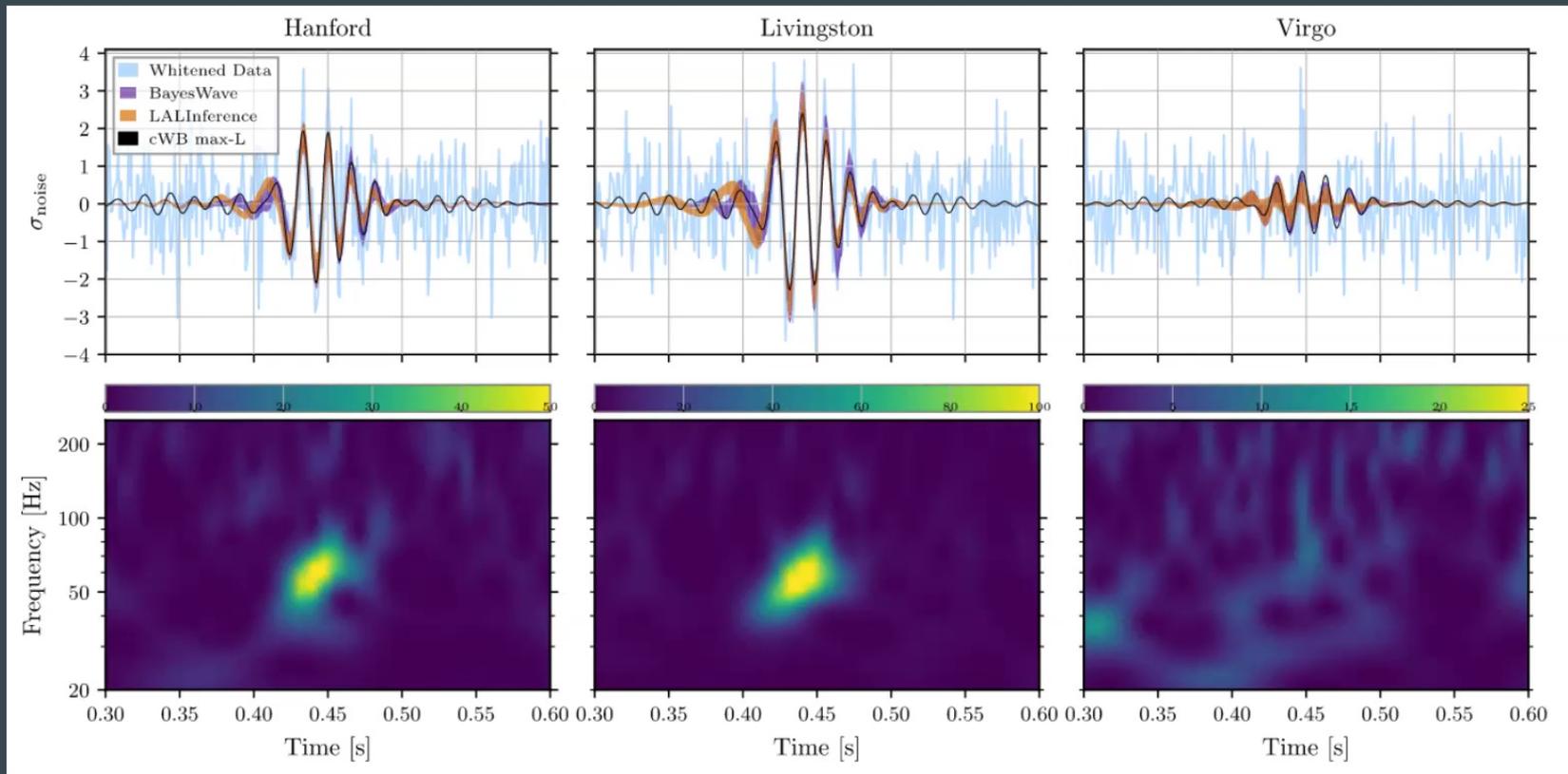
- Truncated Powerlaw is no longer consistent with the data
- We are seeing more heavy BBHs
- There is a feature at 35-40 solar masses that could be described by a peak or a break in the Powerlaw
- Appears to be a dearth of Low-mass black holes <6 solar masses

Mass model	\mathcal{B}	$\log_{10} \mathcal{B}$
POWER LAW + PEAK	1.0	0.0
MULTI PEAK	0.5	-0.3
BROKEN POWER LAW	0.12	-0.92
TRUNCATED	0.01	-1.91



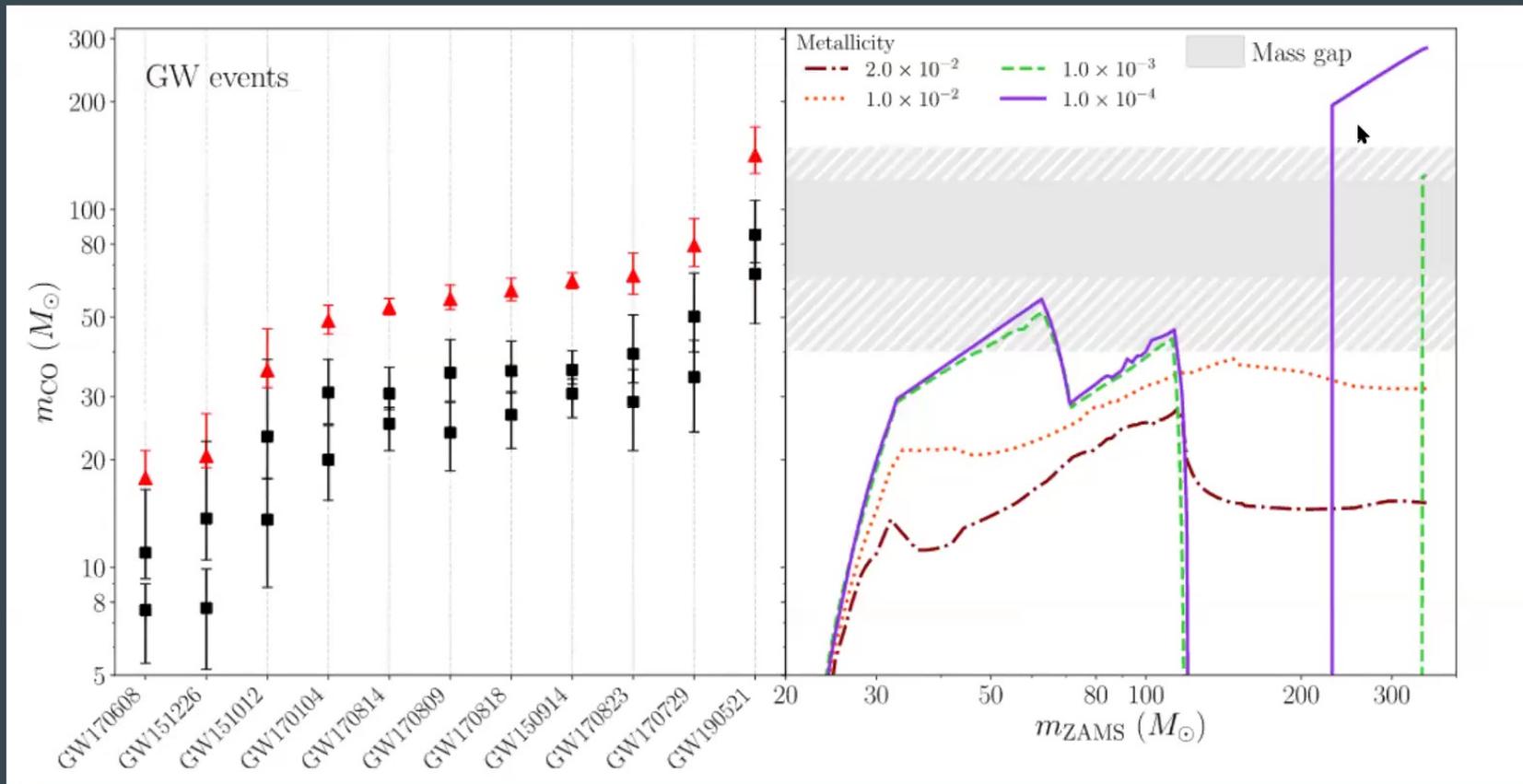
R. Abbott et al 2021 ApJL 913 L7

GW190521 – A Challenge for Pair-Instability



[Phys. Rev. Lett. 125, 101102](#)

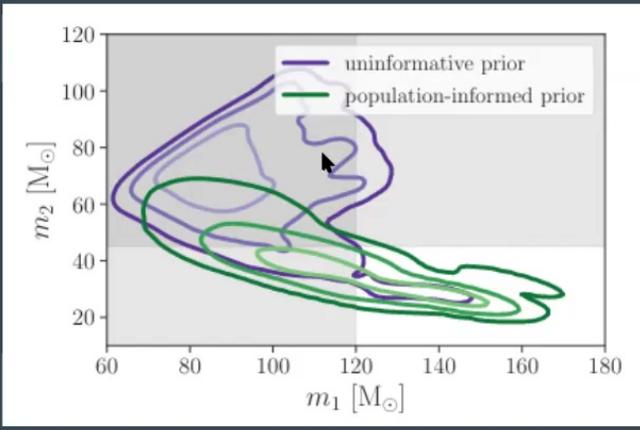
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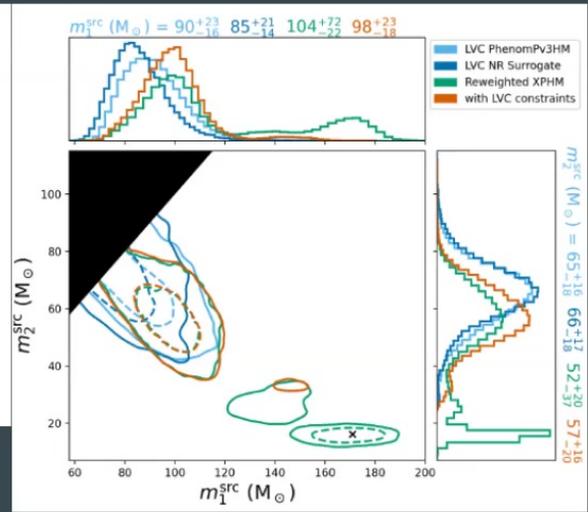
R. Abbott et al 2020 ApJL 900 L13



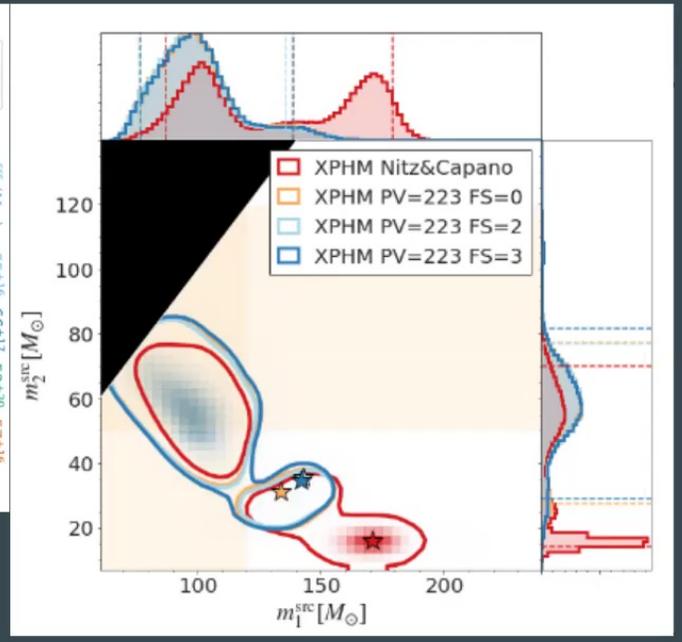
GW190521 – A Challenge for Pair-Instability



[Maya Fishbach and Daniel E. Holz 2020 *ApJL* 904 L26](#)



[Alexander H. Nitz and Collin D. Capano 2021 *ApJL* 907 L9](#)

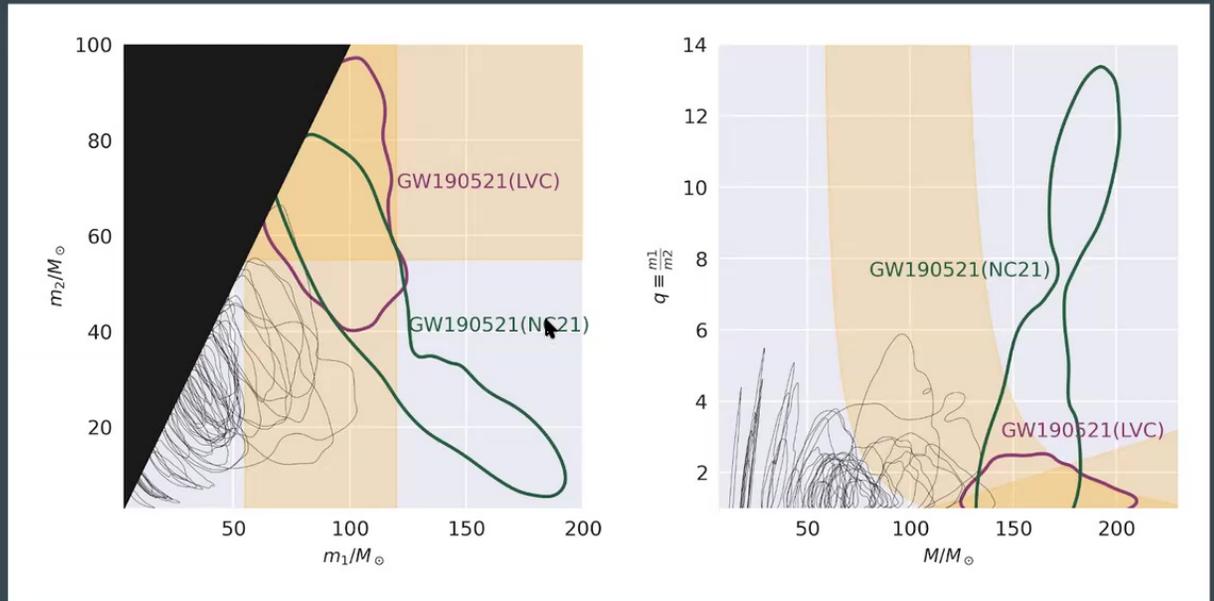


[Héctor Estellés et al 2022 *ApJ* 924 79](#)



Parameterized PISN Mass Gap Model

- GW190521's component masses could straddle the PISN mass gap
- Parameterize mass gap with start, m_g , (M_{sun}) and width, w_g (M_{sun})
- Use Truncated and Powerlaw+Peak model and explicitly enforce zero rate within the gap

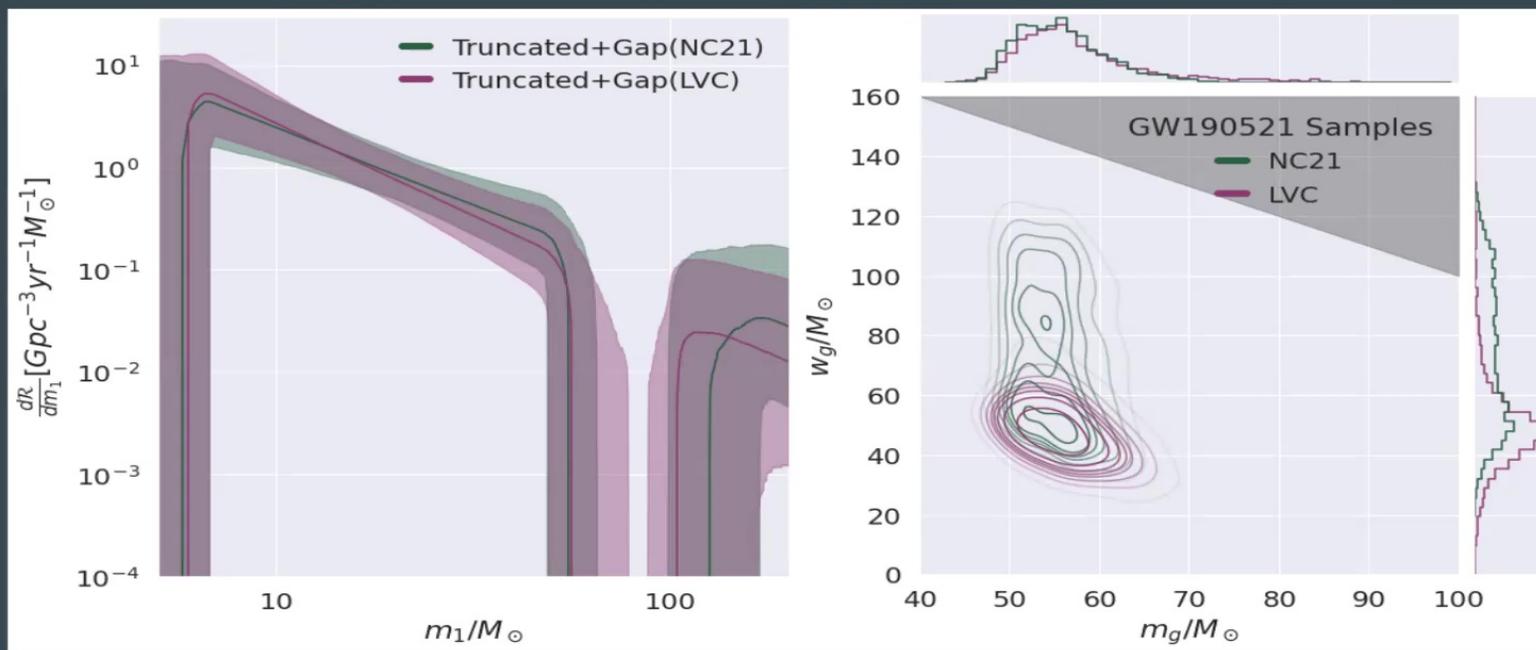


$$p(m_1|\Lambda, m_g, w_g) = \begin{cases} 0 & \text{for } m_g \leq m_1 \leq m_g + w_g \\ p(m_1|\Lambda) & \text{otherwise} \end{cases}$$

$$p(q|\Lambda, m_1, m_g, w_g) = \begin{cases} 0 & \text{for } m_g \leq m_1 * q \leq m_g + w_g \\ p(q|\Lambda) & \text{otherwise} \end{cases}$$

[Edelman, Doctor, Farr 2021](#)

Evidence for an Upper Mass gap in GWTC-2



log-BF for a gap: 2.7

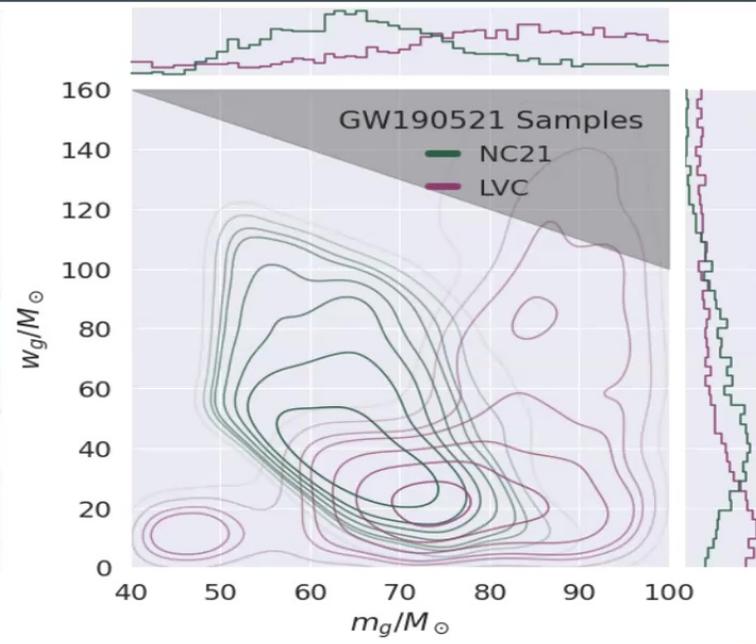
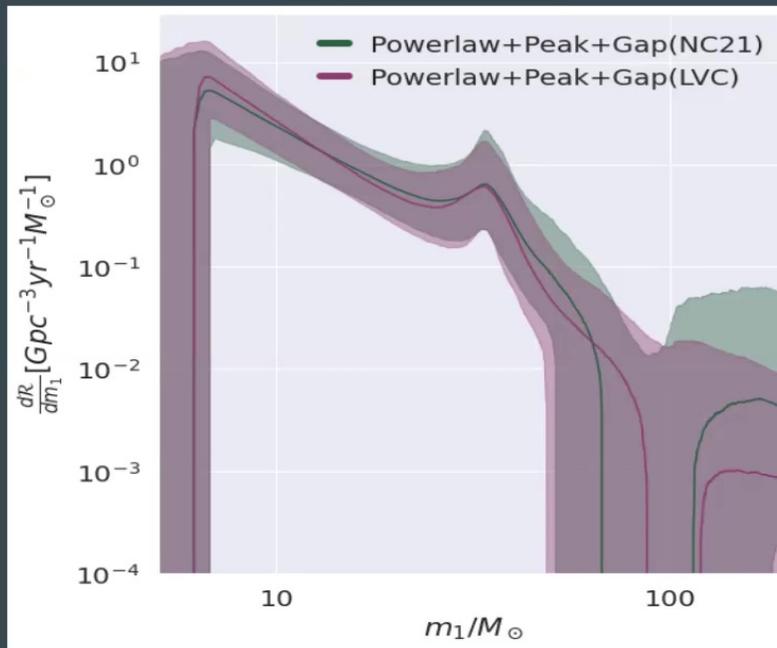
Gap bounds: $55.12^{+7.54}_{-4.38} \text{ ---> } 103.74^{+17.01}_{-6.32}$

log-BF for a gap: 6.5

Gap bounds: $55.33^{+5.21}_{-4.21} \text{ ---> } 126.03^{+30.25}_{-22.65}$

Edelman, Doctor, Farr 2021

Evidence for an Upper Mass gap in GWTC-2



log-BF for a gap: -0.5
(Inconclusive)

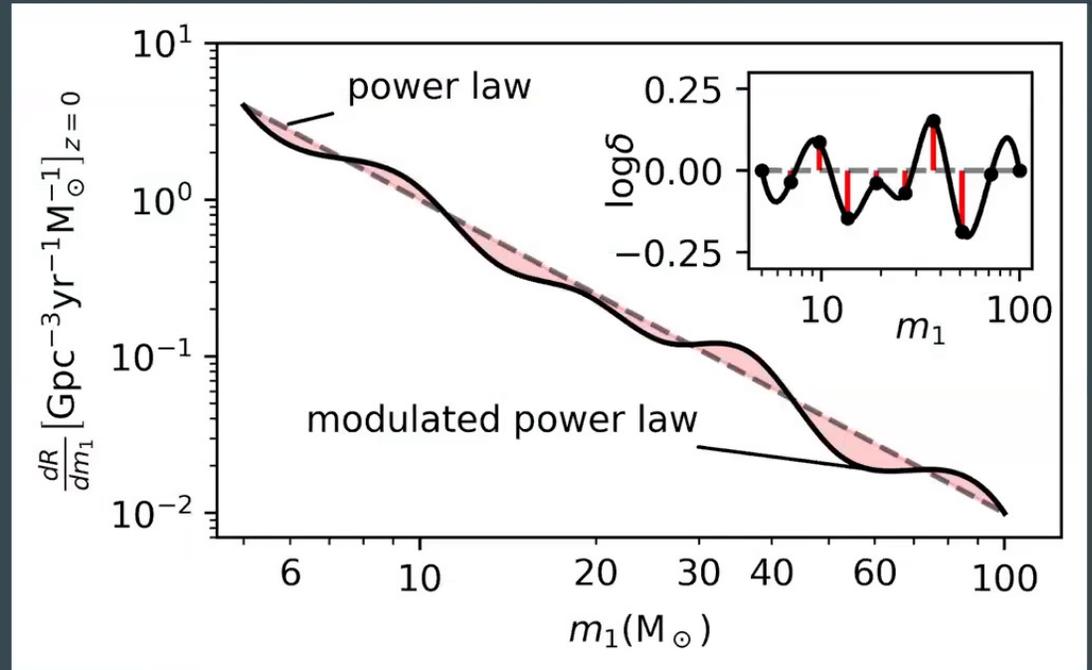
log-BF for a gap: 0.5
(Inconclusive)

[Edelman, Doctor, Farr 2021](#)



Semi-Parametric Modeling of the BBH Mass Distribution

- Introduce a multiplicative perturbation factor to the simple Truncated mass model
- We choose to model the perturbation with cubic splines interpolated from n knots



$$p_{\text{spline}}(m_1 | \Lambda, \{f_i\}) \propto p(m_1 | \Lambda) \exp(f(m_1 | \{f_i\}))$$

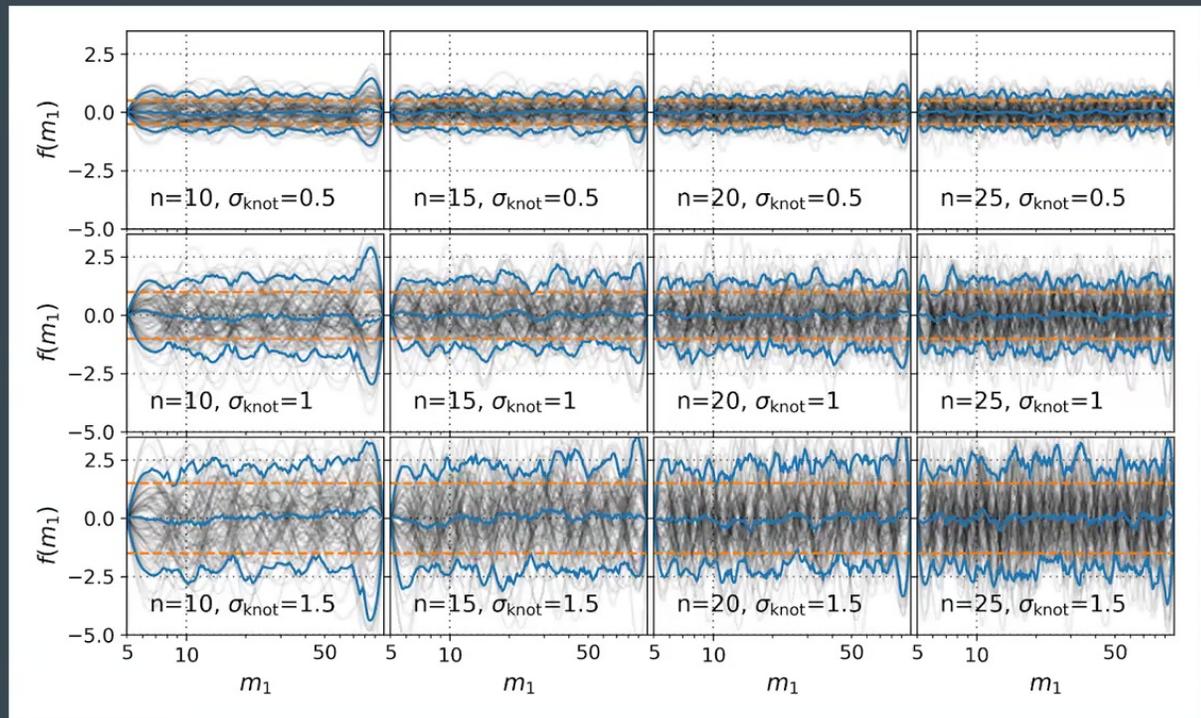
$$f(m_1) = \mathcal{I}_3(m_1; \{m_n, f_n\})$$

[Edelman et al 2022](#)

Semi-Parametric Modeling of the BBH Mass Distribution



- knots linearly spaced in log- m_1 space,
- Gaussian Priors on knot heights centered on zero
- Model Specifications
 - n , the number of knots, sets the resolution of perturbations
 - Width of the gaussian priors sets the a priori magnitude of perturbations



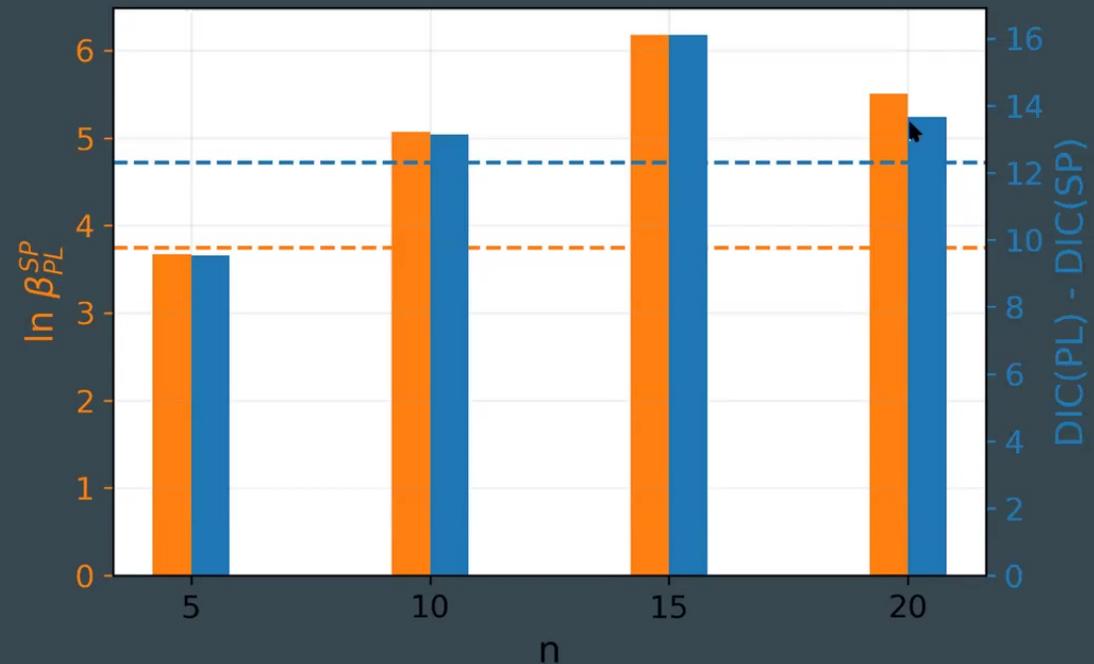
$$p(\{f_i\}) = \mathcal{N}(\mu = 0, \sigma = \sigma_{\text{knot}})$$

[Edelman et al 2022](#)

Semi-Parametric Modeling of the BBH Mass Distribution

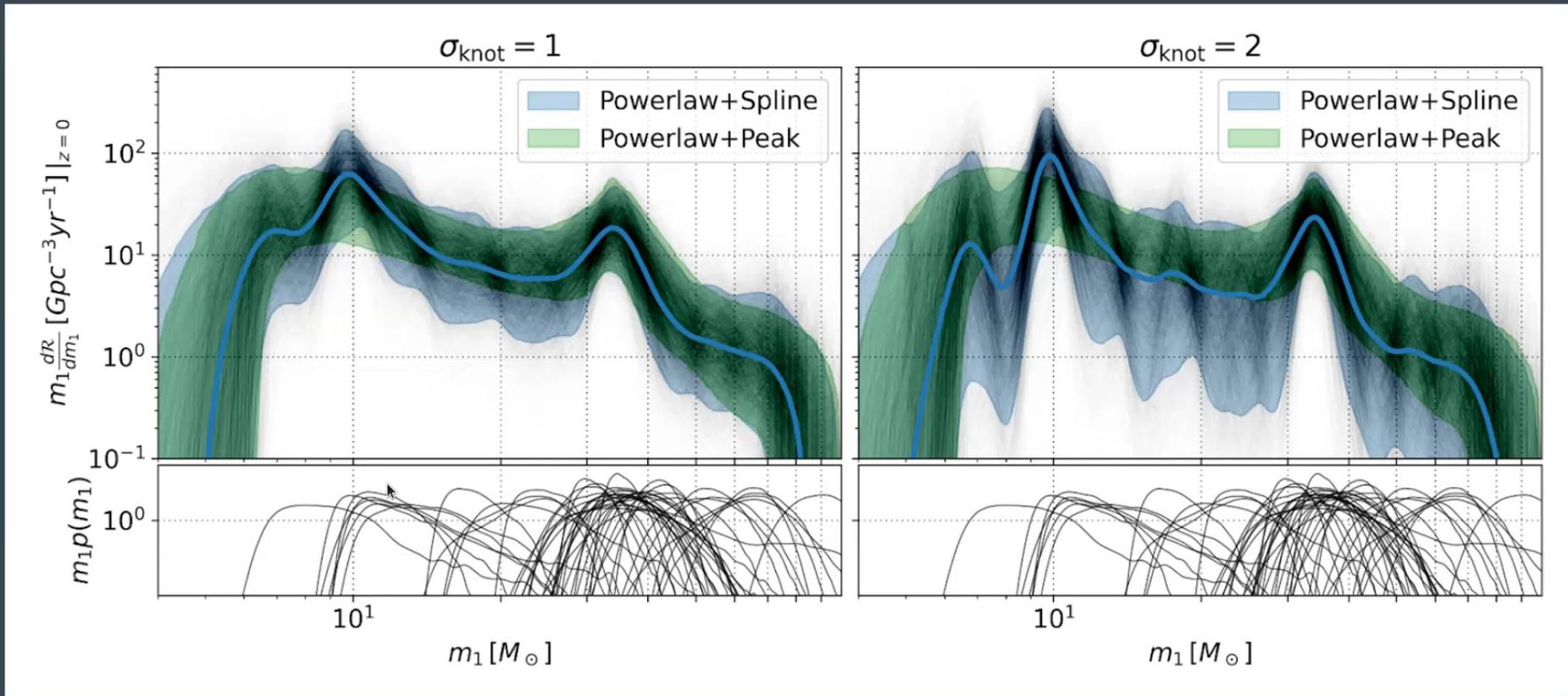


- Run multiple models with variable number of knots
- Use Model Comparison Metrics to evaluate
- Combine posterior samples from each model variant according each BF



[Edelman et al 2022](#)

Signs of Structure in the BBH Mass Distribution

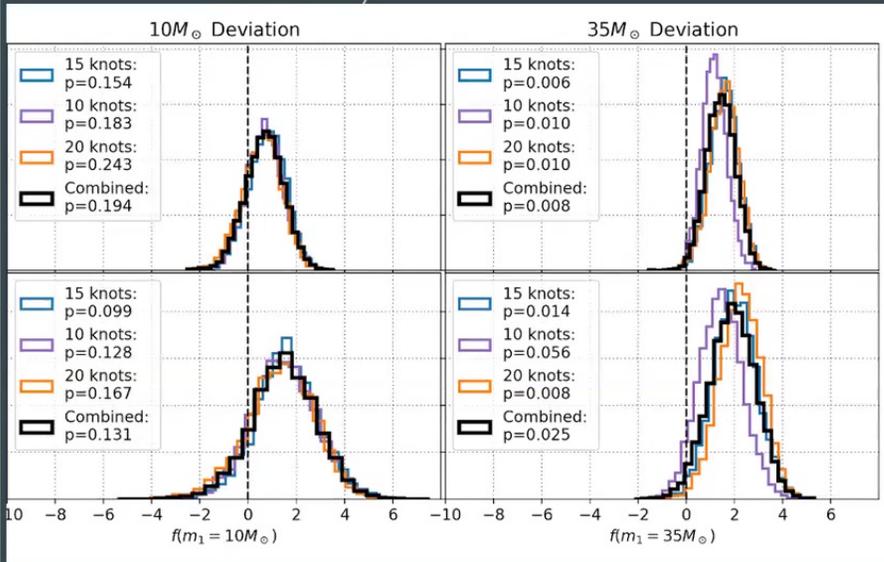


[Edelman et al 2022](#)

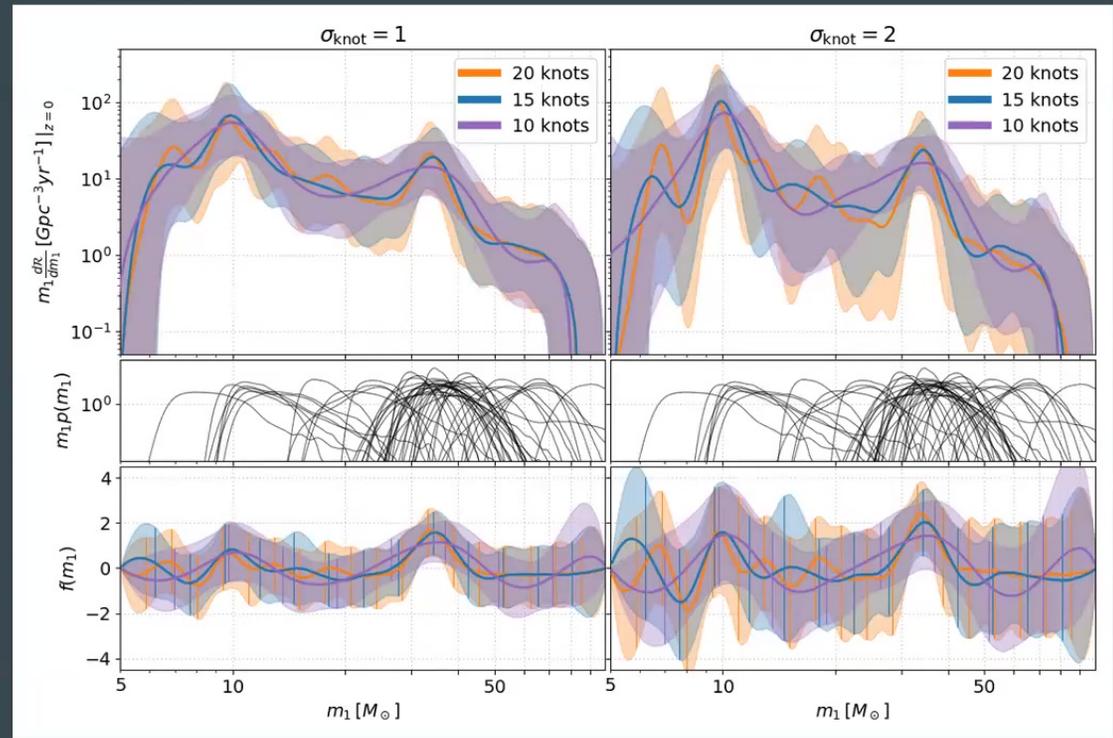
Signs of Structure in the BBH Mass Distribution



- 10 Solar mass excess at 80-87% credibility
- 35 Solar mass excess at 97-99% credibility



[Edelman et al 2022](#)



- Less certain about low-mass end where there are less observations and sensitivity



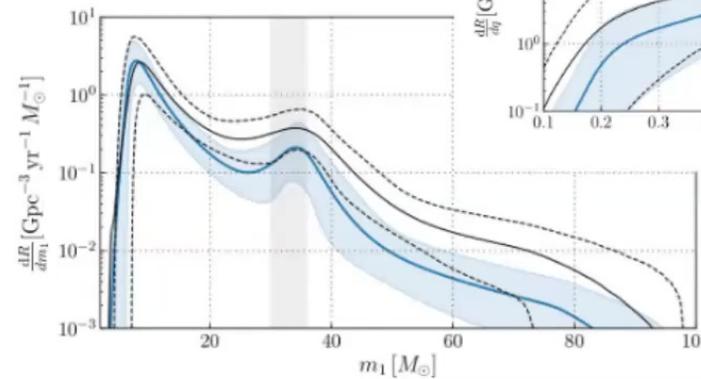
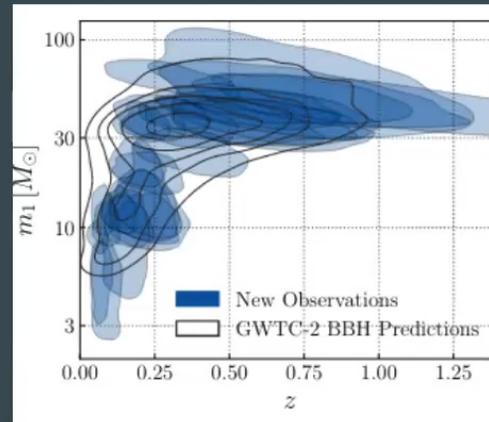
Overview

- Single Source Inference and Parameter Estimation
 - Coherent Waveform Deviation Model ([Edelman et. al 2021](#))
- Hierarchical Inference of a Population of Sources
 - Pair-Instability Mass Gap Model ([Edelman, Doctor, Farr 2021](#))
 - Semi-Parametric Modeling of the BBH Mass Distribution ([Edelman et al 2022](#))
- Population Updates from GWTC-3



What is new in the population with GWTC-3? (BBH Mass Distributions)

- New events broadly consistent with previous predictions
- PL+Peak model still “favorite”
- Steeper primary mass power law
- Shallower mass ratio power law

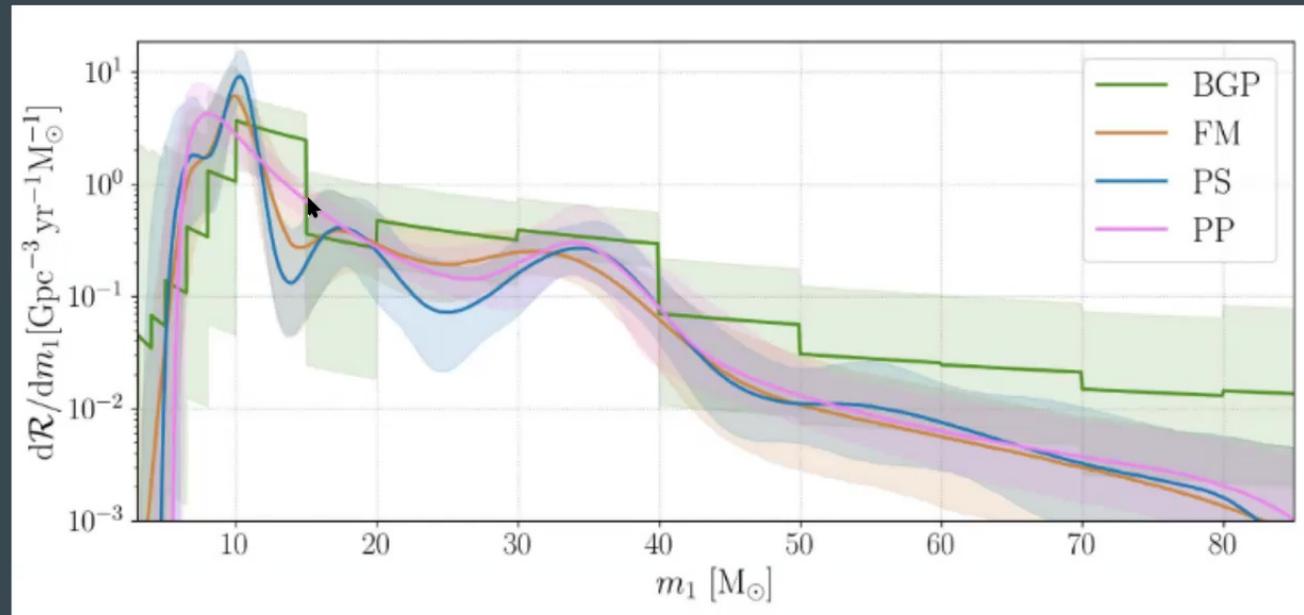


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What is new in the population with GWTC-3? (BBH Mass D

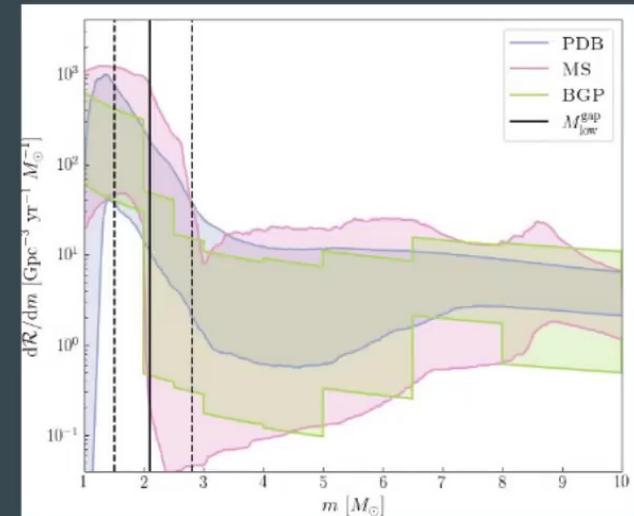
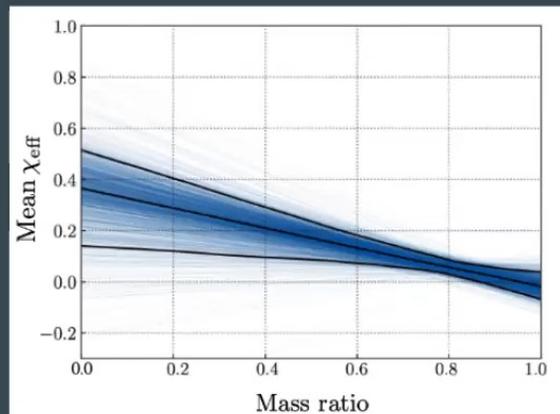
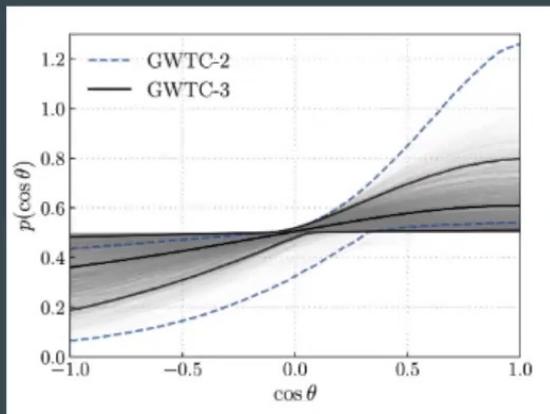
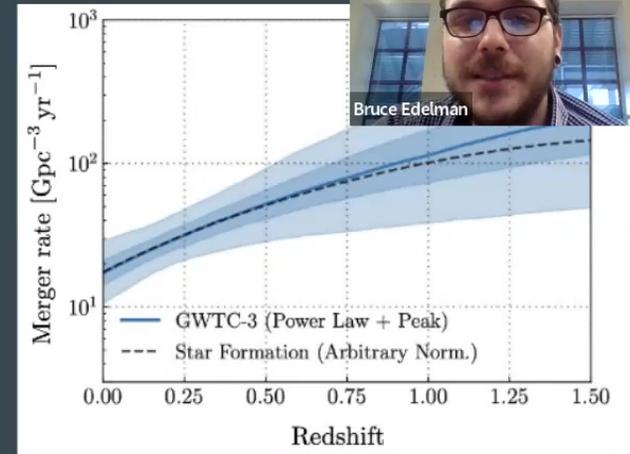
- First application of more “flexible” models
- Three different Non or semi parametric approaches broadly agree
- Signs of Structure in the mass distribution (peaks/dips/etc) are now stronger than in GWTC-2
- Repeat of PISN mass gap model with LVK GW190521 samples in GWTC-3 shows inconclusive evidence



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GWTC-3 Population - Other Lessons Learned

- NS Mass distribution broader than galactic
- First self-consistent measurement of merger rate across all compact object masses
- Merger rate confidently increases with redshift
- Misaligned Spins are present in the population
- Correlation between spin and mass ratio

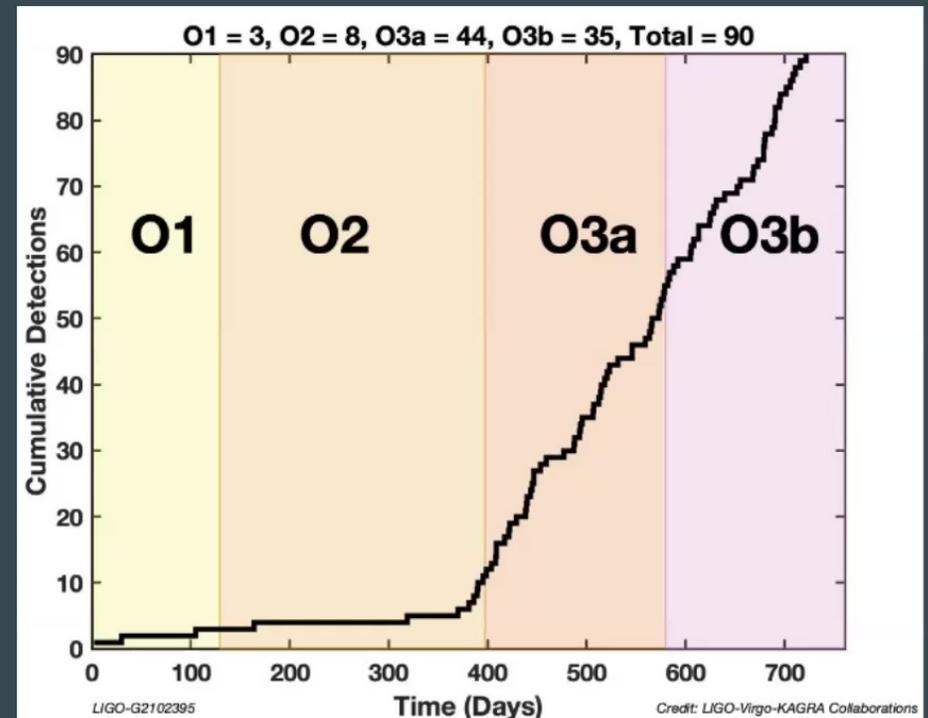


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Conclusions

- Data-Driven Models provide a useful method for testing where our observations agree or disagree with expectations
- The population of compact objects is proving to be a treasure trove of astrophysical information
- O4 coming soon will only increase the fidelity we can probe these properties with
- Semi-parametric population modeling will only become more necessary as we further develop our understanding





Thanks!

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