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Speakers: Beatriz Elizaga

Series: Strong Gravity

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Zoom Link: https://pitp.zoom.us/j/96387248110?pwd=U0xLNldzMldKbmhPc1V0SEhzdkJNZz09

Pre-inflationary effects on the primordial power spectrum: GR vs. LQC

[Based on JCAP 09 (2021) 030]

January 13th, 2022
Perimeter Institute - Strong gravity seminar

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Motivation

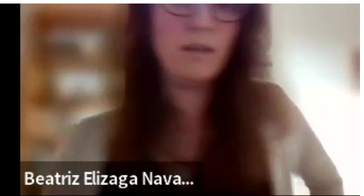
- Standard cosmological model with inflation is extremely successful describing primordial inhomogeneities.
- Is it completely satisfactory, considering the primeval stages of the Universe with very high curvature?
- Theoretically:
  - ★ Big-Bang singularity: loss of predictability.
  - ★ Quantum gravity phenomena?
  - ★ Non-inflationary epoch: State for the perturbations?

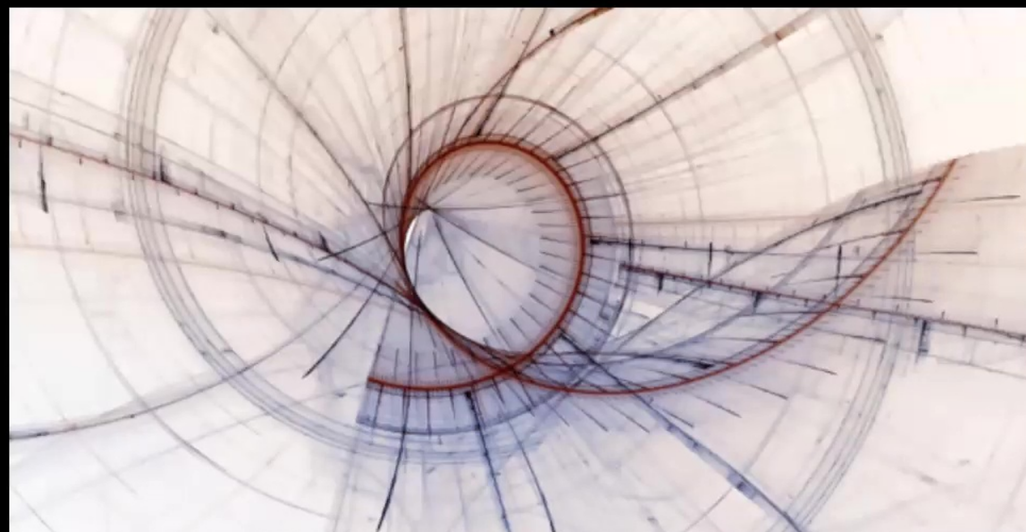
Motivation

- Standard cosmological model with inflation is extremely successful describing primordial inhomogeneities.
- Is it completely satisfactory, considering the primeval stages of the Universe with very high curvature?
- Observationally:
  - ⋆ Angular power spectrum in CMB: Anomalies
  - ⋆ Power suppression $\ell \lesssim 30$, lensing amplitude > 1, …
  - ⋆ Strongly affected by cosmic variance, but could point to new physics ⟶ Planck regime of the Universe?

Motivation

- Standard cosmological model with inflation is extremely successful describing primordial inhomogeneities.
- Is it completely satisfactory, considering the primeval stages of the Universe with very high curvature?
- Theoretical and observational concerns.
- Promising candidate: Loop Quantum Cosmology (LQC).
- Typically includes a classical pre-inflationary epoch.
- Robust predictions **require** disentangling LQC from GR effects on the evolution of the perturbations.





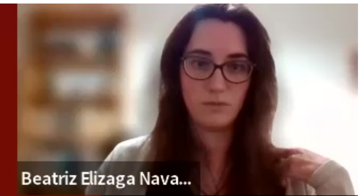
Loop Quantum Cosmology: Mukhanov-Sasaki equations

What is LQC?

- Canonical quantization program for spacetimes with high degree of symmetry: e.g. cosmological spacetimes.
- Techniques from the non-perturbative theory of LQG.
- In e.g. FLRW-type cosmologies with massless scalar field, well-defined Hilbert space with nice physical properties:
  - ★ Energy density operator bounded from above.
  - ★ States of zero-volume are dynamically decoupled.
  - ★ Spectral analysis: Strong evidence for the quantum resolution of the cosmological singularity.

e.g. [A. Ashtekar et al., Phys. Rev. Lett. 96 (2006) 141301;
M. Martín-Benito et al., Phys. Rev. D 80 (2009) 104015]

What is LQC?



- Canonical quantization program for spacetimes with high degree of symmetry: e.g. cosmological spacetimes.

- Techniques from the non-perturbative theory of LQG.

- In e.g. FLRW-type cosmologies with massless scalar field, well-defined Hilbert space with nice physical properties.

- Provides mechanisms to resolve Big-Bang singularity
  → Big Bounce.

- Can be combined with standard quantum field theory techniques to include inhomogeneities.

Effective LQC

- FLRW-type spacetime with massless scalar field ϕ.
- Family of physical states with Gaussian behavior, peaked in the geometry for all values of ϕ.
- Trajectories of peaks are relativistic for low energies, departures close to Planck density ⟶ bounce.
- Modified Friedmann equations, $\rho_c \simeq 0.41$ (Planck units):

$$\left(\frac{a'}{a}\right)^2 = \frac{8\pi}{3} a^2 \rho \left(1 - \frac{\rho}{\rho_c}\right), \qquad \rho = \frac{1}{2}\left(\frac{\phi'}{a}\right)^2,$$

$$\phi'' + 2\frac{a'}{a}\phi' = 0$$

Perturbations in LQC

- Hybrid quantization of perturbed cosmology with inflaton:
  - ★ Background cosmology: LQC techniques.
  - ★ Gauge-invariant perturbations: Fock representation.
- Canonical formulation ⟶ Zero-mode of the Hamiltonian constraint: FLRW + Mukhanov-Sasaki Hamiltonians.
- Physical states should only depend on the background cosmology and the gauge-invariant perturbations.
- Physical states should be annihilated by the Hamiltonian constraint operator, couples background and perturbations.

e.g. [L. Castelló Gomar et al., JCAP 06 (2015) 045]

Mukhanov-Sasaki equations

- Physical states should be annihilated by the constraint.
- Focus on states with small backreaction on background.
- Mean-field approximation on constraint equation

  → Effective constraint for the perturbations, depends on background geometry via expectation values.

- Effective Mukhanov-Sasaki equations:

$$v''_{\vec{k}} + [k^2 + s_{\text{eff}}]v_{\vec{k}} = 0, \qquad s_{\text{eff}} = s_{\text{eff}}(\eta)$$

Mass codifies LQC effects on the background.

e.g. [L. Castelló Gomar et al., JCAP 06 (2015) 045]

Mukhanov-Sasaki equations

- Effective Mukhanov-Sasaki equations:

$$v''_{\vec{k}} + [k^2 + s_{\text{eff}}]v_{\vec{k}} = 0$$

- Restrict to background states with effective LQC behavior:

$$\left(\frac{a'}{a}\right)^2 = \frac{8\pi}{3}a^2\rho\left(1 - \frac{\rho}{\rho_c}\right), \qquad \rho = \frac{1}{2}\left(\frac{\phi'}{a}\right)^2 + V(\phi),$$

$$\phi'' + 2\frac{a'}{a}\phi + a^2 V_{,\phi} = 0$$

- For these states, the mass can be evaluated on the peaks:

$$s_{\text{eff}} = -\frac{4\pi G}{3}a^2(\rho - 3P) + \mathcal{U}$$

Cosmological solutions

- Effective Mukhanov-Sasaki equations:

$$v''_{\vec{k}} + [k^2 + s_{\text{eff}}]v_{\vec{k}} = 0$$

- Restrict to background states with effective LQC behavior.

- Phenomenologically interesting solutions: Large observable scales a/k today were $\sim$ order of curvature at the bounce.

- They are all such that $\rho_B^{kin} \gg V(\phi_B)$,

$$\rho_c = \rho_B^{kin} + V(\phi_B), \qquad \rho_B^{kin} = \frac{1}{2}\left(\frac{\phi'_B}{a_B}\right)^2$$

e.g. [I. Agullo and N.A. Morris, Phys. Rev. D 92 (2015) 124040]

Cosmological solutions

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- They are all such that $\rho_B^{kin} \gg V(\phi_B)$.

- They imply a short-lived inflation ($\gtrsim 65$ e-folds), and a classical deccelerated preinflationary expansion.

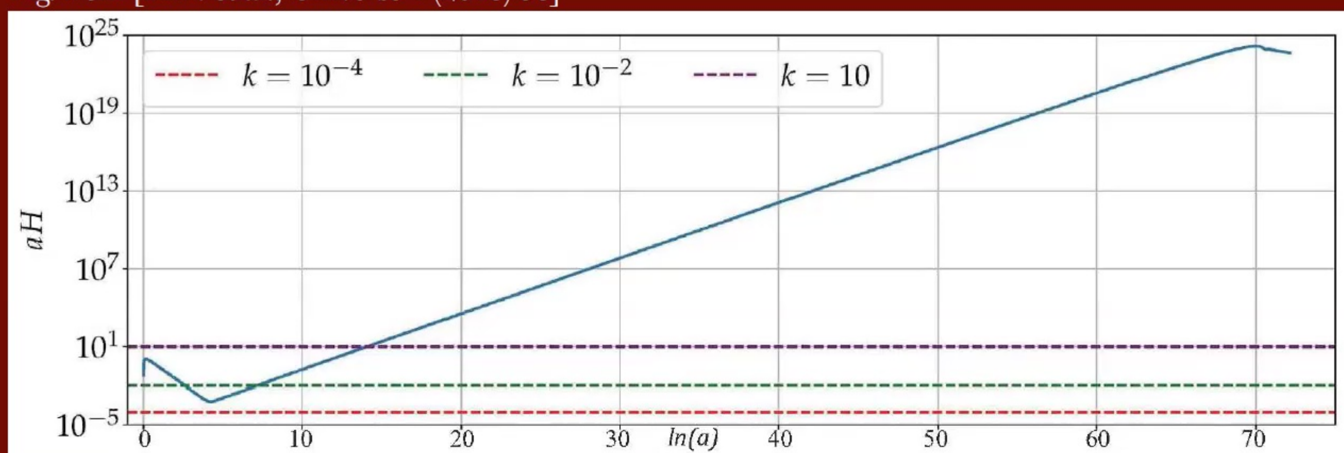
Cosmological solutions

- Effective Mukhanov-Sasaki equations:

$$v''_{\vec{k}} + [k^2 + s_{\rm eff}]v_{\vec{k}} = 0$$

- Focus on phenomenologically interesting effective LQC solutions, e.g. for $V(\phi) = m^2\phi^2/2, \; m = 1.2 \times 10^{-6}, \; \phi_B = 0.97$:

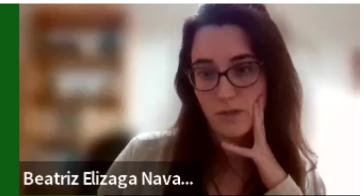
Fig. from [BEN et al., Universe 4 (2018) 98]



LQC GR!! Kinetically dominated (KD) period + Slow-roll inflation

Classical inflation with KD regime

Perturbations in KD period



- Deep in the pre-inflationary epoch, the potential is completely negligible compared with kinetic energy.

- Approximate this classical epoch (η_0, η_i) as a Friedmann universe with a massless scalar field:

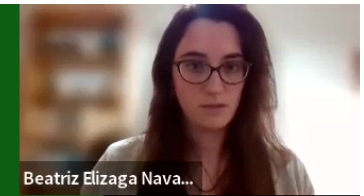
$$a(\eta) = a_0\sqrt{1 + 2a_0H_0(\eta - \eta_0)}, \qquad \rho(\eta) = \rho_0 \left[\frac{a_0}{a(\eta)}\right]^6$$

where $H = \sqrt{8\pi\rho/3}$, $H_0 = H(\eta_0)$, $a_0 = a(\eta_0)$.

★ Time where quantum geometry effects are negligible, for the effective LQC solutions.

★ Alternatively, in a pure GR model η_0 can be arbitrarily close to the singularity.

Perturbations in KD period



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- Mukhanov-Sasaki equations:

$$v''_{\vec{k}} + \left(k^2 + s_{\text{kin}}\right) v_{\vec{k}} = 0, \qquad s_{\text{kin}} = \frac{1}{4}\left(\eta - \eta_0 + \frac{1}{2H_0a_0}\right)^{-2}$$

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- General (isotropic) solutions:

$$\mu_k = C_k \sqrt{\frac{\pi y}{4}} H_0^{(1)}(ky) + D_k \sqrt{\frac{\pi y}{4}} H_0^{(2)}(ky), \qquad y = \eta - \eta_0 + \frac{1}{2H_0 a_0}$$

- They provide normalized positive-frequency solutions if:

$$\mu_k \mu_k^{*\prime} - \mu_k' \mu_k^* = i \quad \Longleftrightarrow \quad |D_k|^2 - |C_k|^2 = 1$$

Perturbations in de Sitter

- The evolution of the Universe during slow-roll inflation is of quasi-de Sitter type.
- For our purposes here, we ignore transition effects and deviations from an exact de Sitter phase.

Perturbations in de Sitter

- The evolution of the Universe during slow-roll inflation is of quasi-de Sitter type.

- Approximate the inflationary period $[\eta_i, \eta_{\rm end}]$ as de Sitter:

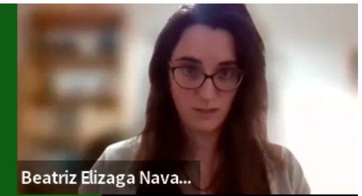
$$a(\eta) = \left[a_i^{-1} - H_\Lambda(\eta - \eta_i)\right]^{-1}, \qquad a_i = a_0\sqrt{1 + 2a_0H_0(\eta_i - \eta_0)}$$

where $H_\Lambda = \sqrt{8\pi\rho_0/3}\,\left(a_0/a_i\right)^3$.

- Mukhanov-Sasaki equations:

$$v''_{\vec{k}} + \left(k^2 + s_{\rm dS}\right) v_{\vec{k}} = 0, \qquad s_{\rm dS} = -\,2H_\Lambda^2\left[a_i^{-1} - H_\Lambda(\eta - \eta_i)\right]^{-2}.$$

Perturbations in de Sitter



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- General (isotropic) solutions:

$$\mu_k = A_k \frac{e^{ik(\eta - \eta_i - a_i^{-1}H_\Lambda^{-1})}}{\sqrt{2k}} \left[1 + \frac{i}{k(\eta - \eta_i - a_i^{-1}H_\Lambda^{-1})}\right]$$

$$+ B_k \frac{e^{-ik(\eta - \eta_i - a_i^{-1}H_\Lambda^{-1})}}{\sqrt{2k}} \left[1 - \frac{i}{k(\eta - \eta_i - a_i^{-1}H_\Lambda^{-1})}\right]$$

- They provide normalized positive-frequency solutions if:

$$|B_k|^2 - |A_k|^2 = 1$$

Inflation with KD epoch in GR

- Approximate pre-inflationary epoch (η_0, η_i) as a Friedmann universe with a massless scalar field.

- Approximate the inflationary period $[\eta_i, \eta_{\text{end}}]$ as de Sitter.

- Instantaneous transition between both periods.

- Approximate Mukhanov-Sasaki equations:

$$v''_{\vec{k}} + \left(k^2 + \tilde{s}_{\text{GR}}\right) v_{\vec{k}} = 0,$$

Discontinuity!

$$\tilde{s}_{\text{GR}} = \begin{cases} \frac{1}{4}\left(\eta - \eta_0 + \frac{1}{2H_0 a_0}\right)^{-2}, & \eta \in (\eta_0, \eta_i) \\ -2H_\Lambda^2 \left[a_i^{-1} - H_\Lambda(\eta - \eta_i)\right]^{-2} & \eta \in [\eta_i, \eta_{\text{end}}] \end{cases}$$

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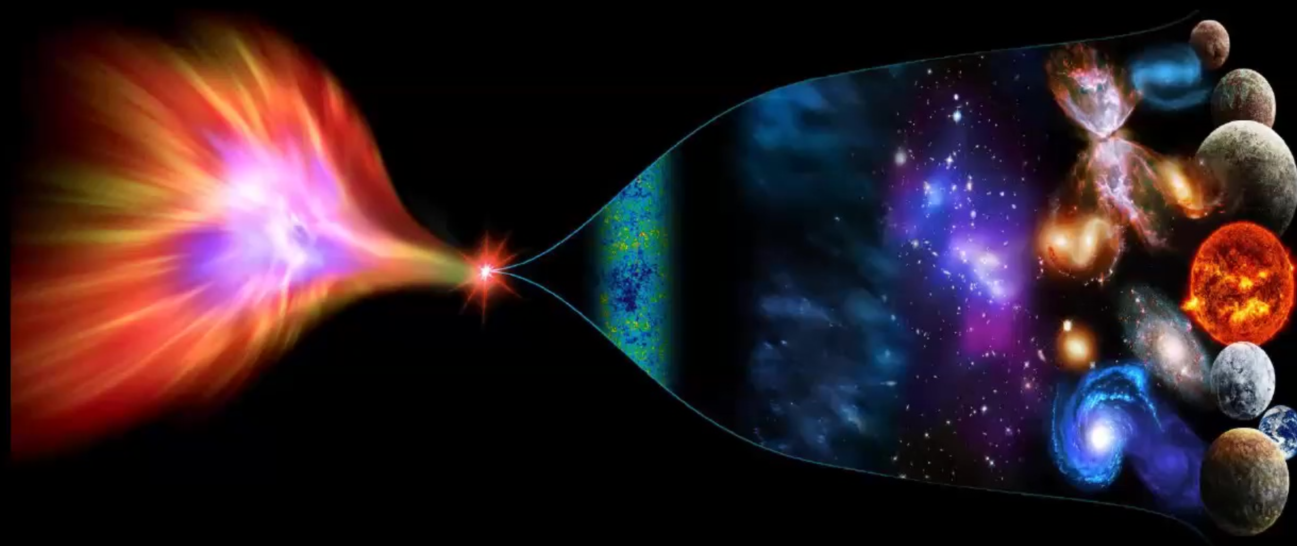
- Solutions in each epoch can be matched by continuity.

- Fixing initial conditions in the KD regime fixes the state of the perturbations during de Sitter inflation (and conversely):

$$A_k = A_k(C_k, D_k), \qquad B_k = B_k(C_k, D_k)$$



LQC with short-lived inflation

Pre-inflationary epoch with KD

- Phenomenologically interesting solutions in effective LQC: Kinetic domination around the bounce: $V(\phi) \simeq 0$.

- Background equations are analytically solvable:

$$\dot{\phi}(t) = \pm\sqrt{2\rho_c}\,a^{-3}(t), \qquad a(t) = \left(1 + 24\pi\rho_c t^2\right)^{1/6},$$

$$\eta - \eta_B = {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; -24\pi\rho_c t^2\right) t$$

- Then, Mukhanov-Sasaki equations with $V(\phi) = 0$:

$$v''_{\vec{k}} + \left(k^2 + s_h^{\text{kin}}\right) v_{\vec{k}} = 0, \qquad s_h^{\text{kin}} = \frac{8\pi\rho_c}{3}\left(1 + 24\pi\rho_c t^2\right)^{-2/3}$$

We do not know their analytic solutions (yet…).

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Cosmological solutions

- Effective Mukhanov-Sasaki equations:

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- Restrict to background states with effective LQC behavior.
- Phenomenologically interesting solutions: Large observable scales a/k today were $\sim$ order of curvature at the bounce.
- They are all such that $\rho_B^{kin} \gg V(\phi_B)$.
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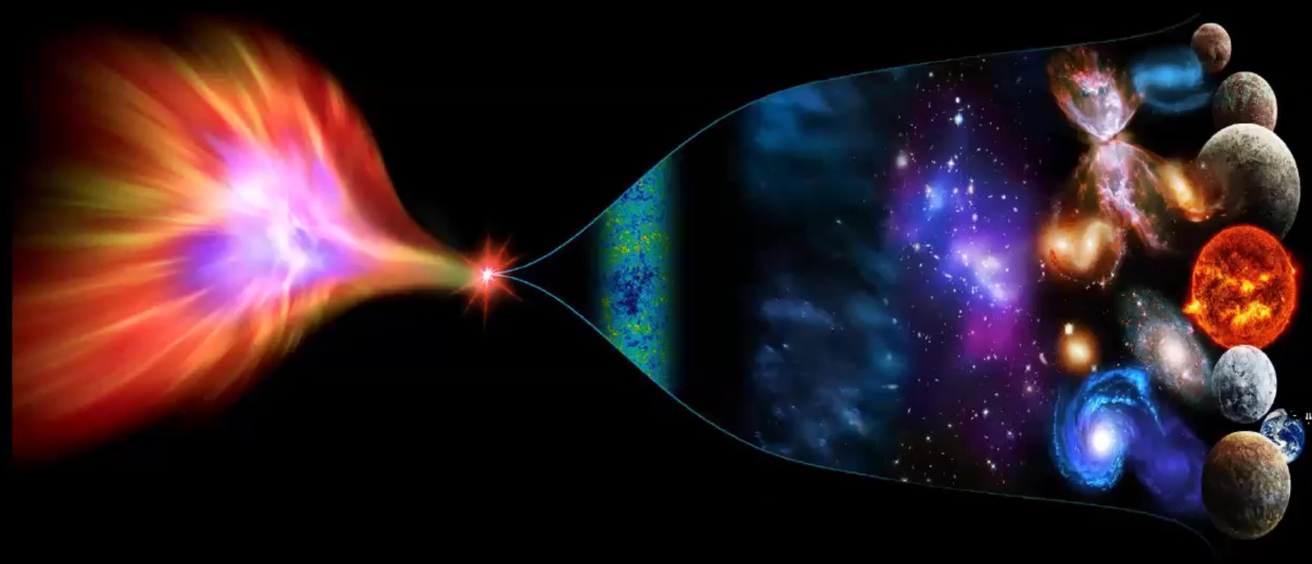

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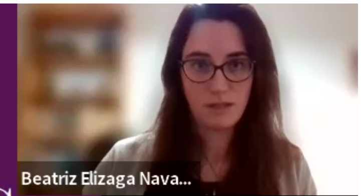
Pre-inflationary epoch with KD



- Phenomenologically interesting solutions in effective LQC: Kinetic domination around the bounce: $V(\phi) \simeq 0$.

- Background equations are analytically solvable:

$$\dot{\phi}(t) = \pm\sqrt{2\rho_c}\, a^{-3}(t), \qquad a(t) = \left(1 + 24\pi\rho_c t^2\right)^{1/6},$$

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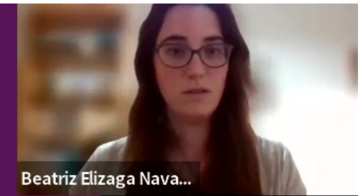
We do not know their analytic solutions (yet…).

Pre-inflationary epoch with KD

- Ignoring transition effects, we can approximate $V(\phi) \simeq 0$ until the onset of inflation at η_i.
- On the other hand, the strong LQC effects are relevant only in a very narrow interval around the bounce.
- We try to find a manageable approximation of s_h^{kin} around the bounce at η_B, until an instant η_0 when GR holds.

Pre-inflationary epoch with KD



- Ignoring transition effects, we can approximate $V(\phi) \simeq 0$ until the onset of inflation at η_i.

- We try to find a manageable approximation of $s_h^{\rm kin}$ around the bounce at η_B, until an instant η_0 when GR holds.

- Then, we can approximate the period (η_0, η_i) by a classical universe with a massless scalar field, with initial data:

$$a_0 = \left(1 + 24\pi\rho_c t_0^2\right)^{1/6}, \qquad \rho_0 = \rho_c a_0^{-6}$$

- Studying the relative difference between $s_h^{\rm kin}$ and $s_{\rm kin}$ we find that for $t_0 \gtrsim 0.4$ the error is no bigger than a $3\,\%$.

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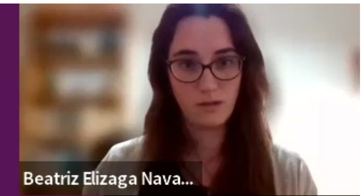


- Ignoring transition effects, we can approximate $V(\phi) \simeq 0$ until the onset of inflation at η_i.
- We try to find a manageable approximation of s_h^{kin} around the bounce at η_B, until an instant η_0 when GR holds.
- Then, we can approximate the period (η_0, η_i) by a classical universe with a massless scalar field, with initial data:

$$a_0 = \left(1 + 24\pi\rho_c t_0^2\right)^{1/6}, \qquad \rho_0 = \rho_c a_0^{-6}$$

- Studying the relative difference between s_h^{kin} and s_{kin} we find that for $t_0 \gtrsim 0.4$ the error is no bigger than a 3 % .

Pöschl-Teller approximation

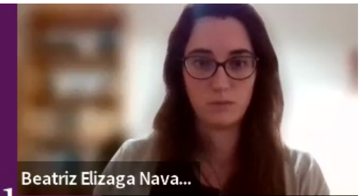
- We try to find a manageable approximation of s_h^{kin} around the bounce at η_B, until an instant η_0 when GR holds.

- Around the bounce, approximate s_h^{kin} by

$$s_{\text{PT}} = \frac{U_0}{\cosh^2[\alpha(\eta - \eta_B)]}, \qquad U_0 = \frac{8\pi\rho_c}{3}, \qquad \alpha = \frac{\text{arcosh}(a_0^2)}{(\eta_0 - \eta_B)}$$

so that $s_{\text{PT}}(\eta_B) = s_h^{\text{kin}}(\eta_B)$ and $s_{\text{PT}}(\eta_0) = s_{\text{kin}}(\eta_0)$?

Pöschl-Teller approximation



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  so that $s_{\text{PT}}(\eta_B) = s_h^{\text{kin}}(\eta_B)$ and $s_{\text{PT}}(\eta_0) = s_{\text{kin}}(\eta_0)$?

- To check its goodness, we define the approximate mass:

$$\tilde{s}_h = \begin{cases} U_0 \cosh^{-2}[\alpha(\eta - \eta_B)], & \eta \in [\eta_B, \eta_0], \\ \tilde{s}_{\text{GR}} & \eta \in (\eta_0, \eta_{\text{end}}] \end{cases}$$

  and compare it with s_h^{kin}.

Pöschl-Teller approximation

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- Around the bounce, approximate s_h^{kin} by

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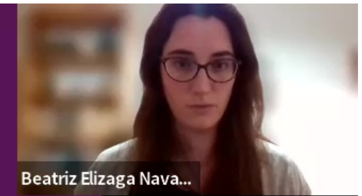
so that $s_{\text{PT}}(\eta_B) = s_h^{\text{kin}}(\eta_B)$ and $s_{\text{PT}}(\eta_0) = s_{\text{kin}}(\eta_0)$?

- Checking for different choices of $t_0 \gtrsim 0.4$:

  ★ Error grows before η_0, negligible afterwards.

  ★ Best situation for $t_0 \simeq 0.4$: Error of at most 15 % .

Mukhanov-Sasaki equations



- Approximate pre-inflationary epoch (η_0, η_i) as a Friedmann universe with a massless scalar field.

- Approximate the inflationary period $[\eta_i, \eta_{\text{end}}]$ as de Sitter.

- For the interval $[\eta_B, \eta_0]$ with strong loop quantum effects, we approximate the mass with the Pöschl–Teller potential.

- Approximate Mukhanov-Sasaki equations:

$$v''_{\vec{k}} + \left(k^2 + \tilde{s}_h\right) v_{\vec{k}} = 0,$$

$$\tilde{s}_h = \begin{cases} s_{\text{PT}}, & \eta \in [\eta_B, \eta_0], \\ \tilde{s}_{\text{GR}} & \eta \in (\eta_0, \eta_{\text{end}}], \end{cases} \qquad \tilde{s}_{\text{GR}} = \begin{cases} \frac{1}{4}\left(\eta - \eta_0 + \frac{1}{2H_0 a_0}\right)^{-2}, & \eta \in (\eta_0, \eta_i) \\ -2H_\Lambda^2 \left[a_i^{-1} - H_\Lambda(\eta - \eta_i)\right]^{-2} & \eta \in [\eta_i, \eta_{\text{end}}] \end{cases}$$

Mukhanov-Sasaki equations

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- Can be solved analytically in each period.
- Solutions can be uniquely matched by continuity.
- We can set initial conditions in the earliest epoch and then know the complete evolution of the perturbations.

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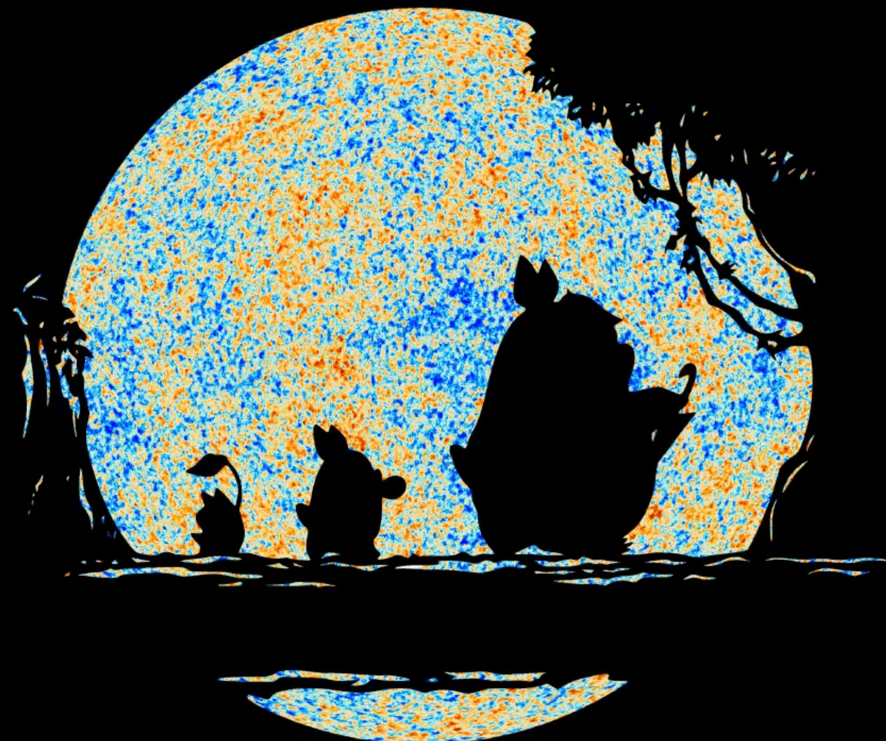


Vacuum state and power spectra

Power spectrum in de Sitter

- In de Sitter, solutions to Mukhanov-Sasaki equations:

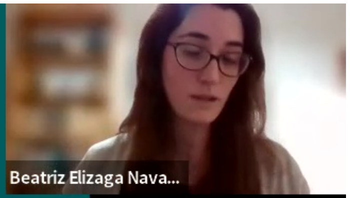
$$\mu_k = A_k \frac{e^{ik(\eta-\eta_i-a_i^{-1}H_\Lambda^{-1})}}{\sqrt{2k}} \left[1 + \frac{i}{k(\eta-\eta_i-a_i^{-1}H_\Lambda^{-1})}\right]$$

$$+B_k \frac{e^{-ik(\eta-\eta_i-a_i^{-1}H_\Lambda^{-1})}}{\sqrt{2k}} \left[1 - \frac{i}{k(\eta-\eta_i-a_i^{-1}H_\Lambda^{-1})}\right]$$

- Primordial power spectrum is well-approximated by:

$$\mathscr{P}(k) = \frac{H_\Lambda^2}{4\pi^2}|B_k - A_k|^2, \qquad |B_k|^2 - |A_k|^2 = 1$$

- Dephasing between constants typically leads to oscillations.



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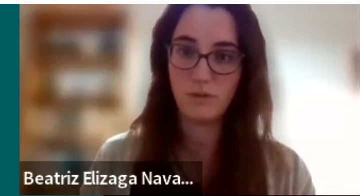
$$\mathscr{P}(k) = \frac{H_\Lambda^2}{4\pi^2} |B_k - A_k|^2, \qquad |B_k|^2 - |A_k|^2 = 1$$

- Dephasing between constants: Oscillations.

⟶ If no interference in previous epoch(s), origin can be traced to instantaneous changes of the mass function.



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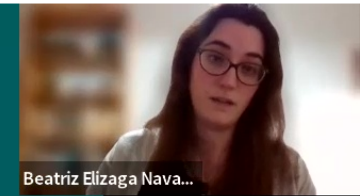
- Dephasing between constants: Oscillations.
- For well-behaved initial state, we remove it in the end.

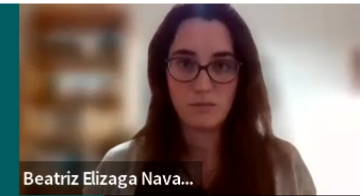
Choice of vacuum state



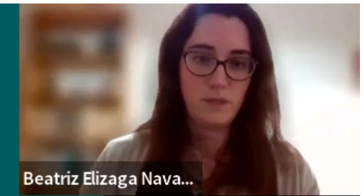
- Vacuum state: Initial conditions for the perturbations.
- In de Sitter, natural choice is Bunch-Davies: $A_k = 0$, $B_k = 1$.
- What if there are observable scales k that are sensitive to the spacetime curvature in the pre-inflationary epoch?

Choice of vacuum becomes an open question

Choice of vacuum state



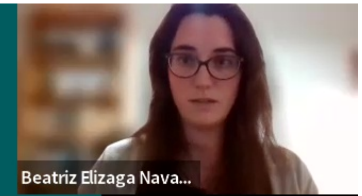
- Vacuum state: Initial conditions for the perturbations.
- In de Sitter, natural choice is Bunch-Davies: $A_k = 0$, $B_k = 1$.
- What if there are observable scales k that are sensitive to the spacetime curvature in the pre-inflationary epoch?
- For a robust comparative study: Criterion of choice should be applicable to different types of cosmological dynamics.
- Ideally, it should also be motivated by fundamental considerations, and lead to positive-frequency solutions that do not present rapid oscillations in time and/or k.



Choice of vacuum state

- Vacuum state: Initial conditions for the perturbations.
- Here, criterion is fixed based on previous investigations:
  - ★ Originates from an ultraviolet diagonalization of the Hamiltonian in quantum cosmology.
  - ★ In the ultraviolet regime, it is the unique one that does not display rapid time oscillations of frequency *k*.
  - ★ Applied to Minkowski and de Sitter spacetimes, leads to Poincaré and Bunch-Davies vacua.

e.g. [BEN, G.A. Mena Marugán, and T. Thiemann, Class. Quant. Grav. 36 (2019) 185010]; [BEN, G.A. Mena Marugán, and S. Prado, Class. Quant. Grav. 38 (2021) 035001]



Choice of vacuum state

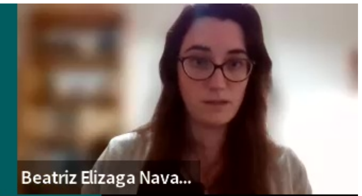
- Vacuum state: Initial conditions for the perturbations.
- Here, criterion is fixed based on previous investigations:

$$\mu_k = \sqrt{-\frac{1}{2\mathrm{Im}(h_k)}} e^{i\int_{\eta_0}^{\eta} \mathrm{Im}(h_k)}, \qquad k h_k^{-1} \sim i \left[1 - \frac{1}{2k^2} \sum_{n=0}^{\infty} \left(\frac{-i}{2k} \right)^n \gamma_n \right],$$

$$\gamma_0 = s, \quad \gamma_{n+1} = -\gamma_n' + 4s \left[\gamma_{n-1} + \sum_{m=0}^{n-3} \gamma_m \gamma_{n-(m+3)} \right] - \sum_{m=0}^{n-1} \gamma_m \gamma_{n-(m+1)}$$

- The smooth mass s can be evaluated on GR or effLQC.
- Formula not applicable with our approximations (due to discontinuities), but can be used to fix initial conditions in the earliest smooth epoch (KD or bouncing regime).

Power spectrum in GR



- In the case of GR with KD, recall that the general solution to the MS equation in the epoch (η_0, η_i) is:

$$\mu_k = C_k\sqrt{\frac{\pi y}{4}}H_0^{(1)}(ky) + D_k\sqrt{\frac{\pi y}{4}}H_0^{(2)}(ky), \qquad y = \eta - \eta_0 + \frac{1}{2H_0 a_0}$$

Power spectrum in GR

- In the case of GR with KD, our criterion fixes the following positive-frequency solutions in the epoch (η_0, η_i):

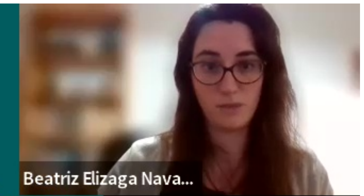
$$\mu_k = \sqrt{\frac{\pi y}{4}} H_0^{(2)}(ky), \qquad y = \eta - \eta_0 + \frac{1}{2H_0 a_0}$$

- By continuity, fixes positive-frequency solutions in de Sitter:

$$A_k = \frac{e^{ik/(a_i H_\Lambda)}}{4} \sqrt{\frac{k\pi}{a_i H_\Lambda}} \left[H_0^{(2)}\left(\frac{k}{2a_i H_\Lambda}\right) - \left(\frac{a_i H_\Lambda}{k} - i\right) H_1^{(2)}\left(\frac{k}{2a_i H_\Lambda}\right)\right],$$

$$B_k = \frac{e^{-ik/(a_i H_\Lambda)}}{4} \sqrt{\frac{k\pi}{a_i H_\Lambda}} \left[H_0^{(2)}\left(\frac{k}{2a_i H_\Lambda}\right) - \left(\frac{a_i H_\Lambda}{k} + i\right) H_1^{(2)}\left(\frac{k}{2a_i H_\Lambda}\right)\right]$$

Power spectrum in GR



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- By continuity, fixes positive-frequency solutions in de Sitter.

- Resulting power spectrum displays artificial oscillations around $k_I = a_i H_\Lambda$, which we remove with the transformation:

$$A_k \rightarrow A_k^{\text{kin}} = |A_k|, \qquad B_k \rightarrow B_k^{\text{kin}} = |B_k|$$

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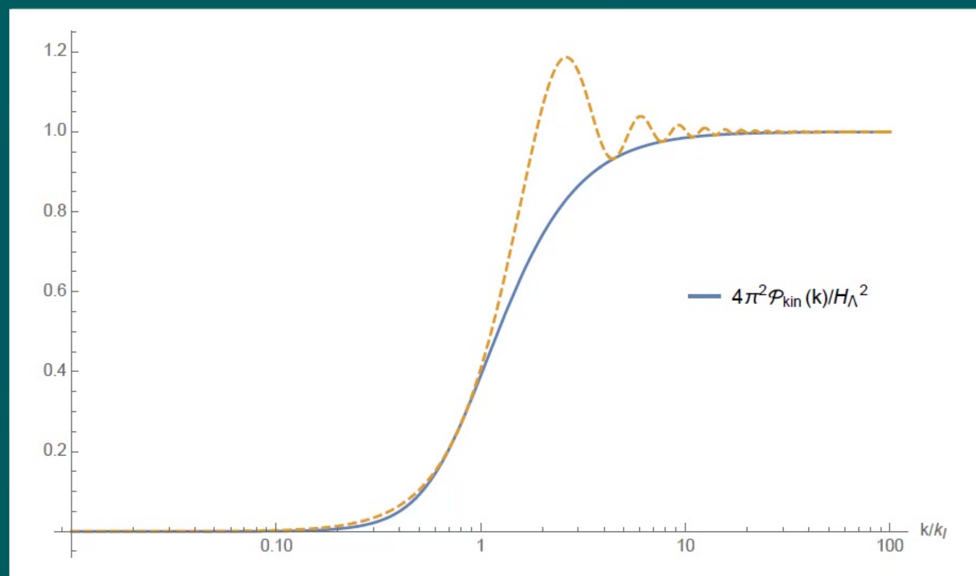
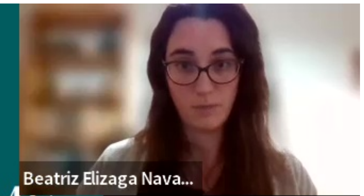


Dashed line in: [C.R. Contaldi, M. Peloso, L. Kofman, and A.D. Linde, JCAP 0307 (2003) 002]

Power spectrum in (hybrid) LQC



- Around the bounce in LQC, the general solution of the MS equations with a Pöschl–Teller potential is:

$$\mu_k = M_k[x(1-x)]^{-i\tilde{k}/2}{}_2F_1\left(b_1^k, b_2^k; b_3^k; x\right)$$

$$+N_k x^{i\tilde{k}/2}(1-x)^{-i\tilde{k}/2}{}_2F_1\left(b_1^k - b_3^k + 1, b_2^k - b_3^k + 1; 2 - b_3^k; x\right),$$

where $x = \left[1 + e^{-2\alpha(\eta-\eta_B)}\right]^{-1}$, $\tilde{k} = k/\alpha$, and:

$$b_1^k = \frac{1}{2}\left(1 + \sqrt{1 + \frac{32\pi\rho_c}{3\alpha^2}}\right) - i\tilde{k}, \qquad b_2^k = \frac{1}{2}\left(1 - \sqrt{1 + \frac{32\pi\rho_c}{3\alpha^2}}\right) - i\tilde{k},$$

$$b_3^k = 1 - i\tilde{k}$$

Power spectrum in (hybrid) LQC

- In the case of LQC, our criterion fixes the following positive-frequency solutions in the epoch $[\eta_B, \eta_0]$ (up to const. phase):

$$\mu_k = \frac{1}{\sqrt{2k}}[x(1-x)]^{-i\tilde{k}/2}{}_2F_1\left(b_1^k, b_2^k; b_3^k; x\right), \qquad \tilde{k} = k/\alpha$$

- By continuity, fixes positive-frequency solutions in the KD classical epoch:

$$C_k = \left[H_0^{(1)}\left(\frac{k}{2H_0a_0}\right)\right]^{-1}\left[\sqrt{\frac{8H_0a_0}{\pi}}\mu_k(\eta_0) - D_kH_0^{(2)}\left(\frac{k}{2H_0a_0}\right)\right],$$

Power spectrum in (hybrid) LQC

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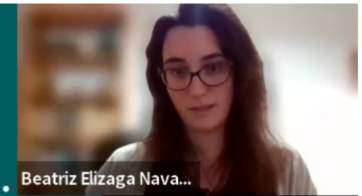
- By continuity, fixes positive-frequency solutions in the KD classical epoch:

$$D_k = i\sqrt{\frac{\pi}{8H_0a_0}}\left[kH_1^{(1)}\left(\frac{k}{2H_0a_0}\right)\mu_k(\eta_0) - H_0^{(1)}\left(\frac{k}{2H_0a_0}\right)H_0a_0\mu_k(\eta_0)\right.$$

$$\left. + H_0^{(1)}\left(\frac{k}{2H_0a_0}\right)\mu_k'(\eta_0)\right]$$

Power spectrum in (hybrid) LQC



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$$\mu_k = \frac{1}{\sqrt{2k}} [x(1-x)]^{-i\tilde{k}/2} {}_2F_1 \left(b_1^k, b_2^k; b_3^k; x \right), \qquad \tilde{k} = k/\alpha$$

- By continuity, fixes positive-frequency solutions in the KD classical epoch and these, in turn, in the de Sitter regime.

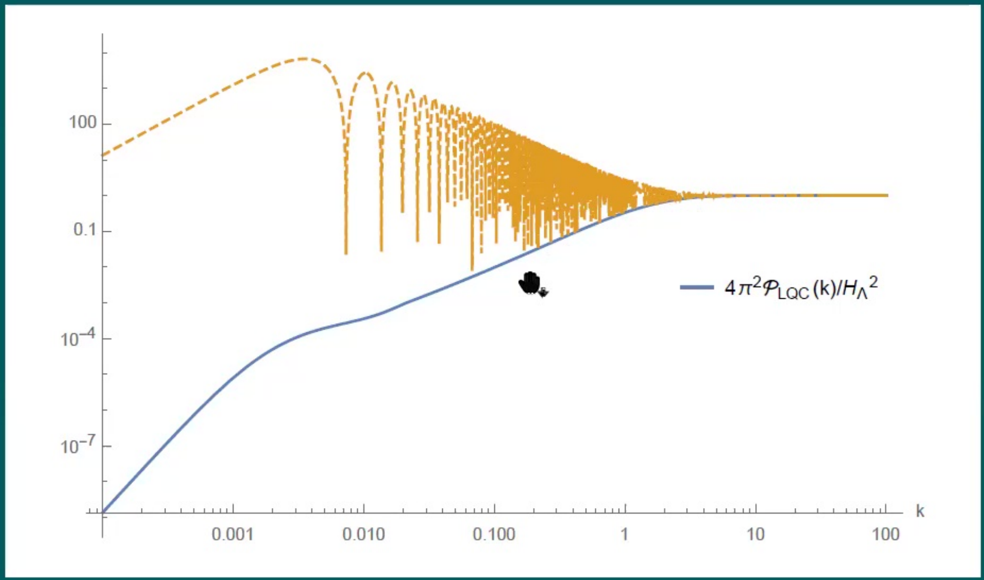
- Resulting power spectrum displays artificial oscillations for $k \lesssim k_{\mathrm{LQC}} = \alpha\,(\sim 3)$, which we remove in analogous way:

$$A_k \to A_k^{\mathrm{LQC}} = |A_k|, \qquad B_k \to B_k^{\mathrm{LQC}} = |B_k|$$

Power spectrum in (hybrid) LQC

- Resulting power spectrum displays artificial oscillations for $k \lesssim k_{\rm LQC} = \alpha\,(\sim 3)$, which we remove:

$$A_k \to A_k^{\rm LQC} = |A_k|, \qquad B_k \to B_k^{\rm LQC} = |B_k|$$



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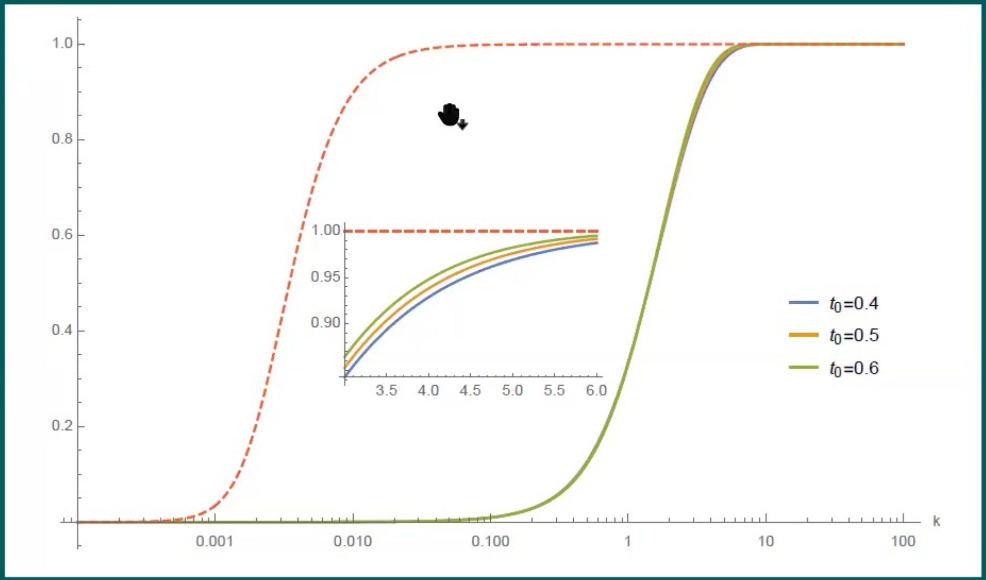
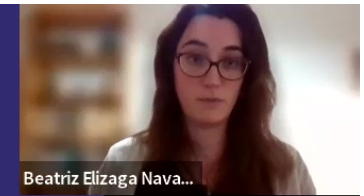
Power spectrum in (hybrid) LQC

- Resulting power spectrum displays artificial oscillations for $k \lesssim k_{\text{LQC}} = \alpha\,(\sim 3)$, which we remove.
- We compare it with the one in the GR with KD model for which inflation starts at the same scale as in LQC: $k_I \sim 10^{-3}$.



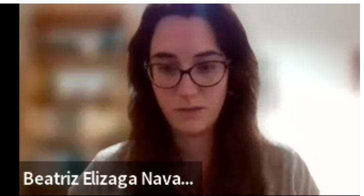
----- GR
— LQC

Conclusions



- Approximative methods to understand **analytically** the main differences between (classical) KD preinflationary and LQC effects leading to suppression in power spectra.
- Differences traceable to existence of two distinct scales:
    - ★ Curvature at onset of inflation (both models).
    - ★ Curvature around the bounce (only in LQC).
- They always differ in 3 orders of magnitude for interesting LQC solutions (phenomenologically speaking).
- Study can be used to compare other preinflationary models.
- Approximations yet rough: Call for further developing the studies about the dynamical behavior of the chosen vacua.

Some ideas for the future



- New LQC-inspired models of spherically symmetric black holes have been investigated over the last few years.

e.g. [Ashtekar, J. Olmedo, and P. Singh, Phys. Rev. D 98, 126003 (2018);
N. Bodendorfer, F.M. Mele, and J. Munch, Class. Quant. Grav. 36, 19015 (2019);
A. García-Quismondo and G.A. Mena Marugán, Front. Astron. Space Sci. 8, 701723 (2021)]

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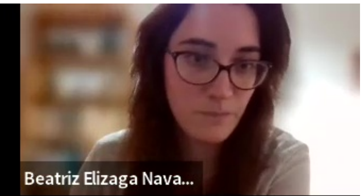

- New LQC-inspired models of spherically symmetric black holes have been investigated over the last few years.
- With a quantum description of the black hole geometry, a hybrid quantization of perturbations around it seems plausible.
- The effective wave equations would contain modifications coming from the quantum corrections of the background:
  - ⋆ Black hole horizon?
  - ⋆ Properties in the asymptotic region?
- The physical choice of initial conditions for the GW and the appropriate definition of their quasinormal modes could vary.
- We should count on the intuition and expertise gained in the context of cosmology about similar issues (e.g. choice of vacuum).