

Title: Towards quantum information processing in gravitating spacetimes"

Speakers: Alex May

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Abstract: Aside from quantum mechanics, other areas in physics constrain information processing. From relativity, we know information cannot move faster than the speed of light. In quantum gravity, we expect but don't fully understand additional constraints, for instance on how densely information can be stored. Can we develop an understanding of information processing in the context of quantum gravity? Towards doing so, we consider quantum information within ``holographic'' spacetimes, which have an alternative, non-gravitational, description. In that context questions about information processing in the presence of gravity can be translated to different questions about quantum information without gravity. As a specific example, we study constraints on computation within a gravitating region, and use the holographic description to argue that gravity constrains the complexity of operations that can happen inside the region. Along the way, we are led to new connections relating quantum gravity, position-based cryptography, quantum secret sharing and complexity theory.

# Towards quantum information processing in gravitational spacetimes

"Quantum tasks  
in holography"

Alex May  
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G. Penington

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AM AM AM, D. Wakeham

## Quantum information processing in spacetime

- Quantum information asks: Given the rules of quantum mechanics, how can information be processed?

- There are additional rules:

Relativity → no-signalling  
Quantum      → covariant entropy bound,  
Gravity                  others?

What can we say about information processing when considering the physics of quantum gravity?

↳ what is the right framework to address this?

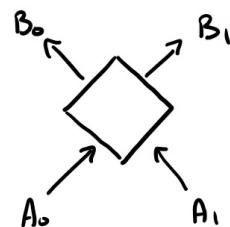
## Outline

- Big picture idea:

For spacetimes described "holographically", we can translate questions about QI with gravity to questions about QI without gravity

In particular:

Does gravity constrain computation?



How much entanglement is required in non-local quantum computation?

complexity

cryptography

secret sharing

Present new results on NLQC today

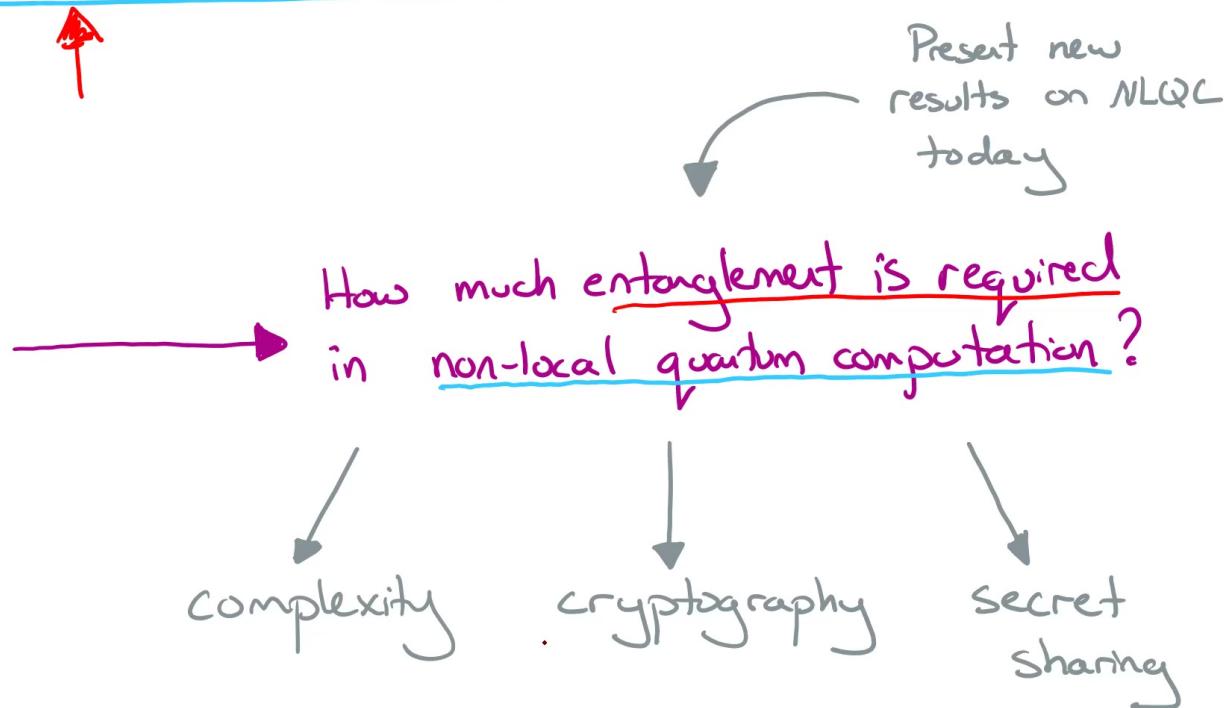
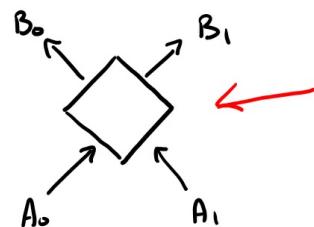
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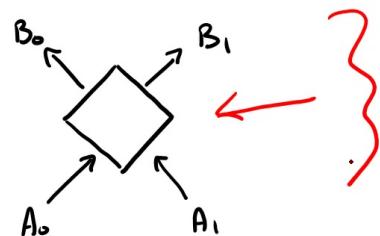
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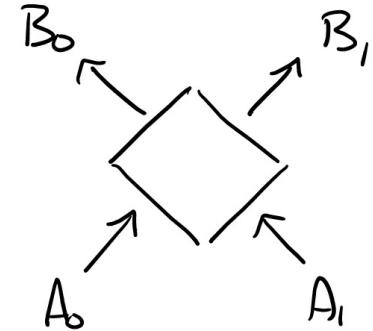
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complexity      cryptography      secret sharing

Quantum information in a  
dynamical spacetime background

## Complexity + gravity?

- One suggestion from Lloyd (2000) is that gravity constrains complexity
- Lloyd argued  $\boxed{\text{Energy} \leq M_{BH}}$  plus ordinary quantum mechanics would upper bound complexity  
↳ But, Jordan (2017) gave a counterexample!
- Jordan's construction ignores no-signalling, etc.  
In full quantum gravity, does gravity constrain complexity?



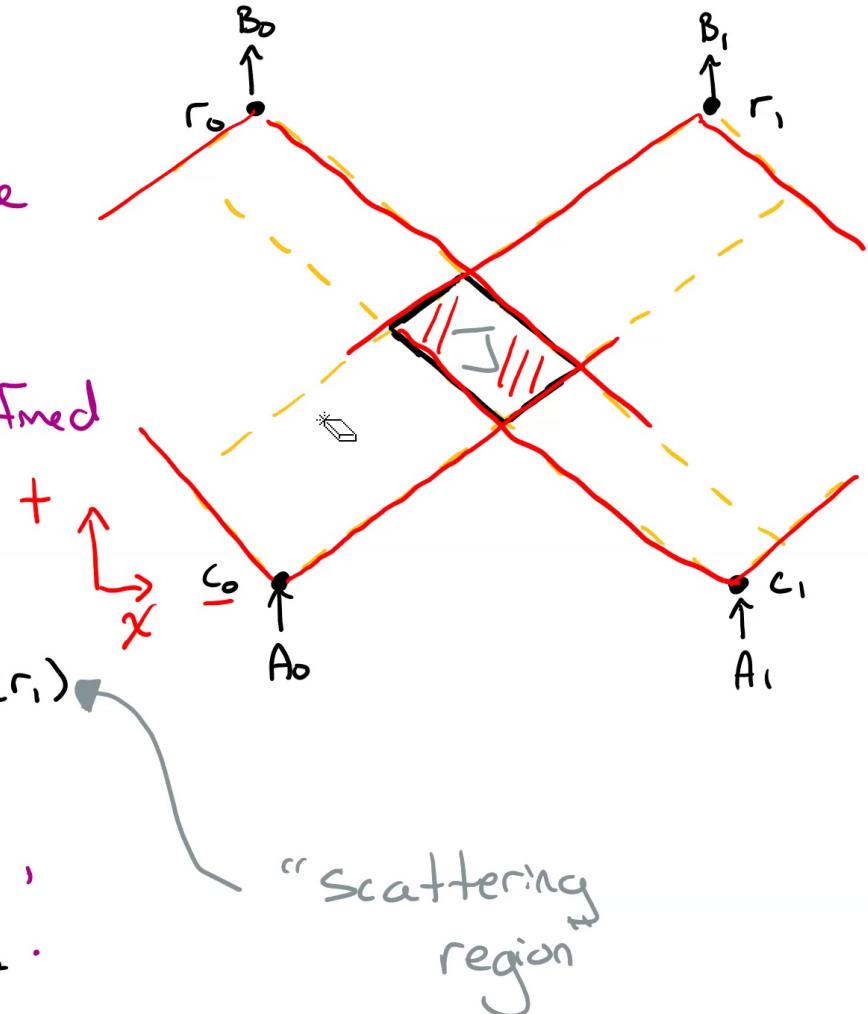
## Defining the region

- Given a spacetime region  $J$  what computations can happen inside the region?

- To specify region  $J$  in a well defined way, pick  $c_0, c_1, r_0, r_1$  s.t.

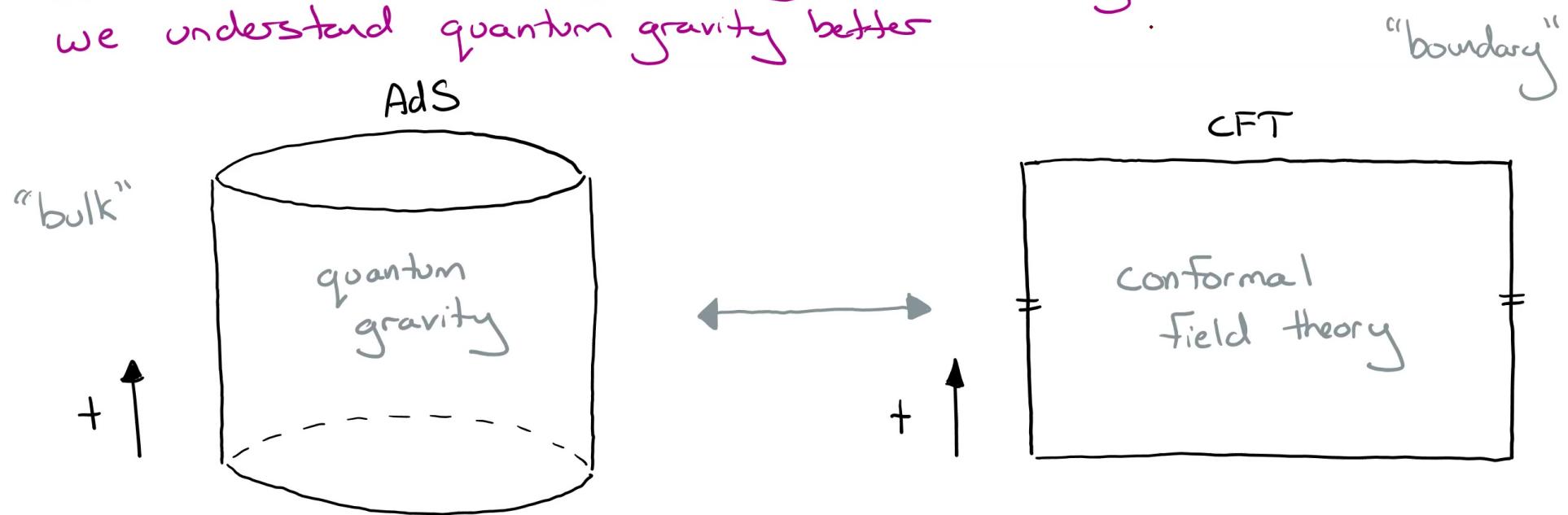
$$J = J^+(c_0) \cap J^+(c_1) \cap J^-(r_0) \cap J^-(r_1)$$

- Give input systems  $A_0, A_1$  at  $c_0, c_1$ , output systems  $B_0, B_1$  at  $r_0, r_1$ .



## AdS/CFT

- To understand QI with gravity, go to a setting where we understand quantum gravity better.

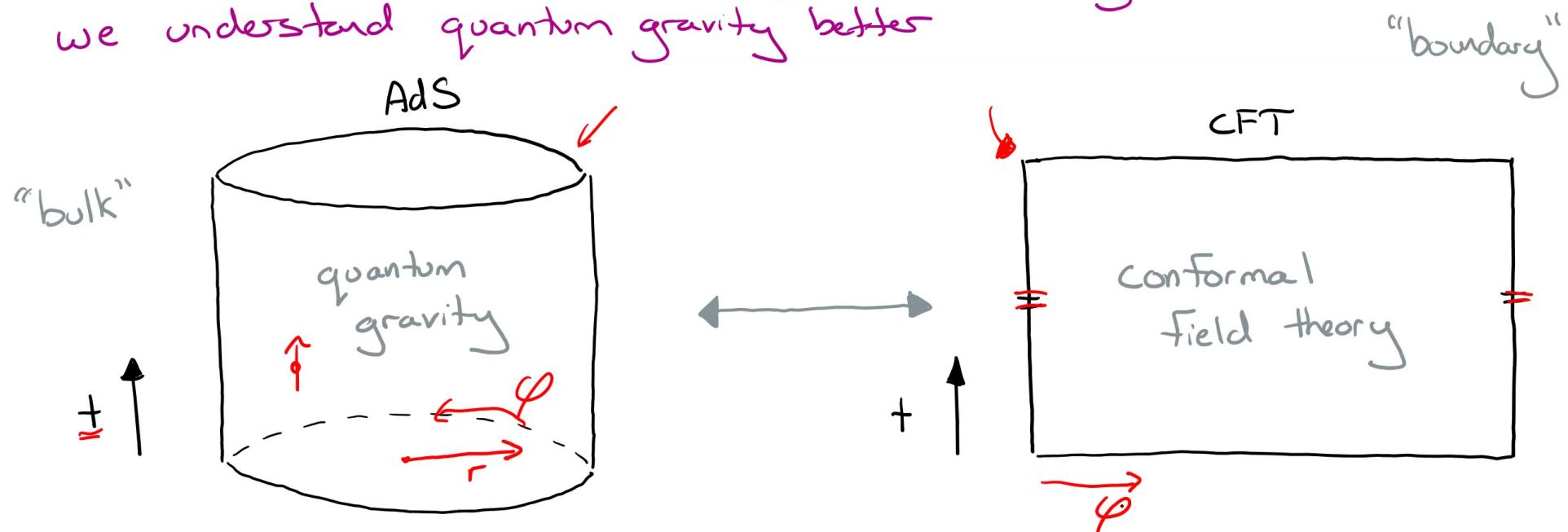


- Dynamical geometry (includes gravity)
- Includes quantum fields
- $d+1$  dimensional

- Fixed geometry (no gravity)
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- $d$  dimensional

## AdS/CFT

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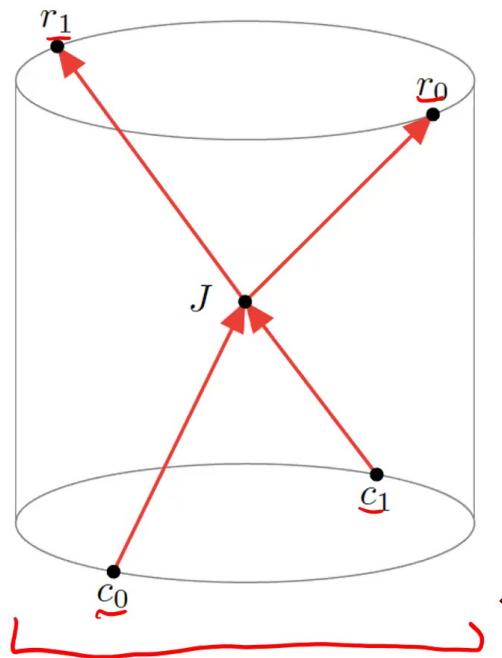


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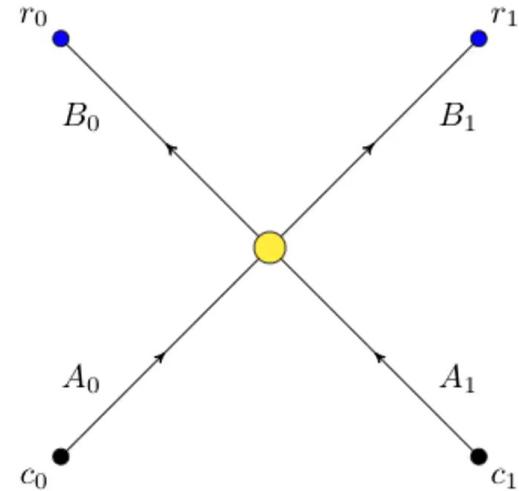
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## Local to non-local computation

- Consider defining a scattering region in an AdS spacetime:

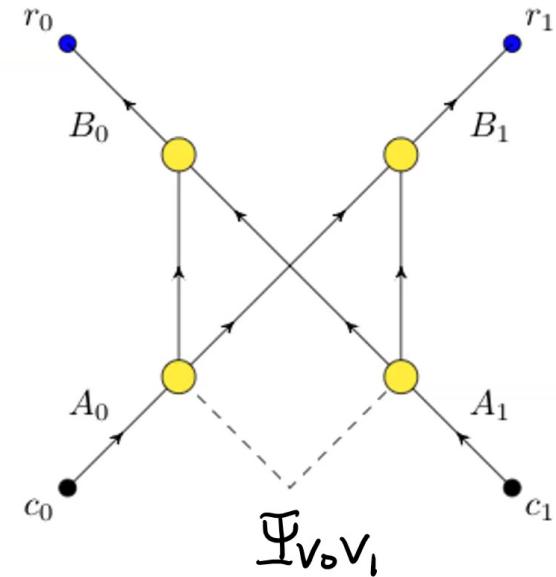
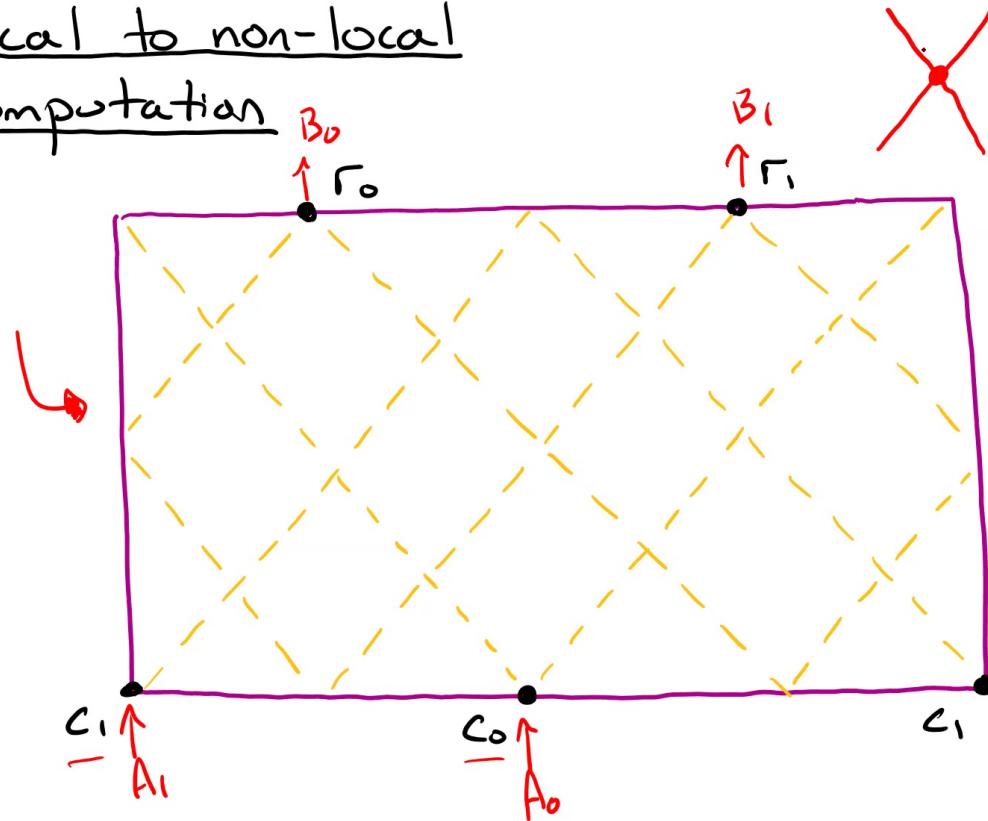


schematic



We'll ask: which computations can be performed inside  $J$ ?

## Local to non-local computation

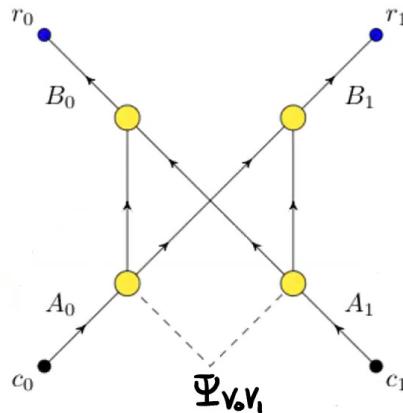
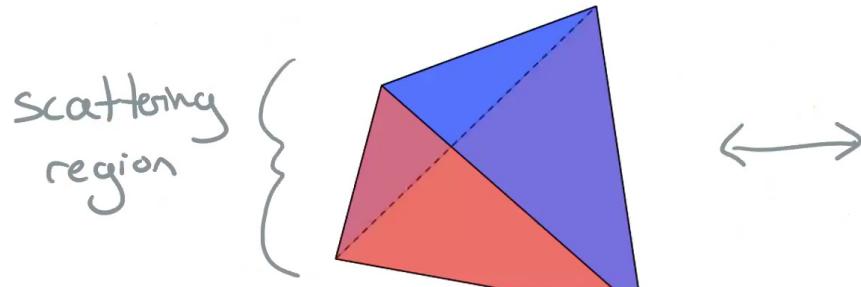


- Any computation which happens locally in the bulk is reproduced non-locally in the boundary

## Entanglement in the boundary

- computable inside scattering region

$\subseteq$  computable non-locally



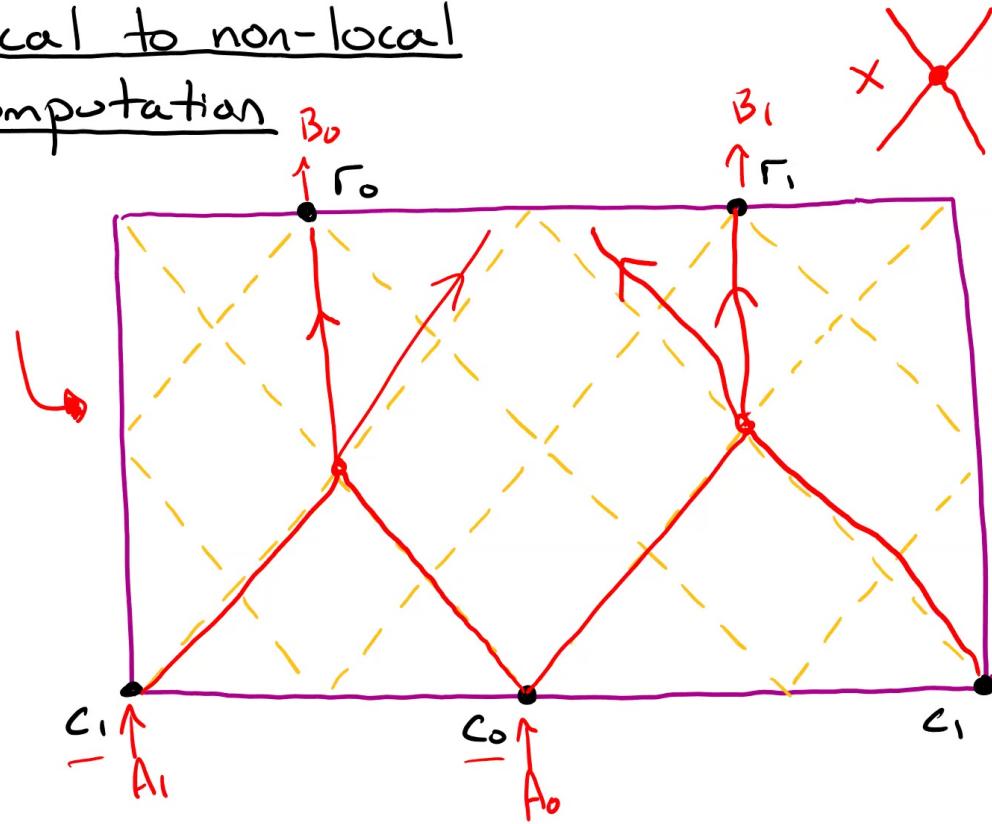
- More quantitatively,

computable inside a scattering region of area  $A_R$

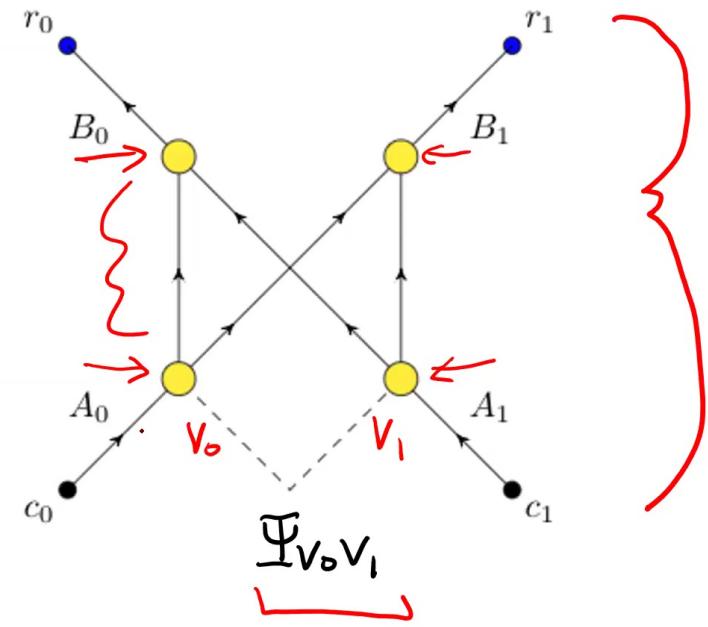
$\subseteq$  computable non-locally using resource state  $\Psi_{V_0 V_1}$

$$I(V_0 : V_1)_{\Psi} = \Theta(A_R)$$

## Local to non-local computation



"non-local"

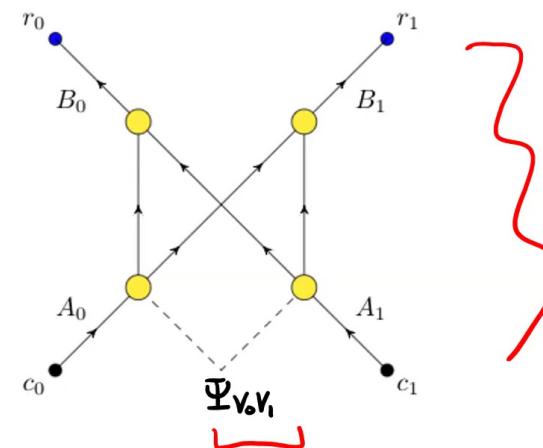
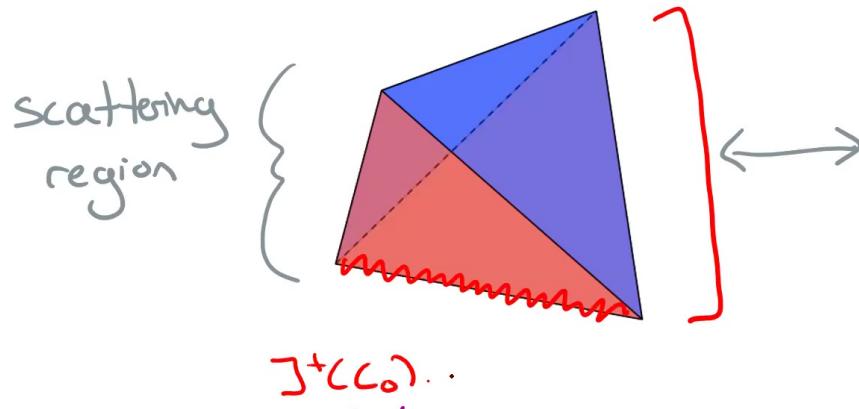


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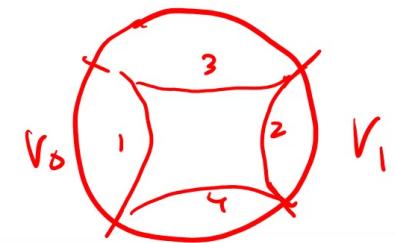
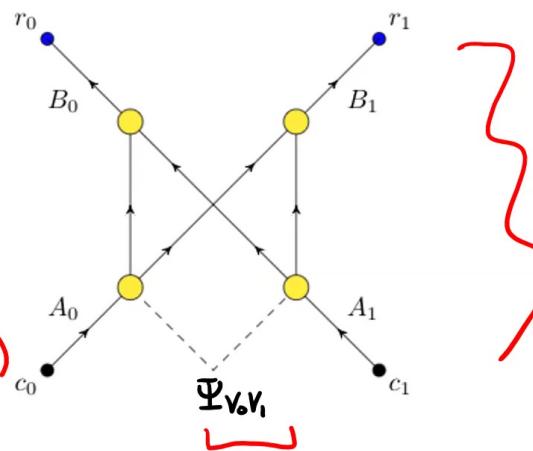
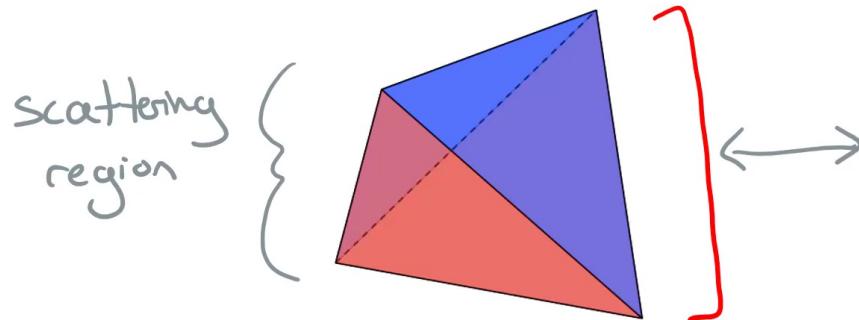
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## Entanglement in the boundary

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$$A(1) + A(2) - A(3) - A(4)$$



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$\subseteq$

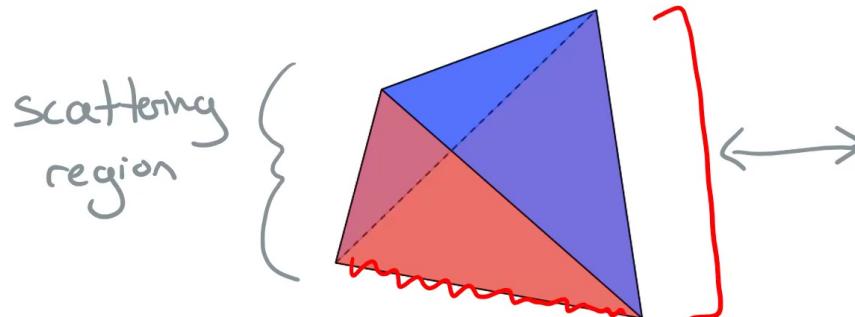
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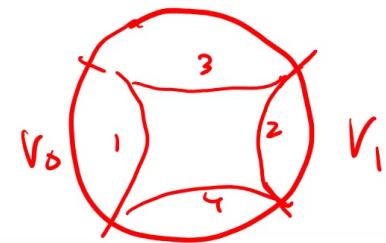
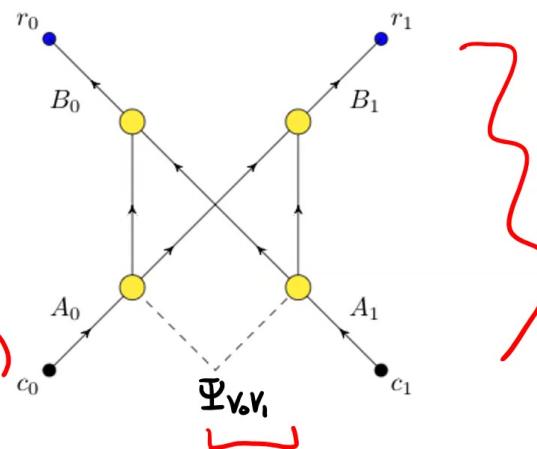
- computable inside scattering region

$\subseteq$  computable non-locally  $A_R \sim [A(1) + A(2) - A(3) - A(4)]$



$$R = \partial J^+ (c_0) \cap \partial J^+ (c_1) \cap J^-(r_0) \cap J^-(r_1)$$

- More quantitatively,



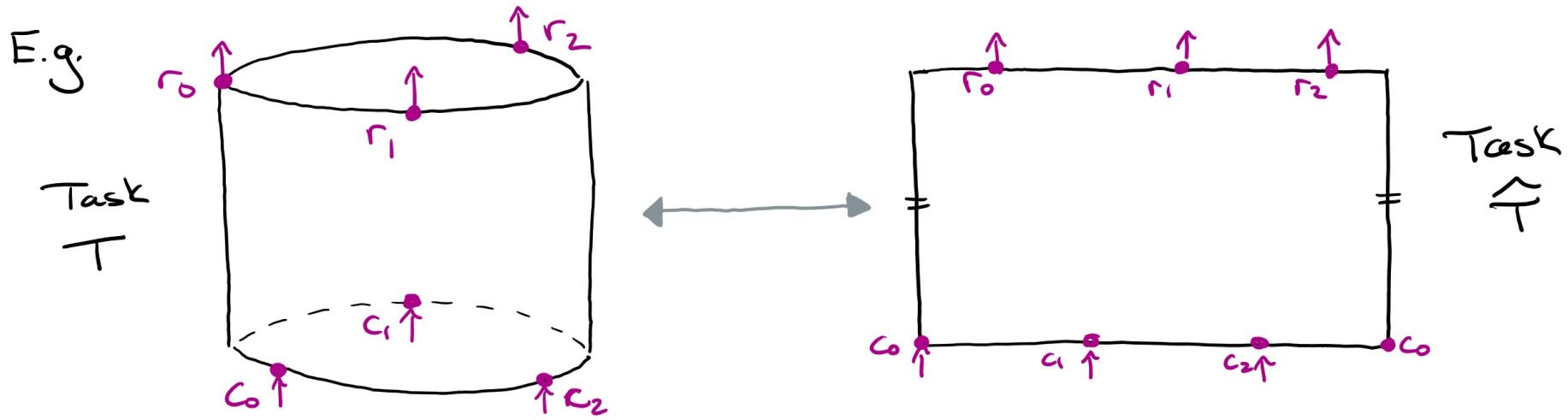
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## Stepping back: bulk + boundary tasks

- More generally, we can relate bulk information processing tasks to boundary ones:

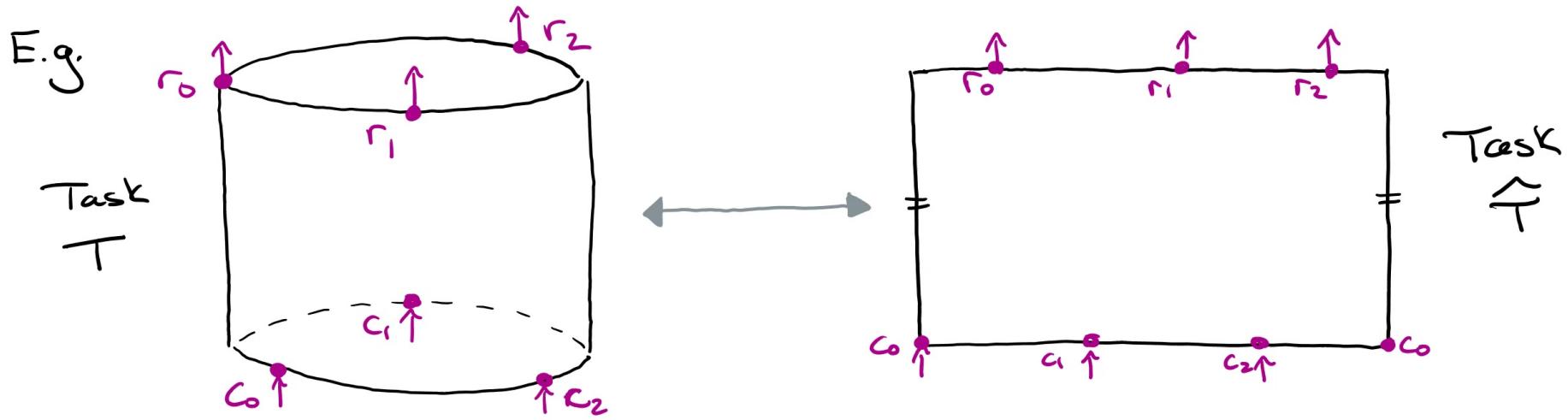


- Many other questions we can ask! (Focus on one for today)

- Can also use this perspective to study:  
gravity in AdS space, entanglement ~ geometry in AdS/CFT

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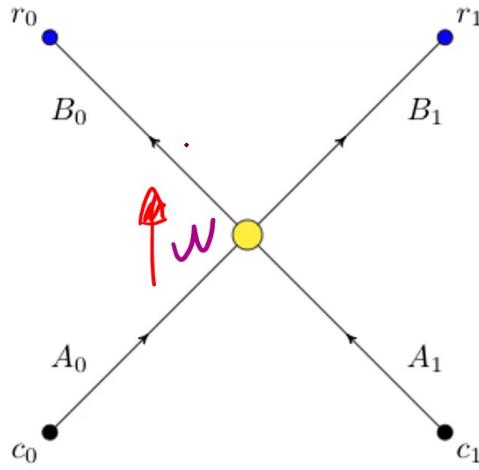
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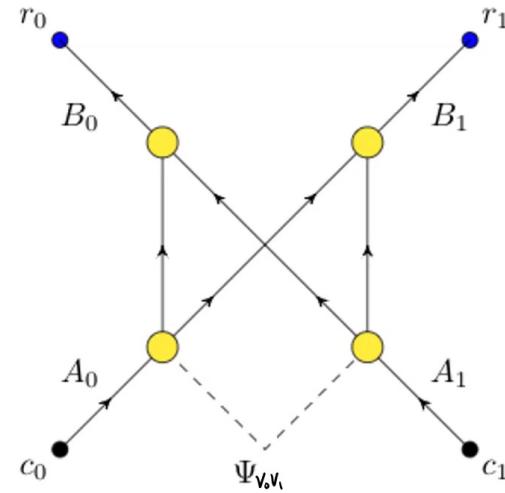
gravity in AdS space, entanglement ~ geometry in AdS/CFT.

## Non-local quantum computation

Local



Non-local



Basic question :

What property of  $N$  controls the entanglement needed to implement it non-locally?

Complexity?

- Suggested by upper bounds
- would imply Lloyds idea

## Does complexity control entanglement?

- One simple example: For Clifford unitaries on  $n$  qubits

Entanglement cost:

$$\frac{1}{2} I(V_0 : V_1) \leq n \log_2 K$$

Complexity:

Mod<sub>K</sub>L

measuring one  
qubit of output,  
note that  
 $L \subseteq \text{Mod}_K L \subseteq P$

Idea: IF complexity  $\sim$  entanglement, then any computation of complexity Mod<sub>K</sub>L can be done non-locally with poly( $n$ ) entanglement

## f-routing

Test this idea using "f-routing":

- For a fixed  $f: \{0,1\}^{2n} \rightarrow \{0,1\}$ ,

Inputs:  $x, y \in \{0,1\}^n$

$$Q \in \mathbb{Z}_d$$

Output: bring  $Q$  to  $r_{f(x,y)}$

$$r_0 \uparrow$$

$$\uparrow r_1$$

$$c_0 \uparrow$$

$$\underline{x}, Q$$

$$\uparrow c_1$$

- This is well studied because especially favourable for use in "position-based cryptography"

↳ Non-local computations = cheating strategies

## Garden-hose + complexity (Buhrman et. al 2013)

- Most efficient known strategy is "garden-hose" protocol:

# EPR pairs used  
by garden-hose

$$\simeq 2^{\text{Memory}(f)}$$

memory needed to  
compute  $f$  on  
a Turing machine

Entanglement cost is controlled  
by (space) complexity (in this protocol)

- Restricting to polynomial entanglement, can do  $f \in L$

$\uparrow$   
 $O(\log n)$  space

## Improvements to garden-hose?

- Can we do better than  $L$ ?  $L \notin \text{Mod} \mathcal{L}$   
Based on complexity  $\sim$  entanglement, would expect yes.
- As well, from gravity perspective, if  $L$  is optimal,  
restricted to  $\log(A_R)$  memory in scattering region of area  $A_R$ .
- How might we do better?
  - ↳ AdS/CFT does NLQEC and quantum error-correction  
plays important role there, let's try QECC?

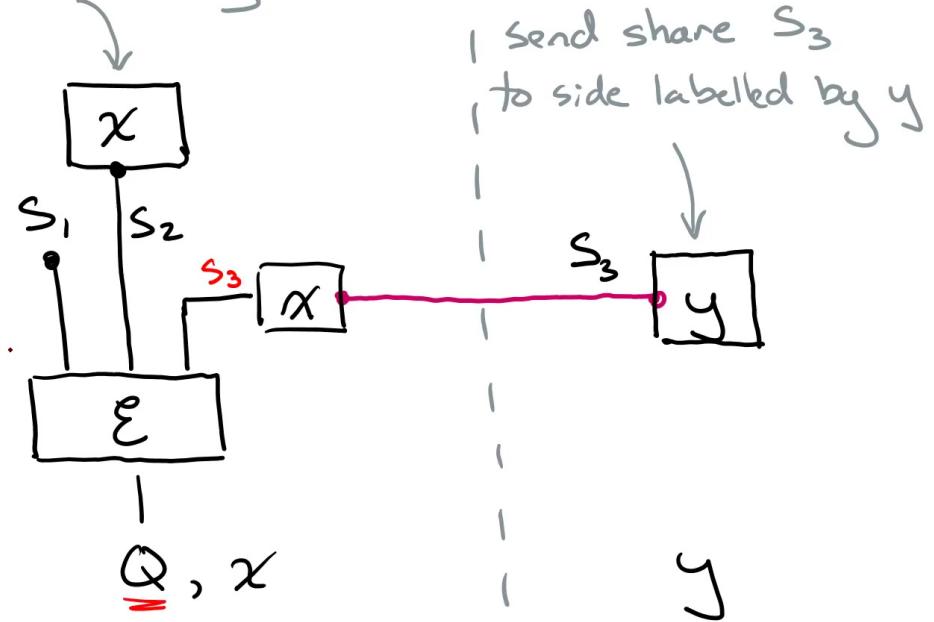
## Code-routing : by example

- Lets complete the routing task for

$$f(x,y) = x \wedge y$$

x	y	Left	Right
0	0	$s_1, s_2, s_3$	
0	1	$s_1, s_2$	$s_3$
1	0	$s_1, s_2, s_3$	$s_2$
1	1	$s_1, s_2$	$s_2, s_3$

Send share  $s_2$  to side labelled by  $x$



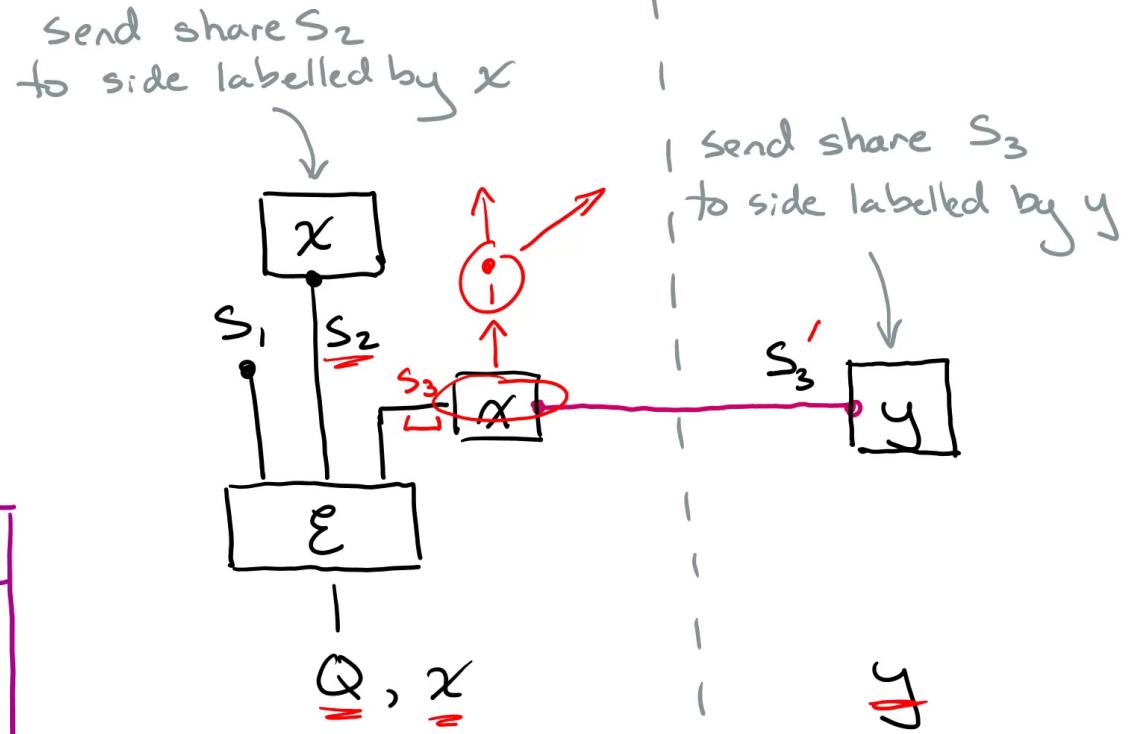
Any two shares recover  $Q$ , so this brings  $Q$  to side  $x \wedge y$

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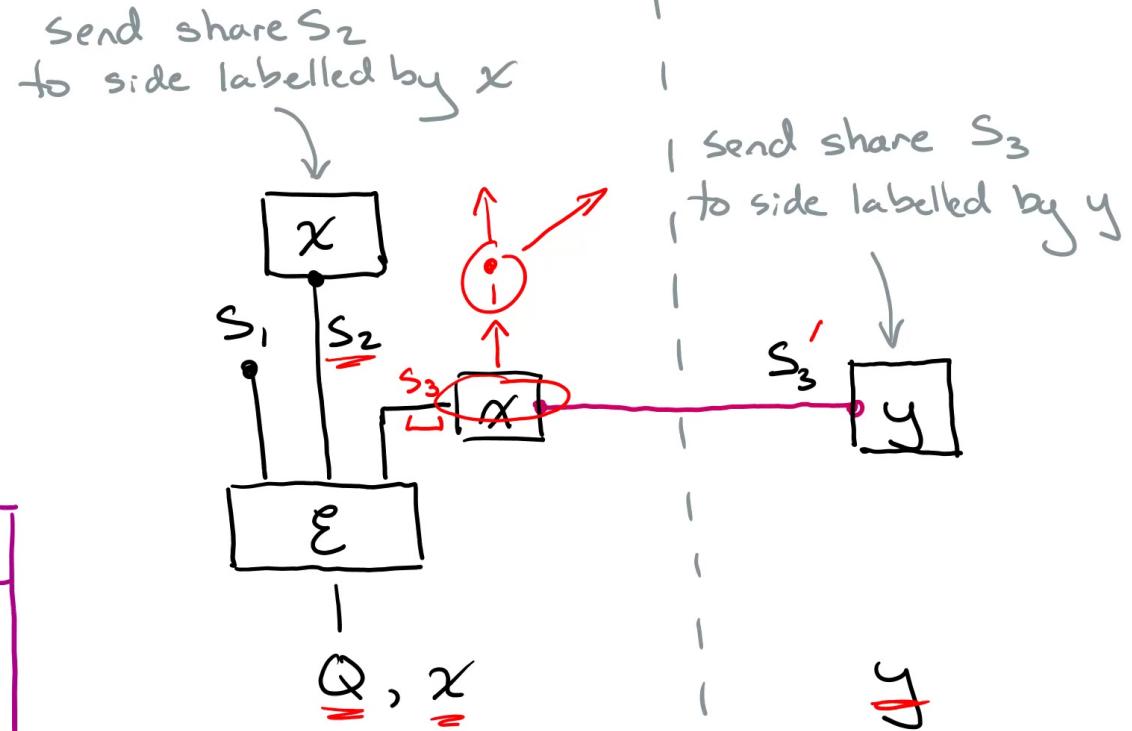
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## Quantum secret sharing

- Consider a quantum secret sharing scheme:

$$Q \xrightarrow{\epsilon} S = \{S_1, S_2, S_3, \dots, S_n\}$$

- Label subsets  $K(z) = \{S_i : z_i = 1\}$ , define:

"Indicator function" →

$$f_I(z) = \begin{cases} 0 & \text{if learn nothing about } Q \text{ from } K(z) \\ 1 & \text{if can recover } Q \text{ from } K(z) \end{cases}$$

- Replace code with an arbitrary QSS:

entanglement to route on  $f = f_I(z)$  ≤ "size" of QSS with indicator  $f_I(z)$

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“Indicator function”  $\rightarrow f_I(z) = \begin{cases} 0 & \text{if learn nothing about } Q \text{ from } \underline{K(z)} \\ 1 & \text{if can recover } Q \text{ from } K(z) \end{cases}$

- Replace code with an arbitrary QSS:

entanglement to route on  $f = f_I(z)$   $\leq$  “size” of QSS with indicator  $f_I(z)$

## Complexity and QSS

- The best known QSS construction Smith (2000)

↳ Given indicator  $f_I$ , constructs QSS with size  $O(SP(f_I))$

size of a "span program" computing  $f$

- Using this, and building on QSS based protocol:

$$E\text{-cost of } f \leq O(SP(f))$$

arbitrary function

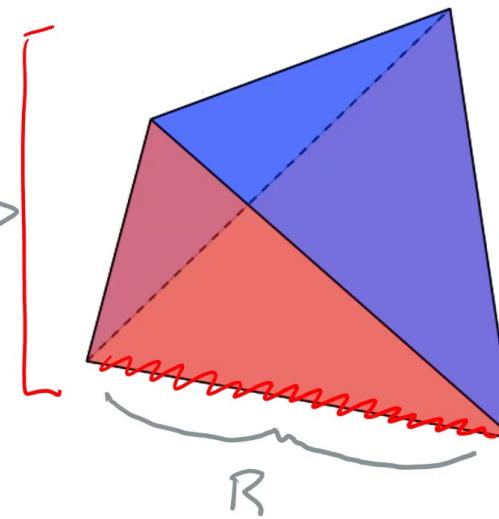
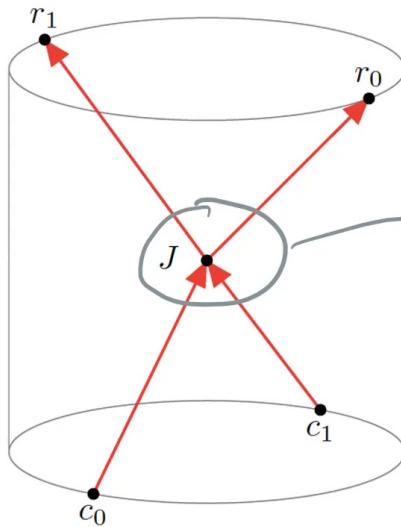
- Functions with poly sized =  $\text{Mod}_k L$   
span programs

→ Aligns with expectation from complexity-wrangling  
→ Does strictly better than garden-hose ( $L \leq \text{Mod}_k L$ )

Consequences of entanglement  
lower bounds

## Covariant entropy bound and scattering regions

- CEB gives an upper bound on how much entropy can pass into a region



CEB:

$$\Delta S \leq \frac{\text{area}(R)}{4G_N}$$

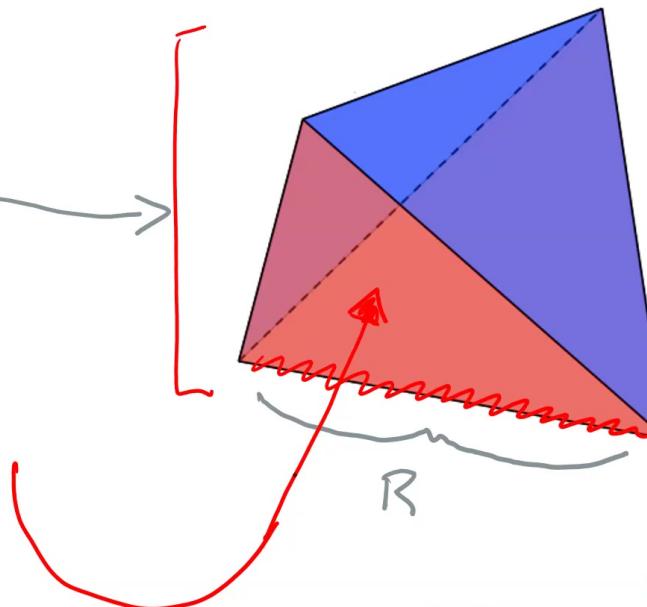
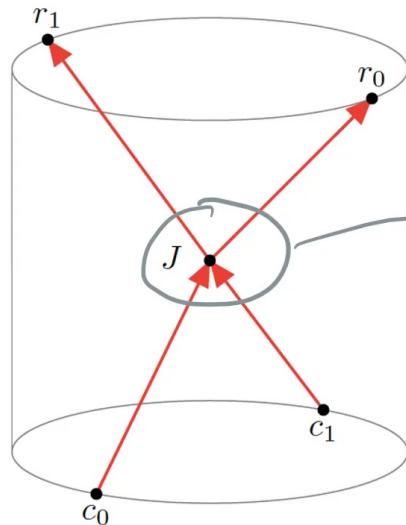
constraint on computation in scattering region

$$\Delta S \leq \frac{1}{2} I(V_0 : V_1)$$

- Using CEB + results from  
1912.05649, 2101.08855

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## Entanglement lower bounds

Can we recover this constraint from the boundary?

- Best Known bound:

$$\frac{1}{2} \Delta S \leq \frac{1}{2} I(V_0 : V_1)$$

- Comes from a "monogamy-of-entanglement" game.

Comment: If there exists a "monogamy" game (or similar) where the optimal strategy is no better than guessing

$$\text{then recover } \Delta S \leq \frac{1}{2} I(V_0 : V_1)$$

## Entanglement lower bounds

Any bound stronger than  $\Delta S \leq \frac{1}{2} I(V_1; V_2)$  would  
be a novel constraint on QI in presence of gravity!

- Exciting, but, keep in mind:

$$\begin{aligned} \textcircled{1} \quad h(n) &\leq E(f) \leq O(\text{size of span program}) \\ \textcircled{2} \quad E(f_I) &\leq \text{size of QSS} \end{aligned} \quad \left. \begin{array}{l} \text{consequences} \\ \text{of code-routing} \end{array} \right\}$$

Has only been limited success in proving explicit lower bounds  
on span programs / QSS, etc.

- Maybe a better idea:

$$\text{"complexity}(f)" \leq E(f)$$

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## Directions

### NLQC

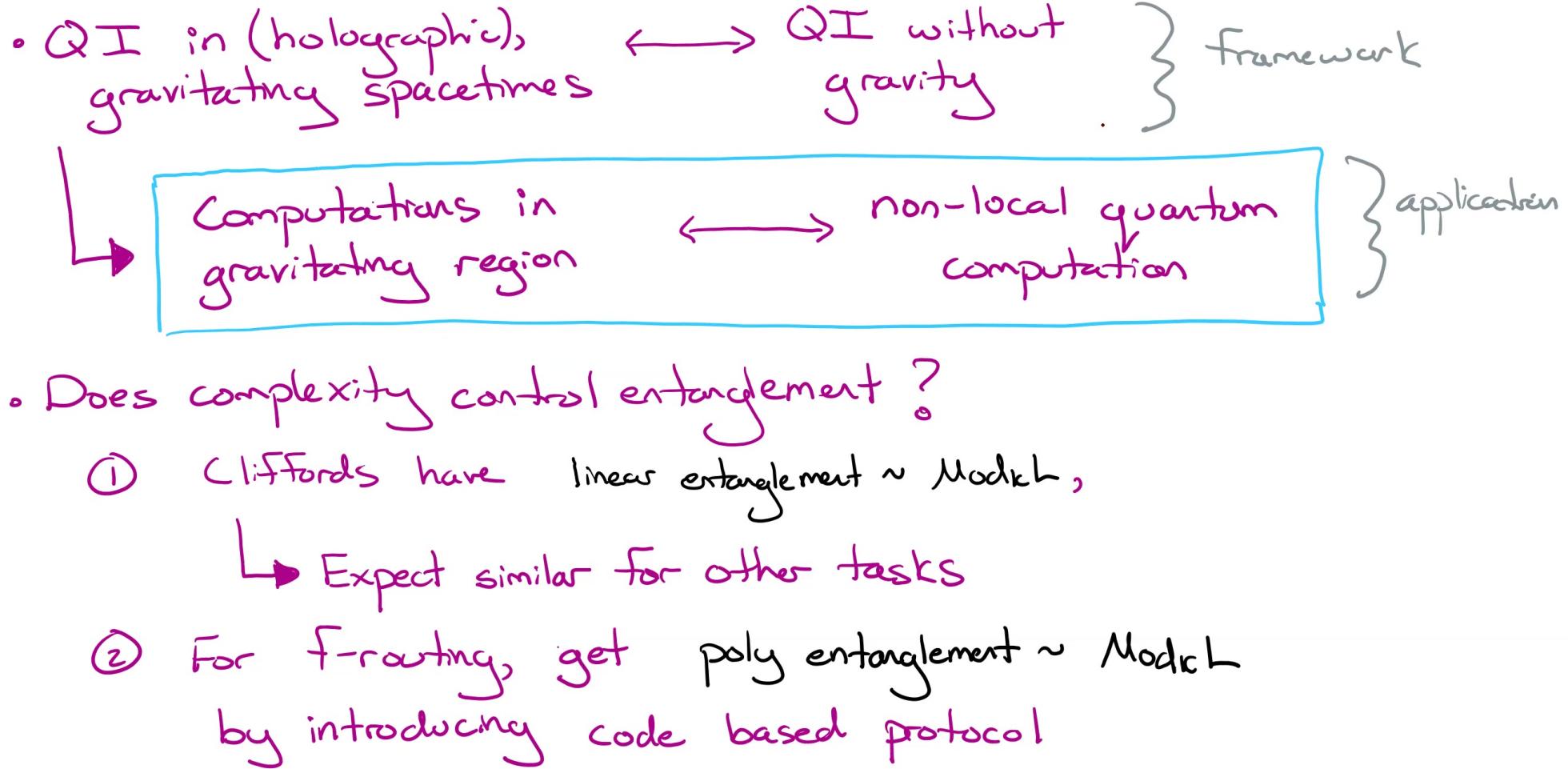
- Better lower bounds on entanglement?
  - ↳ Can we recover the CEB? (monogamy games)
  - ↳ Can we find new constraints on computation in the presence of gravity? (relationship to complexity theory)
- Improved protocols? Can we reach higher complexity with polynomial entanglement?

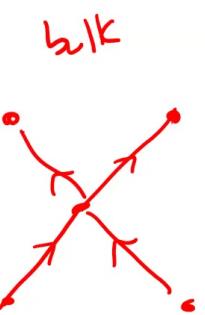
### QI in QG

- What else can we ask about QI in presence of gravity?
  - ↳ what QI/complexity/crypto questions do they raise in the boundary?

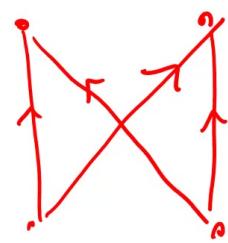
Summary Thanks!

AdS/CFT

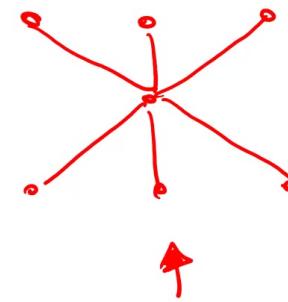
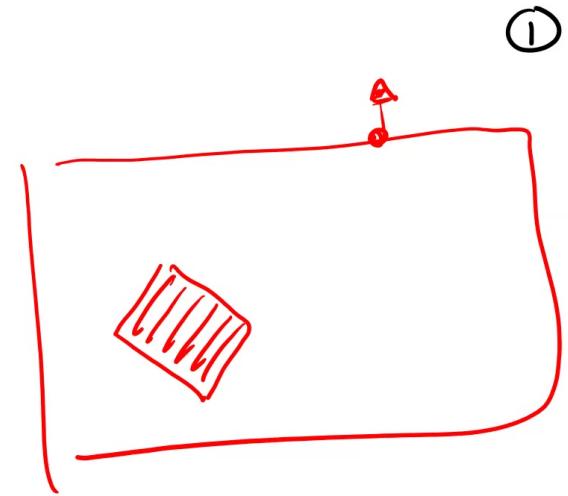
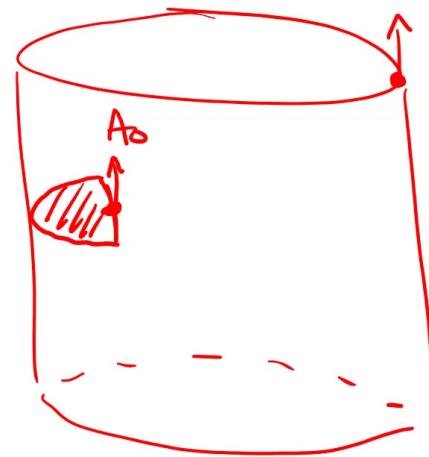




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