Title: Topological quantum codes and quantum computing

Speakers: Aleksander Kubica

Series: Perimeter Institute Quantum Discussions

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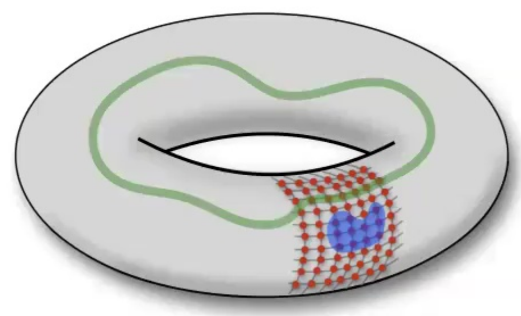
Abstract: Topological quantum codes illustrate a variety of concepts in quantum many-body physics. They also provide a realistic and resource-efficient approach to building scalable quantum computers. In this talk, I will focus on fault-tolerant quantum computing with a new topological code, the 3D subsystem toric code (STC). The 3D STC can be realized on the cubic lattice by measuring small-weight local parity checks. The 3D STC is capable of handling measurement errors while performing reliable quantum error correction and implementing logical gates in constant time. This dramatically reduces the computational time overhead and gives the 3D STC a resilience to time-correlated noise.

Topological quantum codes & quantum computing

Aleksander Kubica

aws Caltech

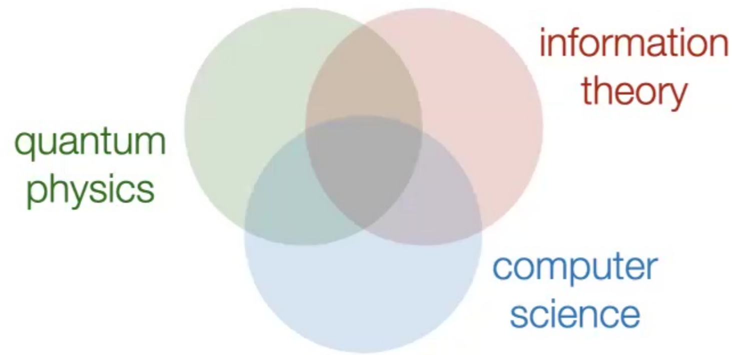


PERIMETER INSTITUTE

Jan 31, 2022

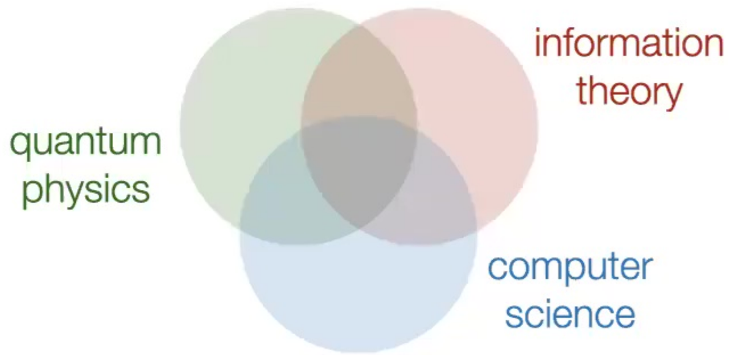
Quantum computer

Quantum information science



- Fundamental differences between classical & quantum information!
- Quantum information ideas enter mainstream physics:
  (i) qubits, (ii) entanglement, (iii) error-correcting code.
- Feynman's idea of quantum computing—"***Nature isn't classical, dammit, and if you want to make a simulation of Nature, you'd better make it quantum mechanical***".

Quantum information science



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- A ~~million~~ billion dollar question—how to build a quantum computer?

Quantum computer

- Quantum information is difficult to control & extremely fragile!

- Healthy skepticism—"***The large scale quantum machine, though it may be the computer scientist's dream, is the experimenter's nightmare***" [Haroche,Raimond96].

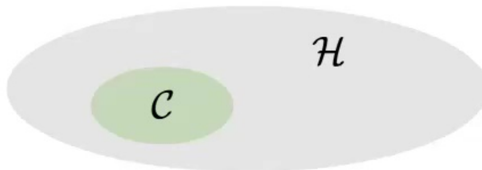
- So… is it even possible, in principle?!

- Shor's breakthrough discoveries:
  (i) quantum error correction, (ii) fault-tolerant methods.

- Why is QEC challenging?
  — Bit flips $|0\rangle \leftrightarrow |1\rangle$ and phase errors $|+\rangle \leftrightarrow |-\rangle$.
  — States cannot be copied [Wootters,Zurek82].

Quantum error correction

- Quantum code = a subspace of the Hilbert space, where we encode information.

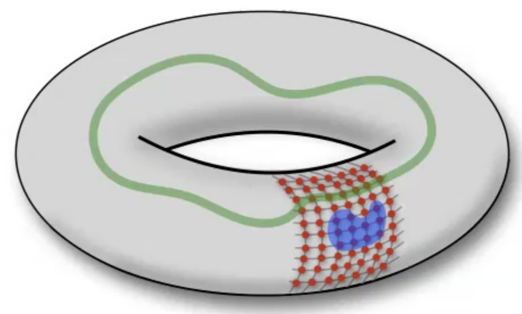


$$= \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \otimes \mathcal{H}_n$$

$$n < \infty \quad \& \quad \dim(\mathcal{H}_*) < \infty$$

- Gottesman's idea—stabilizer codes specified by parity checks.

- Kitaev's idea of topological quantum codes:
  — geometrically-local parity checks,
  — logical information encoded non-locally,
  — leading candidates for quantum computing.



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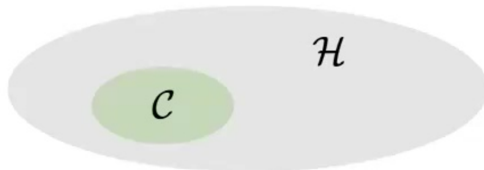

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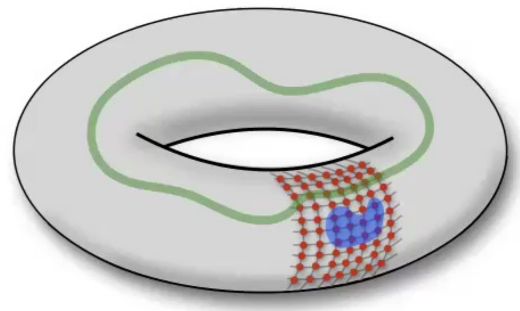
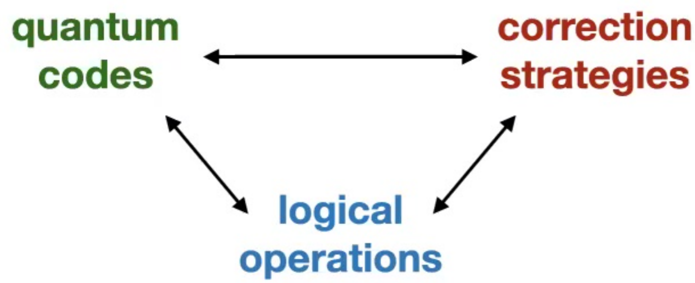


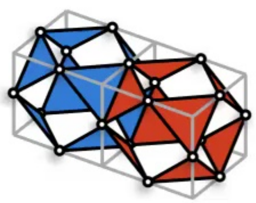
- Exciting new insights in quantum many-body physics:
  (i) topological order, (ii) exotic phases of matter, (iii) self-correction.

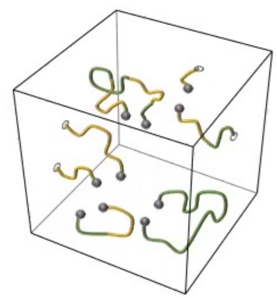
Outline

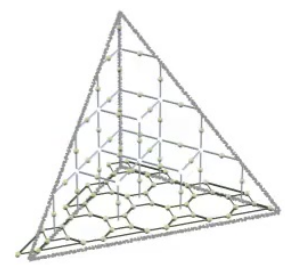
- Challenges of fault-tolerant computing:



- 3D subsystem toric code:
  (i) simple realization, (ii) single-shot QEC, (iii) logical gates in constant time.





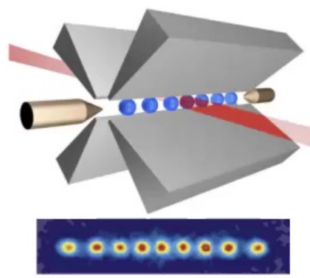


- References: [**AK**,Vasmer21], [Iverson,**AK**].

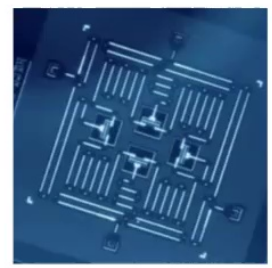
Quantum codes

quantum codes ⟷ correction strategies
logical operations

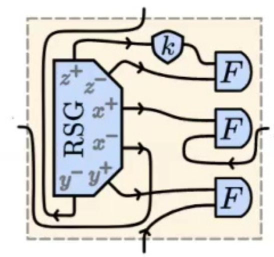
- Good quantum codes exist [Calderbank,Shor96].
- Crucial requirement—parity checks of small weight.
- Different architectures:
  — trapped ions,
  — superconducting qubits,
  — photonic qubits.



Eltony et al., *Quant. Inf. Proc.* (2016)

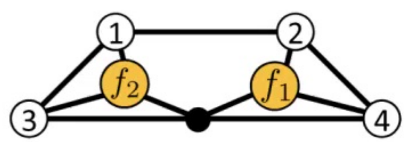


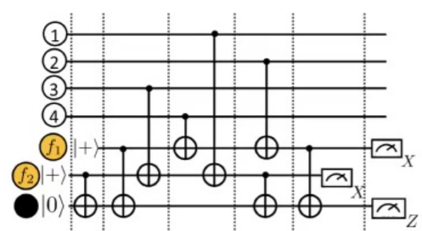
Gambetta et al., *npj Quant. Inf.* (2017)



Bombin+, *arXiv:2103.08612*

- Further limitations and optimizations [Chamberland,**AK**,Yoder,Zhu20].





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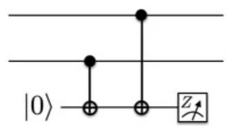
Correction strategies

- Active QEC for stabilizer codes:
  (i) parity-check measurements, (ii) decoders.



(i) → +1 -1 -1 +1 +1 +1… (ii) → Pauli recovery

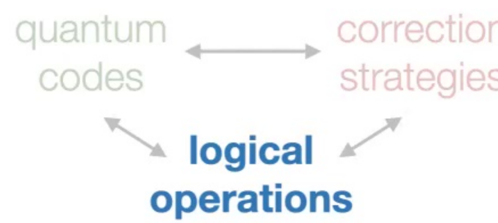
- Finding an optimal recovery is #P-hard [Iyer,Poulin15].

- Decoders need to be fast.

- Unavoidable measurement errors—may complicate & slow down QEC!

- Many imaginative ideas behind decoders:
  — minimum-weight perfect matching [Dennis+02; **AK**,Delfosse19],
  — cellular automata [**AK**,Preskill19; Vasmer,**AK**,Browne21],
  — neural networks [Maskara,**AK**,Jochym-O'Connor19],
  — tensor networks [Bravyi+14].

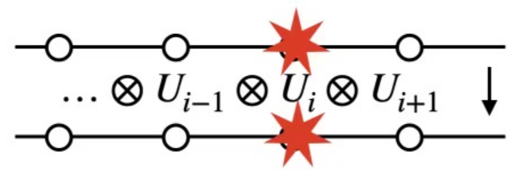
Logical operations



- Goal—process quantum information fault-tolerantly.

protection from noise ⟷ easy logical operations

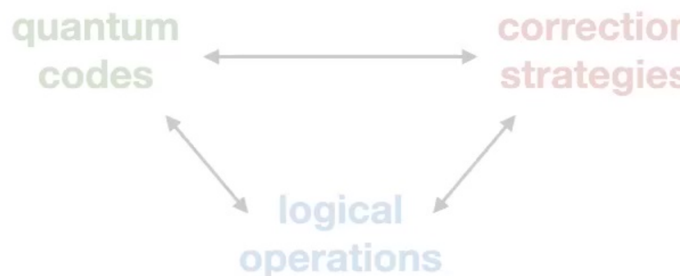
- Transversal gates do not spread errors.



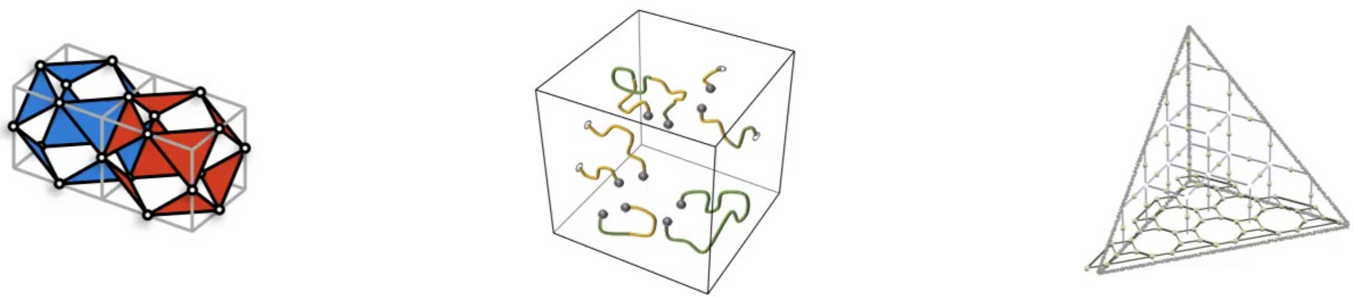
- Can we have a universal set of transversal gates [Eastin,Knill09]?
- Not possible even for approximate QEC [Faist+20; **AK**,Demkowicz-Dobrzański21].
- Making a quantitative statement—disjointness [Jochym-O'Connor,**AK**,Yoder18].
- Finding the disjointness is NP-complete [Bostanci,**AK**21].
- Indirect evidence that finding fault-tolerant logical gates is challenging!

Outline

- Challenges of fault-tolerant computing:



- 3D subsystem toric code:
  (i) simple realization, (ii) single-shot QEC, (iii) logical gates in constant time.



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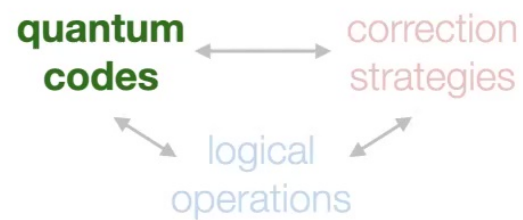
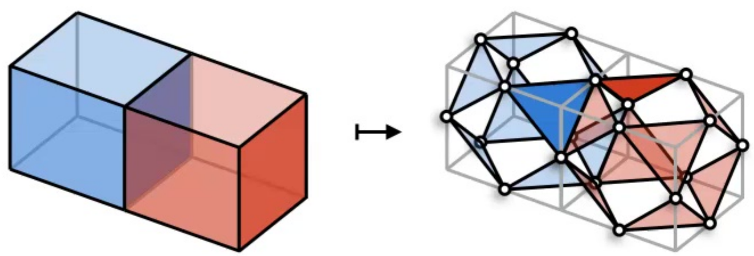
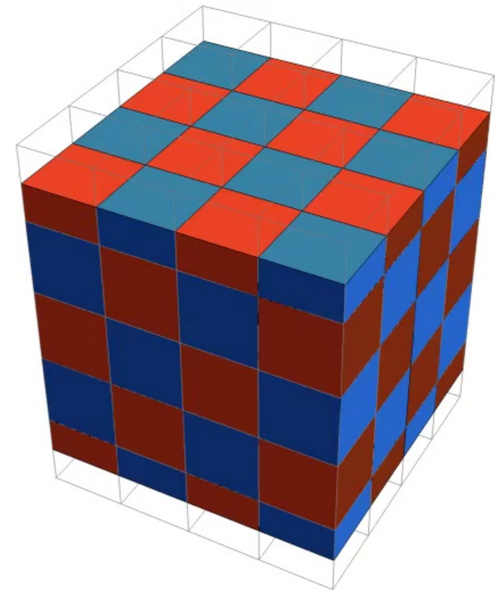
3D subsystem toric code



- Simple realization on the cubic lattice:
  — qubits on the edges.

- Parity checks—Pauli Z (blue) and X (red).





3D subsystem toric code

quantum codes ⟷ correction strategies
logical operations

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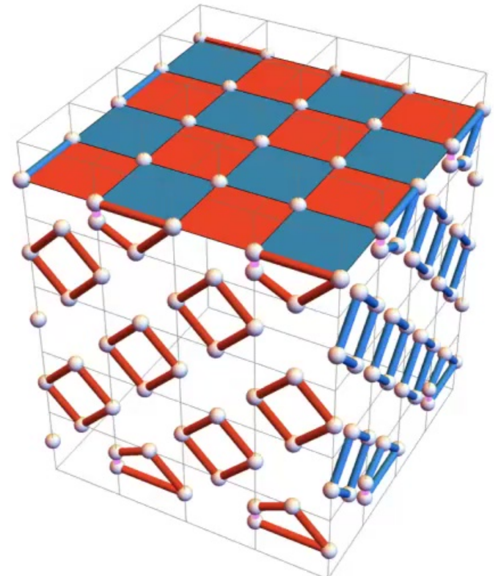

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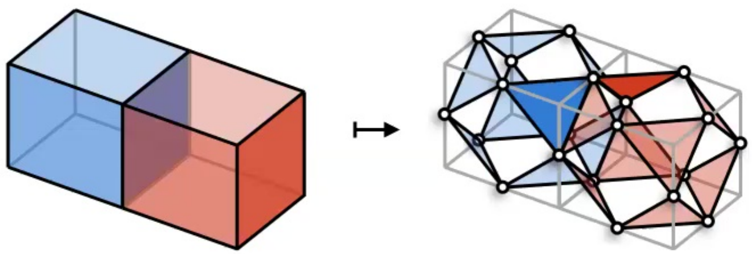
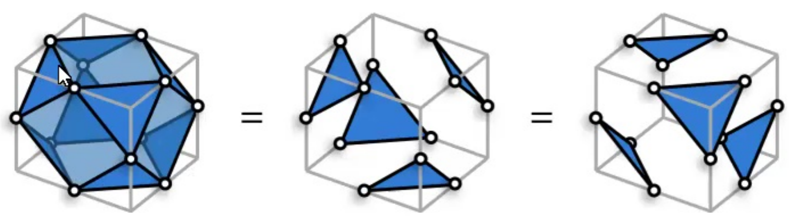


- Boundaries — necessary for a logical qubit!
- Some parity checks do not commute!
- Stabilizer operators commute w/ parity checks:



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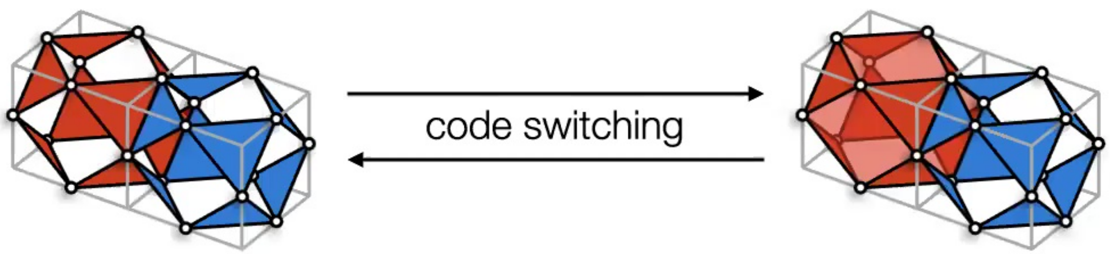
A few remarks

quantum codes ⟷ correction strategies ⟷ logical operations

- Small weight of parity checks matters!

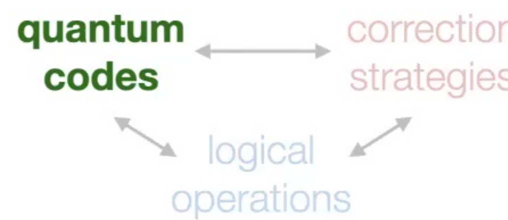
- Does the 3D subsystem toric code have anything to do with to the 3D toric code?



- The first genuine subsystem version of the toric code vs. [Bravyi+13].

- Connection between the toric and color codes [**AK**+15]:
  — the 3D subsystem toric code fundamental vs. the 3D gauge color code derivative.

A few remarks



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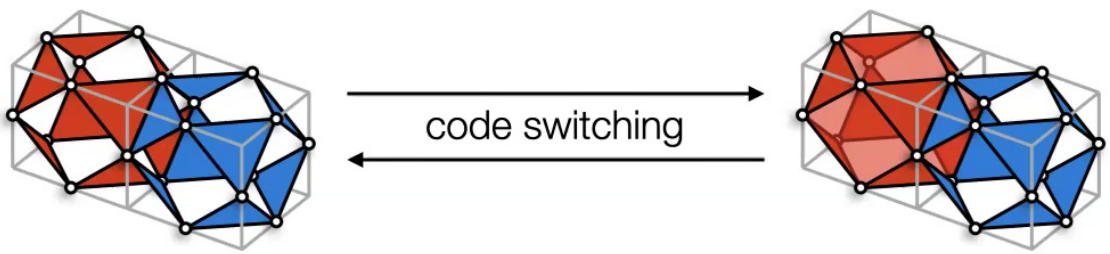


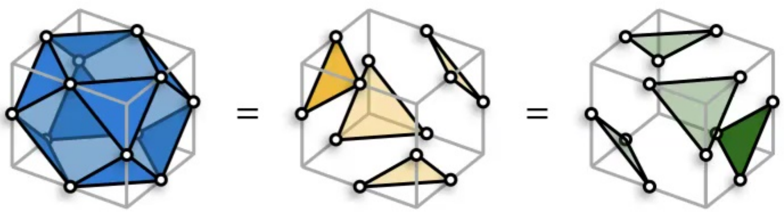
- The first genuine subsystem version of the toric code vs. [Bravyi+13].
- Connection between the toric and color codes [**AK**+15]:
  — the 3D subsystem toric code fundamental vs. the 3D gauge color code derivative.
- The subsystem toric code can be defined in higher dimensions; also for any finite Abelian group [Bridgeman,**AK**,Vasmer].

Decoding problem

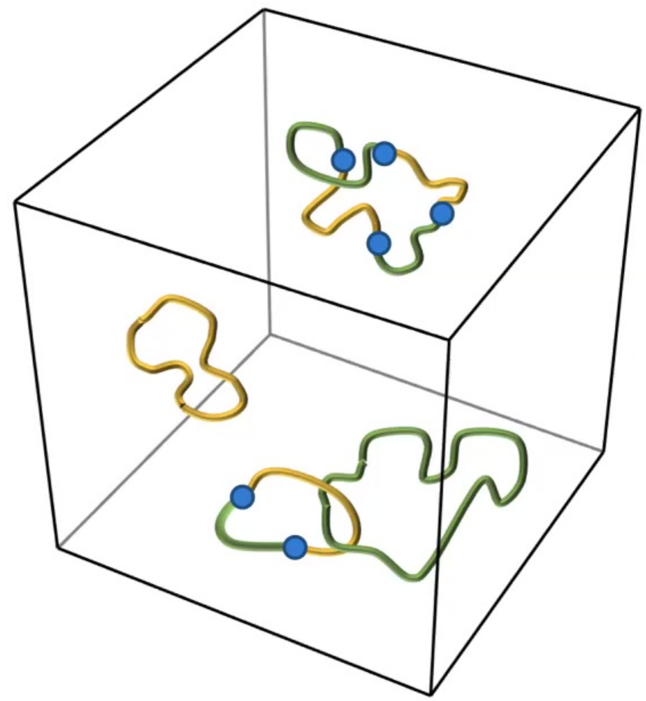
quantum codes ↔ **correction strategies**

logical operations

- Similar to the 2D toric code decoding:
  — string-like errors w/ point-like syndromes.
- We measure more than stabilizer operators:

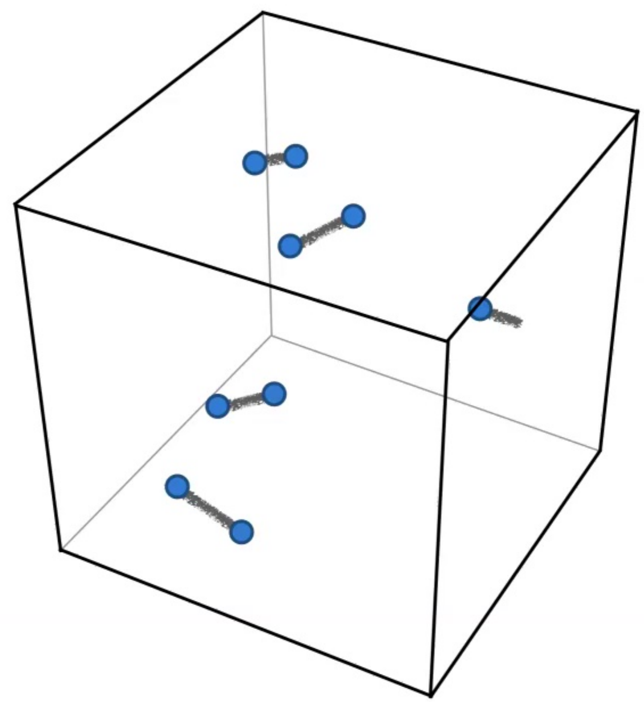


- There are local relations between parity checks!
- Parity checks w/ -1 measurement outcomes form loop-like objects = flux.
- Point-like syndromes = locations where flux changes color.



Single-shot decoder

- In the presence of measurement errors flux does not form loop-like objects!
- No need to repeat parity-check measurements!
- Two-step single-shot decoder:
  — repair flux & estimate point-like syndromes,
  — find an appropriate recovery operator.
- Both steps can be reduced to a problem of pairing up vertices within some graph!
- We use minimum-weight perfect matching:
  — efficient,
  — leads to good performance,
  — needed in the proof of single-shot QEC.



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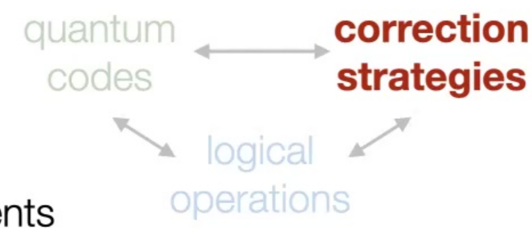
Proving single-shot QEC



- Single-shot QEC—one round of parity-check measurements suffices to perform reliable QEC in the presence of measurement errors [Bombin15].

- Making things precise:
  — $\mathbb{R}_\eta$ = single-shot decoder w/ measurement error rate η,
  — $\mathbb{N}_{\tau,\epsilon}$ = bit-flip noise w/ physical error rate τ and logical error rate ϵ.

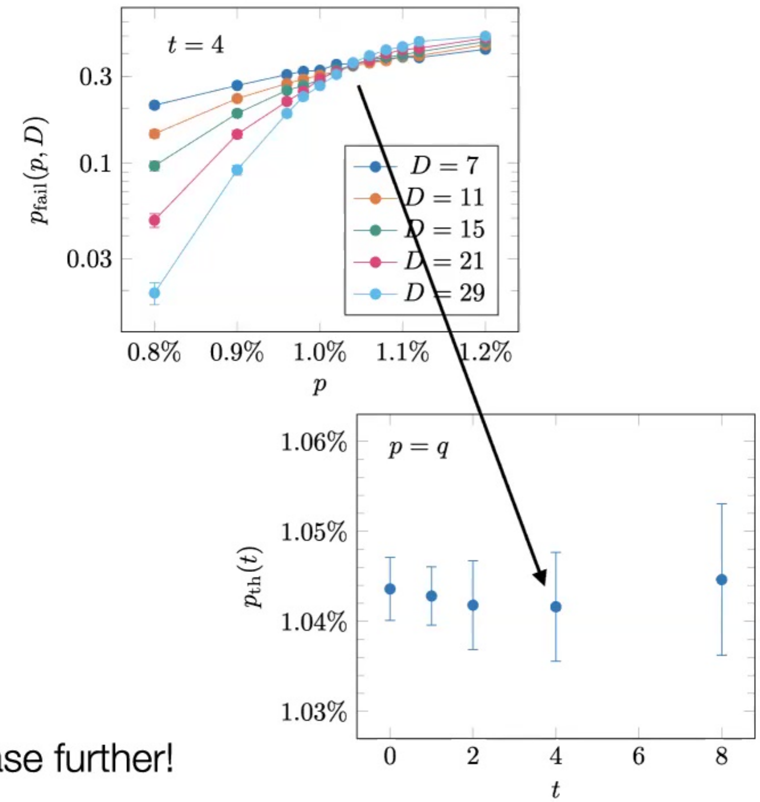
- Theorem: for sufficiently small η & τ we have $-\boxed{\mathbb{N}_{\tau,\epsilon}}-\boxed{\mathbb{R}_\eta}- \; = \; -\boxed{\mathbb{N}_{\tau',\epsilon+\delta}}-$

- Residual noise suppressed if measurement errors suppressed: $\lim_{\eta\to 0} \tau' = 0$.

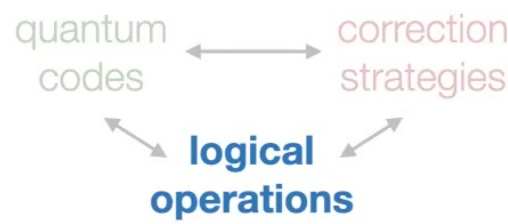
- Increase in logical error rate vanishes as system size increases: $\lim_{L\to\infty} \delta = 0$.

Numerical simulations

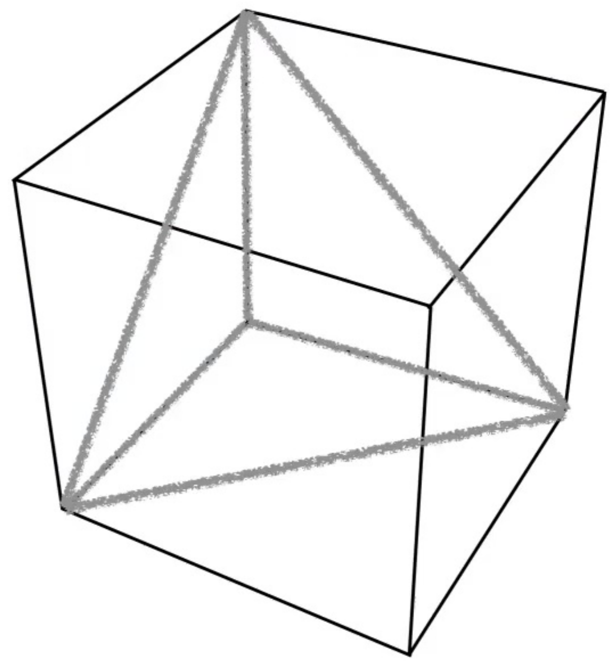
- IID bit-flip & measurement errors w/ rate p.
- We study error accumulation during repeated rounds of single-shot QEC:
  — add new errors,
  — measure parity checks,
  — use single-shot decoder.
- Storage threshold $p_{\mathrm{STC}} = \lim_{t\to\infty} p_{\mathrm{th}}(t)$.
- 3.5x improvement over the threshold of the 3D gauge color code [Brown+15]!
- For circuit noise the advantage will increase further!



Logical gates



- The tetrahedral subsystem code:
  - — based on the 3D subsystem toric code,
  - — three copies fused along the base,
  - — transversal $\overline{\mathrm{H}}$ & $\overline{\mathrm{CNOT}}$ gates.



Logical gates

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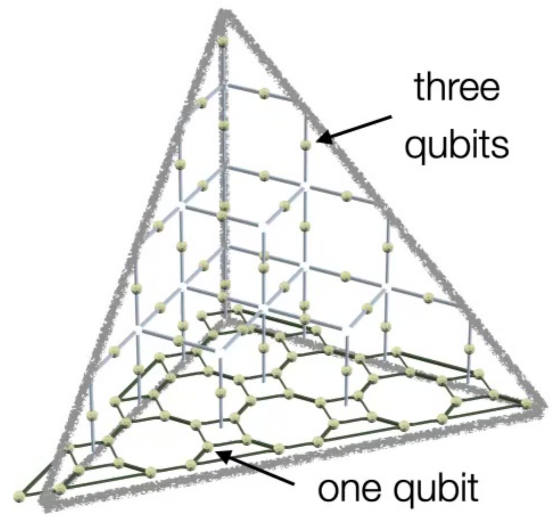
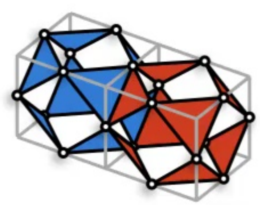
- The tetrahedral stabilizer code:
  — obtained by enforcing Z parity checks,
  — transversal $\overline{\mathrm{T}}$ gate; cf. [Vasmer,**AK**'21].

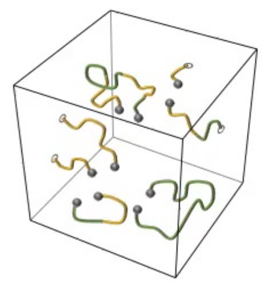
- Code switching done fault-tolerantly & in constant time!

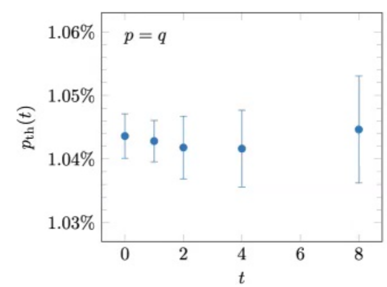


Summary

- The 3D subsystem toric code:
  - — a new beast in the zoo of quantum codes,
  - — the most transparent illustration of single-shot QEC,
  - — good performance.







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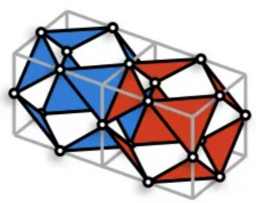
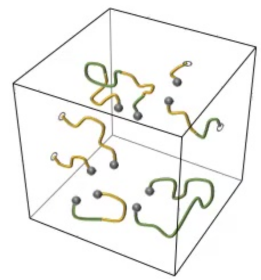
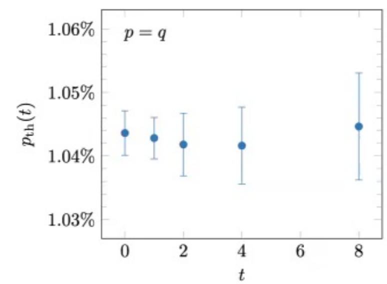






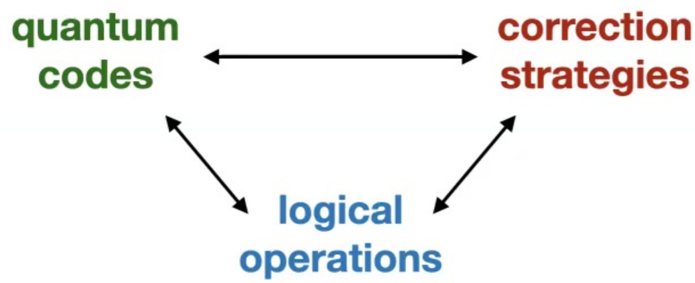
- Time overhead of fault-tolerant universal computation with the 3D subsystem toric code is dramatically reduced compared to the schemes based on the 2D toric code.

- Optimization for the biased noise—Clifford deformations [Dua,**AK**+22].

- To assess validity—a detailed resource analysis; cf. [Beverland,**AK**,Svore21].

So… are we done?

- Challenges of fault-tolerant computing:



- How to demonstrate fault tolerance & QEC with near-term devices?
- Are the assumptions behind fault tolerance & QEC justified?
- Are methods for topological quantum codes applicable to other codes?

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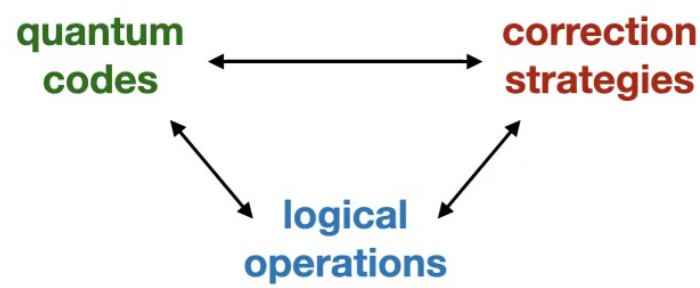

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- New physics: single-shot QEC, self-correction,…