Title: Complexity of the pythonâ€™s lunch and quantum gravity in the lab

Speakers: Hrant Gharibyan

Series: Perimeter Institute Quantum Discussions

Date: January 26, 2022 - 3:30 PM

URL: https://pirsa.org/22010074

Abstract: This talk consists of two parts. At first, I will focus on geometric obstructions to decoding Hawking radiation (python's lunch). Harlow and Hayden argued that distilling information out of Hawking radiation is computationally hard despite the fact that the quantum state of the black hole and its radiation is relatively un-complex. I will trace this computational difficulty to a geometric obstruction in the Einstein-Rosen bridge connecting the black hole and its radiation. Inspired by tensor network models, I will present a conjecture that relates the computational hardness of distilling information to geometric features of the wormhole.

Then, with the long-term goal of studying quantum gravity in the lab, I will discuss a proposal for a holographic teleportation protocol that can be readily executed in table-top experiments. This protocol exhibits similar behavior to that seen in recent traversable wormhole constructions. I will introduce the concept of "teleportation by size" to capture how the physics of operator-size growth naturally leads to information transmission. The transmission of a signal through a semi-classical holographic wormhole corresponds to a rather special property of the operator-size distribution we call "size winding".

Zoom Link: https://pitp.zoom.us/j/93957279481?pwd=eGVTU1MwOGNWNkMyYlRiWGo0QnFldz09

Complexity of the python's lunch and quantum gravity in the lab

Caltech

It from Qubit
Simons Collaboration on
Quantum Fields, Gravity and Information

Hrant Gharibyan
IQIM Caltech
Jan 26, 2022

Overview

- **Part One:** Introduction to holography and quantum information

- **Part Two:** The python's lunch: geometric obstructions to decoding Hawking radiation

  Adam Brown, HG, Geoff Penington, Leonard Susskind; ***arXiv: 1912.00228***

- **Part Three:** Quantum gravity in the lab: teleportation by size and traversable wormholes

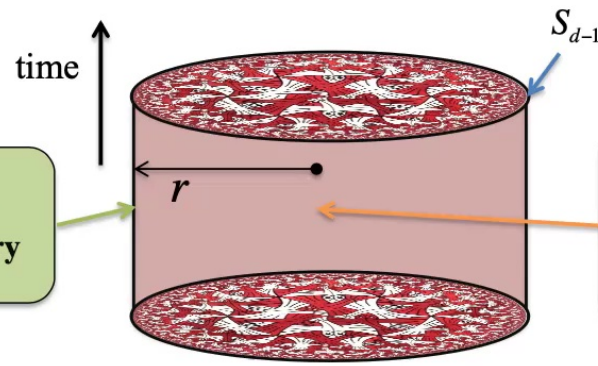
  A. Brown, HG, S. Leichenauer, H. Lin, S. Nezami, G. Salton,
  L. Susskind, B. Swingle, M. Walter; ***arXiv: 1911.06314*** *and* ***2102.01064***

Part One

Introduction to holography and quantum information

Holography: AdS/CFT Correspondence

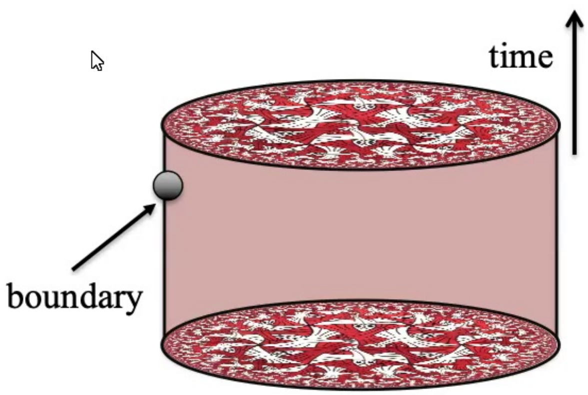
Boundary: d-dim conformal field theory



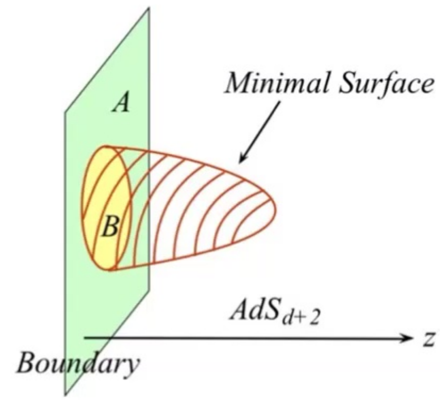
Bulk: (d+1)-dim theory of quantum gravity (asymptotically AdS)

Example: N=4 Super-Yang-Mills = Type IIB string theory in $AdS_5 \times S_5$
[Maldecena'97]

Holography and quantum information

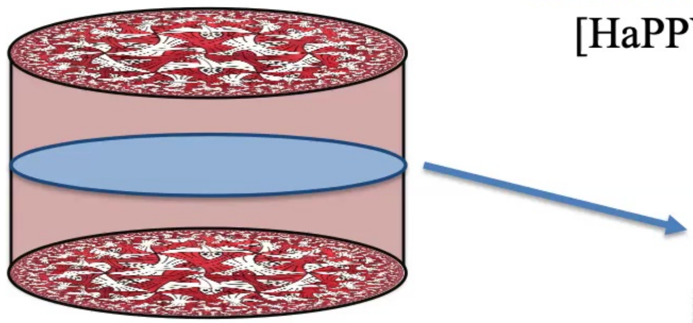


1. Entanglement entropy = Area
[Ryu-Takayanagi 2006]

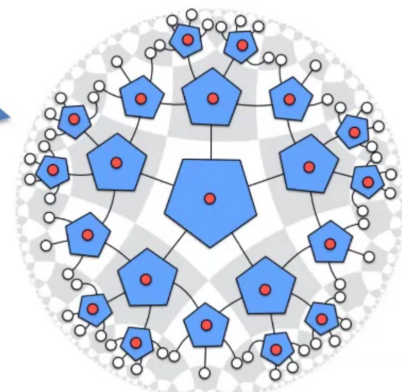


***there is covariant version**
***as well as generalized entanglement, including bulk entanglement**

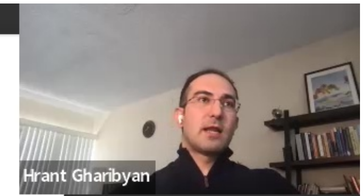
Holography and quantum information



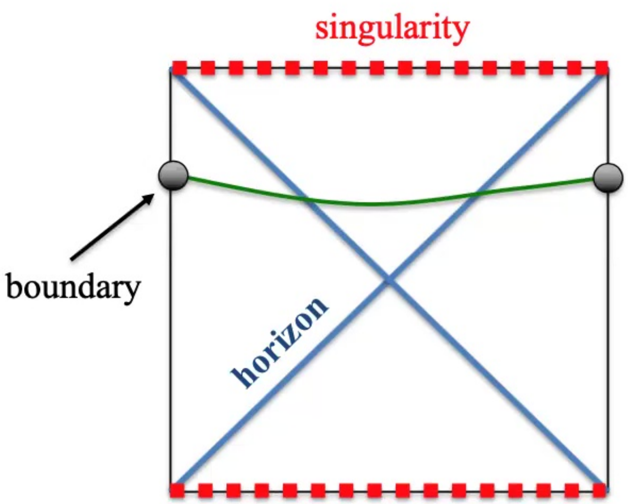
2. Error correction= Space-like slice
[HaPPY paper 2015]



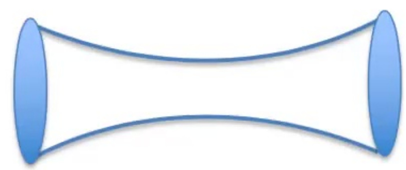
***Operator reconstruction**
***Derivation of the Page curve**



Holography and quantum information



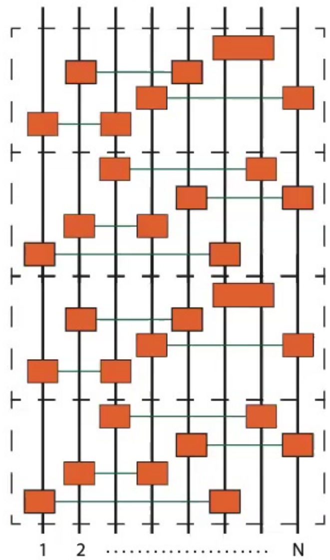
3. Complexity= Volume/Action
[Susskind and collaborators 2014]



$$\mathcal{C}(|\psi(t_L, t_R)\rangle) \propto \frac{V}{\hbar G l_{AdS}}$$

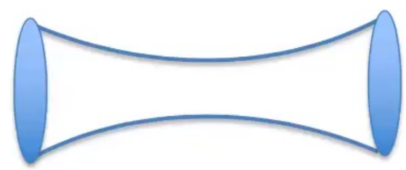
Holography and quantum information

$|\psi(t_L, t_R)\rangle$



$|000 \cdots 000\rangle$

3. Complexity= Volume/Action
[Susskind and collaborators 2014]



$$\boxed{\mathcal{C}(|\psi(t_L, t_R)\rangle) \propto \frac{V}{\hbar G l_{AdS}}}$$

Part Two

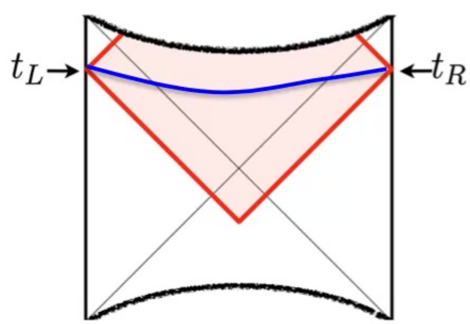
Complexity of evaporating black hole and the python's lunch geometry

[Based on **arXiv: 1912.00228]**

Relationship between complexity and geometry

A. Brown, D. Roberts, L. Susskind, B. Swingle, Y. Zhao
"Complexity Equals Action"; arXiv: 1509.07876

D. Stanford, L. Susskind,
"Complexity and Shock Wave Geometries"; arXiv: 1406.2678



[Bob Myers]

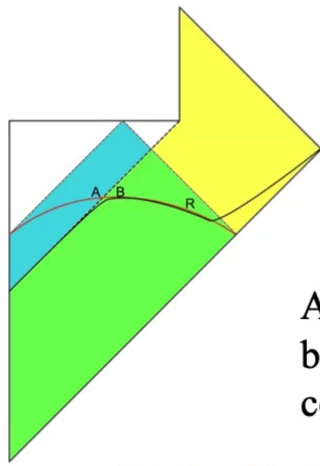
$$|\psi(t_L, t_R)\rangle$$

State Complexity = Volume of ER Bridge

State Complexity = Action of WDW Patch

Decoding Hawking radiation is exponentially hard

Daniel Harlow, Patrick Hayden,
" Quantum Computation vs. Firewalls"; arXiv:1301.4504



Decoding the information that fell into black hole by only action on radiation is hard.

$$\text{Complexity} \sim e^{S}$$

Argument relies on the mixing assumption of black hole and radiation as well as complexity class reduction.

[Harlow-Hayden 2013; Aaronson 2014]
[Kim, Tang, Preskill; 2020]

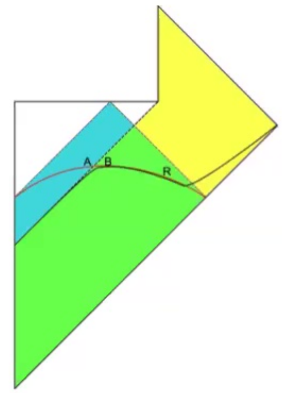
Apparent contradiction

For evaporating black hole volume/action is polynomial and those complexity is polynomial.

ploy(S)

Harlow and Hayden suggested it is exponentially hard to decode the radiation.

exp(S)

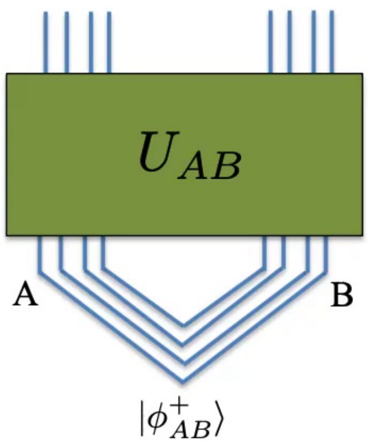


Where is this contradiction coming from?

Restricted vs unrestricted complexity

We are talking about two different notions of complexity

- - - - - - unrestricted vs restricted - - - - - -



Unrestricted complexity

$$U_{AB} = g_1 g_2 \cdots g_C$$

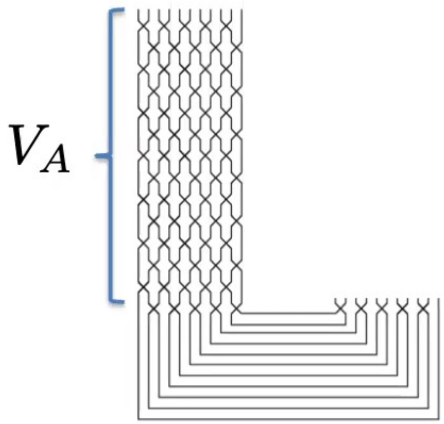
Restricted complexity

$$V_A = g_1 g_2 \cdots g_{C_A}$$

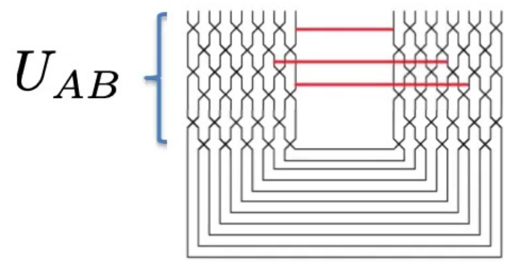
$$V_A|\phi^+_{AB}\rangle \approx U_{AB}|\phi^+_{AB}\rangle$$

Restricted vs unrestricted complexity

Restricted Complexity
(Harlow-Hayden)



Unrestricted Complexity
(Volume/Action)



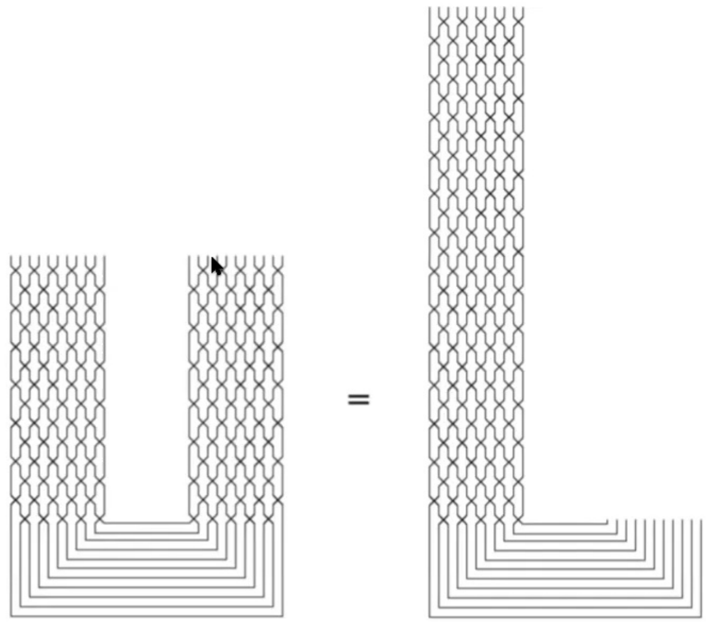
When are restricted and unrestricted complexities very different?

Time evolution of the **thermofield double.**

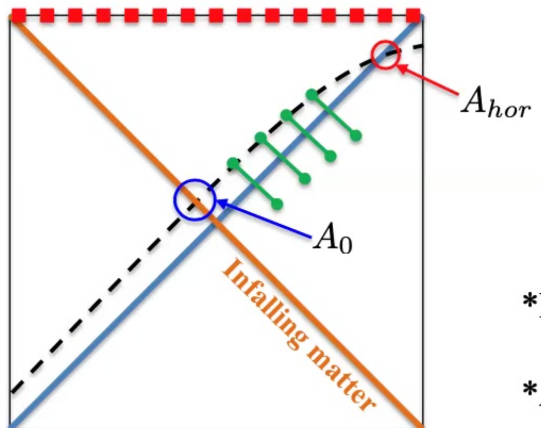
Restricted Complexity

=

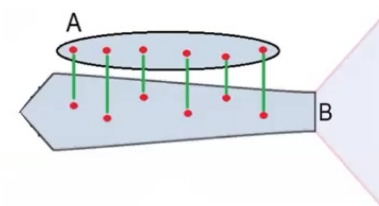
Unrestricted Complexity



Cauchy slice of the evaporating black hole



"Nice" Cauchy slice



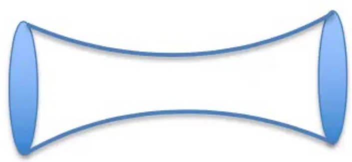
*Radiation and black hole modes are **coupled**!

*A bit after **Page time**, volume/action is polynomial in **S**.

*Restricted complexity is **exponential**.

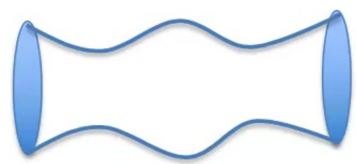
When are restricted and unrestricted complexities very different?

Small (polynomial) restricted complexity



Time evolved thermo field double (TFD)

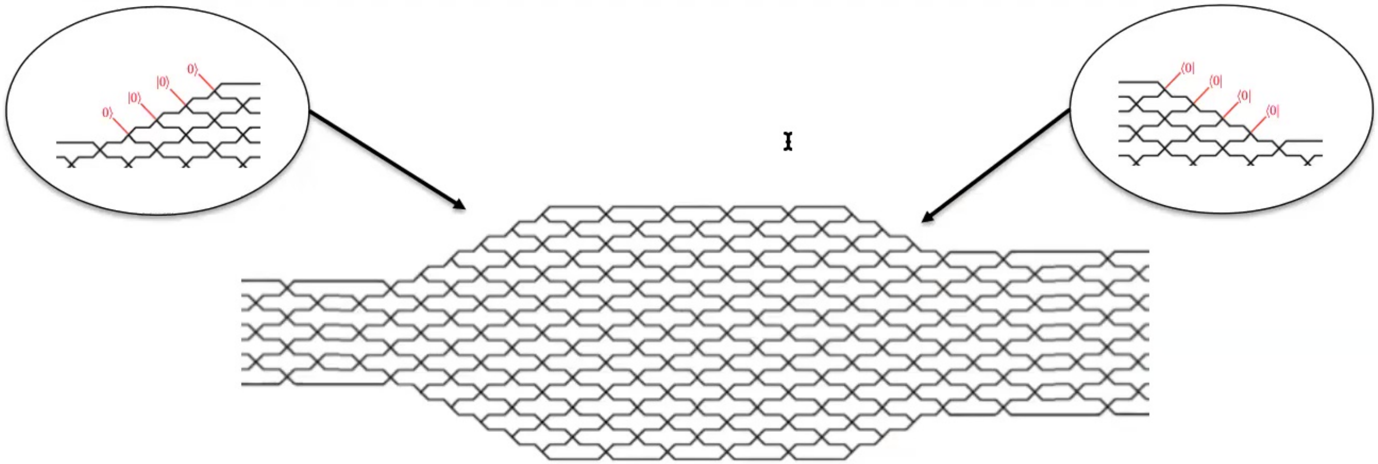
Large (exponential) restricted complexity



Python's lunch geometry

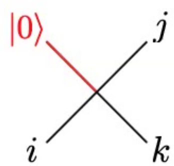
Evaporating Black Hole

The python's lunch in tensor network (toy model)



Tensor network is built off **unitary gates** and has **isometries** in endpoints.

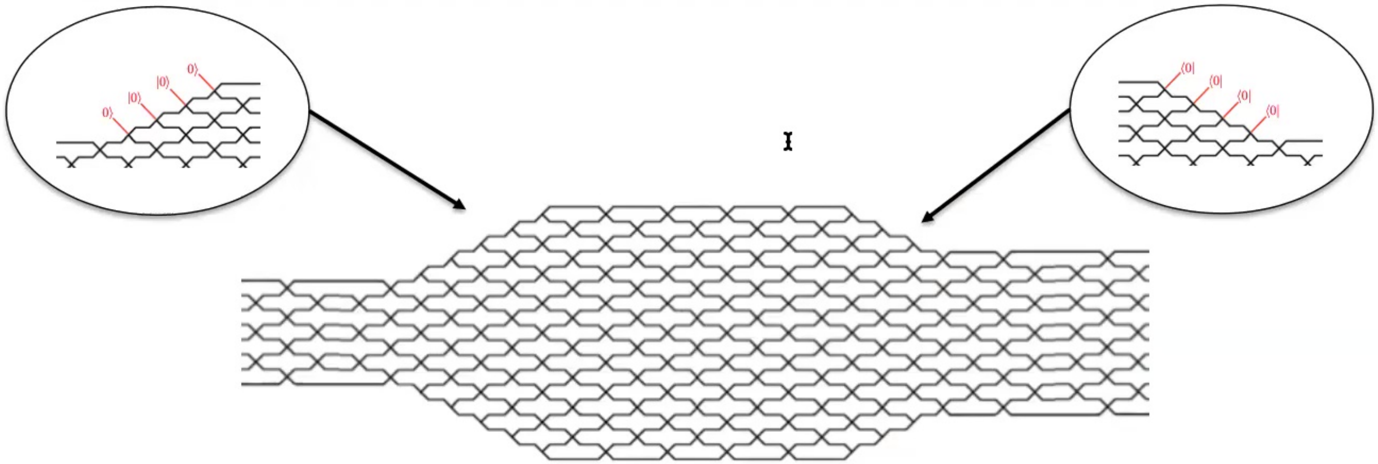
Isometry:



When are restricted and unrestricted complexities very different?

The python's lunch

The python's lunch in tensor network (toy model)



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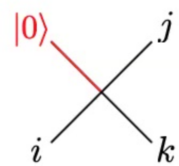
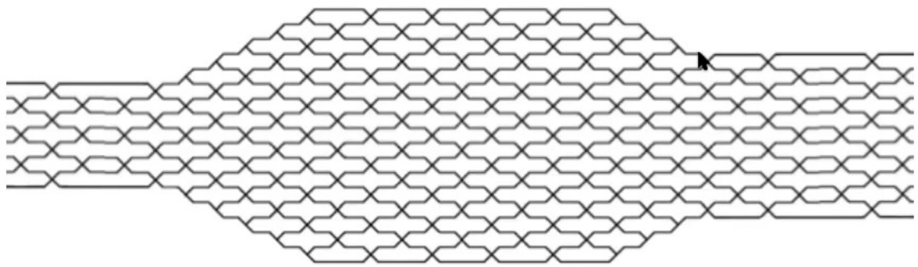
Isometry:



The python's lunch in tensor network

$|\psi_{in}\rangle$  $|\psi_{out}\rangle$

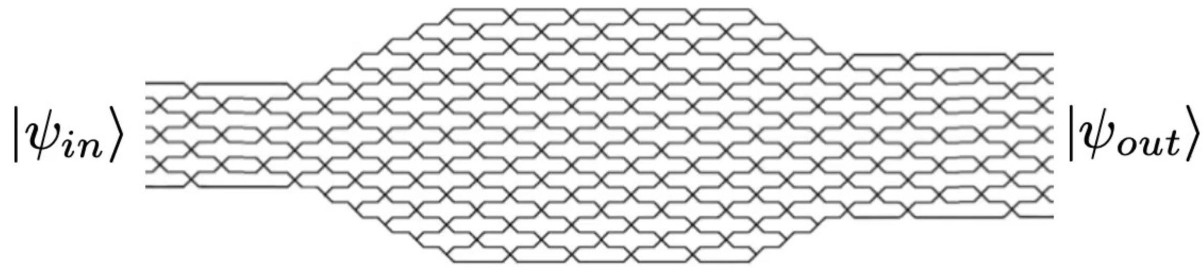
$$|\psi_{out}\rangle \propto \langle 0|^{m_R}\ U_{TN}|\psi_{in}\rangle|0\rangle^{m_L}$$

Post-selection

Polynomial Complexity Unitary

Ancillary Input Qubits

The python's lunch: no post-selection



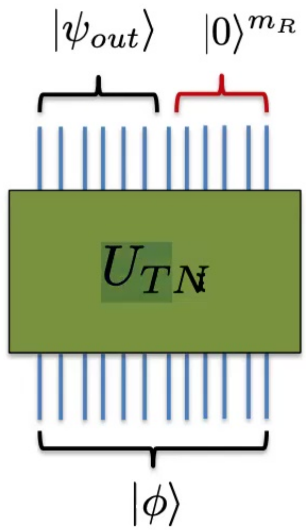
$$|\psi_{out}\rangle|0\rangle^{m_R} = U_{PL}|\psi_{in}\rangle|0\rangle^{m_L}$$

How complex is U_{PL}?

Unitary

Ancillary Input Qubits

Grover-like search: state dependent



[Kitaev - Yoshida 2017]

$$U_\phi = 2|\phi\rangle\langle\phi| - I$$
$$V = U_{TN}(2|0\rangle\langle 0|^{m_R} - I)U_{TN}^\dagger$$

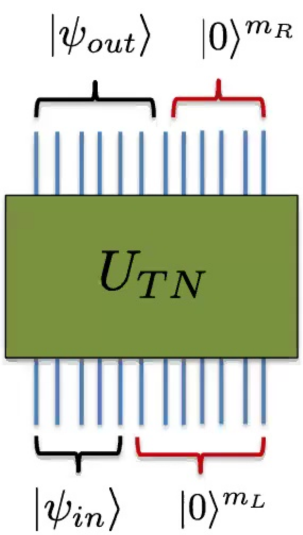
Repeat sequence: $U_\phi V$

$l = \sqrt{2^{m_R}}$ **~ times**

$$U_{PL} = U_{TN}\left[U_\phi V\right]^l$$

This algorithms depends on the state $|\phi\rangle$

Grover-like search: state independent



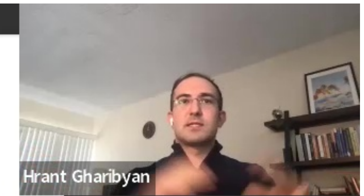
$$U = 2|0\rangle\langle 0|^{m_L} - I$$

$$V = U_{TN}(2|0\rangle\langle 0|^{m_R} - I)U_{TN}^{\dagger}$$

$$U_{PL} = U_{TN}\left[UV\right]^{l}$$

$$\boxed{\mathcal{C}(U_{PL}) \propto \sqrt{2^{m_R}} \cdot C_{TN}}$$

[Berry et. al 2015, Gilyen et. al 2017]

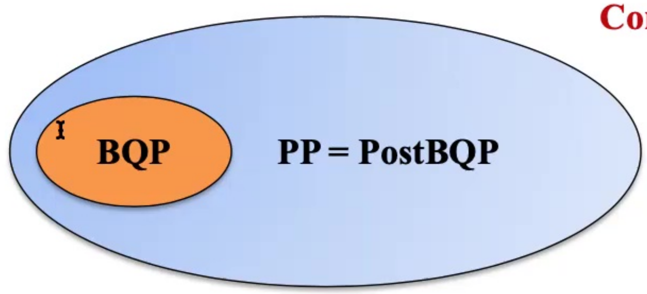


Is Grover search the optimal strategy?

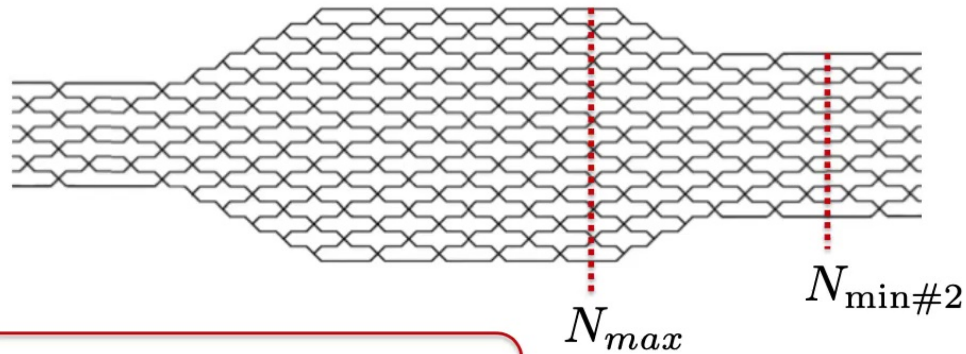
In special cases, for a specific unitary there might be an algorithm that can do better than Grover-like search.

However, for generic, **scrambling tensor network** U_{TN} polynomial algorithm is extremely unlikely to exist.

Complexity class collapse!



Conjecture: restricted complexity in tensor network



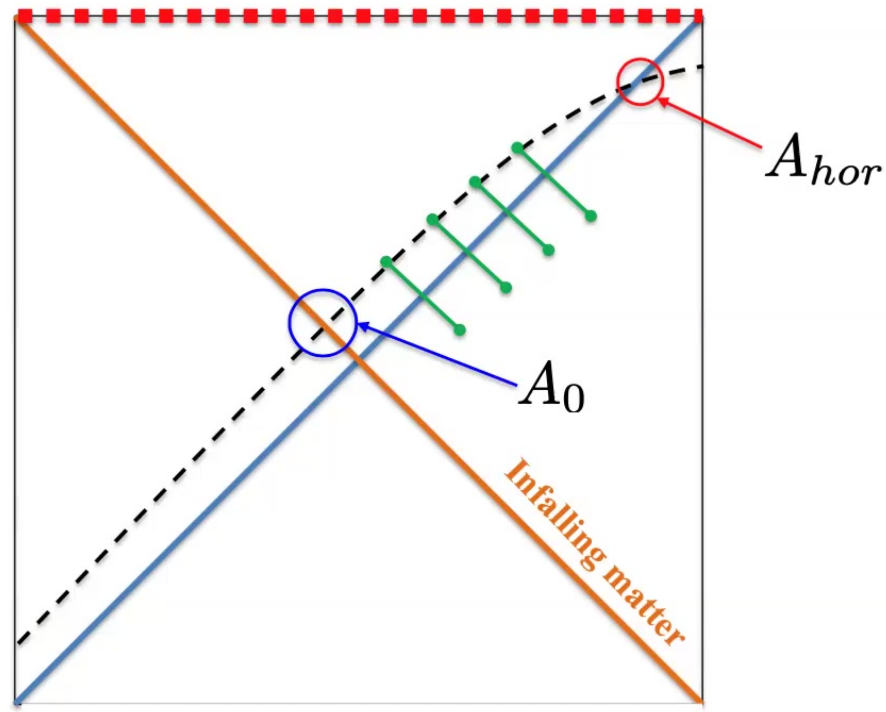
Minimax: minimal of all maximal cuts for all the possible slicing.

Largest minimal cut

The restricted complexity of scrambling tensor network is

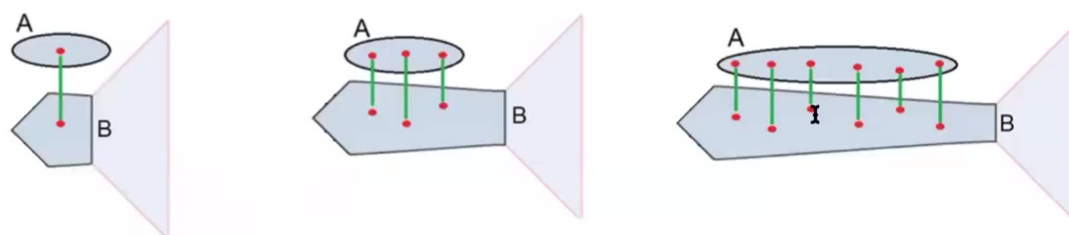
$$\mathcal{C}_R \propto C_{TN} \cdot \exp\left[\frac{1}{2}\left(N_{\max} - N_{\min\#2}\right)\right]$$

The python's lunch in evaporating black hole



The python's lunch in evaporating black hole

Naïve picture of evaporation process

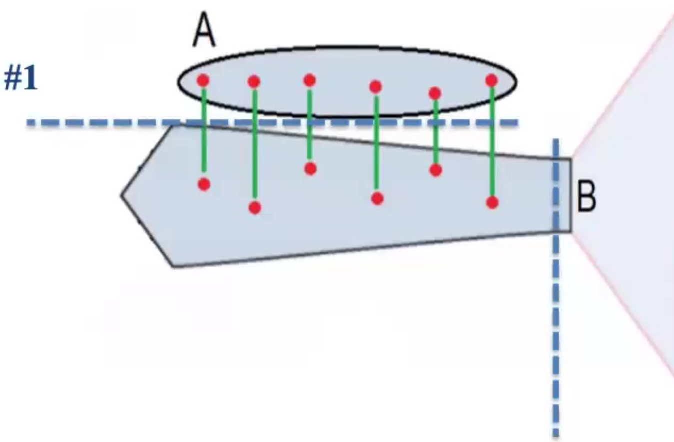


A is the radiation modes. **B** is the remaining black hole, some nice Cauchy slice going into the horizon.

The python's lunch in evaporating black hole

Minimal Cut #1

$$S_{rad}$$



Page time

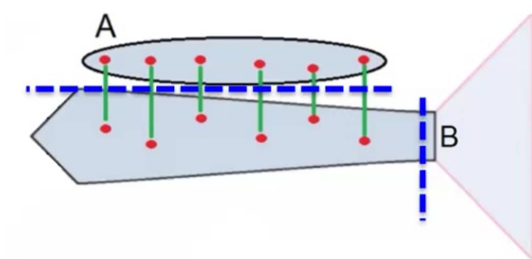
$$S_{rad} = \frac{A_{hor}}{4\hbar G}$$

Minimal Cut #2

$$\frac{A_{hor}}{4\hbar G}$$

What about the maximal cut?

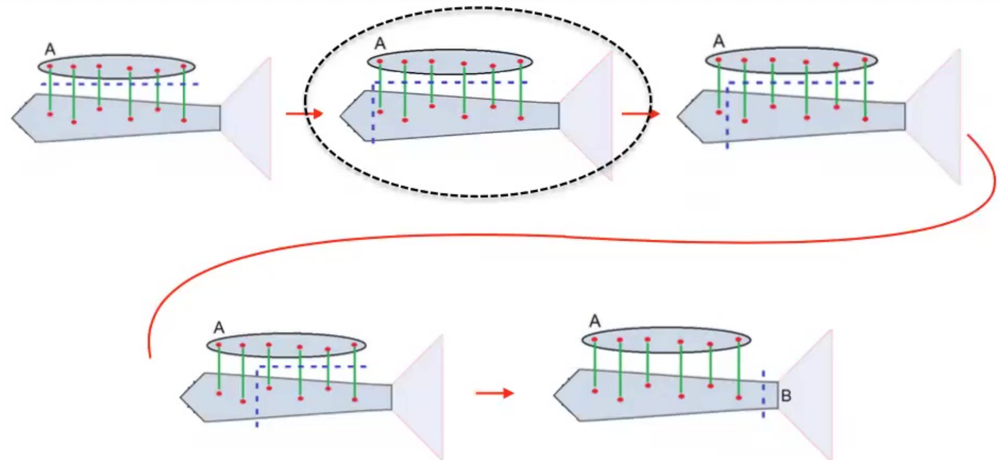
Covariant definition of python's lunch



Both minimal cuts are quantum extremal surfaces (extremize the generalized entropy).
[Engelhardt, Wall 2014]

<u>Maximin prescription:</u> **find the surfaces with smallest and second smallest generalized entropy then maximize over choice of Cauchy surfaces.**
[Wall 2012, AEPU 2019]

Python's lunch search for maximal cut

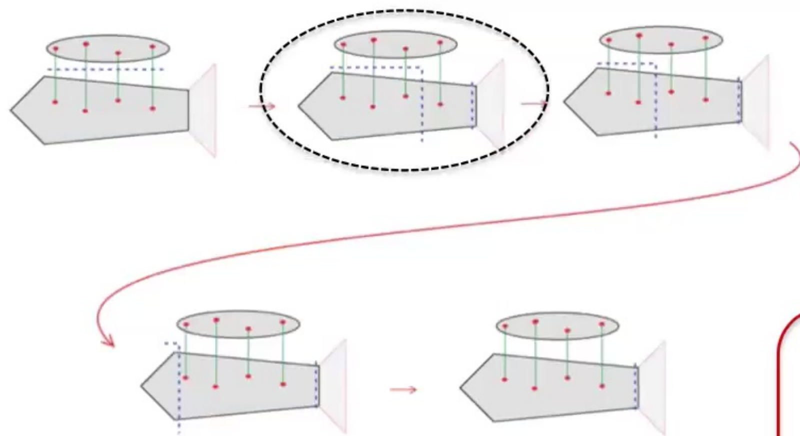


Before half time, we will be to forward slicing. The maximal generalized entropy.

$$S_{max}^{(gen)} = S_{rad} + \frac{A_0}{4\hbar G}$$

***Minimax*: *find* minimal of maximums of all slices.**

Python's lunch search for maximal cut



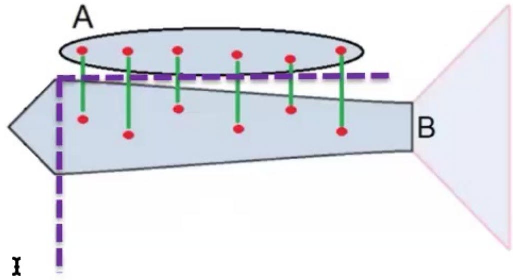
Half time

$$A_{hor} = \frac{A_0}{2}$$

After half time, we will perform reverse slicing.

$$S^{(gen)}_{max} = S_{rad} + 2\frac{A_{hor}}{4\hbar G}$$

Covariant definition of python's lunch



Minimax: **maximal cut for each Cauchy slice find the minimum of the all maximal cuts for all possible slicing methods. Analogous to the tensor network story.**

Maximinimax: **maximize again over all Cauchy surfaces, giving non-minimal quantum extremal surface.**

We find this cuts explicitly for **JT gravity plus non-interacting fermions** and verify the picture from the nice Cauchy slice.

Conjecture: restricted complexity for evaporating black hole

The restricted complexity of evaporating black hole is

$$C_R \propto C_{TN} \cdot \exp\left[\frac{1}{2}\left(S^{(gen)}_{max} - S^{(gen)}_{min}\right)\right]$$

Volume/Action (unrestricted complexity)

Maximinimax prescription

Maximin prescription

Part Three

Quantum gravity in the lab: teleportation by size and traversable wormholes

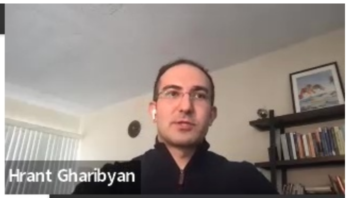
Based on arXiv: 1911.06314 and 2102.01064

Part Three

Quantum gravity in the lab: teleportation by size and traversable wormholes

Based on arXiv: 1911.06314 and 2102.01064

Simulating quantum gravity via holography

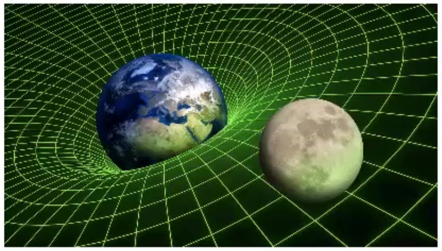


Quantum Devices

Classical Supercomputers

Holography (AdS/CFT)

Quantum Gravity



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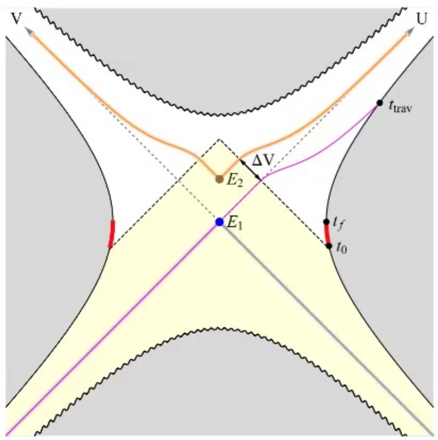
Traversable wormholes in AdS/CFT

P. Gao, D. L. Jafferis, A. Wall

"Traversable Wormholes via a Double Trace Deformation"; arXiv: 1608.05687

J. Maldacena, D. Stanford,

"Diving into Traversable Wormholes"; arXiv: 1704.05333

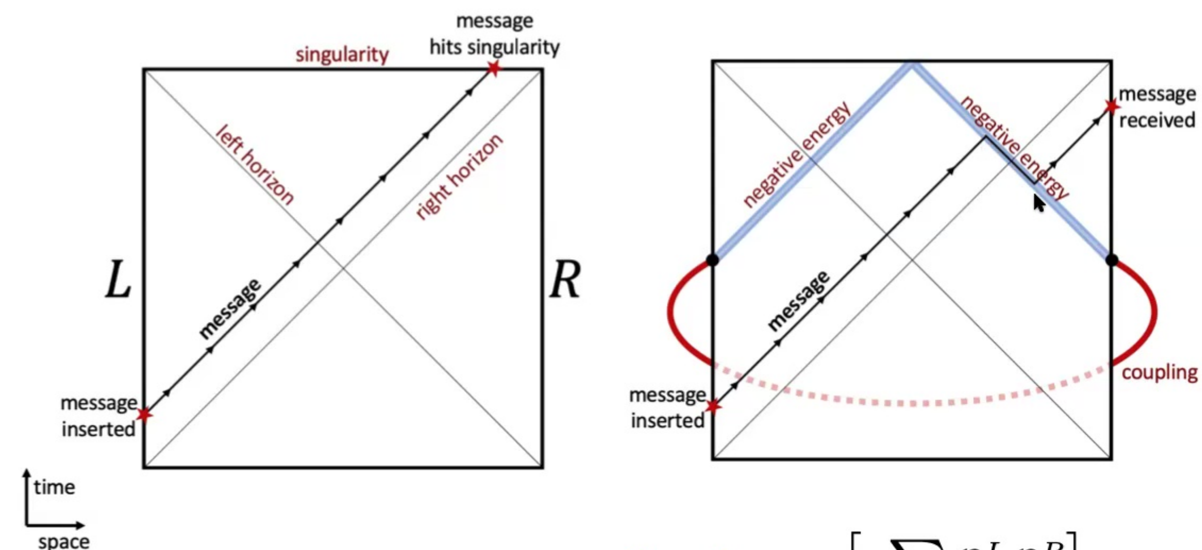


Couple left and right CFT's

$$\exp\left[g\sum_i \mathcal{O}_i^L \mathcal{O}_i^R\right]$$

Sends a negative energy particles in black hole in AdS that makes the wormhole traversable.

Is traveling through the wormhole a new kind of teleportation protocol?

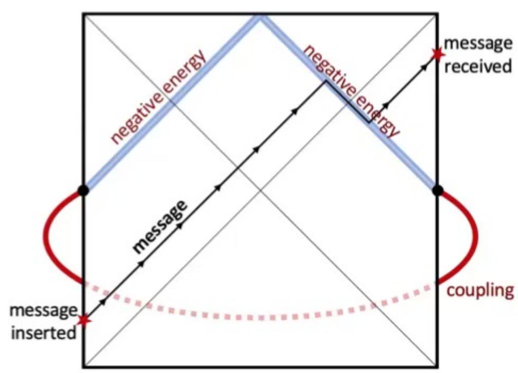


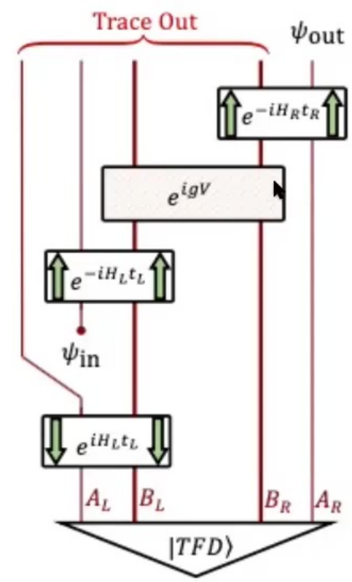
Couple: $\exp\left[g\sum_i \mathcal{O}_i^L \mathcal{O}_i^R\right]$

Understanding the flow of quantum information flow on the boundary

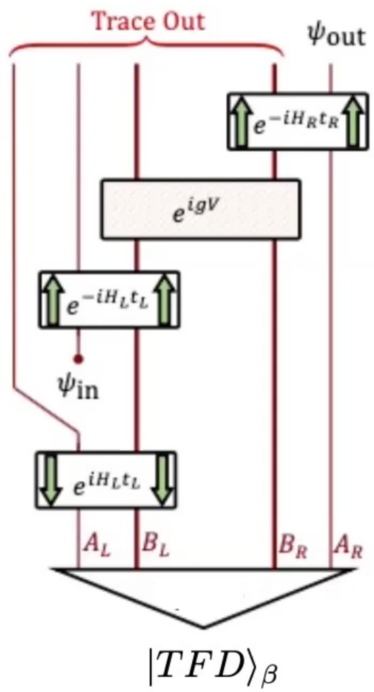
Boundary interpretation?

AdS/CFT set up





Understanding the flow of quantum information flow on the boundary



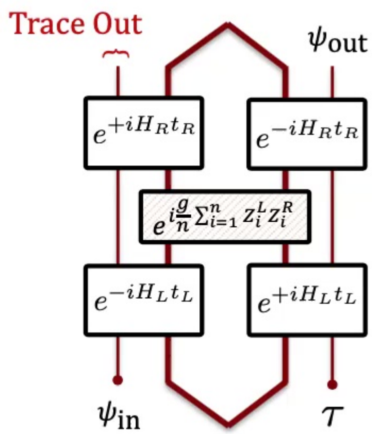
H_L, H_R **:left and right Hamiltonians**

Couples left-right

$$V = \sum_i \mathcal{O}_i^L \mathcal{O}_i^R$$

$$|TFD\rangle_\beta = \frac{1}{\sqrt{\text{Tr } e^{-\beta H}}} \sum_a e^{-\beta E_a/2} |E_a\rangle |E_a\rangle$$

Teleportation by size



$\approx \hat{S}$

Simple transformation of the initial circuit

Approximately implements size operator S: random matrix, chaotic spin chains, SYK model

$$\hat{S}|P\rangle = e^{ig|P|}|P\rangle$$

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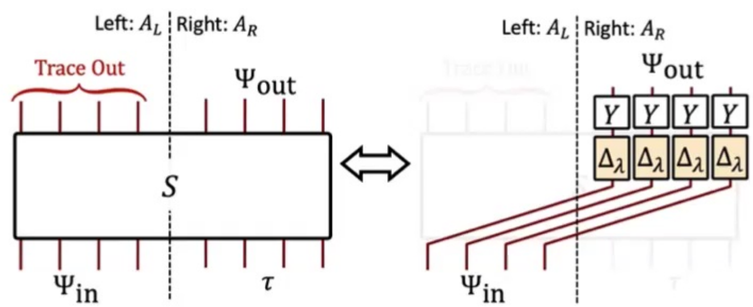
Teleportation by size



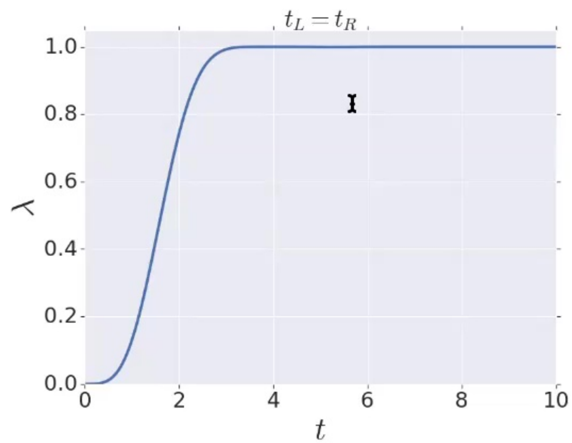
$$\hat{S}|P\rangle = e^{ig|P|}|P\rangle \text{ , where } |P\rangle = P|\phi^+_{Bell}\rangle^{\otimes m}$$

$IIXXIIXYIIZIZ$ ← **Pauli strings**

|P| is the **size of the Pauli operator** (number of non-identity elements)

Regime #1: teleportation of a single qubit

1. Teleports a single qubit with perfect fidelity.

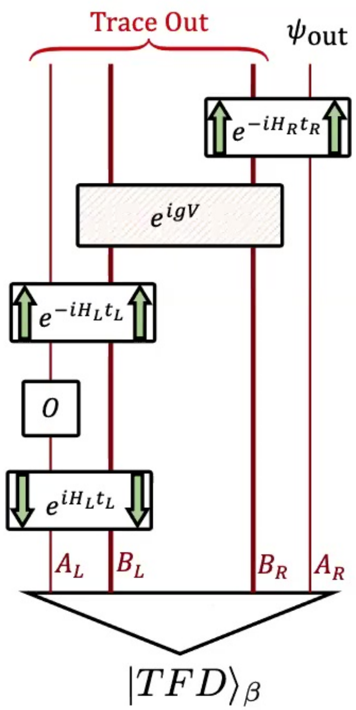


2. Works for all times after scrambling time.

3. Works the best at infinite temperature.

$$|TFD\rangle_{\beta=0} = |\phi^+_{Bell}\rangle^{\otimes n}$$

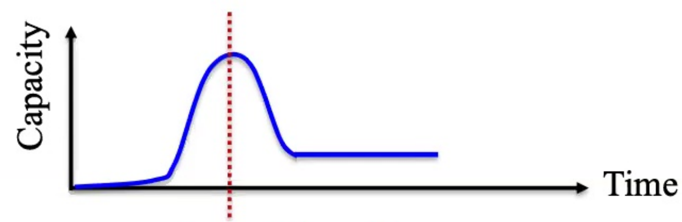
Regime #2: more qubits and low temperatures



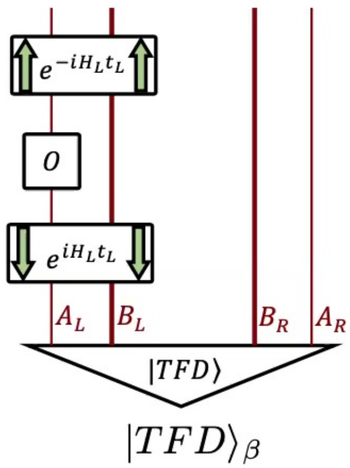
At low temperature a new phenomena appears

We can teleport more then a single qubit

This works for Hamiltonians with holographic duals: example is SYK model



Regime #2: size winding explained



Operator growth in thermal ensemble

$$\mathcal{O}(t)\rho_\beta^{1/2} = e^{-iHt}\mathcal{O}e^{+iH(t+\frac{\beta}{2})} = \sum_P c_P(t)P$$

complex

Size-winding phenomena

$$q(l) = \sum_{|P|=l} c_P^2 = e^{i\theta} \sum_{|P|=l} |c_P|^2$$

The phase of the size-distribution winds near scrambling time:

$$\boxed{\theta \sim l \times (const)}$$

Regime #2: size winding in SYK model

Size-winding is hard to illustrate from the boundary analytically.

Sachdev-Ye-Kitaev

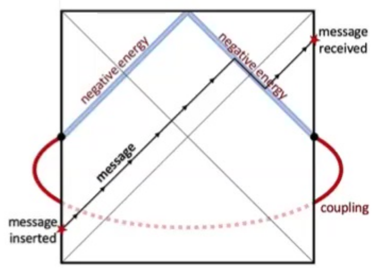
$$H_{SYK} = \sum_{i,j,k,l} J_{ijkl}\chi_i\chi_j\chi_k\chi_l$$

We verify the results noticed earlier in JT gravity

$$\boxed{\text{Moment} \;=\text{Operator Size}}$$

Bulk AdS **Boundary CFT**

Regime #2: size winding and teleportation



Left operator action

$$\mathcal{O}_L(-t_*)|TFD\rangle_\beta = \sum_P e^{+i\alpha|P|/n} r_P |P\rangle$$

Plus the coupling

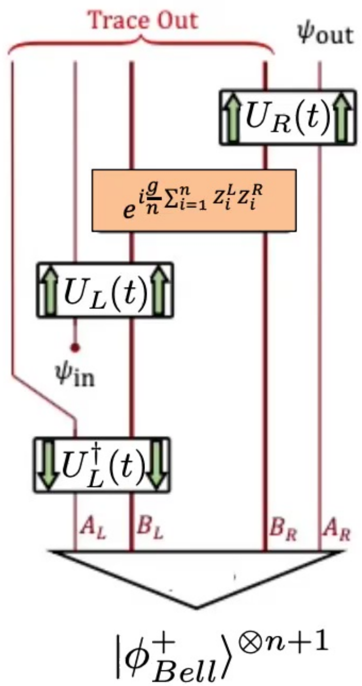
$$e^{igV}|P\rangle \approx e^{\frac{i4g|P|}{3n}}|P\rangle$$

Will equal to an operator acting on the right

$$\mathcal{O}_R(+t_*)|TFD\rangle_\beta = \sum_P e^{-i\alpha|P|/n} r_P |P\rangle$$

[Closely related work by Schuster et.al arXiv:2102.00010]

Experimental realizations



We discuss several experimental realizations that are within reach:

Superconducting qubits,
Rydberg atoms, Trapped ions

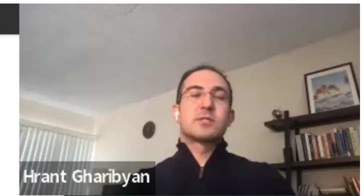
Floquet version of Rydberg dynamics

$$U_R(t) = U_L(t) = U^t$$

where each time step is $U = U_K U_L$

$$U_K = \exp\left[ib\sum_i X_i\right]$$

$$U_L = \exp\left[i\sum_i h_i Z_i + iJ\sum_i Z_i Z_{i+1}\right]$$



What can we learn about quantum gravity (holography) using the tools of quantum information and computation?

- ❖ Complexity =Volume conjecture for evaporating black hole and **the python's lunch** [arXiv:1912.00228]

$$\boxed{C_R \propto C_{TN} \cdot \exp\left[\frac{1}{2}\left(S_{max}^{(gen)} - S_{min}^{(gen)}\right)\right]}$$

- ❖ Holographic complexity in **JT gravity** [arXiv:1810.08741]
- ❖ Onset of chaos in **SFF** from random quantum circuits [arXiv:1803.08050]

Future research:

- ➢ Find other examples of holography where python's lunch appears
- ➢ Test this conjecture in quantum field theories, perhaps with alternative definition of complexity

Can we study deep questions about quantum gravity in the near term quantum devices? (quantum gravity in the lab)

- **Teleportation by size** protocol inspired by traversable wormholes
- The **size winding** phenomenon and connection to semi-classical gravity duals [arXiv:1911.06314 and 2102.01064]
- Simulating **matrix quantum mechanics** on a quantum computer (BFSS/BMN holographic dual to string theory) [arXiv: 2011.06573]

Future research:

- What is the link between maximal chaos (in OTOC) and size winding?
- Teleportation protocol for GHZ-like entangled state
- Construct new protocols and observables that have quantum gravity interpretation and can be measured in near term experiments

The End

The python's lunch

Quantum gravity in the lab