Title: Conservation laws and quantum error correction

Speakers: Benjamin Brown

Series: Perimeter Institute Quantum Discussions

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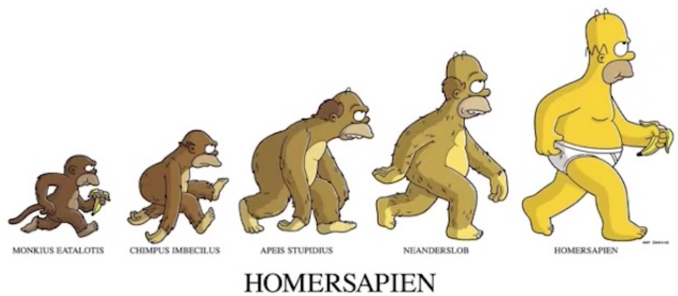
Abstract: A quantum error-correcting code depends on a classical decoding algorithm that uses the outcomes of stabilizer measurements to determine the error that needs to be repaired. Likewise, the design of a decoding algorithm depends on the underlying physics of the quantum error-correcting code that it needs to decode. The surface code, for instance, can make use of the minimum-weight perfect-matching decoding algorithm to pair the defects that are measured by its stabilizers due to its underlying charge parity conservation symmetry. In this talk I will argue that this perspective on decoding gives us a unifying principle to design decoding algorithms for exotic codes, as well as new decoding algorithms that are specialised to the noise that a code will experience. I will describe new decoders for exotic fracton codes we have designed using these principles. I will also discuss how the symmetries of a code change if we focus on restricted noise models, and how we have leveraged this observation to design high-threshold decoders for biased noise models. In addition to these examples, this talk will focus on recent work on decoding the color code, where we found a high-performance decoder by investigating the defect conservation laws at the boundaries of the color code. Remarkably, our results show that we obtain an advantage by decoding this planar quantum error-correcting code by matching defects on a manifold that has the topology of a Moebius strip.

Zoom Link: https://pitp.zoom.us/j/91540245974?pwd=RDkzaVJZZ2tkTldxM2pkdXU5VHlIZz09

Conservation laws and quantum error correction

Ben Brown

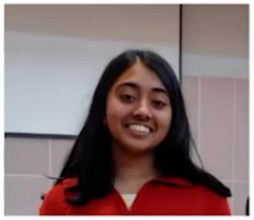


Collaborators

Dom Williamson — PR Research (2020)

Georgia Nixon — IEEE: Trans. Info. Theor. (2021)



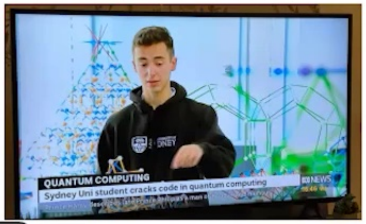



Naomi Nickerson — Quantum (2019)

Kaavya Sahay — To appear in PRX Quantum

D. Tuckett, S. Bartlett, S. Flammia, P. Bonilla Ataides



Phys. Rev. Lett. (2020) + Nat. Comms. (2021)

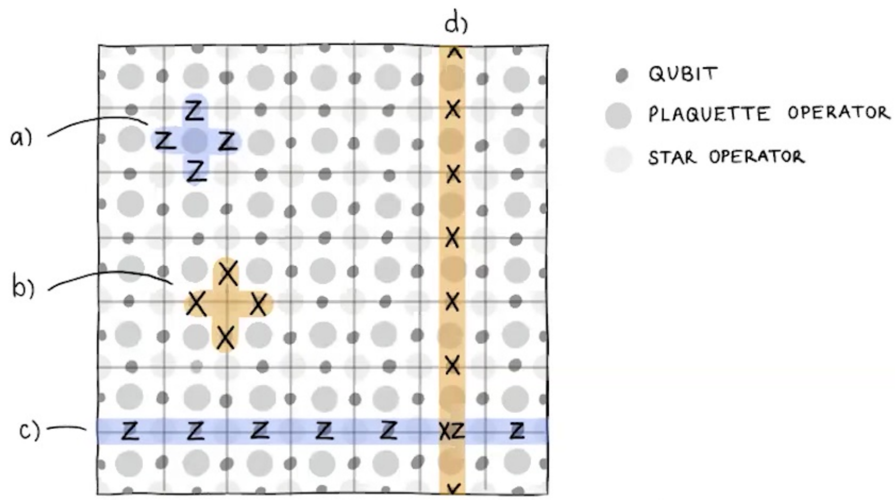
Toric code[1]

The toric code is a stabilizer code

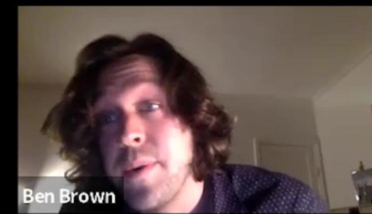
Stabilizers S are elements of the (Abelian) stabilizer group with

$$S|\psi\rangle = |\psi\rangle$$

for code states $|\psi\rangle$.



[1]A. Kitaev, Ann. Phys. 303, 2 (2003)

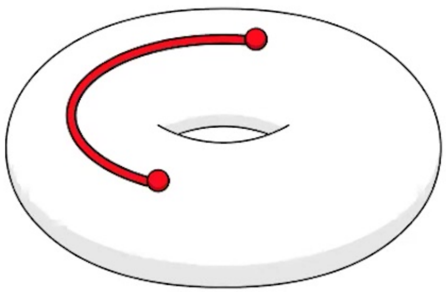


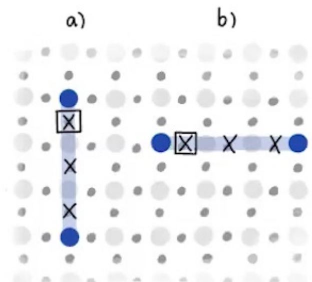
Toric code[2]

The toric code is a stabilizer code

We use stabilizers to measure (Pauli) errors E.

$$SE|\psi\rangle = (-1)E|\psi\rangle$$





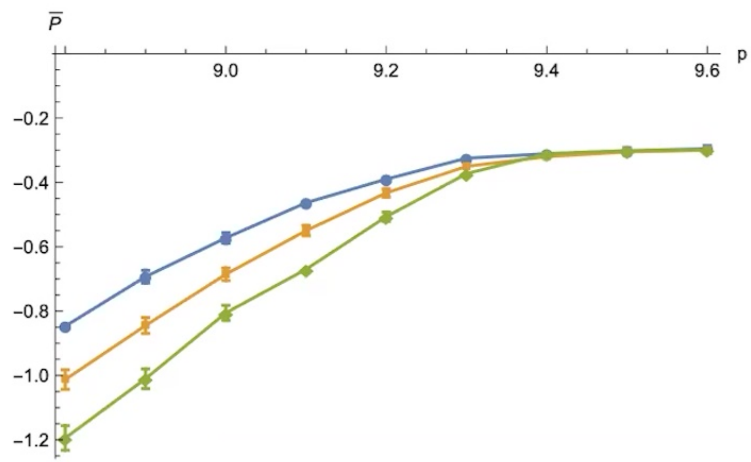
Defects are also excitations of the Hamiltonian

$$H = -\sum_{S} S.$$

[2] A. Kitaev, Ann. Phys. 303, 2 (2003)

Decoding the toric code

Below threshold, we can decrease logical failure rate arbitrarily by increasing the system size.

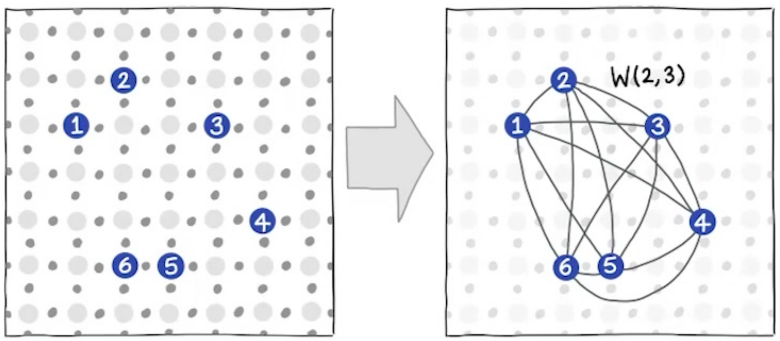


Decoding the toric code

The toric code is decoded using minimum-weight perfect matching[3]

Input: A graph with weighted edges

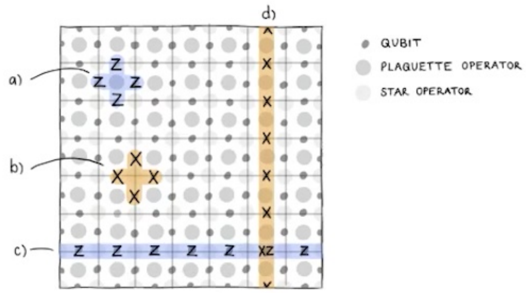
Output: Matching such that the sum of edge weights is minimal

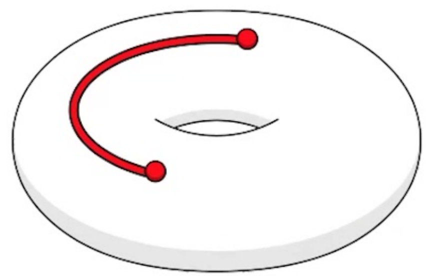


[3]Dennis *et al.* J. Math. Phys. (2002), Wang *et al.* Ann. Phys. (2003), Raussendorf *et al.*(several papers∼2006), Fowler *et al.* [many papers]. . .

Conservations laws

MWPM is possible because the toric code respects a charge conservation symmetry[4]





Mathematically, the symmetry is apparent in the stabilizer group

$$\prod_v A_v = \mathbb{1} \quad \Rightarrow \quad \prod_v a_v = 1$$

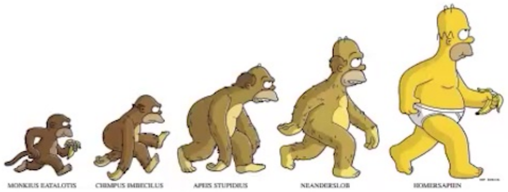
($a_v = \pm 1$ eigenvalues of stabilizers A_v.)

$\Rightarrow$ $\# v$ with $a_v = -1$ is even $\equiv$ defect conservation (mod2)

[4] A. Kitaev, Ann. Phys. **303**, 2 (2003)

Parallelized quantum error correction

We identified a general connection between conservation laws and error correction[5] to be applied to codes beyond the surface code.



$$\prod_v A_v = \mathbb{1} \quad \Rightarrow \quad \prod_v a_v = 1$$

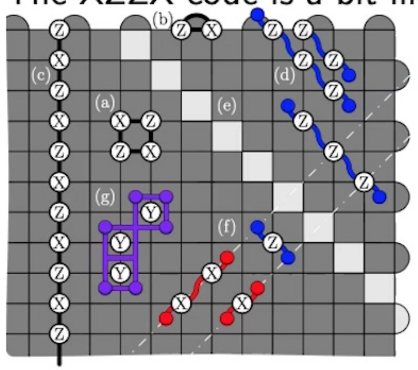
$(a_v = \pm 1$ eigenvalues of stabilizers A_v. $)$

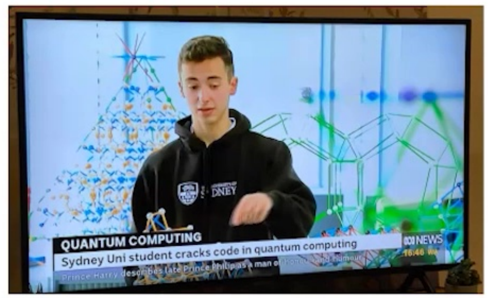
$\Rightarrow \quad \#v$ with $a_v = -1$ is even $\equiv$ defect conservation (mod2)

[5] BJB and D. J. Williamson, Phys. Rev. Research **2**, 013303 (2020)

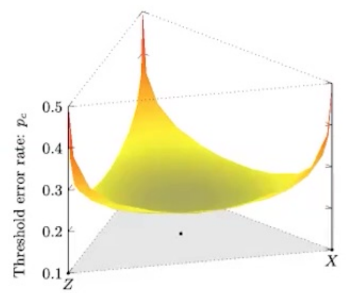
The XZZX surface code and biased noise

The XZZX code is a bit like a 2D fracton code under biased noise.





P. Bonilla Ataides *et al.*, Nat. Commun. **12**, 2172 (2021)



System symmetries

We can define the symmetries of a code **with respect to an error model**

The symmetries (a subgroup of the stabilizer group) are such that

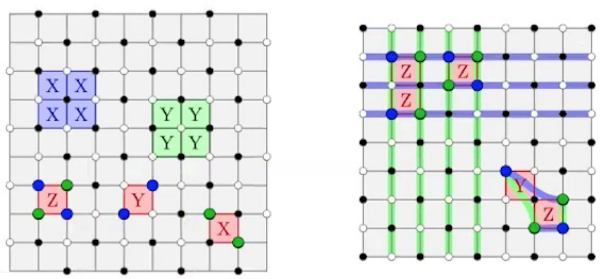
$$SE|\psi\rangle = (+1)E|\psi\rangle$$

for all E in the error model $\mathcal{E}^Z$.

This means, we can match subsets of stabilizer generators S_j where

$$\prod S_j = S$$

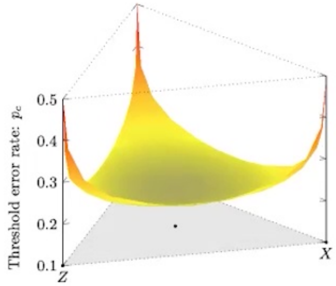
since S_j are constrained to have an even number of defects.

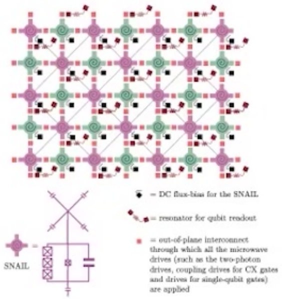


D. K. Tuckett *et al.*, Phys. Rev. Lett. 124, 130501 (2020)

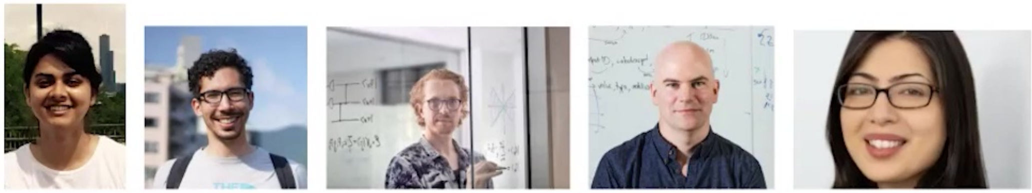
Quantum computing with the XZZX code and cat qubits

Owing to its high biased-noise thresholds, we are now developing a blueprint for quantum computing with the XZZX code.





A. Darmawan *et al.* PRX Quantum **2**, 030345 (2021)



See recent work on high-fidelity magic state prep with biased noise S. Singh *et al.* arXiv:2109.02677

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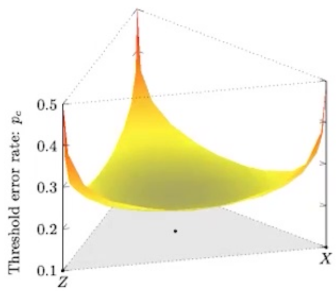
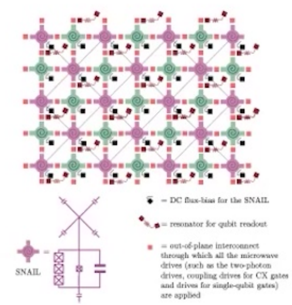
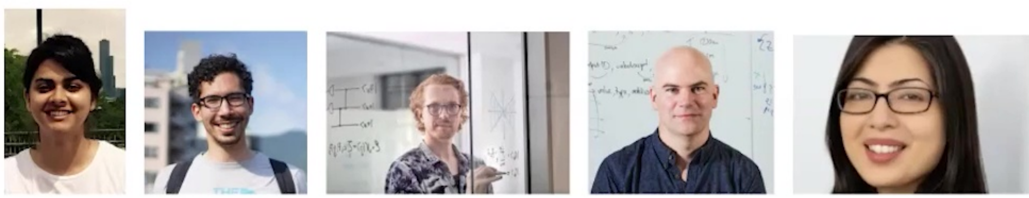




A. Darmawan *et al.* PRX Quantum **2**, 030345 (2021)

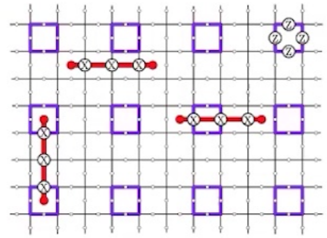


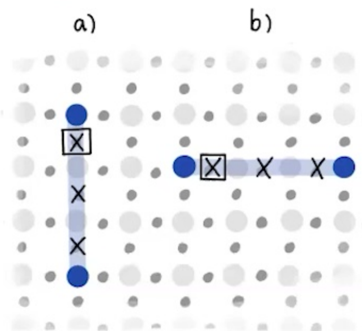
See recent work on high-fidelity magic state prep with biased noise S. Singh *et al.* arXiv:2109.02677

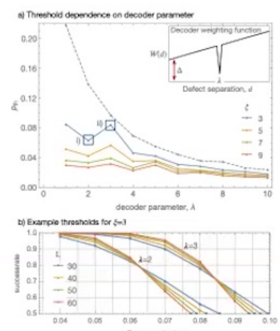
Decoding correlated errors

Correlated errors respect symmetries too

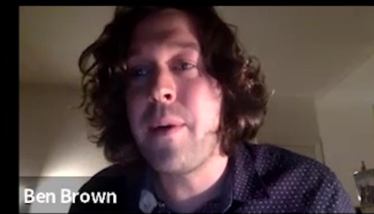
Decoders can be specialised to deal with correlated errors.







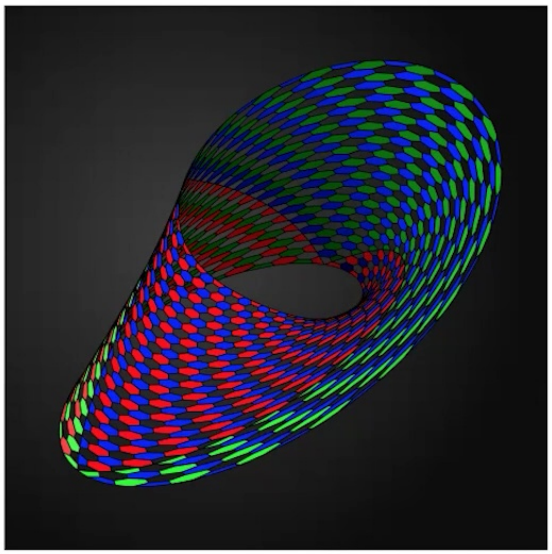
N. Nickerson and BJB, Quantum **3**, 131 (2019)

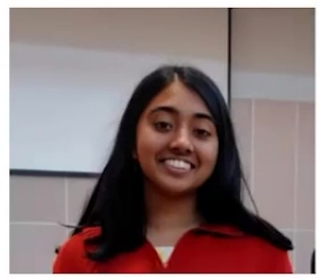


The color code

We found a new symmetry that is embedded on a Möbius strip to decode the color code with boundaries

Work with Kaavya Sahay
arXiv:2108.11395
To appear in PRX Quantum

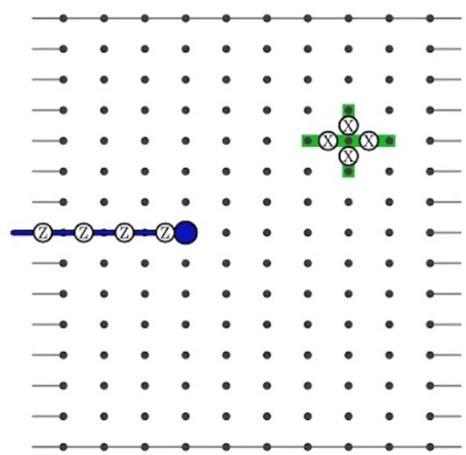




Decoding the surface code with boundaries

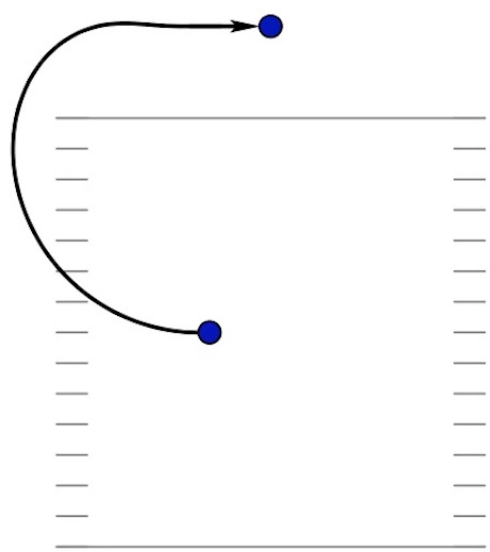
With boundaries, and we can make an odd number of defects

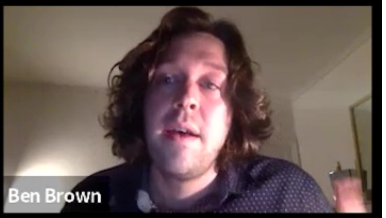
$$A_v = \prod_{\partial e \ni v} X_e$$



Decoding the surface code with boundaries

A defect is added 'outside' the code if the number of defects is odd

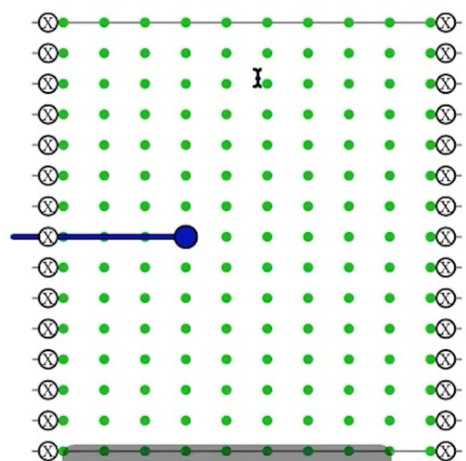




Decoding the surface code with boundaries

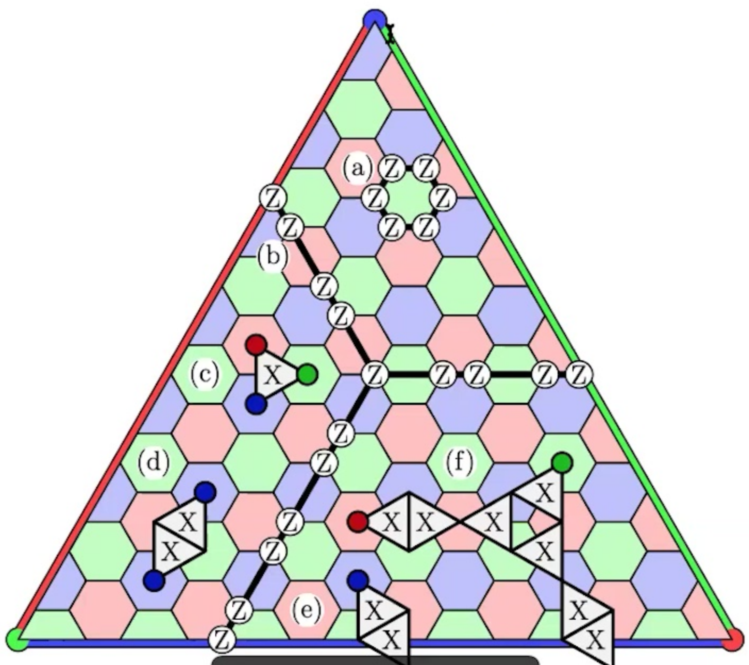
We can view this as over completing the star operators with a boundary operator b.

$$b = \prod_v A_v \quad \Rightarrow \quad b \prod_v A_v = \mathbb{1}$$



The color code

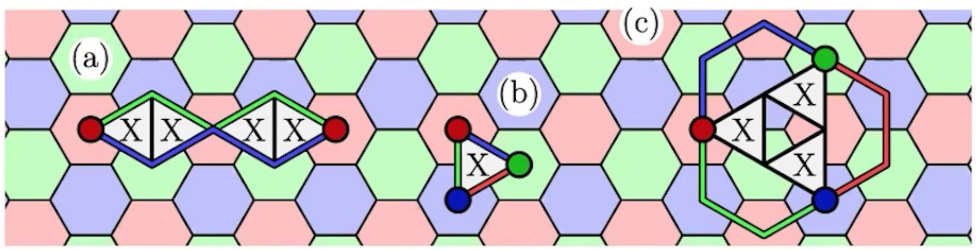
The color code is a stabilizer code with three differently colored stabilizers



Conservation laws for color code

Each (bulk) error makes one red, one green and one blue defect. Therefore:

$$\#\mathbf{r}(\mathrm{mod}2) = \#\mathbf{g}(\mathrm{mod}2) = \#\mathbf{b}(\mathrm{mod}2)$$



This gives rise to a parity conservation law.

It follows that

$$\#\mathbf{g}(\mathrm{mod}2) + \#\mathbf{b}(\mathrm{mod}2) \text{ is even.}$$

(the same follows with any pair of colors)

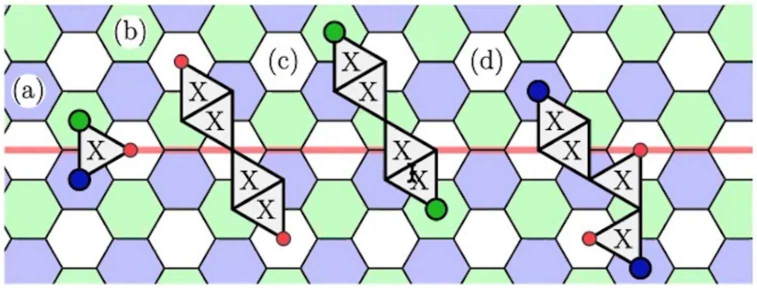
The color code symmetries

The conservation law

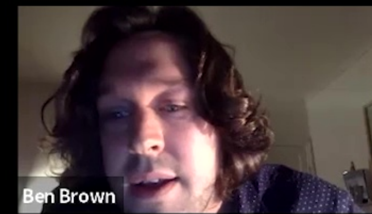
$$\#\mathbf{g}(\text{mod}2) + \#\mathbf{b}(\text{mod}2) \text{ is even.}$$

has a corresponding symmetry

$$\prod_{f:\text{col}(f)\neq \mathbf{r}} S_f = \mathbb{1}$$



Wang *et al.*, Quant. Info. Comput. **10**, 0780 (2010)
Bombin *et al.* New J. Phys. **14**, 073048 (2012)
Delfosse, PRA **89**, 012317 (2014)
Kubica and Delfosse, arXiv:1905.07393

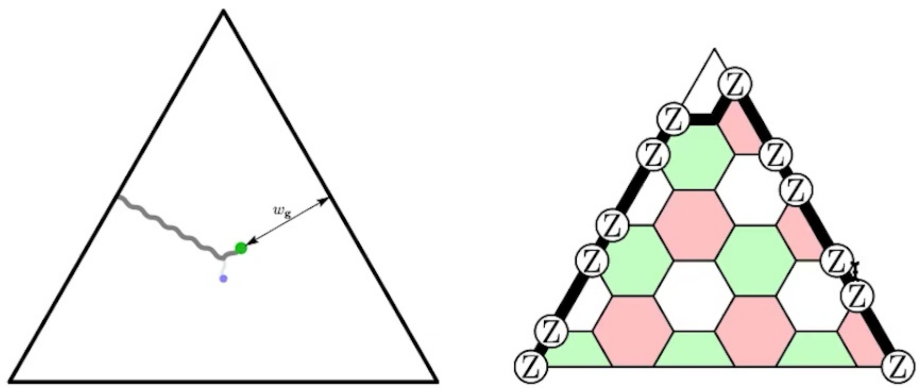


Boundary operators

Boundary operators show us how to pair defects to the boundary

$$b_{\mathbf{bg}} = \prod_{f:\mathrm{col}(f)\neq\mathbf{r}} S_f$$

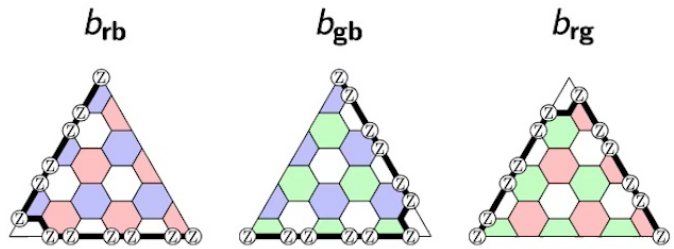
This operator over completes the generating set to give a symmetry.



But a problem is that small errors can cause us to match defects to the wrong boundaries.

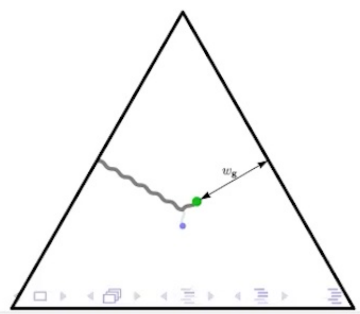
Boundary operators

We can make three color code boundary operators this way

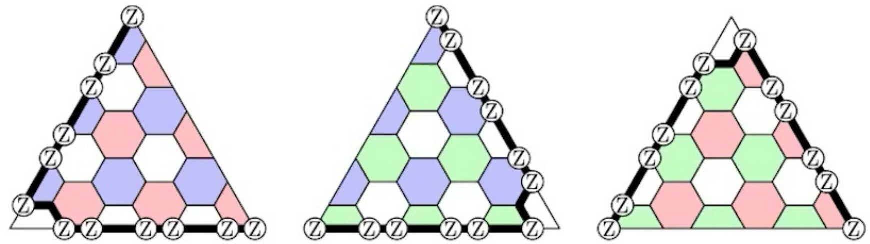


And they respect a symmetry too

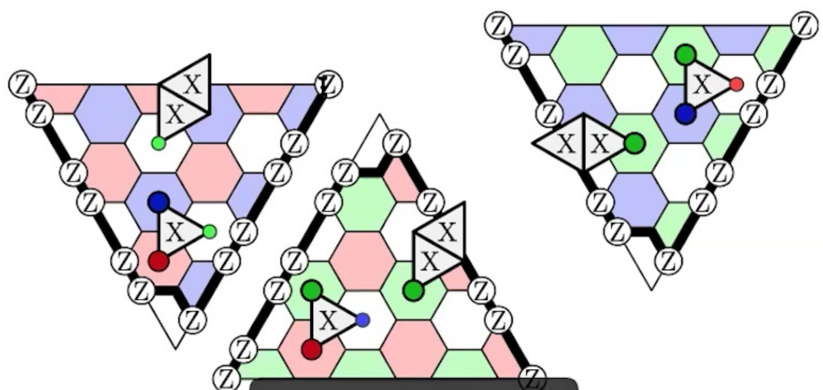
$$b_{\mathbf{rg}} b_{\mathbf{rb}} b_{\mathbf{gb}} = \mathbb{1}$$



Boundary operators

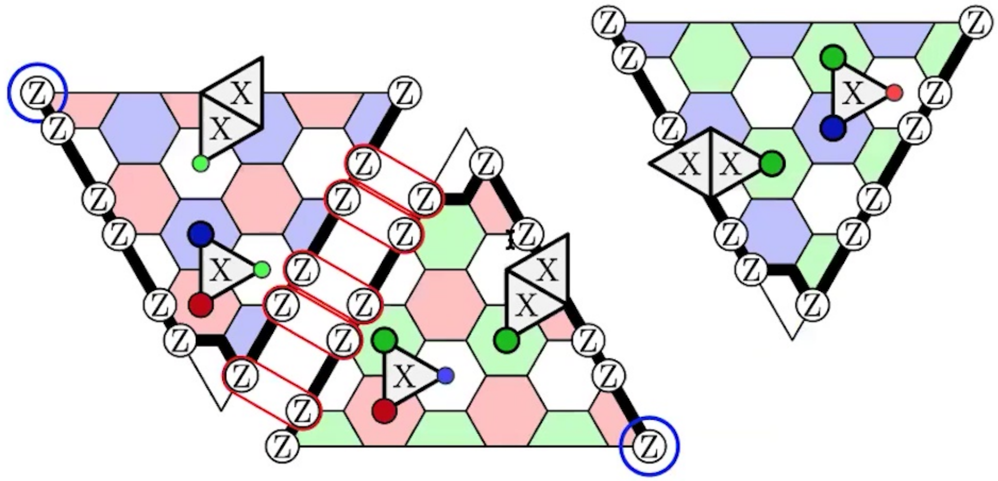


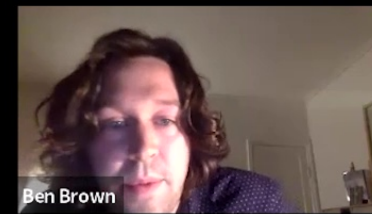
Let us rearrange these operators a bit.



Boundary operators

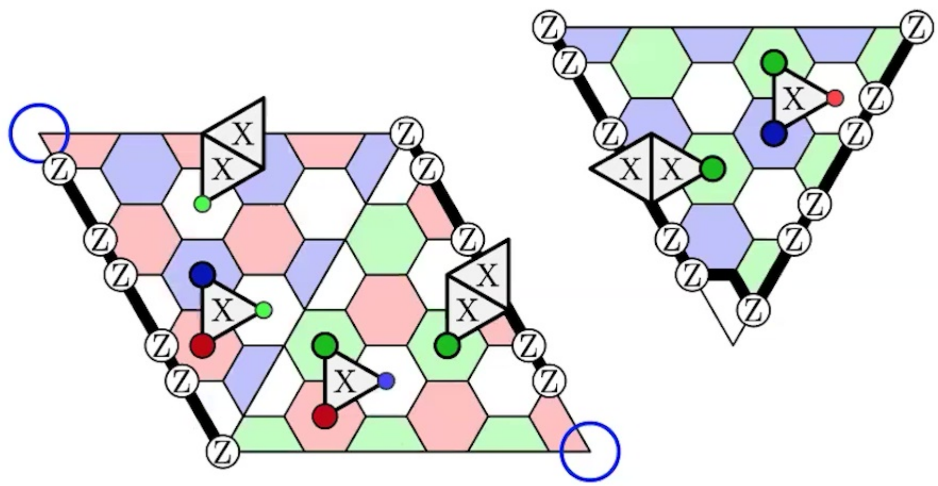
We can unify the qubits along the boundaries where the boundary operators have a common support.





Boundary operators

Here we show the lattice with two merged boundaries



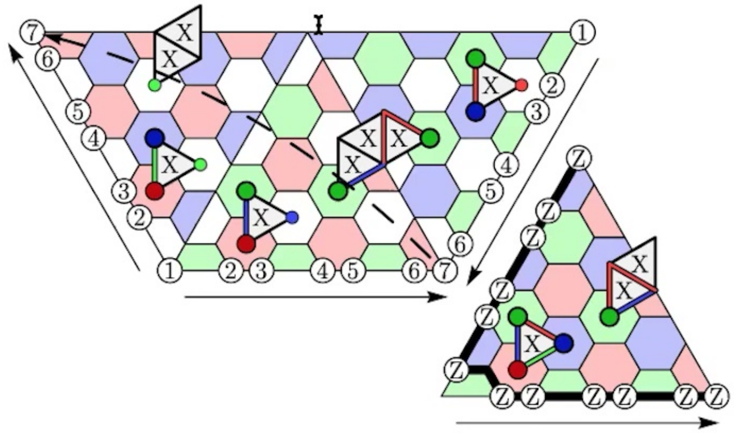
The product of the boundary operators at the merged boundary has no support on the join.

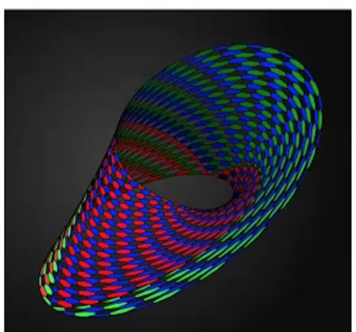
Y. Li, Phys. Rev. A **98**, 012336 (2018)

A. Kubica *et al.*, New J. Phys. **17**, 083026 (2015)

The Möbius strip

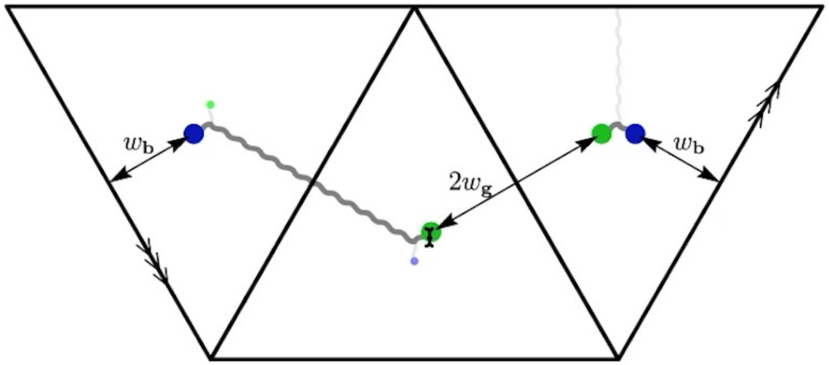
Joining all of the boundaries gives the Möbius strip

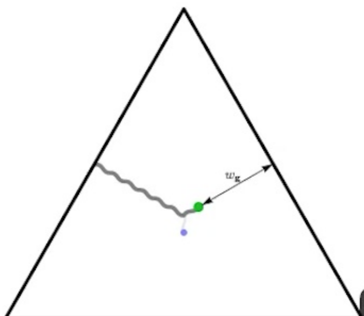


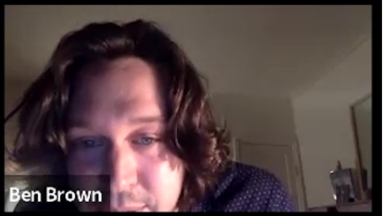


The Möbius strip

Matching on the Möbius strip gives better results because it is harder to match the incorrect way over the non-orientable surface.







The Möbius strip

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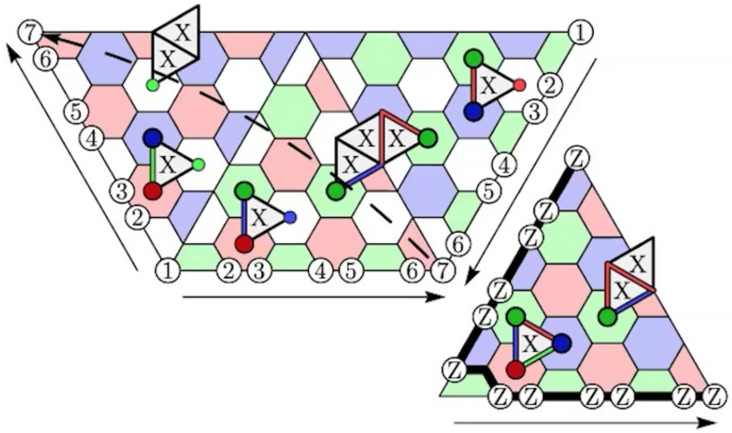
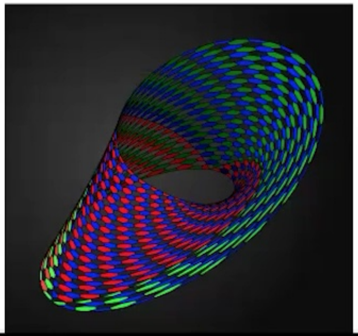


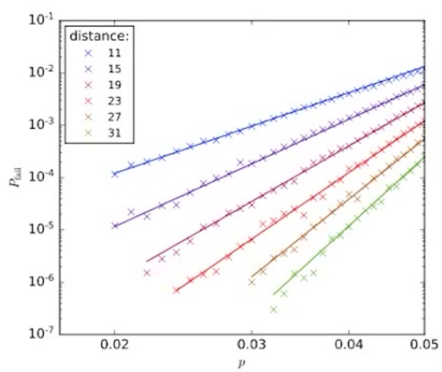


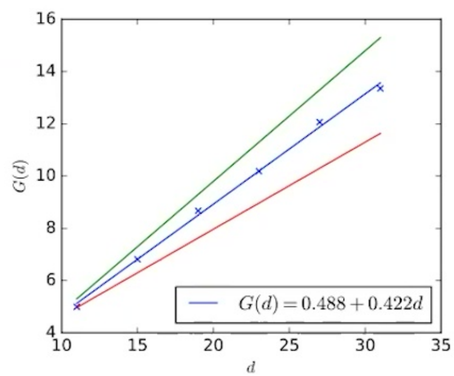
Numerical results

The logical failure rate decays like

$$\overline{P} \sim p^{\alpha d} \quad \text{where} \quad \alpha = 3/7 < 1/2$$

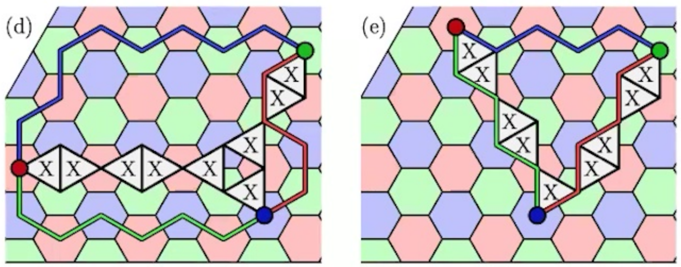
where p is the error rate and d the code distance.



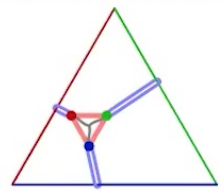


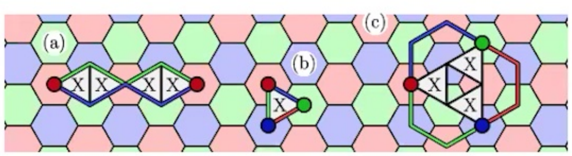
Numerical results

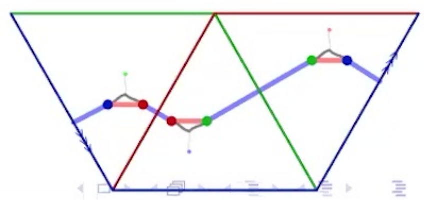
We expected this due to microscopic details of certain branching errors.



So we considered looking for alternative corrections

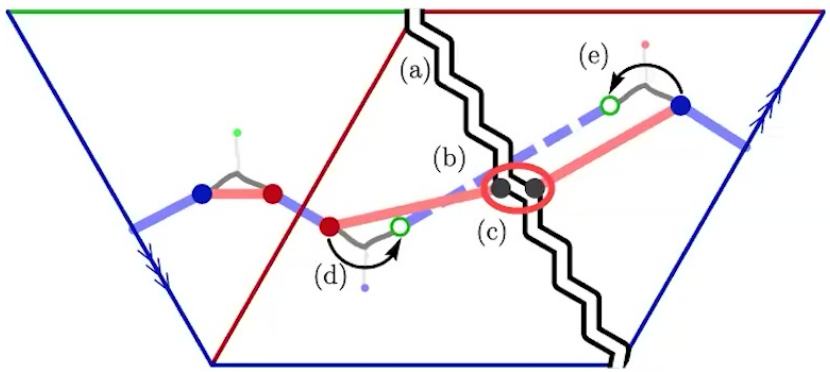






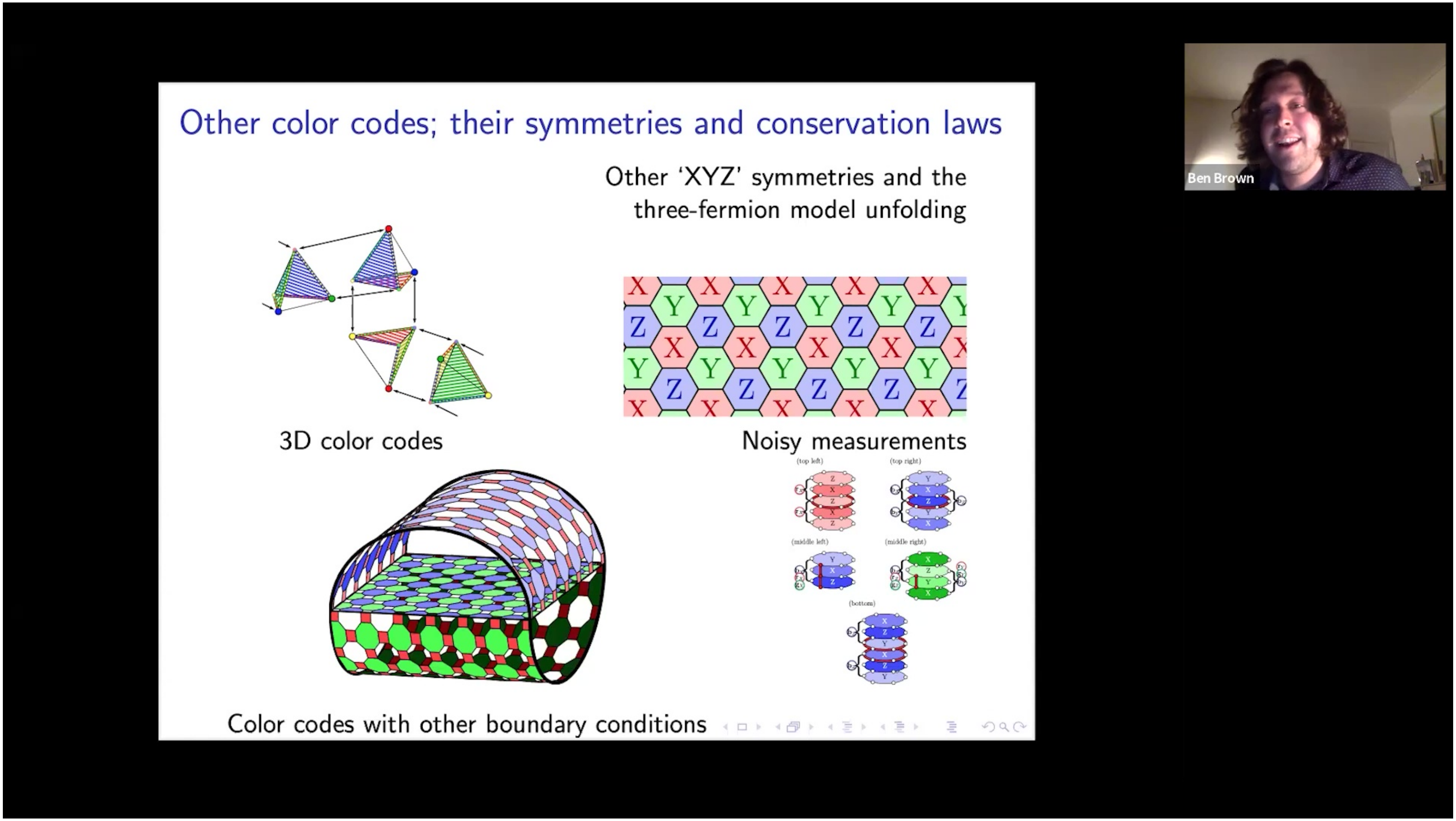
Numerical results

We improved the decoder by finding and comparing inequivalent corrections.



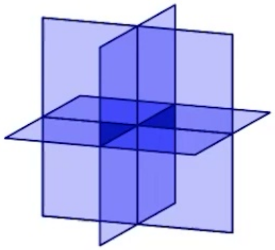
Exhaustive tests showed we can correct errors up to weight $(d-1)/2$ for $d \leq 13$

A. Hutter, D. Loss and J. Wootton, PRA **89** 022326 (2014)

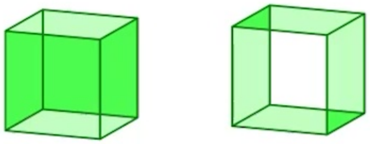

Other color codes; their symmetries and conservation laws
Other 'XYZ' symmetries and the three-fermion model unfolding
3D color codes
Noisy measurements
Color codes with other boundary conditions
Ben Brown

A threshold for X-cube model

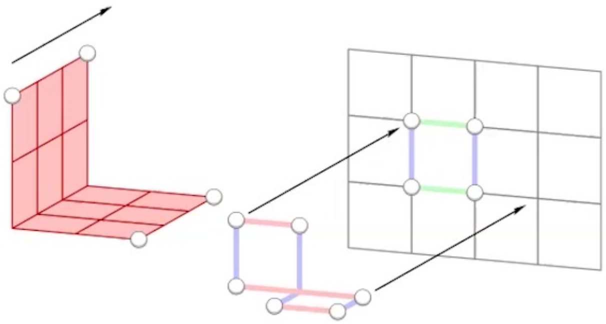
The X-cube model is three-dimensional with qubits on the faces of a cubic lattice



Pauli-X stabilizer



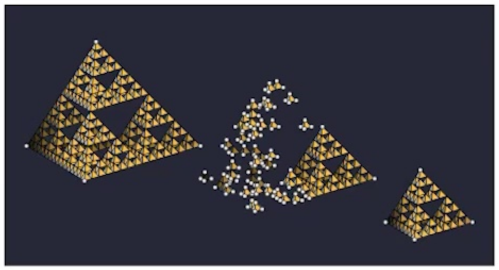
Pauli-Z stabilizer

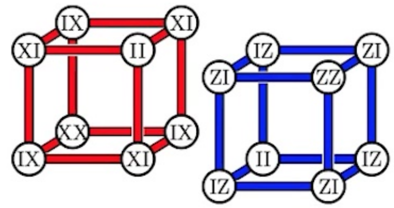


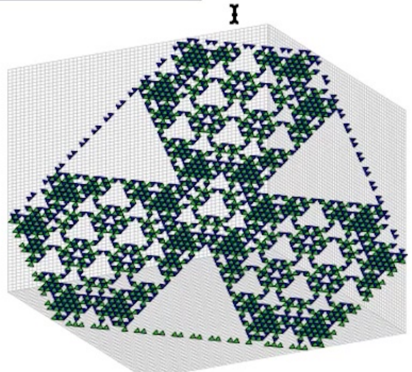
BJB and DJ Williamson, Phys. Rev. Research 2, 013303 (2020)

Fractal quantum error correcting codes

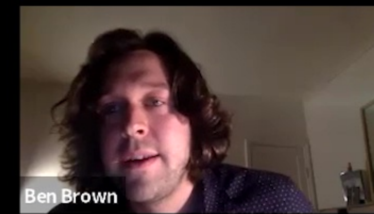
There are more exotic fracton codes that may be useful for quantum error correction





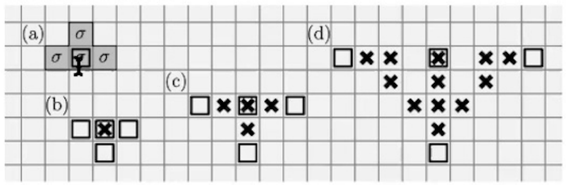


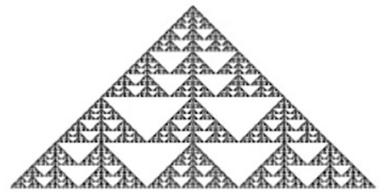
A (layer of a) cubic code fractal symmetry

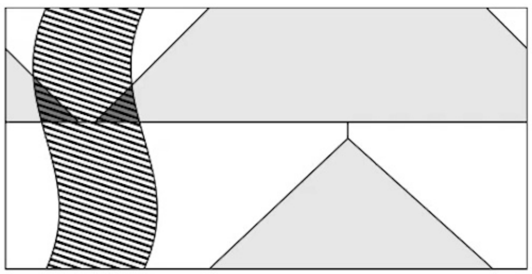


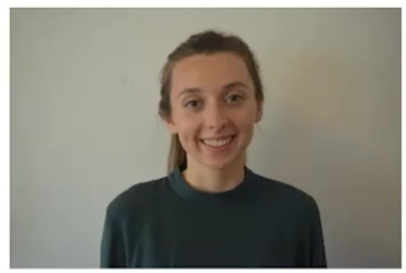
The Fibonacci code

The Fibonacci code is a classical code with similar physics to the cubic code









Work with Georgia Nixon

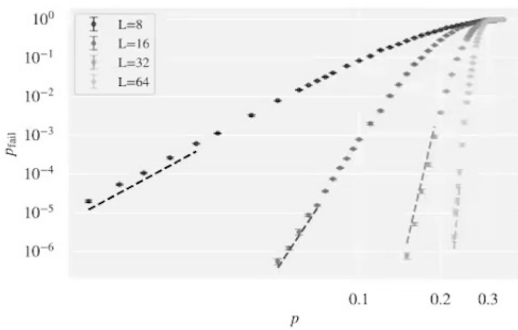
G. Nixon and BJB, IEEE:Trans. Info. Theor. (2021)

The Fibonacci code

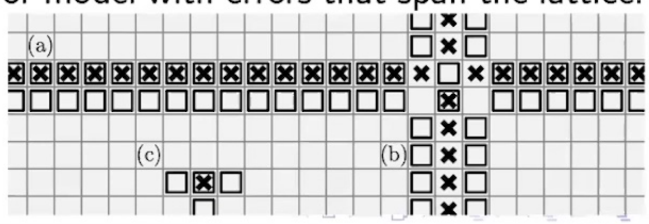
We found:

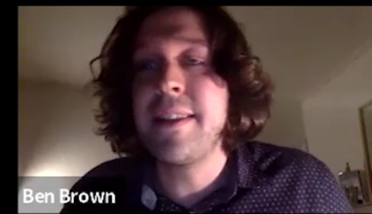
Very good logical failure rate scaling with the number of bits for an iid noise model due to its high distance $d > L$

$\overline{P} \sim \exp(L^{\gamma})$, $\gamma \sim 1.4 > 1$.



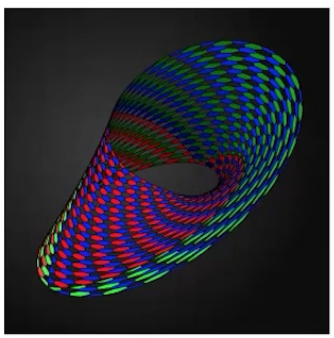
A threshold against an error model with errors that span the lattice.

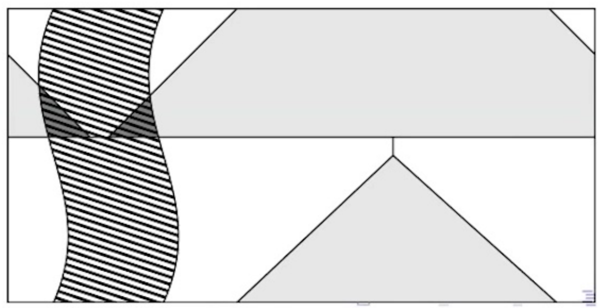


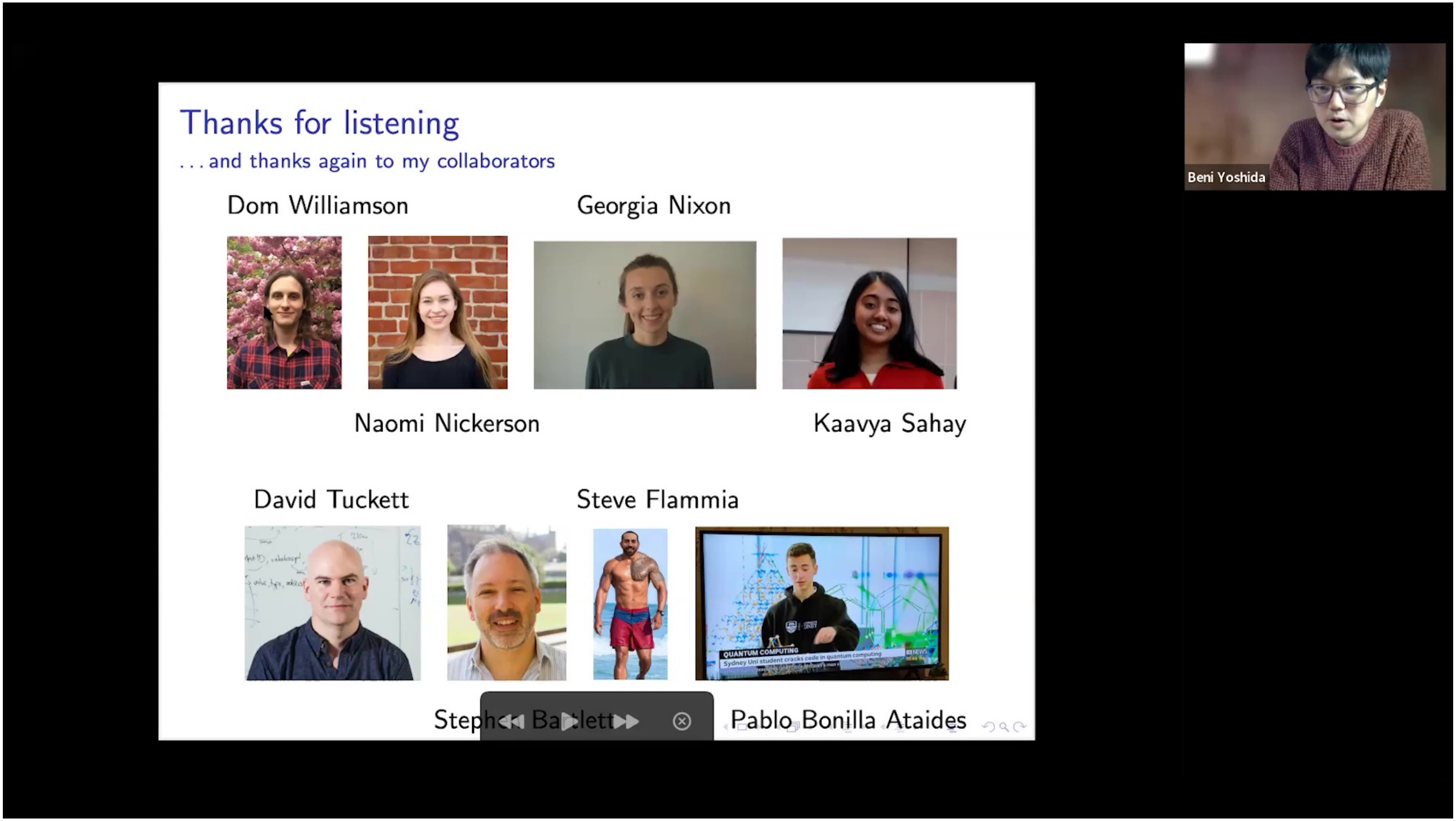


To summarise

- Conservation laws and symmetries give a unifying picture for decoders for topological codes
- They have led to better decoders for codes tailored to correct biased noise as well as color codes
- It has led to the discovery of new decoders for fracton codes and classical fractal codes
- In future I would like to extend this idea further to discover better decoders for more general LDPC codes.

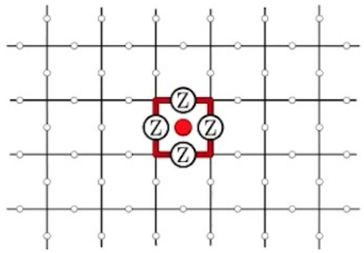




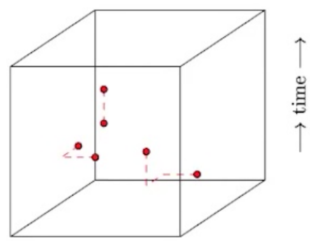

Thanks for listening
. . . and thanks again to my collaborators
Dom Williamson
Georgia Nixon
Naomi Nickerson
Kaavya Sahay
David Tuckett
Steve Flammia
QUANTUM COMPUTING
Sydney Uni student cracks code in quantum computing
Pablo Bonilla Ataides
Beni Yoshida

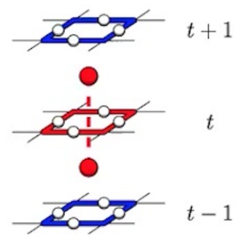
Other examples: measurement errors and symmetries

Measurement errors violate the charge conservation symmetry



We fix this by repeating measurements

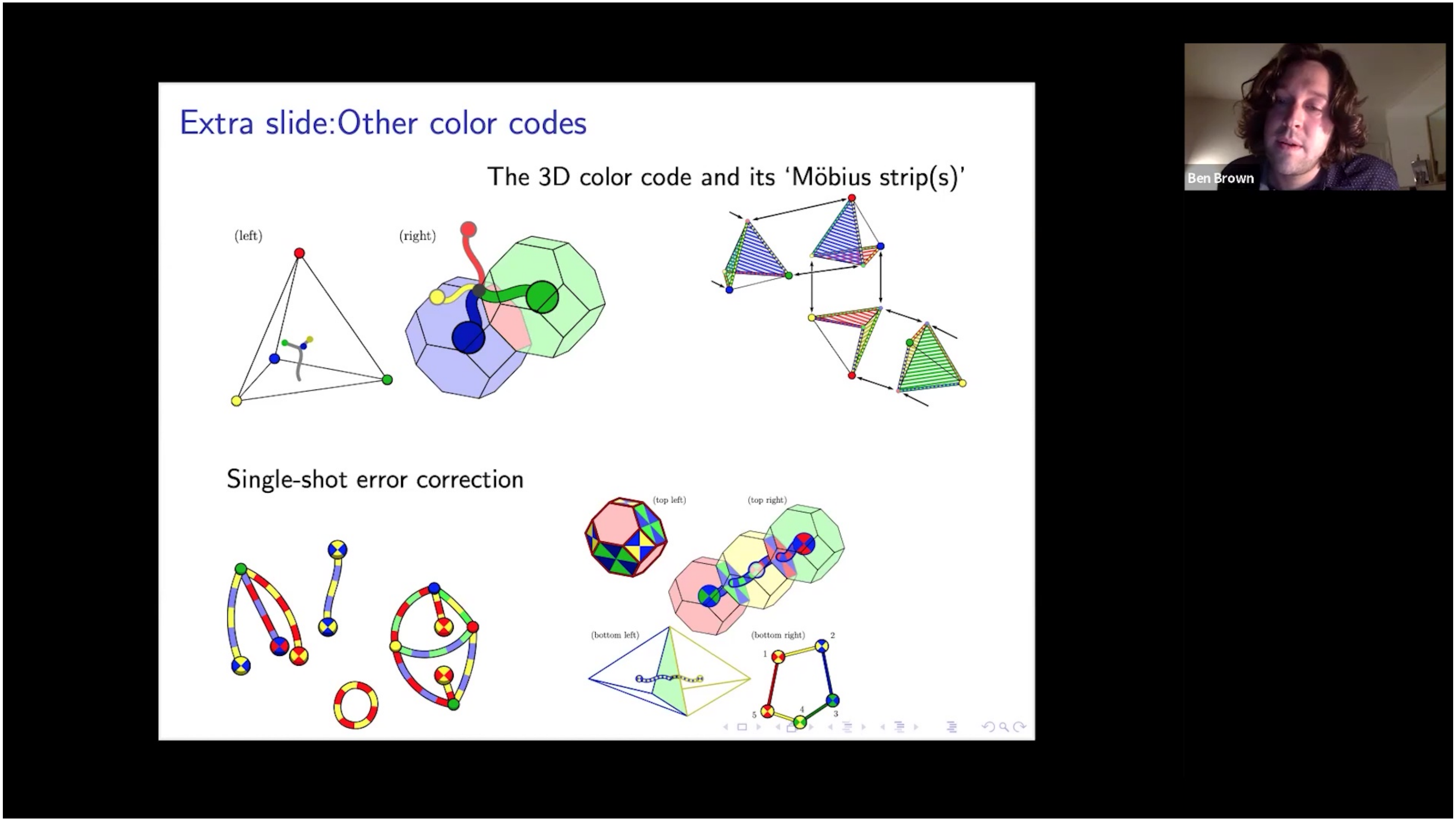




We then check the parity of time adjacent measurements

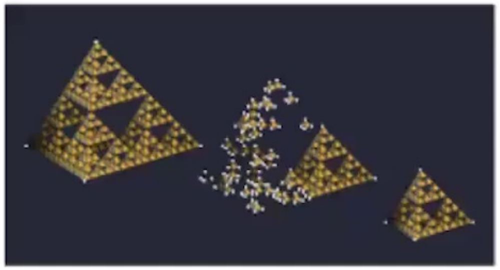
This way, measurement errors give rise to two defects in spacetime.

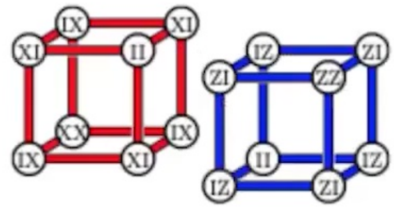
This trick is generic.

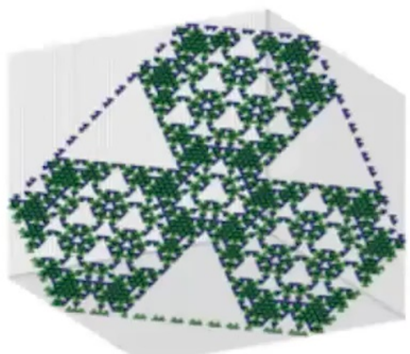

Extra slide:Other color codes
The 3D color code and its 'Möbius strip(s)'
(left)
(right)
Single-shot error correction
(top left)
(top right)
(bottom left)
(bottom right)
1
2
3
4
5
Ben Brown

Fractal quantum error correcting codes

There are more exotic fracton codes that may be useful for quantum error correction







A (layer of a) cubic code fractal symmetry

System symmetries

We can define the symmetries of a code **with respect to an error model**

The symmetries (a subgroup of the stabilizer group) are such that

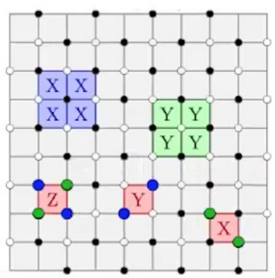
$$SE|\psi\rangle = (+1)E|\psi\rangle$$

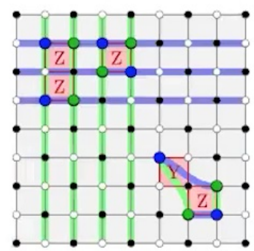
for all E in the error model $\mathcal{E}^Z$.

This means, we can match subsets of stabilizer generators S_j where

$$\prod S_j = S$$

since S_j are constrained to have an even number of defects.

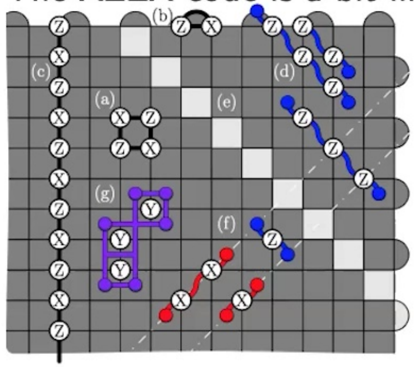


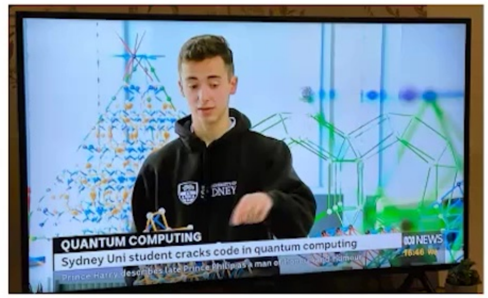


D. K. Tuckett *et al.*, Phys. Rev. Lett. 124, 130501 (2020)

The XZZX surface code and biased noise

The XZZX code is a bit like a 2D fracton code under biased noise.





P. Bonilla Ataides *et al.*, Nat. Commun. **12**, 2172 (2021)