Title: QFT III 2021/2022

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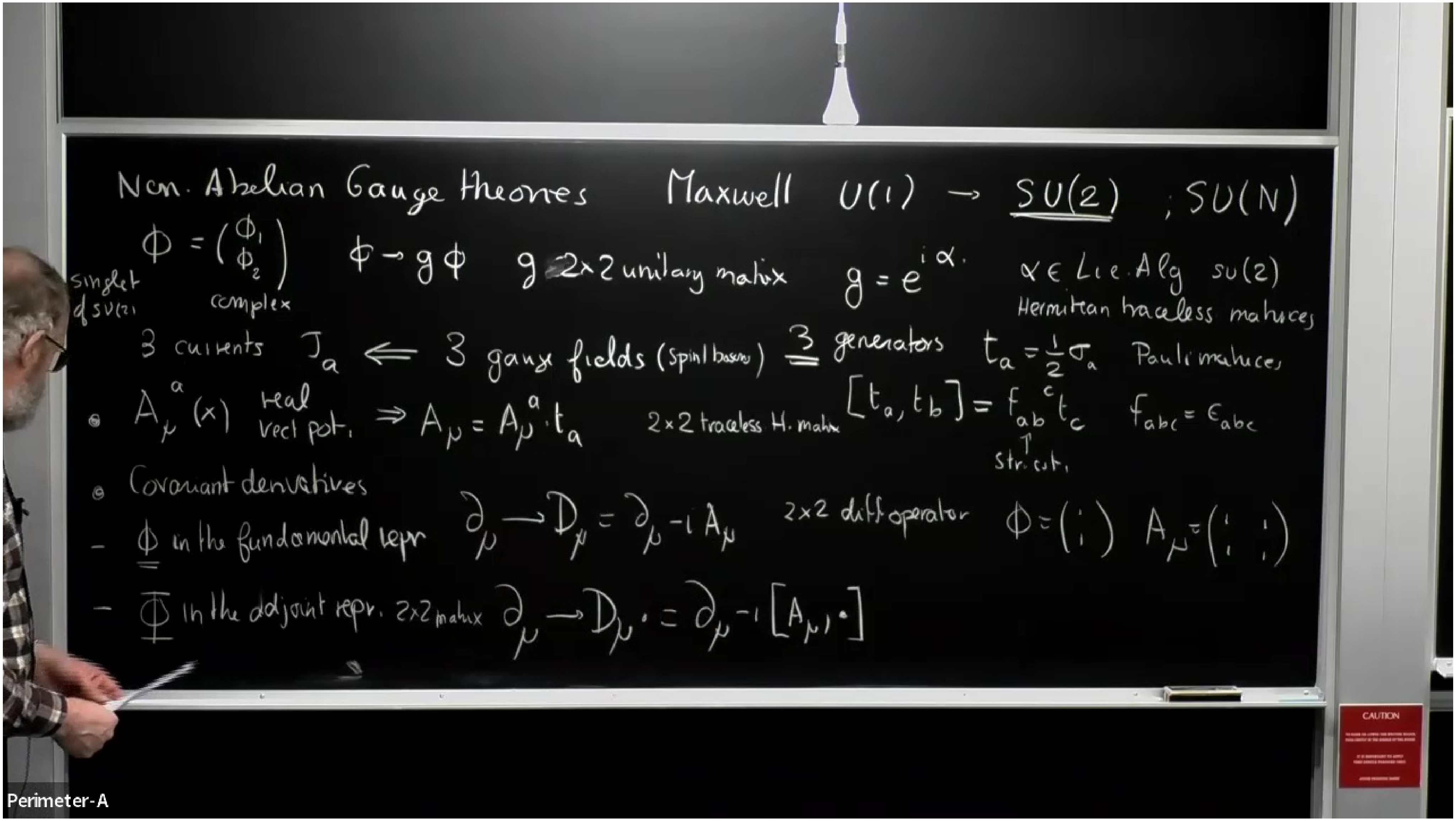
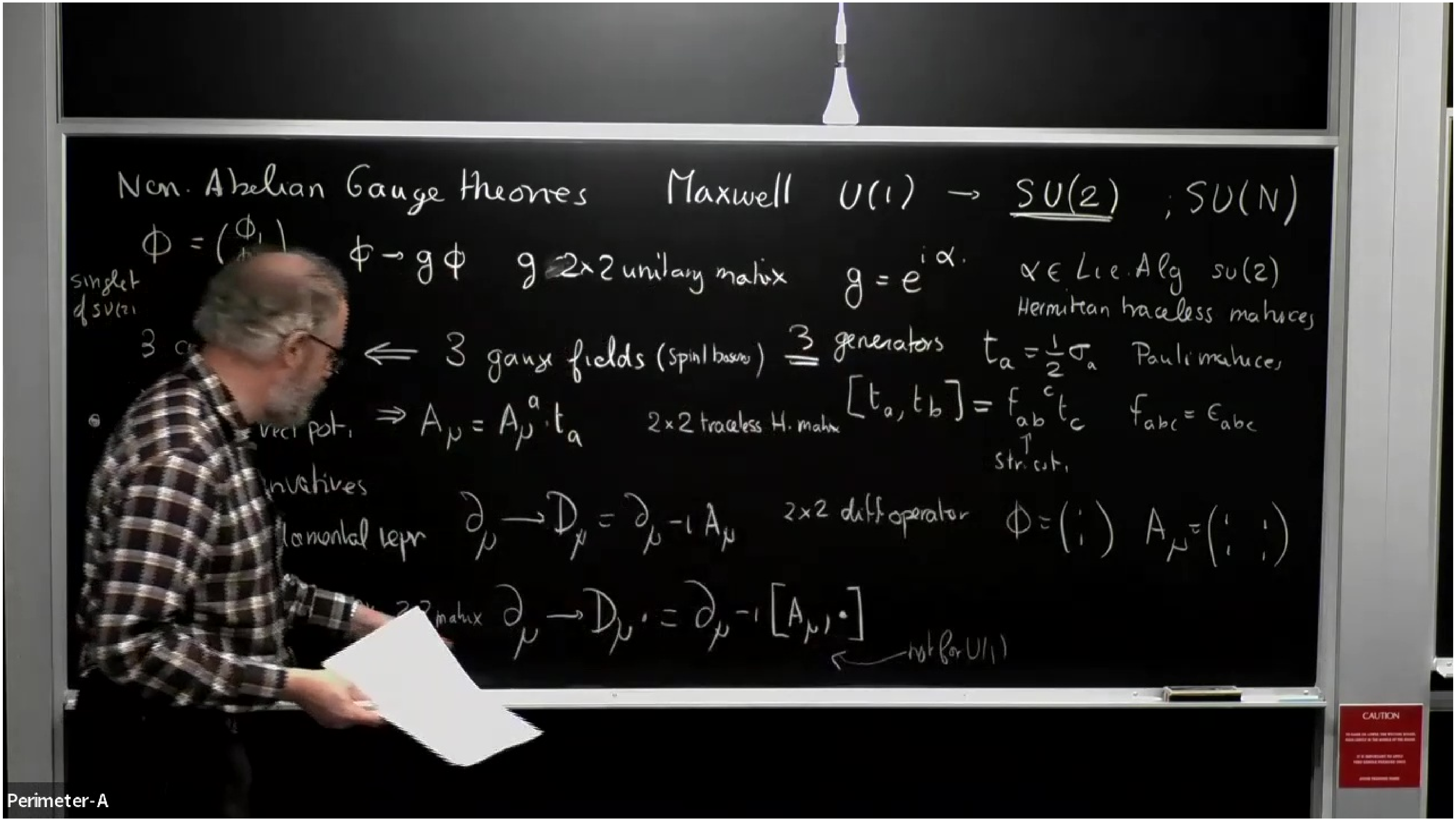
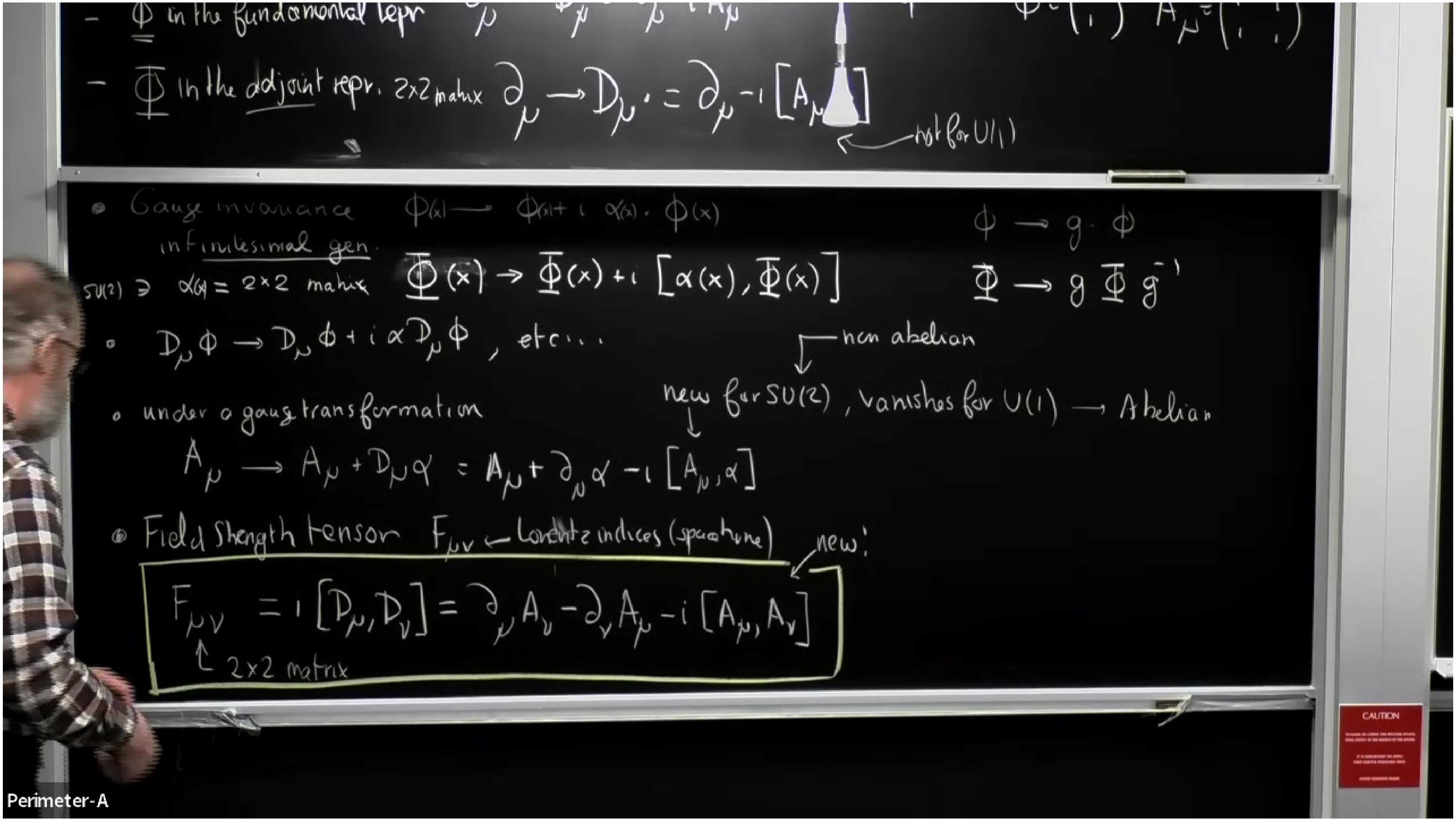


Non. Abelian Gauge theories
Maxwell $U(1) \to SU(2)$; $SU(N)$
$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$
singlet of SU(2)
complex
$\phi \to g\phi$
g 2×2 unitary matrix
$g = e^{i\alpha}$
$\alpha \in$ Lie Alg su(2)
Hermitian traceless matrices
3 currents $J_a \Leftarrow$ 3 gauge fields (spin1 bosons) $\equiv$ 3 generators $t_a = \frac{1}{2}\sigma_a$ Pauli matrices
$A_\mu^a(x)$ real vect pot. $\Rightarrow A_\mu = A_\mu^a \cdot t_a$
2×2 traceless H. matrix
$[t_a, t_b] = f_{ab}^{\ \ c} t_c$
str. const.
$f_{abc} = \epsilon_{abc}$
Covariant derivatives
– Φ in the fundamental repr $\partial_\mu \to D_\mu = \partial_\mu - i A_\mu$
2×2 diff operator
$\Phi = \begin{pmatrix} \cdot \\ \cdot \end{pmatrix}$ $A_\mu = \begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$
– Φ in the adjoint repr. 2×2 matrix $\partial_\mu \to D_\mu \cdot = \partial_\mu - i [A_\mu, \cdot]$
Perimeter-A
CAUTION


Non. Abelian Gauge theories Maxwell U(1) → SU(2) ; SU(N)
φ = (φ1 …) φ → gφ g 2×2 unitary matrix g = e^{iα} α ∈ Lie Alg su(2)
singlet of SU(2)
Hermitian traceless matrices
3 ⇐ 3 gauge fields (spin 1 bosons) 3 generators t_a = ½ σ_a Pauli matrices
[t_a, t_b] = f_ab^c t_c f_abc = ε_abc
str. const.
vect pot. ⇒ A_μ = A_μ^a · t_a 2×2 traceless H. matrix
derivatives
fundamental repr ∂_μ → D_μ = ∂_μ − i A_μ 2×2 diff operator φ = ( ) A_μ = ( )
2×2 matrix ∂_μ → D_μ · = ∂_μ − i [A_μ, ·]
not for U(1)
CAUTION
Perimeter-A


- $\underline{\underline{\phi}}$ in the fundamental repr
- $\underline{\Phi}$ in the adjoint repr. 2x2 matrix $\partial_\mu \to D_\mu \cdot = \partial_\mu - i [A_\mu, \;]$
not for U(1)
• Gauge invariance $\phi(x) \to \phi(x) + i\,\alpha(x)\cdot\phi(x)$
infinitesimal gen.
$\phi \to g \cdot \phi$
$SU(2) \ni \alpha(x) = 2\times2$ matrix $\underline{\Phi}(x) \to \underline{\Phi}(x) + i\,[\alpha(x), \underline{\Phi}(x)]$
$\underline{\Phi} \to g\,\underline{\Phi}\,g^{-1}$
• $D_\mu\phi \to D_\mu\phi + i\alpha D_\mu\phi$, etc...
non abelian
• under a gauge transformation
new for SU(2), vanishes for U(1) → Abelian
$A_\mu \to A_\mu + D_\mu\alpha = A_\mu + \partial_\mu\alpha - i[A_\mu, \alpha]$
• Field Strength tensor $F_{\mu\nu}$ ← Lorentz indices (spacetime)
new!
$\boxed{F_{\mu\nu} = i[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]}$
2x2 matrix
Perimeter-A

- $\underline{\Phi}$ in the fundamental repr $\partial_\mu \quad \phi_\mu \quad \partial_\mu \; A_\mu$ $\quad \phi = \begin{pmatrix} \cdot \\ \cdot \end{pmatrix} \quad A_\mu = \begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$

- Φ in the adjoint repr. 2×2 matrix $\partial_\mu \longrightarrow D_\mu \cdot = \partial_\mu - i\,[A_\mu, \cdot\,]$ ← not for $U(1)$

$SU(2) \ni \alpha(x) = 2\times 2$ matrix $\quad \Phi(x) \to \Phi(x) + i\,[\alpha(x), \Phi(x)] \qquad \Phi \longrightarrow g\,\Phi\,g^{-1}$

- $D_\mu \phi \longrightarrow D_\mu \phi + i\,\alpha D_\mu \phi$, etc...

- under a gauge transformation

new for $SU(2)$, vanishes for $U(1) \longrightarrow$ Abelian

non abelian

$$A_\mu \longrightarrow A_\mu + D_\mu \alpha = A_\mu + \partial_\mu \alpha - i\,[A_\mu, \alpha]$$

$E^a, B^a \qquad a = 1, \cdots 3$

- Field Strength tensor $F_{\mu\nu}$ ← Lorentz indices (spacetime)

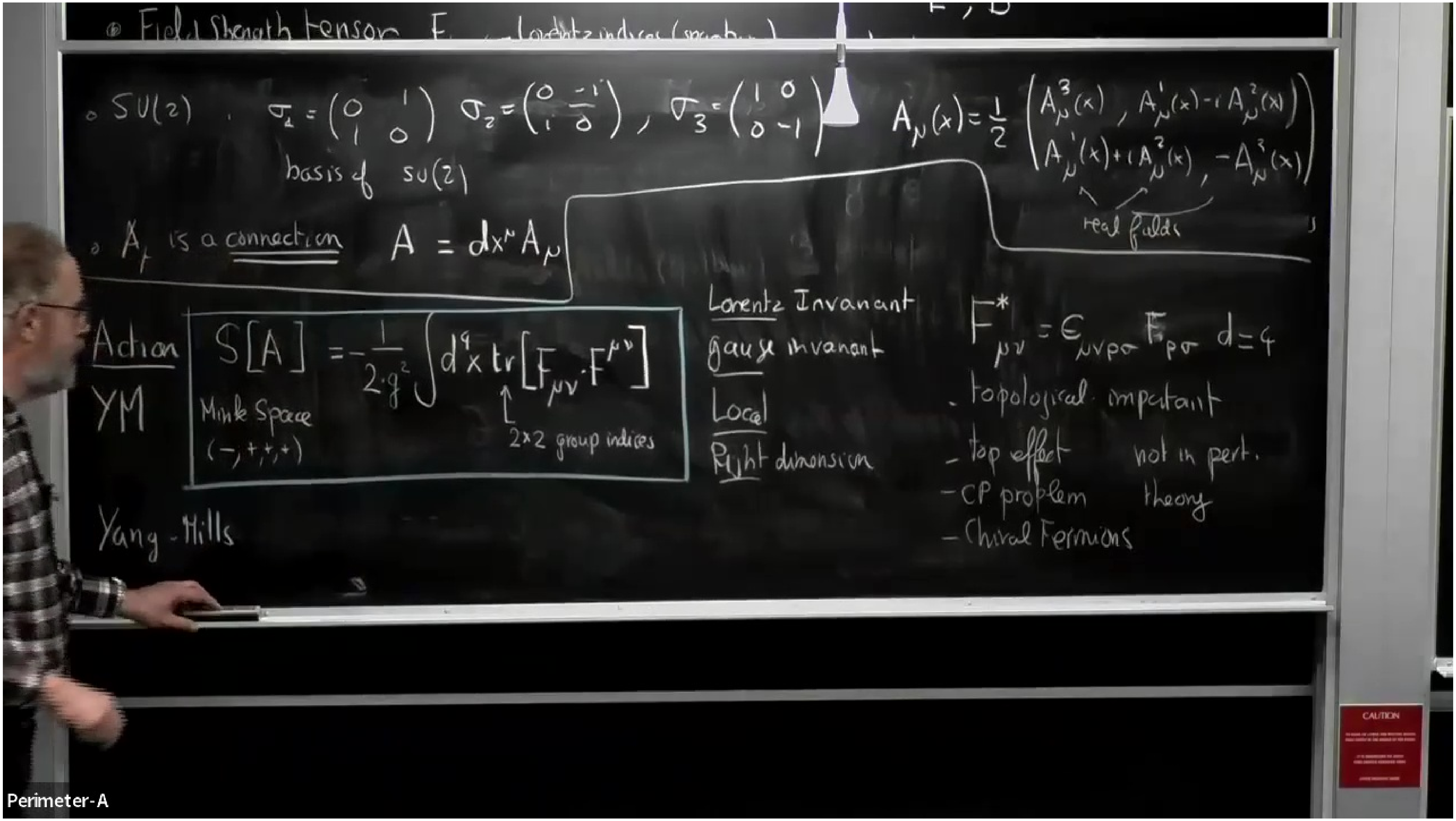
$$\boxed{F_{\mu\nu} = i\,[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - i\,[A_\mu, A_\nu]}$$

$F_{\mu\nu}$ ← 2×2 matrix

new: non-linear in A^a_μ

E^a = standard $A^a - i\, A^b \times A^c$

Perimeter-A



SU(2)
$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
basis of su(2)
$A_\mu(x) = \frac{1}{2}\begin{pmatrix} A^3_\mu(x) & A^1_\mu(x) - i A^2_\mu(x) \\ A^1_\mu(x) + i A^2_\mu(x) & -A^3_\mu(x) \end{pmatrix}$
real fields
A_μ is a connection $A = dx^\mu A_\mu$
Action
YM
$S[A] = -\frac{1}{2 \cdot g^2}\int d^4x \, \mathrm{tr}[F_{\mu\nu} \cdot F^{\mu\nu}]$
Mink Space
(−,+,+,+)
2×2 group indices
Lorentz Invariant
gauge invariant
Local
Right dimension
$F^*_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} F_{\rho\sigma}$ $d=4$
topological important
top effect
not in pert. theory
CP problem
Chiral Fermions
Yang-Mills
Perimeter-A

YM $S[A] = \frac{1}{2 g^2}\int d^4x \, \mathrm{tr}[F_{\mu\nu} F^{\mu\nu}]$

Mink Space $(-,+,+,+)$

2x2 group indices

gauge invariant

Local

Right dimension

- topological important
- top effect
- CP problem
- Chiral Fermions

not in pert. theory

Yang-Mills

pert th: expansion in c constant g $\quad A_\mu \to g A_\mu \quad F_{\mu\nu} \to g F_{\mu\nu}$

infinitesimal gen

$SU(2) \ni \alpha(x) = 2\times 2$ matrix $\quad \Phi(x) \to \Phi(x) + i[\alpha(x), \Phi(x)] \qquad \Phi \to g \Phi g^{-1}$

- $D_\mu \Phi \to D_\mu \Phi + ig\alpha D_\mu \Phi$, etc...

non abelian

new for SU(2), vanishes for U(1) → Abelian

- under a gauge transformation

$$A_\mu \longrightarrow A_\mu + D_\mu \alpha = A_\mu + \partial_\mu \alpha - ig[A_\mu, \alpha]$$

$E^a, B^a \qquad a = 1, \ldots 3$

- Field Strength tensor $F_{\mu\nu}$ ← Lorentz indices (spacetime)

new: non-linear in A_μ^a

$$\boxed{F_{\mu\nu} = i[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]}$$

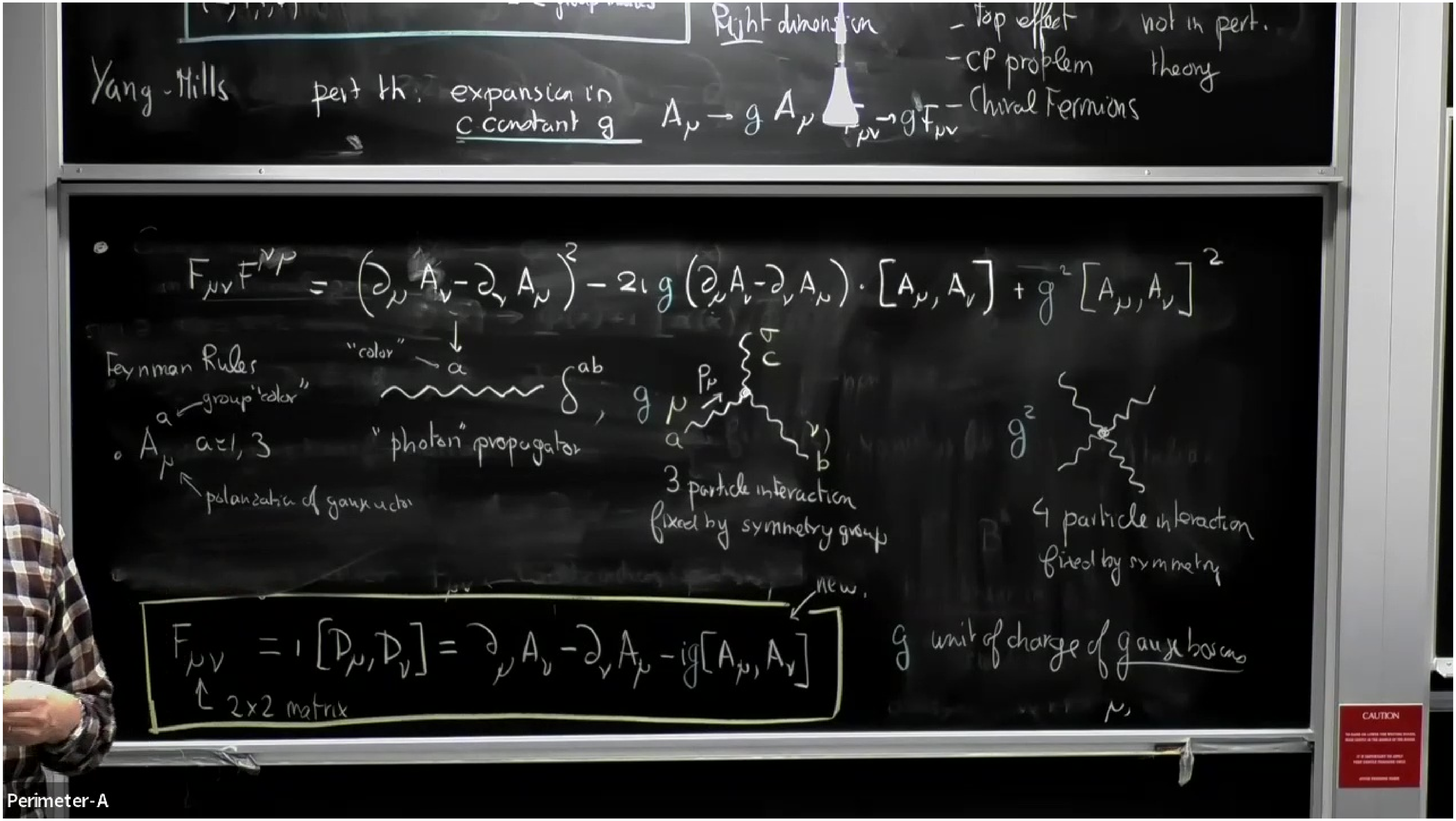
2x2 matrix

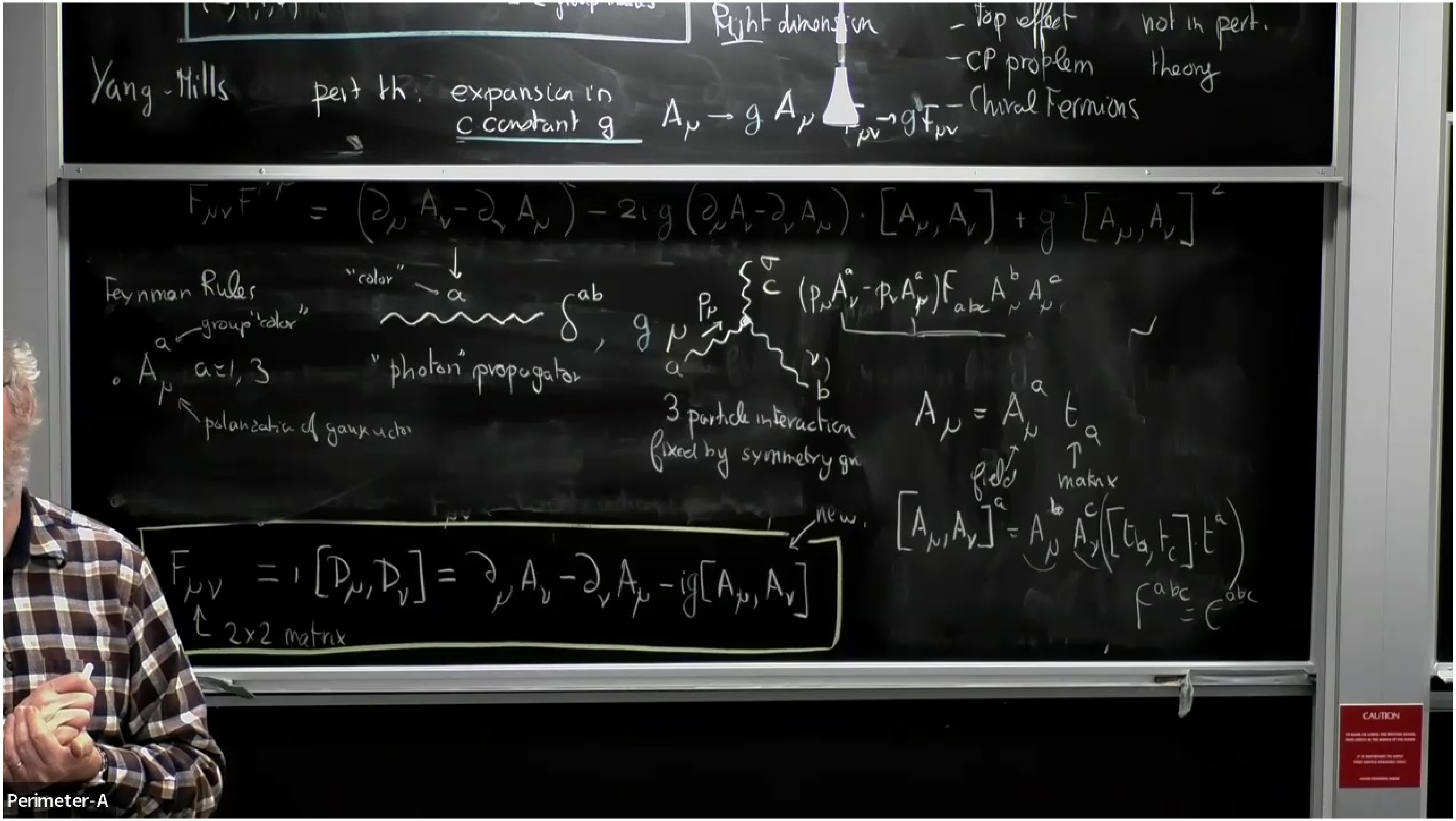
E^a = standard $A^a - i A^b \times A^c$

antisym w.r.t. μ, ν

Perimeter-A

CAUTION


Right dimension
- Top effect
- CP problem
- Chiral Fermions
not in pert. theory
Yang-Mills
pert th: expansion in c constant g
$A_\mu \to g A_\mu$
$F_{\mu\nu} \to g F_{\mu\nu}$
$F_{\mu\nu} F^{\nu\rho} = (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - 2i g (\partial_\mu A_\nu - \partial_\nu A_\mu) \cdot [A_\mu, A_\nu] + g^2 [A_\mu, A_\nu]^2$
Feynman Rules
"color"
a
δ^{ab}
"photon" propagator
group "color"
A_μ^a a=1,3
polarization of gauge vector
$g\,p_\mu$
a b c
3 particle interaction
fixed by symmetry group
g^2
4 particle interaction
fixed by symmetry
new.
$F_{\mu\nu} = i[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$
2x2 matrix
g unit of charge of gauge bosons
Perimeter-A
CAUTION


Right dimension
- Top effect
- CP problem
- Chiral Fermions
not in pert. theory
Yang-Mills
pert th: expansion in c constant g
$A_\mu \to g A_\mu$
$F_{\mu\nu} \to g F_{\mu\nu}$
$F_{\mu\nu}F^{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - 2ig(\partial_\mu A_\nu - \partial_\nu A_\mu)\cdot[A_\mu, A_\nu] + g^2[A_\mu, A_\nu]^2$
Feynman Rules
"color"
group "color"
A^a_μ a=1,3
polarization of gauge vector
δ^{ab}
"photon" propagator
$(p_\mu A^a_\nu - p_\nu A^a_\mu) f_{abc} A^b_\mu A^c_\nu$
3 particle interaction
fixed by symmetry gr
$A_\mu = A^a_\mu t_a$
field
matrix
$[A_\mu, A_\nu]^a = A^b_\mu A^c_\nu ([t_b, t_c] \cdot t^a)$
$f^{abc} = \epsilon^{abc}$
new
$F_{\mu\nu} = i[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$
2x2 matrix
Perimeter-A