Title: QFT III 2021/2022

Speakers: Francois David

Collection: QFT III 2021/2022

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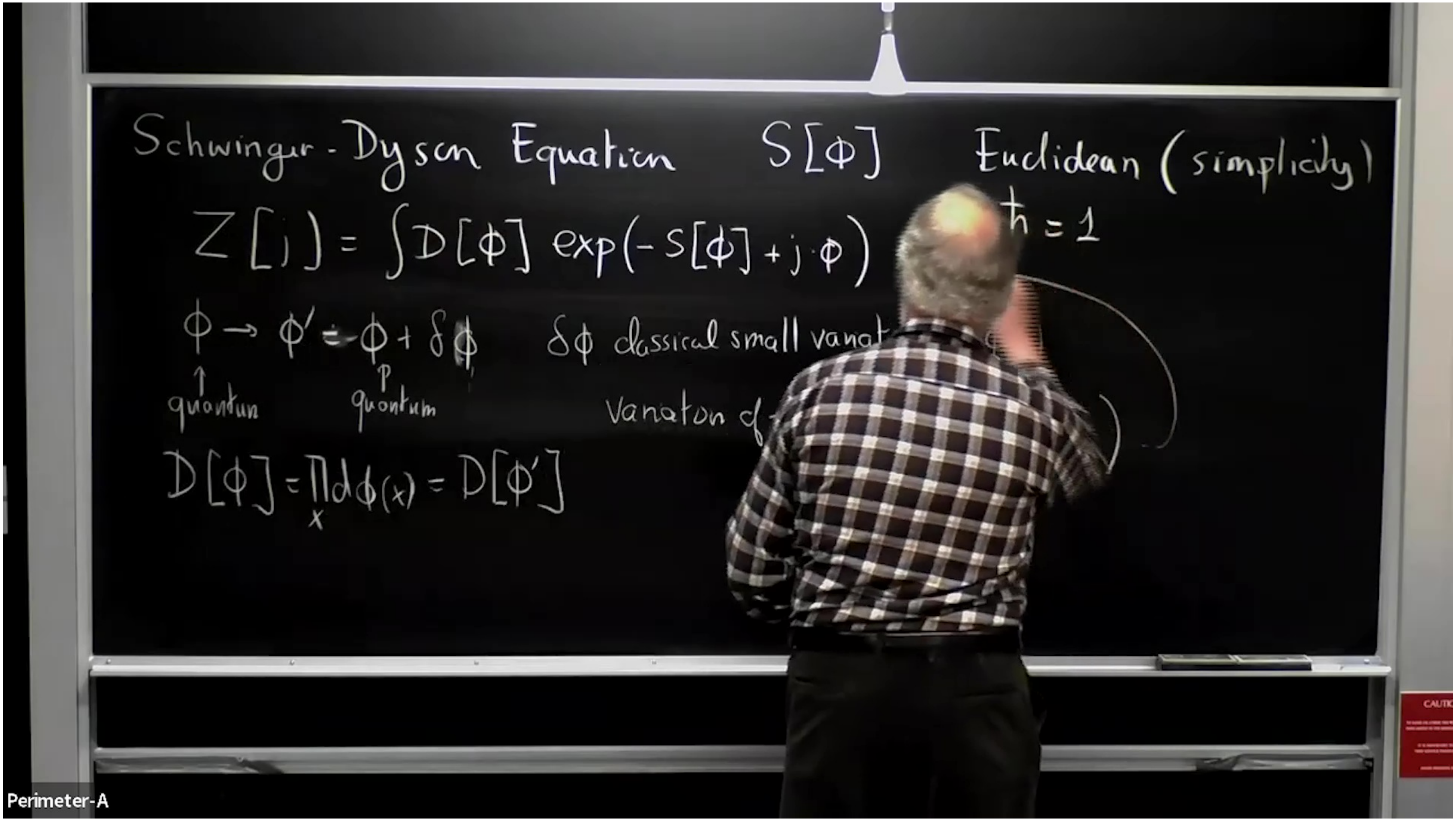
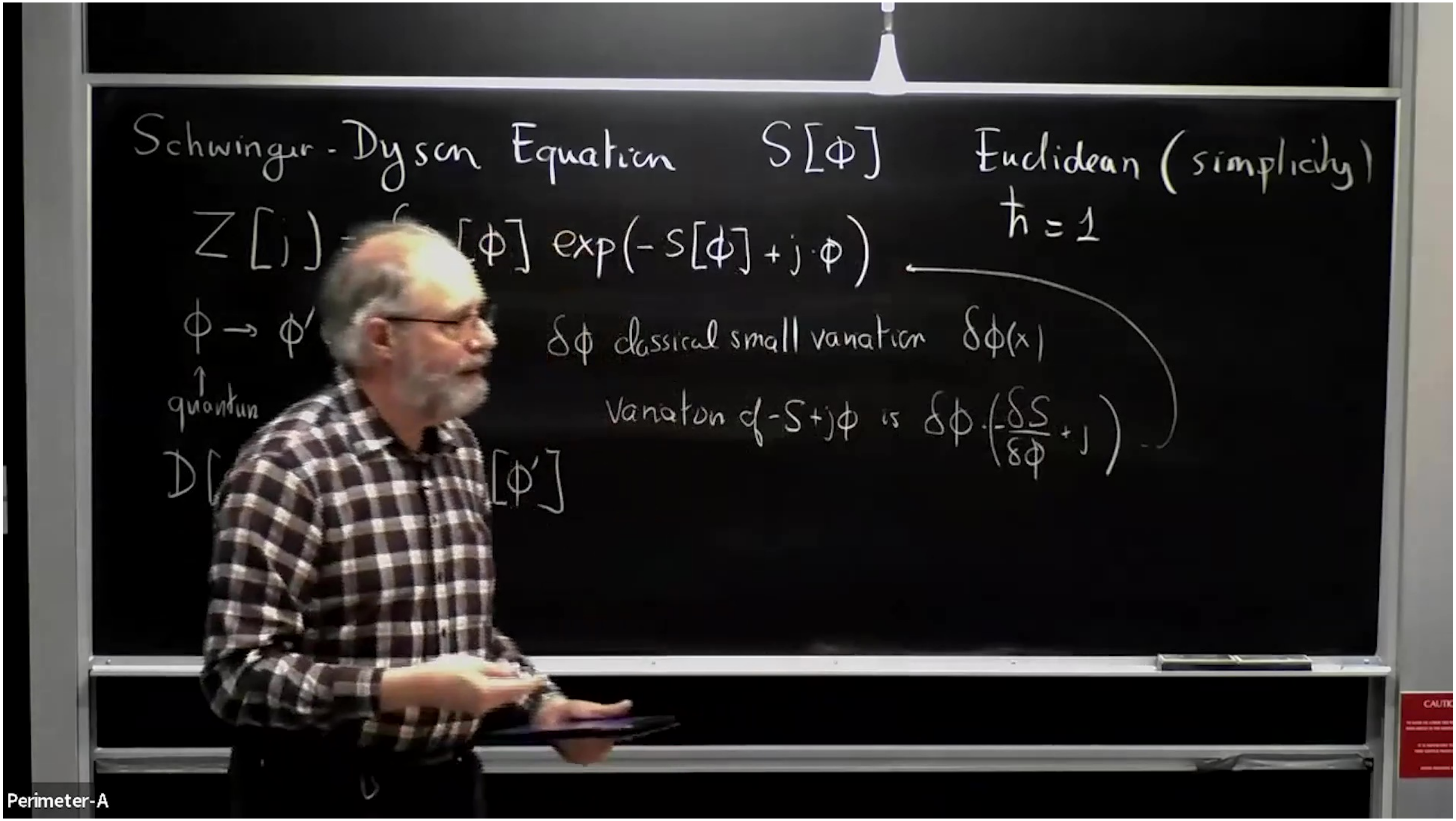
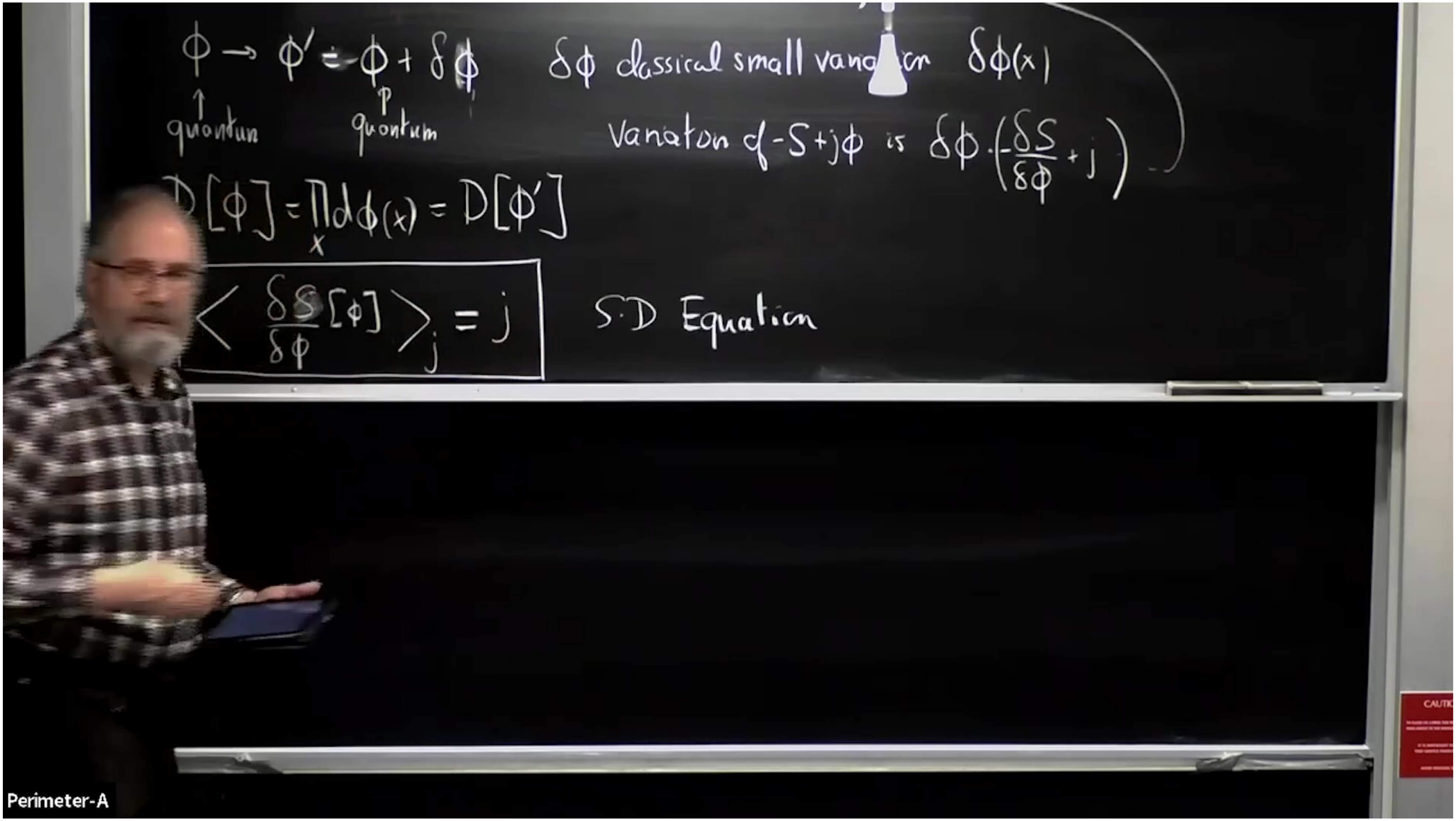
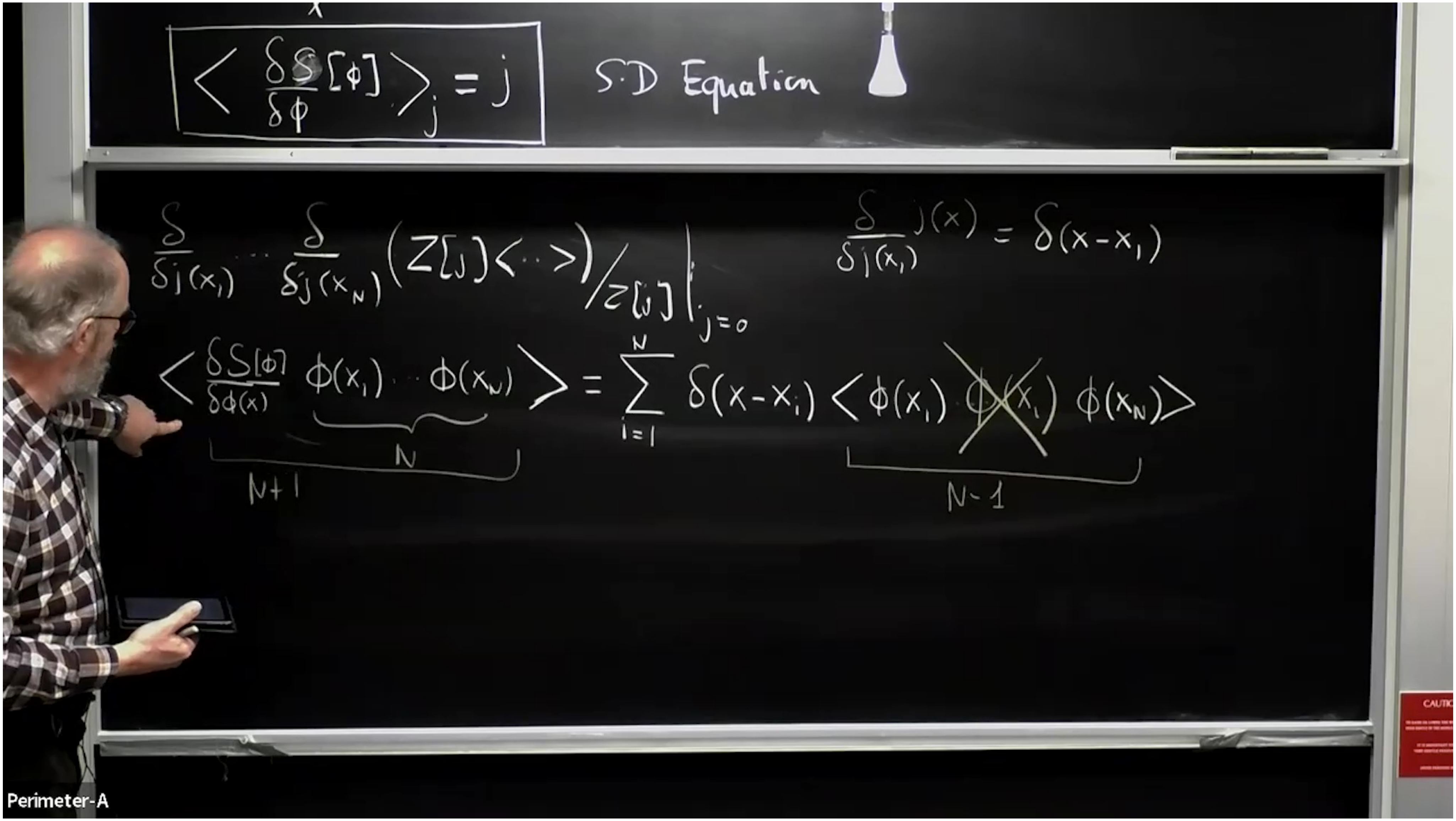
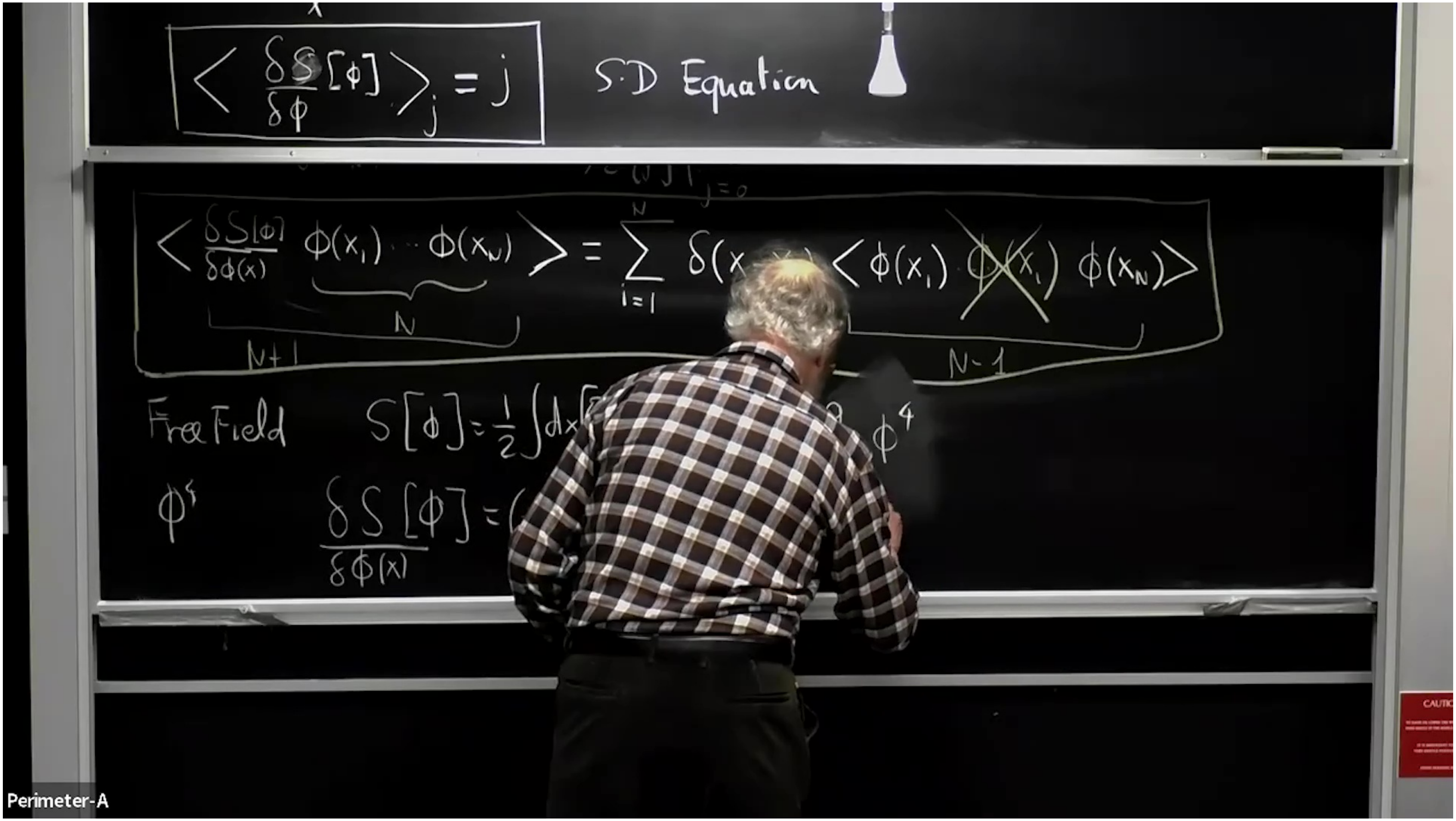
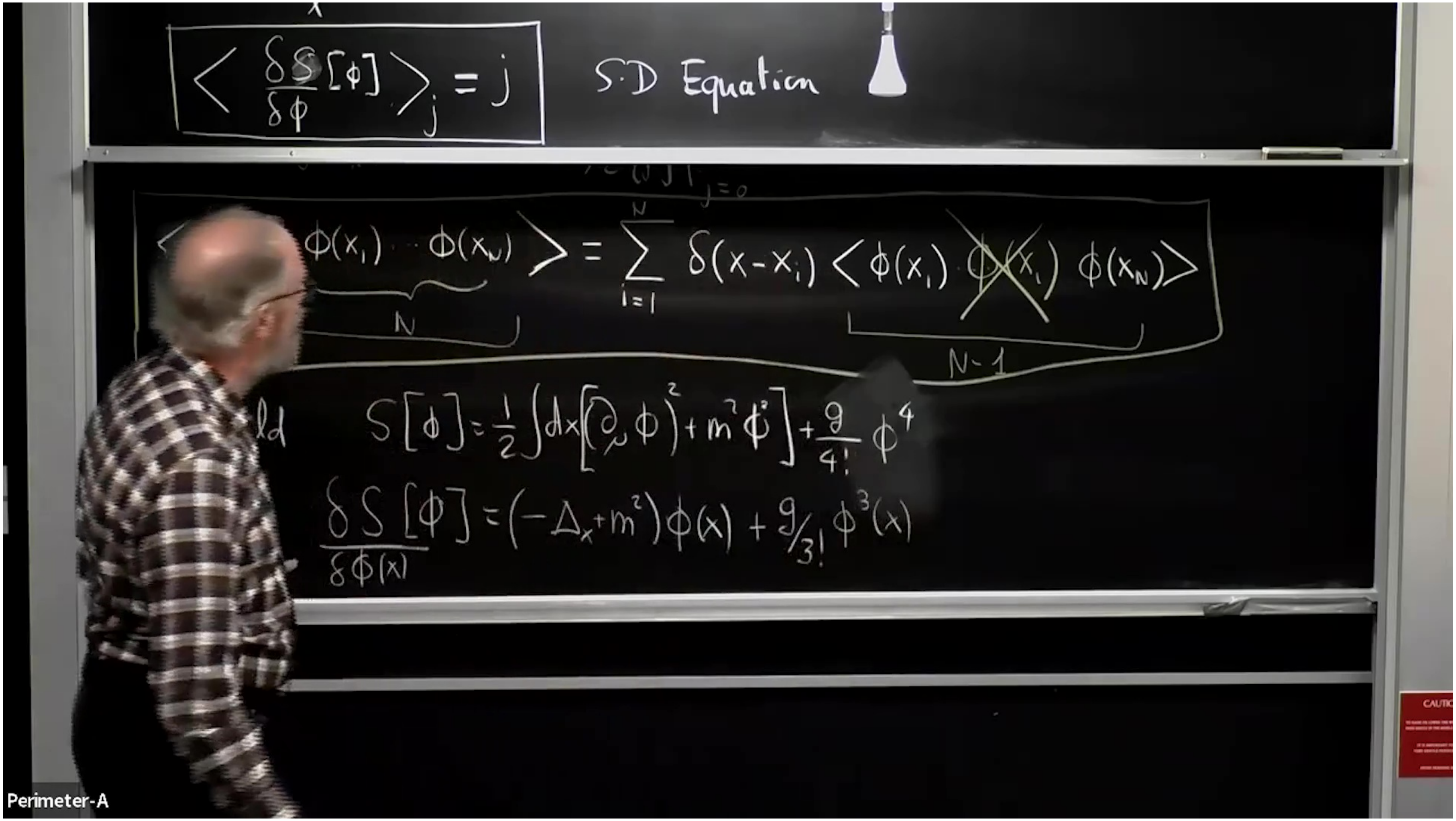

Schwinger-Dyson Equation $S[\phi]$ Euclidean (simplicity)
$\hbar = 1$
$Z[j] = \int D[\phi] \exp(-S[\phi] + j \cdot \phi)$
$\phi \to \phi' = \phi + \delta\phi$
quantum
quantum
$\delta\phi$ classical small variat
Variation of
$D[\phi] = \prod_x d\phi(x) = D[\phi']$
Perimeter-A


Schwinger - Dyson Equation
S[Φ]
Euclidean (simplicity)
ħ = 1
[Φ] exp(- S[Φ] + j·Φ)
φ → φ'
δφ classical small variation δφ(x)
quantum
variation of -S+jφ is δφ·(-δS/δφ + j)
D[
[φ']
Perimeter-A

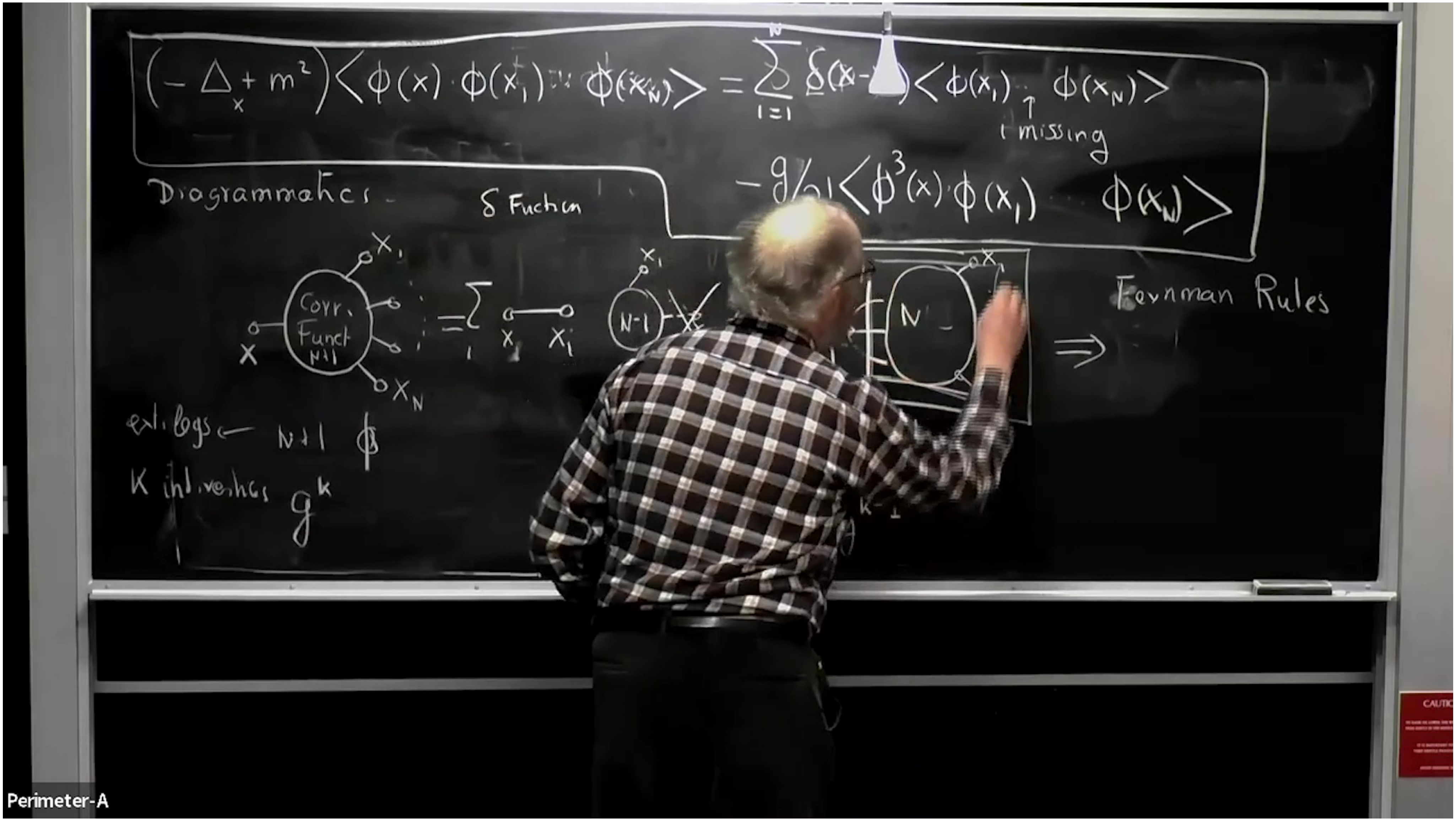

$\phi \rightarrow \phi' = \phi + \delta\phi$
quantum
quantum
$\delta\phi$ classical small
$\delta\phi(x)$
Variation of $-S + j\phi$ is $\delta\phi \cdot \left(-\frac{\delta S}{\delta \phi} + j \right)$
$D[\phi] = \prod_x d\phi(x) = D[\phi']$
$\boxed{\left\langle \frac{\delta S}{\delta \phi}[\phi] \right\rangle_j = j}$
S.D Equation
Perimeter-A


$\boxed{\left\langle \frac{\delta S}{\delta \phi}[\phi] \right\rangle_j = j}$
S.D Equation
$\frac{\delta}{\delta j(x_1)} \cdots \frac{\delta}{\delta j(x_N)} \left(Z[j] \langle \cdots \rangle \right) / Z[j] \Big|_{j=0}$
$\frac{\delta}{\delta j(x_1)} j(x) = \delta(x - x_1)$
$\underbrace{\left\langle \frac{\delta S[\phi]}{\delta \phi(x)} \underbrace{\phi(x_1) \cdots \phi(x_N)}_{N} \right\rangle}_{N+1} = \sum_{i=1}^{N} \delta(x - x_i) \underbrace{\langle \phi(x_1) \cancel{\phi(x_i)} \phi(x_N) \rangle}_{N-1}$
Perimeter-A

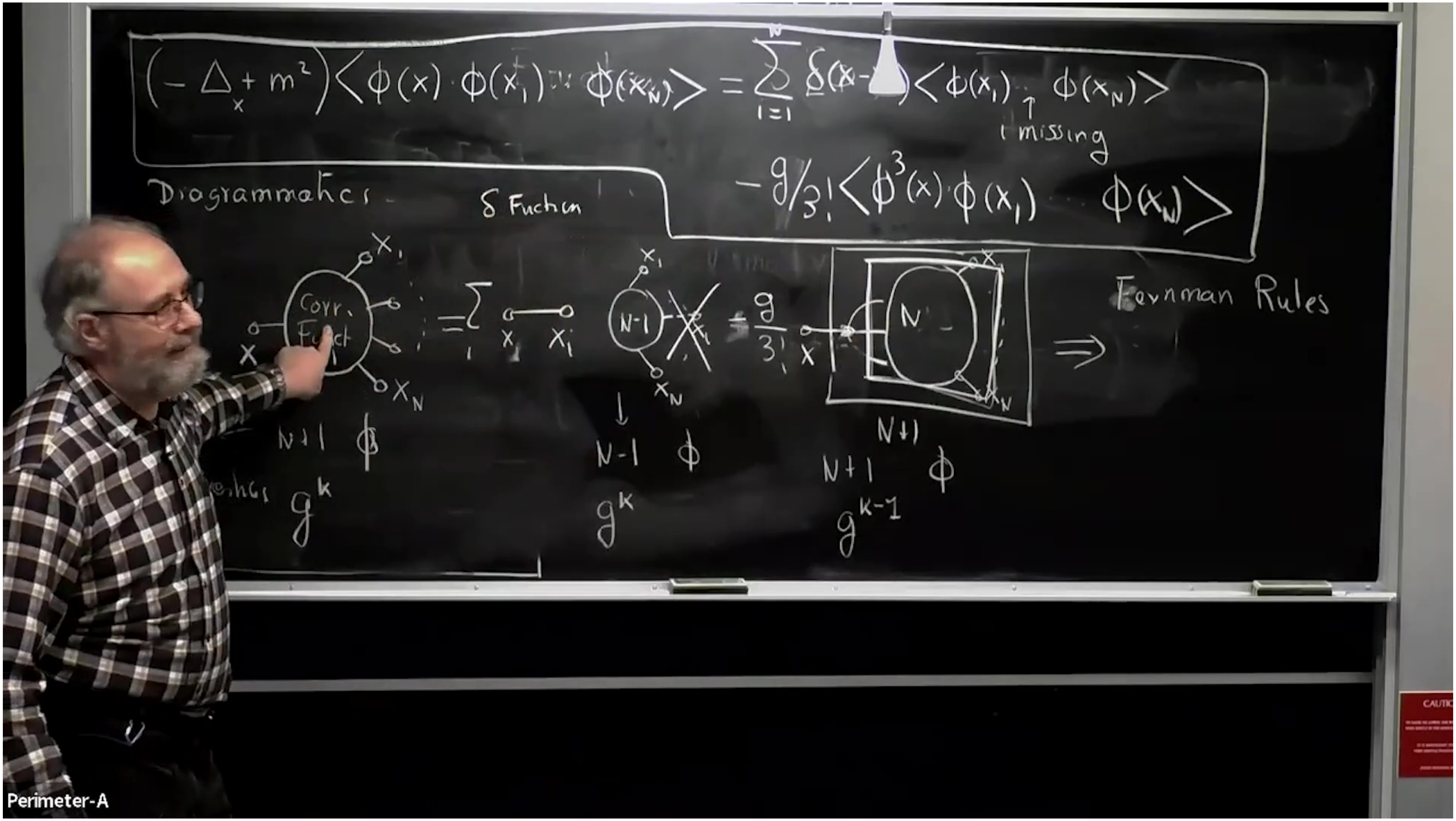

S.D Equation
Free Field
Perimeter-A



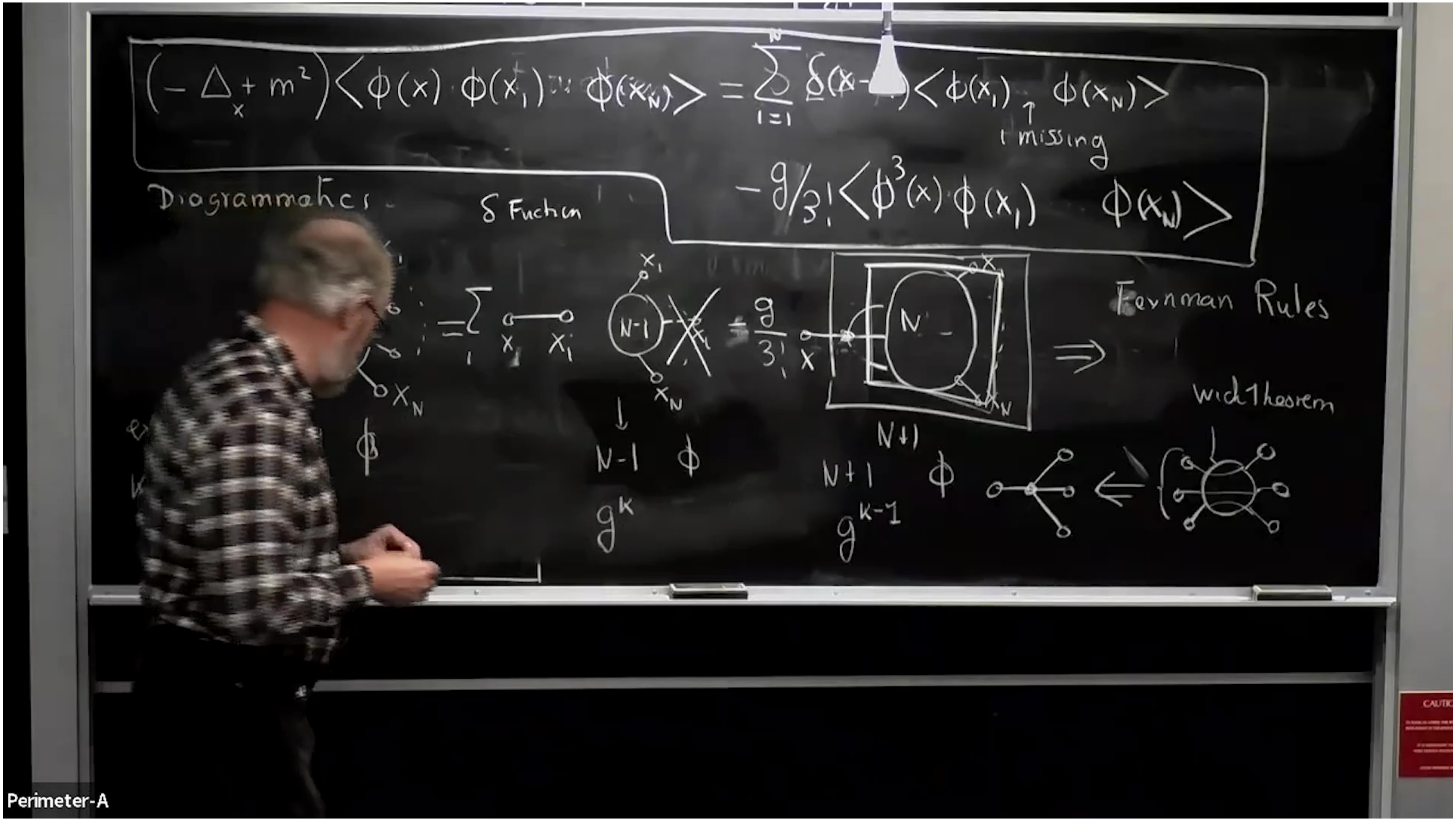
$\boxed{\left\langle \frac{\delta S}{\delta \phi}[\phi] \right\rangle_j = j}$
S.D Equation
$\left\langle \ldots \underbrace{\phi(x_1) \cdots \phi(x_N)}_{N} \right\rangle = \sum_{i=1}^{N} \delta(x - x_i) \left\langle \underbrace{\phi(x_1) \cancel{\phi(x_i)} \phi(x_N)}_{N-1} \right\rangle$
$S[\phi] = \frac{1}{2} \int dx \left[(\partial_\mu \phi)^2 + m^2 \phi^2 \right] + \frac{g}{4!} \phi^4$
$\frac{\delta S[\phi]}{\delta \phi(x)} = (-\Delta_x + m^2) \phi(x) + g/_{3!} \phi^3(x)$
Perimeter-A

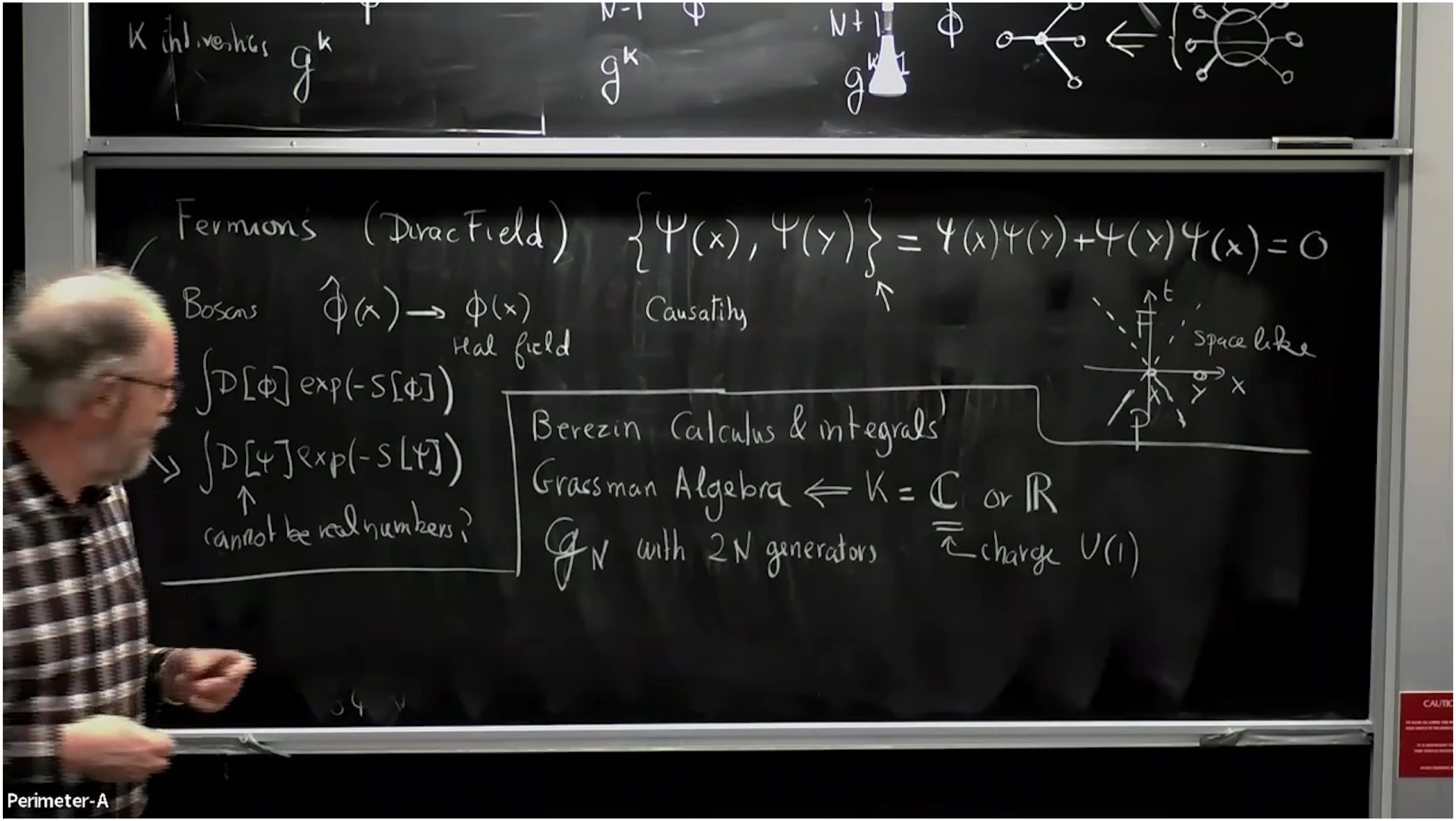


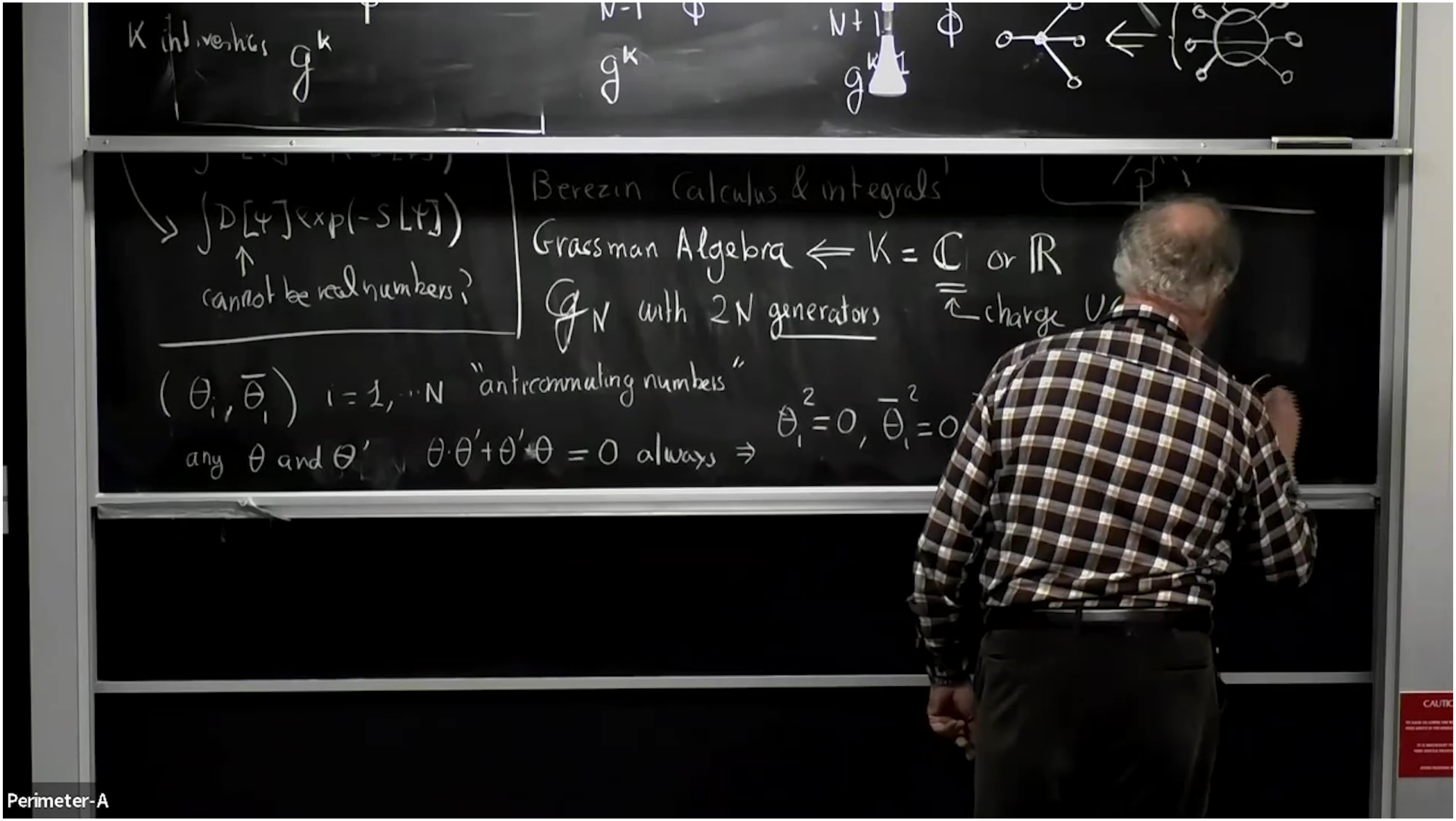
i missing
Diagrammatics
δ Function
Corr. Funct N+1
Feynman Rules
ext. legs ← N+1 ϕ
K interactions g^k
Perimeter-A
CAUTION

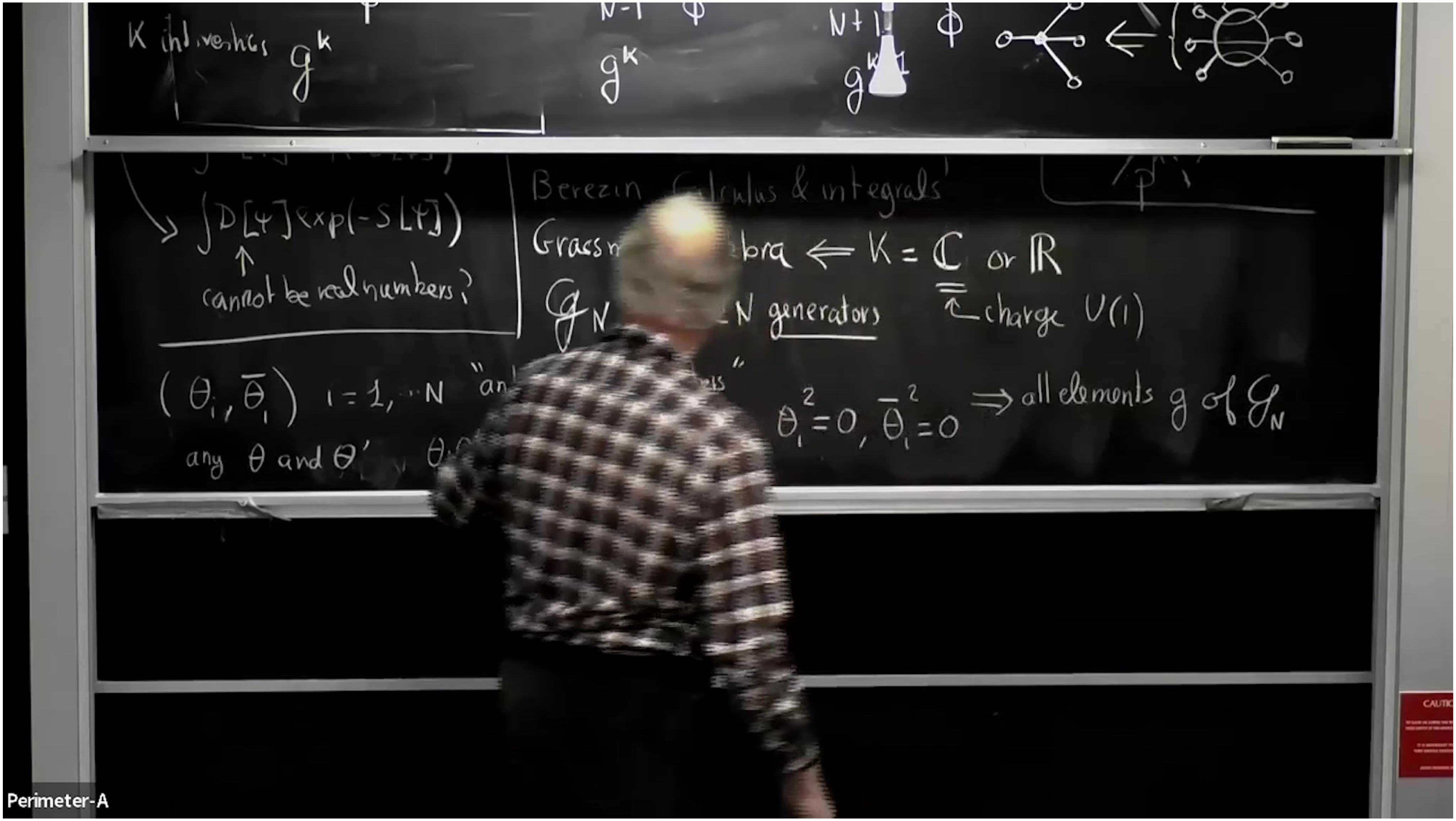


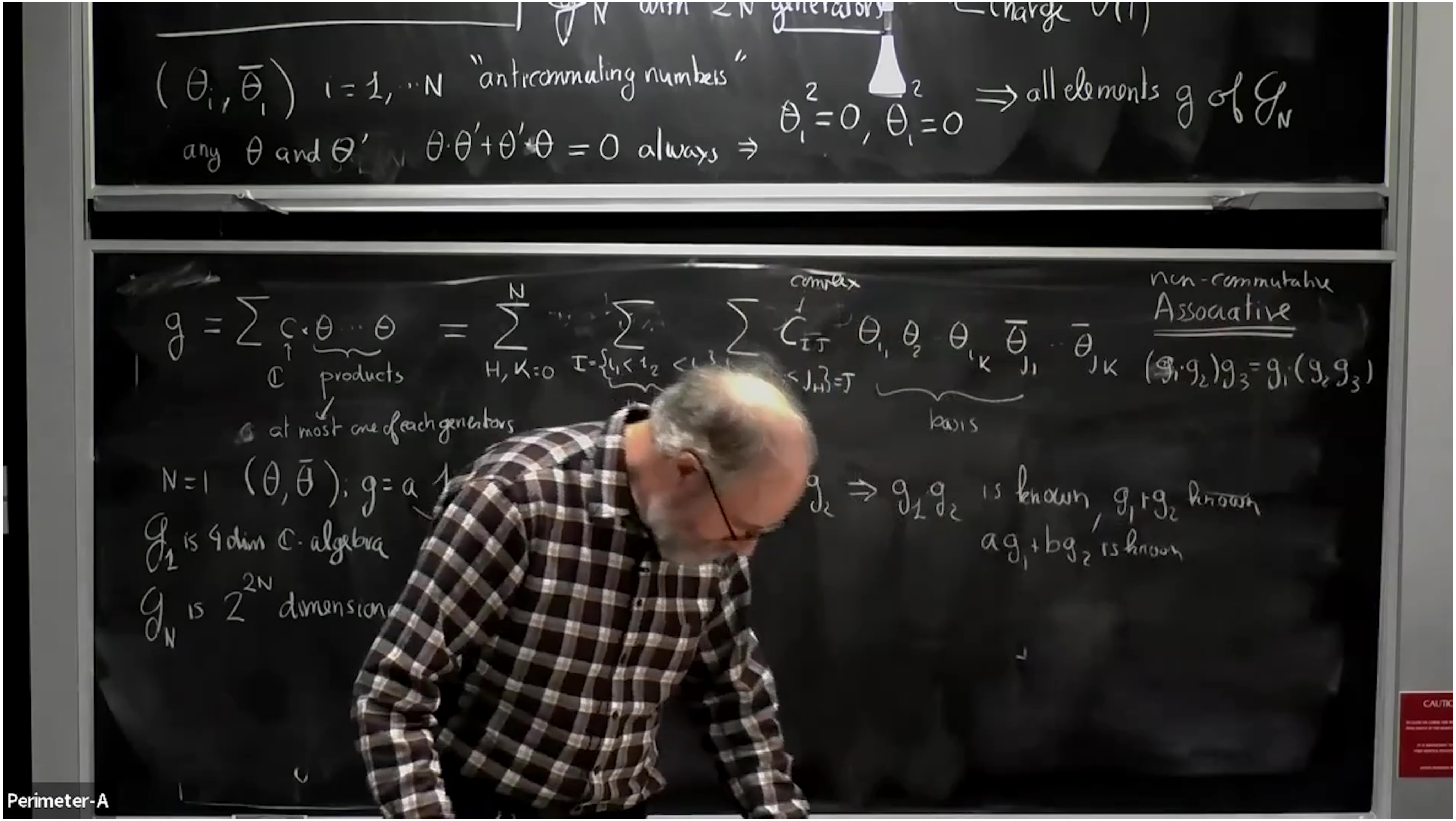
$(-\Delta_x + m^2)\langle \phi(x)\, \phi(x_1) \cdots \phi(x_N) \rangle = \sum_{i=1}^{N} \delta(x - x_i) \langle \phi(x_1) \cdots \phi(x_N) \rangle$
missing
$- g/3! \langle \phi^3(x)\, \phi(x_1) \cdots \phi(x_N) \rangle$
Diagrammatics
δ Function
Corr. Funct
Feynman Rules
N+1 ϕ
g^k
N-1 ϕ
g^k
N+1 ϕ
g^{k-1}
Perimeter-A

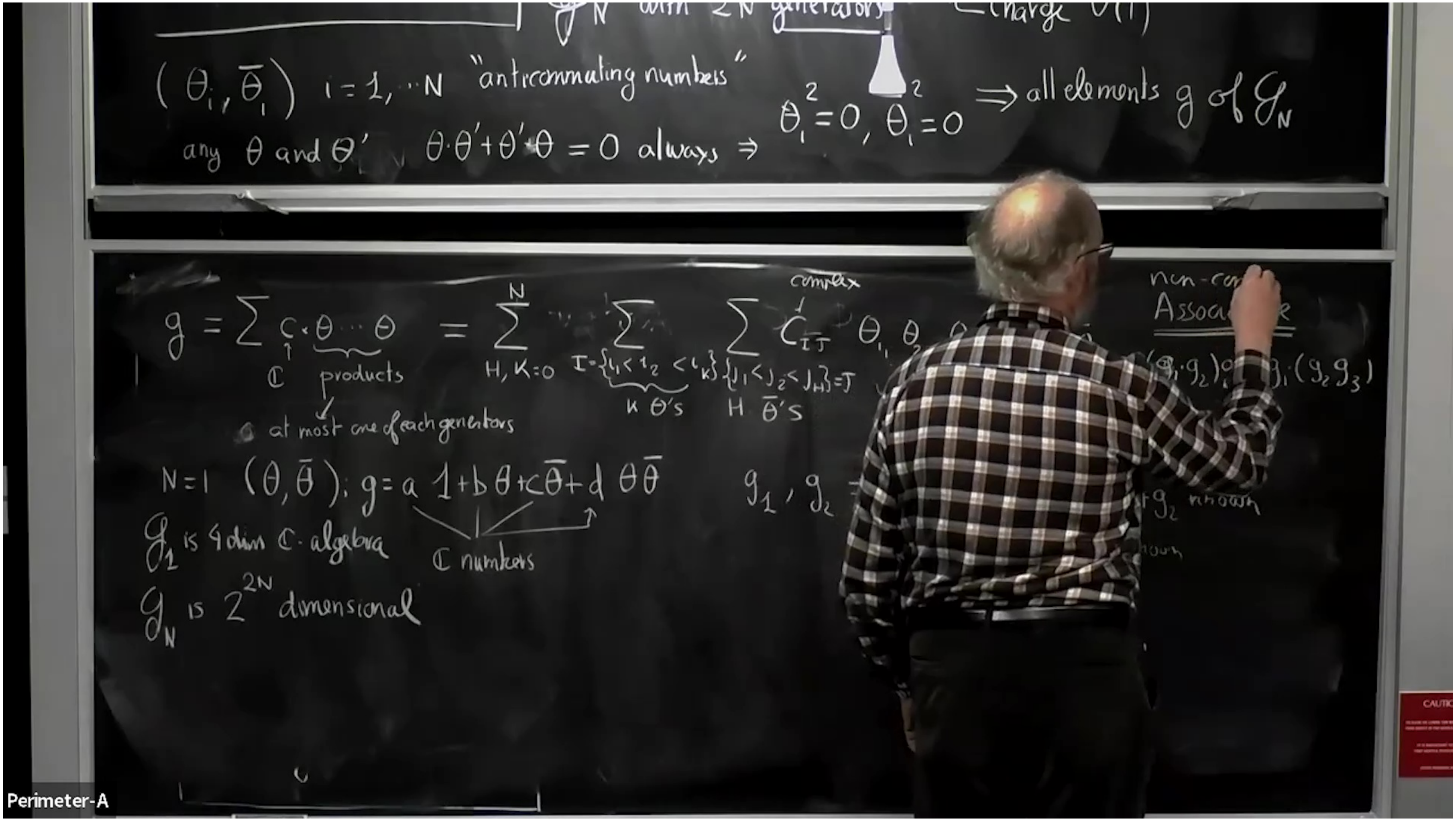

$(-\Delta_x + m^2)\langle \phi(x)\,\phi(x_1)\cdots\phi(x_N)\rangle = \sum_{i=1}^{N} \delta(x-x_i)\langle \phi(x_1) \cdots \phi(x_N)\rangle$
↑ i missing
$- g/3!\,\langle \phi^3(x)\,\phi(x_1) \cdots \phi(x_N)\rangle$
Diagrammatics
δ Fuction
x_1
x_N
$= \sum_i$
x
x_i
N-1
$\frac{g}{3!}$
N
Feynman Rules
$\Rightarrow$
Wick Theorem
N-1 ϕ
g^K
N+1
N+1 ϕ
g^{K-1}
$\Leftarrow$
Perimeter-A

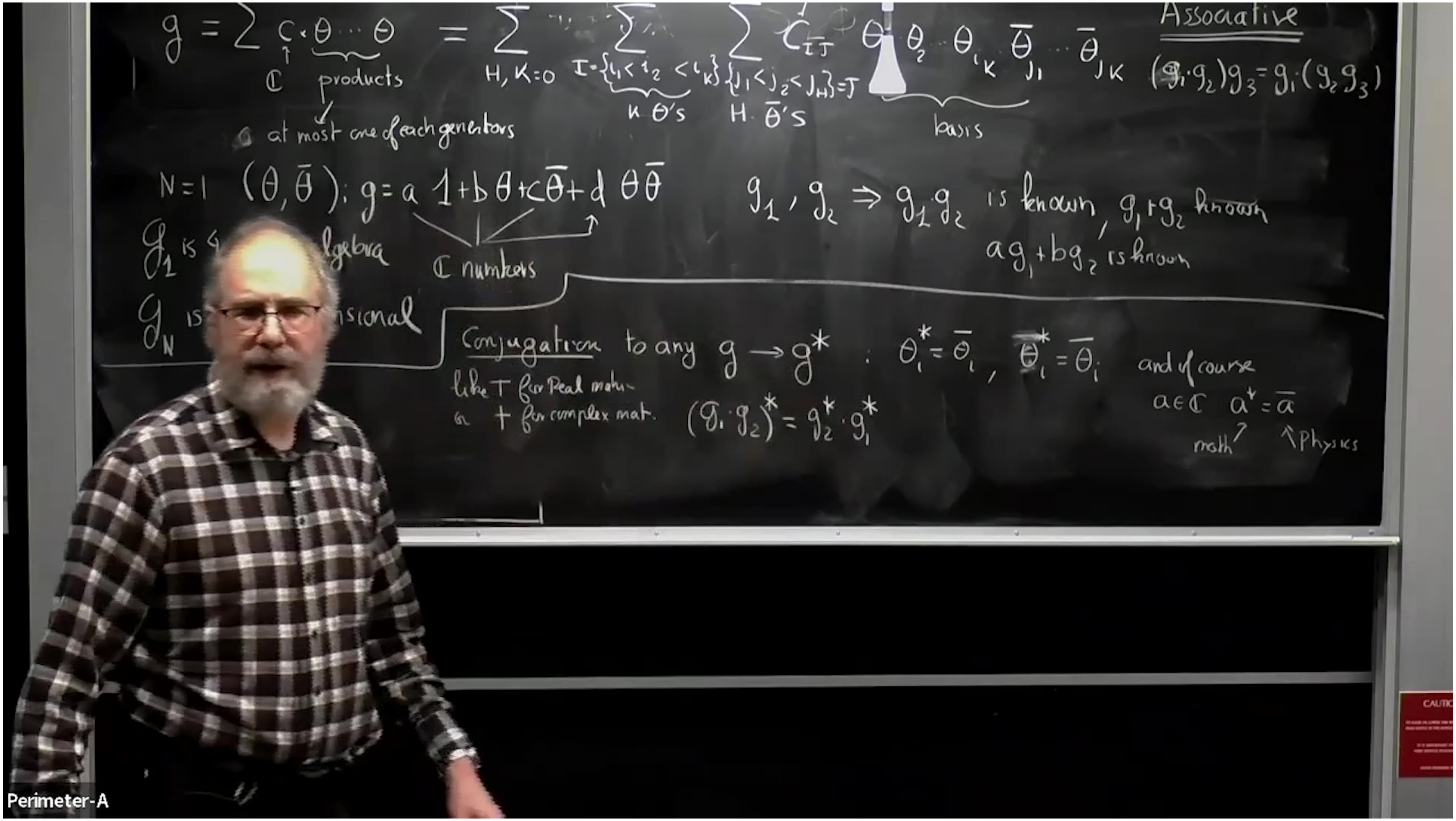

K insertions g^k
$N-1$ ϕ g^k
$N+1$ ϕ g^{k+1}
Fermions (Dirac Field)
$\{\Psi(x), \Psi(y)\} = \Psi(x)\Psi(y) + \Psi(y)\Psi(x) = 0$
Causality
t
space like
x
y
Bosons $\hat{\phi}(x) \longrightarrow \phi(x)$
real field
$\int D[\phi] \exp(-S[\phi])$
$\int D[\Psi] \exp(-S[\Psi])$
cannot be real numbers?
Berezin Calculus & integrals
Grassman Algebra $\Leftarrow K = \mathbb{C}$ or $\mathbb{R}$
$\mathcal{G}_N$ with $2N$ generators
charge $U(1)$
Perimeter-A

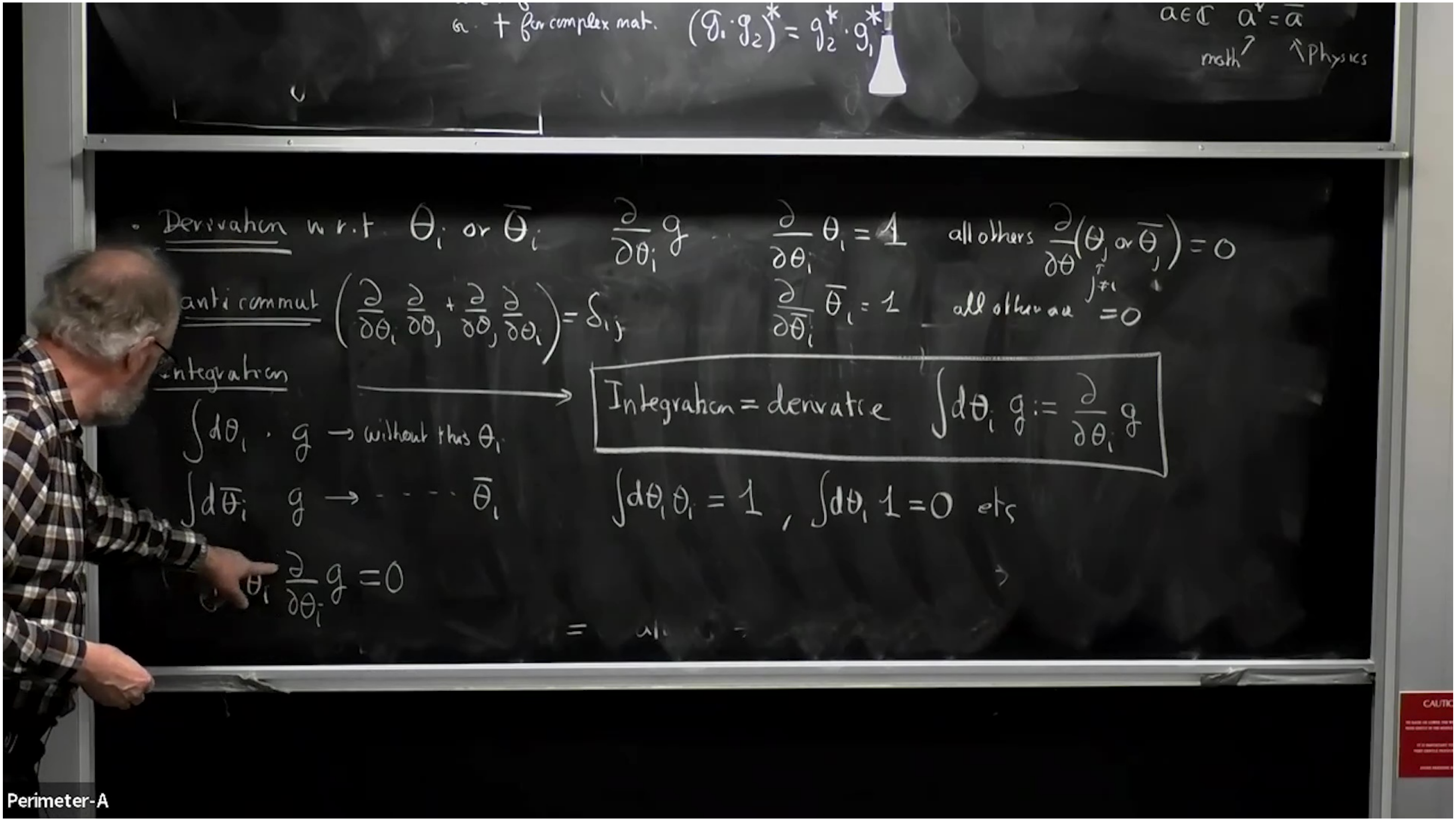

K ind. vertices
Berezin Calculus & integrals
∫D[ψ]exp(-S[ψ])
cannot be real numbers?
Grassman Algebra ⇐ K = ℂ or ℝ
𝒢N with 2N generators
charge
(θi, θ̄i) i = 1, ··· N "anticommuting numbers"
any θ and θ′ θ·θ′+θ′·θ = 0 always ⇒ θi² = 0, θ̄i² = 0
Perimeter-A


K int.vertices
Berezin Calculus & integrals
∫D[ψ]exp(-S[ψ])
cannot be real numbers?
K = ℂ or ℝ
N generators
charge U(1)
(θ_i, θ̄_i) i = 1, ⋯ N
θ_i^2 = 0, θ̄_i^2 = 0 ⇒ all elements g of 𝒢_N
any θ and θ'
Perimeter-A

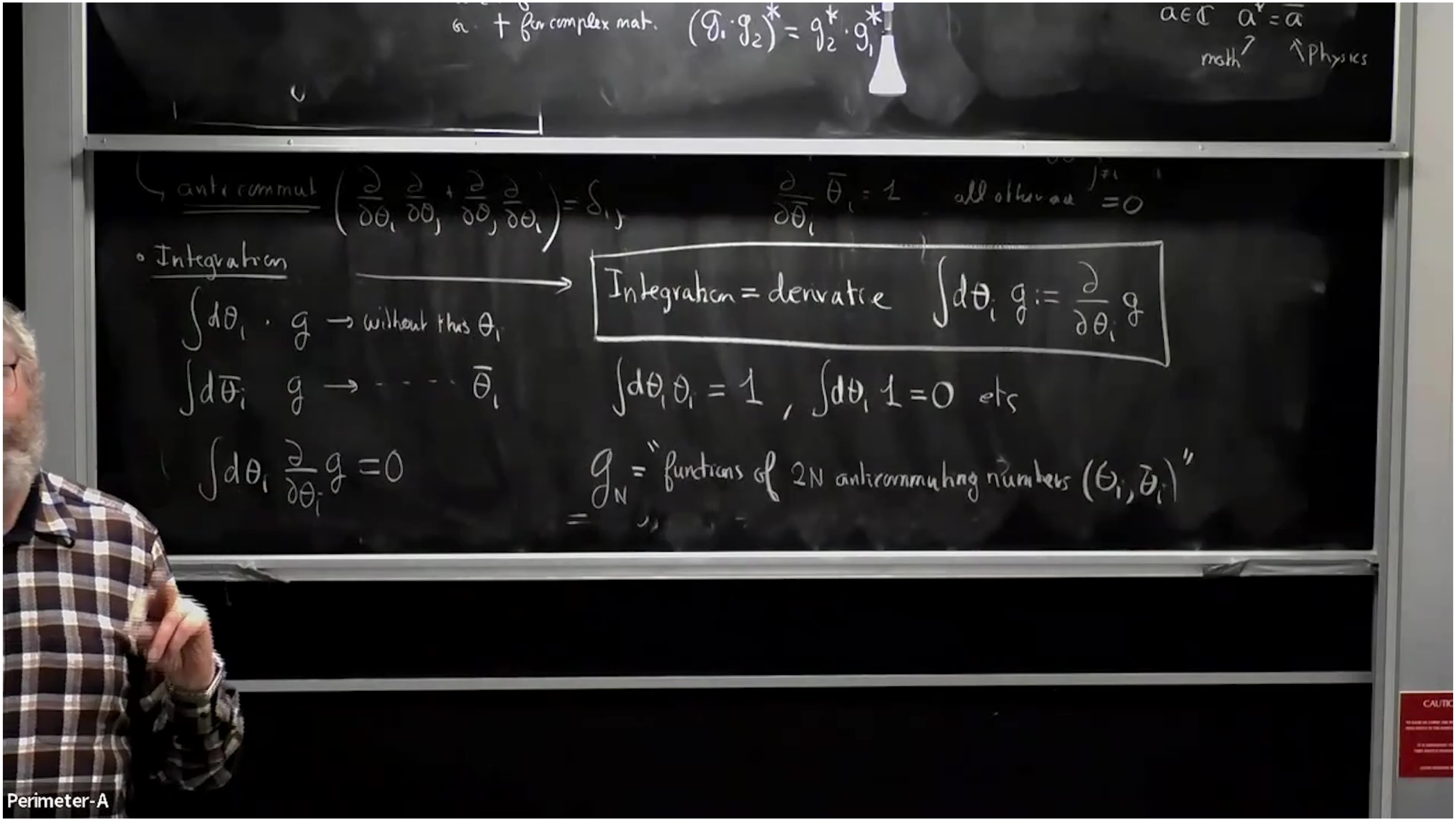

"anticommuting numbers"
any θ and θ' θ·θ'+θ'·θ = 0 always ⇒
⇒ all elements g of
non-commutative
Associative
products
at most one of each generators
complex
basis
is known
is known
Perimeter-A

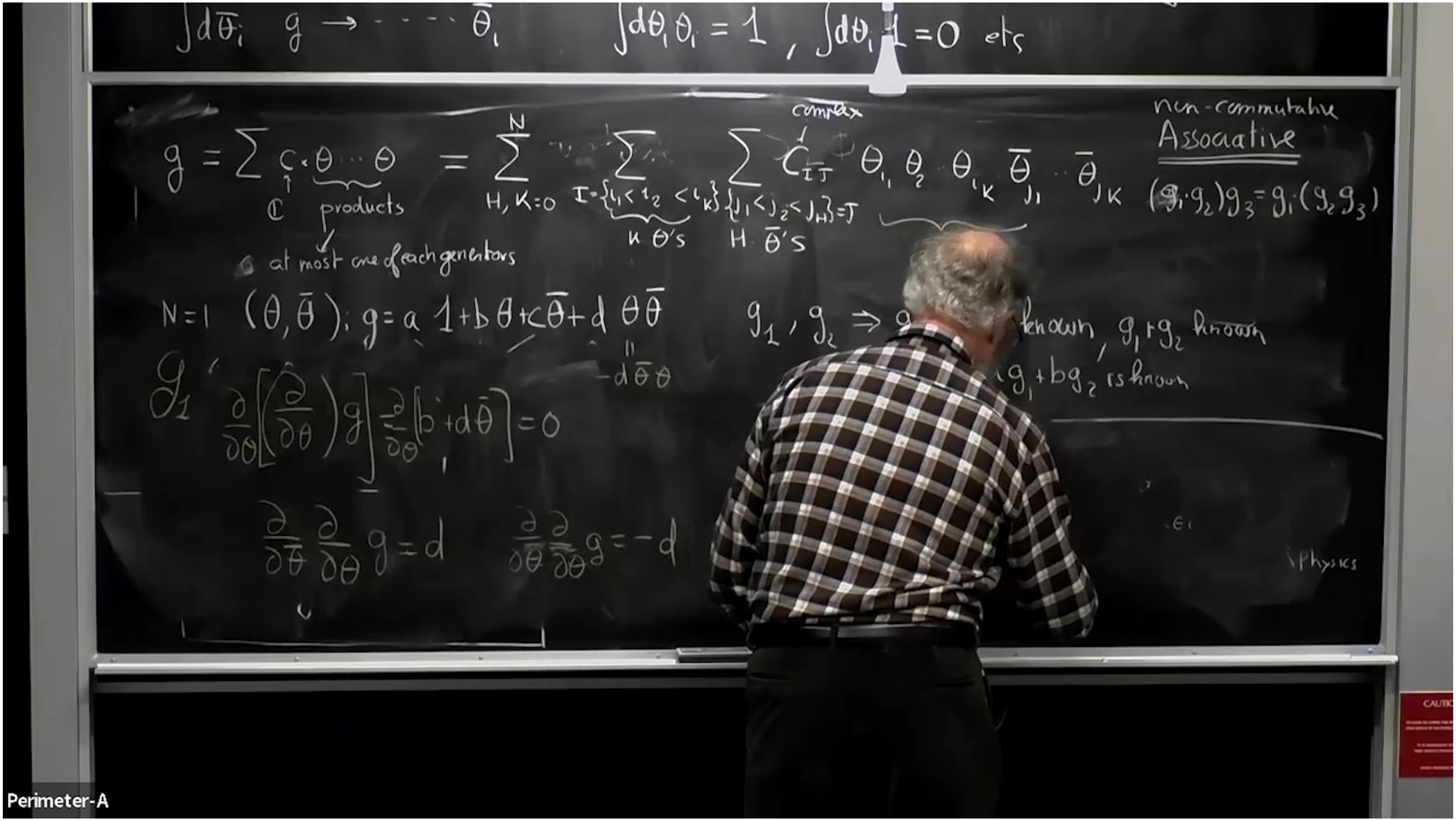

$(\theta_i, \bar\theta_i)$ $i = 1, \cdots N$ "anticommuting numbers"
any θ and θ' $\theta\cdot\theta' + \theta'\cdot\theta = 0$ always $\Rightarrow$ $\theta_i^2 = 0$, $\bar\theta_i^2 = 0$ $\Rightarrow$ all elements g of $\mathcal{G}_N$
complex
$g = \sum c \cdot \underbrace{\theta \cdots \theta}_{\text{products}} = \sum_{H,K=0}^{N} \sum_{I=\{i_1<i_2<\cdots i_K\}} \sum_{\{j_1<j_2<\cdots j_H\}=J} C_{IJ}\, \theta_{i_1}\theta_{i_2}\cdots$
$\mathbb{C}$
at most one of each generators
K θ's
H $\bar\theta$'s
non-com
Associative
$(g_1 \cdot g_2) \cdot g_3 = g_1 \cdot (g_2 g_3)$
$N=1$ $(\theta, \bar\theta)$: $g = a\, 1 + b\,\theta + c\,\bar\theta + d\,\theta\bar\theta$
$\mathbb{C}$ numbers
g_1, g_2
known
$\mathcal{G}_1$ is 4 dim $\mathbb{C}$-algebra
$\mathcal{G}_N$ is 2^{2N} dimensional
Perimeter-A


Associative
$(g_1 \cdot g_2) g_3 = g_1 (g_2 g_3)$
$g = \sum c \cdot \theta \cdots \theta$
products
at most one of each generators
$H, K = 0$
$\kappa\ \theta$'s
$H\ \bar{\theta}$'s
basis
$N = 1\ (\theta, \bar{\theta}):\ g = a\,1 + b\,\theta + c\,\bar{\theta} + d\,\theta\bar{\theta}$
$\mathbb{C}$ numbers
$g_1, g_2 \Rightarrow g_1 g_2$ is known, $g_1 + g_2$ known
$a g_1 + b g_2$ is known
Conjugation to any $g \to g^*$: $\theta_i^* = \bar{\theta}_i$, $\bar{\theta}_i^* = \theta_i$
like T for Real matr.
or † for complex mat.
$(g_1 \cdot g_2)^* = g_2^* \cdot g_1^*$
and of course
$a \in \mathbb{C}$ $a^* = \bar{a}$
math
Physics
Perimeter-A

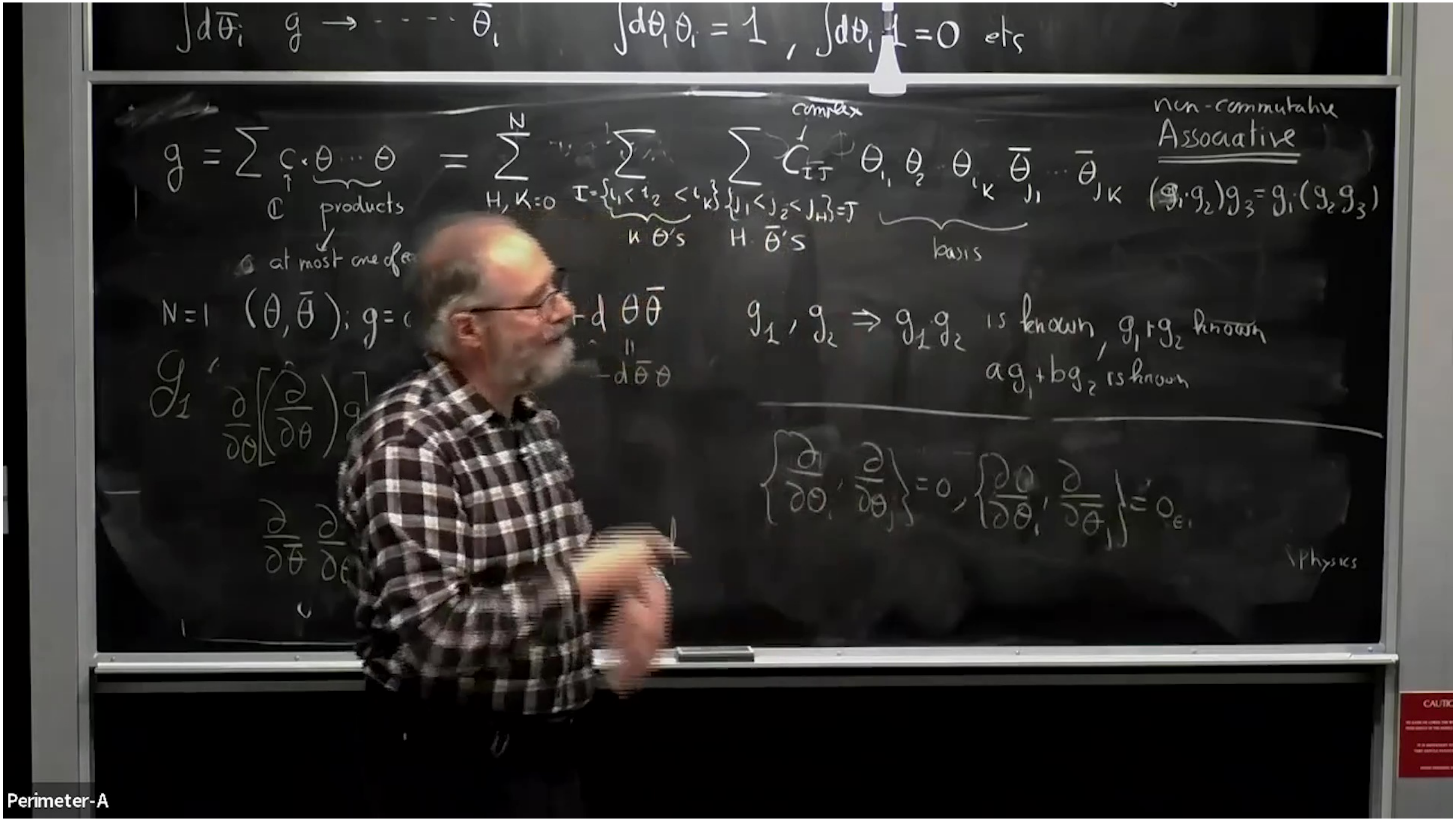


+ for complex mat. $(g_1 \cdot g_2)^* = g_2^* \cdot g_1^*$
$a \in \mathbb{C}$ $a^* = \bar{a}$
math Physics
Derivation w.r.t θ_i or $\bar\theta_i$ $\frac{\partial}{\partial\theta_i} g$
$\frac{\partial}{\partial\theta_i}\theta_i = 1$ all others $\frac{\partial}{\partial\theta}(\theta_j \text{ or } \bar\theta_j) = 0$ $j \neq i$
anticommut $\left(\frac{\partial}{\partial\theta_i}\frac{\partial}{\partial\theta_j} + \frac{\partial}{\partial\theta_j}\frac{\partial}{\partial\theta_i}\right) = \delta_{ij}$
$\frac{\partial}{\partial\bar\theta_i}\bar\theta_i = 1$ all others are $= 0$
Integration
$\int d\theta_i \cdot g \to$ without this θ_i
$\int d\bar\theta_i \; g \to \cdots \bar\theta_i$
$\int \theta_i \frac{\partial}{\partial\theta_i} g = 0$
Integration = derivative $\int d\theta_i \; g := \frac{\partial}{\partial\theta_i} g$
$\int d\theta_i \theta_i = 1$, $\int d\theta_i 1 = 0$ etc
Perimeter-A


+ for complex mat. $(g_1 \cdot g_2)^* = g_2^* \cdot g_1^*$
$a \in \mathbb{C}$ $a^* = \bar{a}$
math
Physics
anticommut $\left(\frac{\partial}{\partial\theta_i}\frac{\partial}{\partial\theta_j} + \frac{\partial}{\partial\theta_j}\frac{\partial}{\partial\theta_i}\right) = \delta_{ij}$
$\frac{\partial}{\partial\bar\theta_i}\bar\theta_i = 1$ all other are $= 0$
Integration
Integration = derivative $\int d\theta_i\, g := \frac{\partial}{\partial\theta_i} g$
$\int d\theta_i \cdot g \to$ without this θ_i
$\int d\bar\theta_i\, g \to \cdots \bar\theta_i$
$\int d\theta_i\, \theta_i = 1$, $\int d\theta_i\, 1 = 0$ etc
$\int d\theta_i \frac{\partial}{\partial\theta_i} g = 0$
g_N = "functions of 2N anticommuting numbers $(\theta_i, \bar\theta_i)$"
Perimeter-A



∫dθ̄ᵢ g → · · · · θ̄ᵢ
∫dθᵢ θᵢ = 1 , ∫dθᵢ 1 = 0 etc
non-commutative
Associative
(g₁·g₂)g₃ = g₁(g₂g₃)
g = Σ c · θ ··· θ = Σ Σ Σ C_IJ θ_i1 θ_i2 θ_iK θ̄_j1 ··· θ̄_jK
complex
ℂ
products
H, K=0
I={i₁< i₂ < i_K}
{j₁<j₂<j_H}=J
K θ's
H θ̄'s
at most one of each generators
N=1 (θ, θ̄): g = a 1 + b θ + c θ̄ + d θ θ̄
= −d θ̄ θ
g₁ , g₂ ⇒
known, g₁+g₂ known
g₁ + bg₂ is known
∂/∂θ [(∂/∂θ) g] = ∂/∂θ [b + d θ̄] = 0
∂/∂θ̄ ∂/∂θ g = d
∂/∂θ ∂/∂θ̄ g = −d
Physics
Perimeter-A


non-commutative
Associative
basis
products
at most one of
is known
known
is known
Physics
Perimeter-A
CAUTION