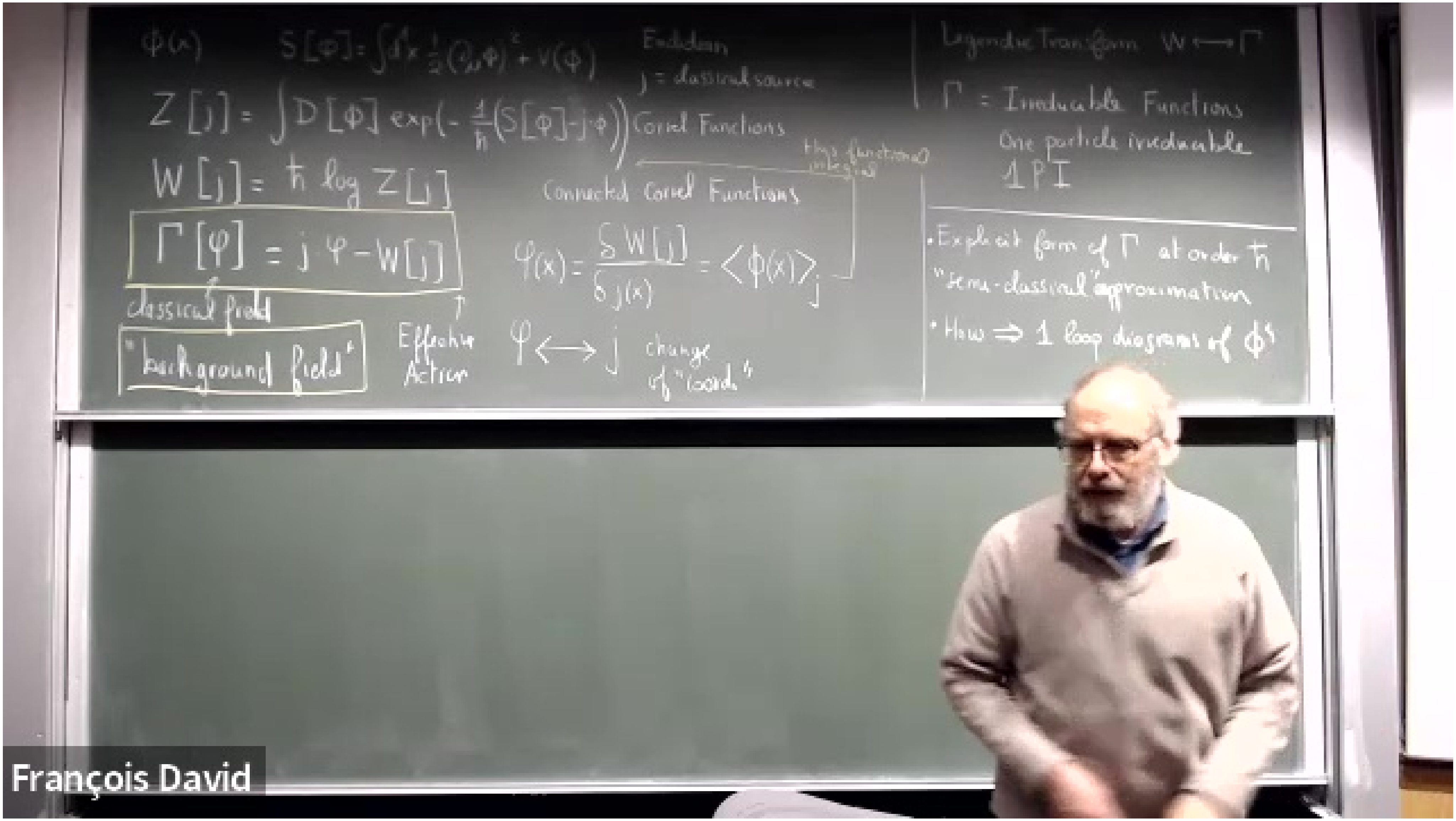
Title: QFT III 2021/2022

Speakers: Francois David

Collection: QFT III 2021/2022

Date: January 21, 2022 - 11:30 AM

URL: https://pirsa.org/22010028


Euclidean
j = classical source
Z[j] = ∫D[Φ] exp(- 1/ħ (S[Φ] - j·Φ))
Correl Functions
W[j] = ħ log Z[j]
Connected Correl Functions
Γ[φ] = j·φ - W[j]
classical field
"background field"
Effective Action
φ(x) = δW[j] / δj(x) = ⟨Φ(x)⟩_j
φ ⟷ j change of "coord."
Legendre Transform W ⟷ Γ
Γ = Irreducible Functions
One particle irreducible
1PI
• Explicit form of Γ at order ħ
"semi-classical" approximation
• How ⇒ 1 loop diagrams of Φ⁴
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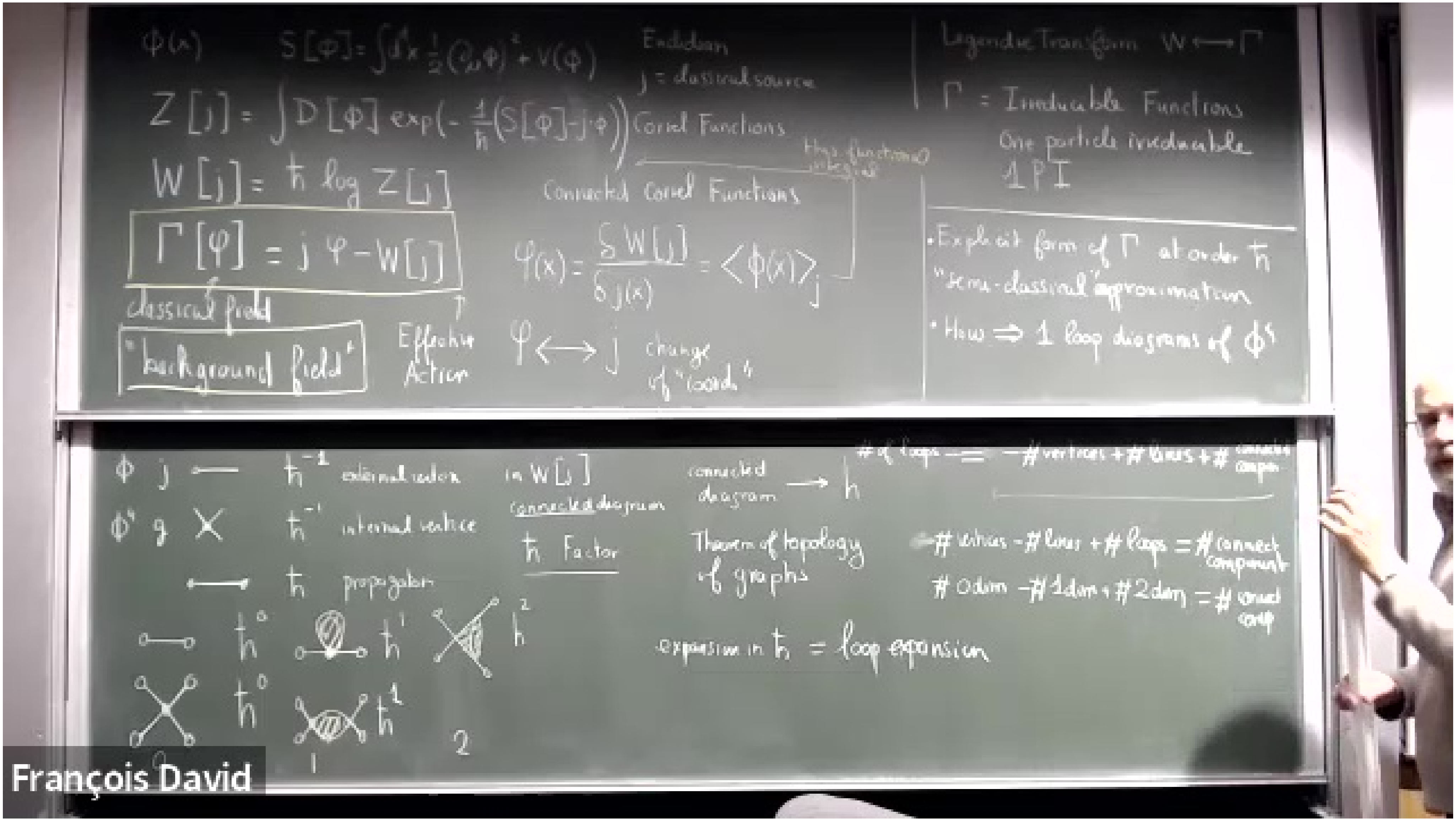

Euclidean
j = classical source
Corrl Functions
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Effective Action
change of "coord."
Legendre Transform W ⟷ Γ
Γ = Irreducible Functions
One particle irreducible
1PI
• Explicit form of Γ at order ħ
"semi-classical" approximation
• How ⇒ 1 loop diagrams of ϕ⁴
external vertex
internal vertex
propagator
Connected diagram
ħ Factor
connected diagram → ħ
Theorem of topology of graphs
# of loops = − # vertices + # lines + # connected compon.
# vertices − # lines + # loops = # connect component
# 0dim − # 1dim + # 2dim = # connect comp
expansion in ħ = loop expansion
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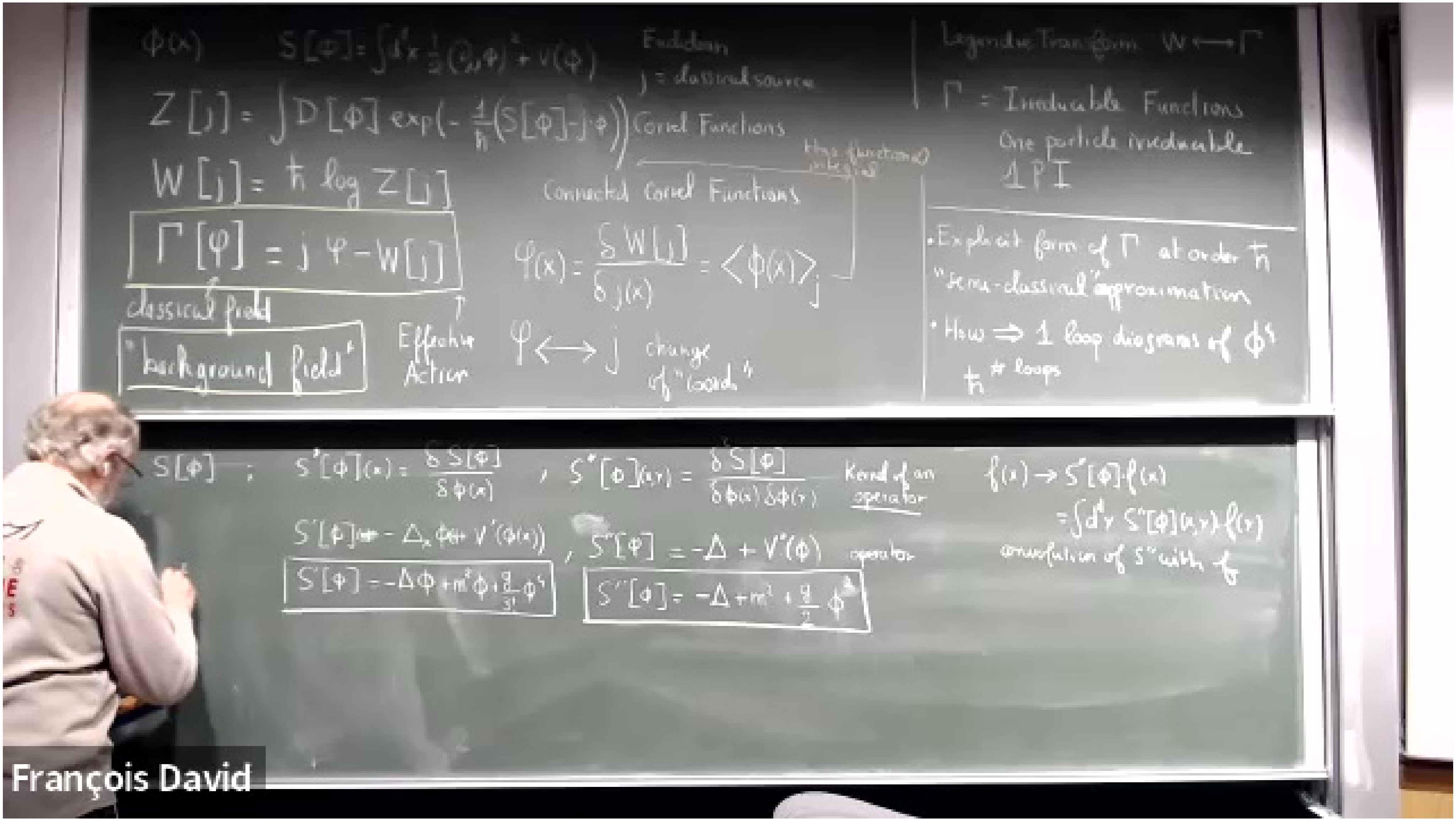
Z[j] = ∫D[Φ] exp(−1/ħ(S[Φ]−j·Φ))
Correl Functions
W[j] = ħ log Z[j]
Connected Correl Functions
Γ[φ] = j φ − W[j]
classical field
"background field"
Effective Action
φ ←→ j
Legendre Transform W ⟷ Γ
Γ = Irreducible Functions
One particle irreducible
1PI
operator
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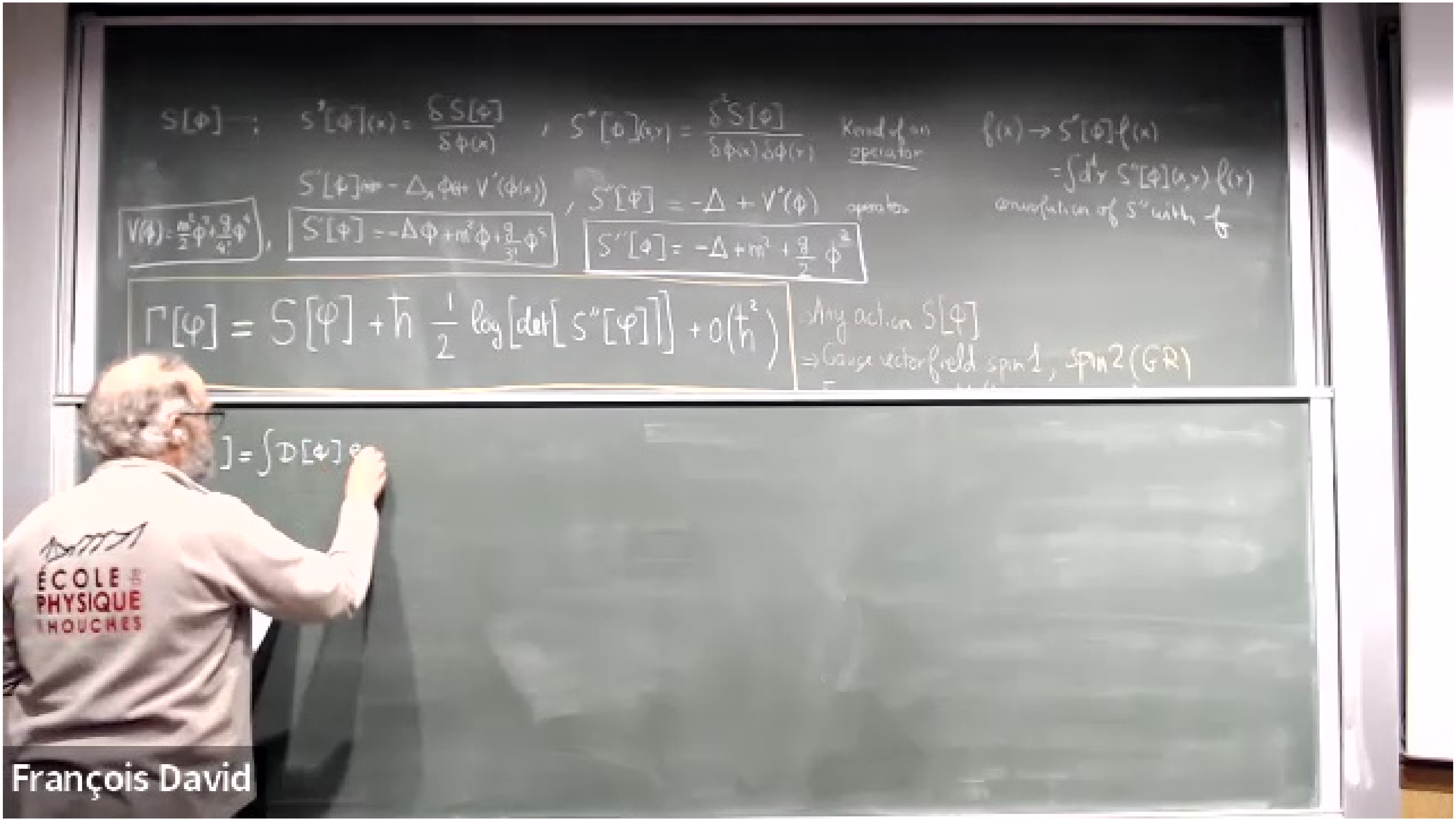
Kernel of an operator
operator
convolution of S'' with f
Any action S[φ]
⇒ Gauge vector field spin 1, spin 2 (GR)
ÉCOLE DE PHYSIQUE DES HOUCHES
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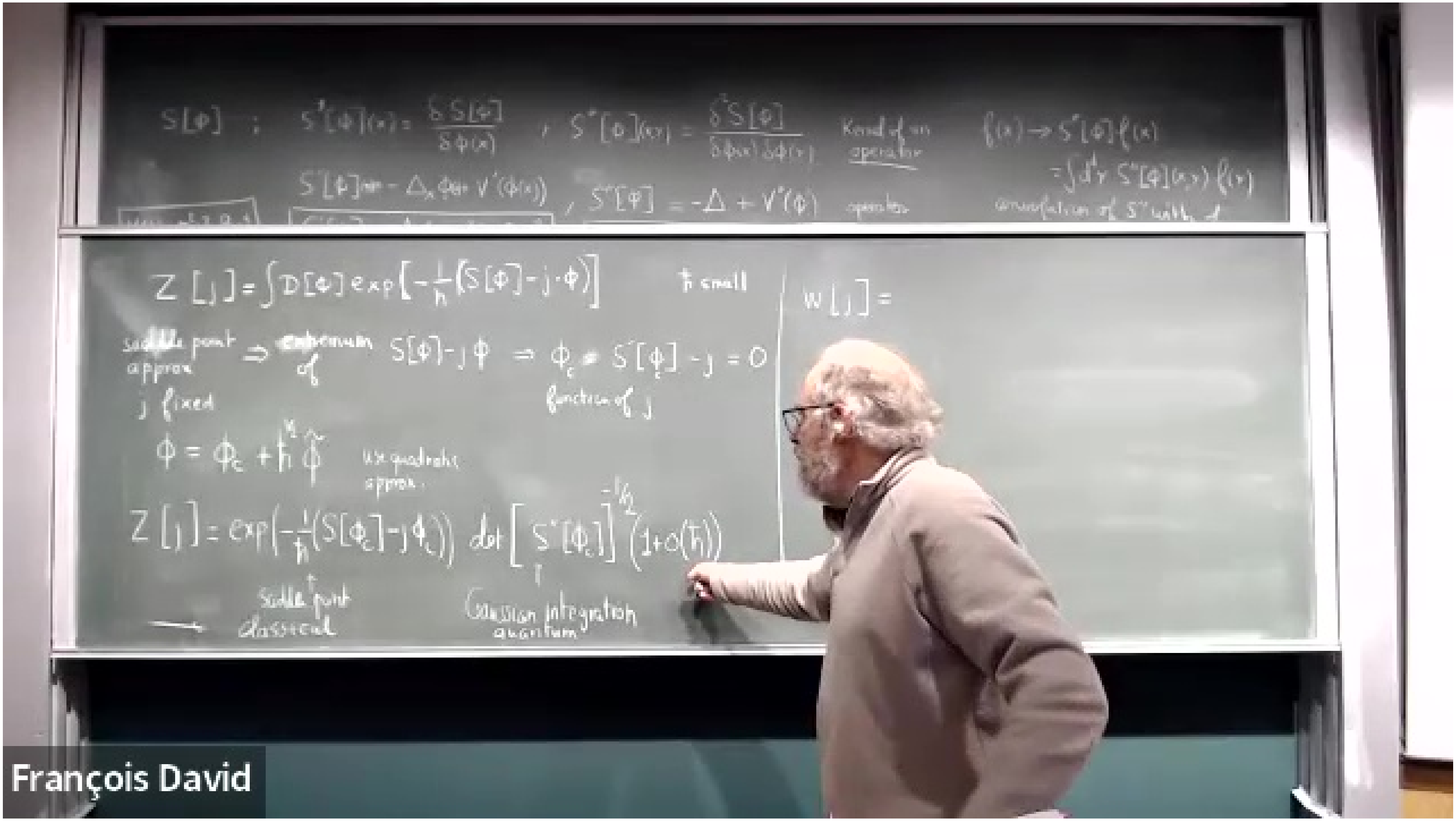
S[φ] ; S'[φ](x) = δS[φ]/δφ(x) , S''[φ](x,y) = δ²S[φ]/δφ(x)δφ(y)
Kernel of an operator
operator
Z[j] = ∫D[φ] exp[-1/ħ (S[φ] - j·φ)]
ħ small
W[j] =
saddle point approx
j fixed
function of j
use quadratic approx.
Z[j] = exp(-1/ħ (S[φ_c] - jφ_c)) det[S''[φ_c]]^{-1/2} (1+O(ħ))
saddle point
classical
Gaussian integration
quantum
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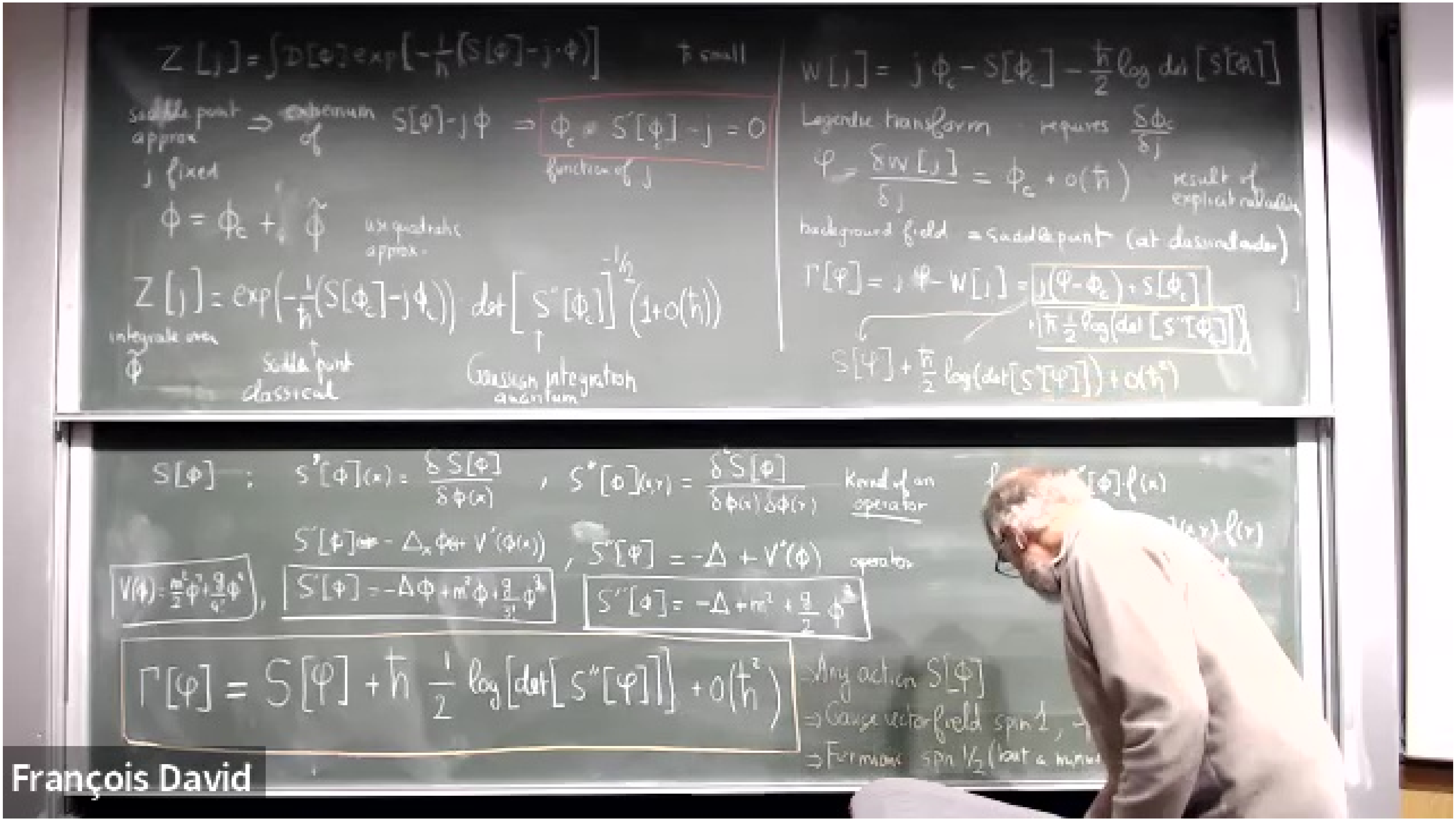
Z[j] = ∫D[φ] exp[-1/ħ (S[φ] - j·φ)]
ħ small
saddle point approx
j fixed
extremum of S[φ] - jφ ⇒ φ_c : S'[φ_c] - j = 0
function of j
φ = φ_c + φ̃
use quadratic approx.
Z[j] = exp(-1/ħ (S[φ_c] - jφ_c)) det[S''[φ_c]]^{-1/2} (1 + O(ħ))
integrate over φ̃
saddle point classical
Gaussian integration quantum
W[j] = jφ_c - S[φ_c] - ħ/2 log det[S''[φ_c]]
Legendre transform
requires δφ_c/δj
φ = δW[j]/δj = φ_c + O(ħ)
result of explicit calculation
background field = saddle point (at classical order)
Γ[φ] = jφ - W[j] = j(φ - φ_c) + S[φ_c] + ħ 1/2 log(det[S''[φ_c]])
S[φ] + ħ/2 log(det[S''[φ]]) + O(ħ²)
S[φ] : S'[φ](x) = δS[φ]/δφ(x) , S''[φ](x,y) = δ²S[φ]/δφ(x)δφ(y)
Kernel of an operator
S'[φ] = -Δ_x φ(x) + V'(φ(x)) , S''[φ] = -Δ + V''(φ)
operator
V(φ) = m²/2 φ² + g/4! φ⁴ , S'[φ] = -Δφ + m²φ + g/3! φ³
S''[φ] = -Δ + m² + g/2 φ²
Γ[φ] = S[φ] + ħ 1/2 log[det[S''[φ]]] + O(ħ²)
⇒ Any action S[φ]
⇒ Gauge vector field spin 1
⇒ Fermions spin 1/2 (but a minus
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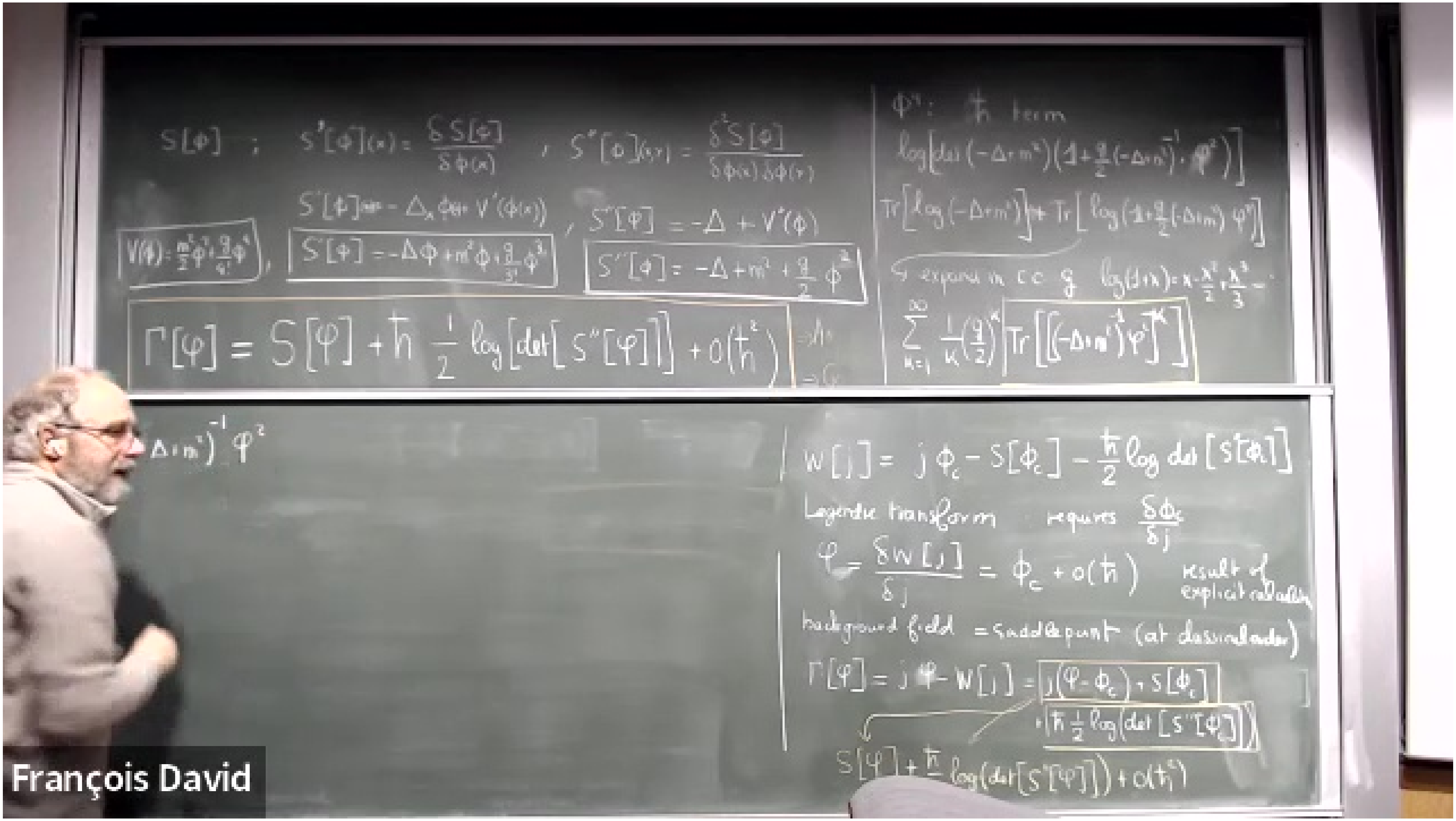

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S[ϕ]
Γ[φ] = S[φ] + ħ 1/2 log[det[S''[φ]]] + o(ħ²)
ϕ⁴: ħ term
expand in cc g
W[j] = j ϕc − S[ϕc] − ħ/2 log det[S''[ϕc]]
Legendre transform requires δϕc/δj
φ = δW[j]/δj = ϕc + o(ħ)
result of explicit calculation
background field = saddle point (at classical order)
Γ[φ] = j φ − W[j] = j(φ − ϕc) + S[ϕc]
S[φ] + ħ/2 log(det[S''[φ]]) + o(ħ²)