Title: QFT III 2021/2022

Speakers: Francois David

Collection: QFT III 2021/2022

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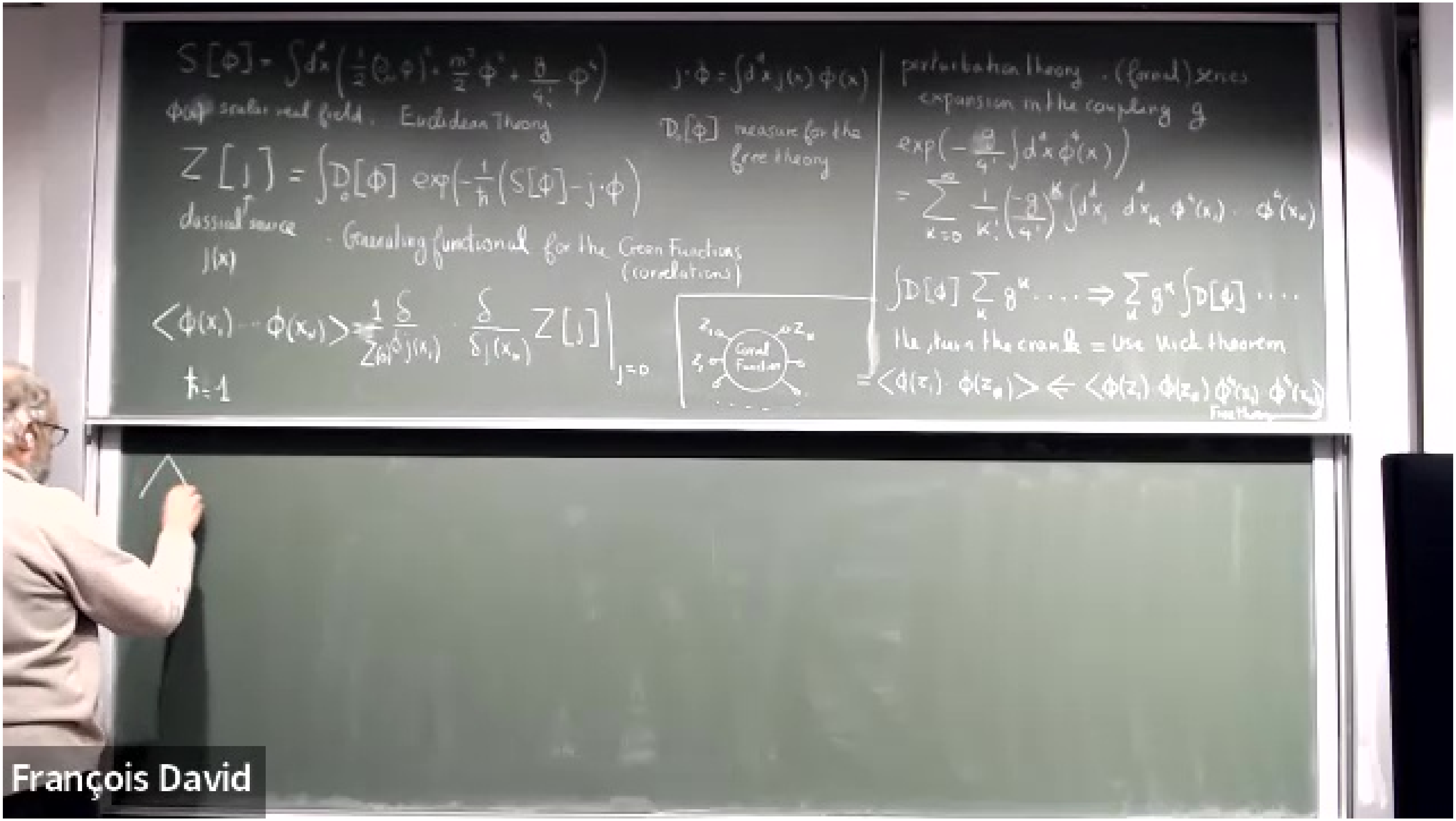

$$S[\phi] = \int d^dx \left(\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right)$$

$\phi(x)$ scalar real field. Euclidean Theory

$$j \cdot \phi = \int d^dx \, j(x) \phi(x)$$

$$Z[j] = \int D_0[\phi] \exp\left(-\frac{1}{\hbar}(S[\phi] - j\cdot\phi)\right)$$

$D_0[\phi]$ measure for the free theory

classical source $j(x)$

· Generating functional for the Green Functions (correlations)

$$\langle \phi(x_1) \cdots \phi(x_N) \rangle = \frac{1}{Z[0]} \frac{\delta}{\delta j(x_1)} \cdots \frac{\delta}{\delta j(x_N)} Z[j] \Big|_{j=0}$$

$\hbar = 1$

perturbation theory. (formal) series expansion in the coupling g

$$\exp\left(-\frac{g}{4!} \int d^dx \, \phi^4(x)\right)$$

$$= \sum_{K=0}^{\infty} \frac{1}{K!} \left(\frac{-g}{4!}\right)^K \int d^dx_1 \cdots d^dx_K \, \phi^4(x_1) \cdots \phi^4(x_K)$$

$$\int D[\phi] \sum_K g^K \cdots \Rightarrow \sum_K g^K \int D[\phi] \cdots$$

then, turn the crank = use Wick theorem

$$= \langle \phi(z_1) \cdots \phi(z_N) \rangle \Leftarrow \langle \phi(z_1) \cdots \phi(z_N) \, \phi^4(x_1) \cdots \phi^4(x_K) \rangle_{\text{Free theory}}$$

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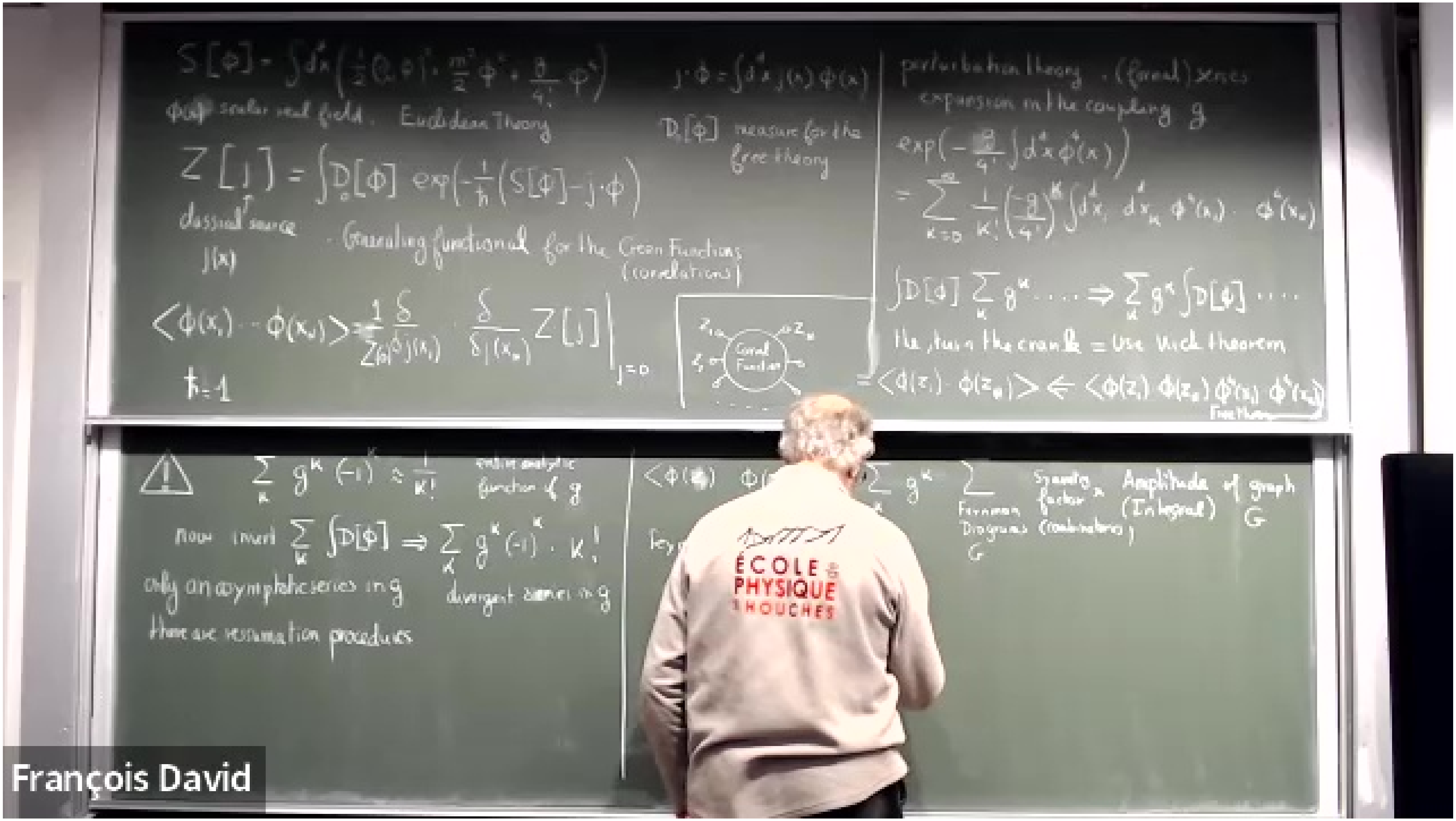

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z_1, z_N, Correl Function

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Free theory

$$\sum_K g^K (-1)^K \simeq \frac{1}{K!}$$

entire analytic function of g

$$\text{now invert } \sum_K \int D[\phi] \Rightarrow \sum_K g^K (-1)^K \cdot K!$$

only an asymptotic series in g

divergent series in g

there are resummation procedures

$$\langle \phi(z_1) \cdots \phi(\ldots \sum_K g^K \sum_{\substack{\text{Feynman}\\ \text{Diagrams } G}} \text{Symmetry factor (combinatorics)} \times \text{Amplitude of graph } G \text{ (Integral)}$$

Feyn

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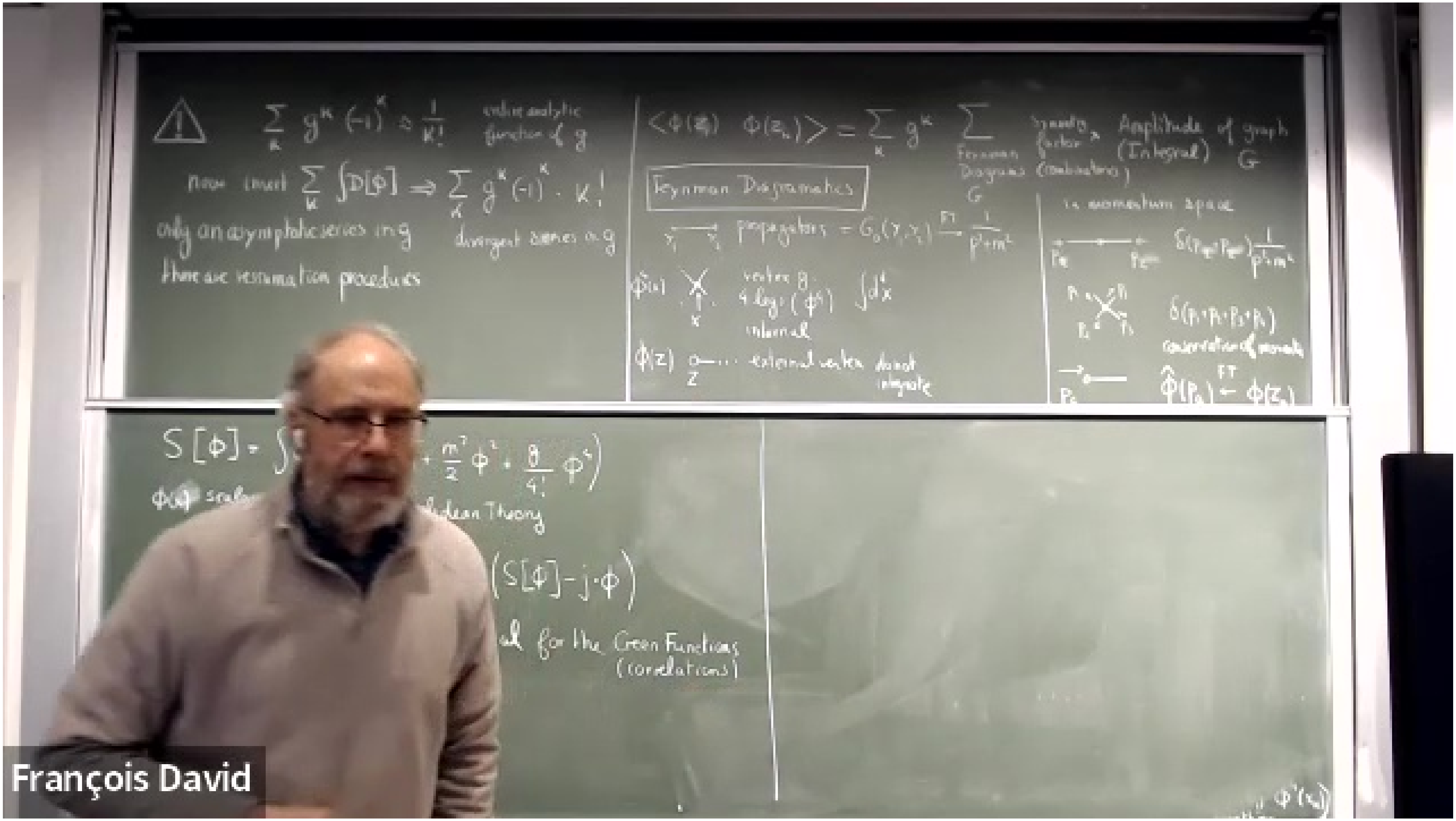


⚠ $\sum_k g^k (-1)^k \frac{1}{k!}$ entire analytic function of g

now insert $\sum_k \int D[\phi] \Rightarrow \sum_k g^k (-1)^k \cdot k!$

only an asymptotic series in g — divergent series in g

there are resummation procedures

$\langle \phi(z_1) \; \phi(z_2) \rangle = \sum_k g^k \sum_{\substack{\text{Feynman Diagrams} \\ G}}$ symmetry factor (combinatorics) × Amplitude of graph (Integral) G

Feynman Diagrammatics

$x_1 \longrightarrow x_2$ propagators $= G_0(x_1,x_2) \xrightarrow{FT} \frac{1}{p^2+m^2}$

$\phi(x)$ vertex g. 4 legs (ϕ^4) $\int d^dx$ internal

$\phi(z)$ external vertex do not integrate

in momentum space

$\delta(p_{in}+p_{out})\frac{1}{p^2+m^2}$

$\delta(p_1+p_2+p_3+p_4)$ conservation of momenta

$\hat{\phi}(p_a) \xleftarrow{FT} \phi(z_a)$

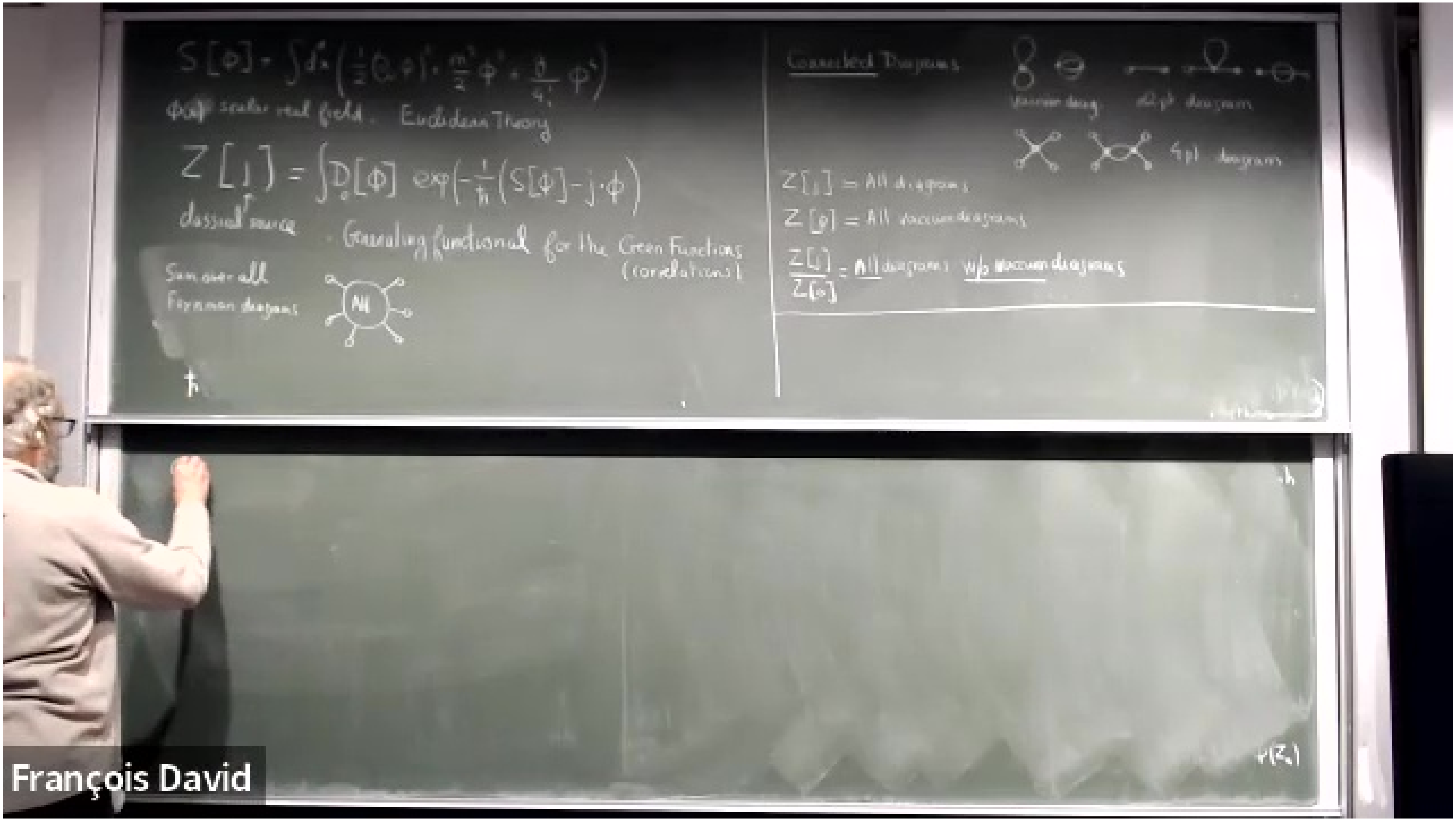
$S[\phi] = \int \ldots + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4)$

$\phi(x)$ scalar … clidean Theory

$(S[\phi] - j\cdot\phi)$

…al for the Green Functions (correlations)

$\phi^4(x_i)$



$$S[\phi] = \int d^dx \left(\frac{1}{2}(\partial\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4\right)$$

$\phi(x)$ scalar real field. Euclidean Theory

$$Z[j] = \int \mathcal{D}[\phi] \exp\left(-\frac{1}{\hbar}\left(S[\phi] - j\cdot\phi\right)\right)$$

classical source

Generating functional for the Green Functions (correlations)

Sum over all Feynman diagrams

$\hbar$

Connected Diagrams

vacuum diag.

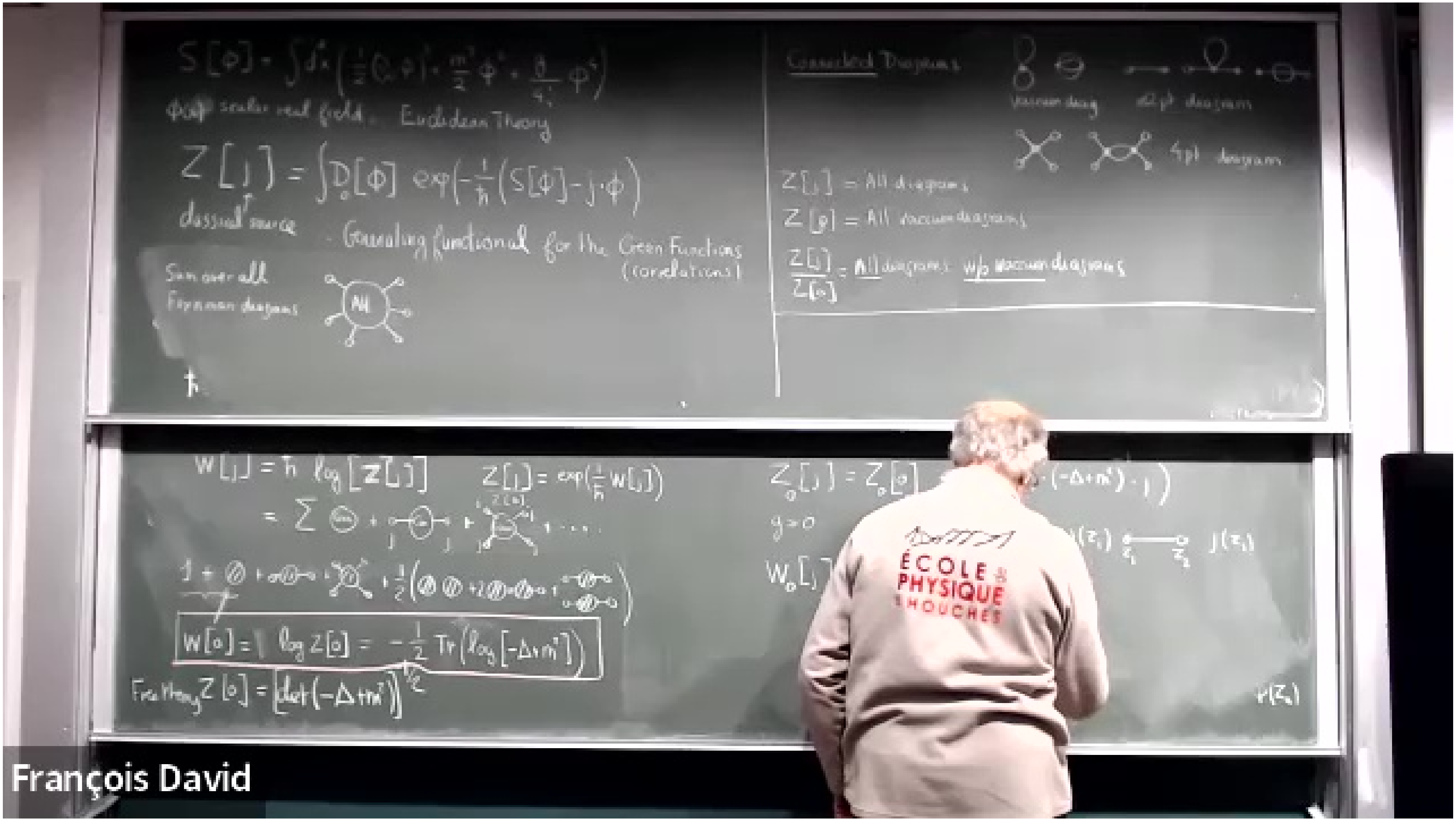
2 pt diagrams

4 pt diagrams

$Z[j]$ = All diagrams

$Z[0]$ = All vacuum diagrams

$\frac{Z[j]}{Z[0]}$ = All diagrams w/o vacuum diagrams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Sum over all Feynman diagrams
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$\hbar$
Connected Diagrams
vacuum diag
2 pt diagrams
4 pt diagram
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$\frac{Z[j]}{Z[0]}$ = All diagrams w/o vacuum diagrams
$W[j] = \hbar \log[Z[j]]$
$Z[j] = \exp(\frac{1}{\hbar}W[j])$
$= \sum$
$Z[0]$
$1 + \ldots + \frac{1}{2}(\ldots)$
$W[0] = \log Z[0] = -\frac{1}{2}\mathrm{Tr}\left(\log[-\Delta+m^2]\right)$
Free theory $Z[0] = \left[\det(-\Delta+m^2)\right]^{-1/2}$
$Z_0[j] = Z_0[0]$
$(-\Delta+m^2)\cdot j)$
$g=0$
$W_0[j]$
$j(z_1)$
z_1
z_2
$j(z_2)$
$j(z_1)$
ÉCOLE de PHYSIQUE LES HOUCHES
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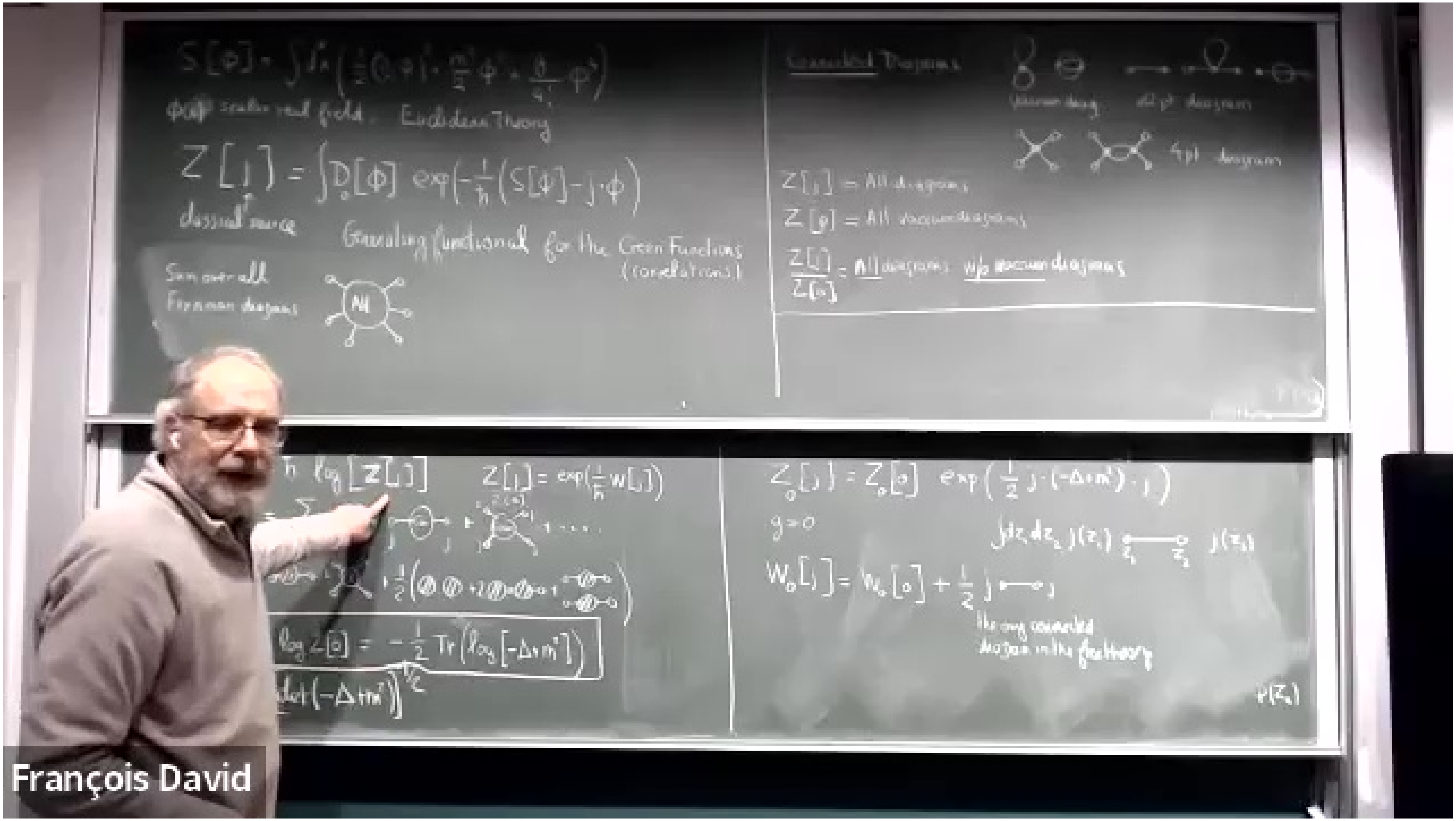

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vacuum diag

2pt diagrams

4pt diagrams

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$\frac{Z[j]}{Z[0]}$ = All diagrams w/o vacuum diagrams

$\hbar \log[Z[j]]$

$Z[j] = \exp\left(\frac{1}{\hbar}W[j]\right)$

$+ \frac{1}{2}\left(\ldots \right)$

$$\boxed{\log Z[0] = -\frac{1}{2}\mathrm{Tr}\left(\log[-\Delta+m^2]\right)}$$

$\det(-\Delta+m^2)^{-1/2}$

$Z_0[j] = Z_0[0] \exp\left(\frac{1}{2} j\cdot(-\Delta+m^2)^{-1}\cdot j\right)$

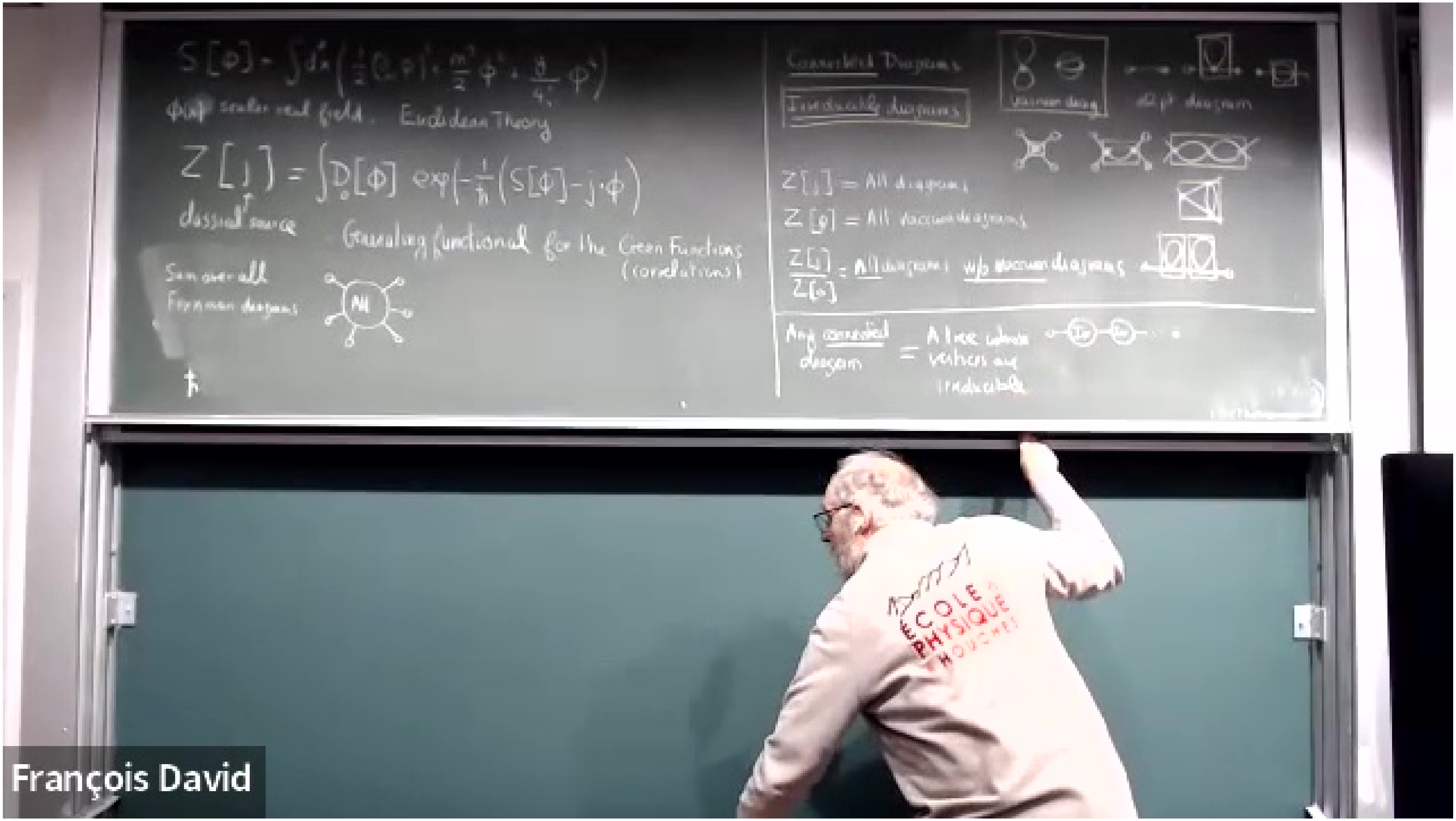
$g=0$

$\int dz_1 dz_2\, j(z_1)$ z_1 z_2 $j(z_2)$

$W_0[j] = W_0[0] + \frac{1}{2} j$ j

the only connected diagram in the free theory

$\phi(z_1)$



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$$Z[j] = \int \mathcal{D}[\phi] \exp\left(-\frac{1}{\hbar}\left(S[\phi] - j\cdot\phi\right)\right)$$

j classical source

Generating functional for the Green Functions (correlations)

Sum over all Feynman diagrams

$\hbar$

Connected Diagrams

Irreducible diagrams

vacuum diag

$g\,\phi^4$ diagram

$Z[j]$ = All diagrams

$Z[0]$ = All vacuum diagrams

$\frac{Z[j]}{Z[0]}$ = All diagrams w/o vacuum diagrams

Any connected diagram = A tree whose vertices are irreducible

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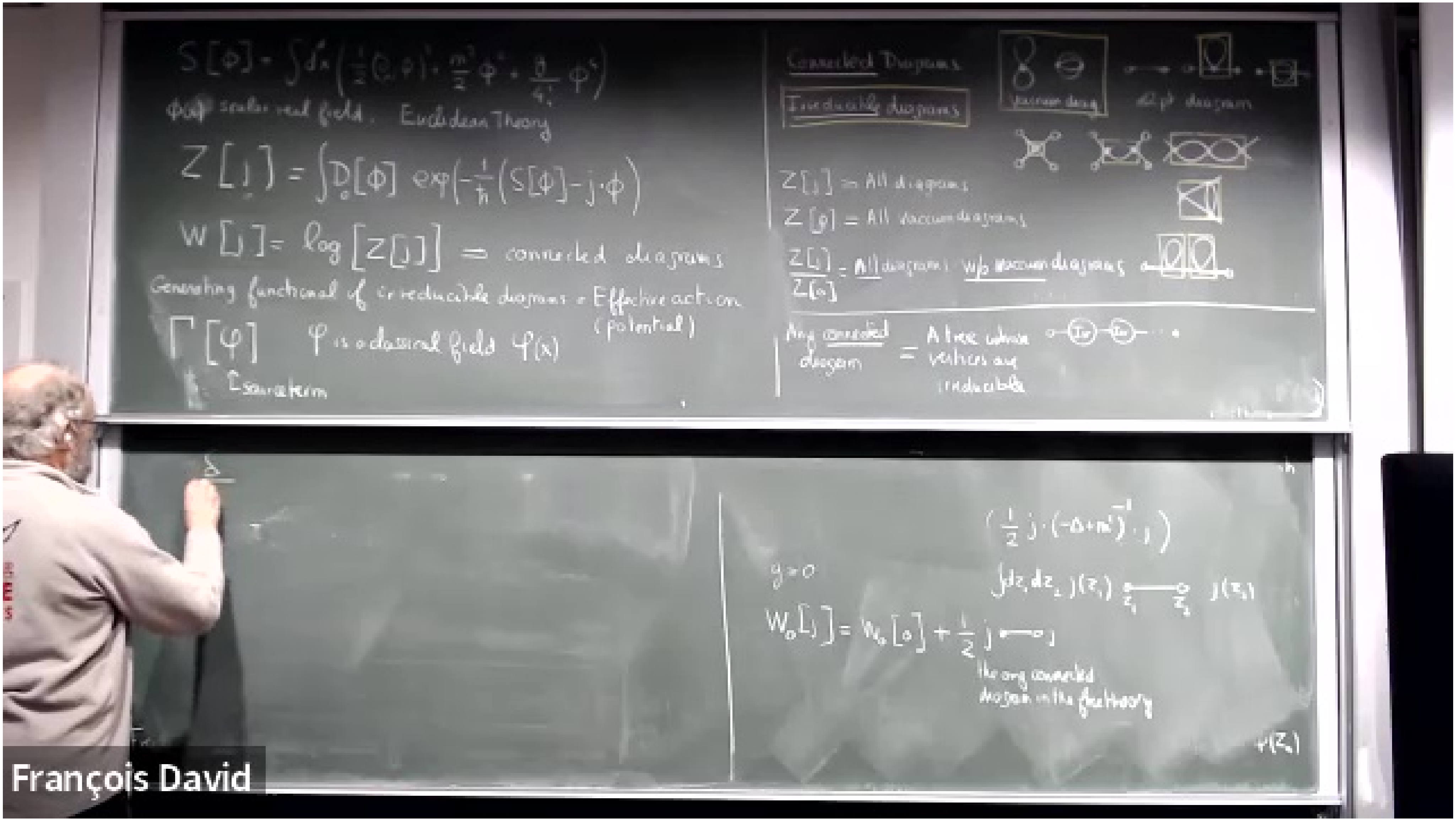

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$W[j] = \log\left[Z[j]\right] \Rightarrow$ connected diagrams

Generating functional of irreducible diagrams = Effective action (potential)

$\Gamma[\varphi]$ φ is a classical field $\varphi(x)$

source term

Connected Diagrams

Irreducible diagrams

vacuum diag

1PI diagram

$Z[j]$ = All diagrams

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Any connected diagram = A tree whose vertices are irreducible

$(\frac{1}{2} j\cdot(-\Delta+m^2)^{-1}\cdot j)$

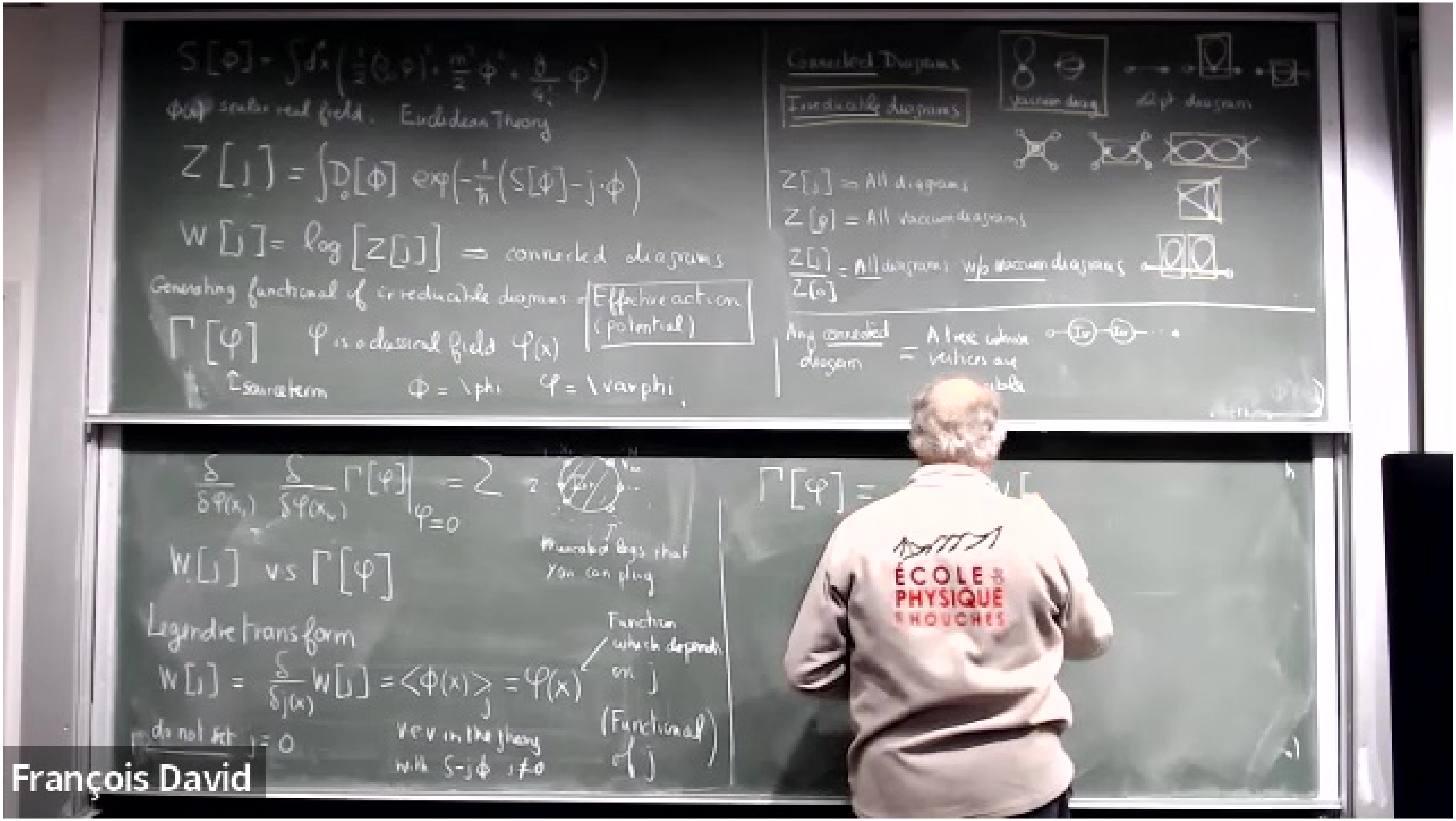
$g=0$

$\int dz_1 dz_2\, j(z_1)$ z_1 — z_2 $j(z_2)$

$W_0[j] = W_0[0] + \frac{1}{2} j$ •—○ j

the only connected diagram in the free theory

$\varphi(z_1)$



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$$Z[j] = \int \mathcal{D}[\Phi] \exp\left(-\frac{1}{\hbar}\left(S[\Phi] - j\cdot\Phi\right)\right)$$

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Generating functional of irreducible diagrams or $\boxed{\text{Effective action (potential)}}$

$\Gamma[\varphi]$ $\quad\varphi$ is a classical field $\varphi(x)$

↑ source term

Φ = \phi $\quad\varphi$ = \varphi

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vacuum diag

1PI diagrams

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Any connected diagram = A tree whose vertices are irreducible

$$\frac{\delta}{\delta\varphi(x_1)} \cdots \frac{\delta}{\delta\varphi(x_N)} \Gamma[\varphi]\Big|_{\varphi=0} = \sum$$

Amputated legs that you can plug

$W[j]$ vs $\Gamma[\varphi]$

Legendre transform

$$W[j] = \quad \frac{\delta}{\delta j(x)} W[j] = \langle\Phi(x)\rangle_j = \varphi(x)$$

Function which depends on j

(Functional of j)

do not set $j=0$

vev in the theory with $S - j\Phi$, $j \neq 0$

$\Gamma[\varphi] =$

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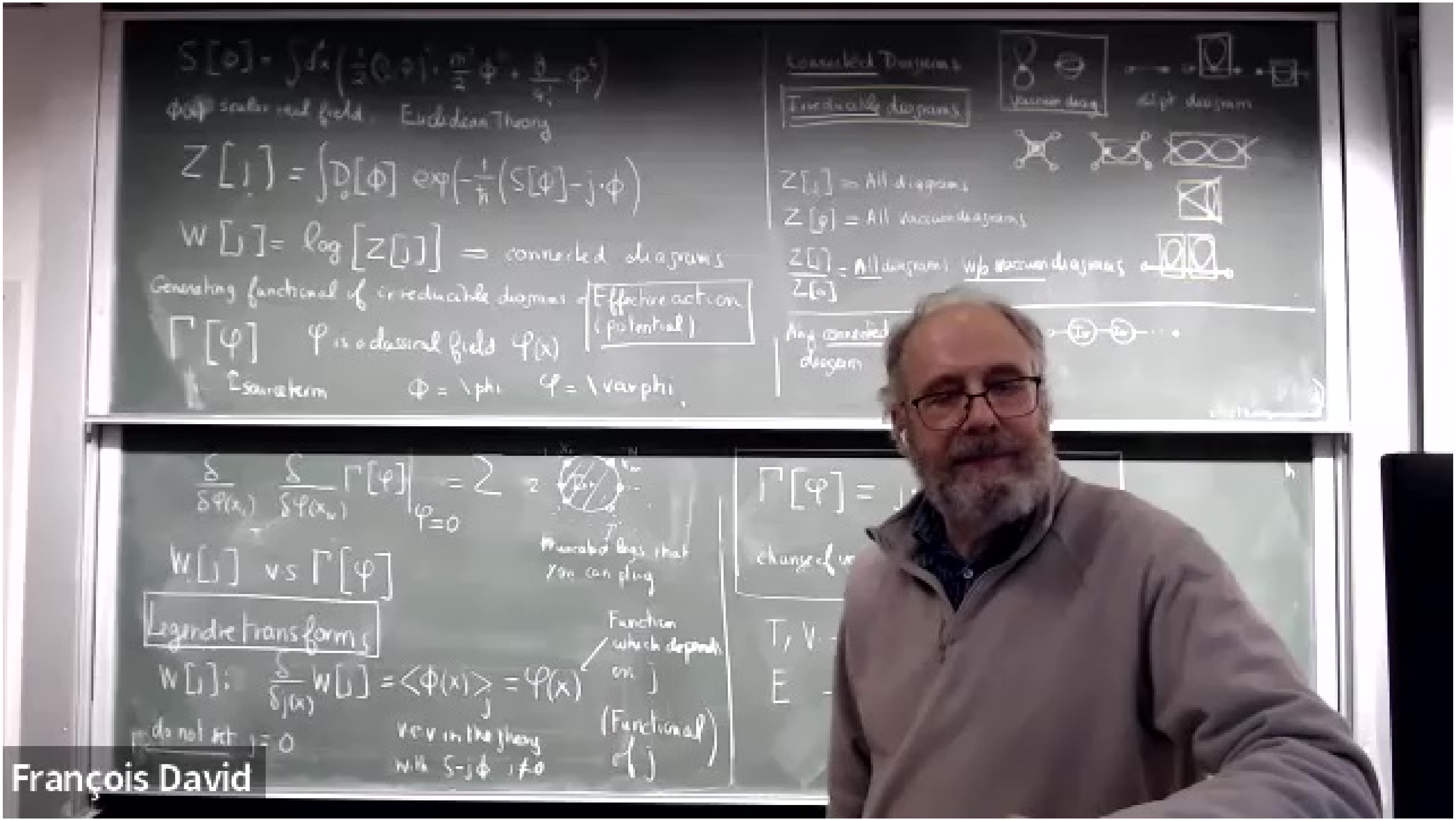
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Generating functional of irreducible diagrams of Effective action (potential)
$\Gamma[\varphi]$ φ is a classical field $\varphi(x)$
source term
Φ = \phi φ = \varphi
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Irreducible diagrams
vacuum diag
1PI diagrams
$Z[j]$ = All diagrams
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Any connected diagram
$\left.\frac{\delta}{\delta\varphi(x_1)} \cdots \frac{\delta}{\delta\varphi(x_N)}\Gamma[\varphi]\right|_{\varphi=0} = \sum$
Truncated legs that you can plug
$W[j]$ vs $\Gamma[\varphi]$
Legendre transforms
$W[j]$: $\frac{\delta}{\delta j(x)} W[j] = \langle\Phi(x)\rangle_j = \varphi(x)$
Function which depends on j
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do not set j = 0
vev in the theory with $S - j\phi$, $j \neq 0$
$\Gamma[\varphi] =$
change of
T, V
E
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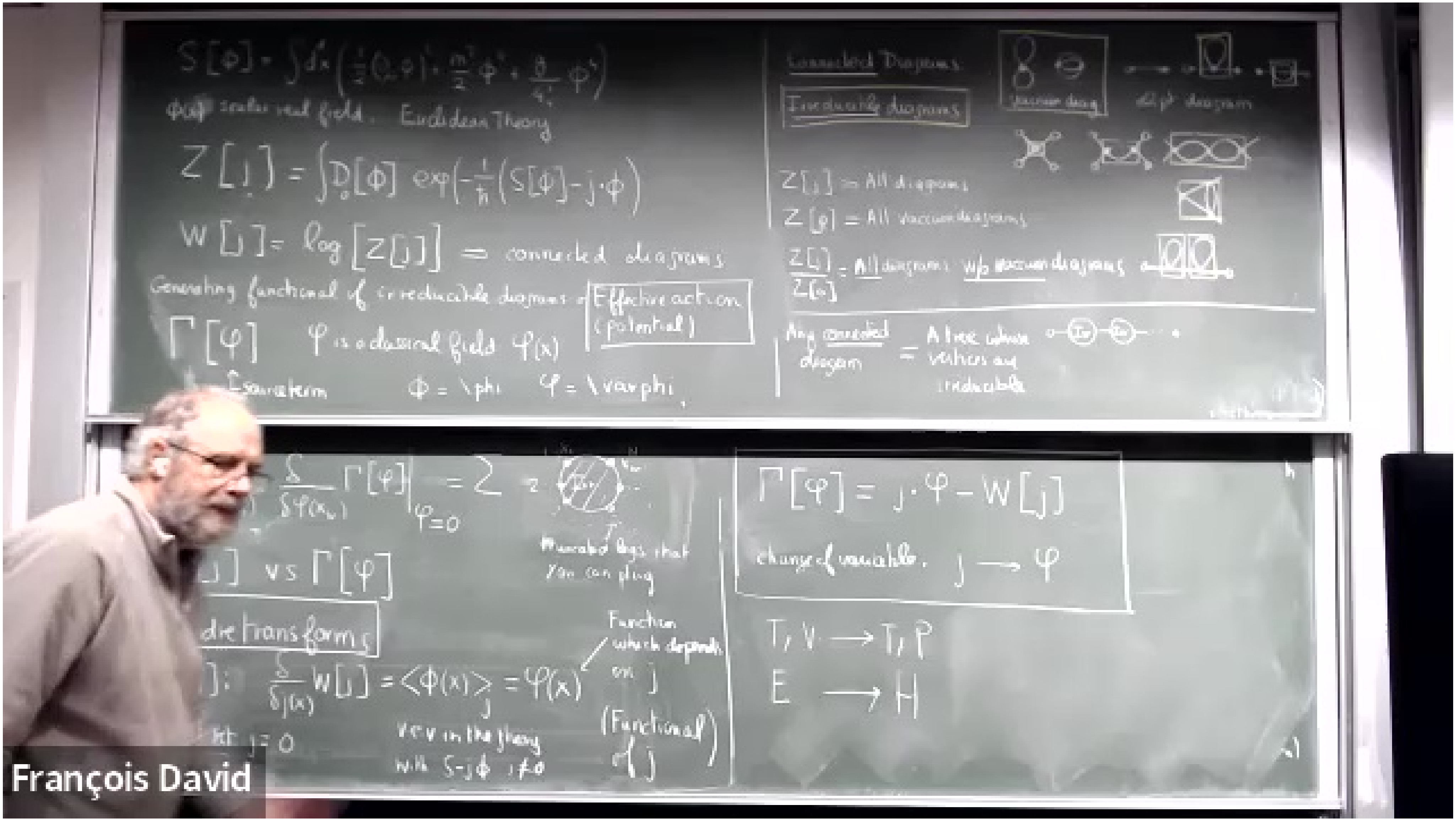

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$W[j] = \log\left[Z[j]\right] \Longrightarrow$ connected diagrams

Generating functional of irreducible diagrams = Effective action (potential)

$\Gamma[\varphi]$ φ is a classical field $\varphi(x)$

source term Φ = \phi φ = \varphi

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Irreducible diagrams

vacuum diag

ϕ^4 diagrams

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$\frac{\delta}{\delta\varphi(x_n)}\Gamma[\varphi]\Big|_{\varphi=0} = \sum$

amputated legs that you can plug

$j] \text{ vs } \Gamma[\varphi]$

dre transforms

$]: \quad \frac{\delta}{\delta j(x)} W[j] = \langle \Phi(x) \rangle_j = \varphi(x)$

Function which depends on j

(Functional of j)

t j = 0

v.e.v in the theory with $S - j\Phi$, $j \neq 0$

$\Gamma[\varphi] = j\cdot\varphi - W[j]$

change of variable. $j \longrightarrow \varphi$

$T, V \longrightarrow T, P$

$E \longrightarrow H$

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