Title: QFT III 2021/2022

Speakers: Francois David

Collection: QFT III 2021/2022

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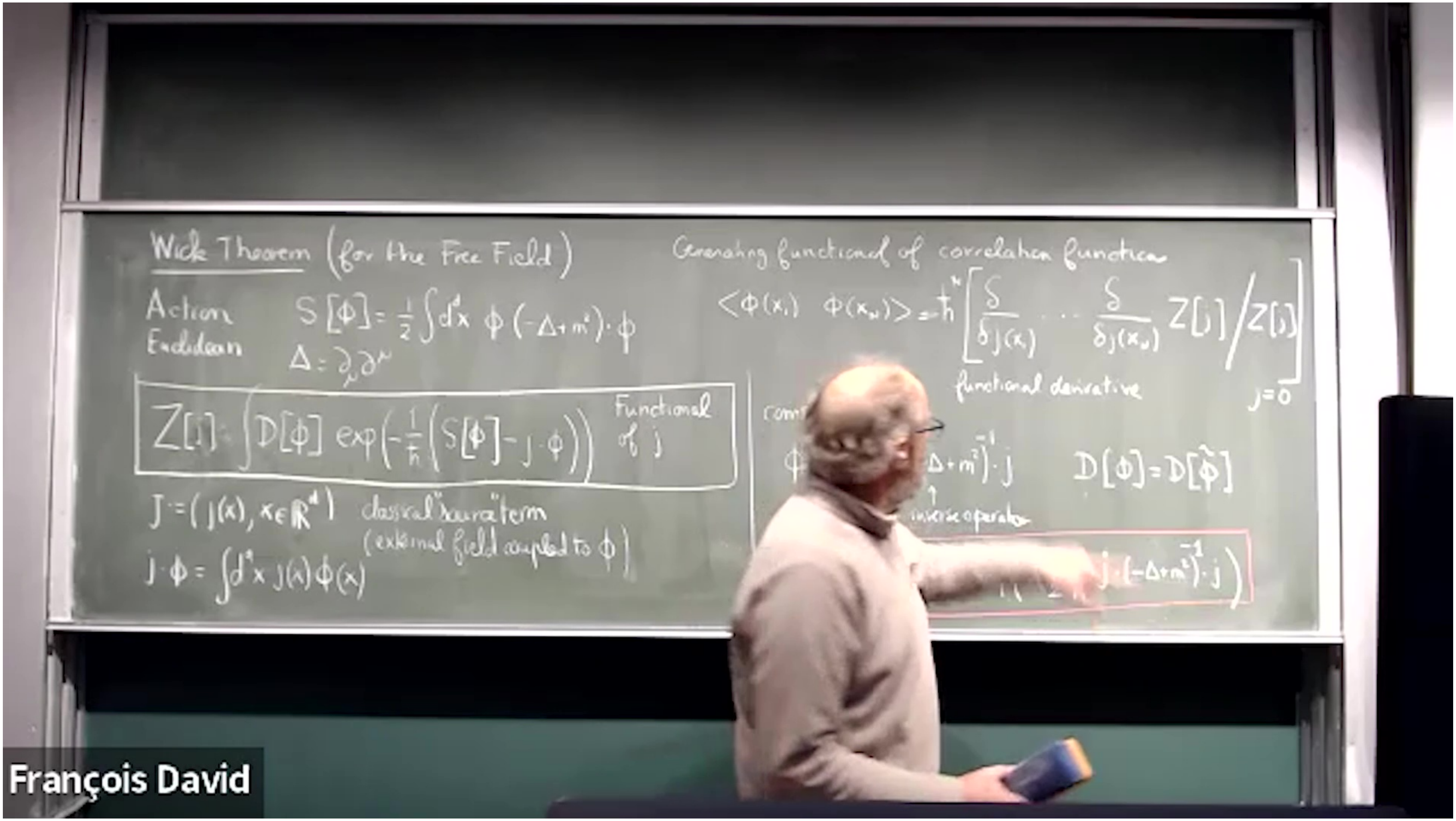
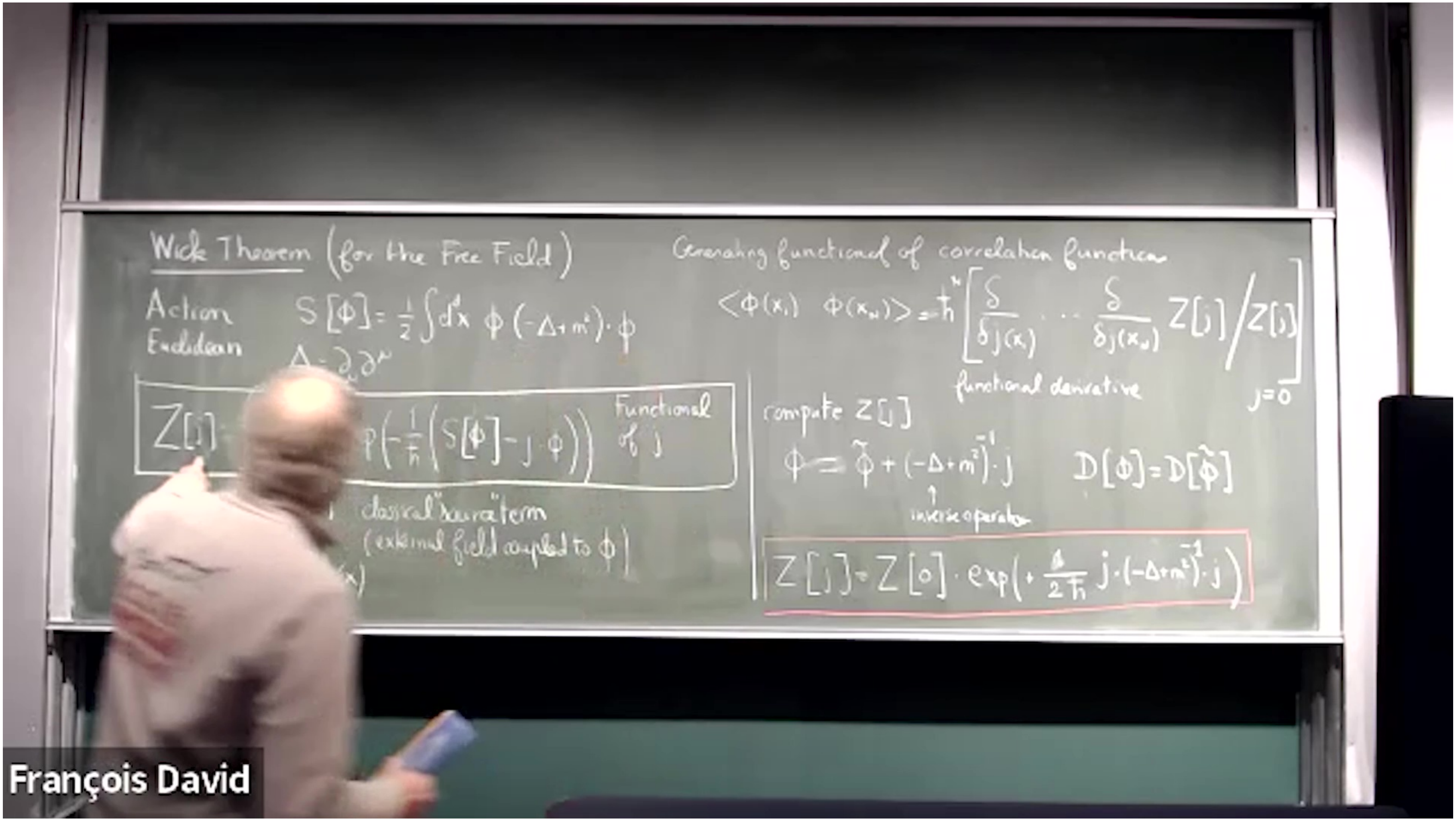

Wick Theorem (for the Free Field)
Action
Euclidean
S[φ] = 1/2 ∫ d^d x φ (-Δ + m²) · φ
Δ = ∂_μ ∂^μ
Z[j] = ∫ D[φ] exp(-1/ħ (S[φ] - j · φ))
Functional of j
j = (j(x), x ∈ ℝ^d) classical "source" term
(external field coupled to φ)
j · φ = ∫ d^d x j(x) φ(x)
Generating functional of correlation functions
<φ(x_1) φ(x_N)> = ħ^N [δ/δj(x_1) … δ/δj(x_N) Z[j] / Z[j]] j=0
functional derivative
D[φ] = D[φ̃]
inverse operator
François David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Δ = ∂_μ ∂^μ
Functional
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classical "source" term
(external field coupled to φ)
Generating functional of correlation functions
<Φ(x_1) Φ(x_N)> = ħ^N [δ/δj(x_1) ... δ/δj(x_N) Z[j] / Z[j]] j=0
functional derivative
compute Z[j]
φ = φ̃ + (-Δ+m²)^{-1} · j
inverse operator
D[Φ] = D[Φ̃]
Z[j] = Z[0] · exp(+ 1/(2ħ) j · (-Δ+m²)^{-1} · j)
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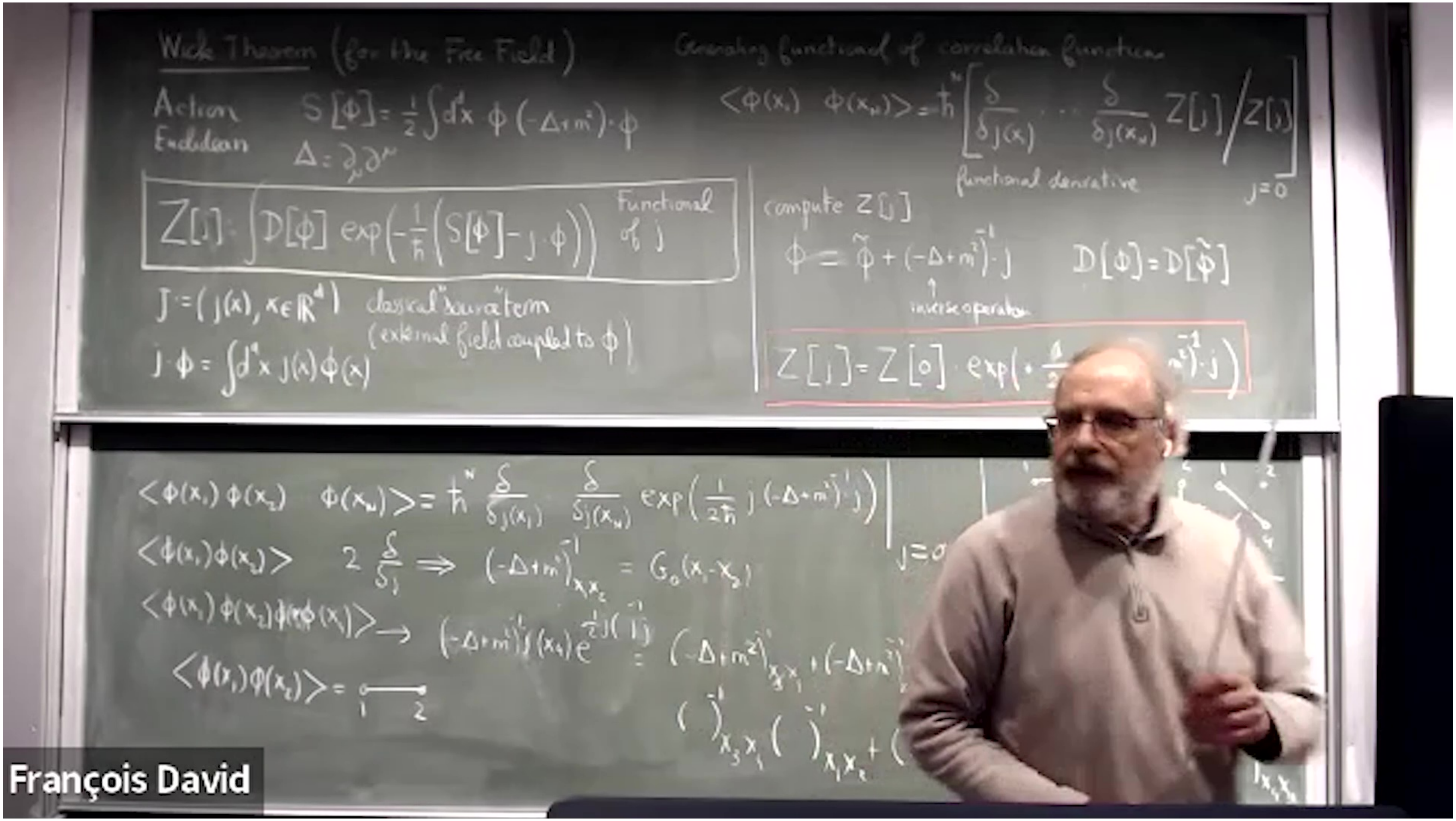
Wick Theorem (for the Free Field)
Generating functional of correlation functions
Action
Euclidean
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classical source term
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functional derivative
compute Z[j]
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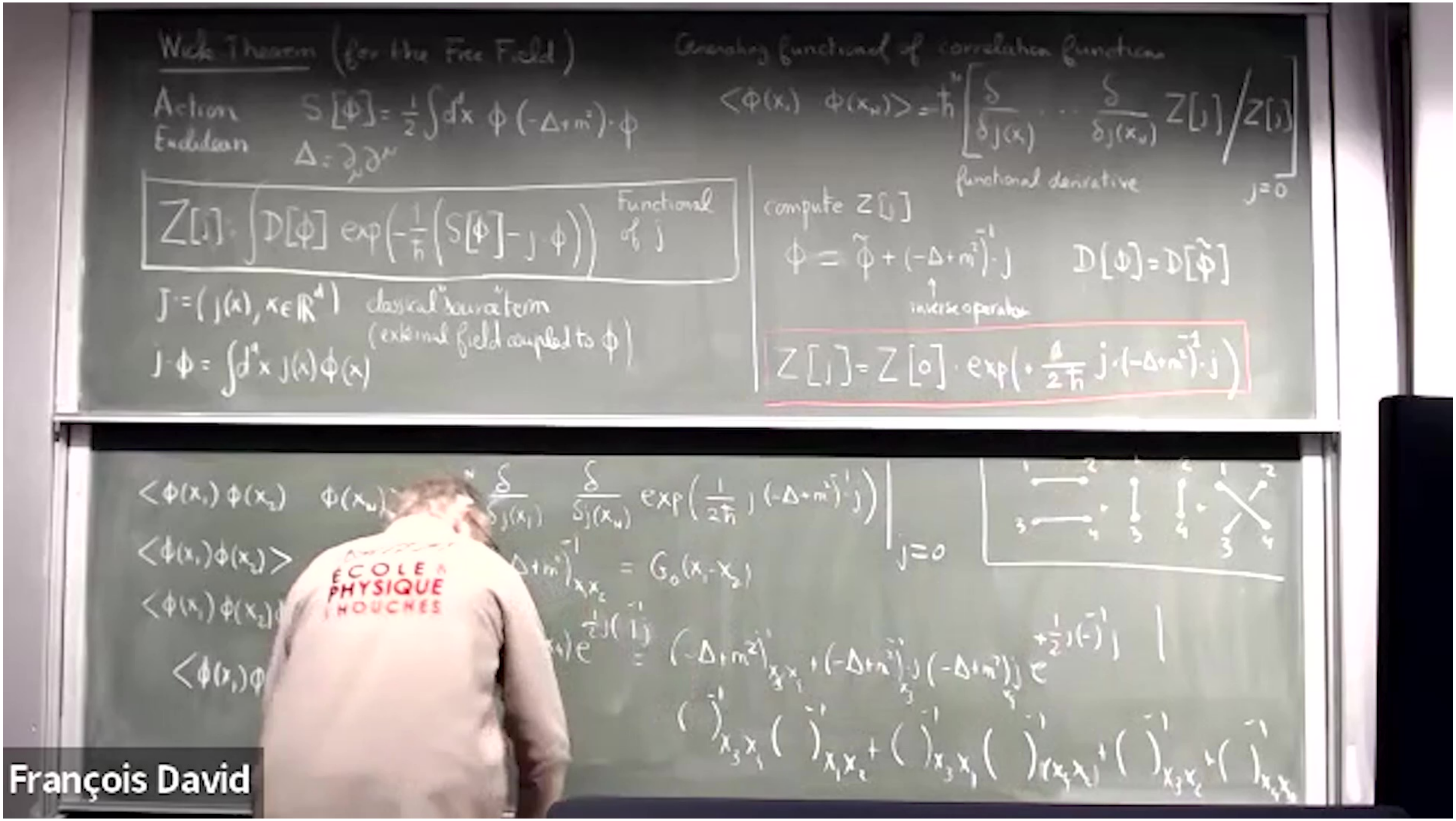
Wick Theorem (for the Free Field)
Action Euclidean
$S[\phi] = \frac{1}{2}\int d^dx\, \phi\,(-\Delta + m^2)\cdot\phi$
$\Delta = \partial_\mu \partial^\mu$
$Z[j] = \int D[\phi]\, \exp\left(-\frac{1}{\hbar}\left(S[\phi] - j\cdot\phi\right)\right)$
Functional of j
$j = (j(x),\, x \in \mathbb{R}^d)$
classical source term
(external field coupled to ϕ)
$j\cdot\phi = \int d^dx\, j(x)\phi(x)$
Generating functional of correlation functions
$\langle \phi(x_1) \dots \phi(x_N)\rangle = \hbar^N \left[\frac{\delta}{\delta j(x_1)} \dots \frac{\delta}{\delta j(x_N)} Z[j] / Z[j]\right]_{j=0}$
functional derivative
compute Z[j]
$\phi = \tilde{\phi} + (-\Delta + m^2)^{-1}\cdot j$
inverse operator
$D[\phi] = D[\tilde{\phi}]$
$Z[j] = Z[0]\cdot \exp\left(+\frac{1}{2\hbar}\, j\cdot(-\Delta + m^2)^{-1}\cdot j\right)$
$G_0(x_1 - x_2)$
$j = 0$
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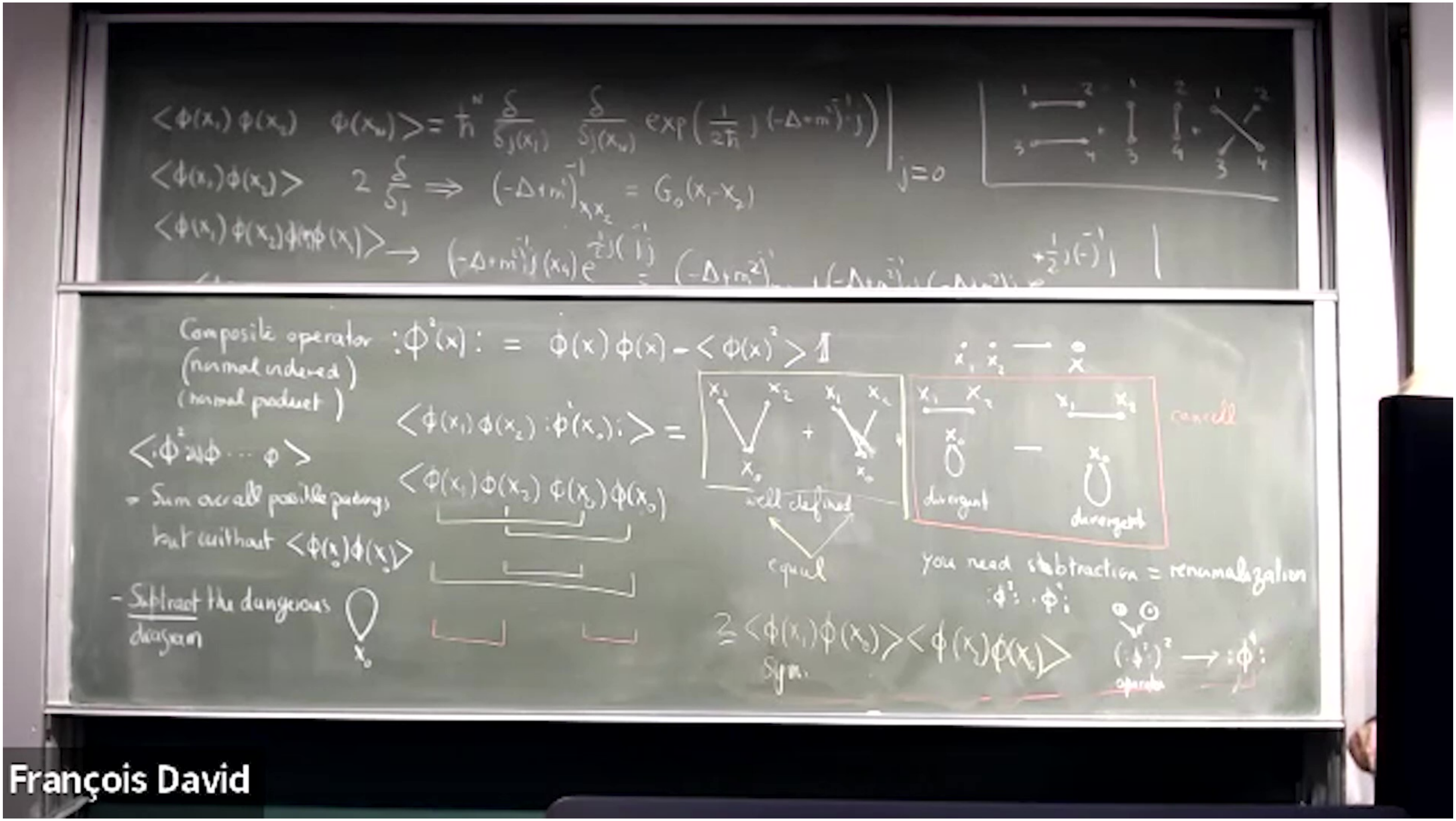
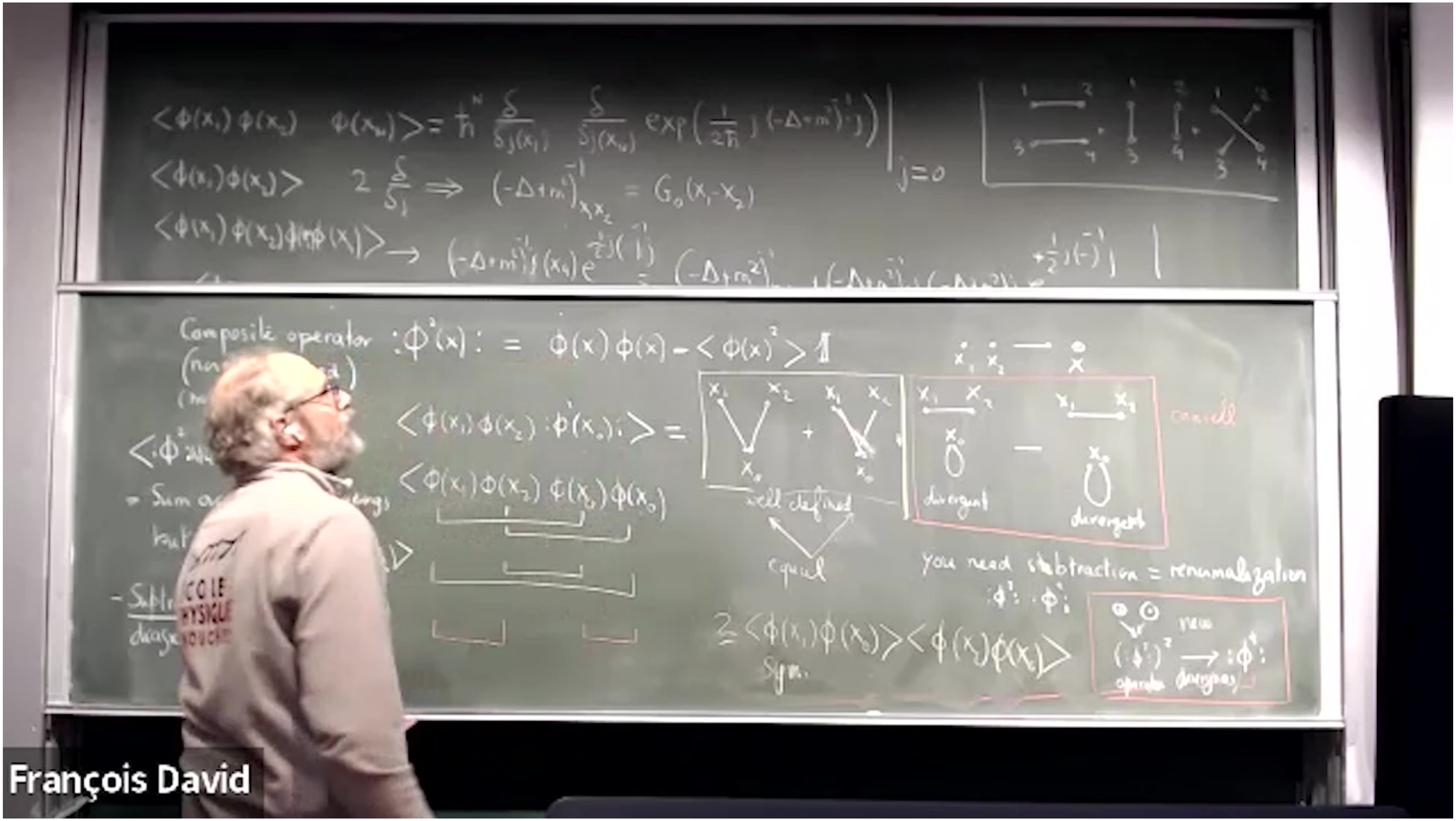

Composite operator
(normal ordered)
(normal product)
well defined
equal
divergent
cancell
you need subtraction = renormalization
Sum over all possible pairings
but without
Subtract the dangerous diagram
Sym.
operator
François David


Composite operator
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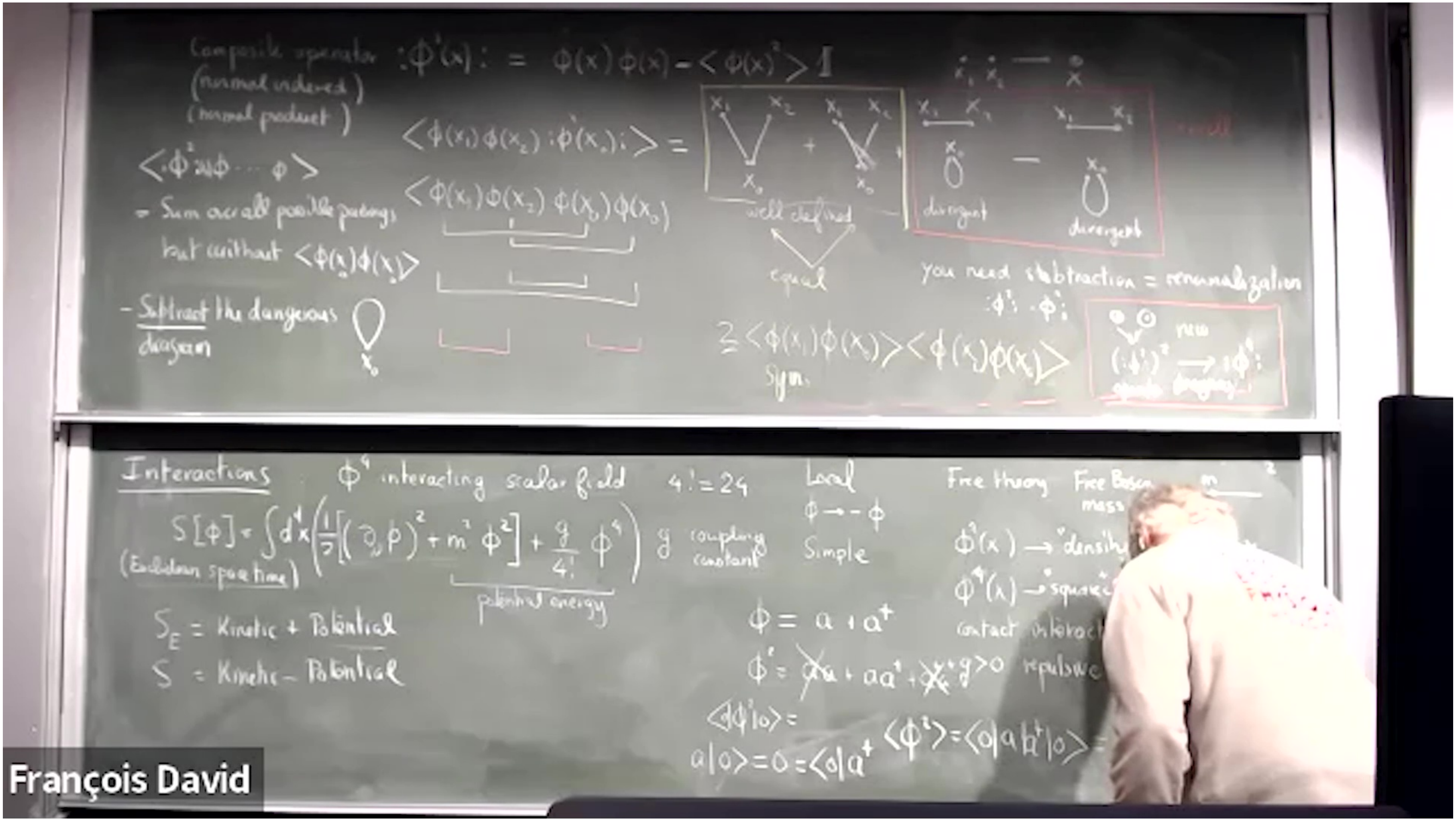
Composite operator
(normal ordered)
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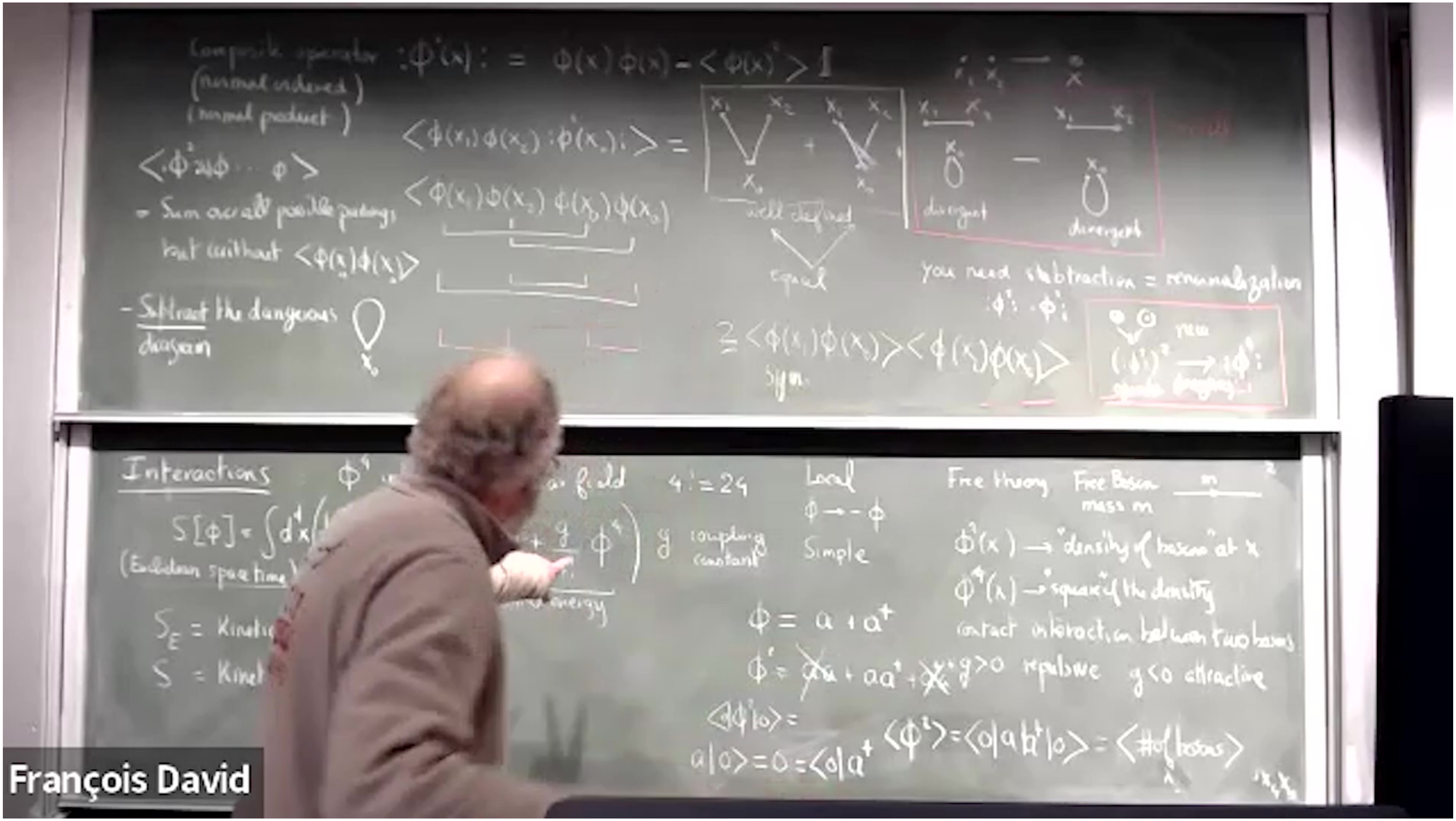
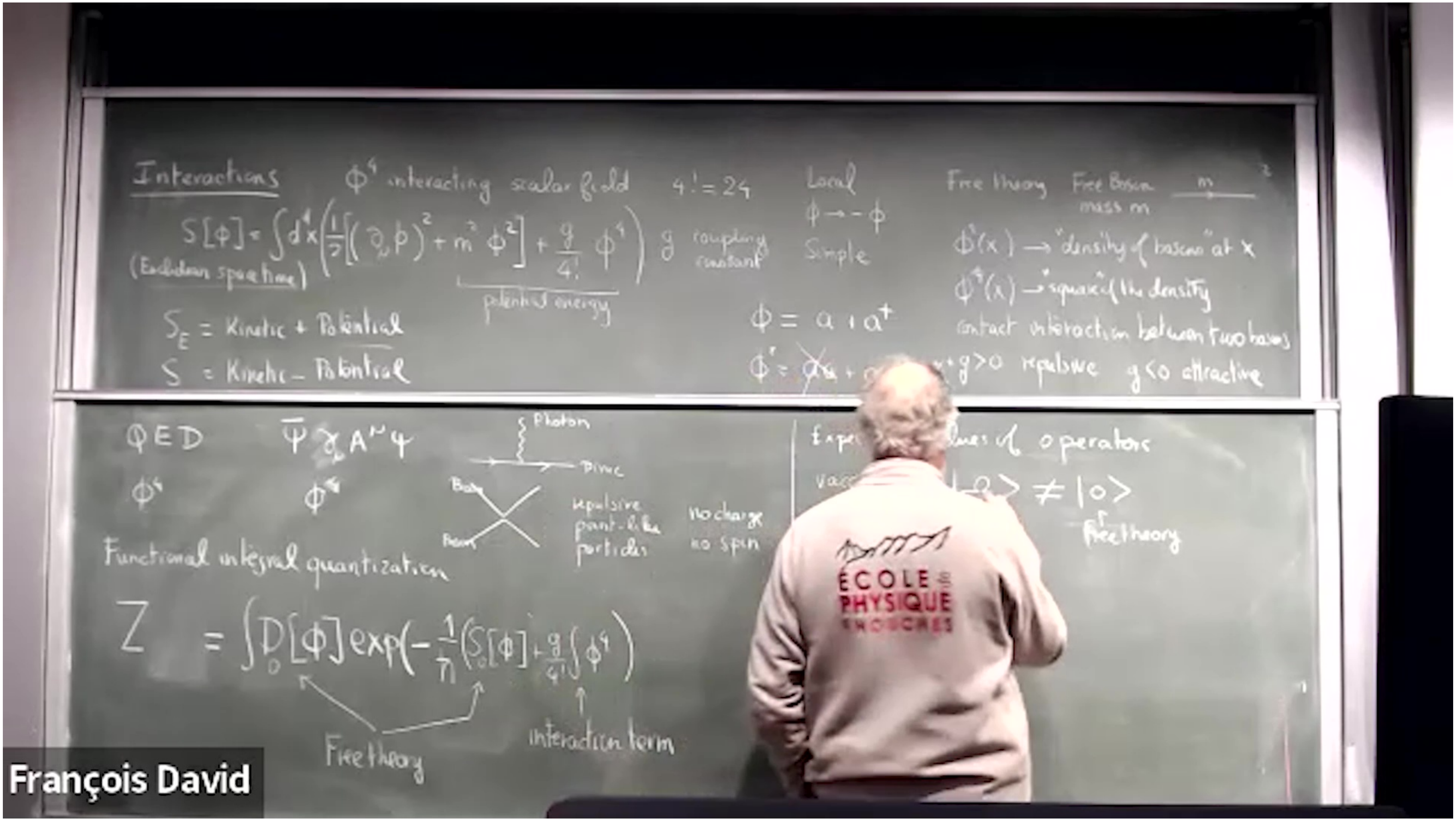
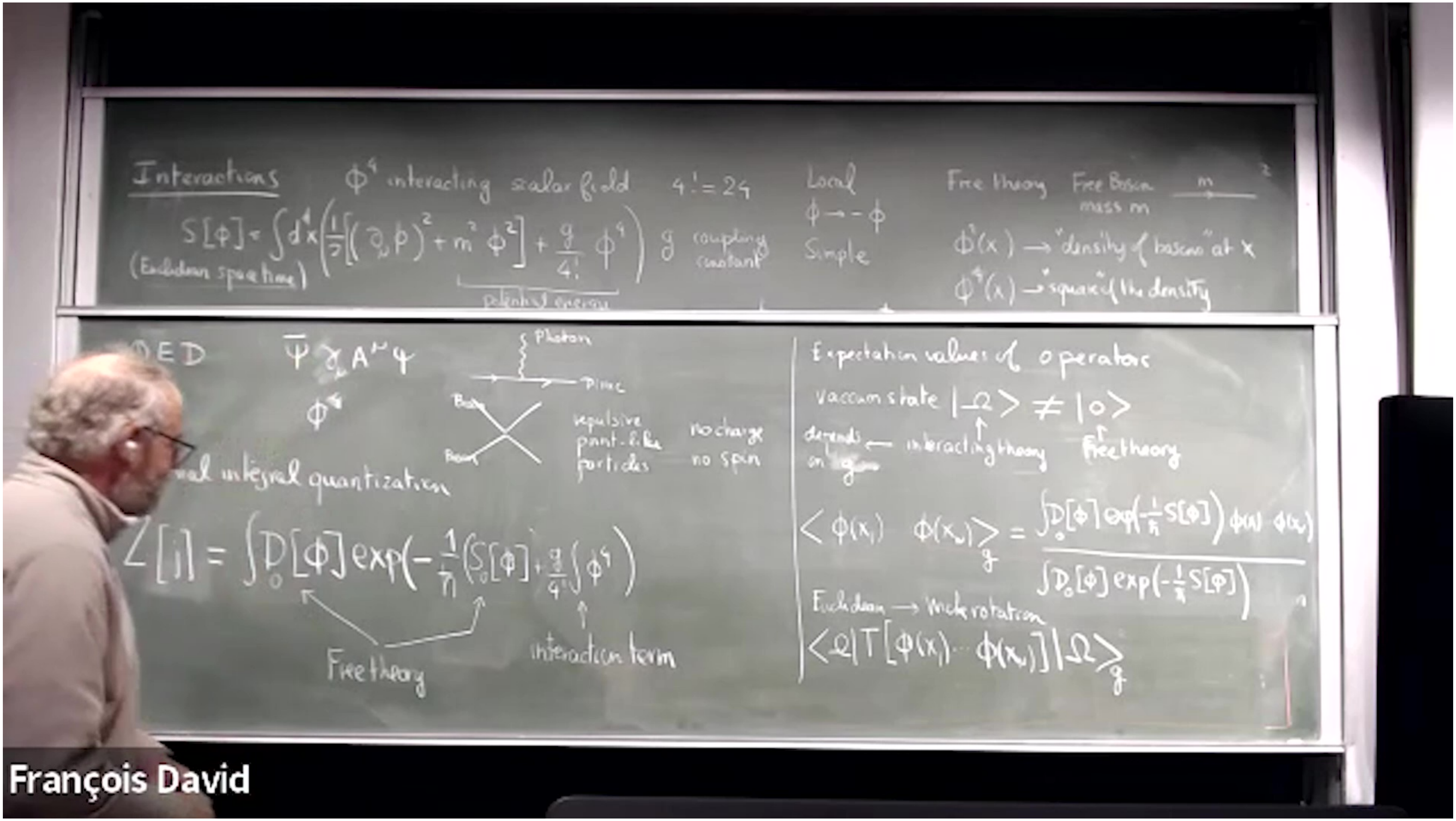
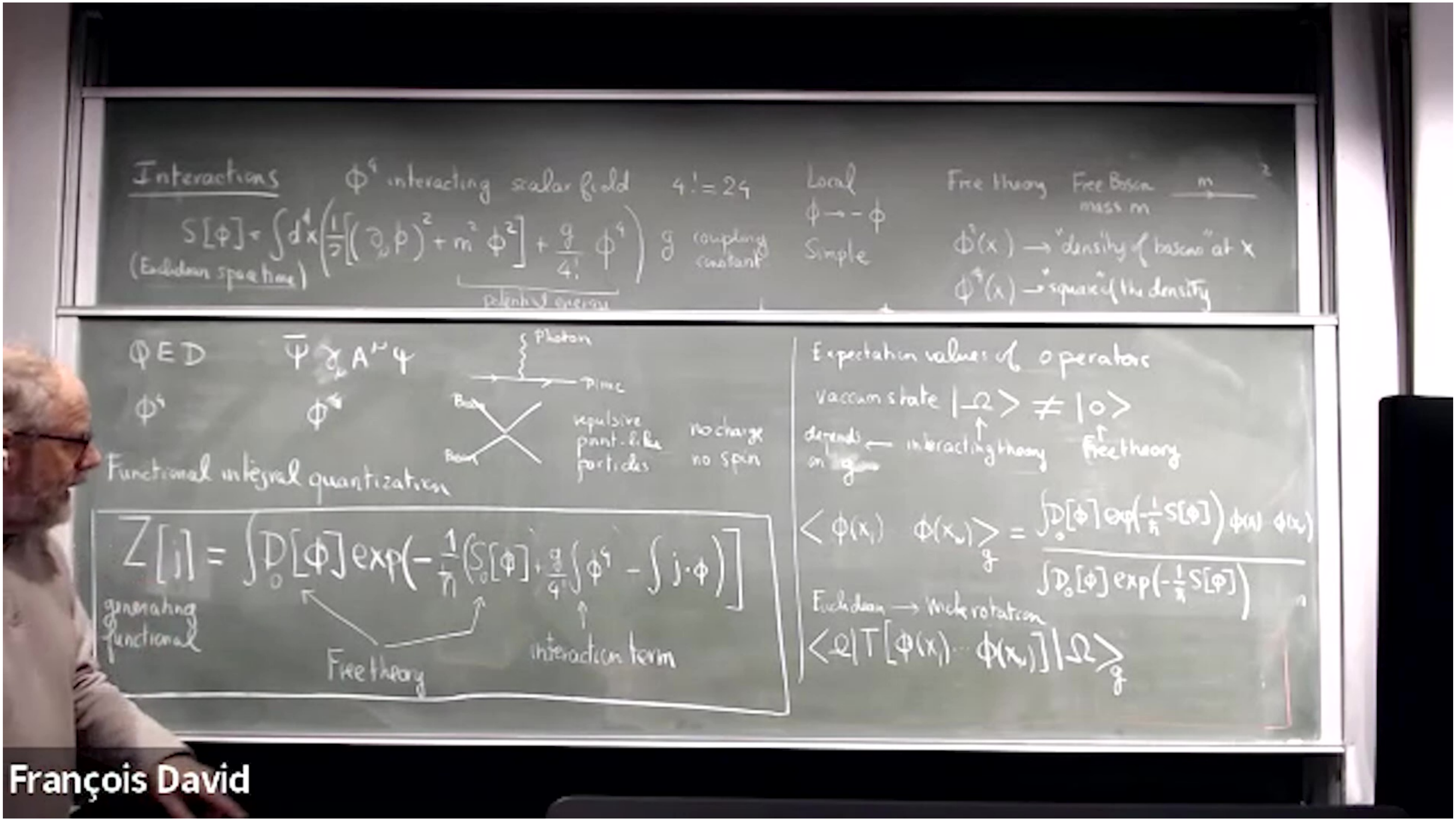
you need subtraction = renormalization
Sum over all possible pairings
but without
Subtract the dangerous diagram
Sym.
Interactions
interacting scalar field
4! = 24
Local
Simple
coupling constant
(Euclidean space time)
potential energy
S_E = Kinetic + Potential
S = Kinetic − Potential
Free theory
Free Boson
mass
contact interact
g>0 repulsive
François David

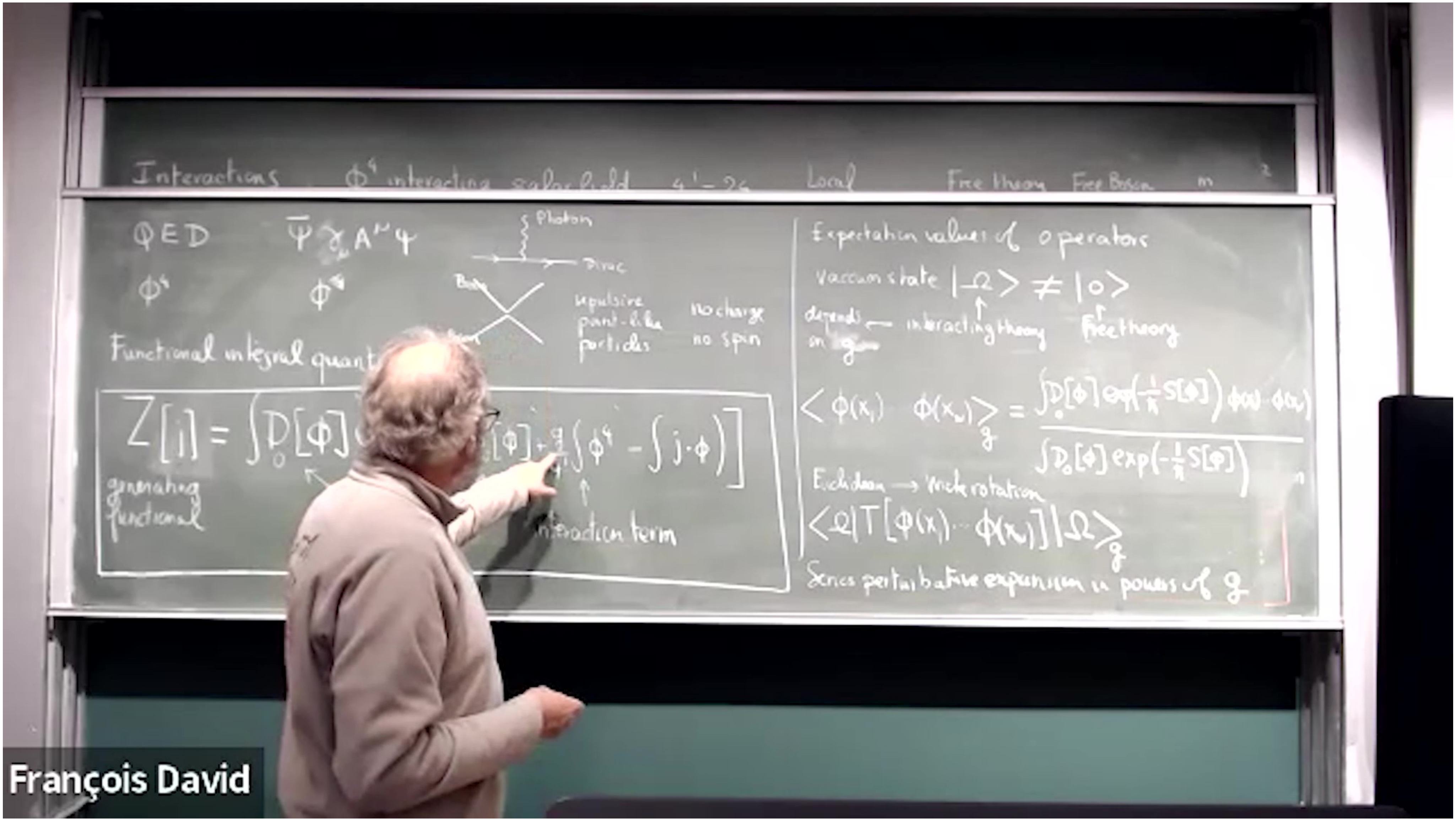

Composite operator
(normal ordered)
(normal product)
well defined
equal
divergent
divergent
You need subtraction = renormalization
- Sum over all possible pairings, but without
- Subtract the dangerous diagram
sym.
new
Interactions
coupling constant
(Euclidean space time)
energy
Local
Simple
Free theory
Free Boson mass m
contact interaction between two bosons
repulsive
attractive
François David

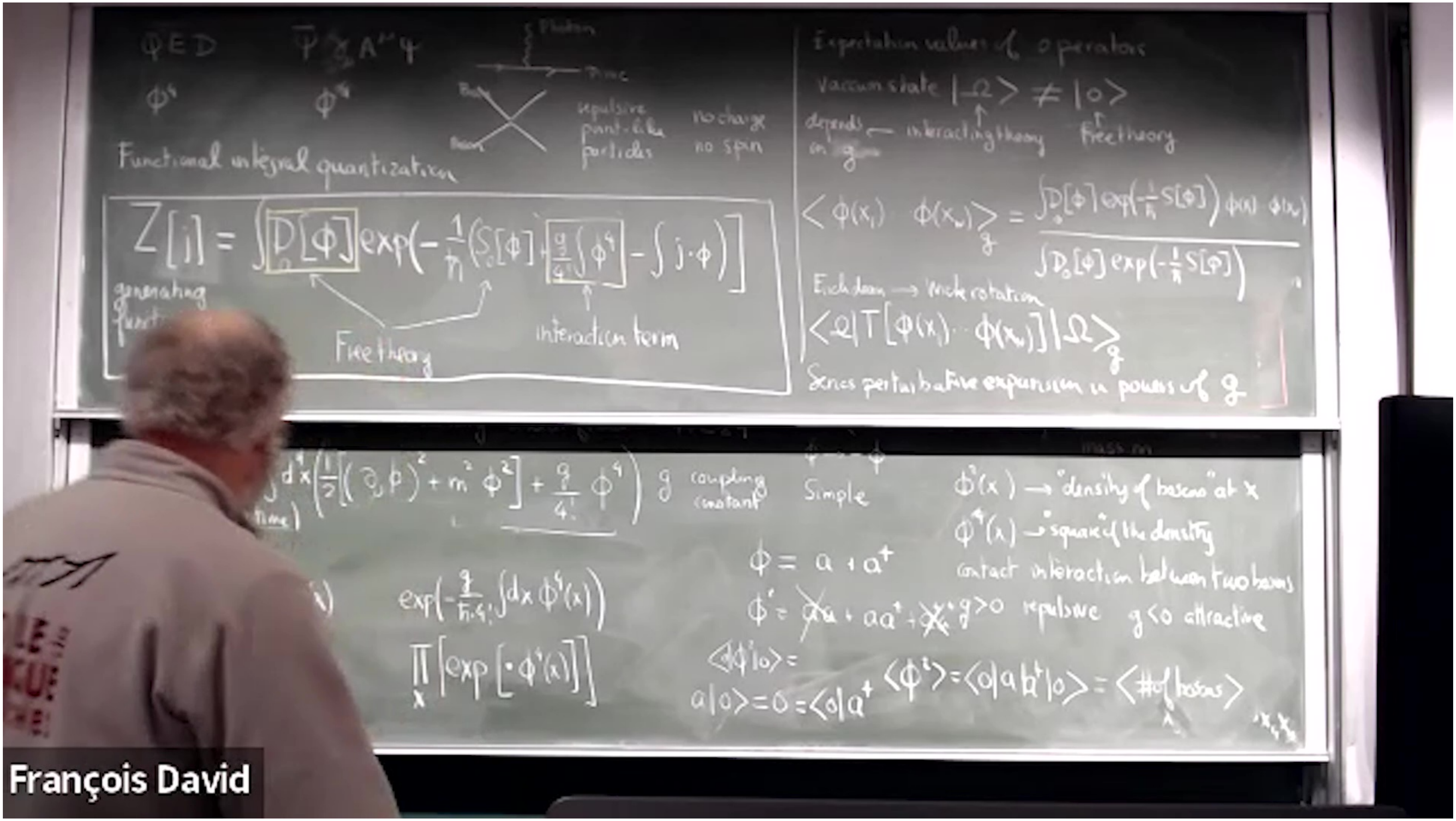


Interactions
ϕ^4 interacting scalar field
4! = 24
Local
ϕ → −ϕ
Simple
Free theory
Free Boson mass m
S[ϕ] = ∫d^4x (1/2[(∂_μϕ)^2 + m^2 ϕ^2] + g/4! ϕ^4)
(Euclidean space time)
g coupling constant
potential energy
ϕ^2(x) → "density of bosons" at x
ϕ^4(x) → "square" of the density
contact interaction between two bosons
g>0 repulsive
g<0 attractive
S_E = Kinetic + Potential
S = Kinetic − Potential
ϕ = a + a^+
QED
Ψ̄ γ_μ A^μ Ψ
Photon
Dirac
ϕ^4
ϕ^4
Boson
Boson
repulsive point-like particles
no charge
no spin
Functional integral quantization
Z = ∫D_0[ϕ] exp(−1/ħ(S_0[ϕ] + g/4! ∫ϕ^4))
Free theory
interaction term
operators
|0⟩ ≠ |0⟩
Free theory
ÉCOLE PHYSIQUE HOUCHES
François David


Interactions
ϕ^4 interacting scalar field
$4! = 24$
$S[\phi] = \int d^4x \left(\frac{1}{2}\left[(\partial_\mu \phi)^2 + m^2 \phi^2 \right] + \frac{g}{4!} \phi^4 \right)$
(Euclidean space time)
potential energy
g coupling constant
Local
$\phi \to -\phi$
Simple
Free theory
Free Boson mass m
$\phi^2(x) \to$ "density of bosons" at x
$\phi^4(x) \to$ "square" of the density
QED
$\bar{\Psi} \gamma_\mu A^\mu \Psi$
ϕ^4
Photon
repulsive point-like particles
no charge
no spin
integral quantization
$Z[j] = \int D_0[\phi] \exp\left(-\frac{1}{\hbar}\left(S_0[\phi] + \frac{g}{4!}\int \phi^4\right)\right)$
Free theory
interaction term
Expectation values of operators
vaccum state $|\Omega\rangle \neq |0\rangle$
depends on g
interacting theory
Free theory
$\langle \phi(x_1) \cdots \phi(x_n) \rangle_g = \frac{\int D_0[\phi] \exp\left(-\frac{1}{\hbar} S[\phi]\right) \phi(x_1) \cdots \phi(x_n)}{\int D_0[\phi] \exp\left(-\frac{1}{\hbar} S[\phi]\right)}$
Euclidean $\to$ Wick rotation
$\langle \Omega | T[\phi(x_1) \cdots \phi(x_n)] | \Omega \rangle_g$
François David


Interactions
φ⁴ interacting scalar field
4! = 24
S[φ] = ∫d⁴x (1/2[(∂μφ)² + m²φ²] + g/4! φ⁴)
(Euclidean space time)
potential energy
g coupling constant
Local
φ → −φ
Simple
Free theory
Free Boson mass m
φ²(x) → "density of bosons" at x
φ⁴(x) → "square" of the density
QED
Ψ̄ γμ Aμ Ψ
Photon
Boson
repulsive point-like particles
no charge
no spin
Functional integral quantization
Z[j] = ∫D₀[φ] exp(−1/ħ(S₀[φ] + g/4! ∫φ⁴ − ∫j·φ))
generating functional
Free theory
interaction term
Expectation values of operators
vaccum state |Ω> ≠ |0>
depends on g
interacting theory
free theory
<φ(x₁) … φ(xₙ)>_g = ∫D₀[φ] exp(−1/ħ S[φ]) φ(x₁) … φ(xₙ) / ∫D₀[φ] exp(−1/ħ S[φ])
Euclidean → Wick rotation
<Ω|T[φ(x₁) … φ(xₙ)]|Ω>_g
François David


Interactions
QED
Photon
Dirac
repulsive
point-like
particles
no charge
no spin
Functional integral quant
generating functional
interaction term
Expectation values of operators
vaccum state $|\Omega\rangle \neq |0\rangle$
interacting theory
Free theory
Euclidean → Wick rotation
Series perturbative expansion in powers of g
François David



QED
ϕ^4
Functional integral quantization
generating
Free theory
interaction term
Expectation values of operators
vacuum state $|\Omega\rangle \neq |0\rangle$
interacting theory
free theory
Euclidean → Wick rotation
Series perturbative expansion in powers of g
coupling constant
Simple
$\phi = a + a^+$
$\phi^2(x)$ → "density of bosons" at x
$\phi^4(x)$ → square of the density
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g>0 repulsive g<0 attractive
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