Title: QFT III 2021/2022

Speakers: Francois David

Collection: QFT III 2021/2022

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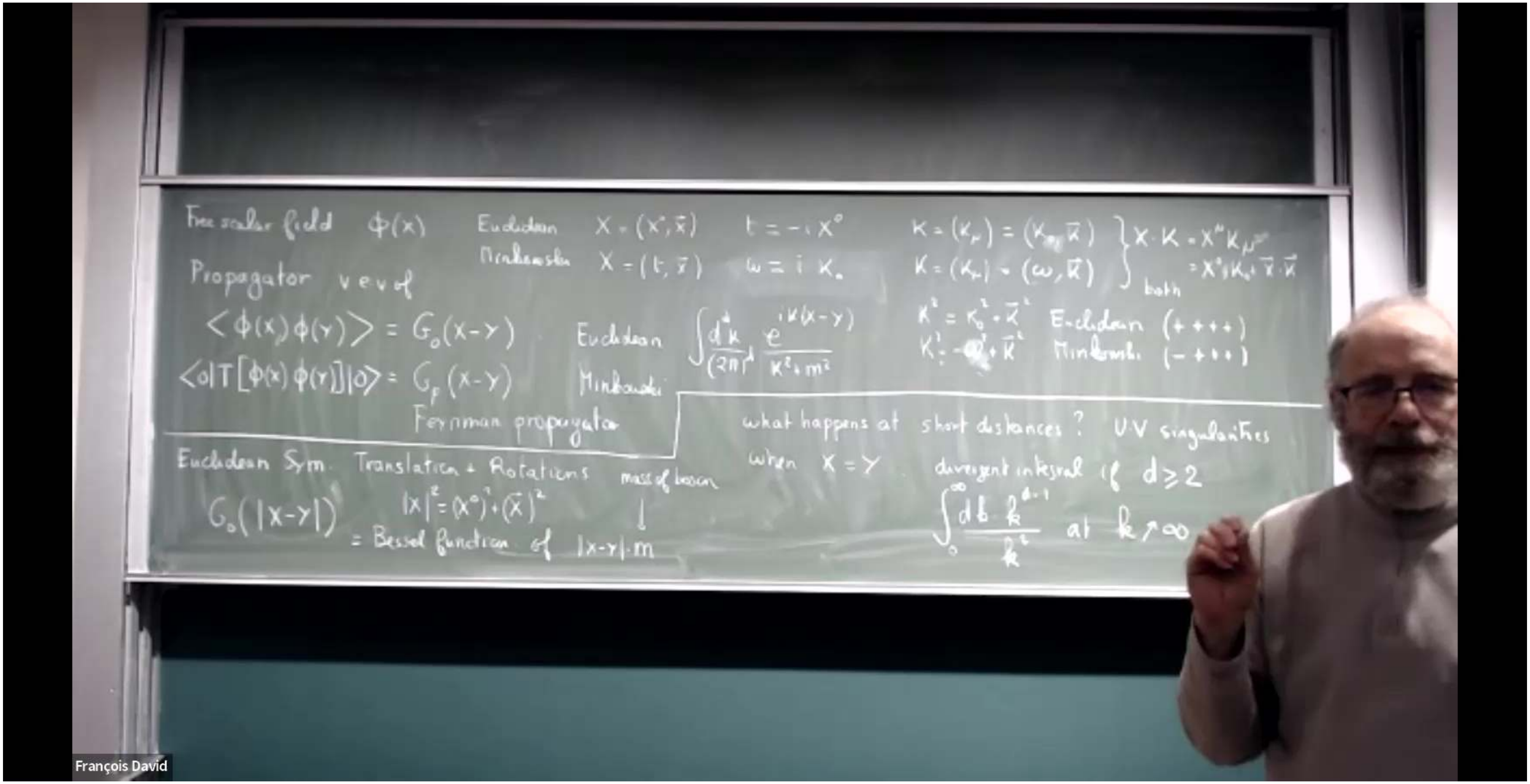
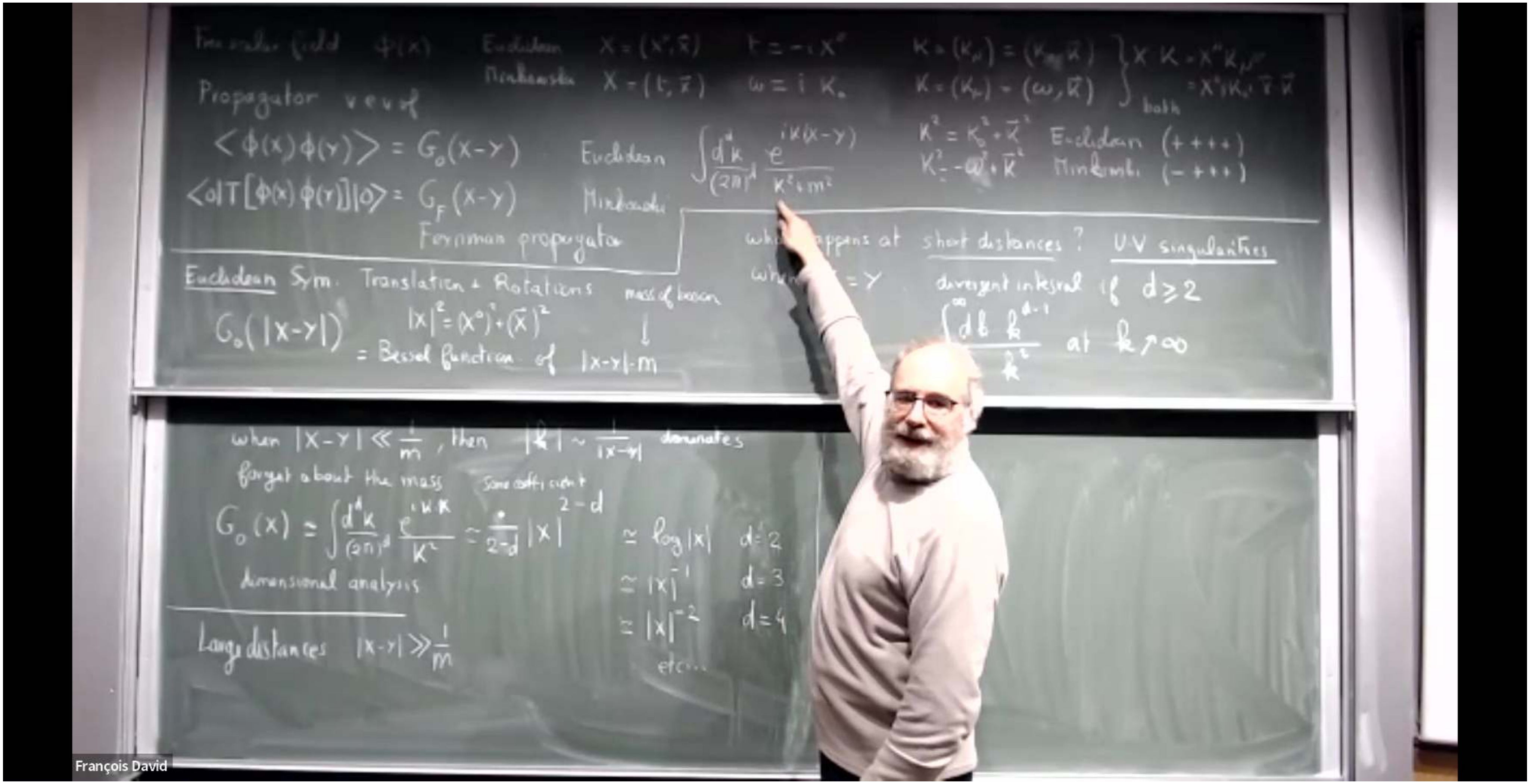
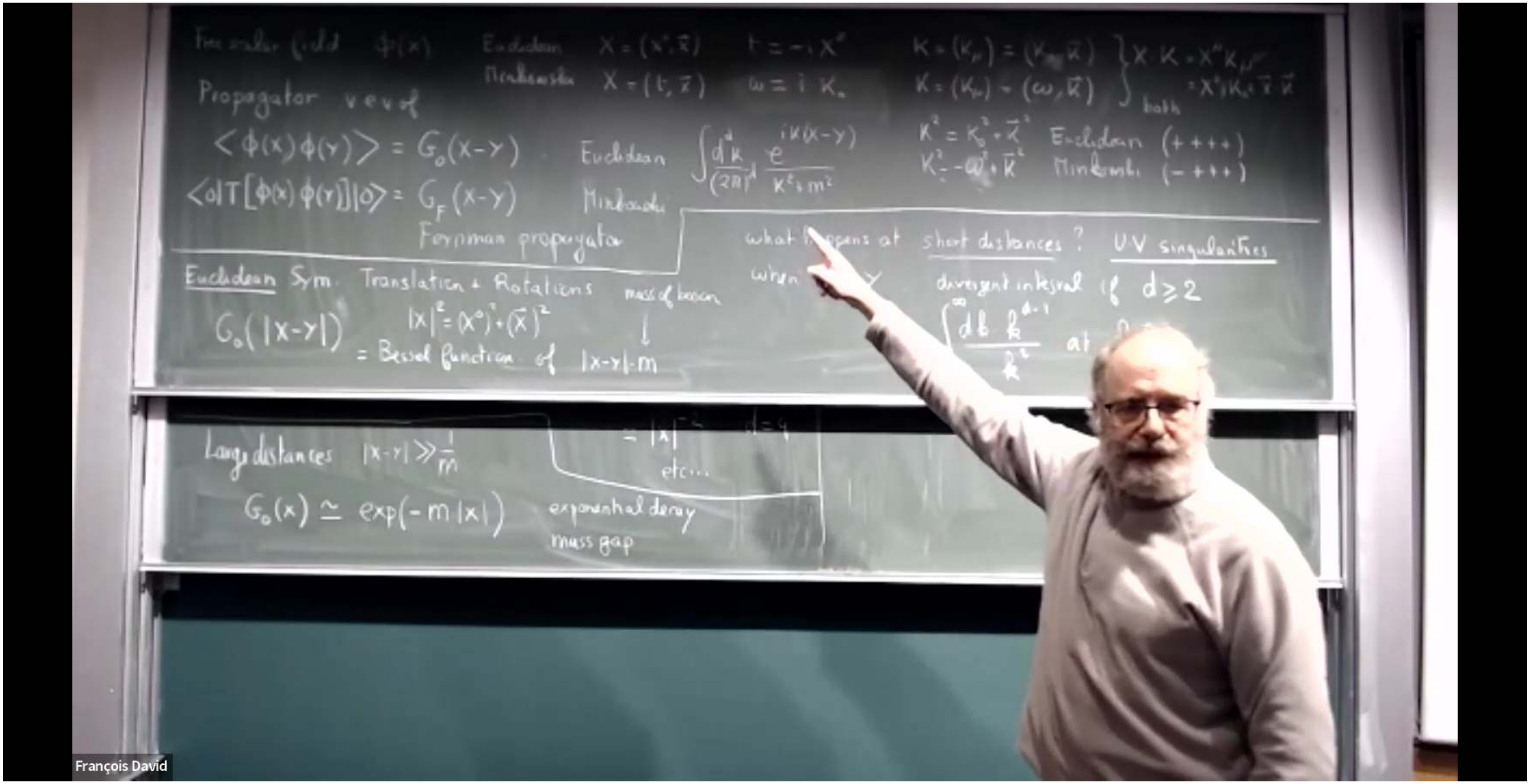
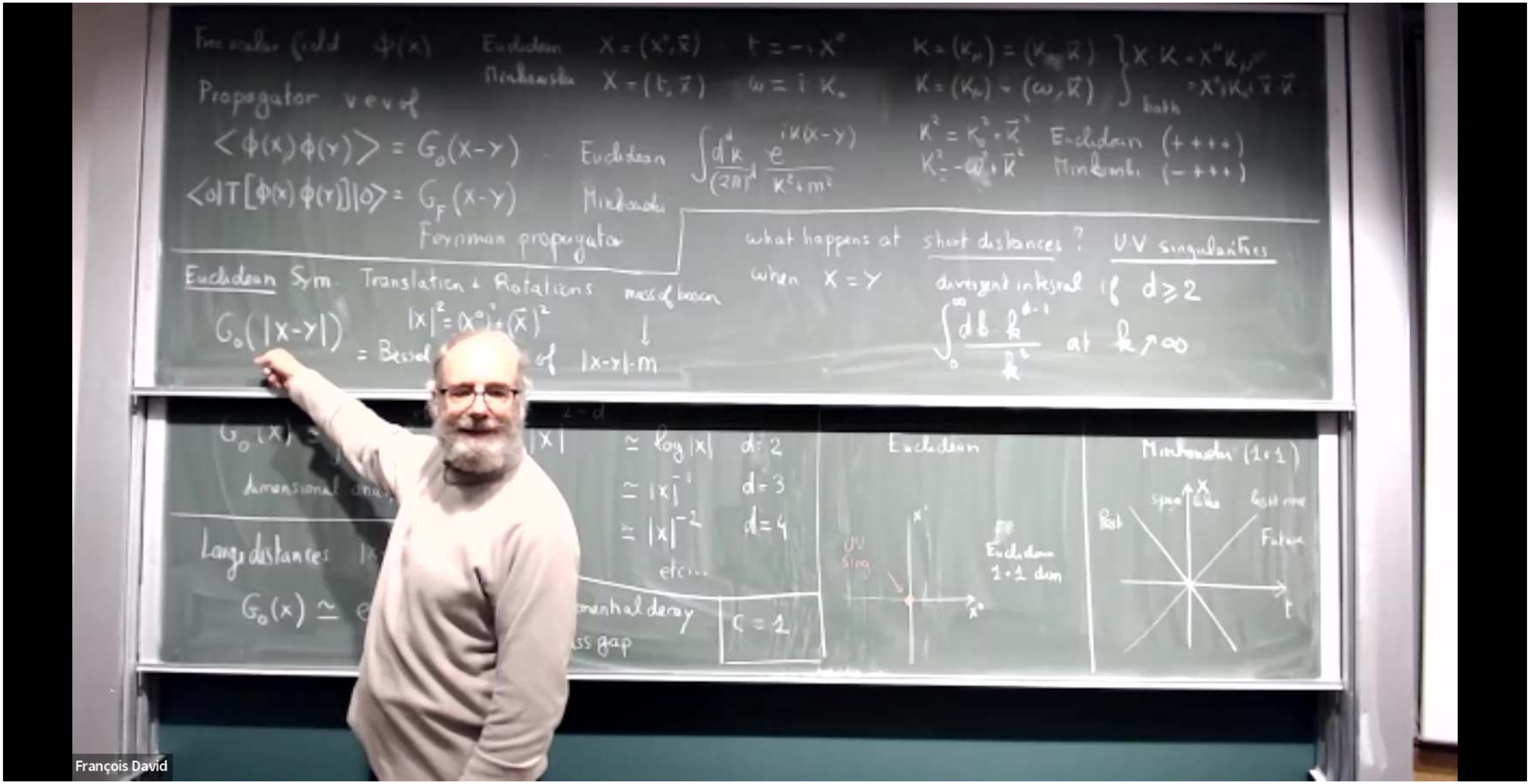
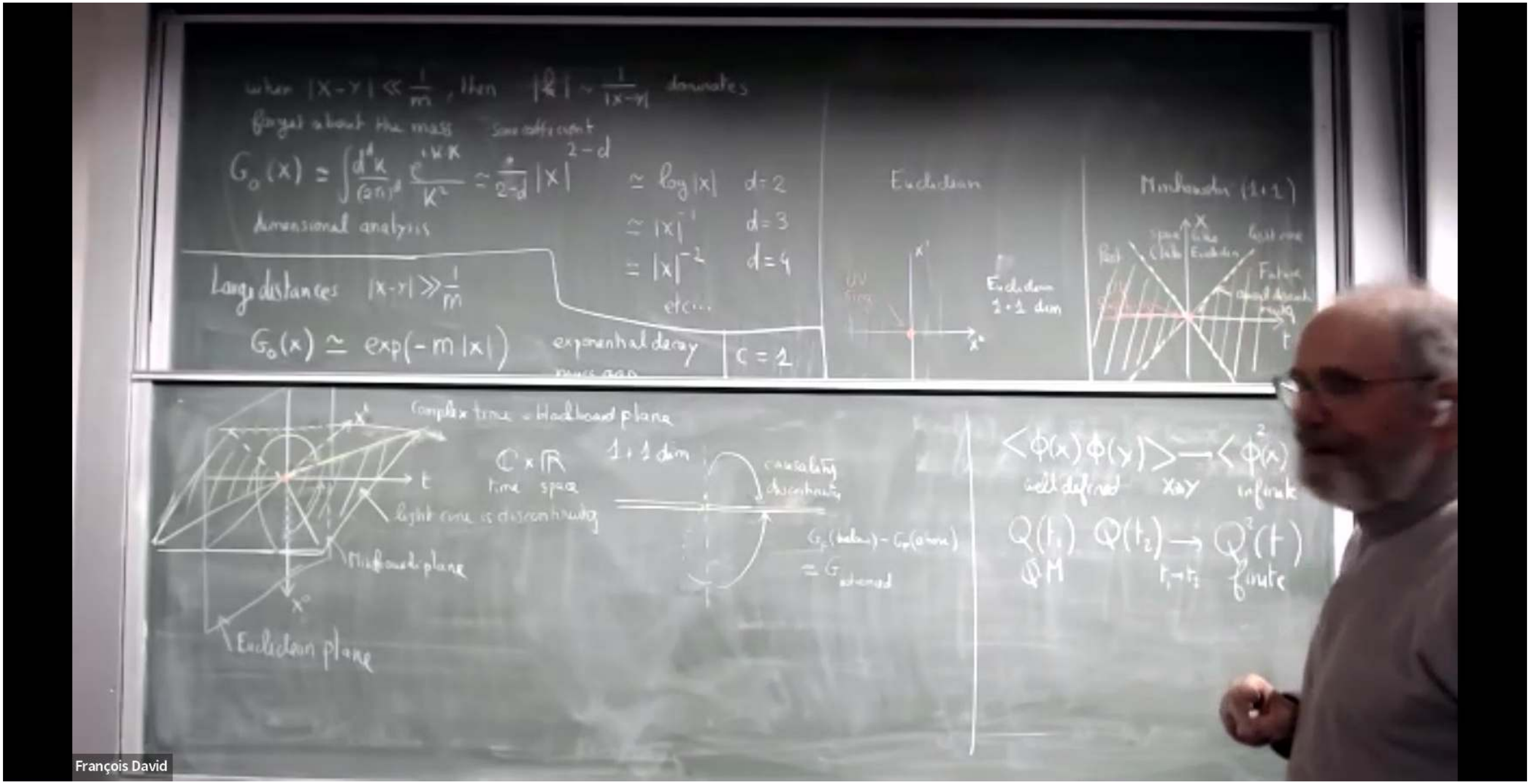

Free scalar field φ(x)
Euclidean x = (x°, x⃗)
t = -i x°
Minkowski x = (t, x⃗)
ω = i K₀
Propagator v.e.v of
<φ(x)φ(y)> = G₀(x-y)
Euclidean
<0|T[φ(x)φ(y)]|0> = G_F(x-y)
Minkowski
Feynman propagator
Euclidean (+ + + +)
Minkowski (− + + +)
both
Euclidean Sym. Translation + Rotations
mass of boson
G₀(|x-y|)
= Bessel function of |x-y|·m
what happens at short distances ? UV singularities
when x = y
divergent integral if d ≥ 2
at k → ∞
François David

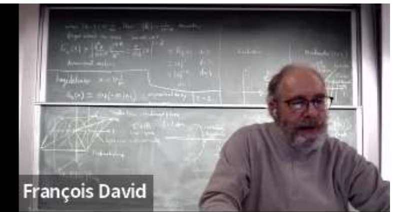

Propagator
Euclidean
Minkowski
Feynman propagator
Euclidean Sym. Translation + Rotations
mass of boson
= Bessel function of |x-y|·m
short distances ?
U.V singularities
divergent integral if d ≥ 2
forget about the mass
some coefficient
dimensional analysis
Large distances
etc...
François David


Propagator v.e.v of
Euclidean
Minkowski
Feynman propagator
Euclidean Sym. Translation + Rotations
mass of boson
= Bessel function of |x-y|·m
what happens at short distances ? UV singularities
divergent integral if d ≥ 2
Large distances |x-y| ≫ 1/m
G₀(x) ≃ exp(-m|x|)
exponential decay
mass gap
etc...
François David


Free scalar field $\phi(x)$
Euclidean $X=(x^0,\vec{x})$ $t=-iX^0$
Minkowski $X=(t,\vec{x})$ $\omega=iK_0$
Propagator v.e.v of
$\langle\phi(x)\phi(y)\rangle = G_0(x-y)$ Euclidean $\int\frac{d^dk}{(2\pi)^d}\frac{e^{ik(x-y)}}{k^2+m^2}$
$\langle 0|T[\phi(x)\phi(y)]|0\rangle = G_F(x-y)$ Minkowski
Feynman propagator
$K^2=K_0^2+\vec{K}^2$ Euclidean (++++)
Minkowski (−+++)
what happens at short distances ? U.V singularities
when $X=Y$
divergent integral if $d\geq 2$
$\int_0^\infty \frac{dk\, k^{d-1}}{k^2}$ at $k\nearrow\infty$
Euclidean Sym. Translation + Rotations
mass of boson
$G_0(|x-y|)$
$|x|^2=(x^0)^2+(\vec{x})^2$
= Bessel ... of $|x-y|\cdot m$
$\simeq \log|x|$ $d=2$
$\simeq |x|^{-1}$ $d=3$
$\simeq |x|^{-2}$ $d=4$
etc...
Long distances
$G_0(x)\simeq$
exponential decay
mass gap
$c=1$
Euclidean
UV sing
Euclidean 1+1 dim
Minkowski (1+1)
Past
Future
light cone
François David

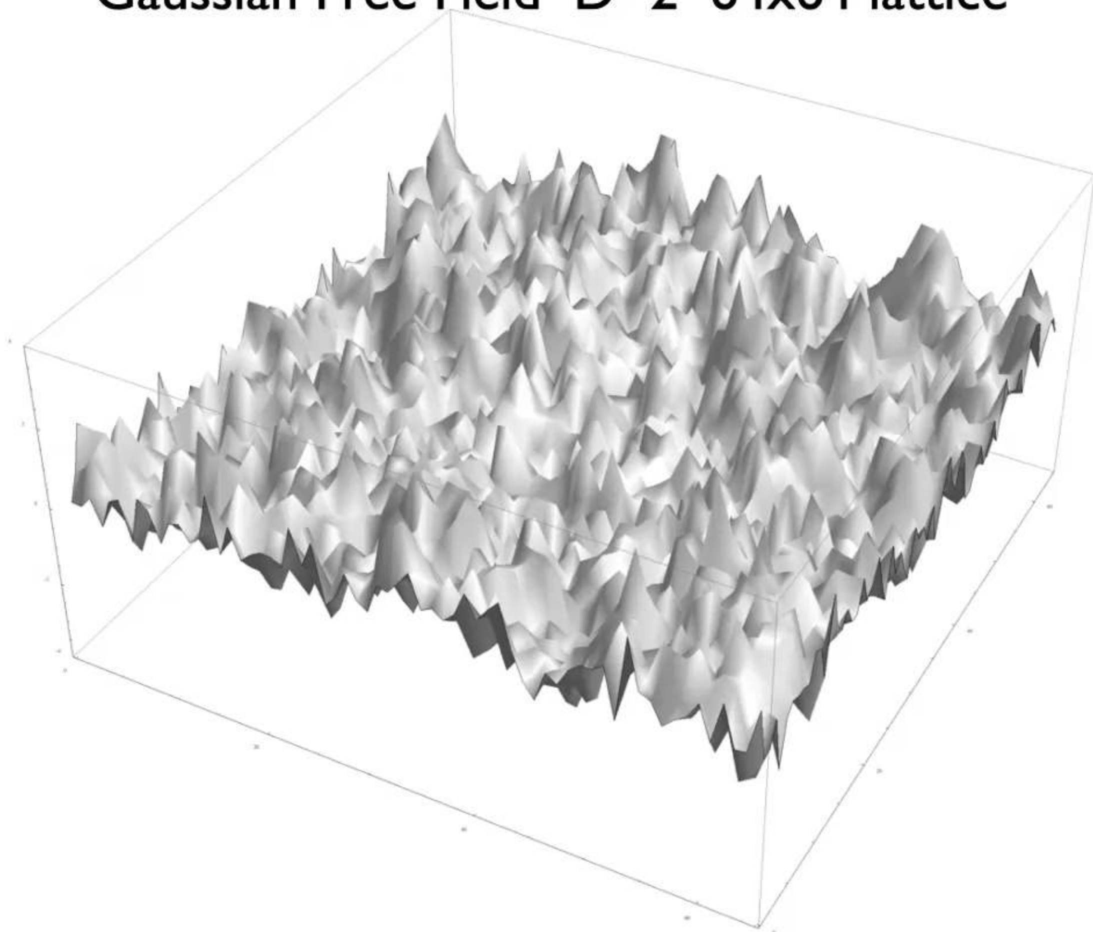

dominates
forget about the mass
dimensional analysis
Long distances
exponential decay
Euclidean
Euclidean 1+1 dim
Complex time = blackboard plane
1+1 dim
time space
light cone is discontinuous
Minkowski plane
Euclidean plane
causality
well defined
infinite
finite
François David

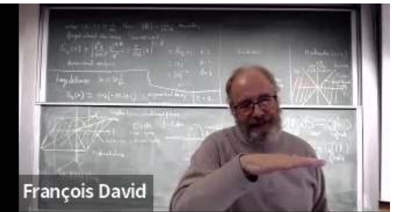
Typical GFF configurations



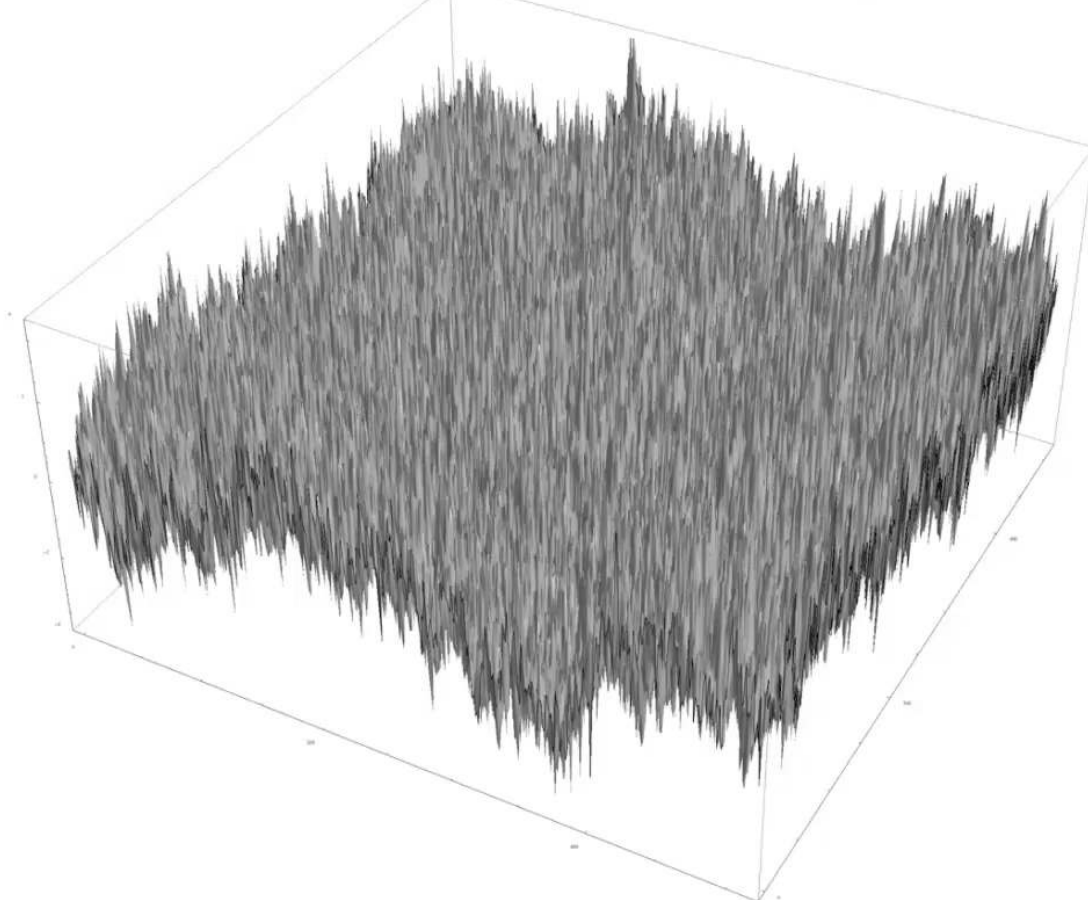
Gaussian Free Field = Massless Scalar Free Fields

Gaussian Free Field D=2 64x64 lattice

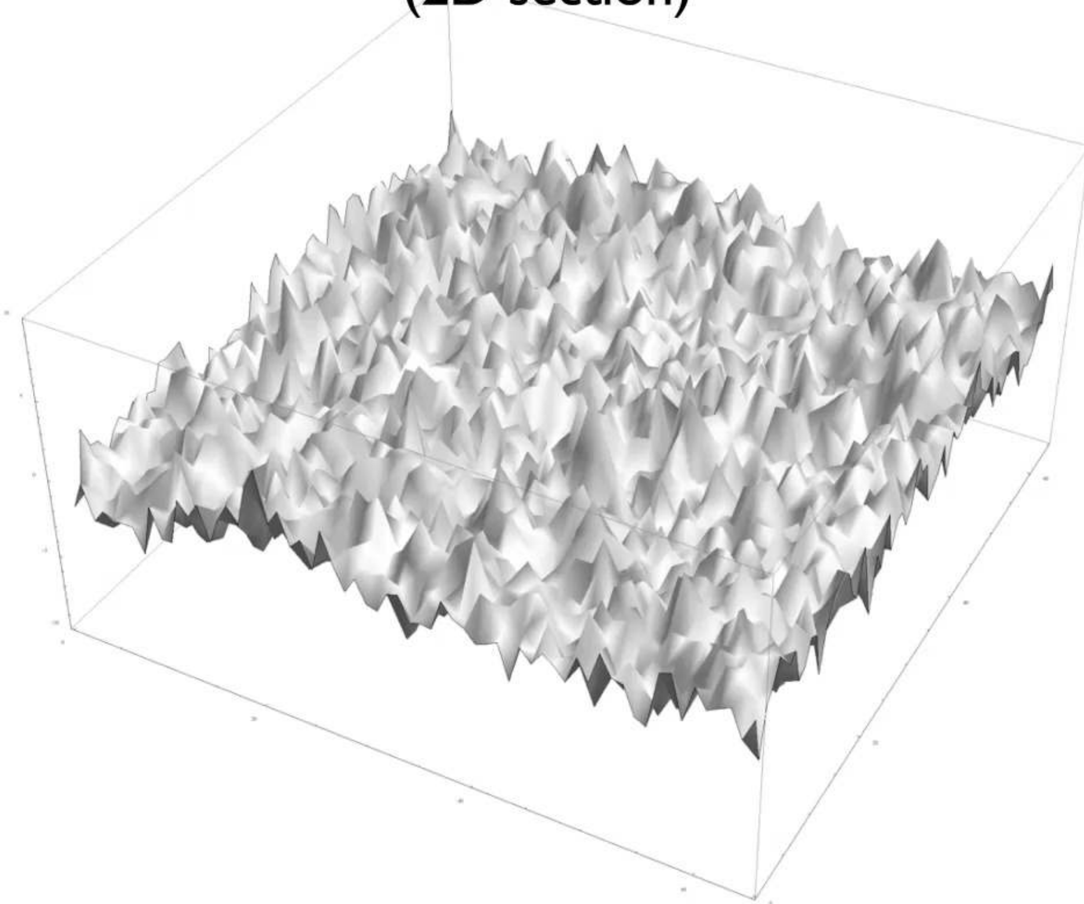




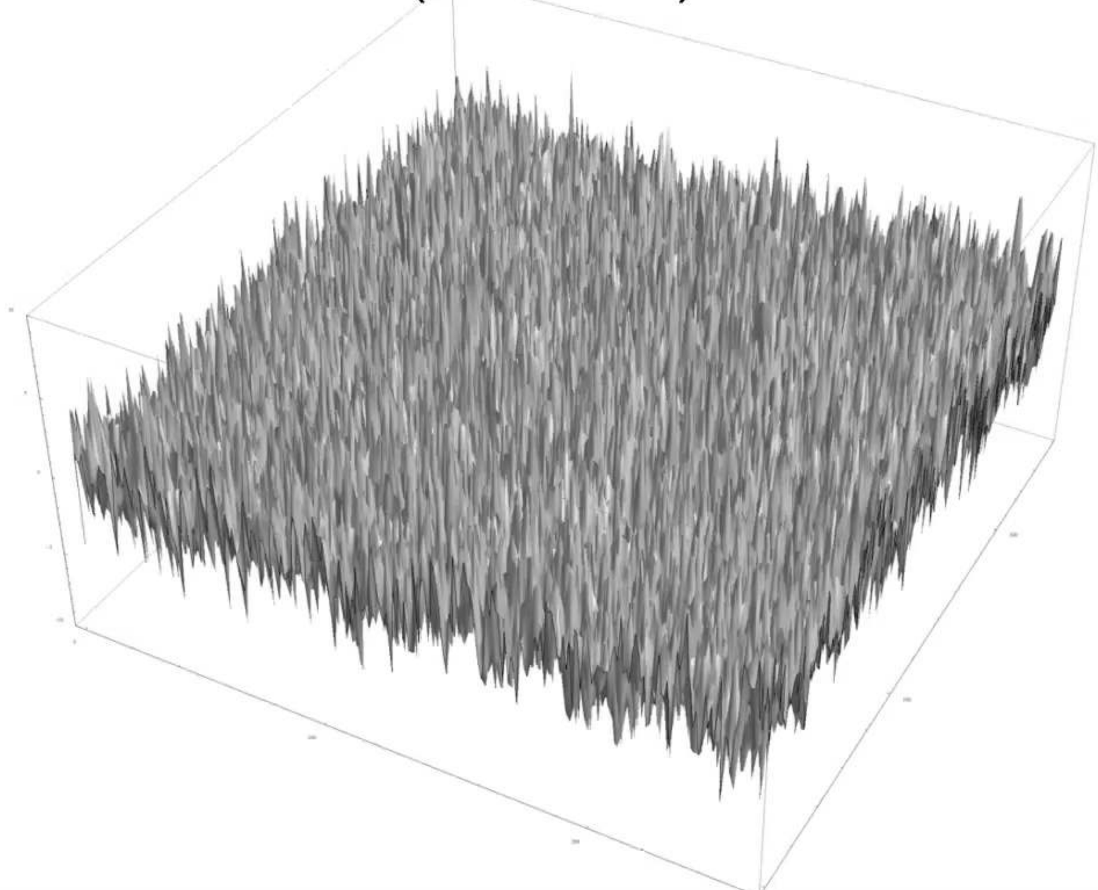
Gaussian Free Field D=2 512x512 lattice

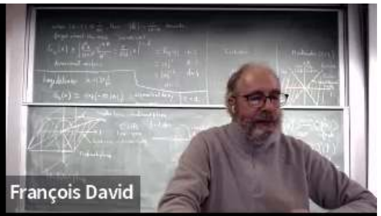


Gaussian Free Field D=3 64x64x64 lattice
(2D section)

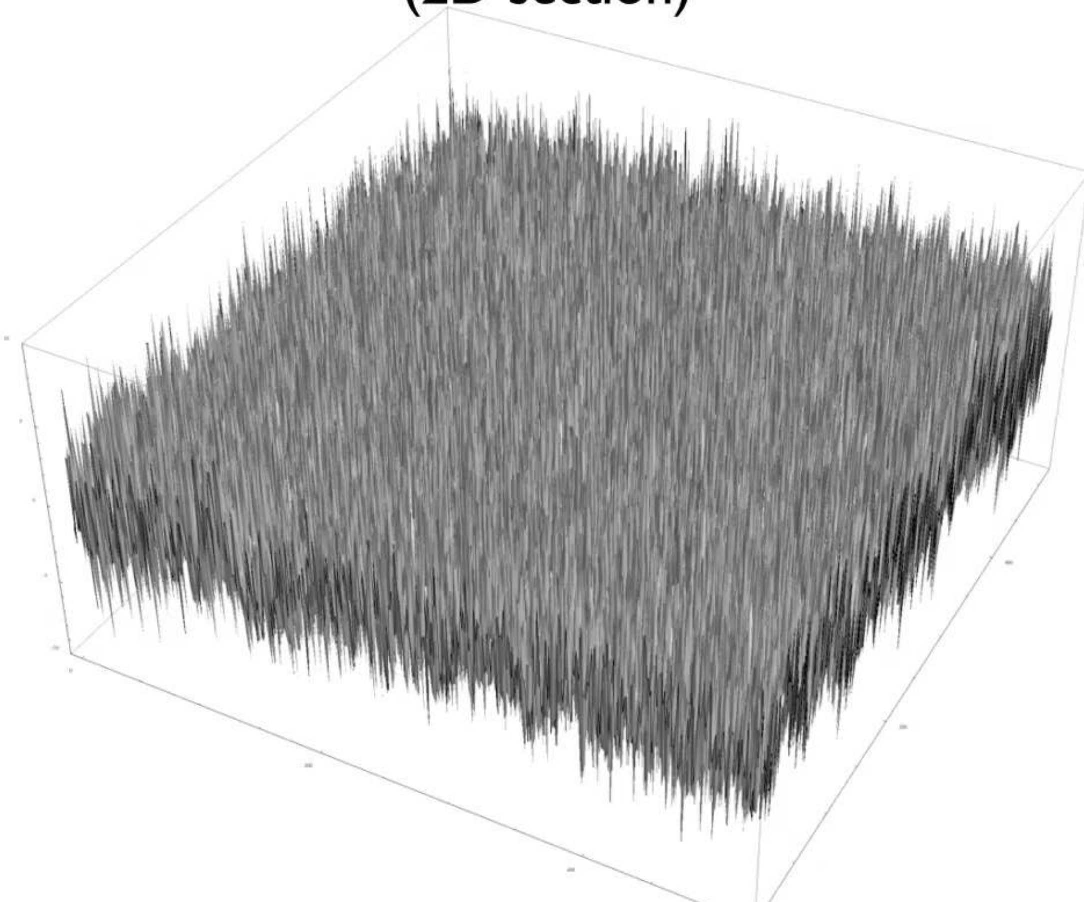


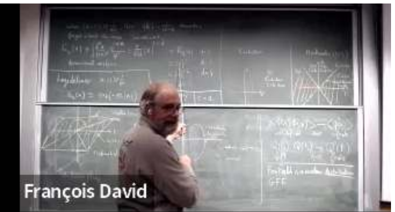
Gaussian Free Field D=3 256x256x256 lattice (2D section)



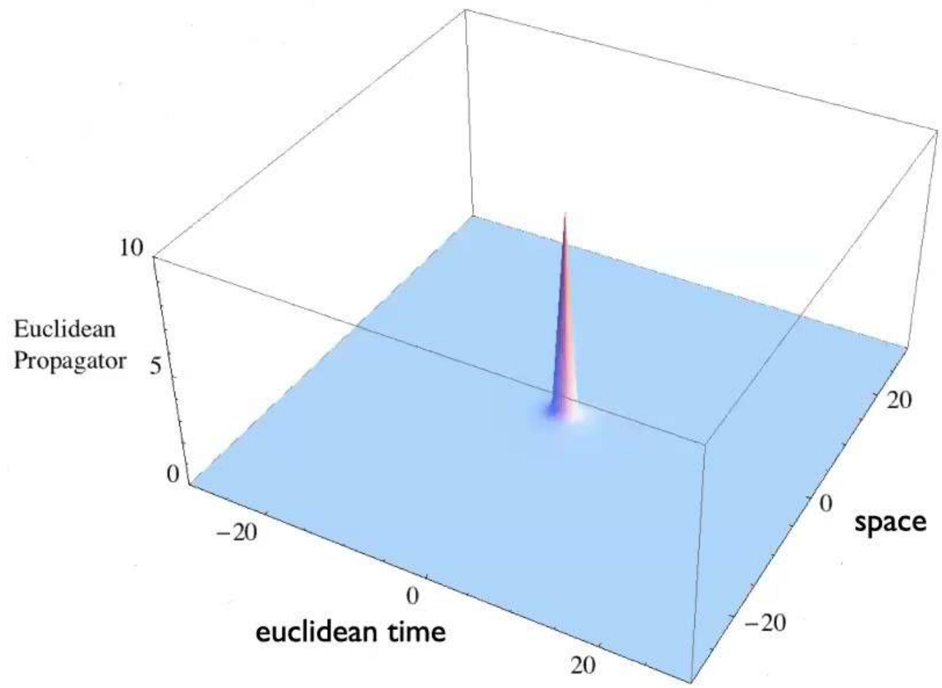


Gaussian Free Field D=3 512x512x512 lattice
(2D section)

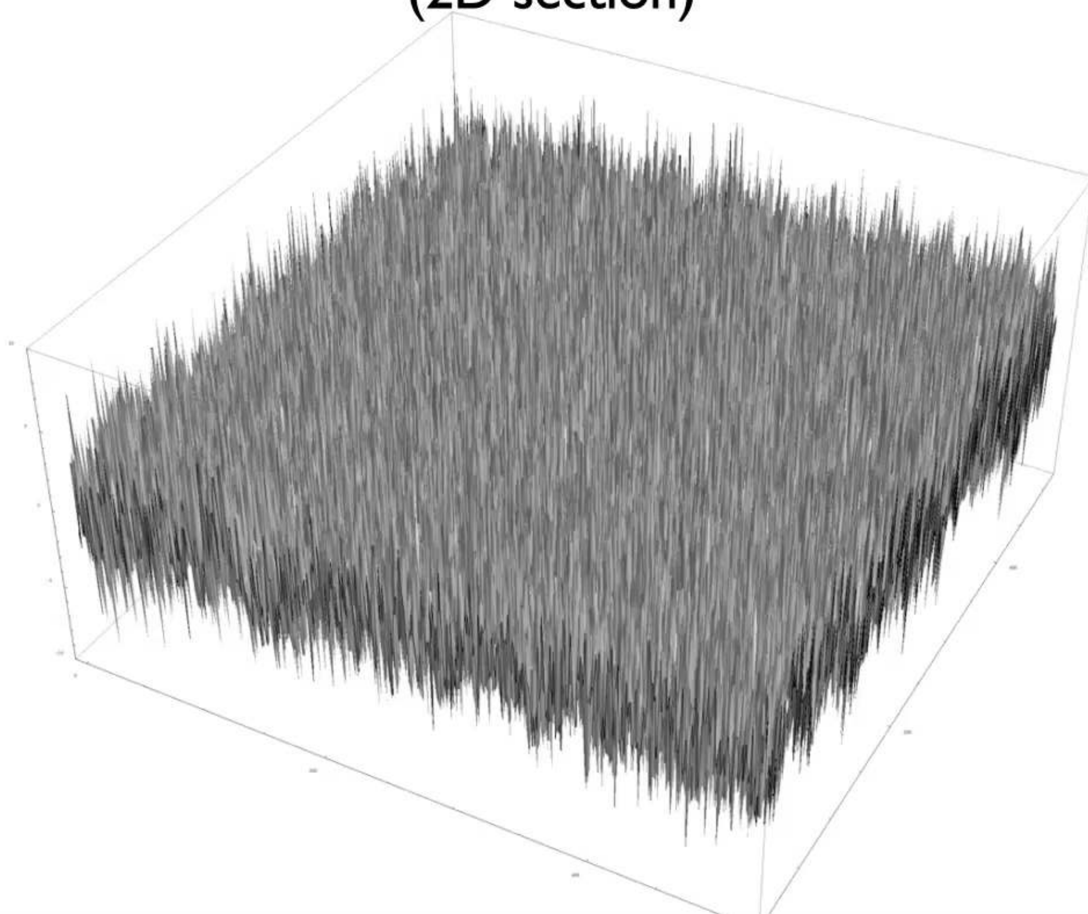


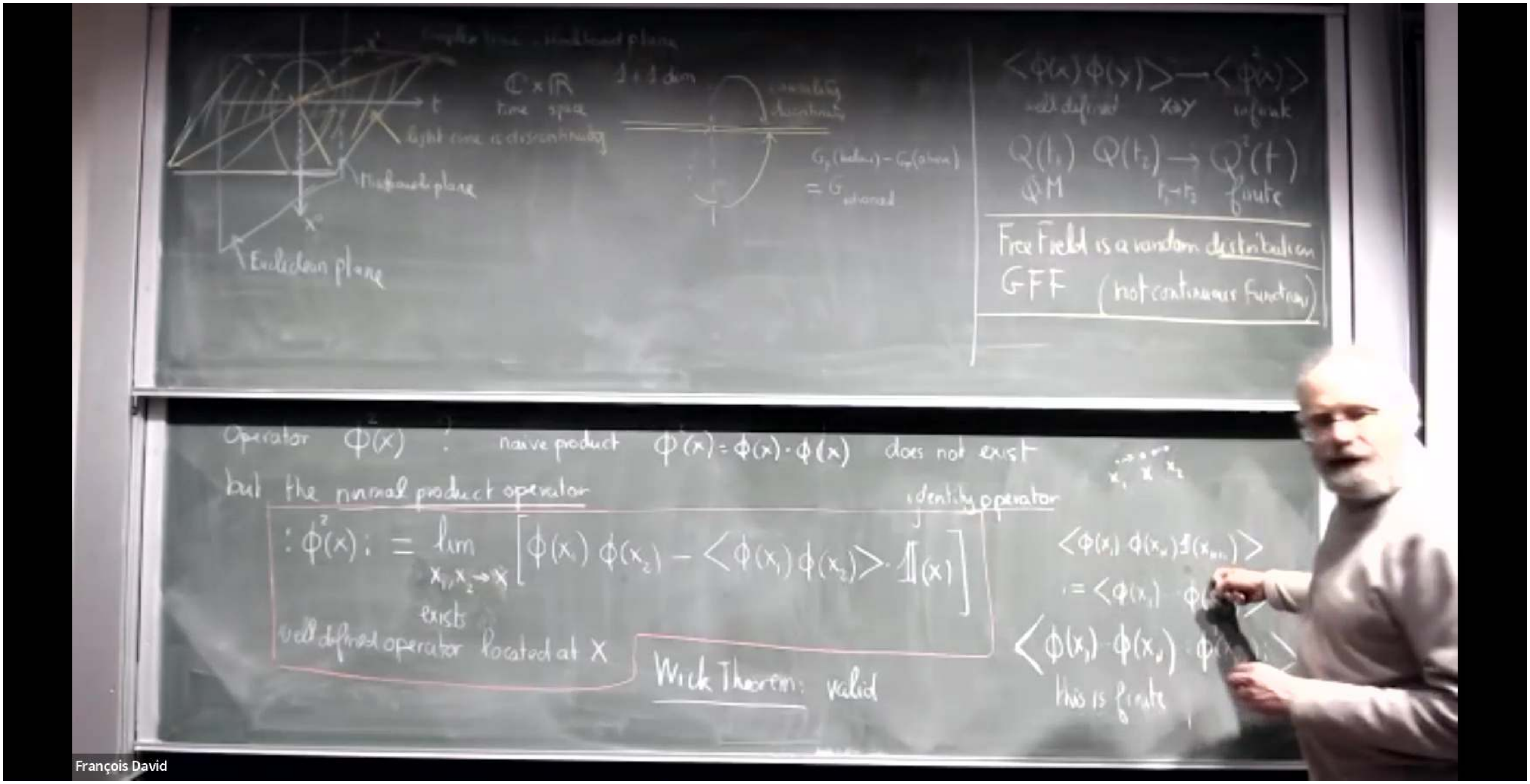


Free field mass m , dimension D=2 propagator in Euclidean space



Gaussian Free Field D=3 512x512x512 lattice
(2D section)




C×R
1+1 dim
time space
light cone is discontinuity
causality discontinuity
Minkowski plane
Euclidean plane
= G advanced
⟨φ(x) φ(y)⟩ → ⟨φ²(x)⟩
well defined x→y infinite
Q(t₁) Q(t₂) → Q²(t)
QM t₁→t₂ finite
Free Field is a random distribution
GFF (not continuous function)
Operator φ²(x) : naive product φ²(x) = φ(x)·φ(x) does not exist
but the normal product operator
identity operator
:φ²(x): = lim x₁,x₂→x [φ(x₁) φ(x₂) − ⟨φ(x₁) φ(x₂)⟩·𝟙(x)]
exists
well defined operator located at x
Wick Theorem: valid
this is finite
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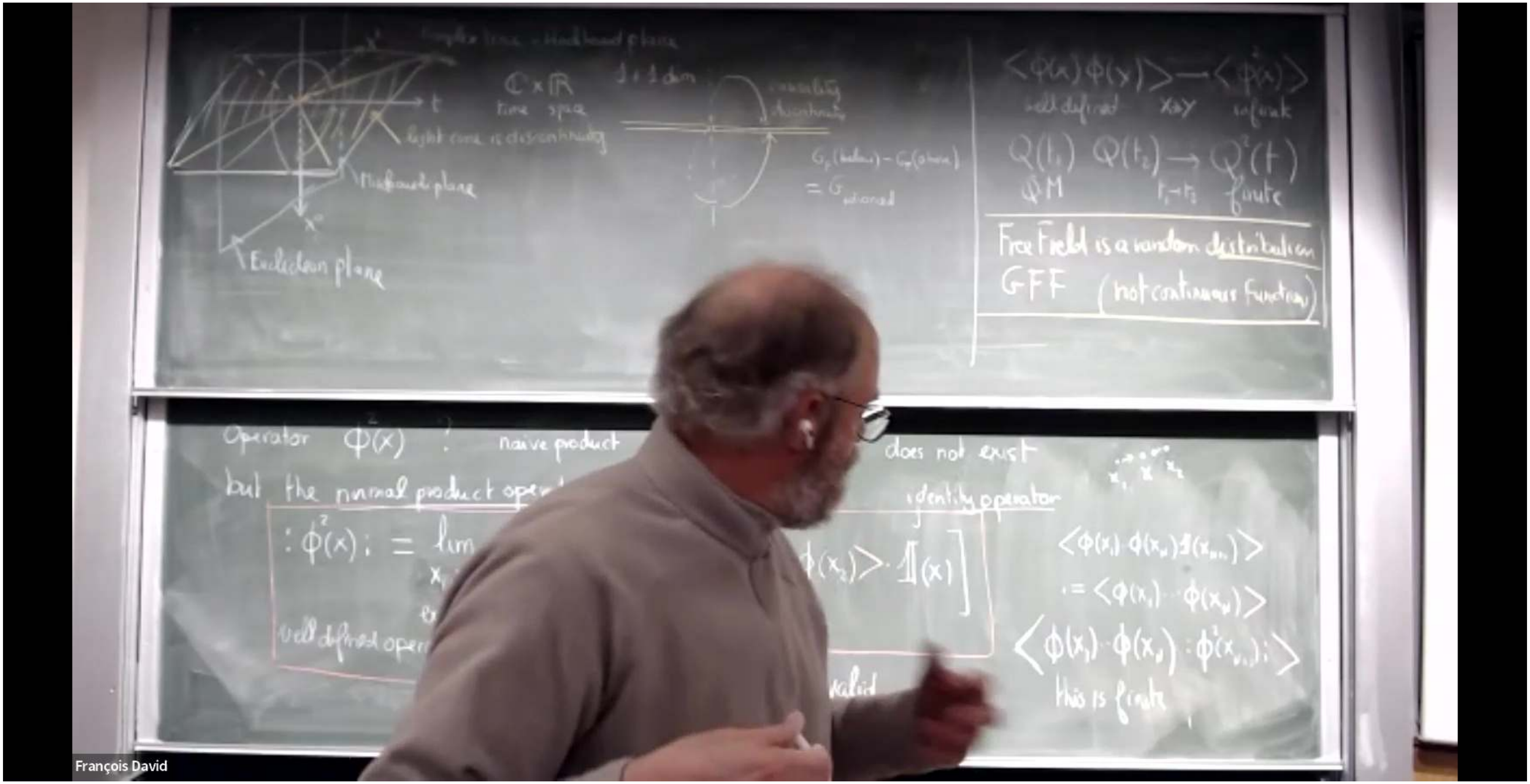
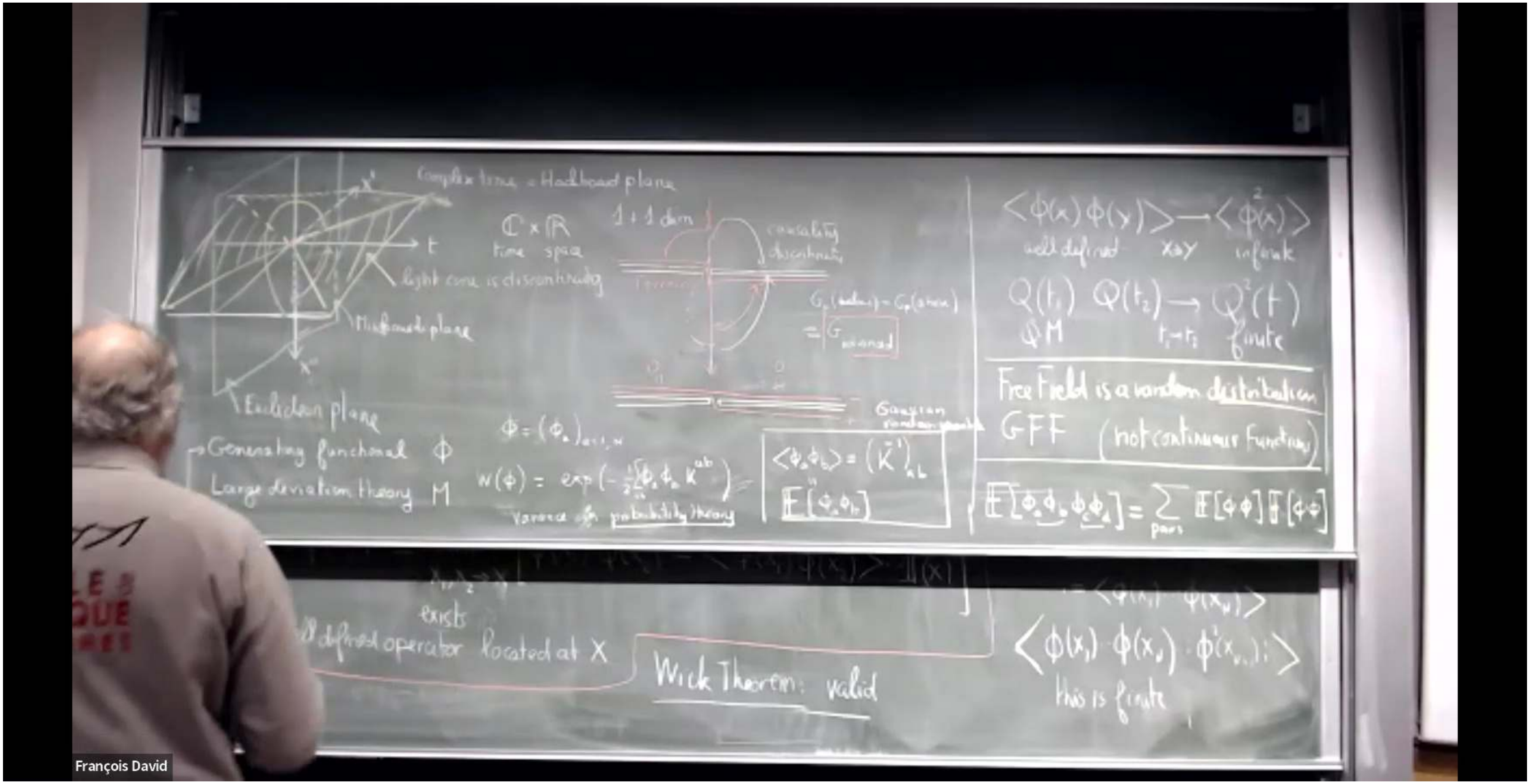

Euclidean plane
light cone is discontinuity
$\mathbb{C} \times \mathbb{R}$
time space
$\langle \phi(x)\phi(y) \rangle \to \langle \phi^2(x) \rangle$
well defined $x \to y$ infinite
$Q(t_1)\ Q(t_2) \to Q^2(t)$
QM $t_1 \to t_2$ finite
Free Field is a random distribution
GFF (not continuous function)
Operator $\phi^2(x)$? naive product
does not exist
but the normal product oper
identity operator
$:\phi^2(x): = \lim$
$\phi(x_2)\rangle \cdot \mathbb{1}(x)]$
well defined oper
$\langle \phi(x_1) \cdots \phi(x_N) \mathbb{1}(x_{N+1}) \rangle$
$:= \langle \phi(x_1) \cdots \phi(x_N) \rangle$
$\langle \phi(x_1) \cdots \phi(x_N) :\phi^2(x_{N+1}): \rangle$
this is finite
François David


Complex time = blackboard plane
$\mathbb{C} \times \mathbb{R}$
time space
light cone is discontinuity
Minkowski plane
Euclidean plane
1+1 dim
causality discontinuity
Generating functional Φ
Large deviation theory M
$\Phi = (\Phi_a)_{a=1..N}$
variance in probability theory
Gaussian random variable
$\langle\Phi_a\Phi_b\rangle = (K^{-1})_{ab}$
$\langle\phi(x)\phi(y)\rangle \to \langle\phi^2(x)\rangle$
well defined
infinite
$Q(t_1)\,Q(t_2) \to Q^2(t)$
QM
finite
Free Field is a random distribution
GFF (not continuous function)
exists
ll defined operator located at X
Wick Theorem: valid
this is finite
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