Title: QFT III 2021/2022

Speakers: Francois David

Collection: QFT III 2021/2022

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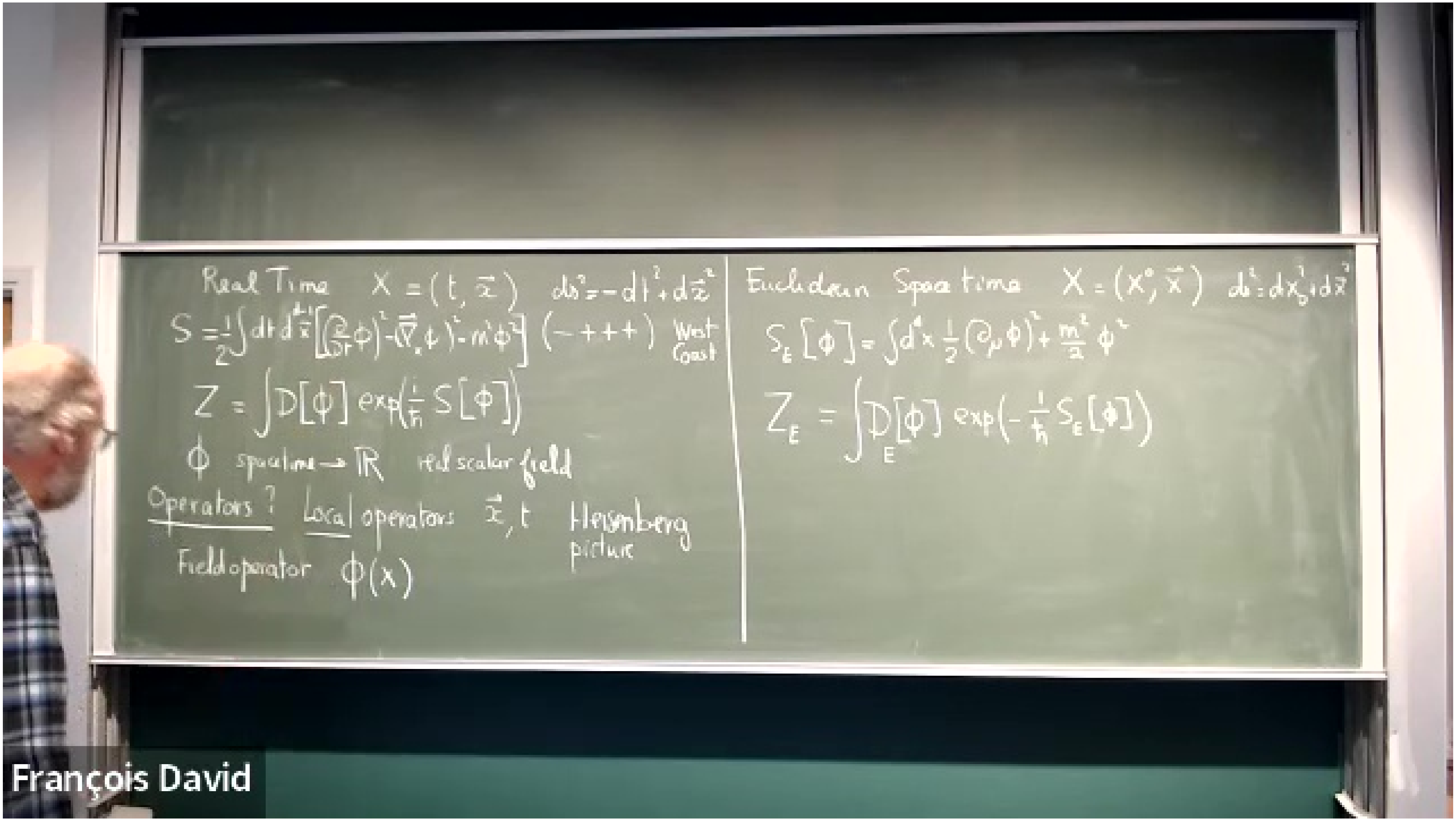
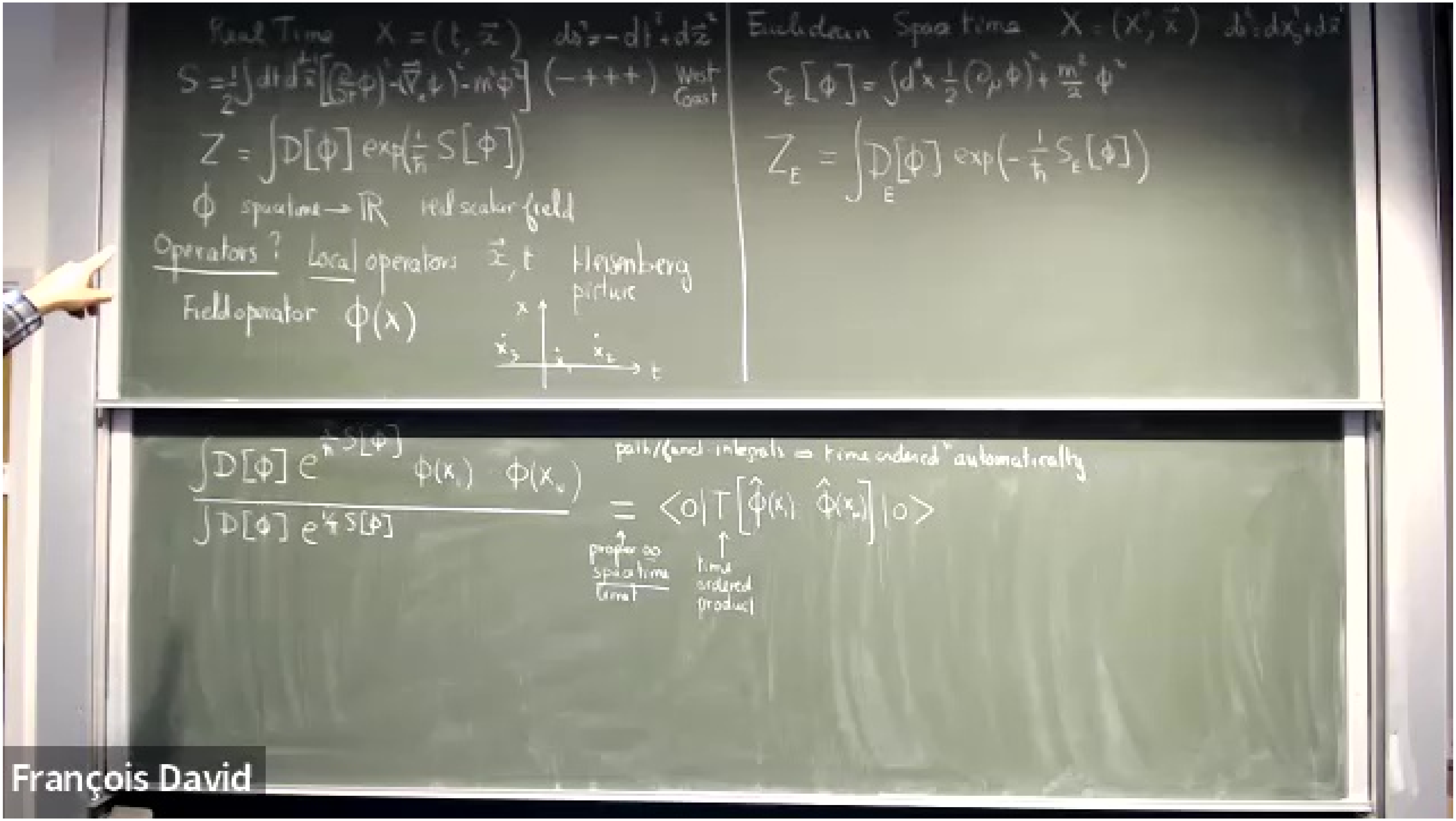
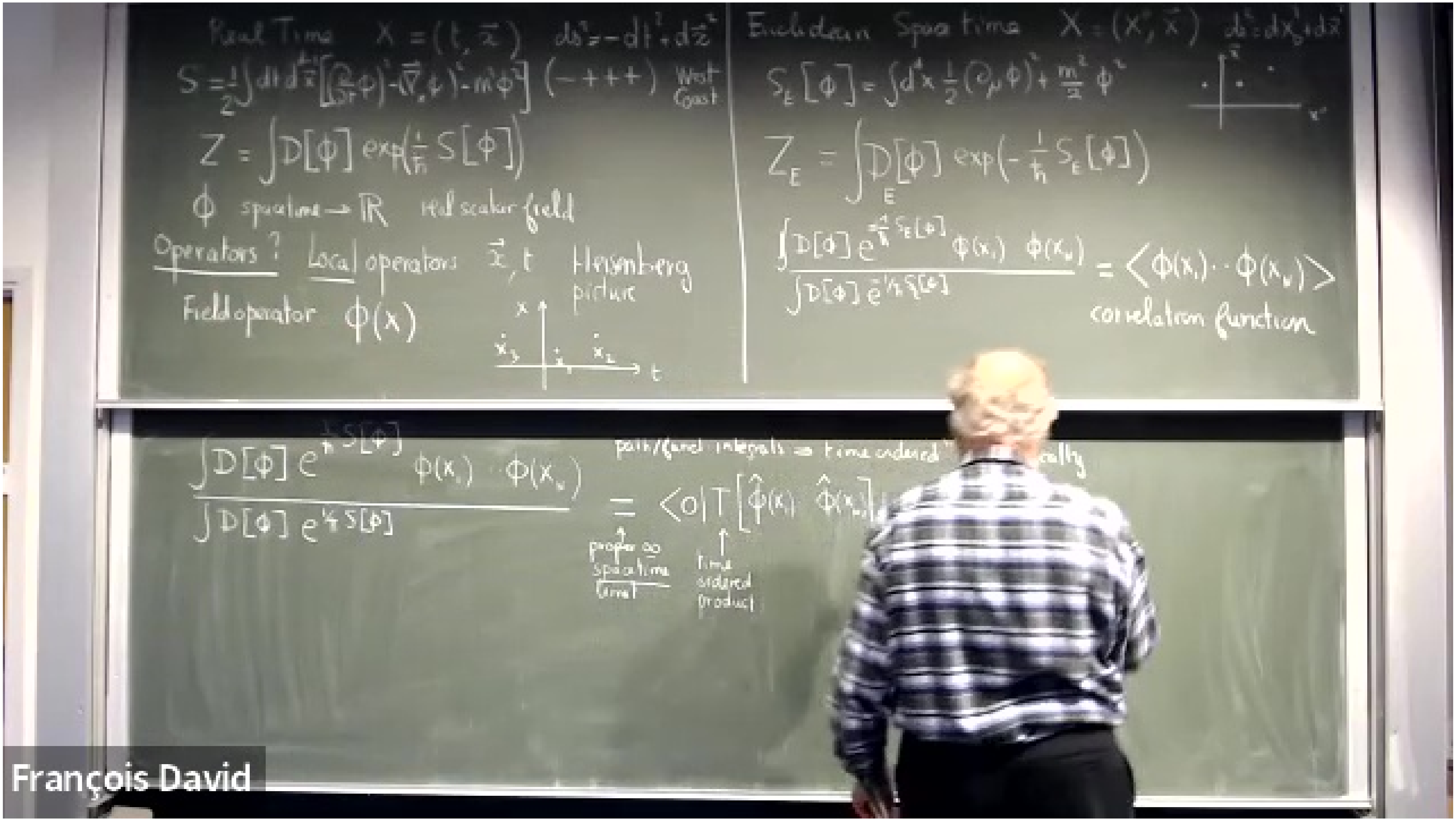
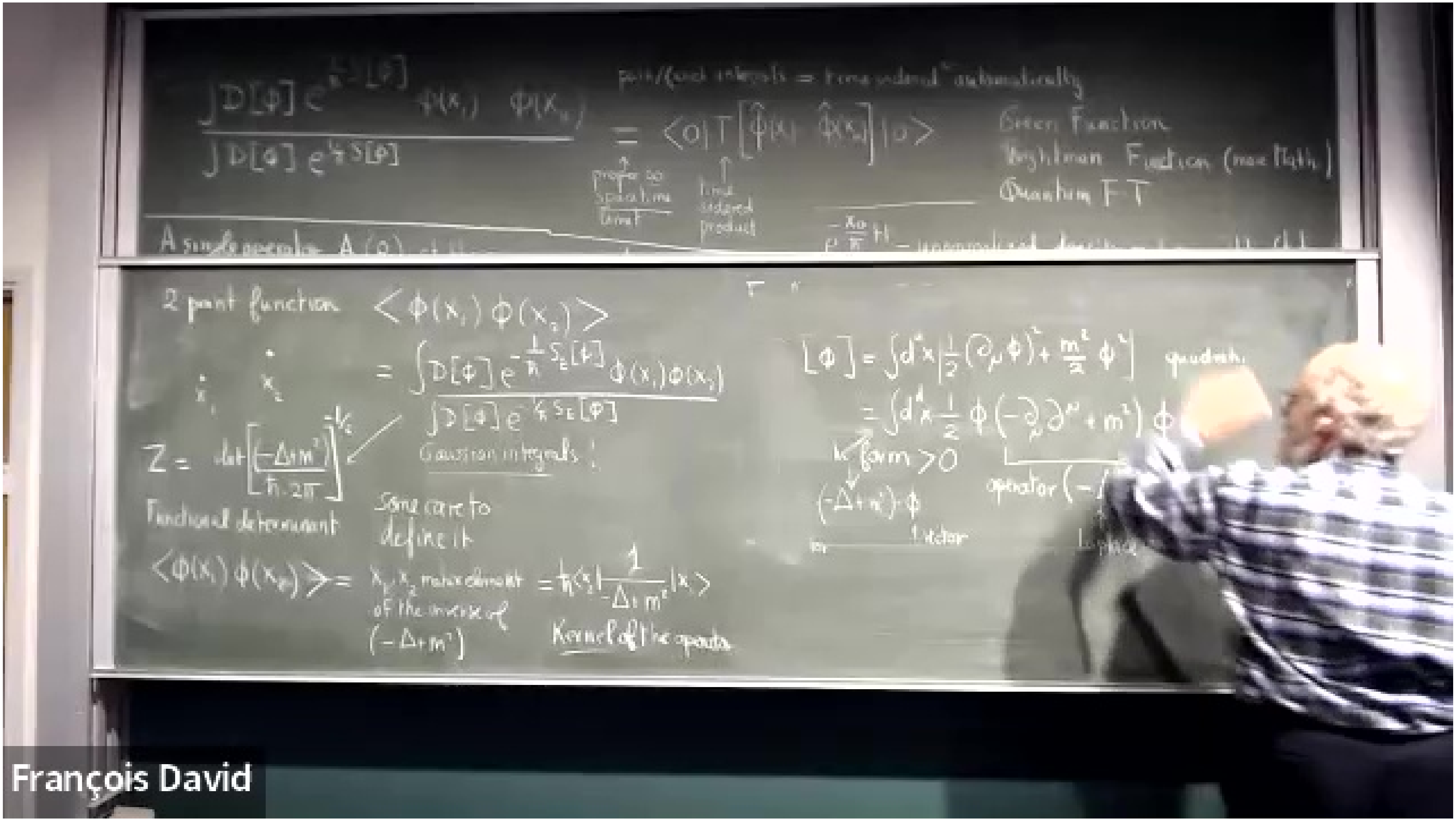

Real Time X = (t, x) ds² = −dt² + dx²
S = 1/2 ∫ dt d^{d−1}x [(∂/∂t φ)² − (∇_x φ)² − m²φ²] (− + + +) West Coast
Z = ∫ D[φ] exp(i/ħ S[φ])
φ spacetime → ℝ real scalar field
Operators ? Local operators x, t Heisenberg picture
Fieldoperator φ(x)
Euclidean Space time X = (x⁰, x) ds² = dx₀² + dx²
S_E[φ] = ∫ d^d x 1/2 (∂_μ φ)² + m²/2 φ²
Z_E = ∫ D_E[φ] exp(− 1/ħ S_E[φ])
François David


Real Time X = (t, x) ds² = -dt² + dx²
(- + + +) West Coast
Z = ∫D[φ] exp(i/ħ S[φ])
φ spacetime → ℝ real scalar field
Operators ? Local operators x, t Heisenberg picture
Field operator φ(x)
Euclidean Spacetime X = (x⁰, x) ds² = dx₀² + dx²
Z_E = ∫D_E[φ] exp(-1/ħ S_E[φ])
path/funct. integrals = time ordered "automatically"
= <0|T[φ̂(x₁) φ̂(x₂)]|0>
proper ∞ spacetime limit
time ordered product
François David


Real Time
Wast Coast
Euclidean Spacetime
φ spacetime → ℝ real scalar field
Operators ? Local operators
Heisenberg picture
Fieldoperator φ(x)
correlation function
path/funct integrals ⇒ time ordered
proper ∞ spacetime (time)
time ordered product
François David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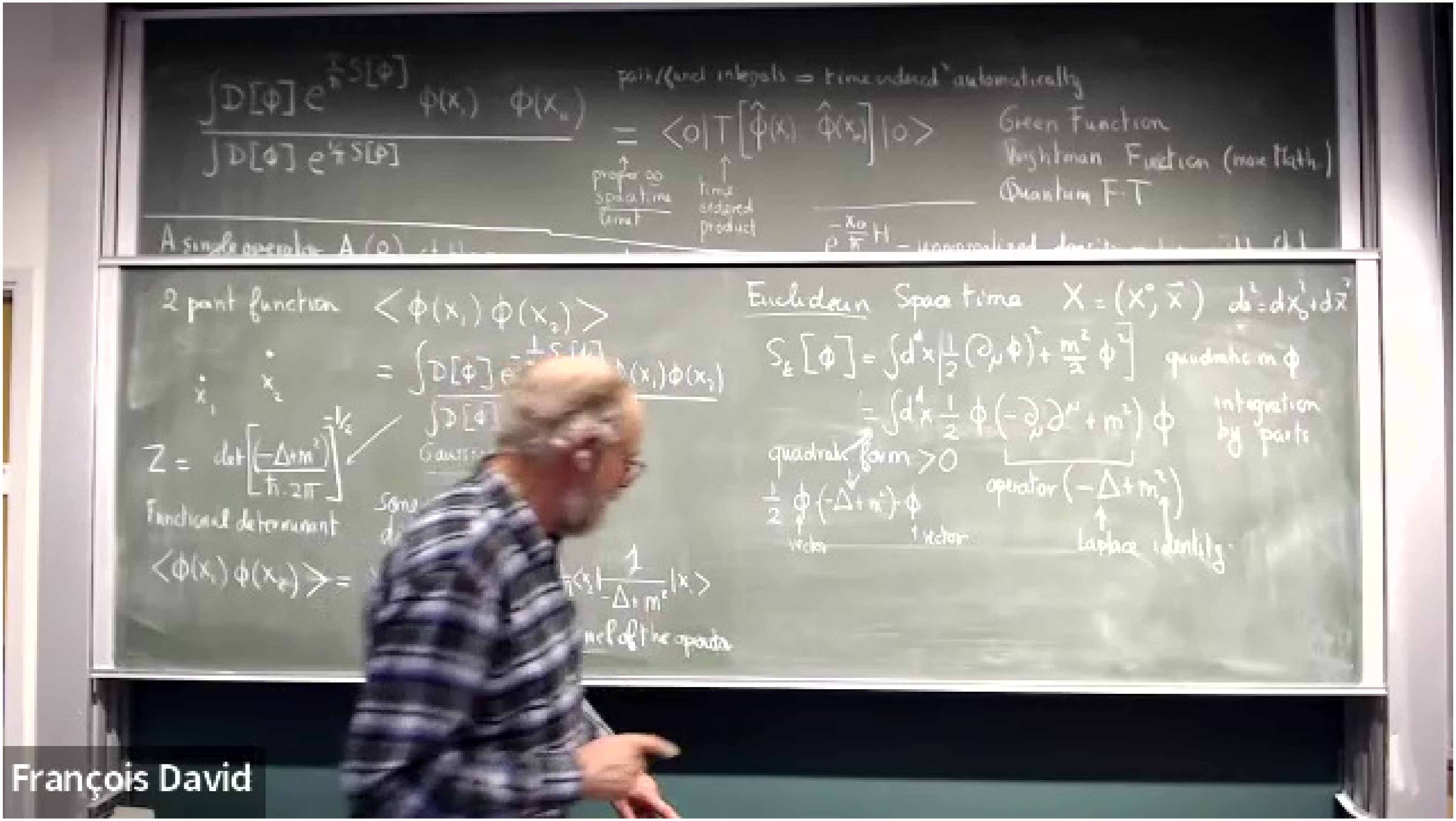
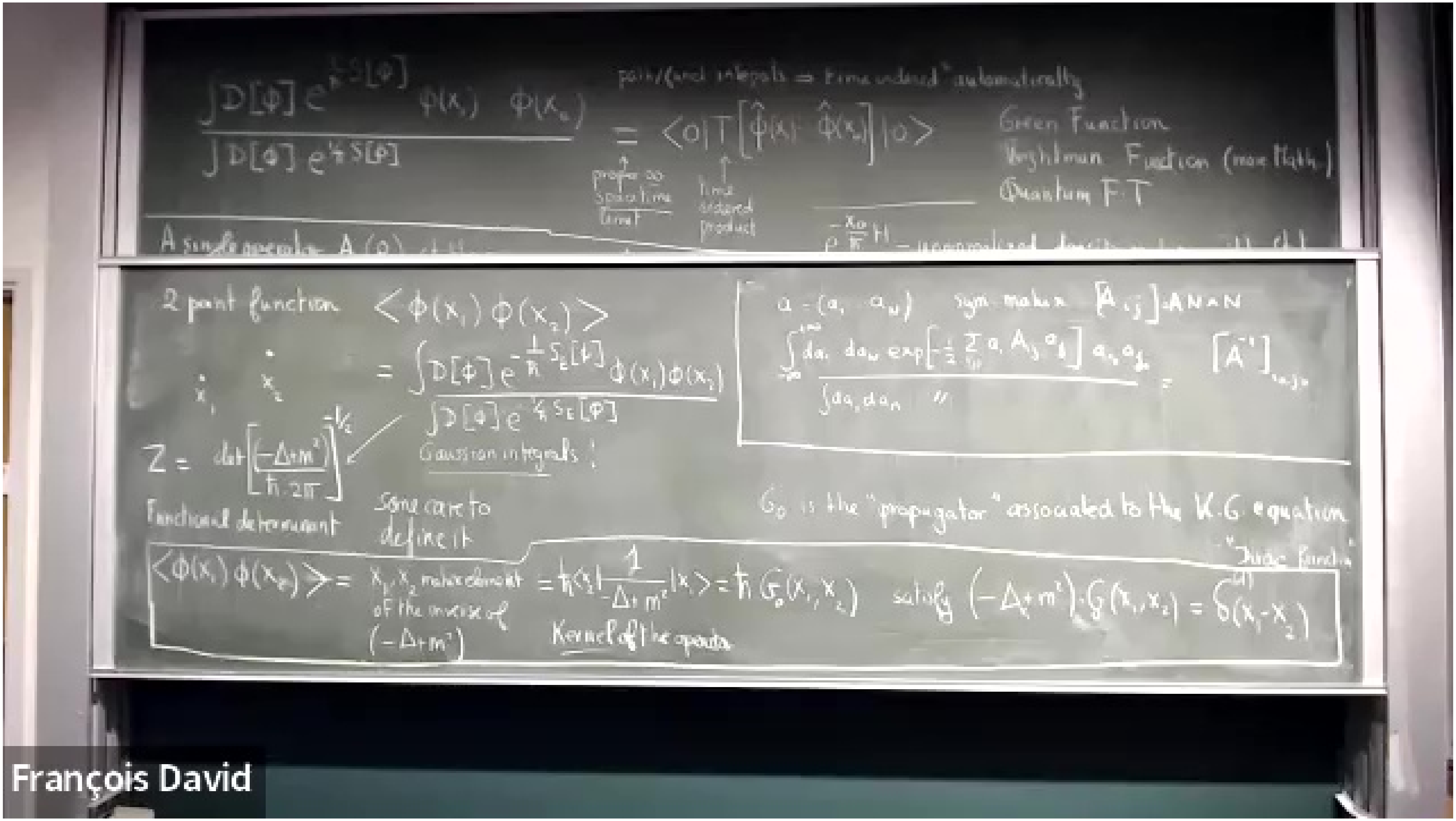
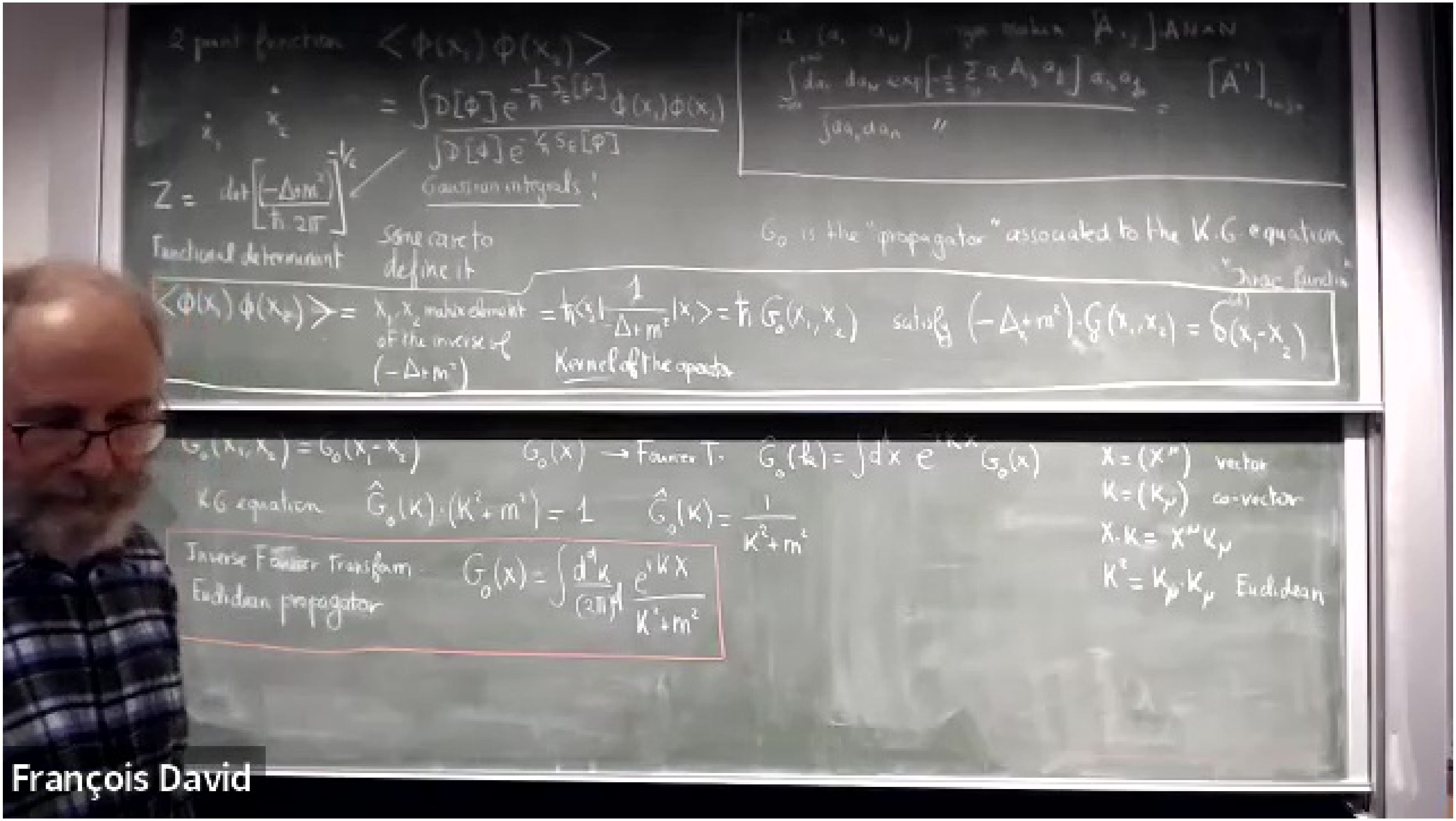
2 point function
Gaussian integrals !
Functional determinant
Some care to define it
of the inverse of
Kernel of the operator
Green Function
Quantum F.T.
proper on space time
time ordered product
operator


François David
Green Function
Quantum F.T
time ordered product
2 point function
Euclidean Space time
quadratic in φ
integration by parts
quadratic form >0
operator
Laplace identity
Functional determinant



path/funct integrals ⇒ time ordered automatically
Green Function
Wightman Function
Quantum F.T
proper as space time limit
time ordered product
2 point function
Gaussian integrals !
Functional determinant
Some care to define it
sym matrix
G_0 is the "propagator" associated to the K.G equation
"Dirac Function"
of the inverse of
Kernel of the operator
François David


2 point function $\langle \phi(x_1)\phi(x_2) \rangle$
$= \dfrac{\int D[\phi]\, e^{-\frac{1}{\hbar}S_E[\phi]}\,\phi(x_1)\phi(x_2)}{\int D[\phi]\, e^{-\frac{1}{\hbar}S_E[\phi]}}$
Gaussian integrals!
$Z = \det\left[\dfrac{(-\Delta+m^2)}{\hbar\, 2\pi}\right]^{-1/2}$
Functional determinant
Some care to define it
$\dfrac{\int da_1 \dots da_N \exp\left[-\frac{1}{2}\sum a_i A_{ij} a_j\right] a_k a_l}{\int da_1 \dots da_N \;''} = [A^{-1}]_{k,l}$
G_0 is the "propagator" associated to the K.G equation
"Dirac function"
$\langle \phi(x_1)\phi(x_2)\rangle =$ x_1, x_2 matrix element of the inverse of $(-\Delta+m^2)$ $= \hbar \langle x_1 | \frac{1}{-\Delta+m^2} | x_1 \rangle = \hbar\, G_0(x_1,x_2)$ satisfy $(-\Delta+m^2)\, G(x_1,x_2) = \delta^{(d)}(x_1-x_2)$
Kernel of the operator
$G_0(x_1,x_2) = G_0(x_1-x_2)$
$G_0(x) \to$ Fourier T. $\hat{G}_0(k) = \int d^dx\, e^{-ikx}\, G_0(x)$
K.G equation $\hat{G}_0(k)\cdot(k^2+m^2) = 1$ $\hat{G}_0(k) = \dfrac{1}{k^2+m^2}$
Inverse Fourier Transform
Euclidean propagator
$G_0(x) = \int \dfrac{d^dk}{(2\pi)^d}\, \dfrac{e^{ikx}}{k^2+m^2}$
$x = (x^\mu)$ vector
$k = (k_\mu)$ co-vector
$x.k = x^\mu k_\mu$
$k^2 = k_\mu k_\mu$ Euclidean
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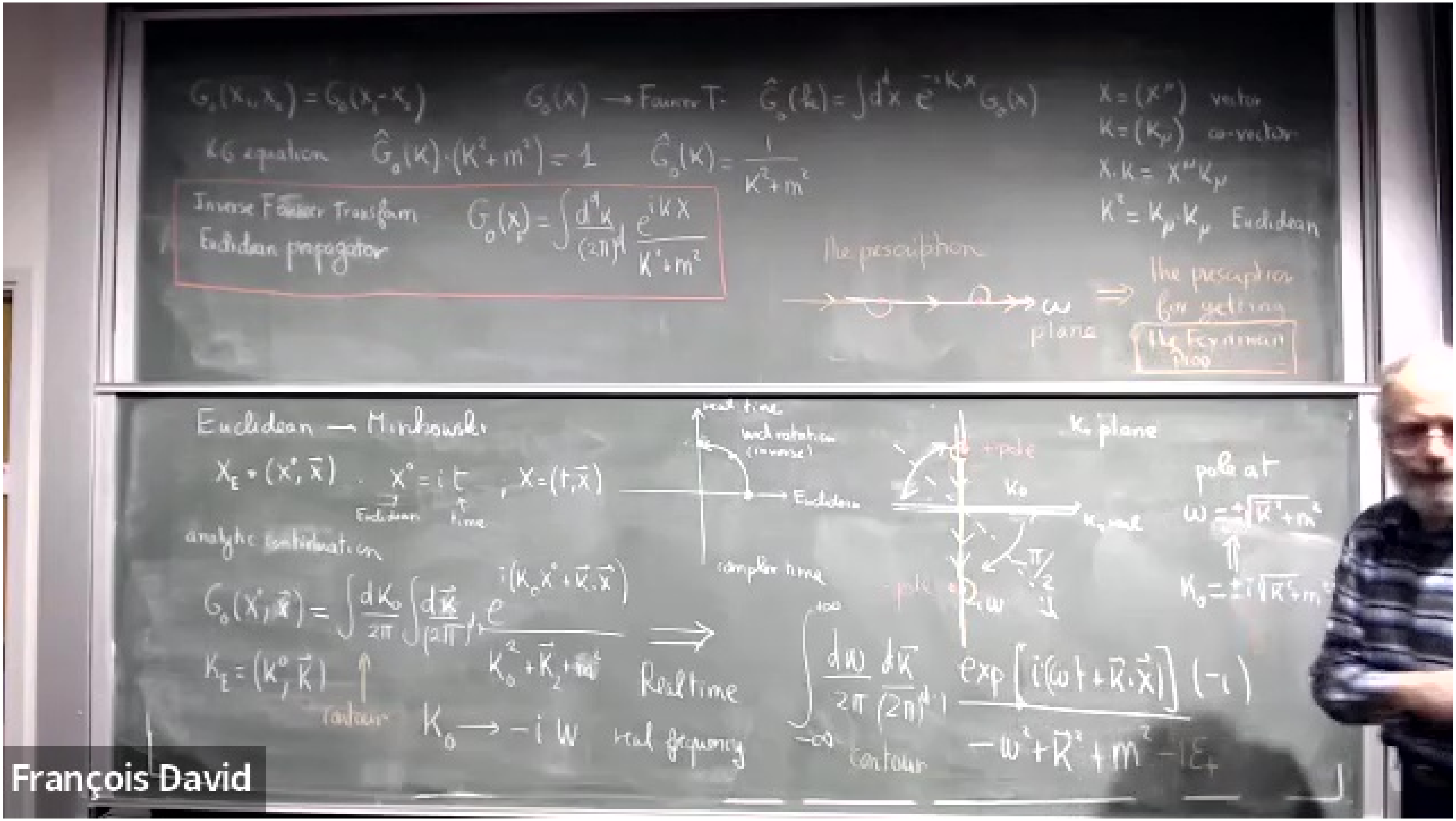
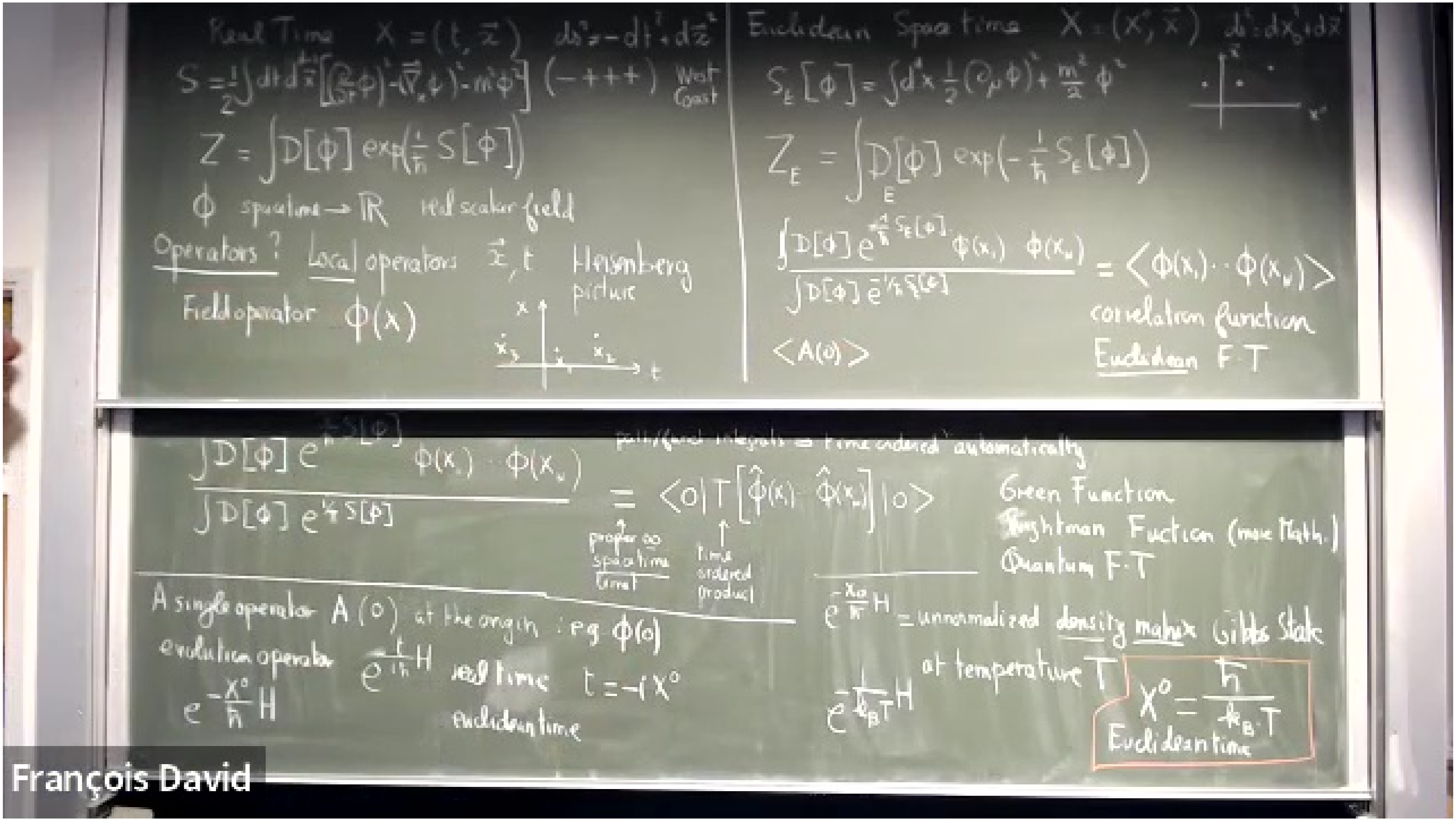

Fourier T.
x = (x^μ) vector
K = (K_μ) co-vector
KG equation
Inverse Fourier Transform
Euclidean propagator
Euclidean
Euclidean → Minkowski
analytic continuation
Real time
real frequency
contour
complex time
K₀ plane
pole at
François David


Real Time
Euclidean Space time
Operators ? Local operators
Heisenberg picture
Field operator
real scalar field
correlation function
Euclidean F.T
Green Function
Quantum F.T
time ordered product
A single operator A(0) at the origin : e.g. ϕ(0)
evolution operator
real time
euclidean time
= unnormalized density matrix Gibbs State
at temperature T
Euclidean time
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