

Title: PSI Lecture - Condensed Matter - Lecture 3

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Collection: PSI Lecture - Condensed Matter

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Today! More second quantization

Last time: Introduced c^\dagger, c using Slater determinants

Correction! Normalization factor should be $\frac{1}{\sqrt{N!}}$ (not $\frac{1}{\sqrt{N}}$)

Many-particle Hilbert space
with basis

$|0\rangle, |\phi_1\rangle, \dots, |\phi_N\rangle, \frac{1}{\sqrt{2}} \begin{vmatrix} |\phi_1\rangle & |\phi_2\rangle \\ |\phi_2\rangle & |\phi_2\rangle \end{vmatrix}, \dots$
Write in terms of c^\dagger .

$$\{c_i^\dagger, c_j^\dagger\} = 0$$

Ok to define basis w/
 c^\dagger operators in any order.
But must be consistent

$|0\rangle, \underbrace{c_1^\dagger|0\rangle, c_2^\dagger|0\rangle, \dots, c_N^\dagger|0\rangle}_{1 \text{ particle}}, \underbrace{c_1^\dagger c_2^\dagger|0\rangle, \dots, c_{N-1}^\dagger c_N^\dagger|0\rangle}_{N \text{-particle}}, \underbrace{c_1^\dagger c_2^\dagger c_3^\dagger|0\rangle, \dots, \dots}_{3 \text{-particle}}, \dots, \underbrace{c_1^\dagger c_2^\dagger \dots c_N^\dagger|0\rangle}_{N \text{-particle state}}$

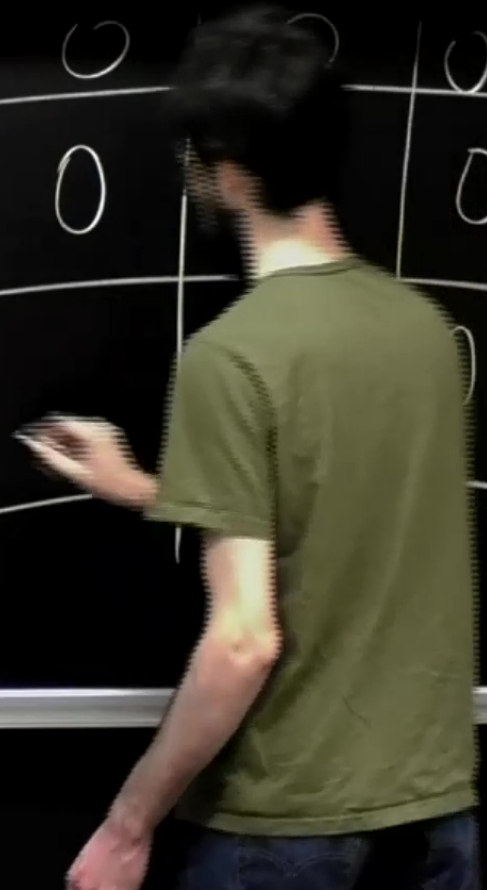
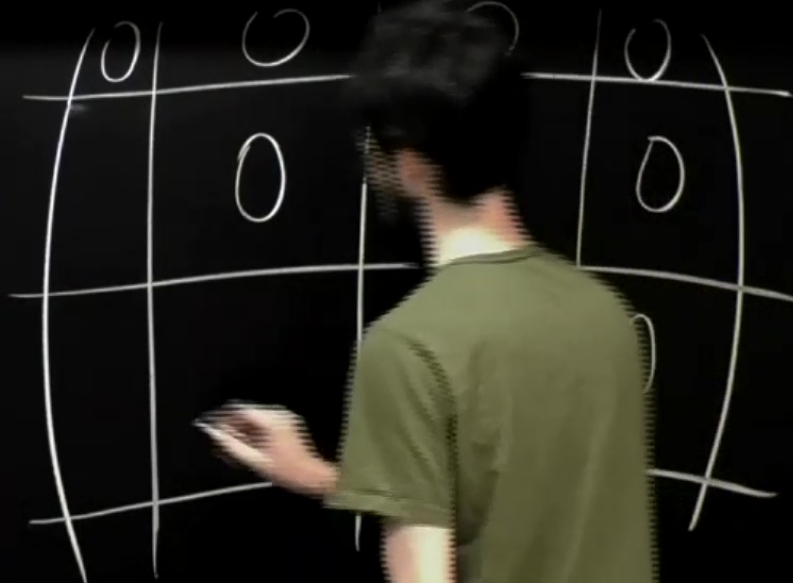
$|0\rangle = \underbrace{c_1^\dagger|0\rangle, c_2^\dagger|0\rangle, \dots, c_N^\dagger|0\rangle}_{1 \text{ particle}}, \underbrace{c_1^\dagger c_2^\dagger|0\rangle, \dots, c_{N-1}^\dagger c_N^\dagger|0\rangle}_{N \text{ particle}}, \underbrace{c_1^\dagger c_2^\dagger \dots c_N^\dagger|0\rangle}_{3 \text{ particle}}, \dots, \underbrace{c_1^\dagger c_2^\dagger \dots c_N^\dagger|0\rangle}_{N \text{ particle state}}$

$$|\mathbf{q}\rangle = \frac{1}{\sqrt{V}} e^{i\mathbf{k}\cdot\mathbf{r}}$$

eg 3-orbital system

Find matrix of c_2^\dagger

- $|0\rangle \rightarrow c_2^\dagger|0\rangle$
- $c_1^\dagger|0\rangle \rightarrow c_2^\dagger c_1^\dagger|0\rangle = -c_1^\dagger c_2^\dagger|0\rangle$
- $c_2^\dagger|0\rangle \rightarrow 0$
- $c_3^\dagger|0\rangle \rightarrow c_2^\dagger c_3^\dagger|0\rangle$
- $c_1^\dagger c_2^\dagger|0\rangle \rightarrow c_2^\dagger c_1^\dagger c_2^\dagger|0\rangle = -c_1^\dagger c_2^\dagger c_2^\dagger|0\rangle$
- $c_1^\dagger c_3^\dagger|0\rangle \rightarrow c_2^\dagger c_1^\dagger c_3^\dagger|0\rangle = -c_1^\dagger c_2^\dagger c_3^\dagger|0\rangle$
- $c_2^\dagger c_2^\dagger|0\rangle \rightarrow 0$
- $c_1^\dagger c_2^\dagger c_3^\dagger|0\rangle \rightarrow 0$



$|0\rangle = \underbrace{c_1^\dagger|0\rangle, c_2^\dagger|0\rangle, \dots, c_N^\dagger|0\rangle}_{1 \text{ particle}}, \underbrace{c_1^\dagger c_2^\dagger|0\rangle, \dots, c_{N-1}^\dagger c_N^\dagger|0\rangle}_{N \text{ particle}}, \underbrace{c_1^\dagger c_2^\dagger c_3^\dagger|0\rangle, \dots, \dots}_{3 \text{ particle}}, \dots, \underbrace{c_1^\dagger c_2^\dagger \dots c_N^\dagger|0\rangle}_{N \text{ particle state}}$

$$|q_{\vec{k}}\rangle = \frac{1}{\sqrt{V}} e^{i\vec{k}\cdot\vec{r}}$$

eg 3-orbital system

Find matrix of c_2^\dagger

- $|0\rangle \rightarrow c_2^\dagger|0\rangle$
- $c_1^\dagger|0\rangle \rightarrow c_2^\dagger c_1^\dagger|0\rangle = -c_1^\dagger c_2^\dagger|0\rangle$
- $c_2^\dagger|0\rangle \rightarrow 0$
- $c_3^\dagger|0\rangle \rightarrow c_2^\dagger c_3^\dagger|0\rangle$
- $c_1^\dagger c_2^\dagger|0\rangle \rightarrow c_2^\dagger c_1^\dagger c_2^\dagger|0\rangle = -c_1^\dagger c_2^\dagger c_2^\dagger|0\rangle$
- $c_1^\dagger c_3^\dagger|0\rangle \rightarrow c_2^\dagger c_1^\dagger c_3^\dagger|0\rangle = -c_1^\dagger c_2^\dagger c_3^\dagger|0\rangle$
- $c_2^\dagger c_2^\dagger|0\rangle \rightarrow 0$
- $c_2^\dagger c_3^\dagger|0\rangle \rightarrow 0$

0	0	0	0
0	0	0	0
-1	0	0	0
0	0	0	0
0	0	-1	0
0	0	0	0

Second quantized Hamiltonian

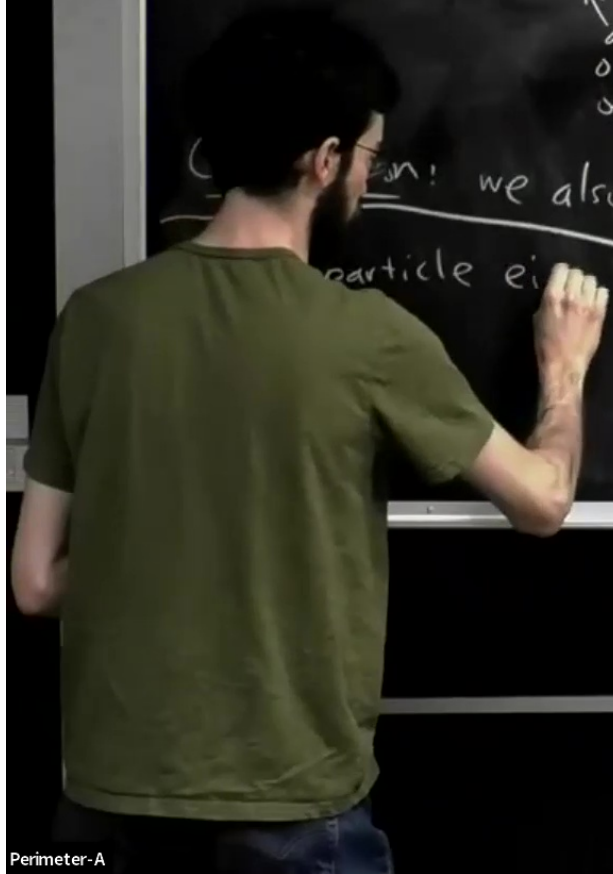
Return to free electron gas, orbitals are $|0_{\vec{k}}\rangle = \frac{1}{\sqrt{V}} e^{i\vec{k}\cdot\vec{r}}$ ↑ quantized

$|0\rangle, \{c_{\vec{k}}^{\dagger}|0\rangle\}, \{c_{\vec{k}_1}^{\dagger}c_{\vec{k}_2}^{\dagger}|0\rangle\}, \dots$

↑ all objects of this form

← many-particle basis

Reason: we also need spin, "spin-orbital" → actually 2 for every \vec{k} ,
 $c_{\vec{k},\uparrow}^{\dagger}, c_{\vec{k},\downarrow}^{\dagger} \rightarrow c_{\vec{k},s}^{\dagger}$ " " $c_{\vec{k},\uparrow}^{\dagger}$ or $c_{\vec{k},\downarrow}^{\dagger}$



$10\rangle, \{c_{\vec{k}}^{\dagger} | 10\rangle\}$ all objects of this form
 $\{c_{\vec{k}_1}^{\dagger}, c_{\vec{k}_2}^{\dagger} | 10\rangle\}, \dots$ ← many-particle basis
 Correction: we also have
 Many-particle eigenstates
 All the basis states above
 "spin-orbital" → actually 2 for every \vec{k}_i
 $c_{\vec{k}_i, \uparrow}^{\dagger}, c_{\vec{k}_i, \downarrow}^{\dagger} \rightarrow c_{\vec{k}_i, \uparrow}^{\dagger} \text{ or } c_{\vec{k}_i, \downarrow}^{\dagger}$
 $\left[\prod_{i=1}^N c_{\vec{k}_i, \sigma_i}^{\dagger} | 0\rangle \right] = \left(\sum_i \frac{\hbar^2 k_i^2}{2m} \right) \left[\prod_{i=1}^N c_{\vec{k}_i, \sigma_i}^{\dagger} \right] | 0\rangle$

of this form
Correction: we also need spin, "spin-orbital" \rightarrow actually 2 for every \vec{k}_i

Many-particle eigenstates

All the basis states above:

$$H \left[\prod_{i=1}^N c_{\vec{k}_i, \sigma_i}^\dagger |0\rangle \right] = \left(\sum_i \frac{\hbar^2 k_i^2}{2m} \right) \left[\prod_{i=1}^N c_{\vec{k}_i, \sigma_i}^\dagger \right] |0\rangle$$

$c_{\vec{k}_i, \uparrow}^\dagger, c_{\vec{k}_i, \downarrow}^\dagger \rightarrow c_{\vec{k}_i, \uparrow}^\dagger$ or $c_{\vec{k}_i, \downarrow}^\dagger$

$c_{\vec{k}_1, \uparrow} |0\rangle = c_{\vec{k}_1, \uparrow} |0\rangle$
 $c_{\vec{k}_1, \downarrow} |0\rangle = c_{\vec{k}_1, \downarrow} |0\rangle$
 $c_{\vec{k}_1, \uparrow} c_{\vec{k}_2, \downarrow} |0\rangle = c_{\vec{k}_2, \downarrow} c_{\vec{k}_1, \uparrow} |0\rangle$
 $c_{\vec{k}_1, \downarrow} c_{\vec{k}_2, \uparrow} |0\rangle = c_{\vec{k}_2, \uparrow} c_{\vec{k}_1, \downarrow} |0\rangle$

0	0	0	1	0	0
0	0	0	-1	0	0

All the basis states above: $H \left[\left(\prod_{i=1}^N c_{\vec{k}_i, \sigma_i}^\dagger \right) |0\rangle \right] = \left(\sum_i \frac{\hbar^2 k_i^2}{2m} \right) \left[\left(\prod_{i=1}^N c_{\vec{k}_i, \sigma_i}^\dagger \right) |0\rangle \right]$

$c_{\vec{k}, \uparrow}, c_{\vec{k}, \downarrow} \rightarrow c_{\vec{k}, \uparrow}^\dagger$ or $c_{\vec{k}, \downarrow}^\dagger$

First guess: use the fact that if $H|v_i\rangle = E_i|v_i\rangle$, $\langle v_i|v_j\rangle = \delta_{ij}$
 Then $H = \sum_i E_i |v_i\rangle \langle v_i|$

$$H = \sum_{\text{sets of occupied orbitals}} \left(\sum_i \frac{\hbar^2 k_i^2}{2m} \right) \left(\prod_{i=1}^N c_{\vec{k}_i, \sigma_i}^\dagger \right) |0\rangle \langle 0| \left(\prod_{i=1}^N c_{\vec{k}_i, \sigma_i} \right)$$

This is correct, but not ideal

Better

$$H = \sum_{\vec{k}, \sigma} E_{\vec{k}} c_{\vec{k}, \sigma}^\dagger c_{\vec{k}, \sigma}$$

- 2 ways to prove:
- Show equality with
 - Show that it has the correct eigenstates & energies

$$H = \sum_{\vec{k}, \sigma} E_{\vec{k}} c_{\vec{k}, \sigma}^\dagger c_{\vec{k}, \sigma}$$

- Show equality with)
- Show that it has the

Proof Want to show! ⊛

$$\left(\sum_{\vec{k}, \sigma} E_{\vec{k}} c_{\vec{k}, \sigma}^\dagger c_{\vec{k}, \sigma} \right) \left(\prod_i c_{\vec{k}_i, \sigma_i}^\dagger \right) |0\rangle = \sum_{\vec{k}, \sigma} \left(E_{\vec{k}} \delta_{\vec{k}, \vec{k}_1} \delta_{\sigma, \sigma_1} c_{\vec{k}_1, \sigma_1}^\dagger + c_{\vec{k}_1, \sigma_1}^\dagger E_{\vec{k}} c_{\vec{k}, \sigma}^\dagger c_{\vec{k}, \sigma} \right) \prod_{i=2}^N c_{\vec{k}_i, \sigma_i}^\dagger |0\rangle$$

↑ replace b/c

$$= c_{\vec{k}_1, \sigma_1}^\dagger \left(\sum_{\vec{k}, \sigma} E_{\vec{k}} (\delta_{\vec{k}, \vec{k}_1} \delta_{\sigma, \sigma_1} + c_{\vec{k}, \sigma}^\dagger c_{\vec{k}, \sigma}) \prod_{i=2}^N c_{\vec{k}_i, \sigma_i}^\dagger |0\rangle \right)$$

$$= c_{\vec{k}_1, \sigma_1}^\dagger c_{\vec{k}_2, \sigma_2}^\dagger \left(\sum_{\vec{k}, \sigma} E_{\vec{k}} (\delta_{\vec{k}, \vec{k}_1} \delta_{\sigma, \sigma_1} + c_{\vec{k}, \sigma}^\dagger c_{\vec{k}, \sigma}) \prod_{i=3}^N c_{\vec{k}_i, \sigma_i}^\dagger |0\rangle \right)$$

$$\left(E_{\vec{k}} c_{\vec{k}, \sigma}^\dagger c_{\vec{k}, \sigma} \right) \left(c_{\vec{k}_1, \sigma_1}^\dagger \right)$$

$$= E_{\vec{k}} c_{\vec{k}, \sigma}^\dagger (\delta_{\vec{k}, \vec{k}_1} \delta_{\sigma, \sigma_1} - c_{\vec{k}_1, \sigma_1}^\dagger c_{\vec{k}, \sigma})$$

$$= E_{\vec{k}} \delta_{\vec{k}, \vec{k}_1} \delta_{\sigma, \sigma_1} c_{\vec{k}, \sigma}^\dagger + c_{\vec{k}_1, \sigma_1}^\dagger c_{\vec{k}, \sigma}^\dagger c_{\vec{k}, \sigma}$$

$$\text{⊛ } H \left[\left(\prod_{i=1}^N c_{\vec{k}_i, \sigma_i}^\dagger \right) |0\rangle \right] = \left(\sum_i \frac{\hbar^2 k_i^2}{2m} \right) \prod_{i=1}^N c_{\vec{k}_i, \sigma_i}^\dagger |0\rangle$$

$$H = \sum_{\vec{k}, \sigma} E_{\vec{k}} c_{\vec{k}, \sigma}^\dagger c_{\vec{k}, \sigma}$$

- Show equality with)
- Show that it has the

Proof Want to show! ⊗

$$\begin{aligned} & \left(\sum_{\vec{k}, \sigma} E_{\vec{k}} c_{\vec{k}, \sigma}^\dagger c_{\vec{k}, \sigma} \right) \left(\prod_{i=1}^N c_{\vec{k}_i, \sigma_i}^\dagger \right) |0\rangle = \sum_{\vec{k}, \sigma} \left(E_{\vec{k}} \delta_{\vec{k}\vec{k}_1} \delta_{\sigma\sigma_1} c_{\vec{k}_1, \sigma_1}^\dagger + c_{\vec{k}_1, \sigma_1}^\dagger E_{\vec{k}} c_{\vec{k}\sigma}^\dagger c_{\vec{k}\sigma} \right) \prod_{i=2}^N c_{\vec{k}_i, \sigma_i}^\dagger |0\rangle \\ & \quad \text{replace b/c of } \delta \text{ for } \\ & = c_{\vec{k}_1, \sigma_1}^\dagger \left(\sum_{\vec{k}, \sigma} E_{\vec{k}} (\delta_{\vec{k}\vec{k}_1} \delta_{\sigma\sigma_1} + c_{\vec{k}\sigma}^\dagger c_{\vec{k}\sigma}) \prod_{i=2}^N c_{\vec{k}_i, \sigma_i}^\dagger |0\rangle \right) \\ & = c_{\vec{k}_1, \sigma_1}^\dagger c_{\vec{k}_2, \sigma_2}^\dagger \left(\sum_{\vec{k}, \sigma} E_{\vec{k}} (\delta_{\vec{k}\vec{k}_1} \delta_{\sigma\sigma_1} + (\vec{k}\vec{k}_2 \delta_{\sigma\sigma_2} + c_{\vec{k}\sigma}^\dagger c_{\vec{k}\sigma})) \prod_{i=3}^N c_{\vec{k}_i, \sigma_i}^\dagger |0\rangle \right) \end{aligned}$$

$$\begin{aligned} & \left(\sum_{\vec{k}, \sigma} E_{\vec{k}} c_{\vec{k}, \sigma}^\dagger c_{\vec{k}, \sigma} \right) \left(\prod_{i=1}^N c_{\vec{k}_i, \sigma_i}^\dagger \right) |0\rangle \\ & = E_{\vec{k}_1} c_{\vec{k}_1, \sigma_1}^\dagger (\delta_{\vec{k}\vec{k}_1} \delta_{\sigma\sigma_1} + c_{\vec{k}\sigma}^\dagger c_{\vec{k}\sigma}) \prod_{i=2}^N c_{\vec{k}_i, \sigma_i}^\dagger |0\rangle \\ & = E_{\vec{k}_1} \delta_{\vec{k}\vec{k}_1} \delta_{\sigma\sigma_1} c_{\vec{k}_1, \sigma_1}^\dagger \prod_{i=2}^N c_{\vec{k}_i, \sigma_i}^\dagger |0\rangle \end{aligned}$$

$$\otimes H \left[\left(\prod_{i=1}^N c_{\vec{k}_i, \sigma_i}^\dagger \right) |0\rangle \right] = \left(\sum_i \frac{\hbar^2 k_i^2}{2m} \right) \left[\left(\prod_{i=1}^N c_{\vec{k}_i, \sigma_i}^\dagger \right) |0\rangle \right]$$

$$\begin{aligned}
& \left(\prod_{i=1}^N c_{k_i, \sigma_i}^+ \right) |0\rangle = \sum_{k_1, \sigma_1} E_{k_1} \delta_{k_1, \sigma_1} c_{k_1, \sigma_1}^+ \left(\prod_{i=2}^N c_{k_i, \sigma_i}^+ \right) |0\rangle \\
& \left(\prod_{i=1}^N c_{k_i, \sigma_i}^+ \right) |0\rangle = \sum_{k_1, \sigma_1} E_{k_1} \delta_{k_1, \sigma_1} \left(c_{k_1, \sigma_1}^+ c_{k_1, \sigma_1} \right) \left(\prod_{i=2}^N c_{k_i, \sigma_i}^+ \right) |0\rangle \\
& \left(\prod_{i=1}^N c_{k_i, \sigma_i}^+ \right) |0\rangle = \sum_{k_1, \sigma_1} E_{k_1} \delta_{k_1, \sigma_1} \left(\delta_{k_1, \sigma_1} - c_{k_1, \sigma_1}^+ c_{k_1, \sigma_1} \right) \left(\prod_{i=2}^N c_{k_i, \sigma_i}^+ \right) |0\rangle \\
& \left(\prod_{i=1}^N c_{k_i, \sigma_i}^+ \right) |0\rangle = \sum_{k_1, \sigma_1} E_{k_1} \delta_{k_1, \sigma_1} c_{k_1, \sigma_1}^+ \left(\prod_{i=2}^N c_{k_i, \sigma_i}^+ \right) |0\rangle \\
& \left(\prod_{i=1}^N c_{k_i, \sigma_i}^+ \right) |0\rangle = \sum_{k_1, \sigma_1} E_{k_1} \delta_{k_1, \sigma_1} \left(\prod_{i=2}^N c_{k_i, \sigma_i}^+ \right) \left(\sum_{k_2, \sigma_2} E_{k_2} \delta_{k_2, \sigma_2} + c_{k_2, \sigma_2}^+ c_{k_2, \sigma_2} \right) \left(\prod_{i=3}^N c_{k_i, \sigma_i}^+ \right) |0\rangle \\
& \left(\prod_{i=1}^N c_{k_i, \sigma_i}^+ \right) |0\rangle = \left(\prod_{i=1}^N c_{k_i, \sigma_i}^+ \right) \left(\sum_{k_1, \sigma_1} E_{k_1} \delta_{k_1, \sigma_1} \left(\prod_{i=2}^N c_{k_i, \sigma_i}^+ \right) \right) |0\rangle \\
& \left(\prod_{i=1}^N c_{k_i, \sigma_i}^+ \right) |0\rangle = \left(\prod_{i=1}^N c_{k_i, \sigma_i}^+ \right) \left(\sum_{k_1, \sigma_1} E_{k_1} \delta_{k_1, \sigma_1} \right) |0\rangle = \left(\prod_{i=1}^N c_{k_i, \sigma_i}^+ \right) \left(\sum_{k_1, \sigma_1} E_{k_1} \right) |0\rangle
\end{aligned}$$

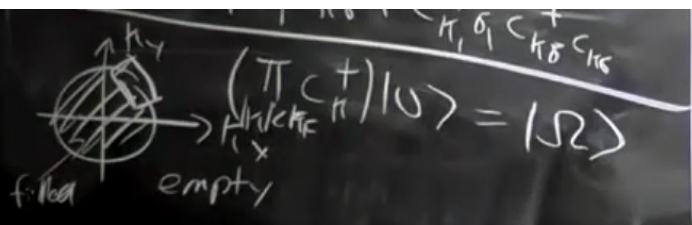
↑ replace b/c of δ for

Perimeter-A

$$\begin{aligned}
& \sum_{\vec{k}, \sigma} E_{\vec{k}} \delta_{\vec{k}, \vec{k}_0} \delta_{\sigma, \sigma_0} c_{\vec{k}\sigma} + c_{\vec{k}_1 \sigma_1} E_{\vec{k}} c_{\vec{k}\sigma} c_{\vec{k}\sigma} \prod_{i=2}^N c_{\vec{k}_i \sigma_i} |0\rangle \\
& = c_{\vec{k}_1 \sigma_1} \left(\sum_{\vec{k}, \sigma} E_{\vec{k}} (\delta_{\vec{k}, \vec{k}_1} \delta_{\sigma, \sigma_1} + c_{\vec{k}\sigma} c_{\vec{k}\sigma}) \prod_{i=2}^N c_{\vec{k}_i \sigma_i} |0\rangle \right) \\
& = c_{\vec{k}_1 \sigma_1} c_{\vec{k}_2 \sigma_2} \left(\sum_{\vec{k}, \sigma} E_{\vec{k}} (\delta_{\vec{k}, \vec{k}_1} \delta_{\sigma, \sigma_1} + \delta_{\vec{k}, \vec{k}_2} \delta_{\sigma, \sigma_2} + c_{\vec{k}\sigma} c_{\vec{k}\sigma}) \prod_{i=3}^N c_{\vec{k}_i \sigma_i} |0\rangle \right) \\
& = \left(\prod_{i=1}^3 c_{\vec{k}_i \sigma_i} \right) \left(\sum_{\vec{k}, \sigma} E_{\vec{k}} (c_{\vec{k}\sigma} c_{\vec{k}\sigma} + \sum_i \delta_{\vec{k}, \vec{k}_i} \delta_{\sigma, \sigma_i}) |0\rangle \right) \\
& = \left(\prod_{i=1}^3 c_{\vec{k}_i \sigma_i} \right) \left(\sum_{\vec{k}, \sigma} \left(\sum_{\vec{k}_0} E_{\vec{k}} \delta_{\vec{k}, \vec{k}_0} \delta_{\sigma, \sigma_0} \right) |0\rangle \right) = \left(\prod_{i=1}^3 c_{\vec{k}_i \sigma_i} \right) \left(\sum_{\vec{k}} E_{\vec{k}} \right) |0\rangle
\end{aligned}$$

replace b/c of δ for

$$\begin{aligned}
& E_{\vec{k}} c_{\vec{k}\sigma} c_{\vec{k}\sigma} c_{\vec{k}\sigma} \\
& = E_{\vec{k}} c_{\vec{k}\sigma} (\delta_{\vec{k}, \vec{k}_1} \delta_{\sigma, \sigma_1} + c_{\vec{k}\sigma} c_{\vec{k}\sigma}) \\
& = E_{\vec{k}} \delta_{\vec{k}, \vec{k}_1} \delta_{\sigma, \sigma_1} + E_{\vec{k}} c_{\vec{k}\sigma} c_{\vec{k}\sigma} c_{\vec{k}\sigma}
\end{aligned}$$



$$\begin{aligned}
 &= \left(\prod_i c_{k_i, \sigma_i}^+ \right) \left(\sum_{k, \sigma} E_k c_{k, \sigma}^+ c_{k, \sigma} + \sum_{k, \sigma} \delta_{k, \sigma} \right) |0\rangle \\
 &= \left(\prod_i c_{k_i, \sigma_i}^+ \right) \left(\sum_{k, \sigma} E_k \delta_{k, \sigma} \right) |0\rangle = \left(\prod_i c_{k_i, \sigma_i}^+ \right) \left(\sum_i E_{k_i} \right) |0\rangle
 \end{aligned}$$

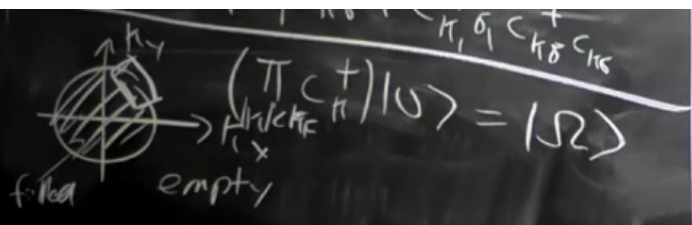
the fact that if $H|v_i\rangle = E_i|v_i\rangle$, $\langle v_i|v_j\rangle = \delta_{ij}$
 Then $H = \sum_i E_i |v_i\rangle \langle v_i|$

$$\left(\sum_i \frac{\hbar^2 k_i^2}{2m} \right) \left(\prod_i c_{k_i, \sigma_i}^+ \right) |0\rangle \langle 0| \left(\prod_i c_{k_i, \sigma_i} \right)$$

This is correct, but not ideal

$$H = \sum_{\vec{k}, \sigma} E_{\vec{k}} c_{\vec{k}, \sigma}^+ c_{\vec{k}, \sigma}$$

- 2 ways to prove:
- Show equality with
 - Show that it has the correct eigenstates & energies



$$= \left(\prod_i c_{k_i, s_i}^+ \right) \left(\sum_{k, s} E_k \left(c_{k, s}^+ c_{k, s} + \sum_i \delta_{k, s} \delta_{s, i} \right) |0\rangle \right)$$

$$= \left(\prod_i c_{k_i, s_i}^+ \right) \left(\sum_{k, s} E_k \delta_{k, s} \delta_{s, i} \right) |0\rangle = \left(\prod_i c_{k_i, s_i}^+ \right) \left(\sum_i E_{k_i} \right) |0\rangle$$

One more definition:

$$n_\alpha = c_\alpha^+ c_\alpha \quad \text{"number operator"}$$

$$n_\alpha \left(\prod_i c_{\alpha_i}^+ \right) |0\rangle = \left(\sum_i \delta_{\alpha, \alpha_i} \right) \left(\prod_i c_{\alpha_i}^+ \right) |0\rangle$$

eig value is 1 if $\alpha \in \{\alpha_i\}$
 0 if $\alpha \notin \{\alpha_i\}$

Useful facts:

- $[n_\alpha, n_\beta] = 0$

- $n_\alpha^2 = n_\alpha, \quad c_\alpha^+ c_\alpha c_\alpha^+ c_\alpha$

Better

$$H = \sum_{\vec{k}, s} E_{\vec{k}} c_{\vec{k}, s}^+ c_{\vec{k}, s}$$

$$H = \sum_i \frac{p_i^2}{2m}$$

N particles,
But also acts
in more than the
physical space
(anti-symmetrized)

$$H = \sum_{\text{configs}} (\sum_i E_i) \pi(t|0) \dots$$

configs - could mean:
→ N-particle configs
→ all configurations

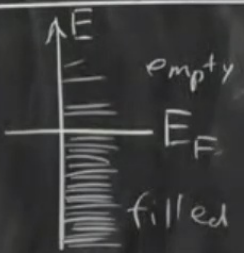
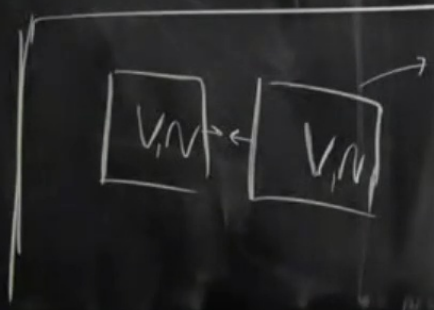
$$\sum E_k n_k$$

no mention of
particles

project
to
physical
space

$$H_{\text{phys}} = H_N$$

project
to only N particle
states



"single-particle picture"
the highest E any one electron has
is E_F
suddenly join, $N \gg N$, $V \gg V$
But max energy of an e- shouldn't jump

$$\frac{1}{\kappa, \beta} \ln \dots$$