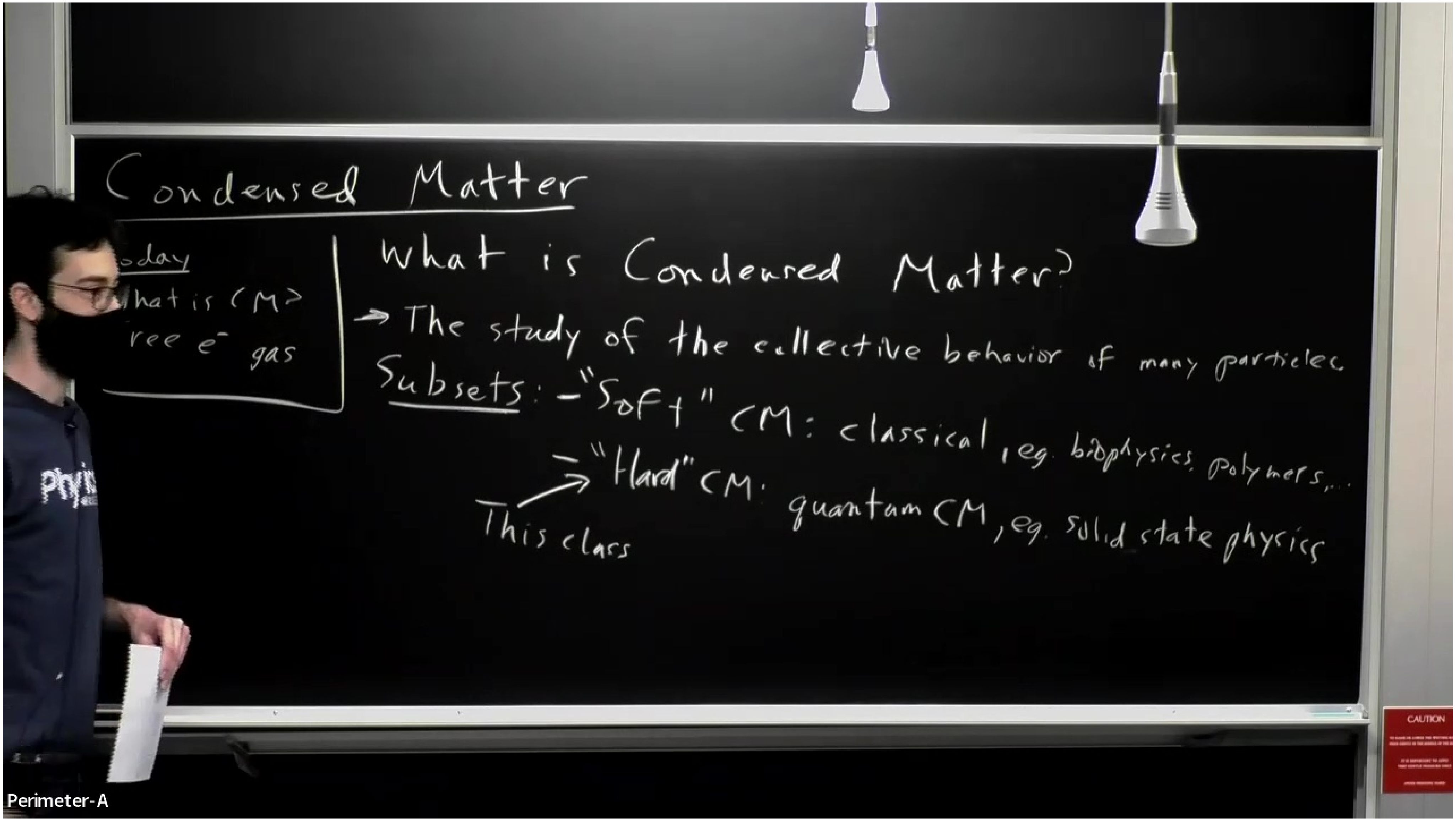
Title: PSI Lecture - Condensed Matter - Lecture 1

Speakers: Aaron Szasz

Collection: PSI Lecture - Condensed Matter

Date: January 10, 2022 - 9:00 AM

URL: https://pirsa.org/22010012


Condensed Matter
oday
hat is CM?
ree e⁻ gas
What is Condensed Matter?
→ The study of the collective behavior of many particles
Subsets: – "Soft" CM: classical, eg. biophysics, polymers, ...
– "Hard" CM: quantum CM, eg. solid state physics
This class
Perimeter-A
CAUTION

# Overview of Hard CM

"Realistic"
"Practical"

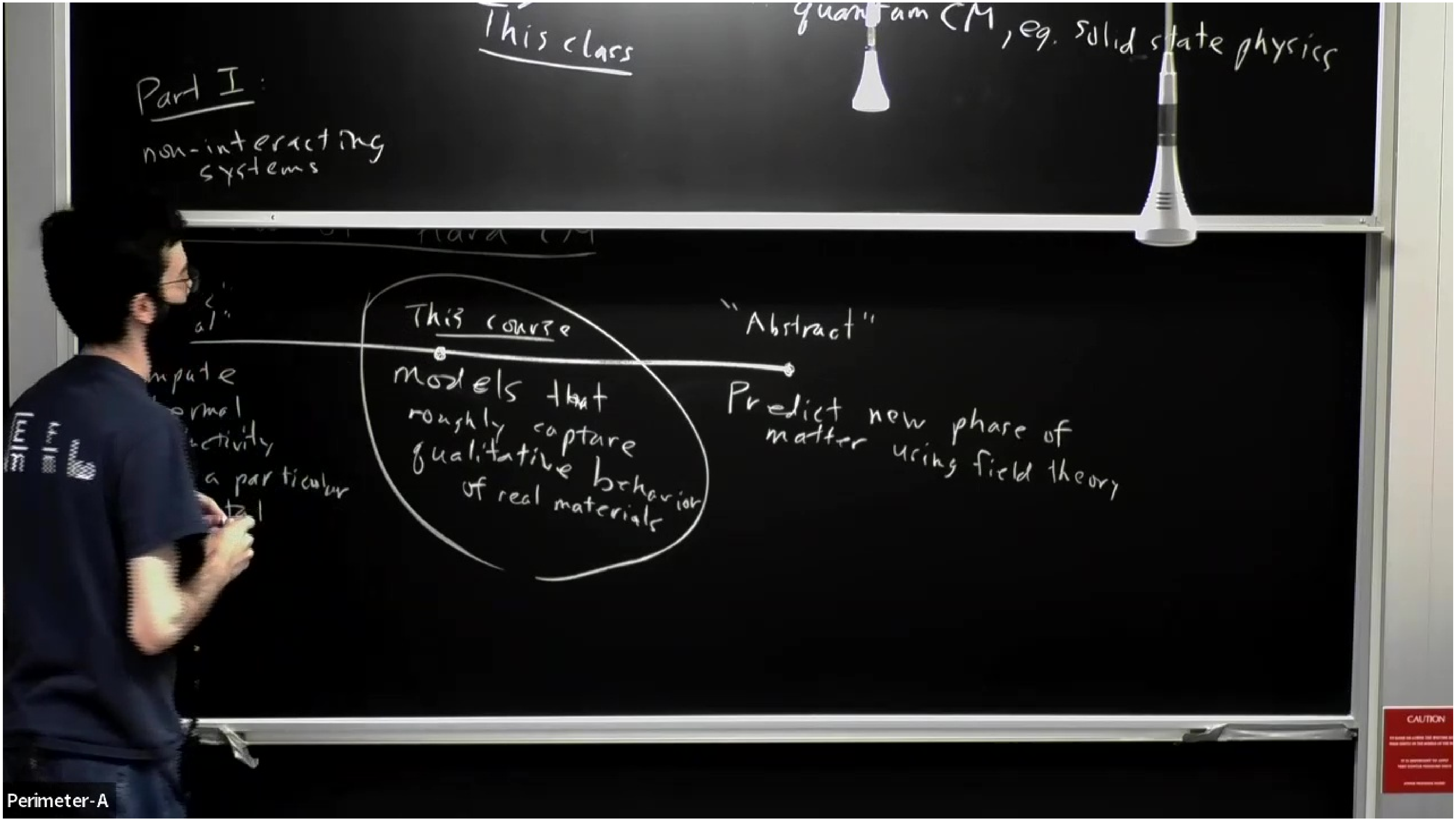
"Abstract"

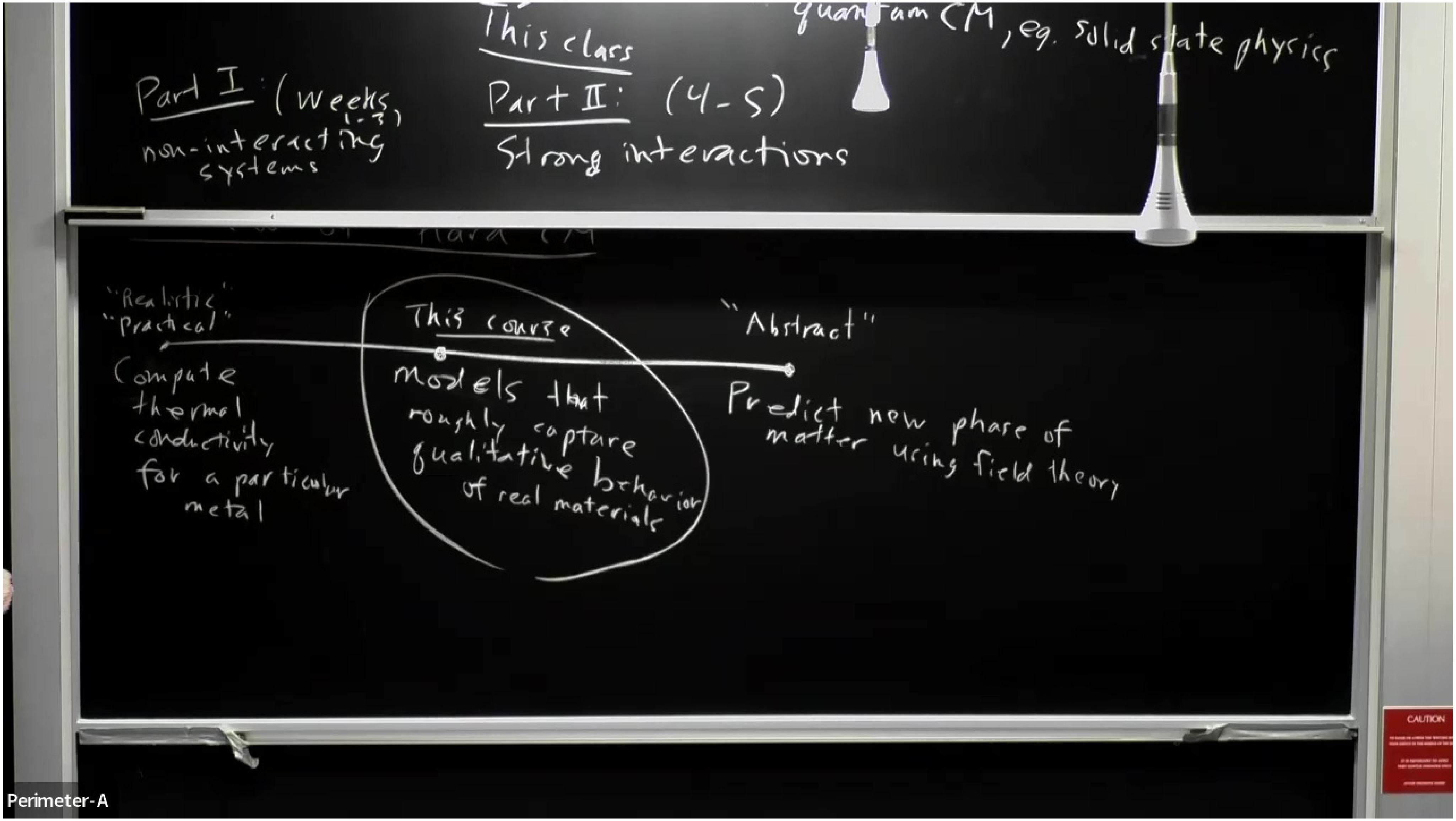
Compute thermal conductivity for a particular metal

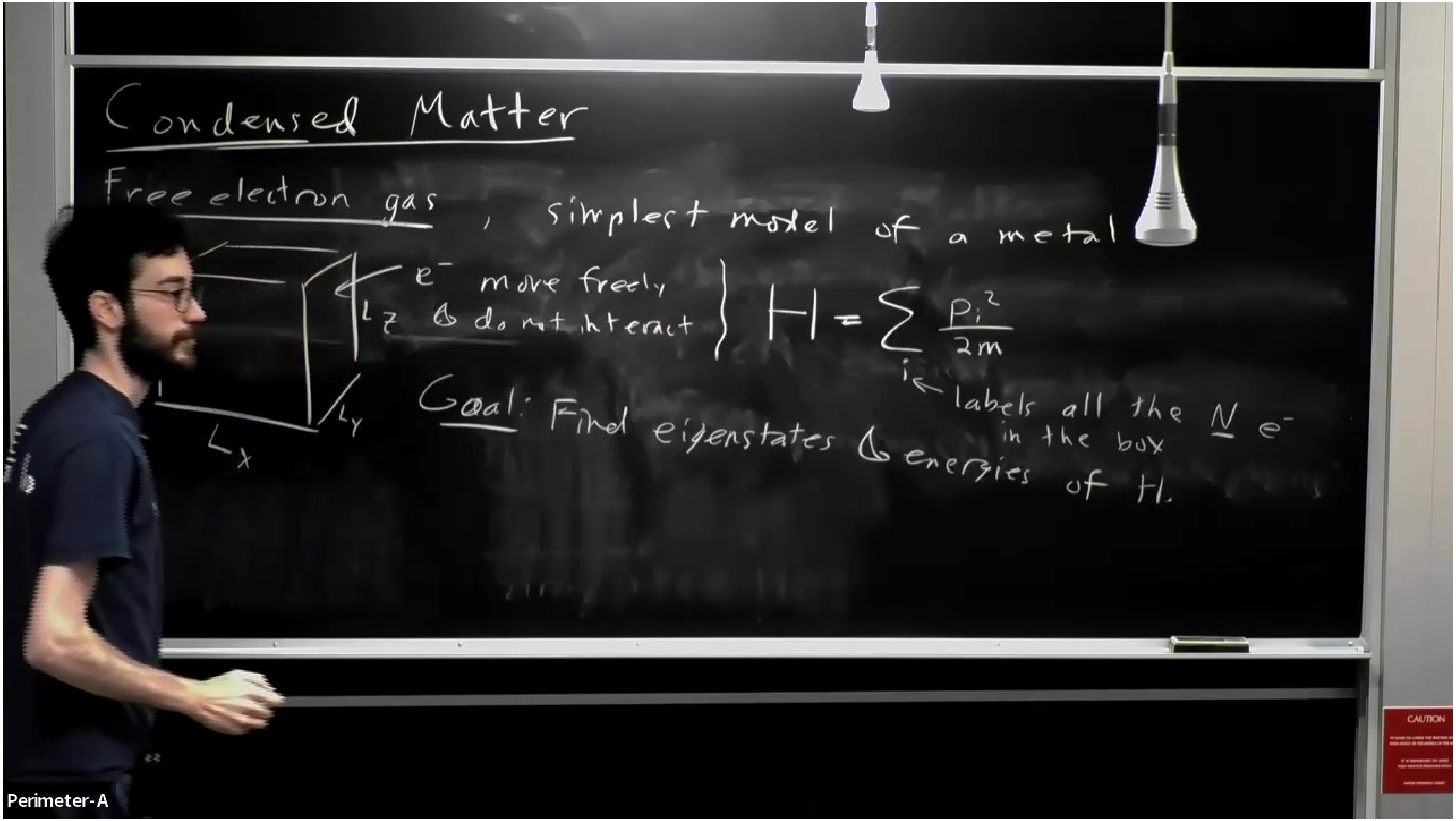
Models that roughly capture qualitative behavior of real materials

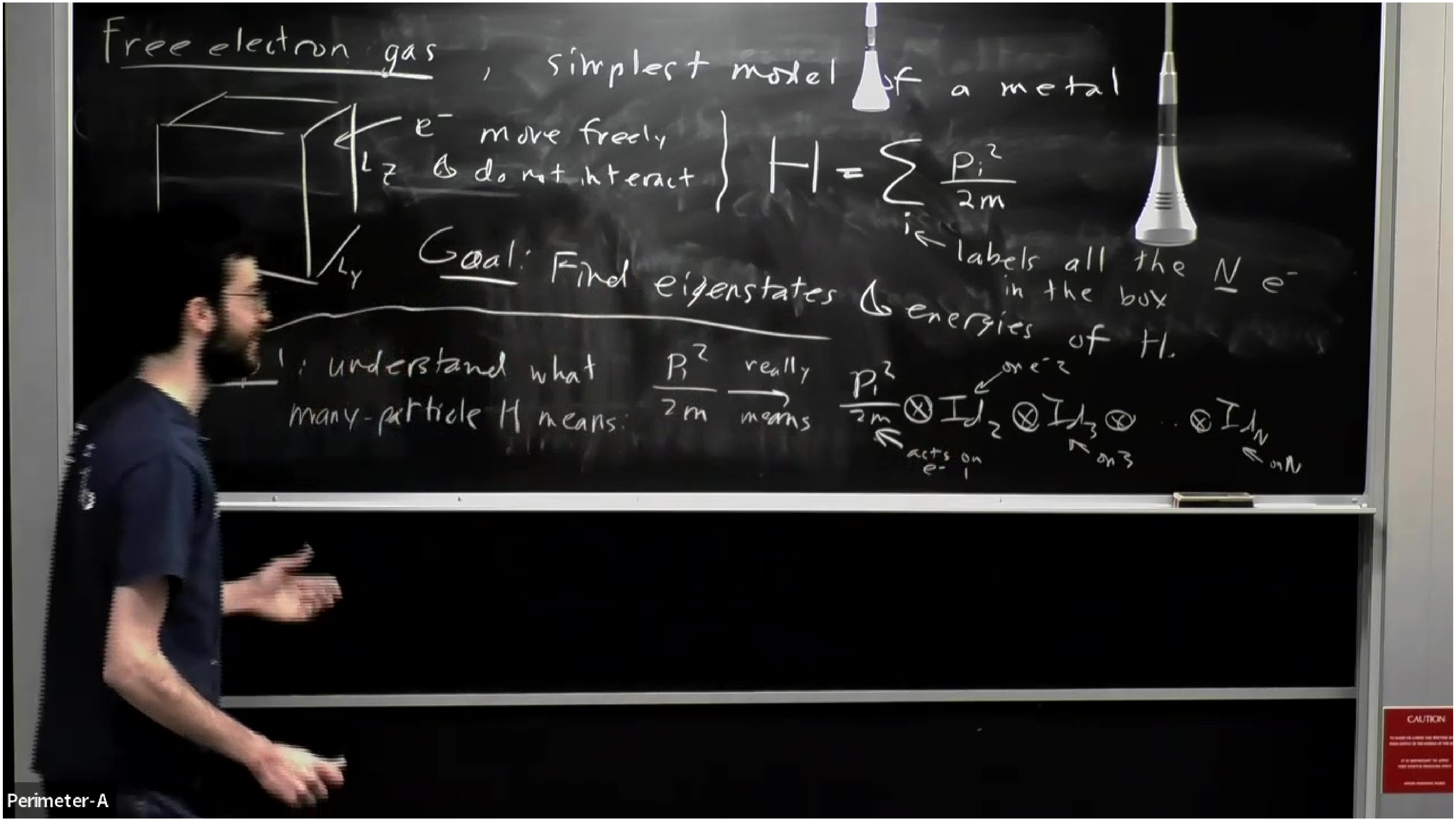
Predict new phase of matter using field theory

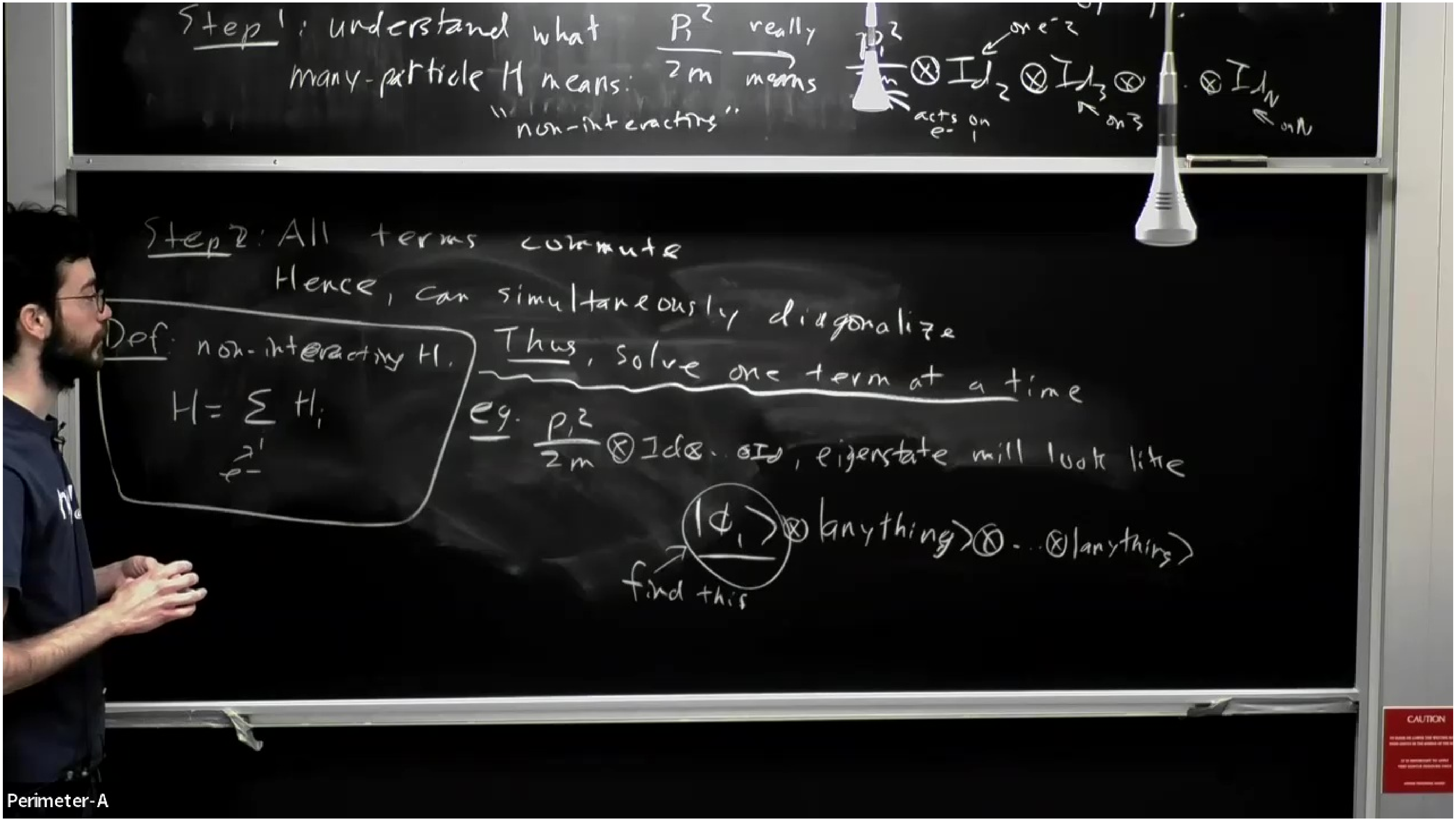
Perimeter-A

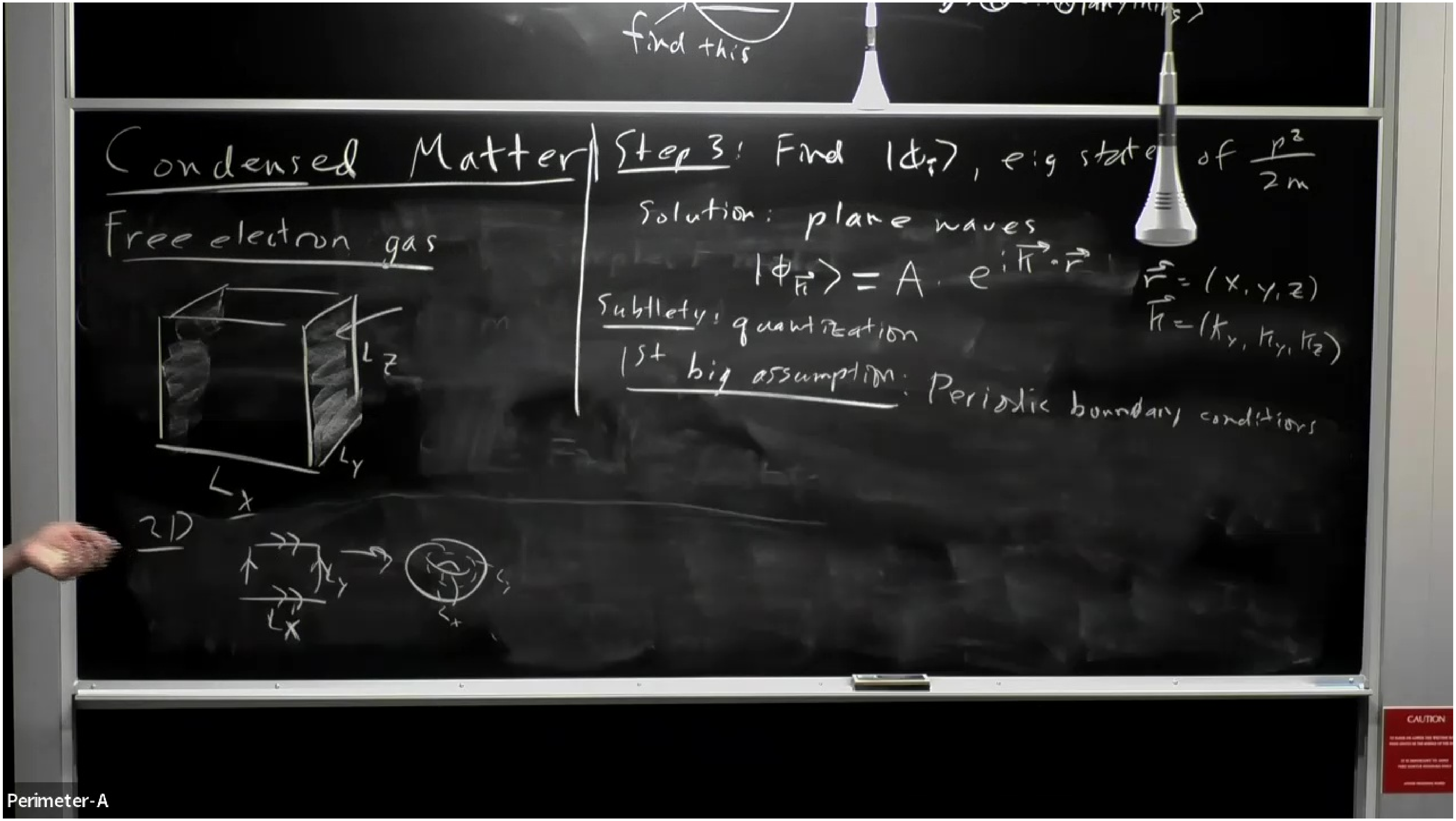

This class
quantum CM, eg. solid state physics
Part I:
non-interacting systems
This course
models that roughly capture qualitative behavior of real materials
"Abstract"
Predict new phase of matter using field theory
a particular
Perimeter-A
CAUTION

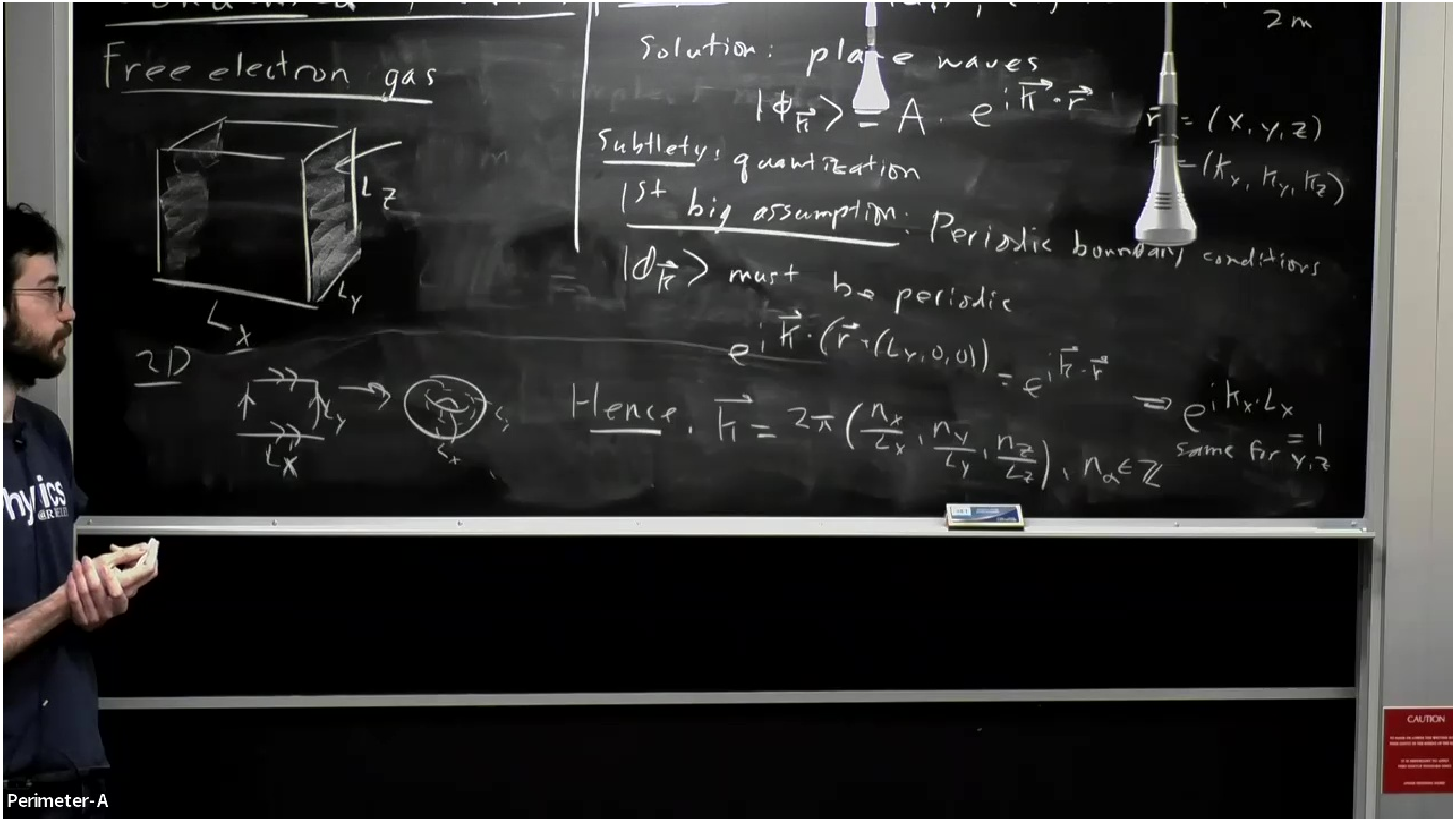

This class
quantum CM, eg. solid state physics
Part I: (weeks 1-3)
non-interacting systems
Part II: (4-5)
Strong interactions
"Realistic"
"Practical"
This course
"Abstract"
Compute thermal conductivity for a particular metal
models that roughly capture qualitative behavior of real materials
Predict new phase of matter using field theory
Perimeter-A

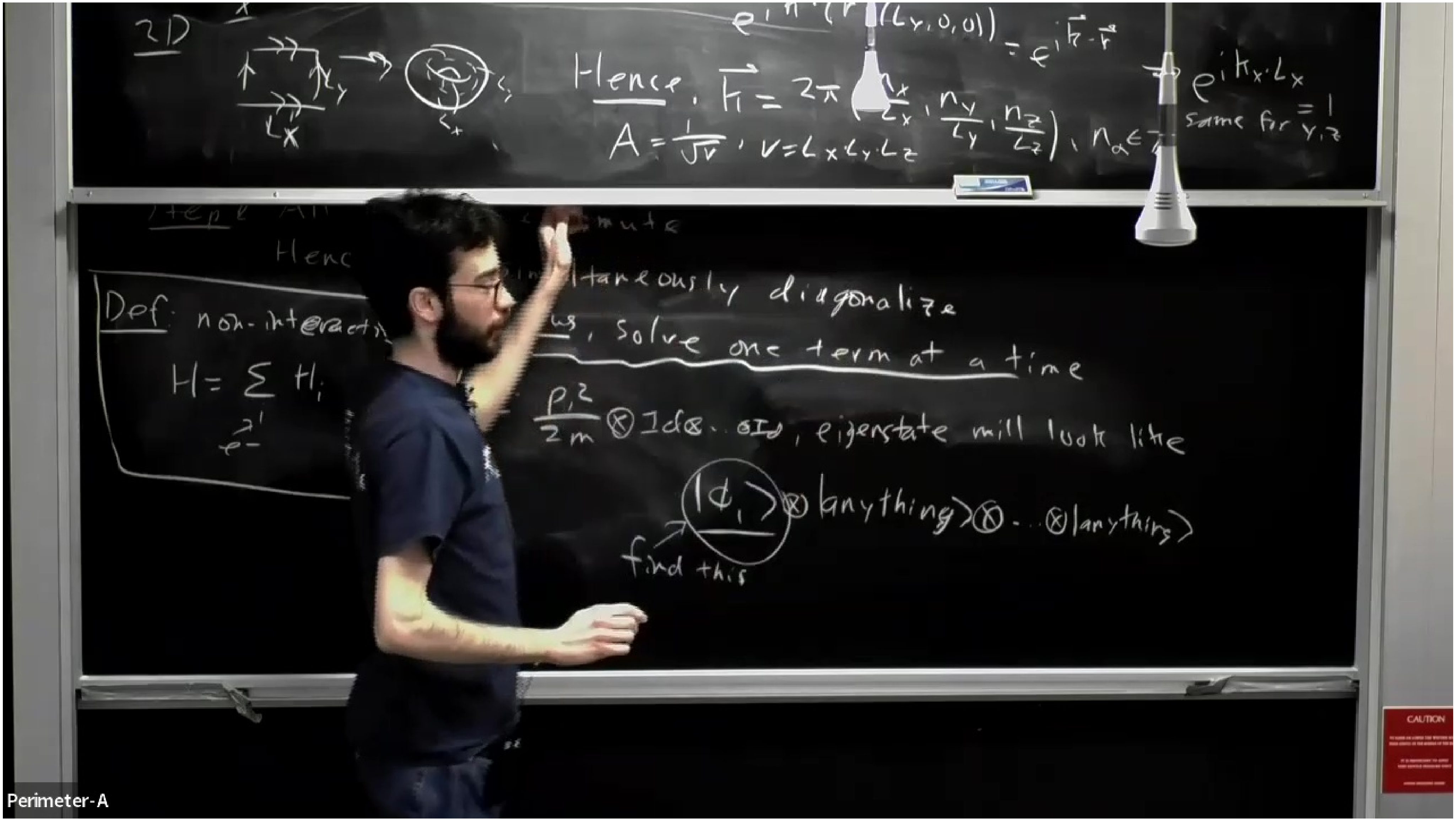

Condensed Matter
Free electron gas , simplest model of a metal
$e^-$ move freely
& do not interact
$H = \sum_i \frac{p_i^2}{2m}$
$L_z$
$L_y$
$L_x$
i ← labels all the N $e^-$ in the box
Goal: Find eigenstates & energies of H.
Perimeter-A
CAUTION

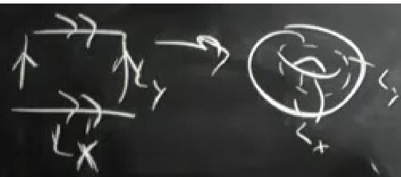

Free electron gas, simplest model of a metal
e⁻ move freely & do not interact
$L_z$
$L_y$
$H = \sum_i \frac{p_i^2}{2m}$
i ← labels all the N e⁻ in the box
Goal: Find eigenstates & energies of H.
1: understand what $\frac{p_1^2}{2m}$ really means
many-particle H means: $\frac{p_1^2}{2m} \otimes I_{d_2} \otimes I_{d_3} \otimes \cdots \otimes I_{d_N}$
acts on e⁻ 1
on e⁻ 2
on 3
on N
Perimeter-A


Step 1: understand what $\frac{P_1^2}{2m}$ really means
many-particle H means: $\frac{p_1^2}{2m} \otimes Id_2 \otimes Id_3 \otimes \ldots \otimes Id_N$
"non-interacting"
one e⁻ 2
acts on e⁻ 1
on 3
on N
Step 2: All terms commute
Hence, can simultaneously diagonalize
Def: non-interacting H.
$H = \sum H_i$
e⁻
Thus, solve one term at a time
e.g. $\frac{p_1^2}{2m} \otimes Id \otimes \ldots \otimes Id$, eigenstate will look like
$|\phi_1\rangle \otimes |anything\rangle \otimes \ldots \otimes |anything\rangle$
find this
Perimeter-A
CAUTION


find this
Condensed Matter
Free electron gas
Step 3: Find |ϕ_k⟩, eig state of p²/2m
Solution: plane waves
|ϕ_k⟩ = A · e^{i k·r}
r = (x, y, z)
k = (k_x, k_y, k_z)
Subtlety: quantization
1st big assumption: Periodic boundary conditions
L_z
L_y
L_x
2D
Perimeter-A
CAUTION


Free electron gas
$L_z$
$L_y$
$L_x$
2D
$L_y$
$L_x$
$L_y$
$L_x$
Solution: plane waves
$|\phi_{\vec{k}}\rangle = A \cdot e^{i\vec{k}\cdot\vec{r}}$
$\vec{r} = (x, y, z)$
$\vec{k} = (k_x, k_y, k_z)$
Subtlety: quantization
1st big assumption: Periodic boundary conditions
$|\phi_{\vec{k}}\rangle$ must be periodic
$e^{i\vec{k}\cdot(\vec{r}+(L_x,0,0))} = e^{i\vec{k}\cdot\vec{r}}$
$\Rightarrow e^{ik_x \cdot L_x} = 1$
same for y, z
Hence, $\vec{k} = 2\pi\left(\frac{n_x}{L_x}, \frac{n_y}{L_y}, \frac{n_z}{L_z}\right)$, $n_\alpha \in \mathbb{Z}$
2m
Perimeter-A


2D
Hence, $\vec{k} = 2\pi\left(\frac{n_x}{L_x}, \frac{n_y}{L_y}, \frac{n_z}{L_z}\right)$, $n_\alpha \in \mathbb{Z}$
$A = \frac{1}{\sqrt{V}}$, $V = L_x L_y L_z$
$e^{ik_x L_x} = 1$
same for y, z
Def: non-interacting
$H = \sum H_i$
simultaneously diagonalize
solve one term at a time
$\frac{p_1^2}{2m} \otimes Id \otimes \ldots \otimes Id$, eigenstate will look like
$|\phi_1\rangle \otimes |anything\rangle \otimes \ldots \otimes |anything\rangle$
find this
Perimeter-A




$= e^{i\vec{k}\cdot\vec{r}}$

Hence, $\vec{k} = 2\pi\left(\frac{n_x}{L_x}, \frac{n_y}{L_y}, \frac{n_z}{L_z}\right)$, $n_\alpha \in \mathbb{Z}$

$A = \frac{1}{\sqrt{V}}$, $V = L_x \cdot L_y \cdot L_z$

$\Rightarrow e^{ik_x \cdot L_x} = 1$

same for $y, z$

## Step 4 many-particle eigenstates

eigenstates of $\frac{p_1^2}{2m} \otimes I_d \otimes \cdots \otimes I_d$, $|\phi_{\vec{k}}\rangle_1 \otimes \_\_ \otimes \cdots \otimes \_\_$

$I_d \otimes \frac{p_2^2}{2m} \otimes \cdots$ $\quad \_\_ \otimes |\phi_{\vec{k}}\rangle_2 \otimes \_\_ \quad \_\_$

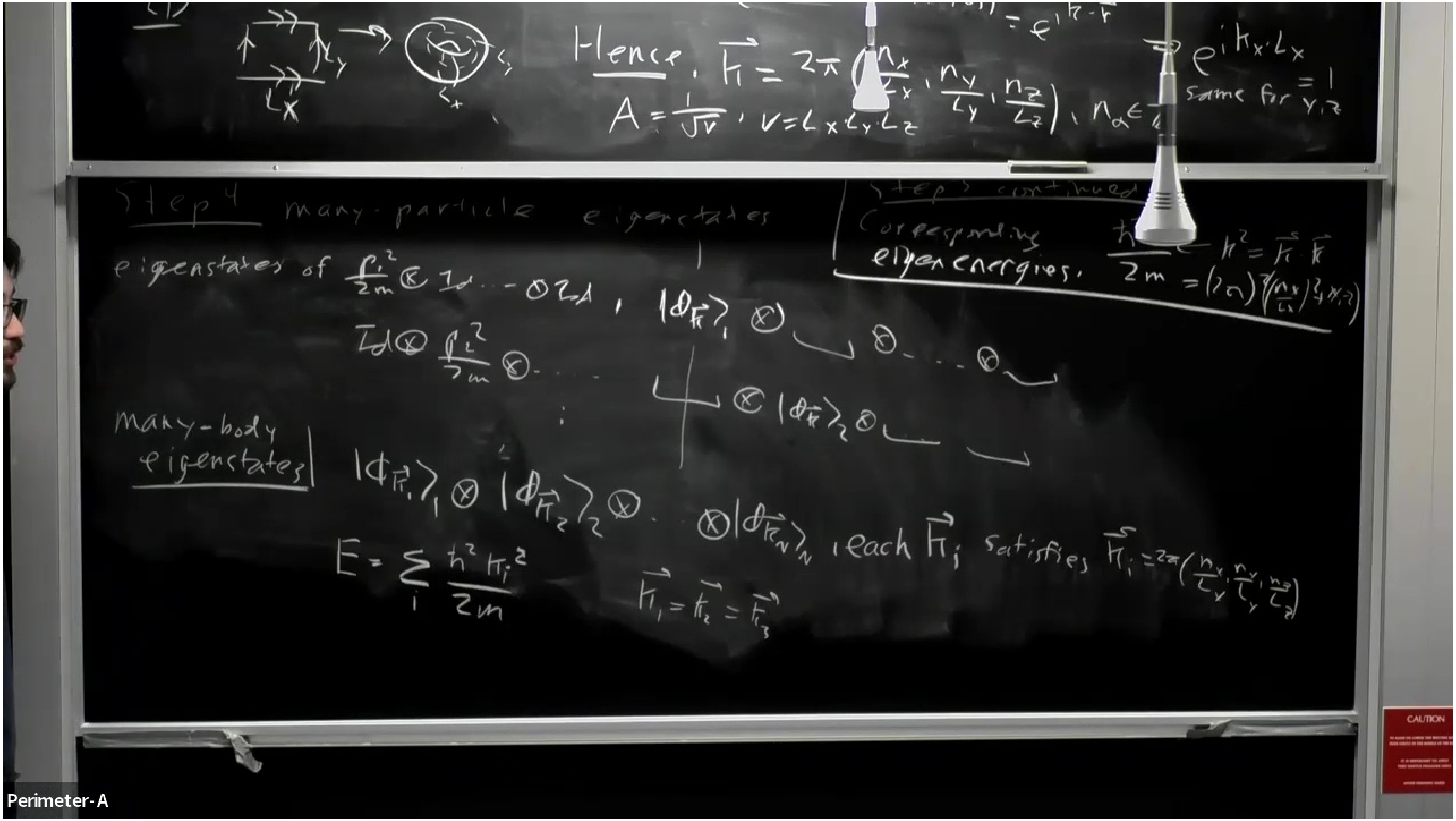
$\vdots$

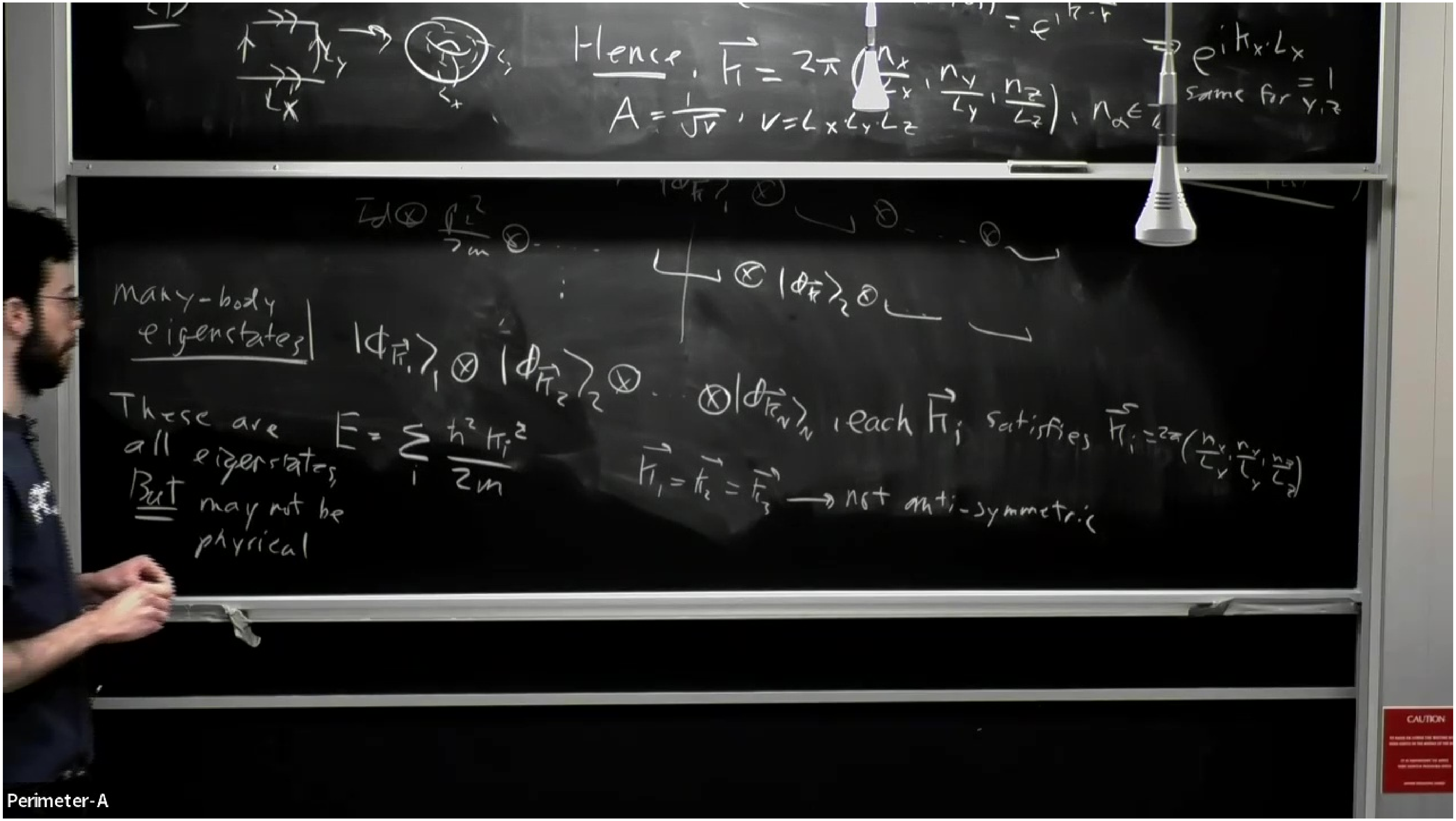
many-body eigenstates: $|\phi_{\vec{k}_1}\rangle_1 \otimes |\phi_{\vec{k}_2}\rangle_2 \otimes \cdots \otimes |\phi_{\vec{k}_N}\rangle_N$, each $\vec{k}_i$ satisfies $\vec{k}_i = 2\pi\left(\frac{n_x}{L_x}, \frac{n_y}{L_y}, \frac{n_z}{L_z}\right)$

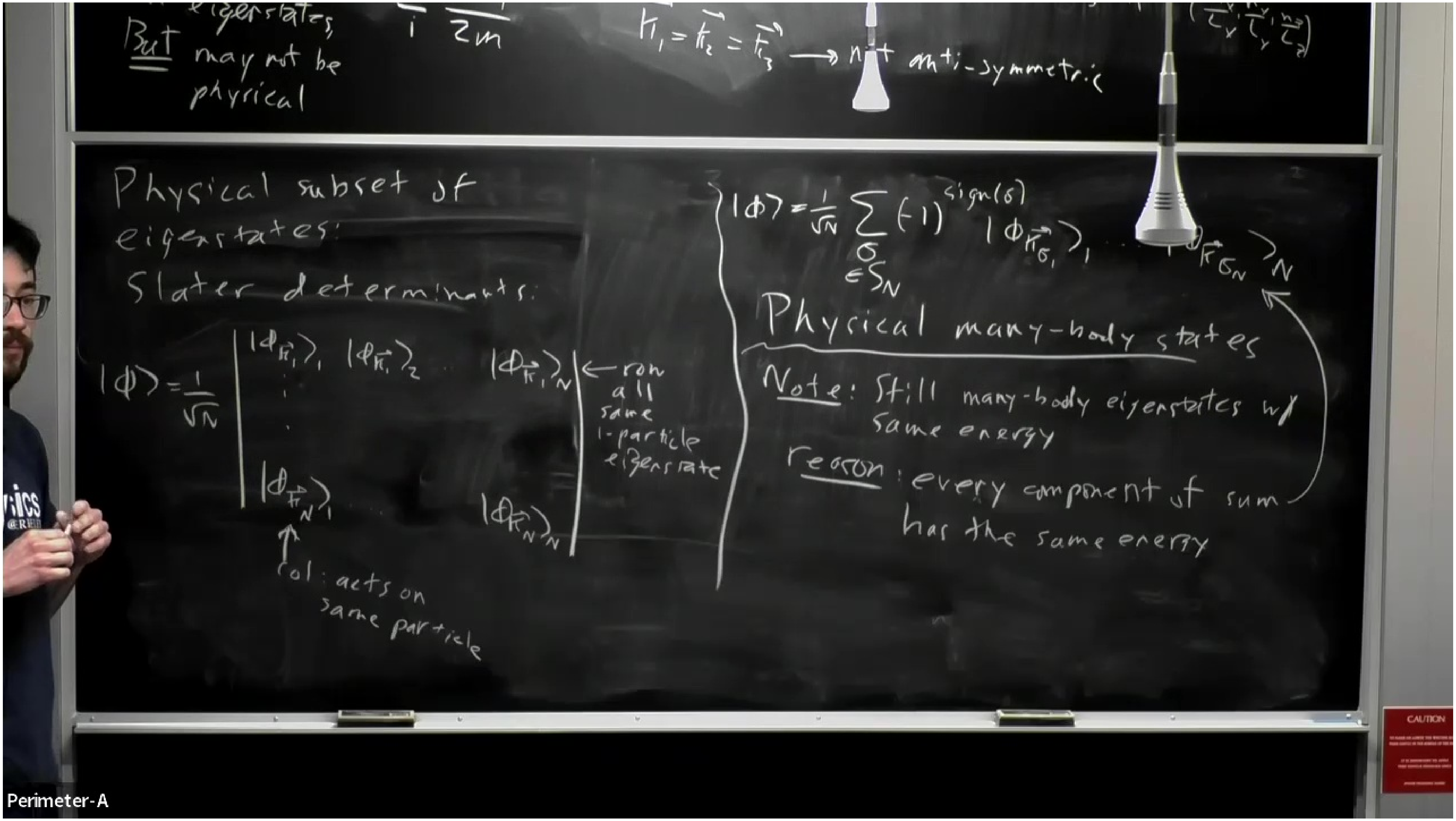
$$E = \sum_i \frac{\hbar^2 k_i^2}{2m}$$

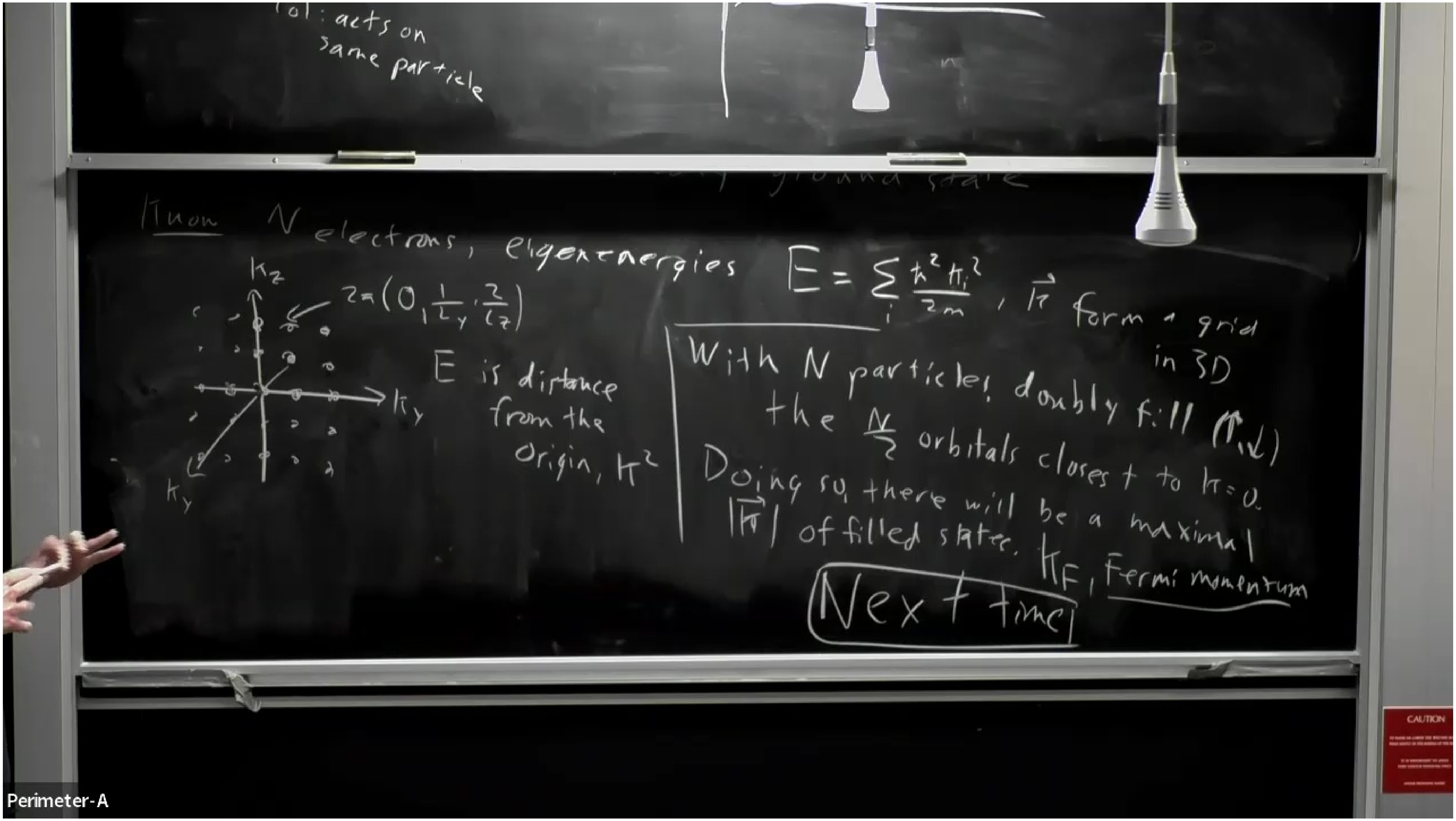
## Step 3 continued

Corresponding eigenenergies: $\frac{\hbar^2 k^2}{2m}$, $k^2 = \vec{k}\cdot\vec{k} = (2\pi)^2\left(\left(\frac{n_x}{L_x}\right)^2 + y, z\right)$


Hence, $\vec{k} = 2\pi\left(\frac{n_x}{L_x}, \frac{n_y}{L_y}, \frac{n_z}{L_z}\right), n_\alpha \in \mathbb{Z}$
$A = \frac{1}{\sqrt{V}}, V = L_x L_y L_z$
$\Rightarrow e^{ik_x L_x} = 1$
same for y, z
Step 4 many-particle eigenstates
Step 3 continued
Corresponding eigenenergies. $\frac{\hbar^2 k^2}{2m}$, $k^2 = \vec{k}\cdot\vec{k}$
eigenstates of $\frac{p_1^2}{2m} \otimes \mathbb{1}_d \otimes \cdots \otimes \mathbb{1}_d$, $|\phi_{\vec{k}}\rangle_1 \otimes \cdots \otimes \cdots$
$\mathbb{1}_d \otimes \frac{p_2^2}{2m} \otimes \cdots$
$\otimes |\phi_{\vec{k}}\rangle_2 \otimes \cdots$
many-body eigenstates
$|\phi_{\vec{k}_1}\rangle_1 \otimes |\phi_{\vec{k}_2}\rangle_2 \otimes \cdots \otimes |\phi_{\vec{k}_N}\rangle_N$, each $\vec{k}_i$ satisfies $\vec{k}_i = 2\pi\left(\frac{n_x}{L_x}, \frac{n_y}{L_y}, \frac{n_z}{L_z}\right)$
$E = \sum_i \frac{\hbar^2 k_i^2}{2m}$
$\vec{k}_1 = \vec{k}_2 = \vec{k}_3$
Perimeter-A
CAUTION


Hence, $\vec{k} = 2\pi\left(\frac{n_x}{L_x}, \frac{n_y}{L_y}, \frac{n_z}{L_z}\right)$, $n_\alpha \in \mathbb{Z}$
$A = \frac{1}{\sqrt{V}}$, $V = L_x \cdot L_y \cdot L_z$
$\Rightarrow e^{ik_x \cdot L_x} = 1$
same for y, z
many-body eigenstates
$|\phi_{\vec{k}_1}\rangle_1 \otimes |\phi_{\vec{k}_2}\rangle_2 \otimes \cdots \otimes |\phi_{\vec{k}_N}\rangle_N$, each $\vec{k}_i$ satisfies $\vec{k}_i = 2\pi\left(\frac{n_x}{L_x}, \frac{n_y}{L_y}, \frac{n_z}{L_z}\right)$
These are all eigenstates
But may not be physical
$E = \sum_i \frac{\hbar^2 k_i^2}{2m}$
$\vec{k}_1 = \vec{k}_2 = \vec{k}_3 \longrightarrow$ not anti-symmetric
Perimeter-A


eigenstates
But may not be physical
$\frac{\hbar}{i}\,\frac{\;}{2m}$
$k_1 = k_2 = k_3 \longrightarrow$ not anti-symmetric
Physical subset of eigenstates:
Slater determinants:
$|\Phi\rangle = \frac{1}{\sqrt{N}} \begin{vmatrix} |\phi_{\vec{k}_1}\rangle_1 & |\phi_{\vec{k}_1}\rangle_2 & \cdots & |\phi_{\vec{k}_1}\rangle_N \\ \vdots & & & \\ |\phi_{\vec{k}_N}\rangle_1 & \cdots & & |\phi_{\vec{k}_N}\rangle_N \end{vmatrix}$
← row all same 1-particle eigenstate
col: acts on same particle
$|\Phi\rangle = \frac{1}{\sqrt{N}} \sum_{\sigma \in S_N} (-1)^{\mathrm{sign}(\sigma)} |\phi_{\vec{k}_{\sigma_1}}\rangle_1 \cdots |\phi_{\vec{k}_{\sigma_N}}\rangle_N$
Physical many-body states
Note: Still many-body eigenstates w/ same energy
reason: every component of sum has the same energy
Perimeter-A


acts on
same particle
Know N electrons, eigenenergies
E is distance
from the
origin, k²
With N particles doubly fill (↑,↓)
the N/2 orbitals closest to k = 0.
Doing so, there will be a maximal
|k| of filled states, k_F, Fermi momentum
Next time
Perimeter-A
CAUTION