Title: Exotic compressible quantum liquids and fractons in coupled wire models

Speakers: Joseph Sullivan

Series: Quantum Matter

Date: December 14, 2021 - 3:30 PM

URL: https://pirsa.org/21120031

Abstract: The coupled wire construction is a powerful method for studying exotic quantum phases of matter. In this talk I will discuss some recent work in which this technique was used to realize new types of 3D compressible quantum phases. These phases possess a U(1) charge conservation symmetry that is weakly broken by rigid string or membrane-like order parameters. No local order parameter is present and the emergent quasiparticles have restricted mobility. I will discuss the unusual symmetry breaking mechanism and its connection to the compressibility. For a particular class of models I will also describe an effective low energy theory given by coupled layers Maxwell-Chern-Simons theories.

Zoom Link: https://pitp.zoom.us/j/95372524441?pwd=UTlVTTZlSmFRK0FmVE5pTHhDRThwdz09

Exotic compressible quantum liquids and fractons in coupled wire models

Joe Sullivan
Yale University

LUX ET VERITAS

Perimeter Institute: Quantum Matter Seminar
12/14/21

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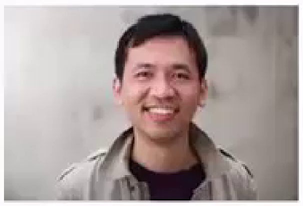
Collaborators



Meng Cheng
Yale

Arpit Dua
Caltech

arXiv:2109.13267

arXiv:2010.15148

Dom Williamson
Stanford -> Sydney

Tom Iadecola
Iowa State

arXiv:2010.15127

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Outline

- - Compressibility & Emergent Symmetries and Fracton Phases

- Coupled wire constructions
  - Intro
  - Examples: Superfluid and Laughlin State
- "Layered" 3D models
  - Gapped Planon Phase example
  - New compressible quantum liquid: "Weak Superfluid"
- Other interesting wire models

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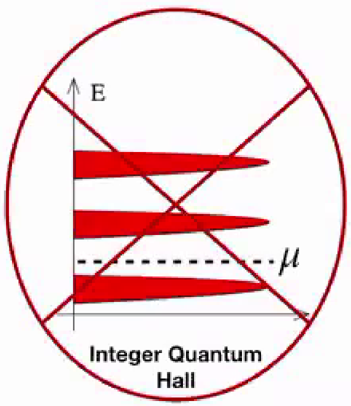
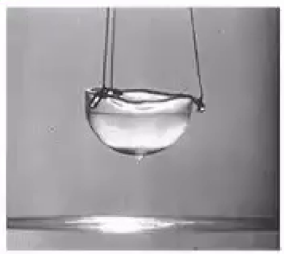
Compressible phases

Compressible $\equiv \dfrac{\partial \rho}{\partial \mu} > 0$

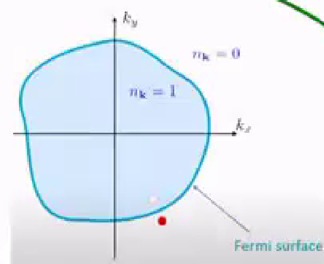
ρ = charge density
μ = chemical potential



Integer Quantum Hall



Superfluid (Bosonic)



Fermi Liquid (Fermionic)

Compressible phases distinguished by emergent symmetry group G_{IR} and t'Hooft anomalies.

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Emergent Symmetries

Else, Thorngren, Senthil 2021

Wang, Hickey, Ying, Burkov 2021

Filling related to mixed t'Hooft anomaly of U(1) and translation : $T_x T_y T_x^{-1} T_y^{-1} = e^{2\pi i \nu}$ $\longrightarrow$ Arbitary ν constrains G_{IR}

6

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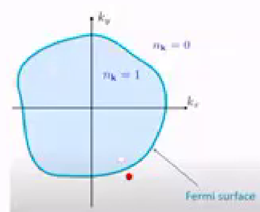
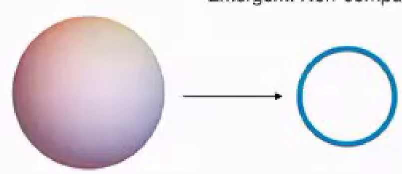
Compressible $\Longrightarrow$ $G_{IR} \neq$ compact 0-form symmetry



$\underbrace{L^{D-1}U(1)}$

Emergent: Non-compact "loop group"



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Done

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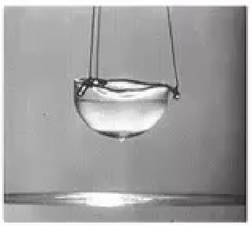
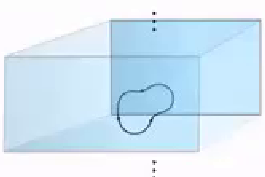
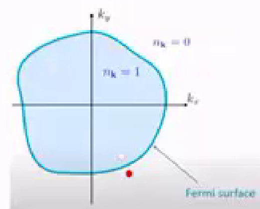


$U(1)^{[0]} \times \underbrace{U(1)^{[D-1]}}$

Emergent: (D-1)−form





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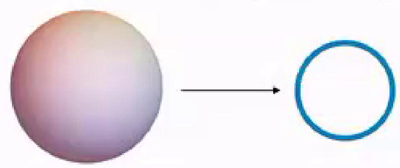

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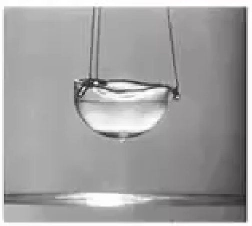
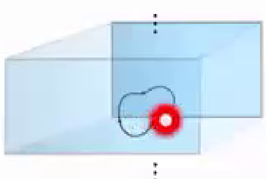
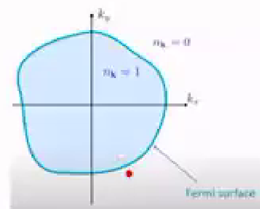




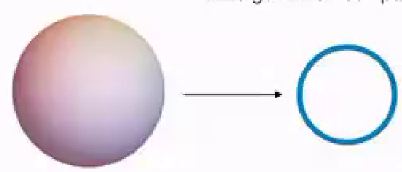
$U(1)^{[0]} \times \underbrace{U(1)^{[D-1]}}$

Emergent: (D-1)—form



$\underbrace{L^{D-1}U(1)}$

Emergent: Non-compact "loop group"



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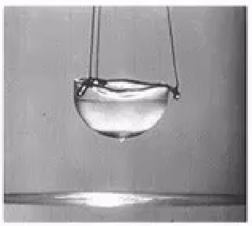
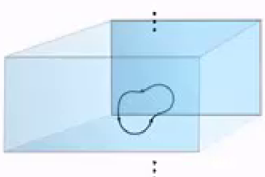
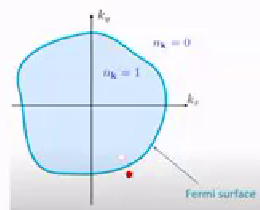




$U(1)^{[0]} \times \underbrace{U(1)^{[D-1]}}$

Emergent: (D-1)—form

New example in a moment!



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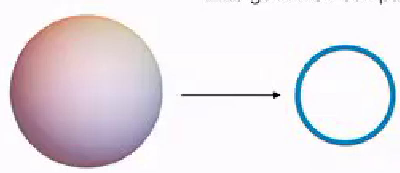

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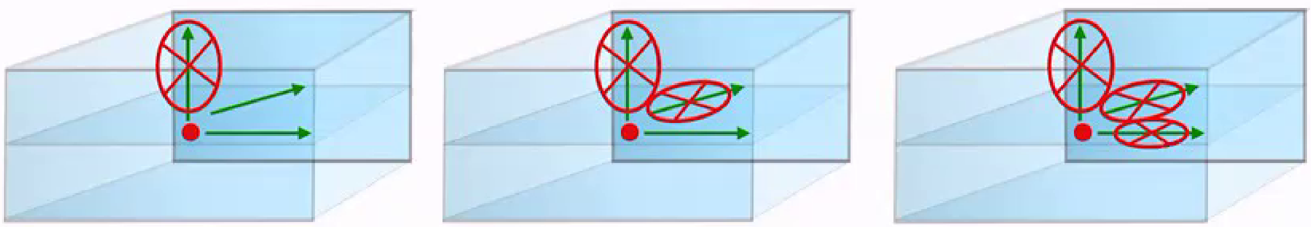
Fractons

Chamon (2005)
Haah (2011)
Yoshida (2013)
Pretko, Chen, You (2020)
Many others…..

"Quasiparticles move on submanifolds"



"Planon" "Lineon" "Fracton"

Applications/Overlap: Quantum Error Correction, Weird QFTs, Quantum Glassiness, Elasticity Theory etc

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Coupled Wire Construction

Kane, Mukhopadhyay, Lubensky (2001)

Meng (2019)

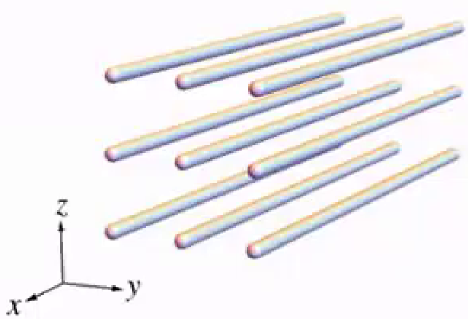
Luttinger Liquid ($\varphi \equiv \varphi + 2\pi, \theta \equiv \theta + 2\pi$)

$$H_0 = \frac{v}{2\pi}\left[\left(\partial_x \varphi\right)^2 + \left(\partial_x \theta\right)^2\right]$$

$$[\partial_x \theta(x), \varphi(x')] = 2\pi i \delta(x - x')$$

$e^{i\varphi}$ (Boson) and $\rho = \partial_x \theta / 2\pi$ (charge density)

$e^{in\varphi}, e^{im\theta}, \partial_x \theta, \partial_x \varphi$ are local ops $(m, n \in \mathbb{Z})$



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Done

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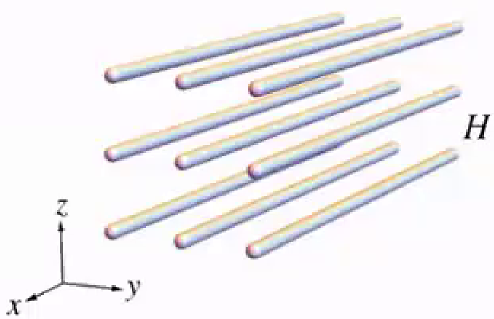

$H = H_0$ —interactions→ $H = H_0 - \sum_{\text{wires}} g\cos\left(\Theta_r\right)$

$$\Theta_r = \sum_{r'} \Lambda^{\varphi}_{rr'}\varphi_{r'} + \Lambda^{\theta}_{rr'}\theta_{r'}$$

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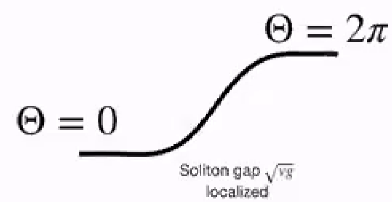
Done

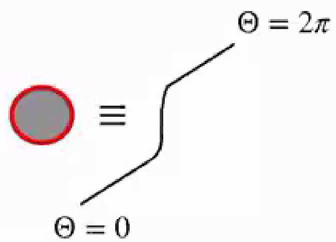
Excitations

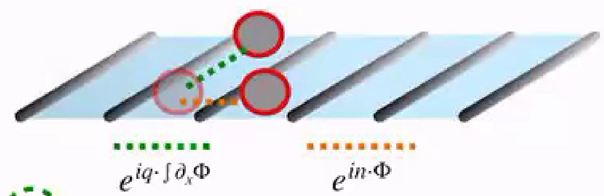
$[\Theta_r, \Theta_r'] = 0$ and $g \gg 1$

Ground State : $\{\Theta_r\} \in 2\pi\mathbb{Z}$



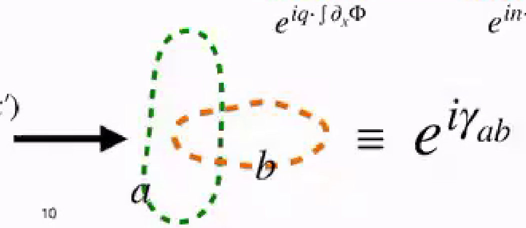
Gapped quasiparticles





$[\partial_x\theta(x), \varphi(x')] = 2\pi i\delta(x - x')$

$e^A e^B = e^B e^A e^{-[A,B]}$



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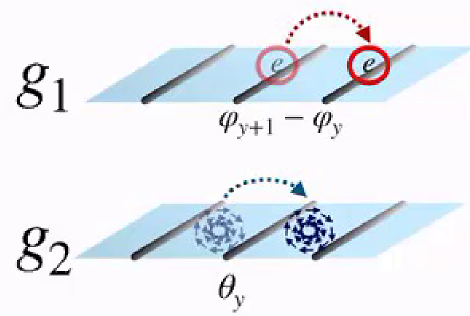
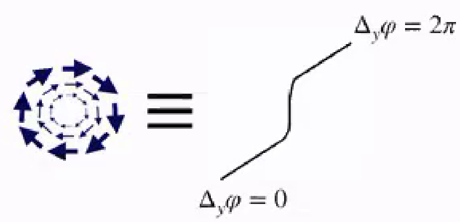
2+1 D Superfluid

$$H = \int_x \sum_y H_0 - \underbrace{g_1 \cos(\varphi_{y+1} - \varphi_y)}_{\text{Josephson coupling}} - \overbrace{g_2 \cos(\theta_y)}^{2\pi \text{ phase slip}}$$



SF phase: $g_1/g_2 \gg 1$: $\Delta\varphi = 0$  $\Delta\varphi = 0$

Gapless

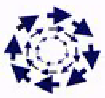


Vortex Soliton (gapped)

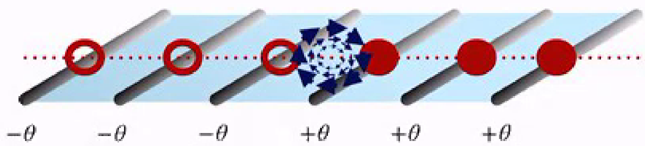
Particle-Vortex Duality in wires

Mross, Alicea and Motrunich 2017
Mross, Alicea and Motrunich 2016

Conjugate to Josephson term



$$\equiv \underbrace{\prod_{y'<y+1/2} e^{-i\theta} \prod_{y'>y+1/2} e^{+i\theta}}_{e^{i\tilde{\varphi}_{y+1/2}}}$$



$e^{i\tilde{\varphi}}$ (vortex) and $\partial_x \tilde{\theta}/2\pi$ (vortex density)

$Z[\varphi, \theta]$ → H-S tranformation → $Z[\tilde{\varphi}, \tilde{\theta}, a_\mu]$

"charge basis" "vortex basis"

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Done

Laughlin State

Kane, Mukhopadhyay, Lubensky (2001)

$$H_{int} = -g\cos\Big(\underbrace{\varphi_{y+1} - \varphi_y + m(\theta_y + \theta_{y+1})}_{\Theta_{Laughlin}}\Big)$$

$\Theta = 0$  $\Theta = 0$

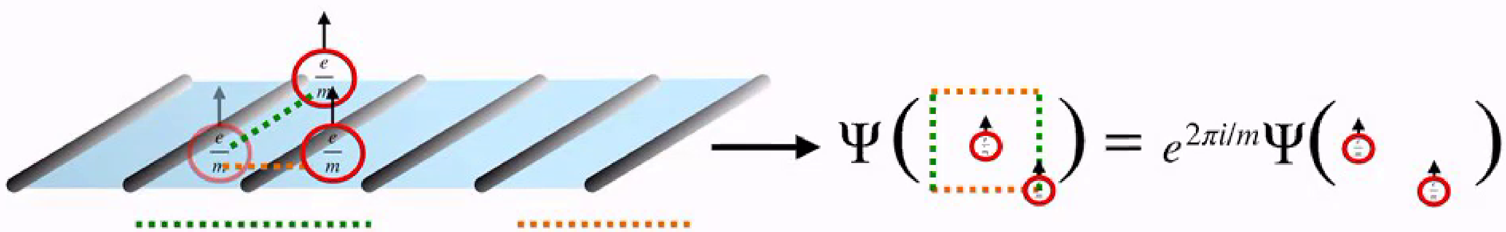
Gapped

$$\Theta_{Laughlin} = \Delta_y \underbrace{(\varphi + m\tilde{\varphi})}$$

e + m 

(flux attachment)

Laughlin (cont'd)



$$\exp\left(\frac{i}{2m}\int \partial_x[\varphi_{y+1}+\varphi_y+m(\theta_y-\theta_{y+1})]\right) \qquad e^{i\theta}$$

In vortex language: $\tilde{\varphi}, \tilde{\theta}$ and a $\xrightarrow{\int \text{out matter}}$ $L=\frac{m}{4\pi}a\wedge da+L_{Maxwell}$

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3D models

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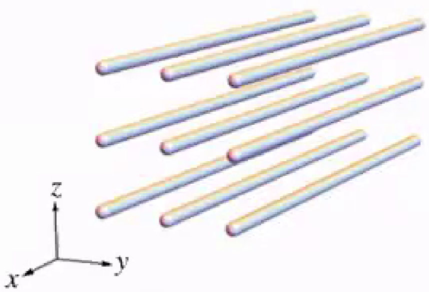
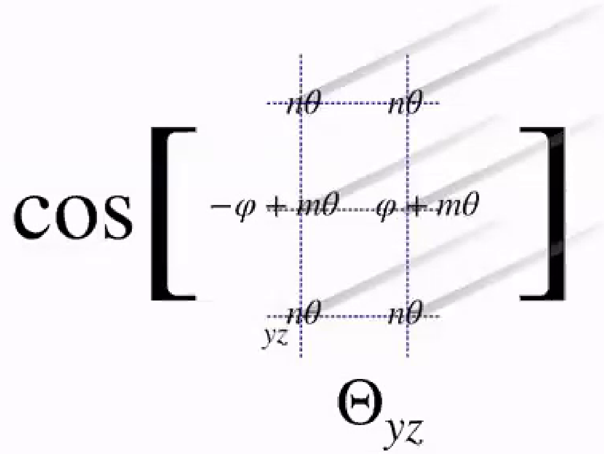
Six-Wire Model

JS, Dua, Cheng 2021



$$H_0 = \sum_r \frac{v}{2\pi}\left((\partial_x\varphi_r)^2 + (\partial_x\theta_r)^2\right)$$



$$\Theta_{yz} = -\varphi_{yz} + m\theta_{yz} + \varphi_{y+1,z} + m\theta_{y+1,z} + \left(n\theta_{y,z-1} + \theta_{y,z+1} + n\theta_{y+1,z-1} + n\theta_{y+1,z+1}\right)$$

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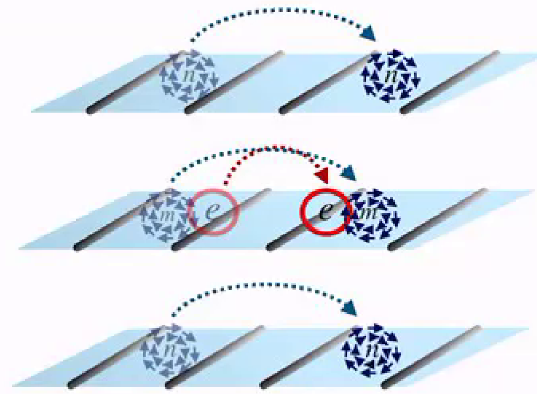
Six-Wire Model (cont'd)

$$\cos\begin{bmatrix} n\theta & n\theta \\ -\varphi + m\theta & \varphi + m\theta \\ n\theta & n\theta \end{bmatrix}_{yz}$$

$$\Theta_{yz}$$



Excitations:

Ground State : $\{\Theta_r\} \in 2\pi\mathbb{Z}$

$\Theta = 0$  $\Theta = 0$

Gapped when $|m| > |2n|$

$\Theta = 0$  $\Theta = 2\pi$

"Vortex" with gap $\sqrt{vg}$ localized

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Gapped Example

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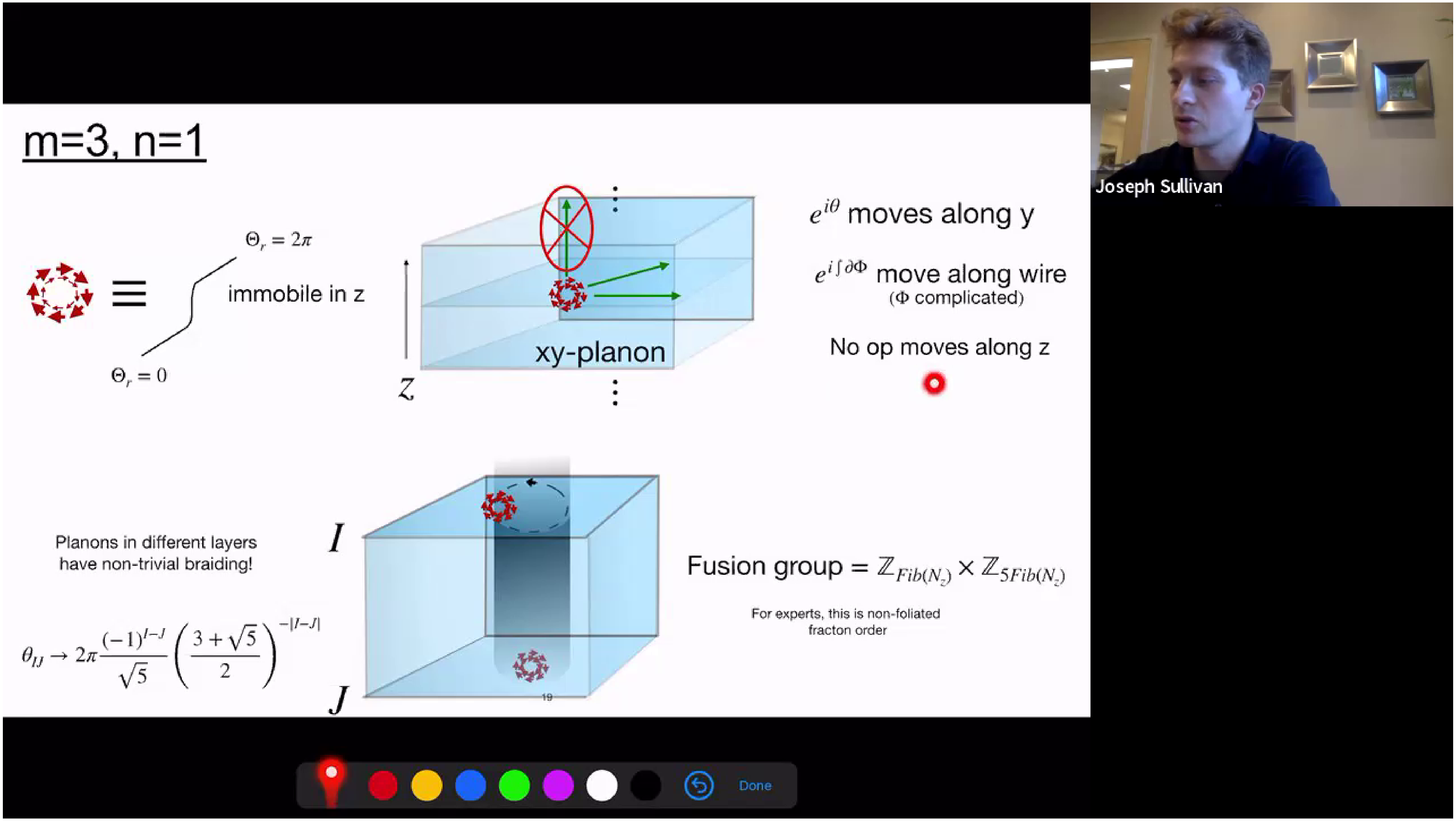
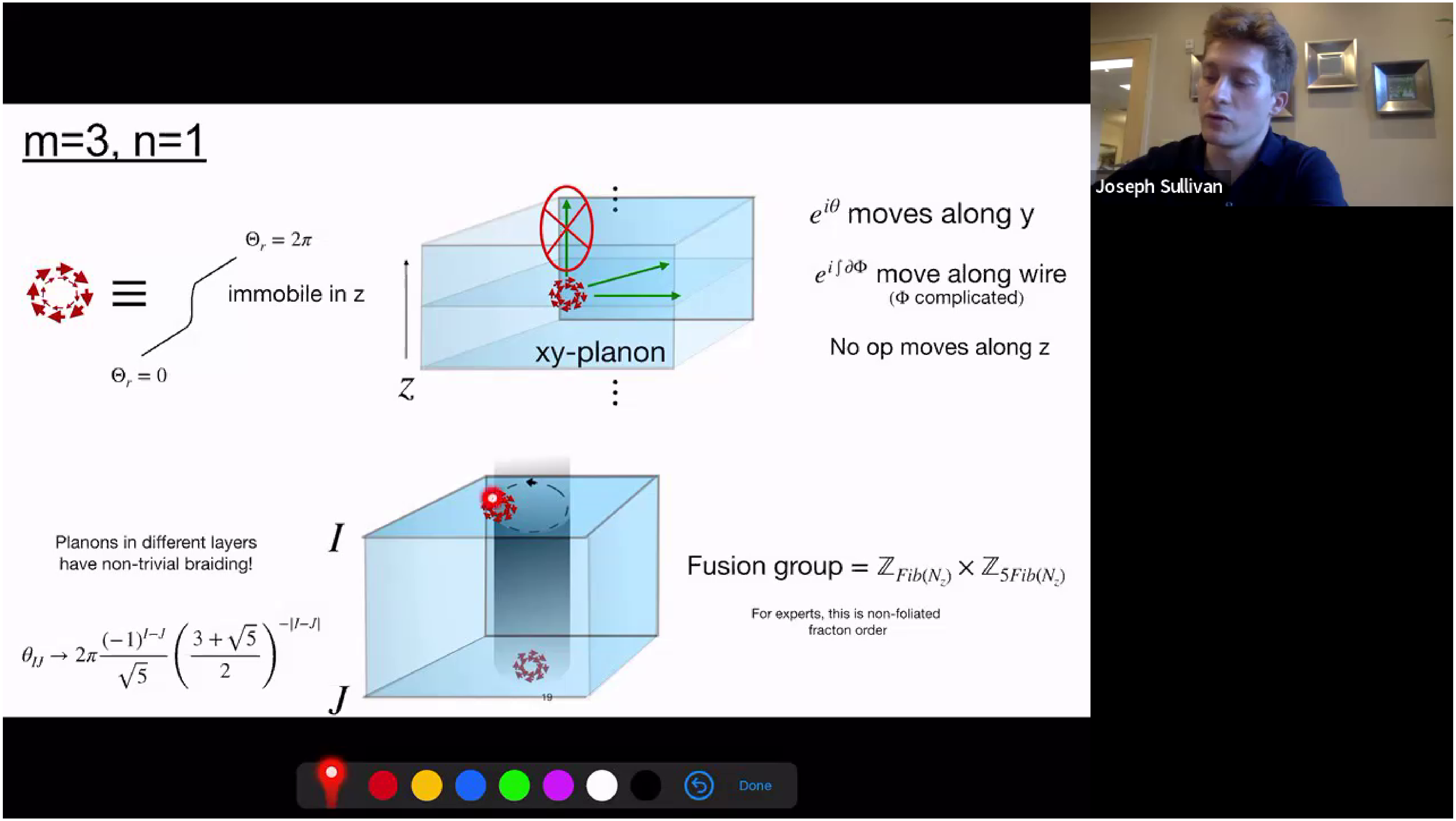
Done


m=3, n=1
$\Theta_r = 2\pi$
≡
immobile in z
$\Theta_r = 0$
z
xy-planon
$e^{i\theta}$ moves along y
$e^{i\int\partial\Phi}$ move along wire
(Φ complicated)
No op moves along z
Planons in different layers
have non-trivial braiding!
$\theta_{IJ} \to 2\pi \frac{(-1)^{I-J}}{\sqrt{5}} \left(\frac{3+\sqrt{5}}{2}\right)^{-|I-J|}$
I
J
Fusion group = $\mathbb{Z}_{Fib(N_z)} \times \mathbb{Z}_{5Fib(N_z)}$
For experts, this is non-foliated
fracton order
19
Joseph Sullivan
Done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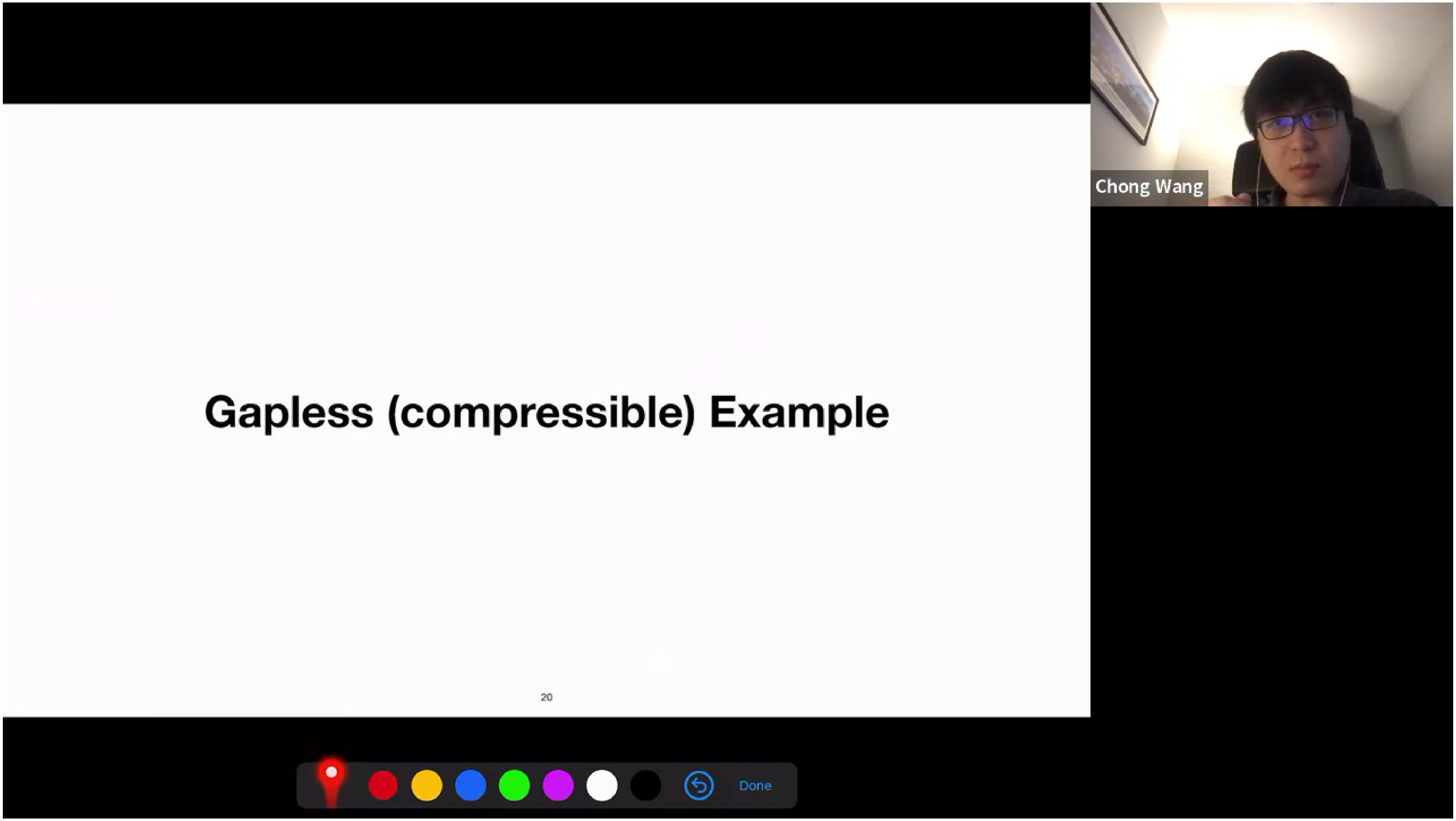
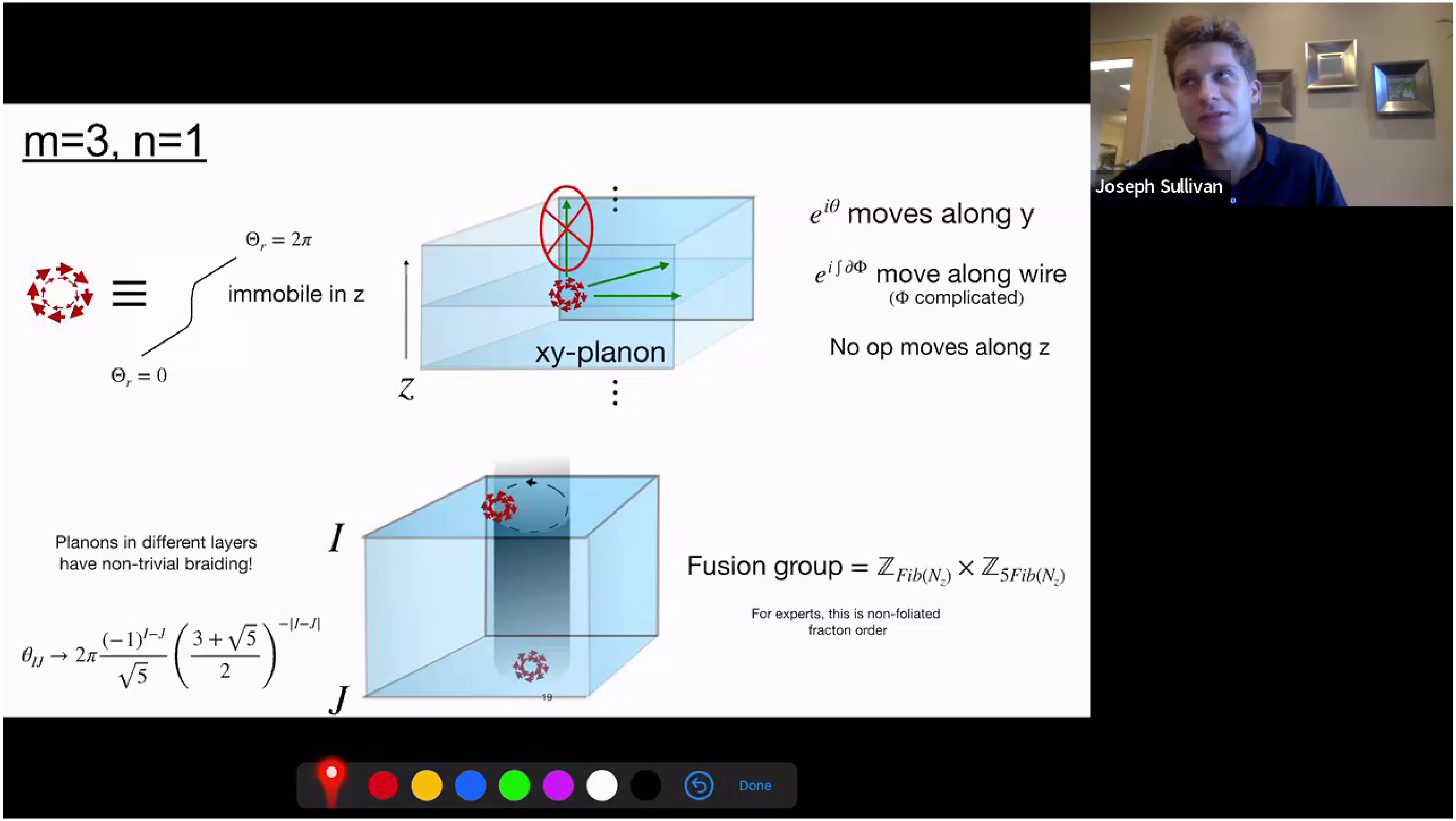

Gapless (compressible) Example
20
Chong Wang
Done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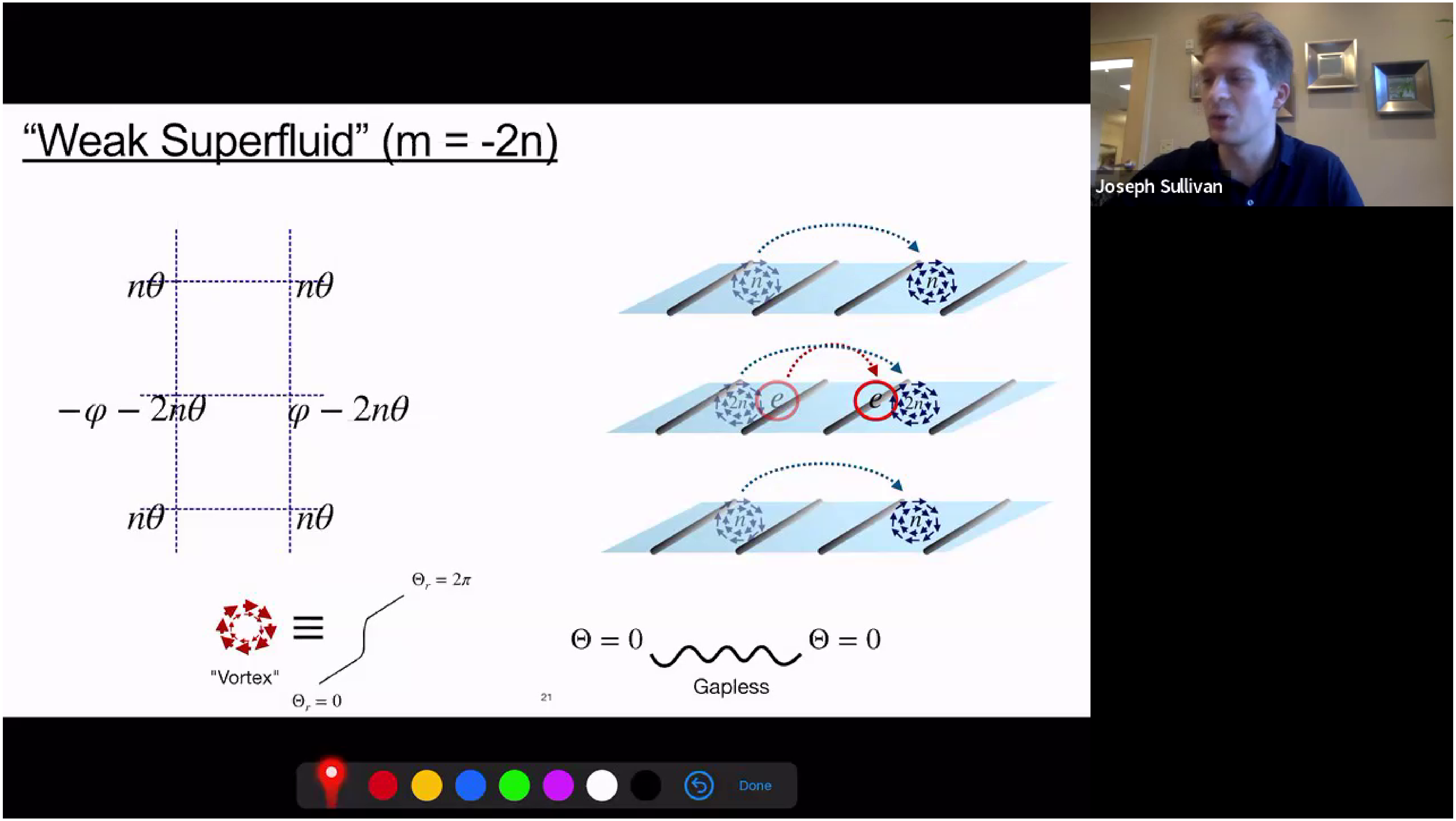
Done

Gapless (compressible) Example

20



"Weak Superfluid" (m = -2n)
$n\theta$
$n\theta$
$-\varphi - 2n\theta$
$\varphi - 2n\theta$
$n\theta$
$n\theta$
n
n
2n
e
e
2n
n
n
$\Theta_r = 2\pi$
$\equiv$
"Vortex"
$\Theta_r = 0$
21
$\Theta = 0$
$\Theta = 0$
Gapless
Joseph Sullivan
Done

Compressibility

Translation along wire $\theta(x) = \int_{x_0}^{x} \partial_x \theta = \pi \int_{x_0}^{x} \rho \implies$ $\boxed{T_x : \theta \to \theta + \pi\nu}$

Luttinger Liquid

$$H = \int_x \frac{v}{2\pi} \left[(\partial_x \varphi)^2 + (\partial_x \theta)^2 \right]$$

$U(1)_\varphi : \varphi \to \varphi + \alpha$ 👍

$U(1)_\theta : \theta \to \theta + \beta$ 👍

$$T_x : H \to H$$

ν unconstrained $\implies$ compressible

Laughlin State

$$\Theta_{Laughlin} = \varphi_{y+1} - \varphi_y + m(\theta_y + \theta_{y+1})$$

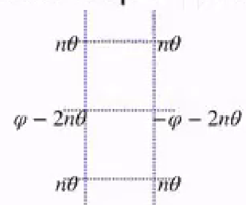
$U(1)_\varphi : \varphi \to \varphi + \alpha$ 👍

$U(1)_\theta : \theta \to \theta + \beta$ 👎

$$T_x : \Theta \to \Theta + 2\pi m\nu$$

filling constraint $\implies \nu = \frac{1}{m}$

Weak Superfluid



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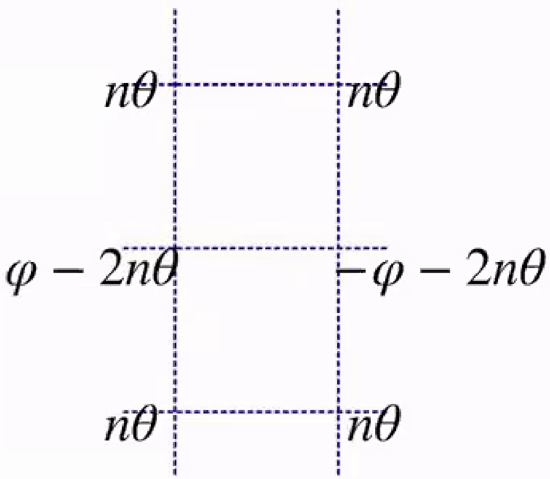
$U(1)_\theta : \theta \to \theta + \beta$ 👍

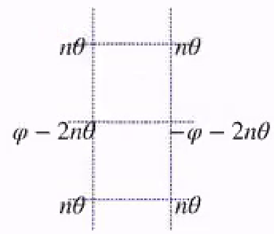
$$T_x : \Theta \to \Theta$$

ν unconstrained $\implies$ compressible

22

Stability





and $e^{i\varphi}, e^{i\theta}$ don't commute

$e^{i\varphi}, e^{i\theta}$ gapped

↓

$\cos\left(\Lambda_\varphi \varphi + \Lambda_\theta \theta\right)$ is irrelevant

Can't condense any local process to drive transition!

Done

(Non-local) order parameter

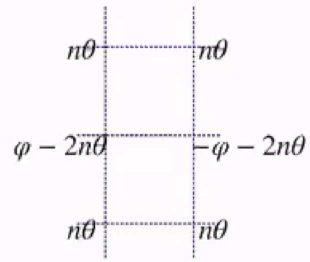
2+1 SF

Josephson term condensed phase difference

$$\underbrace{\langle e^{i\varphi(r)} e^{-i\varphi(0)} \rangle}_{\text{SSB of charge } U(1)^{[0]}} \sim e^{\frac{1}{\rho_s}\left(\frac{1}{r}-\frac{1}{a_0}\right)} \rightarrow \text{const}$$

SSB of charge $U(1)^{[0]}$

Weak SF

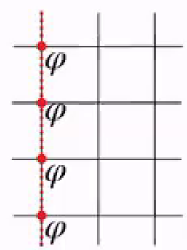


$e^{i\varphi}, e^{i\theta}$ gapped

$\downarrow$

$$\langle e^{i\varphi(r)} e^{-i\varphi(0)} \rangle = 0$$

No local order parameter



Instead: Non-local $\Phi(r) = \prod_z e^{i\varphi_{xz}(x)}$

$$\langle \Phi(r)\Phi(0)^\dagger \rangle \sim e^{\frac{N_z}{\rho_s}\left(\frac{1}{r}-\frac{1}{a_0}\right)} \rightarrow \text{const}$$

"Weak" SB of charge $U(1)^{[0]}$

Done

Emergent Symmetry

2+1 SF

$U(1)^{[1]}$ charge : $Q_C = \oint_C \partial\varphi$ measures vorticity

Weak SF

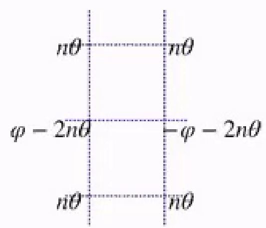
$U(1)^{[2]}$ charge : Q_C "vorticity any plane"

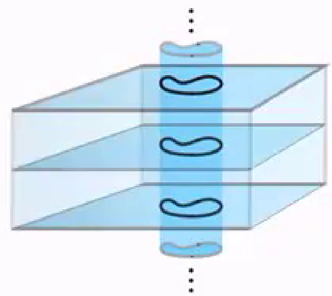
Emergent Symmetry

2+1 SF

$U(1)^{[1]}$ charge : $Q_C = \oint_C \partial\varphi$ measures vorticity

Weak SF





$U(1)^{[2]}$ charge : Q_C "vorticity any plane"

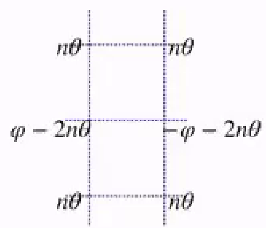
Emergent Symmetry

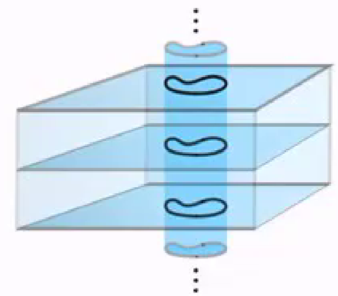
2+1 SF

vortex gapped

$U(1)^{[1]}$ charge : $Q_C = \oint_C \partial\varphi$ measures vorticity

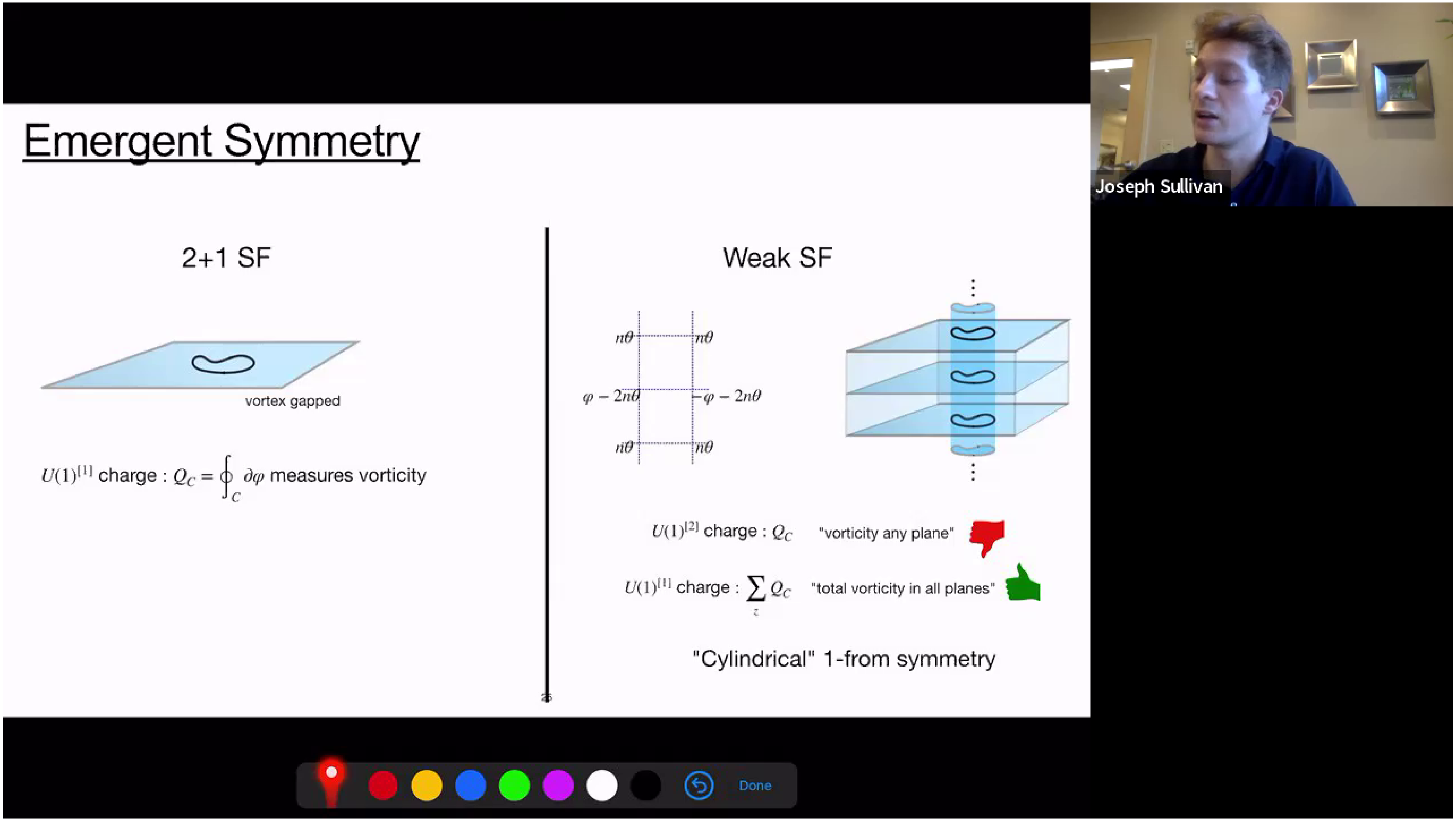
Weak SF



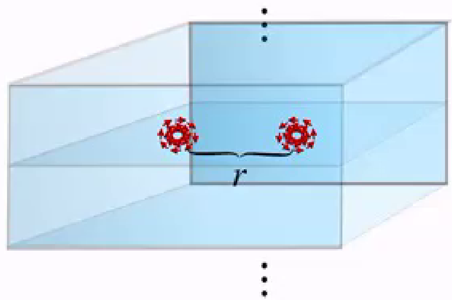


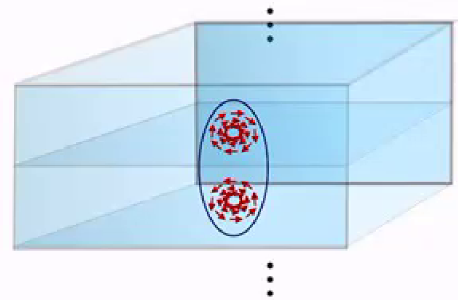
$U(1)^{[2]}$ charge : Q_C "vorticity any plane"

$U(1)^{[1]}$ charge : $\sum_z Q_C$ "total vorticity in all planes"


Emergent Symmetry
2+1 SF
vortex gapped
$U(1)^{[1]}$ charge : $Q_C = \oint_C \partial\varphi$ measures vorticity
Weak SF
$n\theta$
$\varphi - 2n\theta$
$U(1)^{[2]}$ charge : Q_C "vorticity any plane"
$U(1)^{[1]}$ charge : $\sum_z Q_C$ "total vorticity in all planes"
"Cylindrical" 1-from symmetry
Joseph Sullivan
Done

Gapped Excitations





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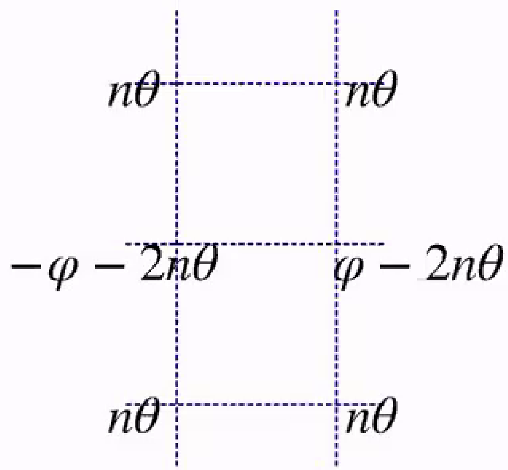
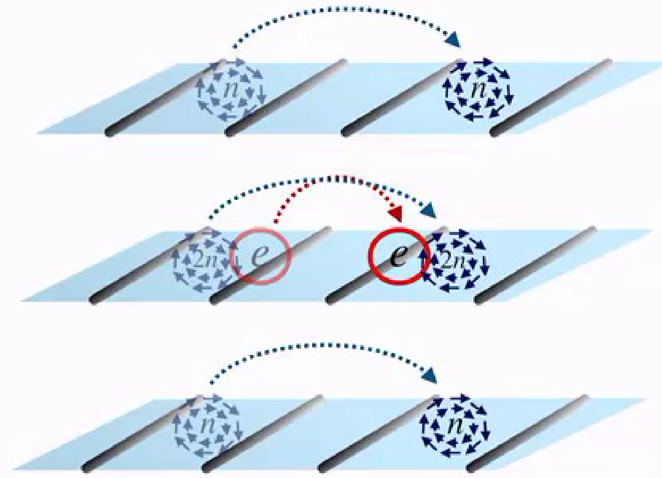
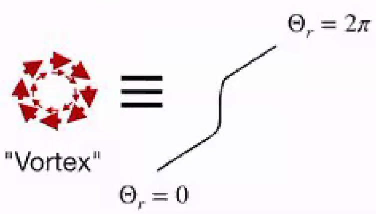
“Weak Superfluid” (m = -2n)







21

Done

Compressibility

Translation along wire $\theta(x) = \int_{x_0}^{x} \partial_x \theta = \pi \int_{x_0}^{x} \rho \implies T_x : \theta \to \theta + \pi\nu$

Luttinger Liquid

$$H = \int_x \frac{\nu}{2\pi} \left[(\partial_x \varphi)^2 + (\partial_x \theta)^2 \right]$$

$U(1)_\varphi : \varphi \to \varphi + \alpha$

$U(1)_\theta : \theta \to \theta + \beta$

$$T_x : H \to H$$

ν unconstrained $\implies$ compressible

22

Done

(Non-local) order parameter

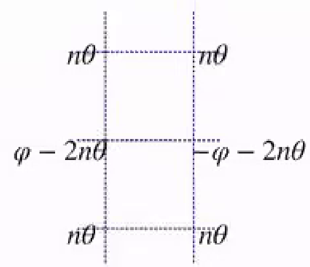
2+1 SF

Josephson term condensed phase difference

$$\underbrace{\langle e^{i\varphi(r)}e^{-i\varphi(0)}\rangle}_{\text{SSB of charge } U(1)^{[0]}} \sim e^{\frac{1}{\rho_s}\left(\frac{1}{r}-\frac{1}{a_0}\right)} \to \text{const}$$

SSB of charge $U(1)^{[0]}$

Weak SF

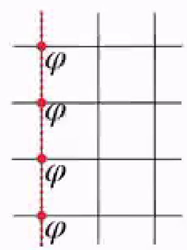


$e^{i\varphi}, e^{i\theta}$ gapped

↓

$$\langle e^{i\varphi(r)}e^{-i\varphi(0)}\rangle = 0$$

No local order parameter



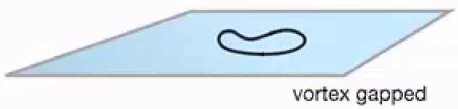
Instead: Non-local $\Phi(r) = \prod_z e^{i\varphi_z(x)}$

$$\langle \Phi(r)\Phi(0)^\dagger\rangle \sim e^{\frac{N_z}{\rho_s}\left(\frac{1}{r}-\frac{1}{a_0}\right)} \to \text{const}$$

"Weak" SB of charge $U(1)^{[0]}$

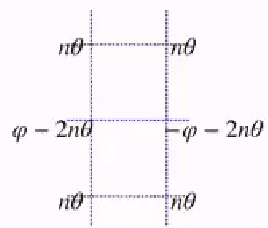
Emergent Symmetry

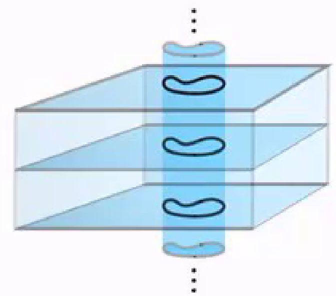
2+1 SF



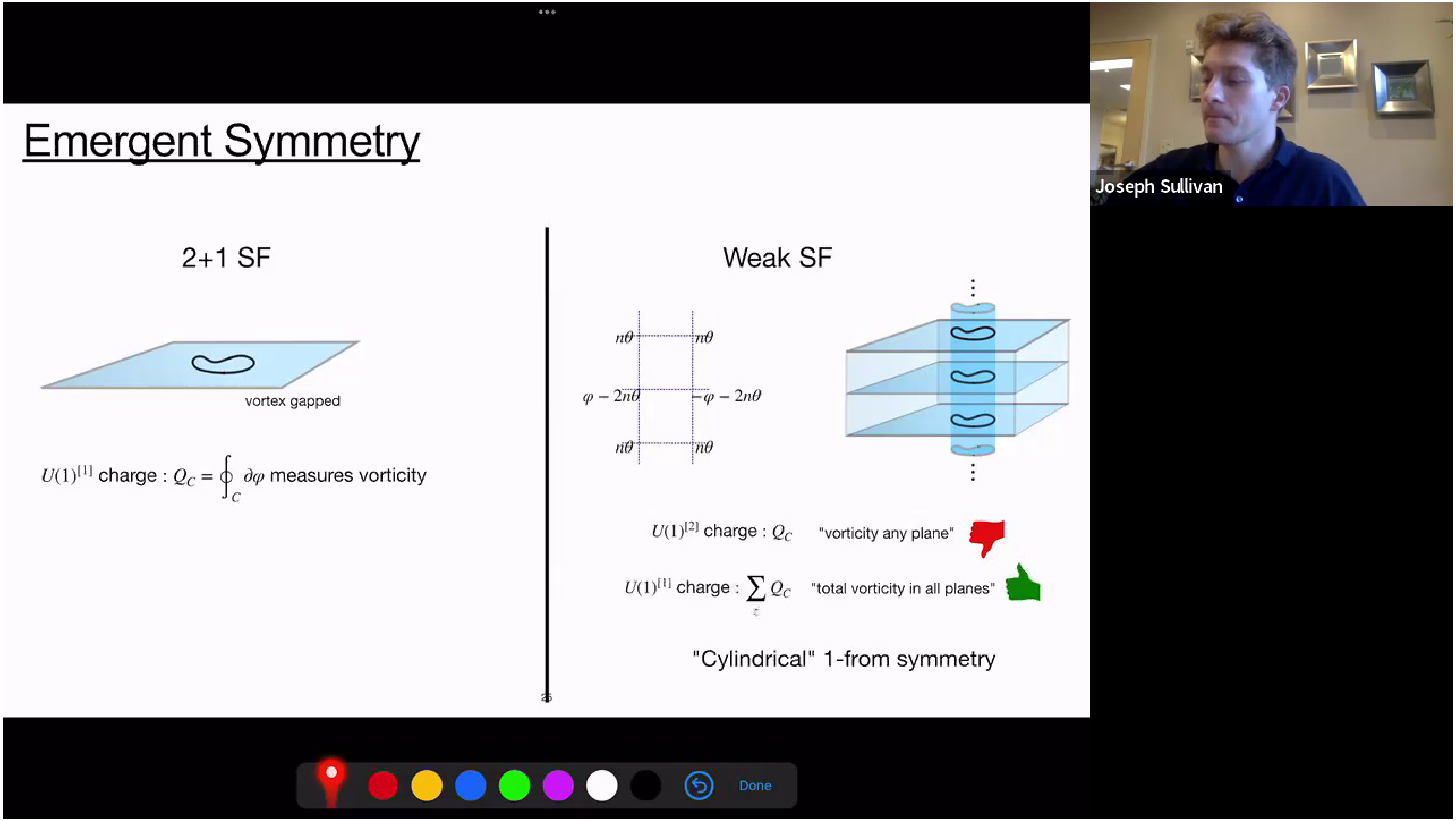
$U(1)^{[1]}$ charge : $Q_C = \oint_C \partial\varphi$ measures vorticity

Weak SF





$U(1)^{[2]}$ charge : Q_C "vorticity any plane"



Joseph Sullivan
Emergent Symmetry
2+1 SF
vortex gapped
$U(1)^{[1]}$ charge : $Q_C = \oint_C \partial\varphi$ measures vorticity
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$U(1)^{[2]}$ charge : Q_C "vorticity any plane"
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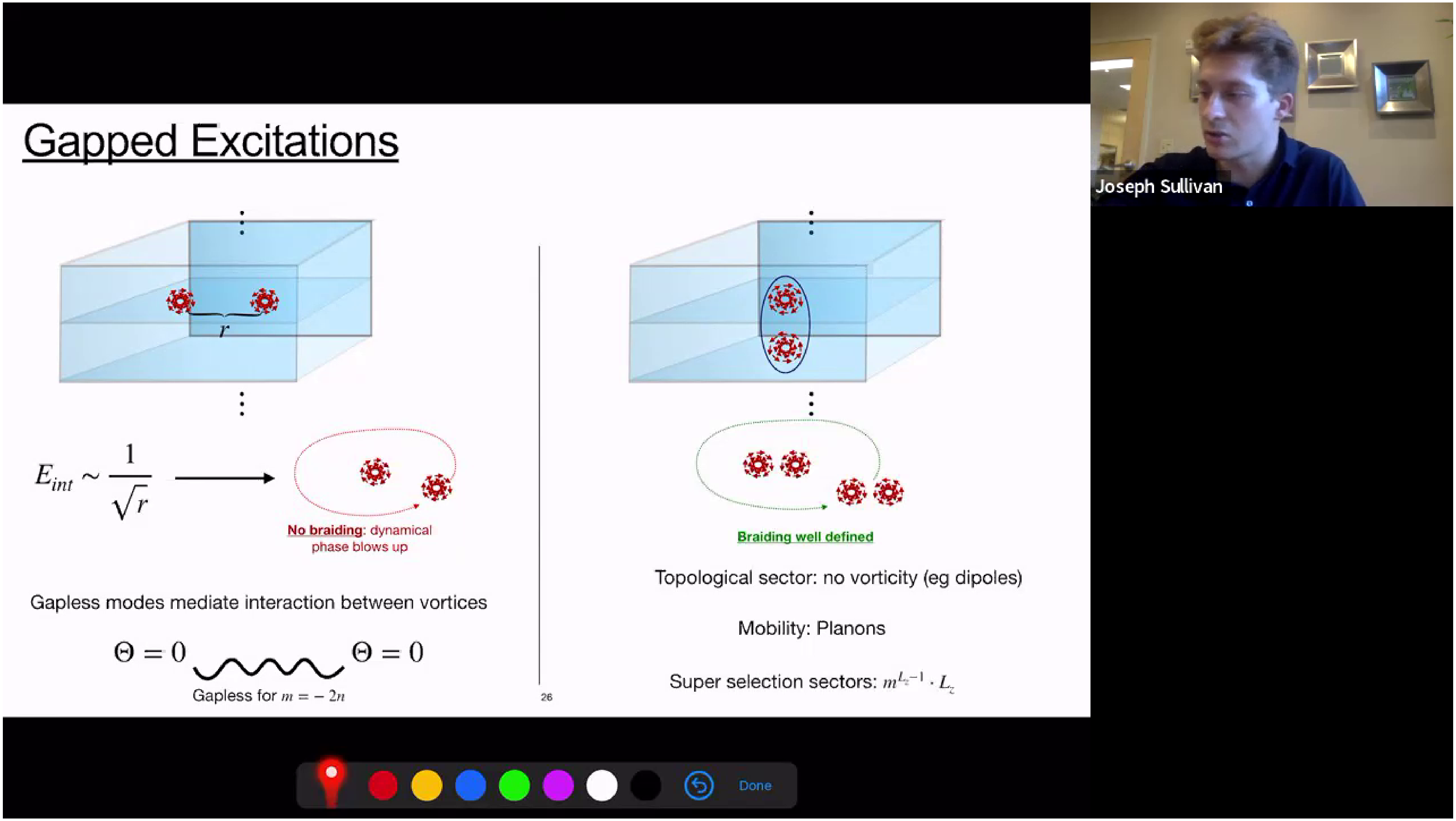
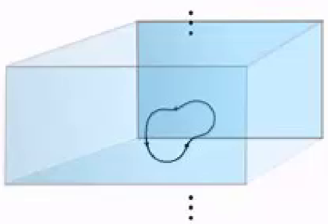
"Cylindrical" 1-from symmetry
Done


Gapped Excitations
r
$E_{int} \sim \frac{1}{\sqrt{r}}$
No braiding: dynamical phase blows up
Gapless modes mediate interaction between vortices
$\Theta = 0$
$\Theta = 0$
Gapless for $m = -2n$
26
Braiding well defined
Topological sector: no vorticity (eg dipoles)
Mobility: Planons
Super selection sectors: $m^{L_z - 1} \cdot L_z$
Joseph Sullivan
Done

3D Compressible Phases



3D Superfluid

$$U(1)^{[0]} \times \underbrace{U(1)^{[2]}}$$

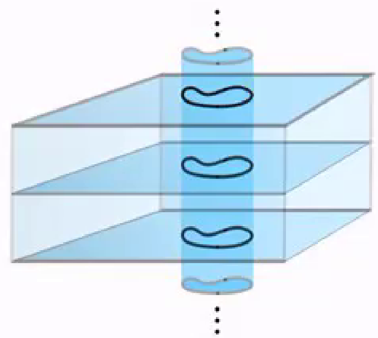
Emergent: 2-form in 3D

3D Fermi Liquid

$$\underbrace{L^2 U(1)}$$

Emergent: Smooth maps from 2-sphere to U(1)

3D Weak Superfluid



$$U(1)^{[0]} \times \underbrace{U(1)^{[1]}}$$

Emergent: 1-form in 3D

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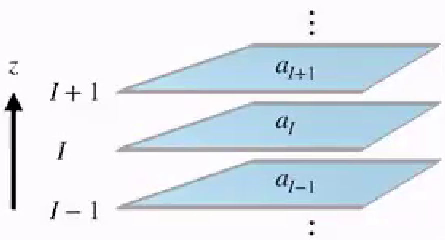
Done

Physics of these models



Ma, Shirley, Cheng, Levin, McGreevy Chen 21
Qui, Joynt, MacDonald 89. 90

$S = \int \frac{K_{IJ}}{4\pi} a_I \wedge da_J$ + Maxwell term

"gauge charges = planons"

$B_I = K_{IJ}^{-1}\rho_J$ flux attachment is "long range"

w/ $K = \begin{pmatrix} m & n & & & \\ n & m & n & & \\ & \ddots & \ddots & \ddots & \\ & & n & m & n \\ & & & n & m \end{pmatrix}$

Gapped $(|m| > |2n|)$: Fracton (Planon) Phase

"Long range braiding"

Gapless $(|m| \leq |2n|)$: Confusing from field theory perspective

Stable!

String-like order parameters

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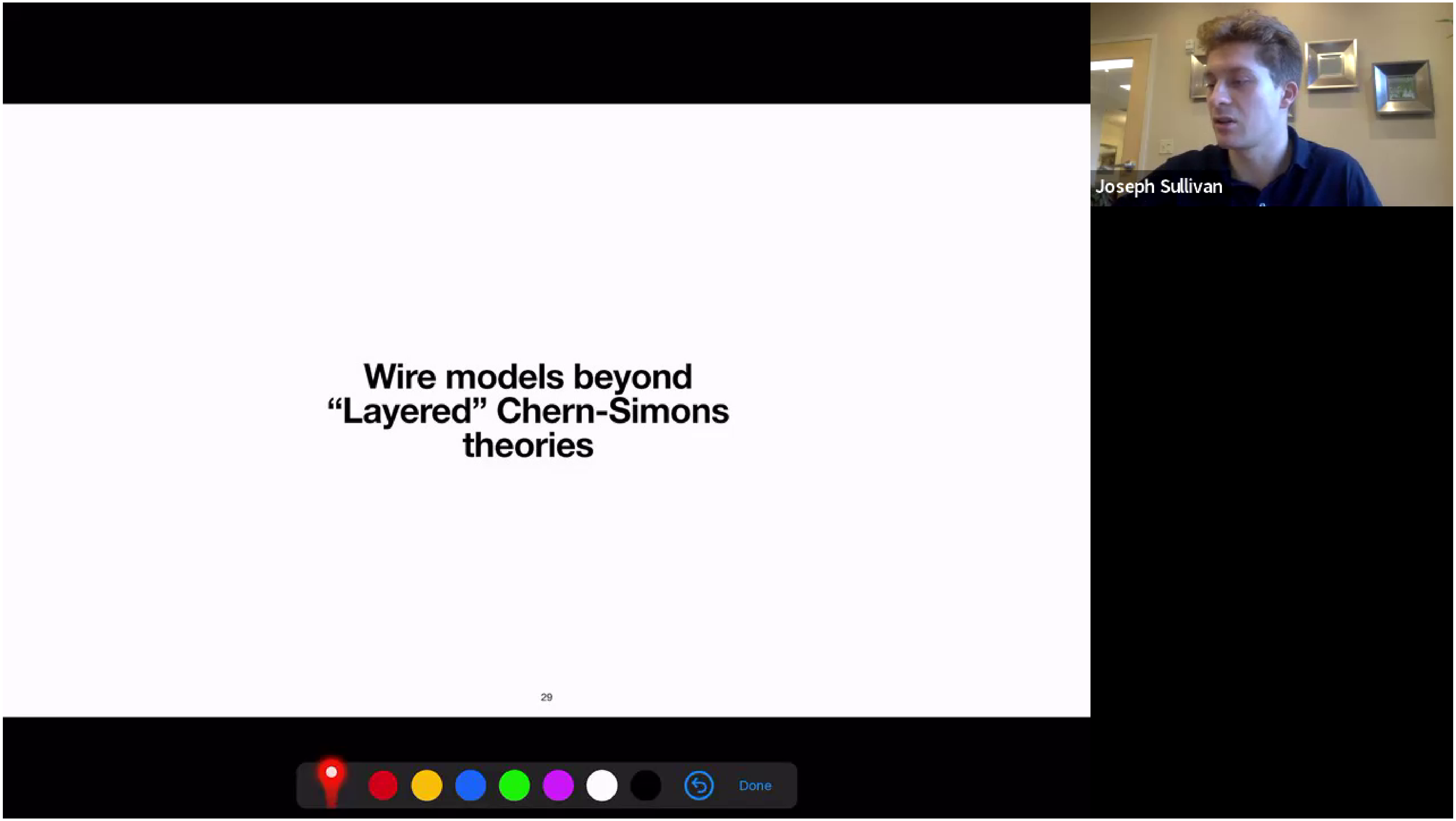
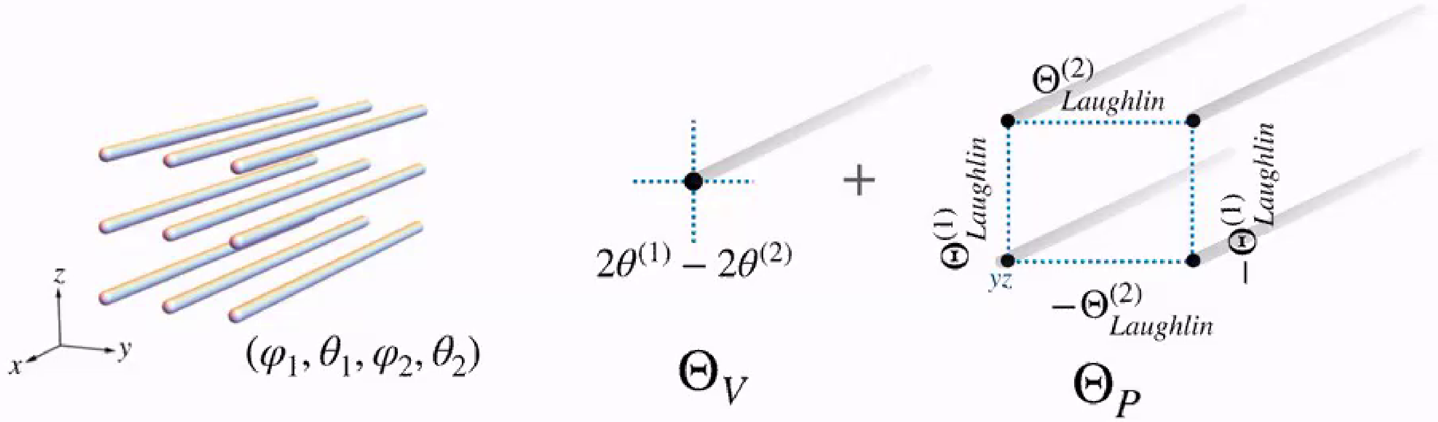
Done


Wire models beyond
“Layered” Chern-Simons
theories
29
Joseph Sullivan
Done

"Fractonal" Quantum Hall State



$$\Theta_{Laughlin} = \varphi_{y+1} - \varphi_y + m(\theta_y + \theta_{y+1})$$

$$T_x : \Theta_{V/P} \to \Theta_{V/P} \implies \text{compressible}$$

JS, Iadecola, Williamson (2020)

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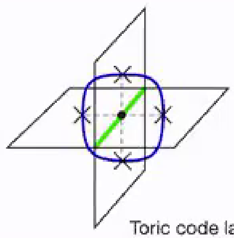
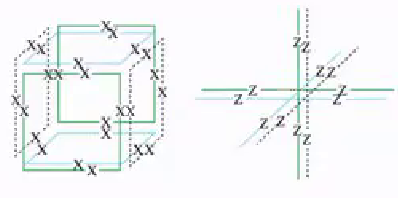
"Fractonal" Quantum Hall State

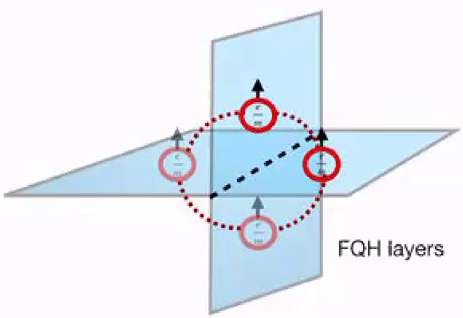


Condense m particles →



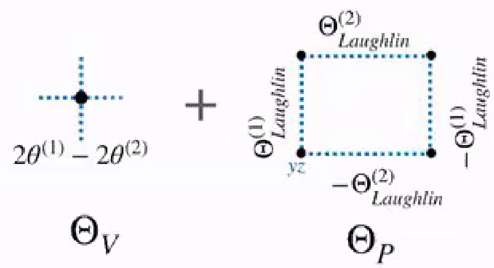
Ma, Lake, Chen, Hermele (2017)

What about FQH states?



condense anyon string →

Use wire model to get explicit microscopic model



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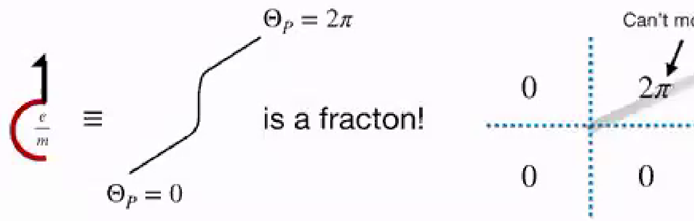
JS, Iadecola, Williamson (2020)

Done

Properties



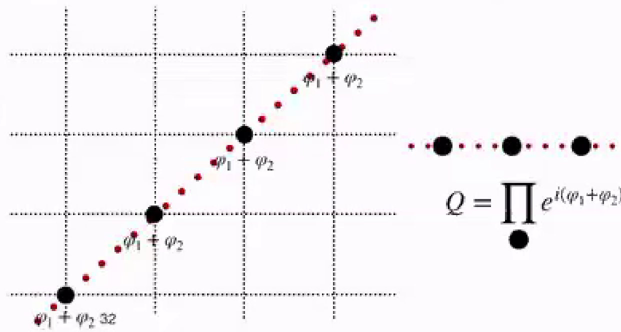
Can't move this in any direction w/out creating additional excitations

Lineons and planons as well

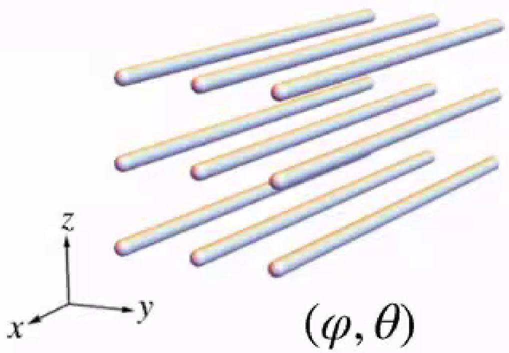
Stable: All local vertex ops create gapped excitations

Emergent $U(1)_\theta$ "cylindrical" 1-form symmetry

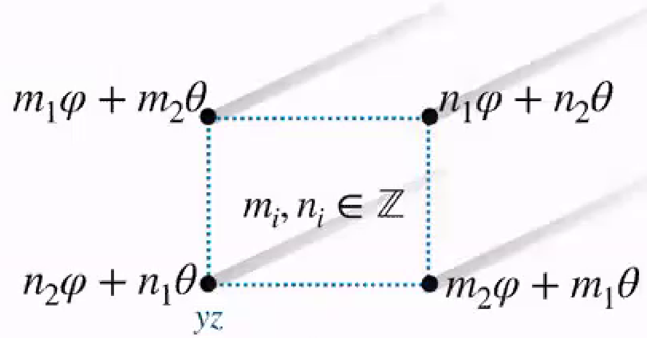
Mixed t'Hooft Anomaly between global charge symmetry and Q



Square plaquette models



(φ, θ)



Θ_{yz}

$$m_1 + m_2 + n_1 + n_2 = 0 \implies T_x : \Theta \to \Theta \implies \text{compressible}$$

JS, Cheng, Dua (2020)

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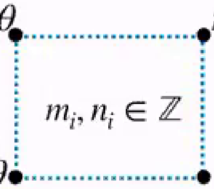
Membrane operator example

$m_1\varphi + m_2\theta$ $\quad$ $n_1\varphi + n_2\theta$



$m_i, n_i \in \mathbb{Z}$

$n_2\varphi + n_1\theta$ $\quad$ $m_2\varphi + m_1\theta$

yz

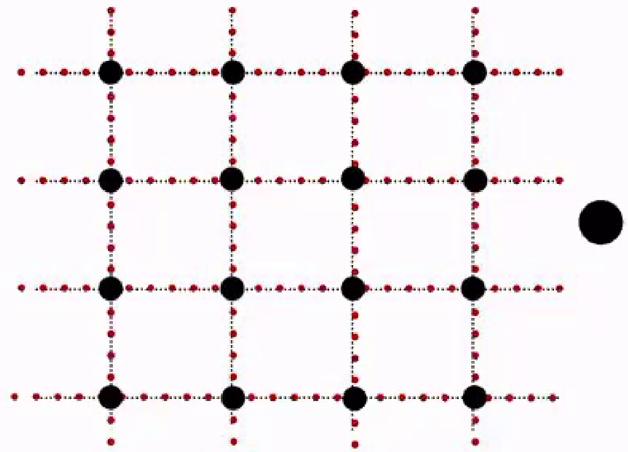
Θ_{yz}

$$m_1 + m_2 + n_1 + n_2 = 0$$

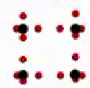
$$m_1 \neq n_1, n_2, m_2$$

$$n_1 \neq n_2, m_2$$

$$n_2 \neq m_2$$



$e^{i\varphi}$



$Q = \prod e^{i\varphi}$

$$G_{IR} = U(1)^{[0]}_{\varphi} \times U(1)^{[0]}_{\theta} \implies$$ "Weak" Luttinger Liquid?

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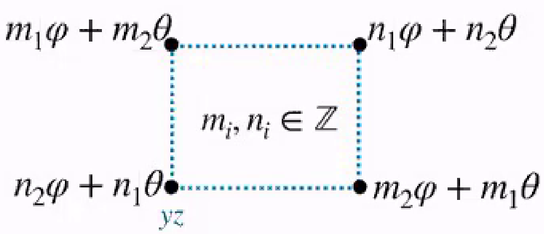
Membrane operator example



Θ_{yz}

$$m_1 + m_2 + n_1 + n_2 = 0$$

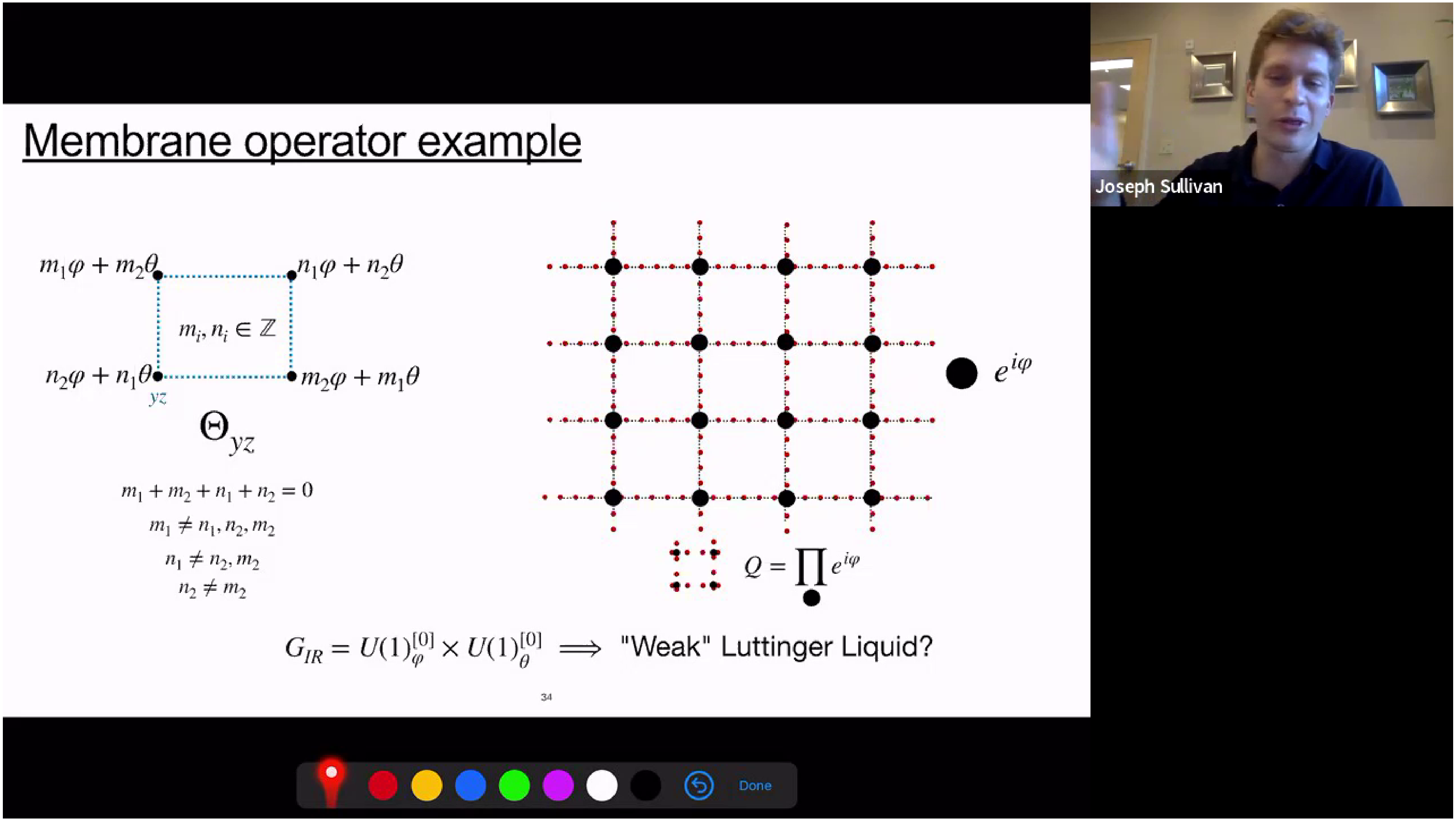
$$m_1 \neq n_1, n_2, m_2$$

$$n_1 \neq n_2, m_2$$

$$n_2 \neq m_2$$

34

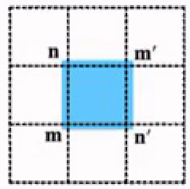
Done



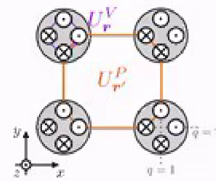
Membrane operator example
$m_1\varphi + m_2\theta$
$n_1\varphi + n_2\theta$
$m_i, n_i \in \mathbb{Z}$
$n_2\varphi + n_1\theta$
$m_2\varphi + m_1\theta$
yz
Θ_{yz}
$m_1 + m_2 + n_1 + n_2 = 0$
$m_1 \neq n_1, n_2, m_2$
$n_1 \neq n_2, m_2$
$n_2 \neq m_2$
$e^{i\varphi}$
$Q = \prod e^{i\varphi}$
$G_{IR} = U(1)^{[0]}_{\varphi} \times U(1)^{[0]}_{\theta} \implies$ "Weak" Luttinger Liquid?
34
Joseph Sullivan
Done

Future Directions

- Effective theories for other wire models



JS, M Cheng, A Dua 21



JS, T Iadecola, D Williamson 21

- Wire Models for other effective theories
  - ⋆ General K-Matrices
  - ⋆ Gapless models

- 2D Weak Superfluid?
  - ⋆ 2D compressibly state with no local order parameter
  - ⋆ Probably outside wire framework: Wire model $\implies$ CS theory

gapped $K \implies \nu$ constrained

gapless $K \implies$ local order parameter in 2D

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Thank you!

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