

Title: The Story of Anyons

Speakers: Steve Simon

Series: Colloquium

Date: December 15, 2021 - 2:00 PM

URL: <https://pirsa.org/21120030>

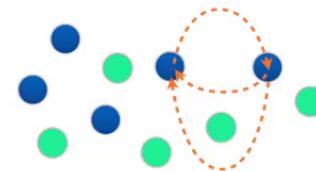
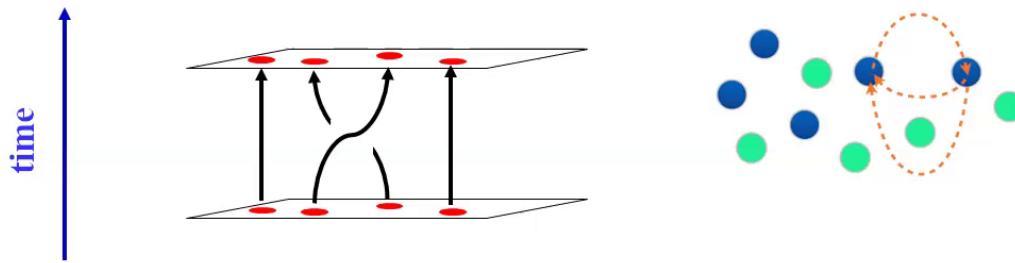
Abstract: I will review the history of anyons, particles that are neither bosons nor fermions, starting with their theoretical proposal all the way to their definitive experimental observation over 40 years later. I will further discuss why the more general idea of non-abelian anyons is of intense interest for quantum computation. If time permits I will give a status report on some of the current non-abelian anyon experiments.

Zoom Link: <https://pitp.zoom.us/j/98698779123?pwd=SWIyak9OZ0dud2ZGcWdoazdkVURHQT09>

The Story of Anyons

Caution: This is a selective history

Steven Simon



Steve Simon

EPSRC

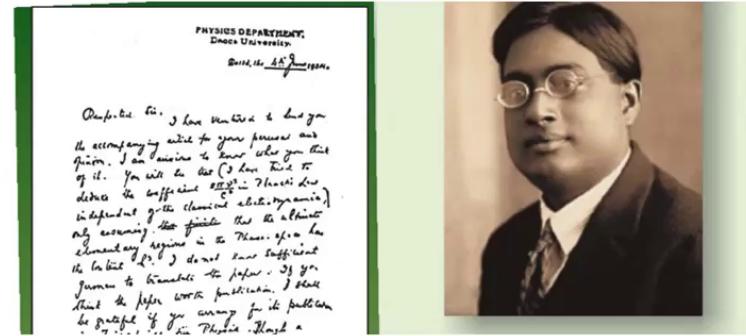
UNIVERSITY OF
OXFORD

What happens when you exchange two identical particles?

BOSONS

1924: Bose-Einstein Statistics

"Respected Sir,
I have ventured to send you
the accompanying article
for your perusal and opinion..."



FERMIONS

1925: Pauli Exclusion

Über den Zusammenhang des Abschlusses der Elektronengruppen im Atom mit der Komplexstruktur der Spektren.

Von W. Pauli jr. in Hamburg.

(Eingegangen am 16. Januar 1925.)

Resulting in Fermi-Dirac Statistics ... discovered first by Jordan 1925..

... then Fermi and Dirac 1926

1930: **Basics of Quantum Mechanics Finished** (QFT by ~1948)



Are there other types of particles besides bosons and fermions?

There *were* attempts to define “parastatistics” but eventually these were found to be essentially bosons and fermions in disguise. (Doplicher, Haag, Roberts ~ 1970)

No one asked about lower dimensions until...

IL NUOVO CIMENTO

VOL. 37 B, N. 1

11 Gennaio 1977

“...in one and two dimensions a continuum of possible intermediate cases connects the boson and fermion cases...”

On the Theory of Identical Particles.

J. M. LEINAAS and J. MYRAEIM

Department of Physics, University of Oslo - Oslo

(ricevuto il 16 Agosto 1976)

Clockwise exchange gets phase



$$e^{i\theta}$$

Counterclockwise exchange gets phase

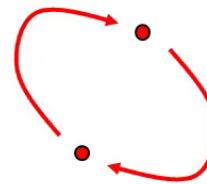


$$e^{-i\theta}$$



Dogma:

Exchanging twice should be identity



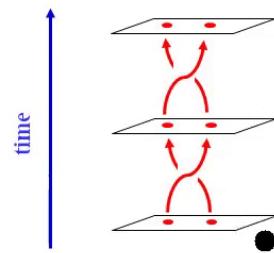
$$\sqrt{1} = \pm 1$$

- Bosons $\Psi(\mathbf{r}_1, \mathbf{r}_2) = \Psi(\mathbf{r}_2, \mathbf{r}_1)$
- Fermions $\Psi(\mathbf{r}_1, \mathbf{r}_2) = -\Psi(\mathbf{r}_2, \mathbf{r}_1)$

And That's All ?



In 2+1 Dimensions: Two Exchanges \neq Identity



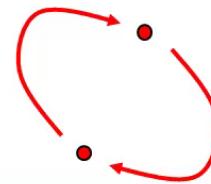


Kareem Hassaan



Dogma:

Exchanging twice should be identity



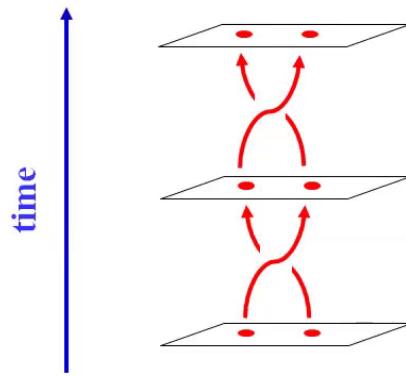
$$\sqrt{1} = \pm 1$$

- Bosons $\Psi(\mathbf{r}_1, \mathbf{r}_2) = \Psi(\mathbf{r}_2, \mathbf{r}_1)$
- Fermions $\Psi(\mathbf{r}_1, \mathbf{r}_2) = -\Psi(\mathbf{r}_2, \mathbf{r}_1)$

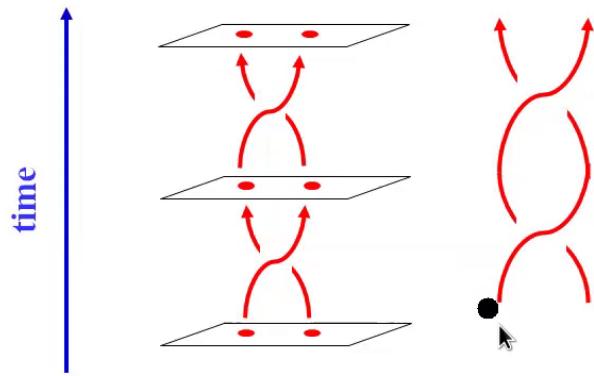


And That's All ?

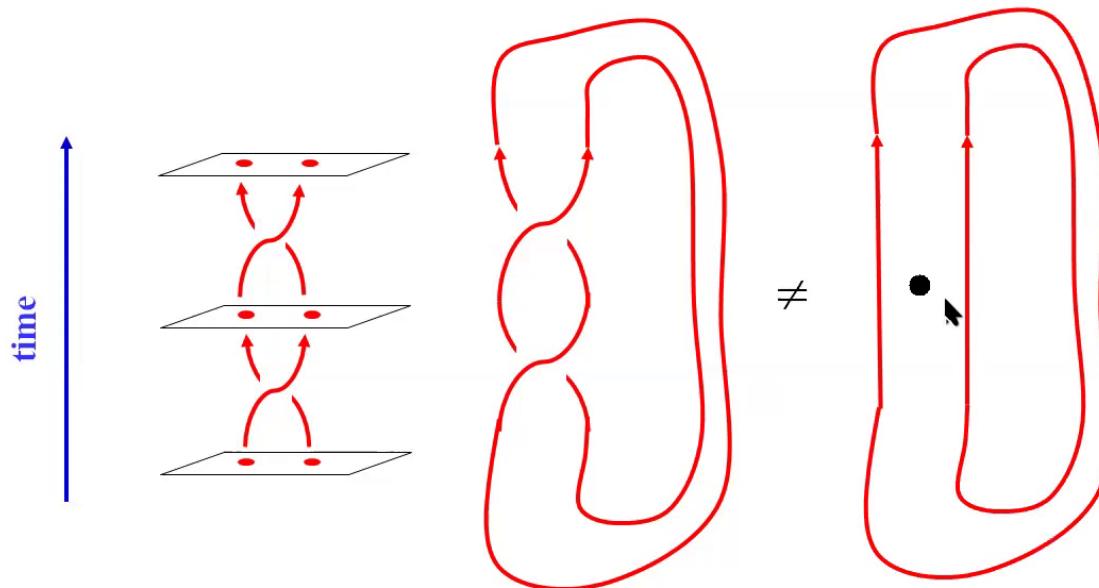
In 2+1 Dimensions: Two Exchanges \neq Identity



In 2+1 Dimensions: Two Exchanges \neq Identity

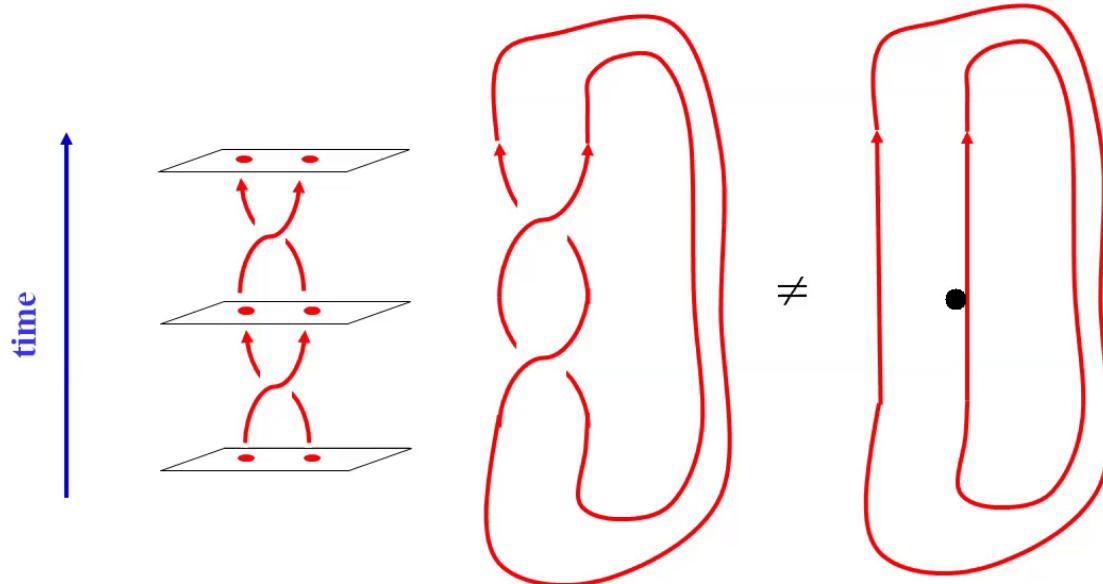


In 2+1 Dimensions: Two Exchanges \neq Identity





In 2+1 Dimensions: Two Exchanges \neq Identity



In 3+1 Dimensions: Two Exchanges = Identity

No Knots in (one dimensional) World Lines in 3+1 D !



Why are there no knots in 3+1 dimensions?

1+1 d point particles



No way to cross without crashing



Why are there no knots in 3+1 dimensions?

1+1 d point particles



No way to cross without crashing

Why are there no knots in 3+1 dimensions?



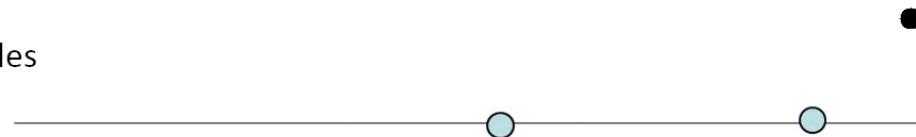
2+1 d point particles



Why are there no knots in 3+1 dimensions?



2+1 d point particles





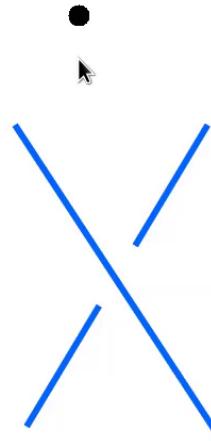
Why are there no knots in 3+1 dimensions?

2+1 d point particles



Can get to the other side without touching

2+1 d world lines



No way to change over-crossing to under-crossing without crashing



Why are there no knots in 3+1 dimensions?

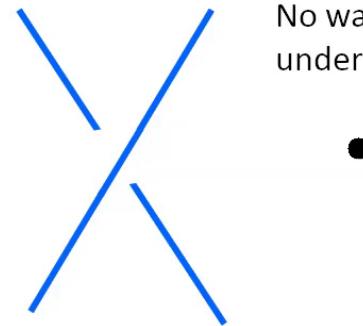


2+1 d point particles



Can get to the other side without touching

2+1 d world lines



No way to change over-crossing to under-crossing without crashing

But in 3+1d, can get to other side without touching

1977 J. M. LEINAAS and J. MYRHEIM

1982 Quantum Mechanics of Fractional-Spin Particles
Frank Wilczek

coins word “Anyon”
5th citation of Leinaas + Myrheim



1982 Two-Dimensional Magnetotransport in the Extreme Quantum Limit
D. C. Tsui,^{(a), (b)} H. L. Stormer,^(a) and A. C. Gossard

Discovery of
FQHE!

1983 Anomalous Quantum Hall Effect: An Incompressible Quantum Fluid
with Fractionally Charged Excitations
R. B. Laughlin

Laughlin
Theory

Nobel Prize 1998

1977	J. M. LEINAAS and J. MYRHEIM	
1982	Quantum Mechanics of Fractional-Spin Particles Frank Wilczek	coins word “Anyon” 5 th citation of Leinaas + Myrheim
1982	Two-Dimensional Magnetotransport in the Extreme Quantum Limit D. C. Tsui, ^{(a), (b)} H. L. Stormer, ^(a) and A. C. Gossard	Discovery of FQHE!
1983	Anomalous Quantum Hall Effect: An Incompressible Quantum Fluid with Fractionally Charged Excitations R. B. Laughlin	Laughlin Theory
1984	Statistics of Quasiparticles and the Hierarchy of Fractional Quantized Hall States B. I. Halperin	FQHE quasiparticles are anyons!
	Fractional Statistics and the Quantum Hall Effect Daniel Arovas J. R. Schrieffer and Frank Wilczek	



Nobel Prize 1998

1977	J. M. LEINAAS and J. MYRHEIM	
1982	Quantum Mechanics of Fractional-Spin Particles Frank Wilczek	coins word “Anyon” 5 th citation of Leinaas + Myrheim
1982	Two-Dimensional Magnetotransport in the Extreme Quantum Limit D. C. Tsui, ^{(a), (b)} H. L. Stormer, ^(a) and A. C. Gossard	Discovery of FQHE!
1983	Anomalous Quantum Hall Effect: An Incompressible Quantum Fluid with Fractionally Charged Excitations R. B. Laughlin	Laughlin Theory
1984	Statistics of Quasiparticles and the Hierarchy of Fractional Quantized Hall States B. I. Halperin	FQHE quasiparticles are anyons!
	Fractional Statistics and the Quantum Hall Effect Daniel Arovas J. R. Schrieffer and Frank Wilczek	●
:	
:	
2020		



Nobel Prize 1998



(Fractional) Quantum Hall Primer

- Occurs in 2D electron systems in strong B-field when filling $v \approx p/q$

$$\nu = \frac{\text{electron density}}{\text{flux density}} \approx p/q$$

- We consider $v=1/3$:

Low energy excitations:

Charge $e^* = e/3$

Statistics $\theta = 2\pi/3$



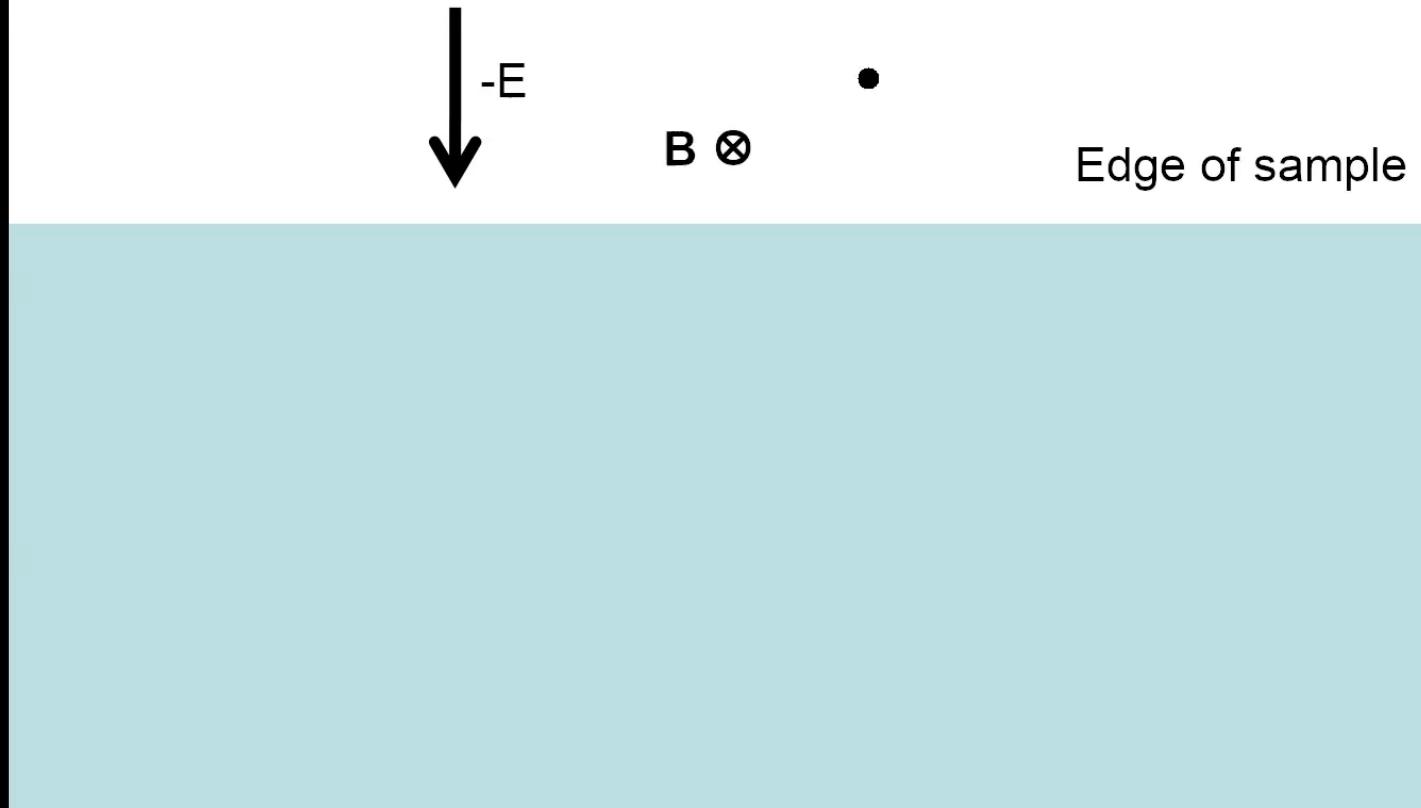
Clockwise exchange gets phase



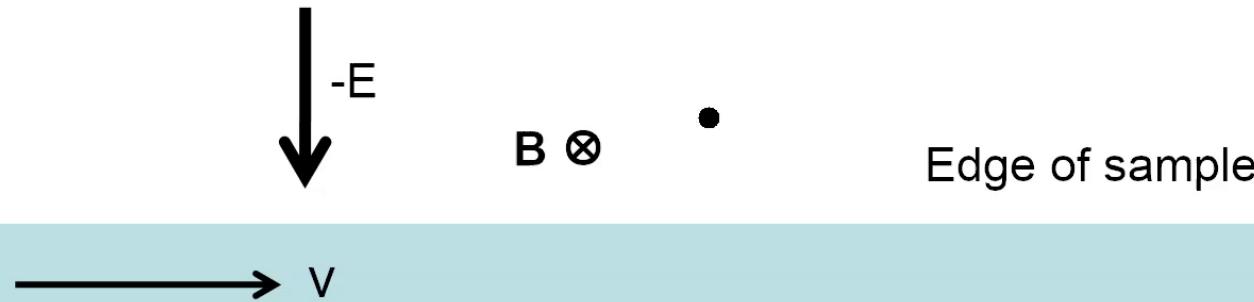
Counterclockwise exchange gets phase



Quantum Hall Edge States



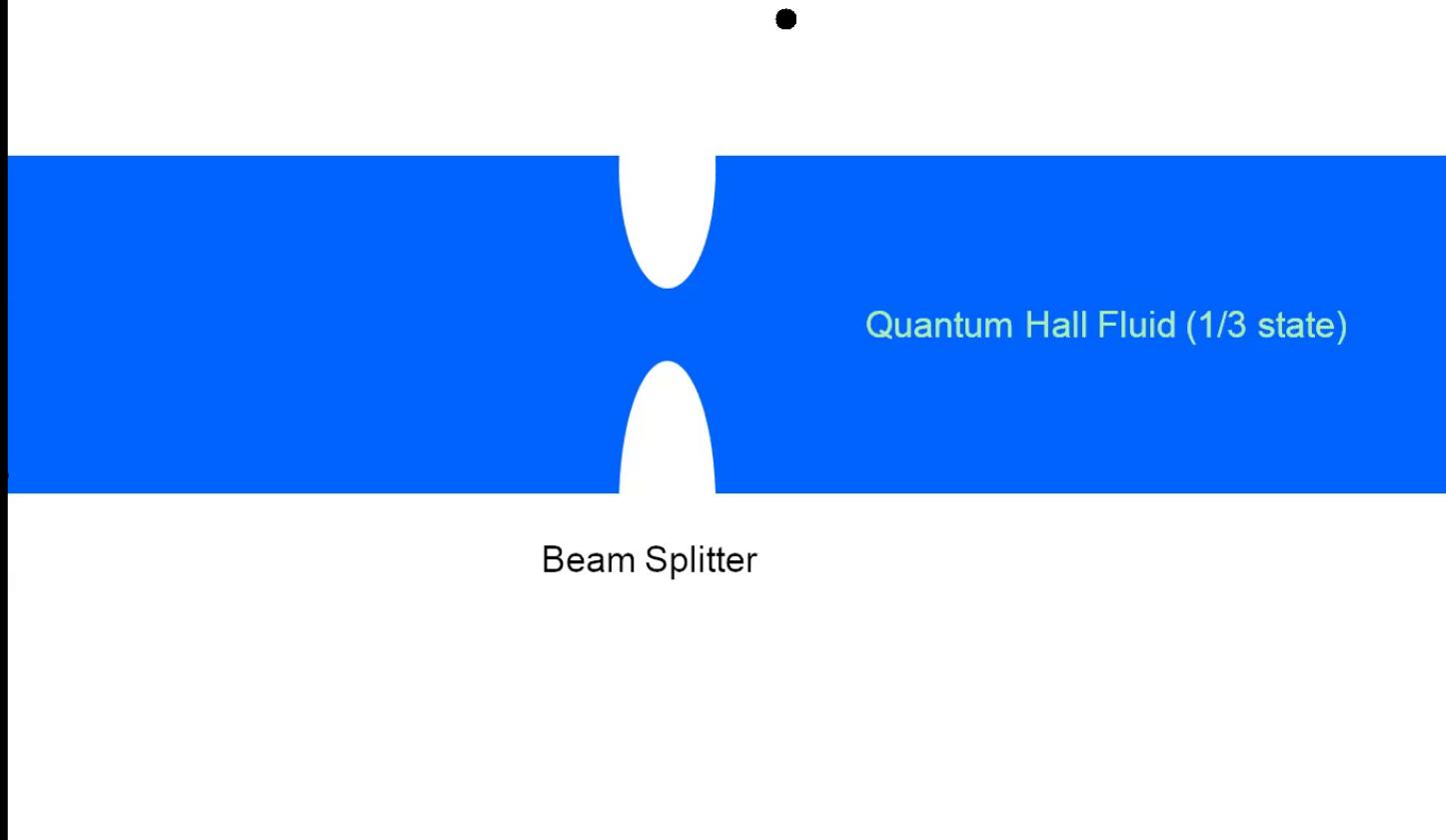
Quantum Hall Edge States



Edge States Carry Electrical Current



QPC=Quantum Point Contact
= Half-Silvered Mirror



Two Successful Observation of Anyons in 2020

RESEARCH

MESOSCOPIC PHYSICS

Bartolomei *et al.*, *Science* **368**, 173–177 (2020) 10 April 2020

Fractional statistics in anyon collisions

H. Bartolomei^{1*}, M. Kumar^{1*†}, R. Bisognin¹, A. Marguerite^{1‡}, J.-M. Berroir¹, E. Bocquillon¹, B. Plaçais¹, A. Cavanna², Q. Dong², U. Gennser², Y. Jin², G. Fève^{1§}

nature
physics

ARTICLES

<https://doi.org/10.1038/s41567-020-1019-1>

NATURE PHYSICS | VOL 16 | SEPTEMBER 2020 | 931–936

Direct observation of anyonic braiding statistics

J. Nakamura^{1,2}, S. Liang^{1,2}, G. C. Gardner  ^{2,3} and M. J. Manfra  ^{1,2,3,4,5} 

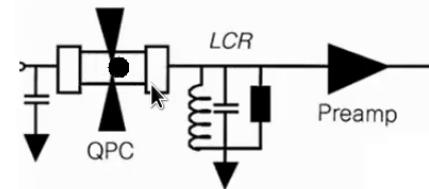
... Plus I will report on progress observing nonabelions



Nonequilibrium Noise and Fractional Charge in the Quantum Hall Effect

C. L. Kane Matthew P. A. Fisher

Shot noise measurements of fractional charge



Nonequilibrium Noise and Fractional Charge in the Quantum Hall Effect

C. L. Kane Matthew P. A. Fisher

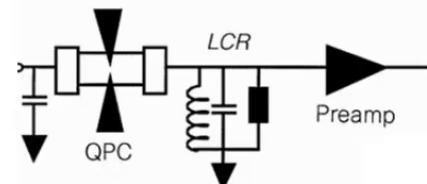


Shot noise measurements of fractional charge

VOLUME 79, NUMBER 13 PHYSICAL REVIEW LETTERS 29 SEPTEMBER 1997

Observation of the $e/3$ Fractionally Charged Laughlin Quasiparticle

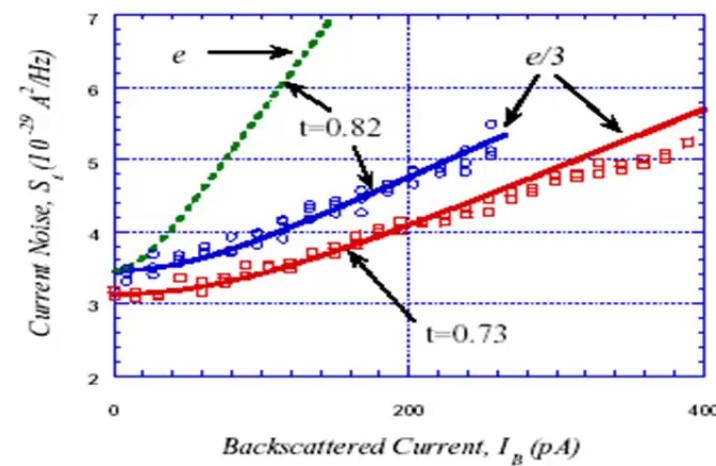
L. Saminadayar and D. C. Glattli
Y. Jin and B. Etienne



Direct observation of a fractional charge

R. de-Picciotto, M. Reznikov, M. Heiblum, V. Umansky,
G. Bunin & D. Mahalu

NATURE | VOL 389 | 11 SEPTEMBER 1997



Nonequilibrium Noise and Fractional Charge in the Quantum Hall Effect

C. L. Kane Matthew P. A. Fisher

Shot noise measurements of fractional charge



VOLUME 79, NUMBER 13

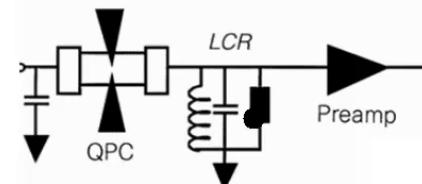
PHYSICAL REVIEW LETTERS

29 SEPTEMBER 1997

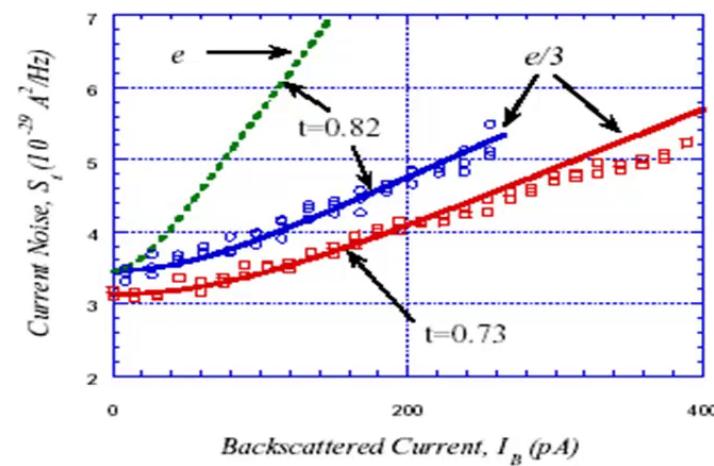
Observation of the $e/3$ Fractionally Charged Laughlin Quasiparticle

L. Saminadayar and D. C. Glattli

Y. Jin and B. Etienne

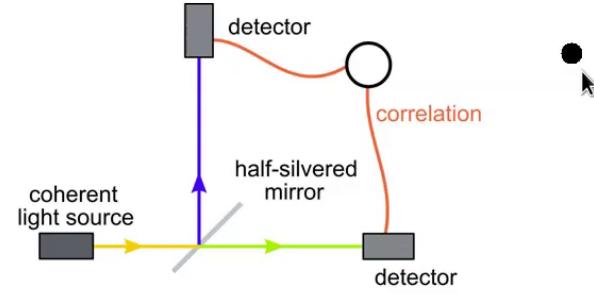
Direct observation of a
fractional chargeR. de-Picciotto, M. Reznikov, M. Heiblum, V. Umansky,
G. Bunin & D. Mahalu

NATURE | VOL 389 | 11 SEPTEMBER 1997

This became a technology!

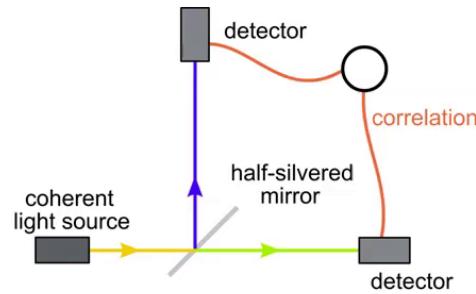
Bosons bunch...
Fermions antibunch...

Hanbury Brown and Twiss (1954)



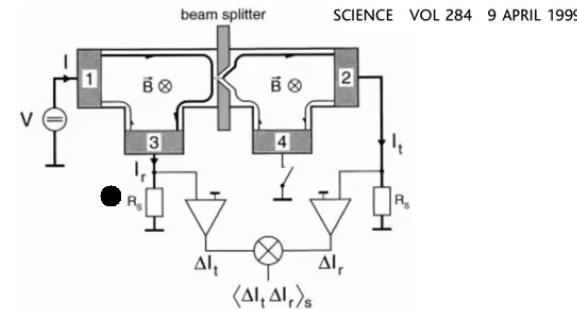
Bosons bunch...
Fermions antibunch...

Hanbury Brown and Twiss (1954)



The Fermionic Hanbury Brown and Twiss Experiment

M. Henny,¹ S. Oberholzer,¹ C. Strunk,¹ T. Heinzel,² K. Ensslin,²
M. Holland,³ C. Schönenberger^{1*}



PHYSICAL REVIEW B

VOLUME 46, NUMBER 19

15 NOVEMBER 1992-I

Scattering theory of current and intensity noise correlations in conductors and wave guides

M. Büttiker

Bosons bunch...
Fermions antibunch...

Hanbury Brown and Twiss (1956)

VOLUME 92, NUMBER 2

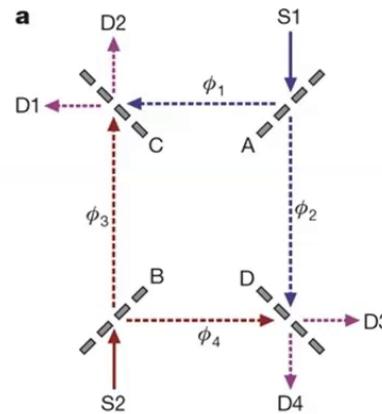
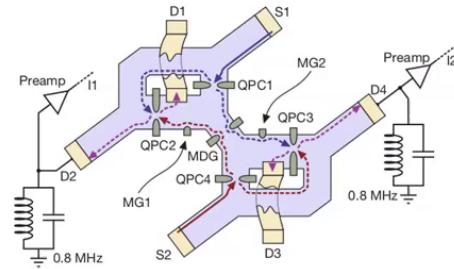
PHYSICAL REVIEW LETTERS

week ending
16 JANUARY 2004



Two-Particle Aharonov-Bohm Effect and Entanglement in the Electronic Hanbury Brown-Twiss Setup

P. Samuelsson, E. V. Sukhorukov, and M. Büttiker

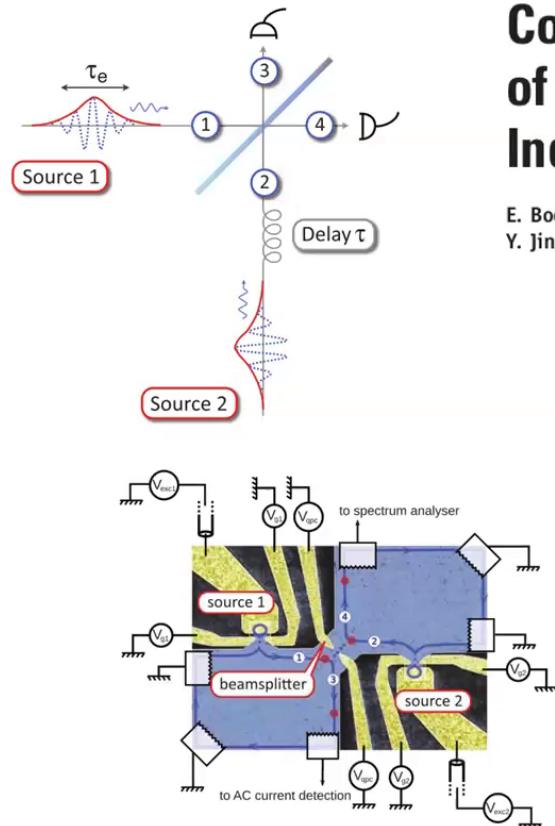


Interference between two indistinguishable electrons from independent sources

I. Neder¹, N. Ofek¹, Y. Chung², M. Heiblum¹, D. Mahalu¹ & V. Umansky¹ 2007 **Nature**

Bosons bunch...
Fermions antibunch...

Hong-Ou-Mandel (1987)

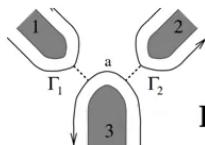


Coherence and Indistinguishability of Single Electrons Emitted by Independent Sources

SCIENCE VOL 339 1 MARCH 2013

E. Bocquillon,¹ V. Freulon,¹ J.-M. Berroir,¹ P. Degiovanni,² B. Plaçais,¹ A. Cavanna,³
Y. Jin,³ G. Féve^{1*}



Partition Noise and Statistics in the Fractional Quantum Hall EffectI. Safi,¹ P. Devillard,^{1,2} and T. Martin^{1,3}

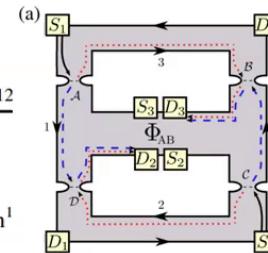
Fractional Statistics has important effects on current correlation (noise) measurements

Fractional statistics, Hanbury-Brown and Twiss correlations and the quantum Hall effectRodolphe Guyon^{a,b}, Thierry Martin^{a,b*}, Inès Safi^{a,c}, Pierre Devillard^{a,d} C. R. Physique 3 (2002)**Revisiting the Hanbury Brown–Twiss Setup for Fractional Statistics**

Smitha Vishveshwara

Signatures of Fractional Statistics in Noise Experiments in Quantum Hall Fluids

Eun-Ah Kim, Michael Lawler, Smitha Vishveshwara, and Eduardo Fradkin

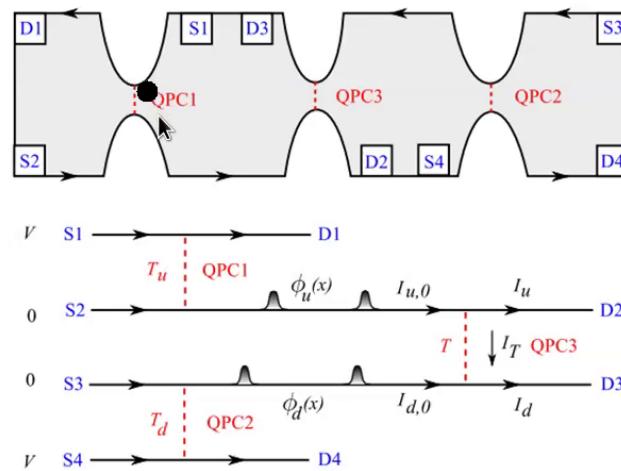
Hanbury Brown–Twiss Interference of AnyonsGabriele Campagnano,¹ Oded Zilberberg,¹ Igor V. Gornyi,^{2,3} Dmitri E. Feldman,⁴ Andrew C. Potter,⁵ and Yuval Gefen¹

Steven Simon



Current Correlations from a Mesoscopic Anyon Collider

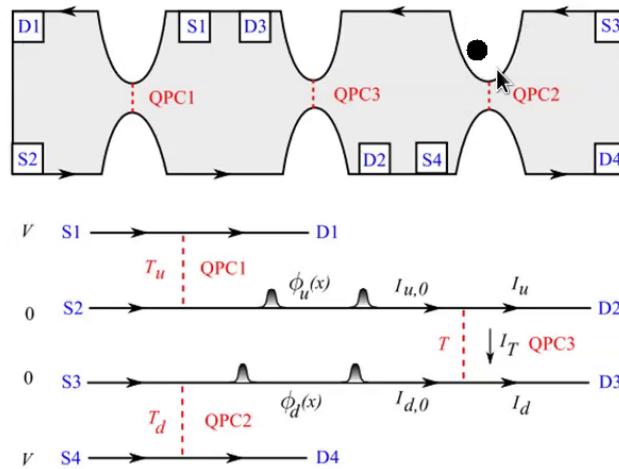
Bernd Rosenow,^{1,2} Ivan P. Levkivskyi,^{3,2} and Bertrand I. Halperin²





Current Correlations from a Mesoscopic Anyon Collider

Bernd Rosenow,^{1,2} Ivan P. Levkivskyi,^{3,2} and Bertrand I. Halperin²



$$\langle \delta I_d \delta I_u \rangle_{\omega=0} = \text{(appropriately normalized and with } T_u = T_d \text{)}$$

$$= 1 - \frac{\tan \pi \lambda}{\tan \pi \delta} \frac{1}{1 - 2\delta} \quad \lambda = \delta = 1/3 \quad \longrightarrow \quad -2$$

λ = Edge “screening parameter”
(noninteracting anyons = 1/3)

Negative correlations =
Bunching Behavior

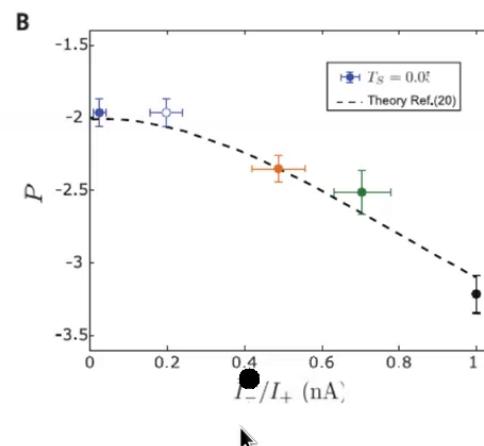
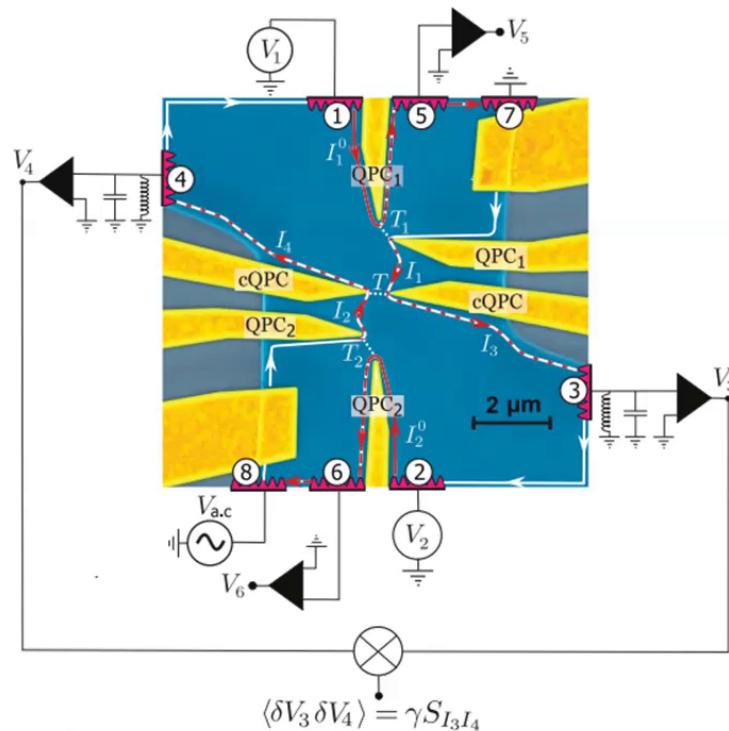
δ = Statistical angle (noninteracting anyons = 1/3)

Fractional statistics in anyon collisions

H. Bartolomei^{1*}, M. Kumar^{1*}†, R. Bisognin¹, A. Marguerite¹‡, J.-M. Berroir¹, E. Bocquillon¹, B. Plaçais¹, A. Cavanna², Q. Dong², U. Gennser², Y. Jin², G. Fève¹§



Bartolomei *et al.*, *Science* **368**, 173–177 (2020) 10 April 2020



Two Successful Observation of Anyons in 2020



RESEARCH

MESOSCOPIC PHYSICS

Bartolomei *et al.*, *Science* **368**, 173–177 (2020) 10 April 2020

Fractional statistics in anyon collisions

H. Bartolomei^{1*}, M. Kumar^{1*†}, R. Bisognin¹, A. Marguerite^{1‡}, J.-M. Berroir¹, E. Bocquillon¹, B. Plaçais¹, A. Cavanna², Q. Dong², U. Gennser², Y. Jin², G. Fève^{1§}

nature
physics

ARTICLES

<https://doi.org/10.1038/s41567-020-1019-1>

NATURE PHYSICS | VOL 16 | SEPTEMBER 2020 | 931–936

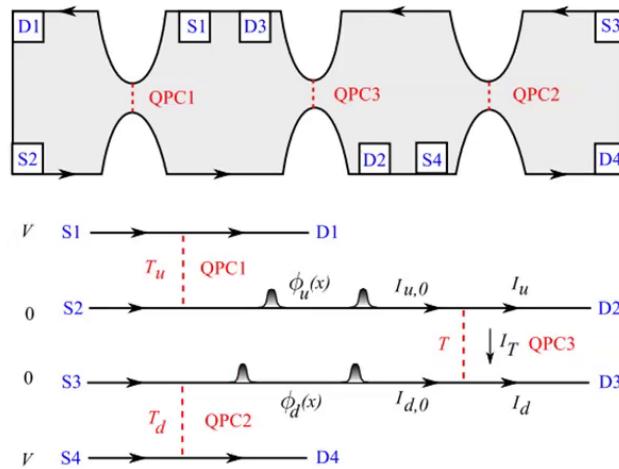
Direct observation of anyonic braiding statistics

J. Nakamura^{1,2}, S. Liang^{1,2}, G. C. Gardner  ^{2,3} and M. J. Manfra  ^{1,2,3,4,5} 



Current Correlations from a Mesoscopic Anyon Collider

Bernd Rosenow,^{1,2} Ivan P. Levkivskyi,^{3,2} and Bertrand I. Halperin²



$$\langle \delta I_d \delta I_u \rangle_{\omega=0} = \text{(appropriately normalized and with } T_u = T_d \text{)}$$

$$= 1 - \frac{\tan \pi \lambda}{\tan \pi \delta} \frac{1}{1 - 2\delta} \quad \lambda = \delta = 1/3 \longrightarrow -2$$

λ = Edge "screening parameter"
(noninteracting anyons = 1/3)



Negative correlations =
Bunching Behavior

δ = Statistical angle (noninteracting anyons = 1/3)

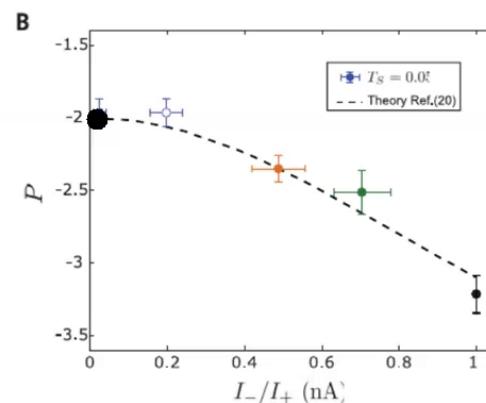
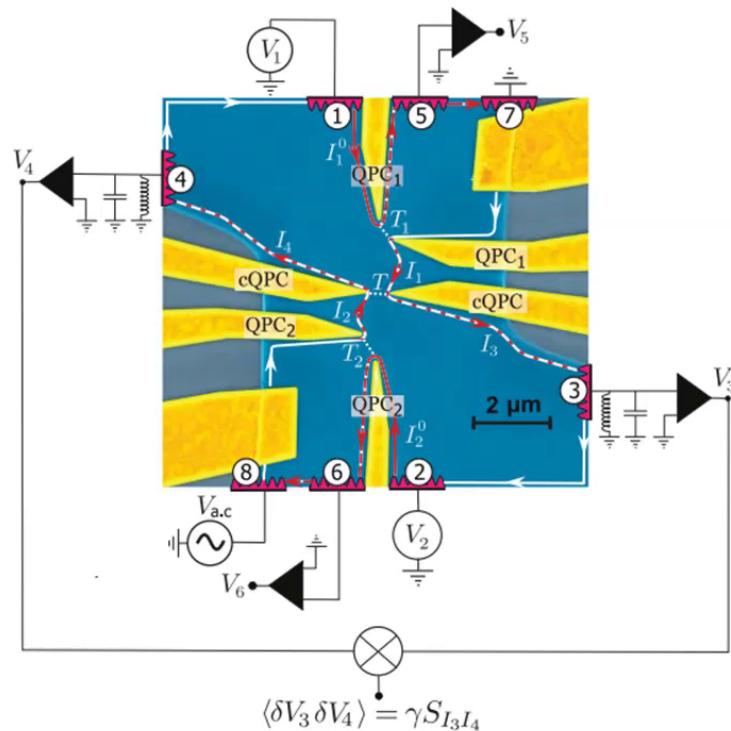


Fractional statistics in anyon collisions

H. Bartolomei^{1*}, M. Kumar^{1*}†, R. Bisognin¹, A. Marguerite¹‡, J.-M. Berroir¹, E. Bocquillon¹, B. Plaçais¹, A. Cavanna², Q. Dong², U. Gennser², Y. Jin², G. Fève^{1§}



Bartolomei *et al.*, *Science* **368**, 173–177 (2020) 10 April 2020



Two Successful Observation of Anyons in 2020



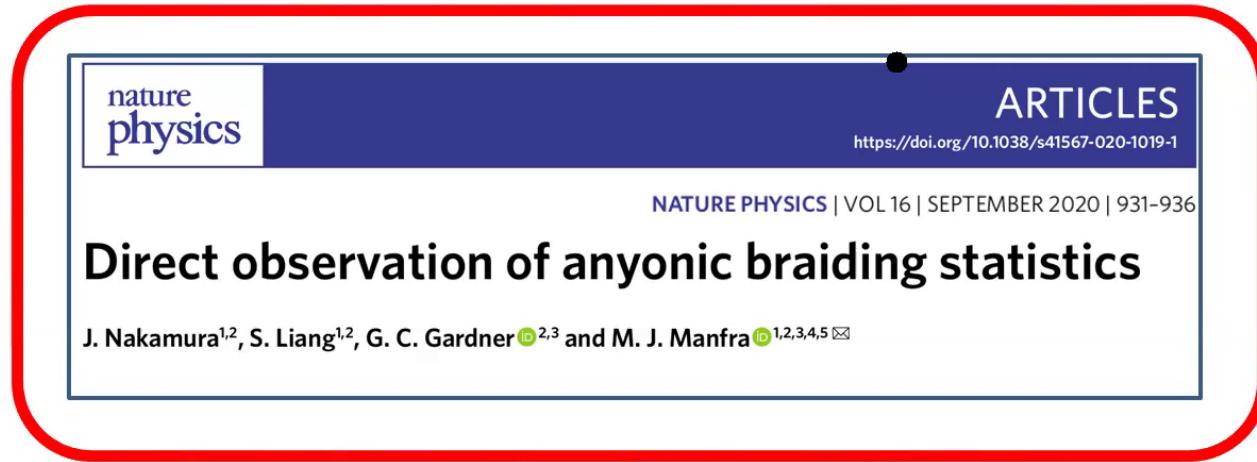
RESEARCH

MESOSCOPIC PHYSICS

Bartolomei *et al.*, *Science* **368**, 173–177 (2020) 10 April 2020

Fractional statistics in anyon collisions

H. Bartolomei^{1*}, M. Kumar^{1*†}, R. Bisognin¹, A. Marguerite^{1‡}, J.-M. Berroir¹, E. Bocquillon¹, B. Plaçais¹, A. Cavanna², Q. Dong², U. Gennser², Y. Jin², G. Fève^{1§}

A red rounded rectangle highlights the following article from Nature Physics:

nature physics ARTICLES
<https://doi.org/10.1038/s41567-020-1019-1>
NATURE PHYSICS | VOL 16 | SEPTEMBER 2020 | 931-936

Direct observation of anyonic braiding statistics

J. Nakamura^{1,2}, S. Liang^{1,2}, G. C. Gardner  ^{2,3} and M. J. Manfra  ^{1,2,3,4,5} 

Interferometry:

1990:

Semiclassical Theory of Localized Many-Anyon States

Steven Kivelson

Interference around a dot
can measure statistics

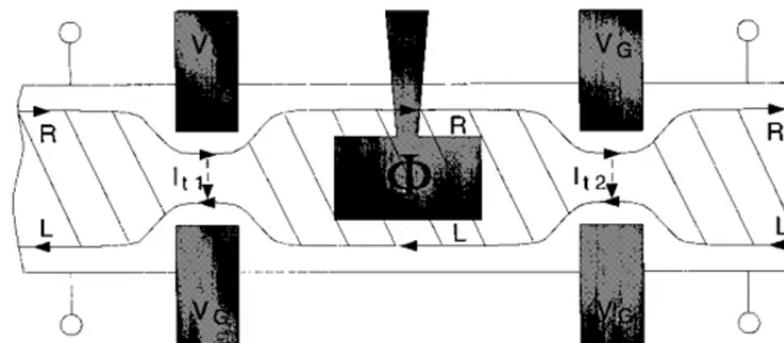


Steven Simon

1997:

Two point-contact interferometer for quantum Hall systems

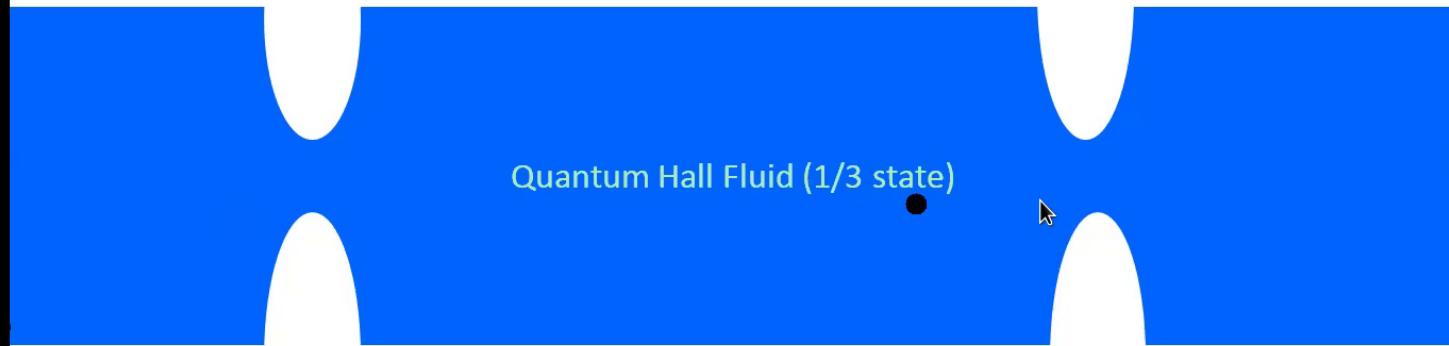
C. de C. Chamon, D. E. Freed, S. A. Kivelson, S. L. Sondhi, and X. G. Wen



Fabry-Perot Interferometer



Interference between two paths = Fabry Perot Interferometer



Beam Splitter

Mirror

Interference of two partial waves



Fabry-Perot Interferometer



Side gate changes phase

$$R = R_0 + R_1 \cos(\theta)$$

$$\theta = 2\pi e^*(\Phi + \beta V_G)$$

Flux

Gate voltage

Resistance R

side gate voltage



Fabry-Perot Interferometer

Side gate changes phase



$$R = R_0 + R_1 \cos(\theta)$$

$$\theta = 2\pi e^*(\Phi + \beta V_G)$$

Flux

Gate voltage

Resistance R

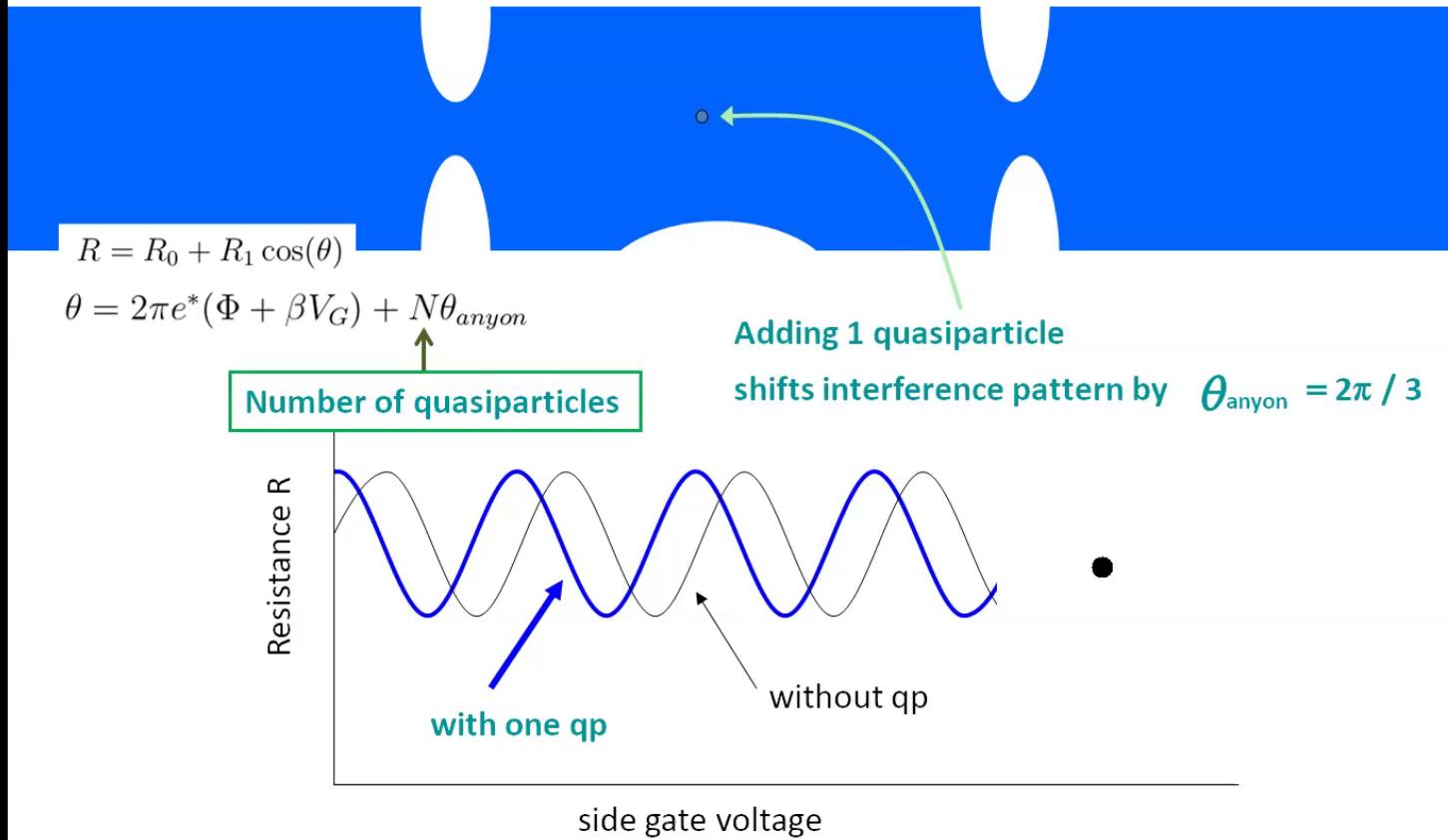
side gate voltage

Fabry-Perot Interferometer



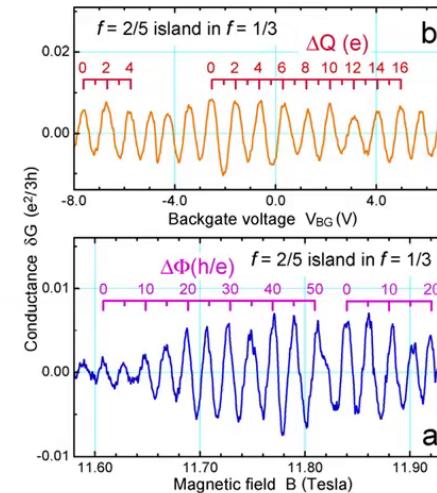
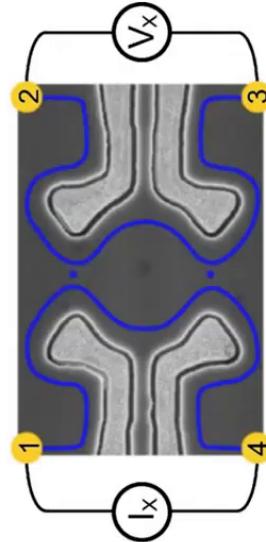
Steven Simon

Side gate changes phase



Realization of a Laughlin quasiparticle interferometer: Observation of fractional statistics

F. E. Camino, Wei Zhou, and V. J. Goldman



Lots of confusion....



Interferometry:

1990:

Semiclassical Theory of Localized Many-Anyon States

Steven Kivelson

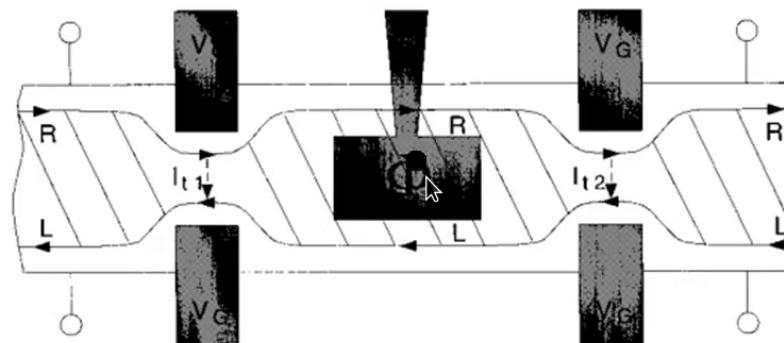
Interference around a dot
can measure statistics



1997:

Two point-contact interferometer for quantum Hall systems

C. de C. Chamon, D. E. Freed, S. A. Kivelson, S. L. Sondhi, and X. G. Wen



Fabry-Perot Interferometer

Side gate changes phase



$$R = R_0 + R_1 \cos(\theta)$$

$$\theta = 2\pi e^*(\Phi + \beta V_G)$$

Flux

Gate voltage

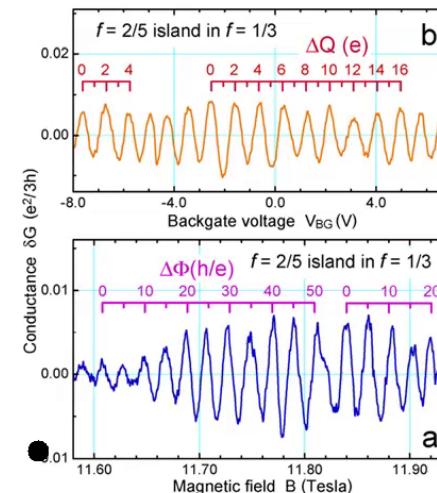
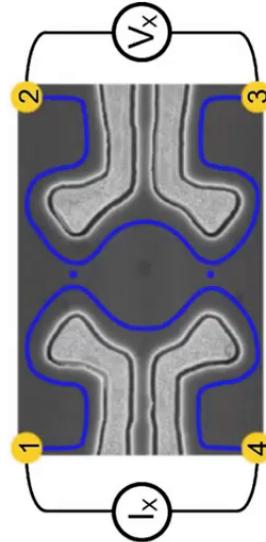
Resistance R

side gate voltage

Realization of a Laughlin quasiparticle interferometer: Observation of fractional statistics

F. E. Camino, Wei Zhou, and V. J. Goldman

Steven Simon



Lots of confusion....



Influence of Interactions on Flux and Back-Gate Period of Quantum Hall Interferometers

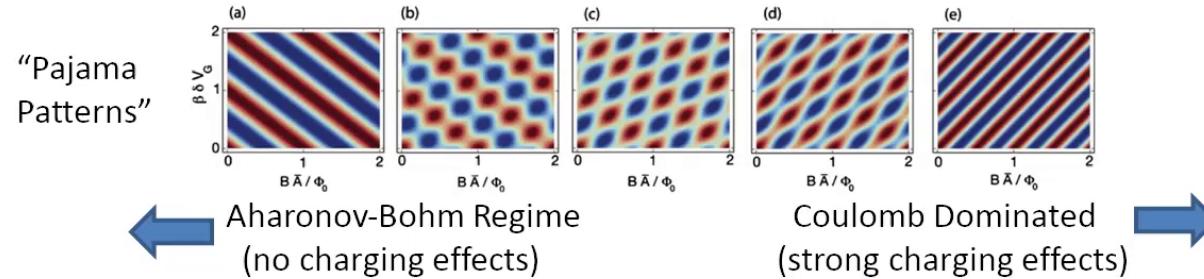
B. Rosenow* and B. I. Halperin

PHYSICAL REVIEW B 83, 155440 (2011)

**Charging effects
of the enclosed dot
can be crucial!**

Steven Simon

Theory of the Fabry-Pérot quantum Hall interferometer

Bertrand I. Halperin,¹ Ady Stern,² Izhar Neder,³ and Bernd Rosenow³

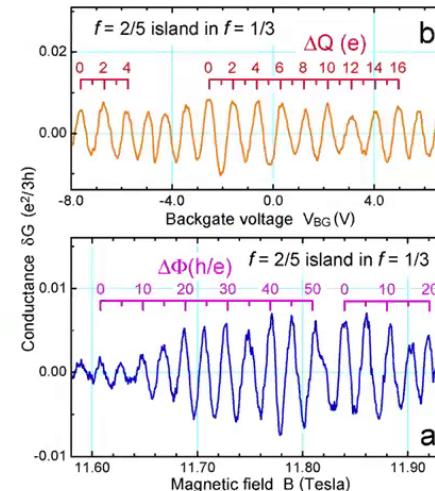
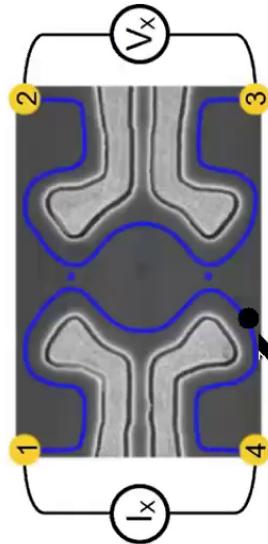
$$R = R_0 + R_1 \cos(\theta)$$

$$\theta = 2\pi e^*(\Phi + \beta V_G) + N\theta_{anyon} + \underbrace{\theta_C(\Phi, V_G, N)}_{\text{Coulomb correction}}$$



Realization of a Laughlin quasiparticle interferometer: Observation of fractional statistics

F. E. Camino, Wei Zhou, and V. J. Goldman



Lots of confusion....





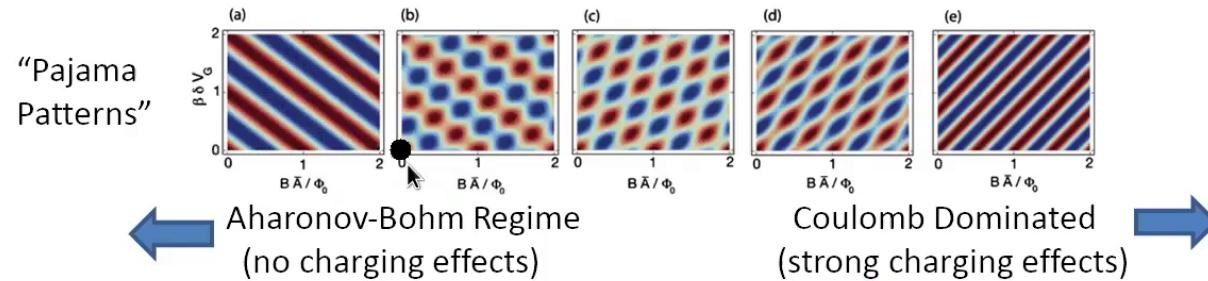
Influence of Interactions on Flux and Back-Gate Period of Quantum Hall Interferometers

B. Rosenow* and B. I. Halperin

PHYSICAL REVIEW B 83, 155440 (2011)

Theory of the Fabry-Pérot quantum Hall interferometer

Bertrand I. Halperin,¹ Ady Stern,² Izhar Neder,³ and Bernd Rosenow³



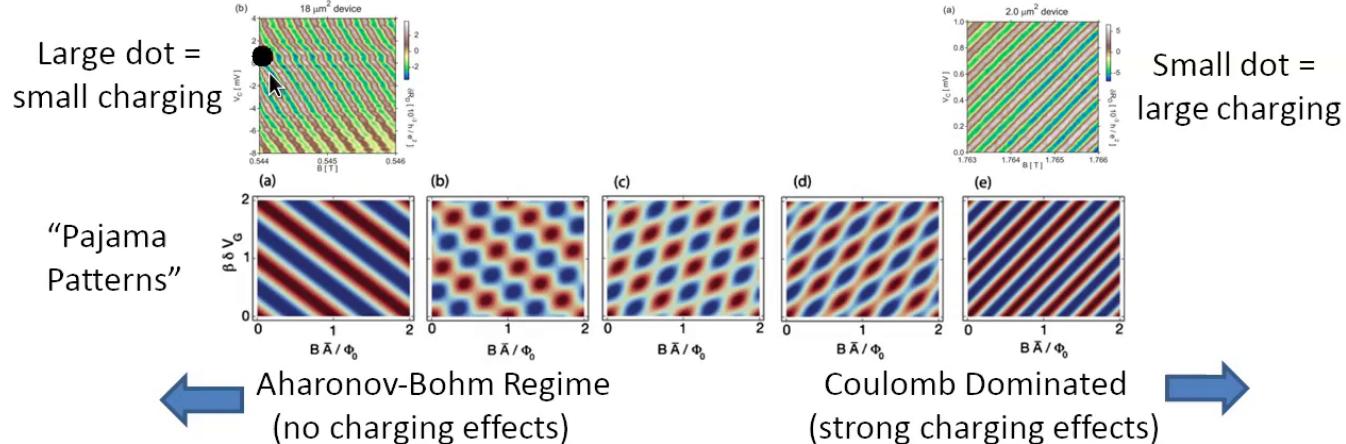
$$R = R_0 + R_1 \cos(\theta)$$

$$\theta = 2\pi e^*(\Phi + \beta V_G) + N\theta_{anyon} + \underbrace{\theta_C(\Phi, V_G, N)}_{\text{Coulomb correction}}$$



Distinct signatures for Coulomb blockade and Aharonov-Bohm interference in electronic Fabry-Perot interferometers

Yiming Zhang, D. T. McClure, E. M. Levenson-Falk, and C. M. Marcus L. N. Pfeiffer and K. W. West



$$R = R_0 + R_1 \cos(\theta)$$

$$\theta = 2\pi e^*(\Phi + \beta V_G) + N\theta_{anyon} + \underbrace{\theta_C(\Phi, V_G, N)}_{\text{Coulomb correction}}$$



PHYSICAL REVIEW B 80, 125310 (2009)

**Electron interferometry in the quantum Hall regime:
Aharonov-Bohm effect of interacting electrons**

Ping V. Lin,¹ F. E. Camino,² and V. J. Goldman¹

Aharonov-Bohm-Like Oscillations in Quantum Hall Corrals

M.D. Godfrey¹, P. Jiang¹, W. Kang¹, S.H. Simon², K.W. Baldwin², L.N. Pfeiffer², and K.W. West²



Steven Simon

**Role of interactions in an electronic Fabry-Perot
interferometer operating in the quantum
Hall effect regime**

5276–5281 | PNAS | March 23, 2010 | vol. 107 | no. 12

Nissim Ofek¹, Aavek Bid, Moty Heiblum, Ady Stern, Vladimir Umansky, and Diana Mahalu

PRL 108, 256804 (2012)

PHYSICAL REVIEW LETTERS

week ending
22 JUNE 2012

Fabry-Perot Interferometry with Fractional Charges

D.T. McClure,¹ W. Chang,¹ C.M. Marcus,¹ L.N. Pfeiffer,² and K.W. West²

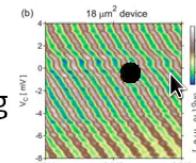
But No Aharonov-Bohm Pajama-Patterns in Fractional Quantum Hall Regime

Distinct signatures for Coulomb blockade and Aharonov-Bohm interference in electronic Fabry-Perot interferometers

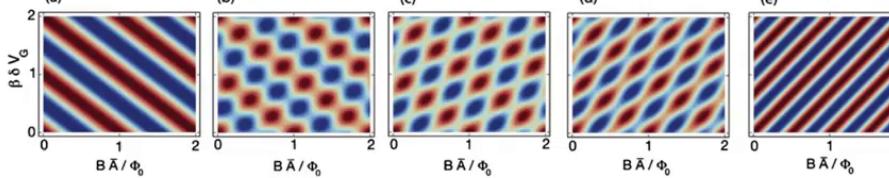
Yiming Zhang, D. T. McClure, E. M. Levenson-Falk, and C. M. Marcus L. N. Pfeiffer and K. W. West



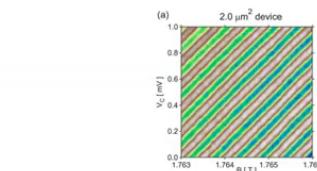
Large dot =
small charging



“Pajama
Patterns”



← Aharonov-Bohm Regime
(no charging effects)



Small dot =
large charging

→ Coulomb Dominated
(strong charging effects)

$$R = R_0 + R_1 \cos(\theta)$$

$$\theta = 2\pi e^*(\Phi + \beta V_G) + N\theta_{\text{anyon}} + \underbrace{\theta_C(\Phi, V_G, N)}_{\text{Coulomb correction}}$$



PHYSICAL REVIEW B 80, 125310 (2009)

**Electron interferometry in the quantum Hall regime:
Aharonov-Bohm effect of interacting electrons**

Ping V. Lin,¹ F. E. Camino,² and V. J. Goldman¹

Aharonov-Bohm-Like Oscillations in Quantum Hall Corrals

M.D. Godfrey¹, P. Jiang¹, W. Kang¹, S.H. Simon², K.W. Baldwin², L.N. Pfeiffer², and K.W. West²



**Role of interactions in an electronic Fabry-Perot
interferometer operating in the quantum
Hall effect regime**

5276–5281 | PNAS | March 23, 2010 | vol. 107 | no. 12

Nissim Ofek¹, Aavek Bid, Moty Heiblum, Ady Stern, Vladimir Umansky, and Diana Mahalu

PRL 108, 256804 (2012)

PHYSICAL REVIEW LETTERS

week ending
22 JUNE 2012

Fabry-Perot Interferometry with Fractional Charges

D. T. McClure,¹ W. Chang,¹ C. M. Marcus,¹ L. N. Pfeiffer,² and K. W. West²

But No Aharonov-Bohm Pajama-Patterns in Fractional Quantum Hall Regime

PHYSICAL REVIEW B 80, 125310 (2009)

**Electron interferometry in the quantum Hall regime:
Aharonov-Bohm effect of interacting electrons**

Ping V. Lin,¹ F. E. Camino,² and V. J. Goldman¹

**Role of interactions in an electronic Fabry-Perot
interferometer operating in the quantum
Hall effect regime**

5276–5281 | PNAS | March 23, 2010 | vol. 107 | no. 12

Nissim Ofek¹, Aavek Bid, Moty Heiblum, Ady Stern, Vladimir Umansky, and Diana Mahalu

PRL 108, 256804 (2012)

PHYSICAL REVIEW LETTERS

week ending
22 JUNE 2012

Fabry-Perot Interferometry with Fractional Charges

D. T. McClure,¹ W. Chang,¹ C. M. Marcus,¹ L. N. Pfeiffer,² and K. W. West²

But No Aharonov-Bohm Pajama-Patterns in Fractional Quantum Hall Regime

PLUS: Mach-Zehnder interferometry
never saw interference of
fractional quantum Hall

letters to nature

**An electronic Mach-Zehnder
interferometer**

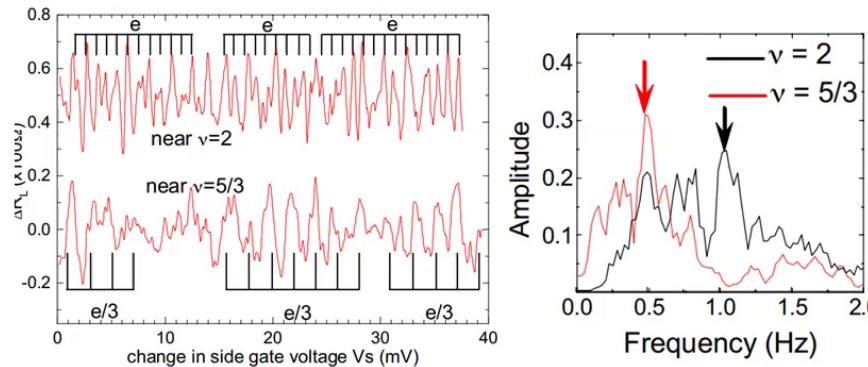
Yang Ji, Yunchul Chung, D. Sprinzak, M. Heiblum, D. Mahalu
& Hadas Shtrikman

NATURE | VOL 422 | 27 MARCH 2003 | www.nature.com/nature



Some rays of hope (or confusion?)

Data from Willett – suggesting Aharonov-Bohm Regime (even in very small dots)



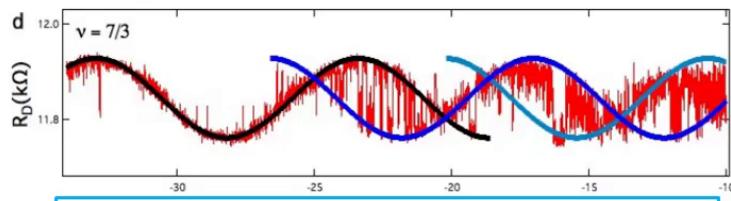
Problems:

- Not pajama patterns
- Not very periodic even
- Not measuring statistics (yet)

Measurement of filling factor 5/2 quasiparticle interference with observation of charge $e/4$ and $e/2$ period oscillations

R. L. Willett¹, L. N. Pfeiffer, and K. W. West
PNAS | June 2, 2009 | vol. 106 | no. 22 | 8853–8858

Data from Kang – suggesting fractional phase slips



Braiding of Abelian and Non-Abelian Anyons
in the Fractional Quantum Hall Effect

Sanghun An¹, P. Jiang^{1,2}, H. Choi¹, W. Kang¹, S.H. Simon³, L.N. Pfeiffer⁴, K.W. West⁴,
and K.W. Baldwin⁴

Problem:

Coulomb Dominated Regime

Telegraph noise and the Fabry-Perot quantum Hall interferometer

PHYSICAL REVIEW B 85, 201302(R) (2012)

B. Rosenow¹ and Steven H. Simon²

Gives a scenario that phase slips could still be
close to quantized

Problem:

Data did not reproduce...



The big breakthrough:

nature
physics

ARTICLES

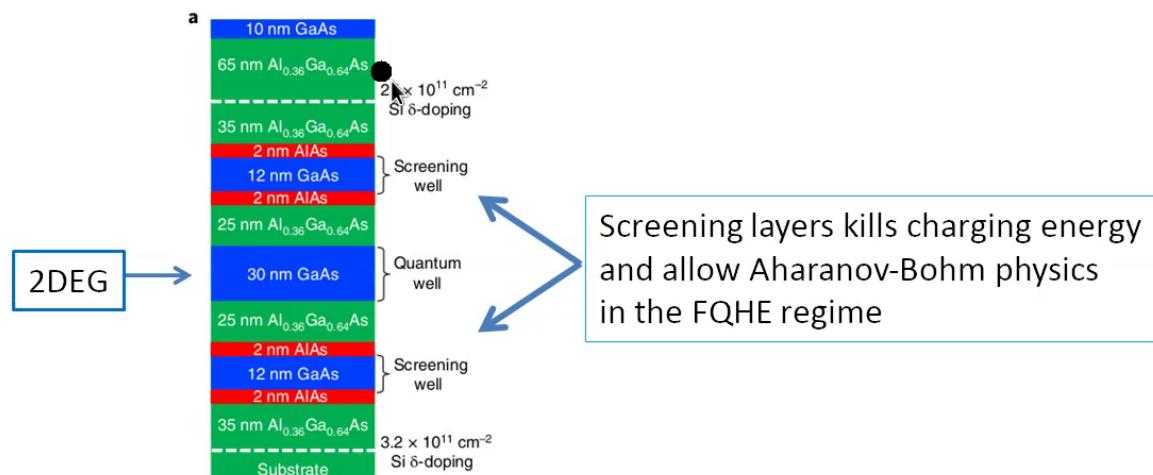
<https://doi.org/10.1038/s41567-019-0441-8>



NATURE PHYSICS | VOL 15 | JUNE 2019 | 563–569 |

Aharonov-Bohm interference of fractional quantum Hall edge modes

J. Nakamura^{1,2}, S. Fallahi^{1,2}, H. Sahasrabudhe¹, R. Rahman¹, S. Liang^{1,2}, G. C. Gardner^{2,4} and M. J. Manfra^{1,2,3,4,5*}



Fabry-Perot Interferometer

FRACTIONAL STATISTICS !

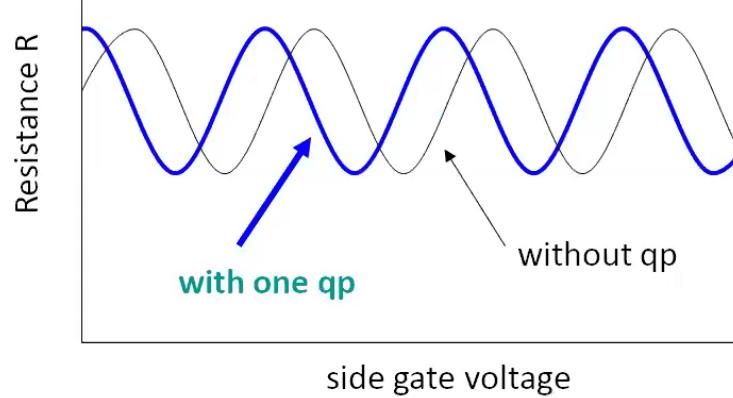


$$R = R_0 + R_1 \cos(\theta)$$

$$\theta = 2\pi e^*(\Phi + \beta V_G) + N\theta_{anyon}$$

Number of quasiparticles

Adding 1 quasiparticle
shifts interference pattern by $\theta_{anyon} = 2\pi / 3$



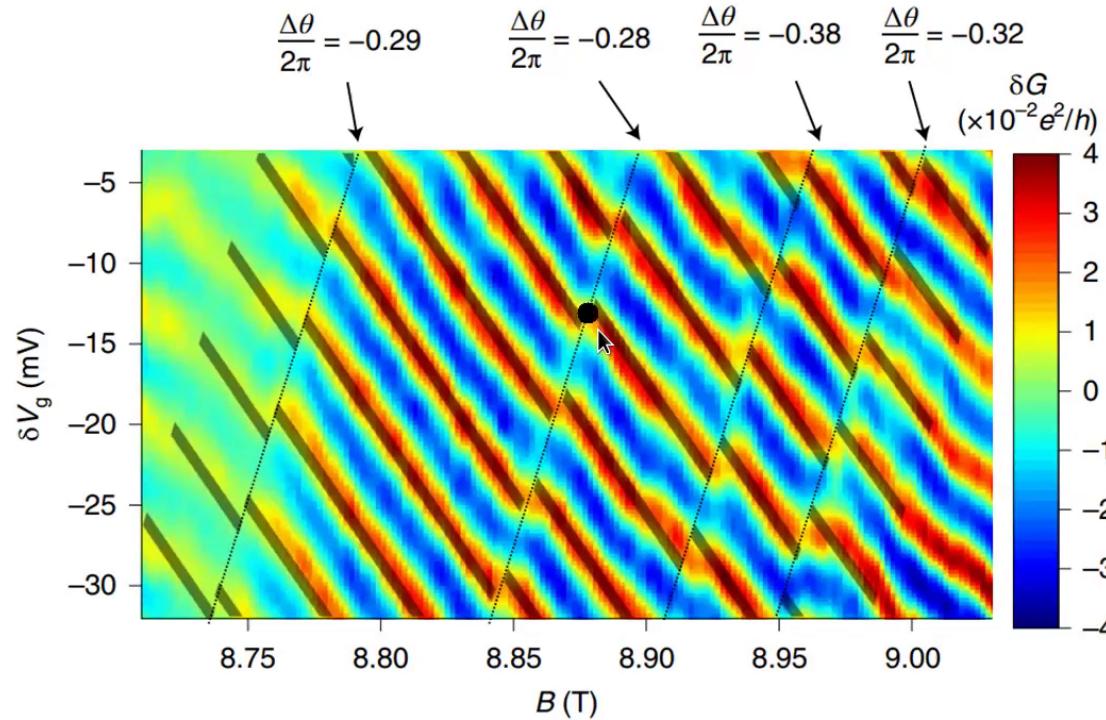
Direct observation of anyonic braiding statistics



Steven Simon

J. Nakamura^{1,2}, S. Liang^{1,2}, G. C. Gardner^{1,2,3} and M. J. Manfra^{1,2,3,4,5}

NATURE PHYSICS | VOL 16 | SEPTEMBER 2020 | 931–936 | www.nature.com/naturephysics



The big breakthrough:

nature
physics

ARTICLES

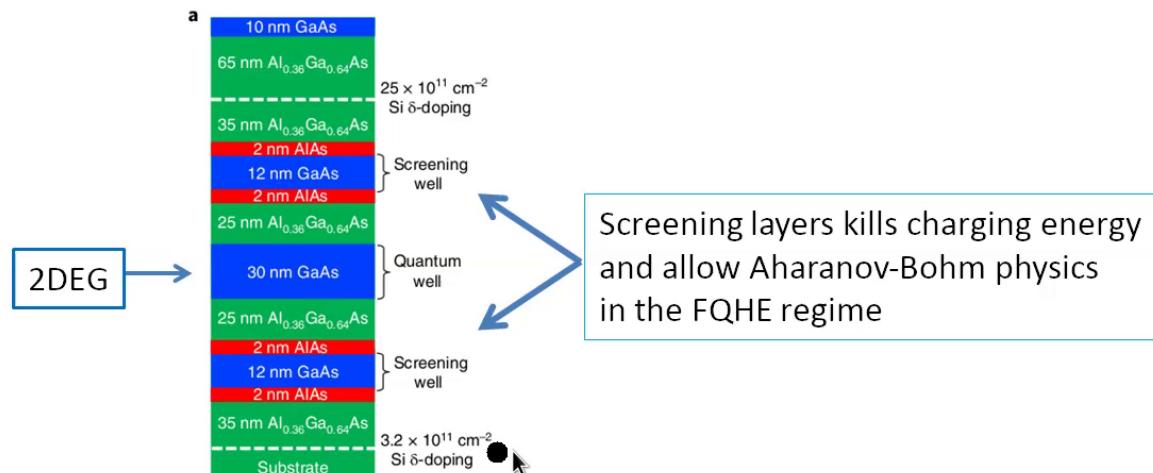
<https://doi.org/10.1038/s41567-019-0441-8>



NATURE PHYSICS | VOL 15 | JUNE 2019 | 563–569 |

Aharonov-Bohm interference of fractional quantum Hall edge modes

J. Nakamura^{1,2}, S. Fallahi^{1,2}, H. Sahasrabudhe¹, R. Rahman¹, S. Liang^{1,2}, G. C. Gardner^{2,4} and M. J. Manfra^{1,2,3,4,5*}

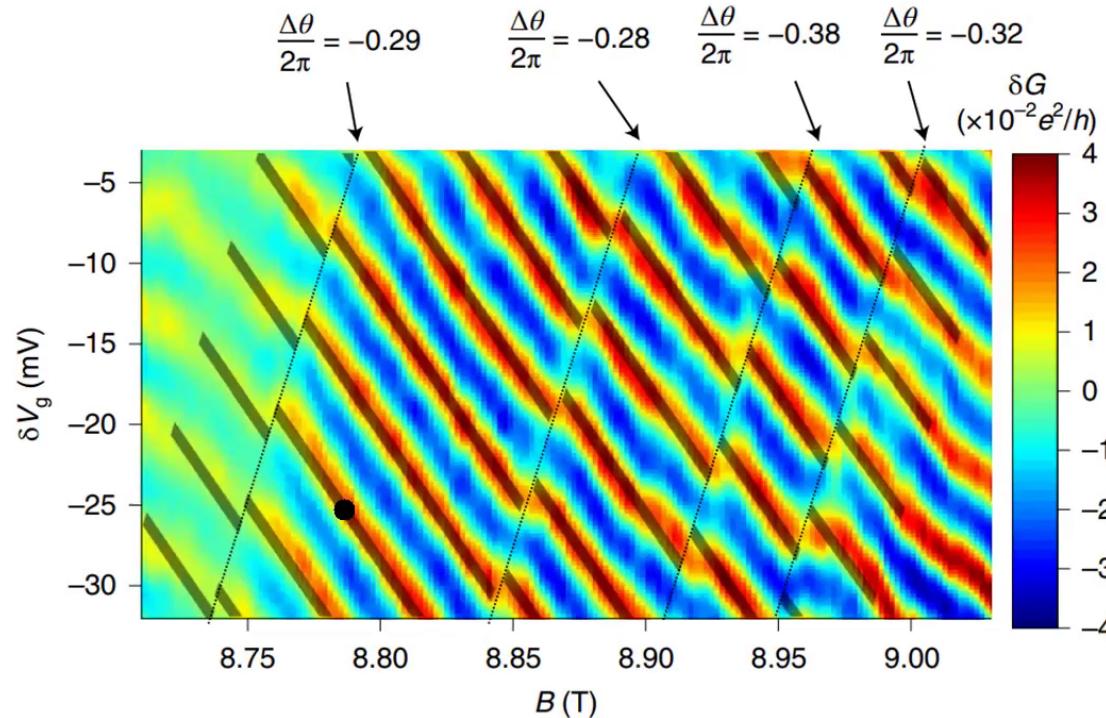


Direct observation of anyonic braiding statistics



J. Nakamura^{1,2}, S. Liang^{1,2}, G. C. Gardner^{1,2,3} and M. J. Manfra^{1,2,3,4,5}

NATURE PHYSICS | VOL 16 | SEPTEMBER 2020 | 931-936 | www.nature.com/naturephysics



Impact of bulk-edge coupling on observation of anyonic braiding statistics in quantum Hall interferometers

J. Nakamura,^{1,2} S. Liang,^{1,2} G. C. Gardner,^{2,3} and M. J. Manfra^{1,2,3,4,5,*}



arXiv:2107.02136v1

Quantum Hall interferometers have been used to probe fractional charge, and more recently, fractional statistics of quasiparticles. Theoretical predictions have been made regarding the effect of electrostatic coupling on interferometer behavior and observation of anyonic phases. Here we present measurements of a small Fabry-Perot interferometer in which these electrostatic coupling constants can be determined experimentally, facilitating quantitative comparison with theory. At the $\nu = 1/3$ fractional quantum Hall state, this device exhibits Aharonov-Bohm interference near the center of the conductance plateau interrupted by a few discrete phase jumps, and Φ_0 oscillations at higher and lower magnetic fields, consistent with theoretical predictions for detection of anyonic statistics. We estimate the electrostatic parameters K_I and K_{IL} by two methods: by the ratio of oscillation periods in compressible versus incompressible regions, and from finite-bias conductance measurements, and these two methods yield consistent results. We find that the extracted K_I and K_{IL} can account for the deviation of the values of the discrete phase jumps from the theoretically predicted anyonic phase $\theta_a = 2\pi/3$. In the integer quantum Hall regime, we find that the experimental values of K_I and K_{IL} can account for the observed Aharonov-Bohm and Coulomb dominated behavior of different edge states.

What next?



- All of these experiments are on $\nu=1/3$.
- There is a zoo of other FQHE states
 $\nu = 2/3, 2/5, 3/5, 3/7, 4/7, \dots$
- None of these have shown the expected results (yet)
- And of course the rare non-abelian FQHE
 $\nu = 5/2, 7/2, 12/5$

Two Successful Observation of Anyons in 2020

RESEARCH

MESOSCOPIC PHYSICS

Bartolomei *et al.*, *Science* **368**, 173–177 (2020) 10 April 2020

Fractional statistics in anyon collisions

H. Bartolomei^{1*}, M. Kumar^{1*†}, R. Bisognin¹, A. Marguerite^{1‡}, J.-M. Berroir¹, E. Bocquillon¹, B. Plaçais¹, A. Cavanna², Q. Dong², U. Gennser², Y. Jin², G. Fève^{1§}



nature
physics

ARTICLES

<https://doi.org/10.1038/s41567-020-1019-1>

NATURE PHYSICS | VOL 16 | SEPTEMBER 2020 | 931–936

Direct observation of anyonic braiding statistics

J. Nakamura^{1,2}, S. Liang^{1,2}, G. C. Gardner ^{2,3} and M. J. Manfra ^{1,2,3,4,5}

... Plus I will report on progress observing nonabelions

The idea of Nonabelian Anyons



Nonabelion = A Particle Obeying *Nonabelian Statistics*

-

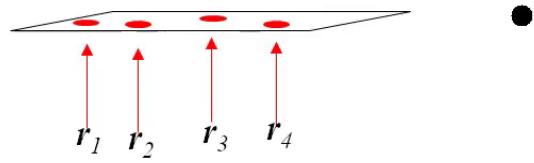
NonAbelian Statistics



Steven Simon

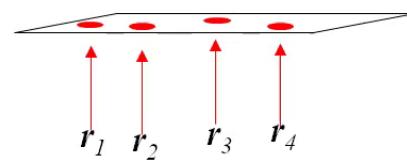


NonAbelian Statistics



NonAbelian Statistics

Suppose 2 Degenerate
Orthogonal States $|\psi_a\rangle, |\psi_b\rangle$

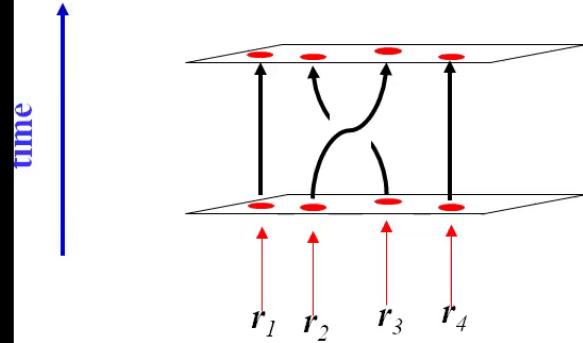


Vector Represents State

$$\Psi_i = a_i |\psi_a\rangle + b_i |\psi_b\rangle = \begin{pmatrix} a_i \\ b_i \end{pmatrix}$$



NonAbelian Statistics



Suppose 2 Degenerate
Orthogonal States

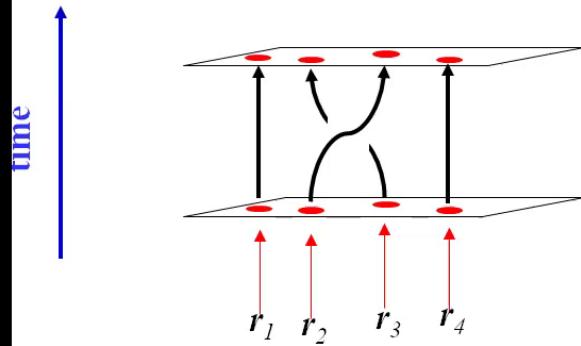
$$|\psi_a\rangle, |\psi_b\rangle$$



Vector Represents State

$$\Psi_i = a_i |\psi_a\rangle + b_i |\psi_b\rangle = \begin{pmatrix} a_i \\ b_i \end{pmatrix} \bullet$$

NonAbelian Statistics



Suppose 2 Degenerate
Orthogonal States $|\psi_a\rangle, |\psi_b\rangle$



Vector Represents State

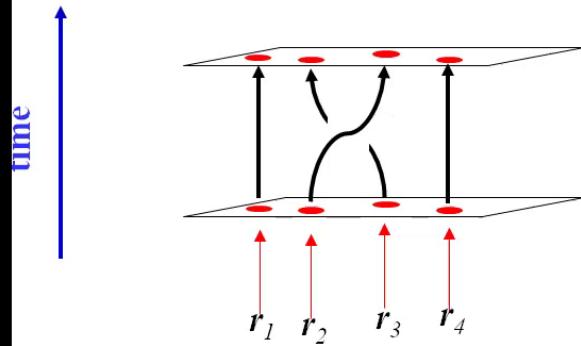
$$\Psi_i = a_i |\psi_a\rangle + b_i |\psi_b\rangle = \begin{pmatrix} a_i \\ b_i \end{pmatrix}$$

$$\Psi_f = a_f |\psi_a\rangle + b_f |\psi_b\rangle = \begin{pmatrix} a_f \\ b_f \end{pmatrix}$$

$$\begin{pmatrix} a_f \\ b_f \end{pmatrix} = U \begin{pmatrix} a_i \\ b_i \end{pmatrix}$$

Unitary Matrix From Braid Operation
Depends only on the Topology of the braid

NonAbelian Statistics



Makes a great
Qubit !!

Suppose 2 Degenerate
Orthogonal States

$$|\psi_a\rangle, |\psi_b\rangle$$



Vector Represents State

$$\Psi_i = a_i |\psi_a\rangle + b_i |\psi_b\rangle = \begin{pmatrix} a_i \\ b_i \end{pmatrix}$$

$$\Psi_f = a_f |\psi_a\rangle + b_f |\psi_b\rangle = \begin{pmatrix} a_f \\ b_f \end{pmatrix}$$

$$\begin{pmatrix} a_f \\ b_f \end{pmatrix} = U \begin{pmatrix} a_i \\ b_i \end{pmatrix}$$

Unitary Matrix From Braid Operation
Depends only on the Topology of the braid



Steven Simon

- In Nonabelian Topological Theories, the ground state in the presence of quasiparticles/quasiholes is degenerate (multiple ground states).
 - The *ONLY* “dynamics” in bulk is that braiding quasiparticles makes transitions between degenerate ground states.
 - No *Local* operator can mix the multiple ground states

Almost all noise processes are local !

Qubits are *HIGHLY* protected from decoherence/error

Nonabelian Anyons?



Steven Simon

1980: Bais (in context of gauge theories)
1989: Moore+Seiberg (in context of CFTs)
1989: Witten (in context of Chern-Simons theories)
1989: Fredenhagen, Rehren, Schroer
1989: Froelich + Gabbiani
1989: Chen, Wilczek, Witten, Halperin

1987: Discovery of 5/2 FQHE
Willett, Eisenstein, Tsui
Gossard, and English

1990: Moore+Read: How Nonabelions can arise in FQHE

....



1998: Morf: 5/2 FQHE is Nonabelian
1997: Kitaev "Fault-Tolerant Quantum Computation by Anyons"
2000: Kitaev "Unpaired Majorana Fermions in Quantum Wires"
...

~2004: Microsoft Station Q formed... > 10^8 \$ Spent

Nonabelian Anyons?



1980: Bais (in context of gauge theories)
1989: Moore+Seiberg (in context of CFTs)
1989: Witten (in context of Chern-Simons theories)
1989: Fredenhagen, Rehren, Schroer
1989: Froelich + Gabbiani
1989: Chen, Wilczek, Witten, Halperin

1987: Discovery of 5/2 FQHE
Willett, Eisenstein, Tsui
Gossard, and English

1990: Moore+Read: How Nonabelions can arise in FQHE

....

1998: Morf: 5/2 FQHE is Nonabelian
1997: Kitaev "Fault-Tolerant Quantum Computation by Anyons"
2000: Kitaev "Unpaired Majorana Fermions in Quantum Wires"
...

~2004: Microsoft Station Q formed... > 10^8 \$ Spent

Research in several different systems thought to (maybe) have nonabelions

LETTER

doi:10.1038/nature26142

Quantized Majorana conductance

Hao Zhang^{1*}, Chun-Xiao Liu^{2*}, Sasa Ozibegovic^{3*}, Di Xu¹, John A. Logan⁴, Guanzhong Wang¹, Nick van Loo¹, Jouri D. S. Bonn¹, Michiel W. A. de Moor¹, Diana Car³, Roy L. M. Op het Veld³, Petrus J. van Veldhoven³, Sebastian Koeling³, Marcel A. Verheijen^{3,5}, Mihir Pendharkar⁶, Daniel J. Pennachio⁴, Borzoyeh Shojaei^{4,7}, Joon Sue Lee⁷, Chris J. Palmstrom^{4,6,7}, Erik P. A. M. Bakkers³, S. Das Sarma² & Leo P. Kouwenhoven^{1,8}

Majorana zero-modes—a true great promise for topological spectroscopy in electrical transport—offer a way of identifying the presence of Majoranas by looking for them as a zero-bias peak in differential conductance. The universal conductance value e is the charge of an electron, so the direct consequence of the fact that a Majorana particle is its own antiparticle is the quantization of the tunnel coupling^{3–5}. Previous work has mostly shown zero-bias peaks in recent observation⁷ of a peak height close to $2e^2/h$. Here we report a quantized conductance plateau at $2e^2/h$ in the zero-bias conductance measured in indium antimonide semiconductor nanowires covered with an aluminium superconducting shell. The height of our zero-bias peak remains constant despite changing parameters such as the magnetic field and tunnel coupling, indicating that it is a quantized conductance plateau. We distinguish this quantized Majorana signal from possible non-Majorana origins by investigating its robustness to electric and magnetic fields as well as its temperature dependence. The observation of a quantized conductance plateau strongly supports the existence of Majorana zero-modes in the system, consequently paving the way for future braiding experiments that could lead to topological quantum computing.

A semiconductor nanowire coupled to a superconductor can be tuned into a topological superconductor with two Majorana zero-modes localized at the wire ends^{8,9}. Tunnelling¹⁰ to a Majorana mode will show a zero-energy state in the tunneling density-of-states, that is, a zero-bias peak (ZBP) in the differential conductance (dI/dV)^{2,6}. This tunnelling process is an Andreev reflection, in which an incoming electron is reflected as a hole. Particle-hole symmetry dictates that the zero-energy tunnelling amplitudes of electrons and holes are equal, resulting in perfect coherent transmission with a ZBP height quantized at $2e^2/h$ (refs 3, 4, 10), irrespective of the precise tunnelling strength^{1,3}. The robustness of this perfect Andreev reflection is a direct result of the well-known Majorana symmetry property ‘particle equals anti-particle’.

The observation of robust conductance quantization has not yet been observed^{11,12,13,14}. Instead, most of the ZBPs have a height considerably less than $2e^2/h$. This discrepancy was first explained by thermal averaging^{15–18}, but that explanation does not hold when the peak width exceeds the thermal broadening (about $3.5k_B T$)^{13,14}. In that case, other averaging mechanisms, such as dissipation¹⁹, have been invoked. The main source of dissipation is a finite quasiparticle density-of-states

(yellow) and the Al shell. The right contact is used to drain the current to ground. The chemical potential in the segment covered with Al can be varied by applying voltages to the two long ‘super-gates’ (purple). Transport spectroscopy is shown in Fig. 1b, which displays dI/dV as a function of voltage V (mV) and Zeeman energy μ (meV). The plot shows a sharp peak at $V = 0$ mV. The peak height is quantized at $2e^2/h$ (red arrow). The peak width is much smaller than the thermal broadening at $T = 20$ mK. The peak height is also constant over a wide range of Zeeman energy μ (black arrow).

and stimulated currents in Fig. 2a and d. An exact quantitative agreement is not feasible as we precise experimental values for the parameters going into the theory (for example, chemical potential, tunnel coupling, Zeeman splitting, spin-orbit coupling) are unknown for our hybrid wire-superconductor structure.

Next, we fix μ at 0.8 T and investigate the robustness of the quantized ZBP against variations in transmission by varying the voltage on the tunnel-gate. Figure 2a shows dI/dV while varying V and tunnel-gate voltage. Figure 2b shows that the ZBP height remains close to the quantized value. Importantly, the above-gap conductance measured at $|V| = 0.2$ mV varies by more than 50% (Fig. 2c and d), implying that the transmission is changing considerably over this range while the ZBP remains quantized. The minor conductance switches in Fig. 2a–c are due to unstable jumps of trapped charges in the surroundings.

Figure 2d (red curves) shows several line-cuts of the quantized ZBP. The extracted height and width are plotted in Fig. 2e (upper panel) as a function of above-gap conductance $G_N = T \times e^2/h$ where T is the transmission probability for a spin-resolved channel. Although the ZBP

and simulated curves in Fig. 2e and d are in excellent quantitative agreement, it is not feasible to precisely experimental values for the parameters going into the theory (for example, chemical potential, tunnel coupling, Zeeman splitting, spin-orbit coupling) are unknown for our hybrid wire-superconductor structure. Nevertheless, at 0.8 T and investigate the robustness of the quantized ZBP against variations in transmission by varying the voltage on the tunnel-gate. Figure 2a shows dI/dV while varying V and tunnel-gate voltage. Figure 2b shows that the ZBP height remains close to the quantized value. Importantly, the above-gap conductance measured at $|V| = 0.2$ mV varies by more than 50% (Fig. 2c and d), implying that the transmission is changing considerably over this range while the ZBP remains quantized. The minor conductance switches in Fig. 2a–c are due to unstable jumps of trapped charges in the surroundings. The simulated height and width are plotted in Fig. 2e (upper panel) as a function of above-gap conductance $G_N = T \times e^2/h$ where T is the transmission probability for a spin-resolved channel. Although the ZBP

height and width depend on the potential profile in the proximitized wire part near the barrier, the quantized height is robust to such variations.

A more negative tunnel-gate voltage (< -8 V) splits the quantized ZBP, which may be explained by an overlapping of the two Majorana wavefunctions from the two wire ends. This splitting not only tunes the transmission of the barrier but also changes the potential profile in the proximitized wire part near the barrier, thus reducing the length of the effective topological gap and the wavefunction overlap between the two Majorana modes. The ZBP to split¹⁶ (black curve in Fig. 2d). The splitting is captured in our simulations shown in Fig. 2f, where we show that the splitting originates from Majorana wavefunction overlap. The simulated ZBP height (red curve in middle panel) remains close to the $2e^2/h$ plateau over a large range of above-gap conductance (black curve in lower panel) and remains substantially. Also, the height and width dependence is in qualitative agreement with our experimental data.

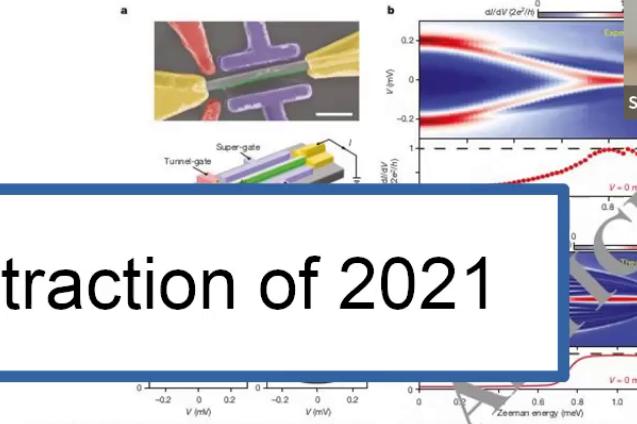


Figure 1 | Quantized Majorana zero-bias peak. a, False-colour scanning



Steven Simon

The Majorana Retraction of 2021

1. What are Majoranas. Why are we interested.
2. What was the supposed smoking gun
3. What was reported. What was retracted
4. Was there foul play or just stupidity?

*Gulnihal Karli Institute of Nanoscience, Delft University of Technology, 2600 GA Delft, The Netherlands. †Condensed Matter Theory Center and Joint Quantum Institute, Department of Physics, University of Maryland, College Park, Maryland 20742, USA. ‡Department of Applied Physics, Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands. §Materials Engineering, University of California Santa Barbara, Santa Barbara, California 93106, USA. ¶Philips Innovation Services Eindhoven, High Tech Campus 11, 5656AE Eindhoven, The Netherlands. #Electrical and Computer Engineering, University of California Santa Barbara, Santa Barbara, California 93106, USA. #Microsoft Station Q Delft, 2600 GA Delft, The Netherlands.

¹These authors contributed equally to this work.

Nonabelian Interferometer and the $\nu=5/2$ FQHE

Theory: Stern, Halperin
Bonderson, Shtengel, Kitaev
Das Sarma, Nayak, Freedman

Chamon, Wen, et al
Nayak, Wilczek, et al
... and many others.



Experiment: Willett Group (Also Kang Group, Marcus Group, Heiblum Group)

The Quantum Hall Fabry-Pérot Interferometer

