Title: Fractons: Perspectives from Quantum Information Theory, Condensed Matter Physics, and High Energy Physics

Speakers: Brandon Rayhaun

Series: Quantum Fields and Strings

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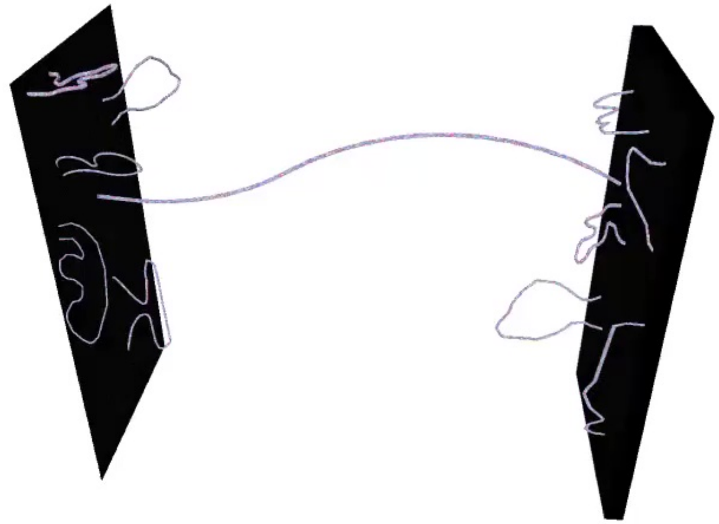
Abstract: Fractons are relatively new types of quasiparticles which have recently been inspiring activity within several branches of physics. I will offer some motivations and perspectives from quantum information theory, condensed matter physics, and high energy physics, focusing mainly on my work in the latter two subjects. This talk is primarily based on https://arxiv.org/abs/2108.08322 and a paper to appear shortly with Dominic Williamson.



# Fractons

## Perspectives from Quantum Information Theory, Condensed Matter Physics, and High Energy Physics

**Brandon Rayhaun**
**Stanford Institute for Theoretical Physics**

**On work with H. Geng, S. Kachru, A. Karch, R. Nally, and separately with D. Williamson.**

**Perimeter Institute — Quantum Fields and Strings Seminar Series**

# What are fractons?

For the purposes of this talk, a fracton model is typically a quantum lattice model (though sometimes a quantum field theory) which has any of the following related properties:

- Quasi-particle excitations with limited mobility (e.g. planons, lineons, fractons, etc.)
- Subsystem symmetries
- Ground state degeneracy which grows subextensively with the system size

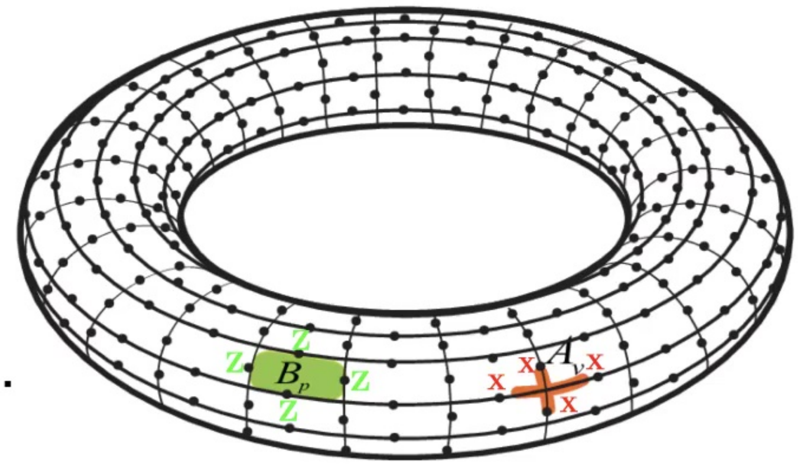
# A motivation from quantum information:

Fractons are a solution to the problem of self-correcting quantum memory.

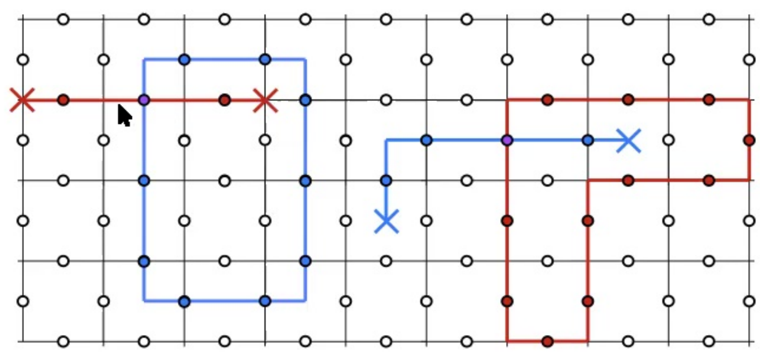
# Reminder: fault tolerance in quantum computing

- Kitaev: consider a gapped quantum lattice model with a degenerate ground space. Demand that ground states are all locally indistinguishable from one another.
- Encode information in the ground space. Rest of the Hilbert space serves as a source of redundancy to protect your information from irreversible corruption.
- Errors in this picture occur when you're taken out of the ground space. In (2+1)D topological orders, errors are physically interpreted as anyonic quasi-particle excitations.

# Example: Toric Code




- Hamiltonian: $H_{\mathrm{TC}} = -\sum_v A_v - \sum_p B_p$
- Commuting projector model: ground states obtained by imposing $A_v = B_p = 1$ for all $v$ and $p$.
- Excited states: $A_v = -1$ or $B_p = -1$ .
  Can be obtained by applying open string operators.
- Logical gates obtained by wrapping closed string operators around cycles of the torus.



# Limitations of toric code

- Gates are not universal; cannot perform arbitrary quantum computation.
    - Quantum double model associated to an arbitrary finite group *G* is a quantum error-correcting code. [S. Cui, D. Ding, X. Han, G. Penington, D. Ranard, BR, Z. Shangnan]
    - For suitable *G,* admits universal gates.
- Not *self-correcting* quantum memory.
    - If quantum computer is coupled to a heat bath, requires active monitoring and continuous correction to avoid proliferation of errors.

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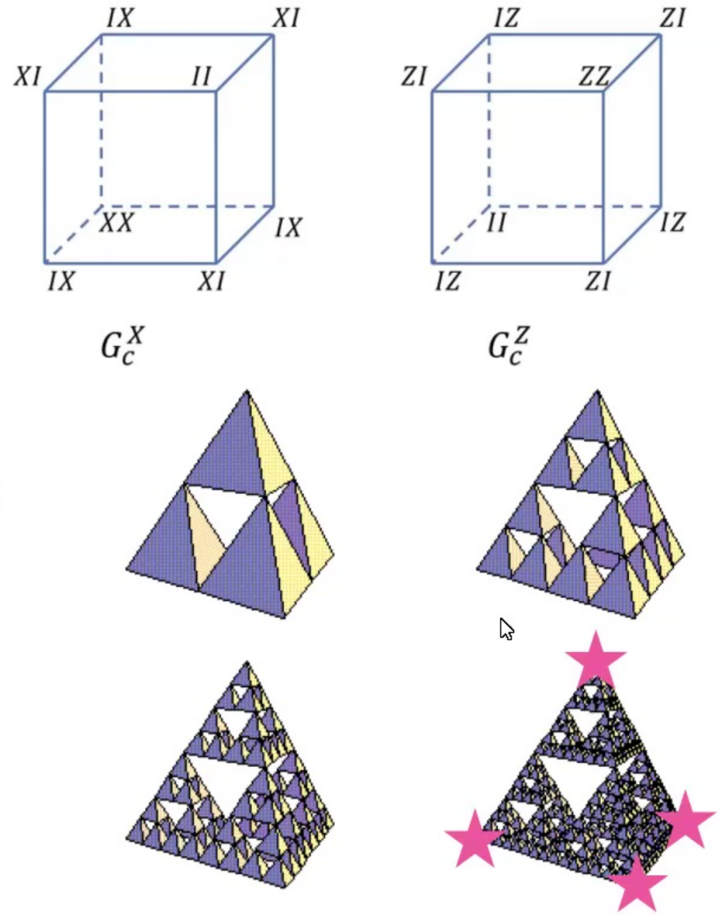
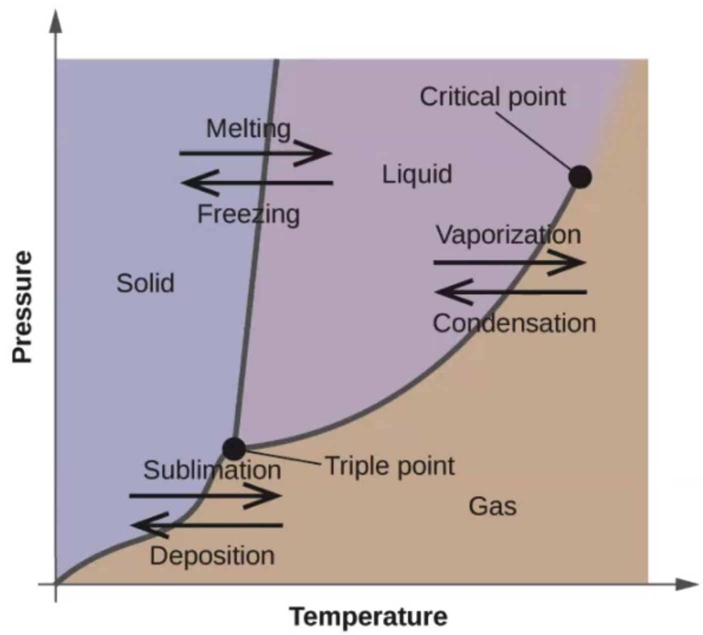
- Not *self-correcting* quantum memory.
  - If quantum computer is coupled to a heat bath, requires active monitoring and continuous correction to avoid proliferation of errors.
  - Before fractons, only systems known with self-correction were in (4+1)D.

# Haah’s cubic code: a quantum hard-drive

- Two qubits per vertex.
  - Hamiltonian: $H = -\sum_{\substack{\text{cubes} \\ c}} (G_c^X + G_c^Z)$
- Fracton particles (errors) created at the corners of fractal operators.
- Immobile in the sense that no local operator is capable of moving a fracton from one location to another. Any local operator applied will create additional fractonic particles.
- Errors stay localized as opposed to wandering the system.

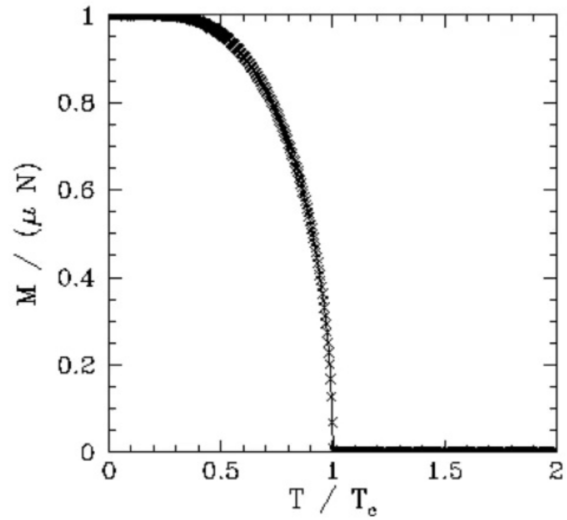


# A motivation from condensed matter:



# Landau paradigm

- Organize phases by the symmetries they break, as measured by order parameters.



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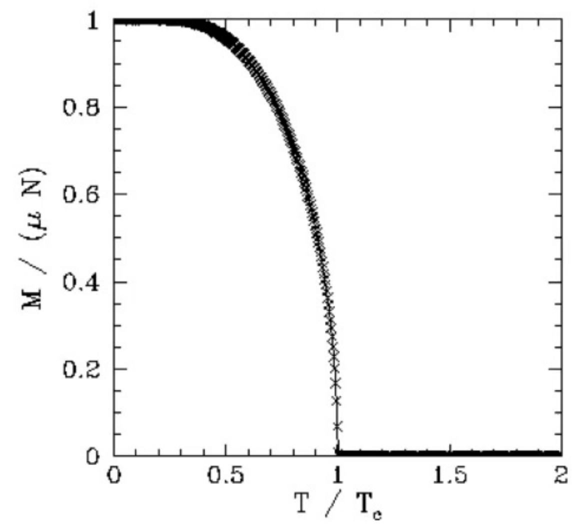



- Topological orders: beyond the Landau paradigm? No!

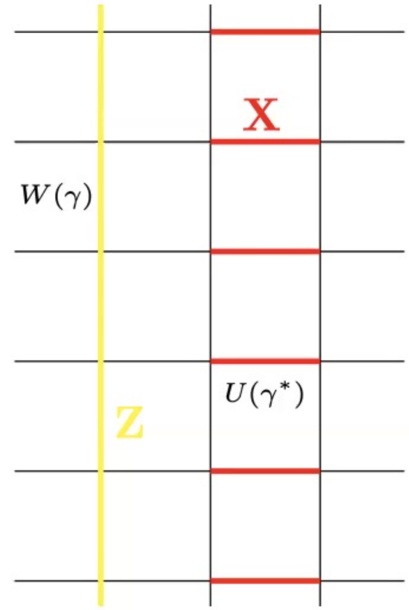
# Higher-form symmetries

- A theory with $(d+1)$ spacetime dimensions is said to admit a *q*-form symmetry $G^{(q)}$ if it admits operators $U_g(M^{(d-q)})$ for each $g \in G$ and each *(d-q)*-dimensional manifold $M^{(d-q)}$ which satisfy:
  - The operators are topological: correlation functions $\langle U_g(M^{(d-q)}) \cdots \rangle$ are insensitive to deformations of $M^{(d-q)}$ provided one does not deform through a charged *q*-dimensional operator.
  - The operators furnish the group multiplication:

$$U_g(M^{(d-q)})U_h(M^{(d-q)}) = U_{gh}(M^{(d-q)})$$

- Since working on the lattice, will mostly consider operators inserted at a fixed instant of time, so that they act on the Hilbert space of the theory.
- Higher form symmetries are not that exotic: even free electromagnetism has higher-form symmetries.

# Confinement/deconfinement transition breaks one-form symmetries




- Consider $\mathbb{Z}_2$ lattice gauge theory in (2+1)D. Qubits on edges.
  - Hamiltonian: $H_{\text{LGT}} = -\sum_e \mathbf{X}_e - g \sum_p \prod_{e \in \partial p} \mathbf{Z}_e$
  - Gauss's law: $G_v = \prod_{e \in \text{star}(v)} \mathbf{X}_e = 1$
  - One-form symmetry operators: $U(\gamma^*) = \prod_{e \in \gamma^*} \mathbf{X}_e$
  - Charged Wegner-Wilson loop: $W(\gamma) = \prod_{e \in \gamma} \mathbf{Z}_e$
- $g \ll 1$ phase: confined, one-form symmetry unbroken, Wilson loop has area law.
- $g \gg 1$ phase: deconfined/toric code phase, one-form symmetry broken, Wilson loop has perimeter law.

# Foliated (Zero-form) Subsystem Symmetries

- Use $\mathcal{F} = \{L^{(d-k)}\}$ to denote a foliation of space by leaves of codimension-*k*. On the lattice, we think of this as a set of sublattices of dimension *(d-k)*.
- Will use notation like $\mathcal{F}^{\|}_{\mathrm{xy}}$ to denote a foliation of space by planes parallel to the x and y directions.
- We focus on operators at a fixed instant of time. A theory has a foliated (zero-form) subsystem symmetry $G^{(0,k)}(\mathcal{F}_1, \mathcal{F}_2, \dots)/\sim$ of codimension-*k* if for each $g \in G$ and each $L^{(d-k)} \in \mathcal{F}_i$ there are operators $U_g(L^{(d-k)})$ which commute with the Hamiltonian.
- The quotient by $\sim$ indicates that there may be non-trivial relations satisfied between symmetry operators supported on the leaves of different foliations. If there are no relations, we refer to the symmetry as *relation-free.*

# Subgroups by coarsening

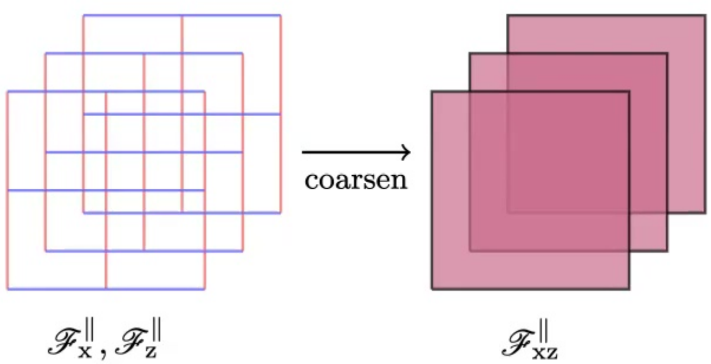
- Say that a foliation $\mathcal{F}_{\mathrm{C}}$ of codimension-*(k-m)* is a coarsening of a foliation $\mathcal{F}$ of codimension-*k* if each leaf $L \in \mathcal{F}$ sits inside some leaf $L_{\mathrm{C}}$ of $\mathcal{F}_{\mathrm{C}}$, and if $\{L \in \mathcal{F} \mid L \subset L_{\mathrm{C}}\}$ is a foliation of $L_{\mathrm{C}}$.

- If foliations $\mathcal{F}_1, \ldots, \mathcal{F}_n$ admit a common coarsening $\mathcal{F}_{\mathrm{C}}$, then can obtain a subgroup

$$G^{(0,k-m)}(\mathcal{F}_{\mathrm{C}})/\sim \; \subset G^{(0,k)}(\mathcal{F}_1, \ldots, \mathcal{F}_n)$$

whose symmetry operators are defined as

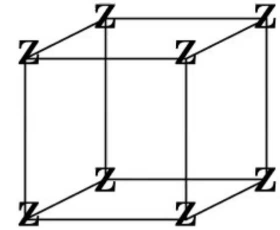
$$U_{\mathrm{C}}(L_{\mathrm{C}}) = \prod_{i=1}^{n} \prod_{\substack{L \in \mathcal{F}_i \\ L \subset L_{\mathrm{C}}}} U(L)$$

- Example: 

leads to $\mathbb{Z}_2^{(0,1)}(\mathcal{F}_{\mathrm{xz}}^{\parallel}) \subset \mathbb{Z}_2^{(0,2)}(\mathcal{F}_{\mathrm{x}}^{\parallel}, \mathcal{F}_{\mathrm{y}}^{\parallel})$

# Example: (3+1)D Cubic Ising Model (CIM)

- Take a cubic lattice with qubits on vertices. Hamiltonian:




$$H_{\text{CIM}} = -\sum_{\substack{\text{vertices}\\ v}} \mathbf{X}_v - g \sum_{\substack{\text{cubes}\\ c}} \prod_{v \in \text{corner}(c)} \mathbf{Z}_v$$

- Call e.g. $\mathcal{F}_{\text{x}}^{\parallel}$ the foliation of the lattice by all of its 1d sublattices which are parallel to the *x*-direction. CIM supports a $\mathbb{Z}_2^{(0,2)}(\mathcal{F}_{\text{x}}^{\parallel}, \mathcal{F}_{\text{y}}^{\parallel}, \mathcal{F}_{\text{z}}^{\parallel})/\sim$ symmetry

$$U_{j,k}^{\text{x}} = \prod_i \mathbf{X}_{i,j,k} \quad U_{i,k}^{\text{y}} = \prod_j \mathbf{X}_{i,j,k} \quad U_{i,j}^{\text{z}} = \prod_k \mathbf{X}_{i,j,k}$$

- To describe the relations, note that the planar subsystem symmetry subgroup $\mathbb{Z}_2^{(0,1)}(\mathcal{F}_{\text{xy}}^{\parallel}, \mathcal{F}_{\text{xz}}^{\parallel}, \mathcal{F}_{\text{yz}}^{\parallel})/\mathbb{Z}_2^{(0)}$ generated by the operators

$$U_j^{\text{xz}} = \prod_i U_{i,j}^{\text{z}} \prod_k U_{j,k}^{\text{x}} \quad U_k^{\text{xy}} = \prod_i U_{i,k}^{\text{y}} \prod_j U_{j,k}^{\text{x}} \quad U_i^{\text{yz}} = \prod_k U_{i,k}^{\text{y}} \prod_j U_{i,j}^{\text{z}}$$

acts trivially in the CIM. Total global symmetry can be described as

$$\mathbb{Z}_2^{(0,2)}/(\mathbb{Z}_2^{(0,1)}/\mathbb{Z}_2^{(0)})$$

- Aside: the $\mathbb{Z}_2^{(0)}$ is a relation of a relation $\prod_k U_k^{\text{xy}} \prod_i U_i^{\text{yz}} \prod_j U_j^{\text{xz}} = 1$

# Example: (3+1)D Cubic Ising Model (CIM) Part 2

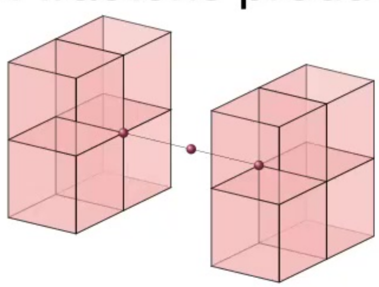
- Hamiltonian:

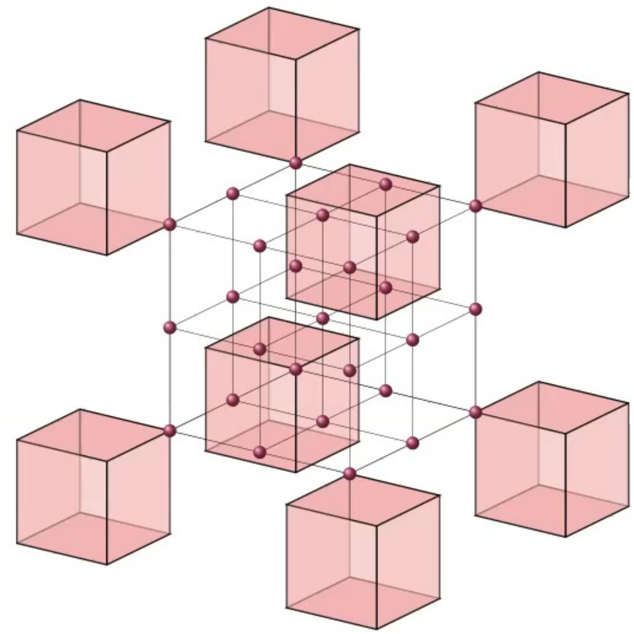
$$H_{\mathrm{CIM}} = -\sum_{\substack{\text{vertices} \\ v}} \mathbf{X}_v - g \sum_{\substack{\text{cubes} \\ c}} \prod_{v \in \mathrm{corner}(c)} \mathbf{Z}_v$$

- When $g \gg 1$, ground state degeneracy $2^{N_x N_y + N_y N_z + N_x N_z - N_x - N_y - N_z + 1}$ arises from spontaneous breaking of linear subsystem symmetry. Robust to perturbations which respect this symmetry.

- Fractons are excitations of the cube term of the Hamiltonian and can be created at the 8 corners of a cuboid operator $\prod_{v \in C} \mathbf{X}_v$. Immobile in isolation.

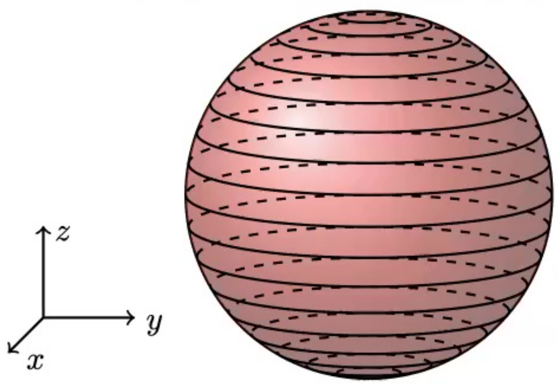
- Quadrupole of fractons produces a lineon.



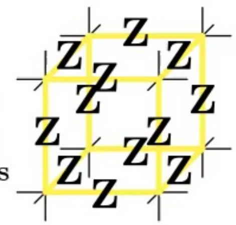


# Higher-form subsystem symmetry

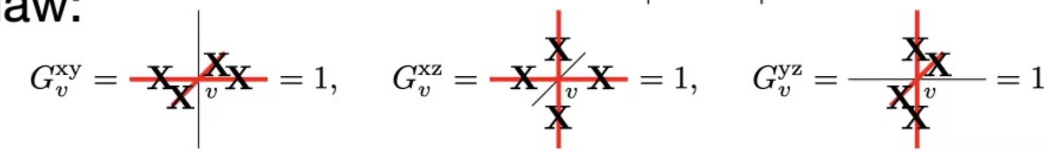
- Intuition: a theory has a higher-form subsystem symmetry if it has an ordinary higher-form symmetry within each leaf of its foliations.
- More precisely: a theory has a $G^{(q,k)}(\mathcal{F}_1, \mathcal{F}_2, \dots)/\sim$ symmetry if for each $g \in G$, each leaf $L^{(d-k)} \in \mathcal{F}_i$ and each $M^{(d-k-q)} \subset L^{(d-k)}$, there are symmetry operators $U_g(M^{(d-k-q)})$ which are topological in the sense that correlation functions are insensitive to deformations of $M^{(d-k-q)}$ within the leaf $L^{(d-k)}$.
- Again, can define subgroups by coarsening. For example, a $G^{(1,1)}(\mathcal{F}^{\parallel}_{\mathrm{xy}})$ planar one-form subsystem symmetry group admits an ordinary $G^{(1)}$ one-form symmetry subgroup.



# Example: (3+1)D X-cube model

- Qubits on the edges.
  - Hamiltonian: $H_{\mathrm{XC}} = -U \sum_{\substack{\text{edges} \\ e}} \mathbf{X}_e - t \sum_{\substack{\text{cubes} \\ c}}$ 

  [Vijay, Haah, Fu]
  - Gauss's law:




- Within each plane, has toric-code like symmetry operators: $U(\gamma^*) = \prod_{e \in \gamma^*} \mathbf{X}_e$

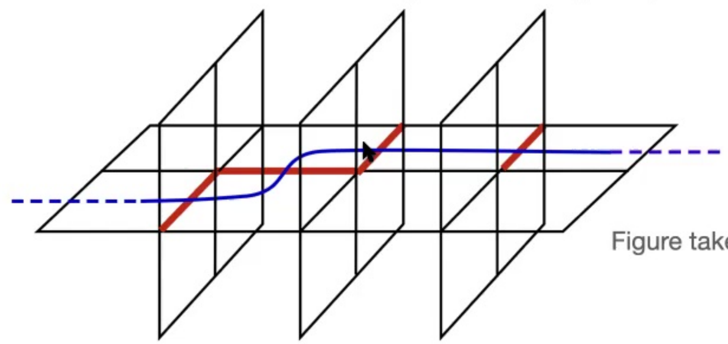
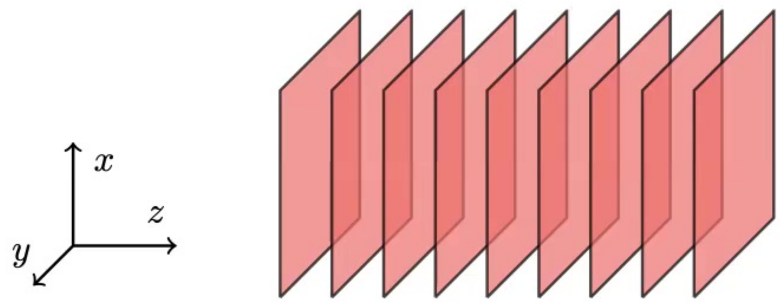


Figure taken from Ma, Lake, Chen, Hermele.

# Coupled wire/layer constructions

- It is often possible to realize fracton order by foliating space with stacks of lower-dimensional systems, and coupling the leaves of these stacks together.
[Ma, Lake, Chen, Hermele; Slagle, Aasen, Williamson; Prem, Huang, Song, Hermele, …]



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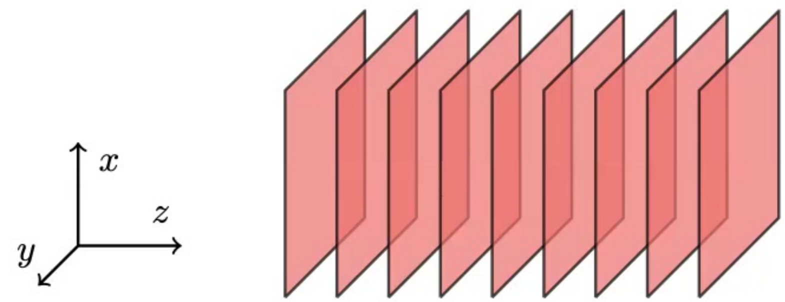



- Useful to revisit some of these constructions with higher-form subsystem symmetry principles in mind. [BR, D. Williamson]

- Symmetry considerations reveal additional structure and logic which underlies such models, and helps predict new examples.

# X-cube Model from Coupled Layers: Mechanism 1

- Consider 3 decoupled stacks of toric code/$\mathbb{Z}_2$ gauge theory layers placed on the leaves of the foliations $\mathcal{F}^{\parallel}_{\mathrm{xy}}, \mathcal{F}^{\parallel}_{\mathrm{yz}}, \mathcal{F}^{\parallel}_{\mathrm{xz}}$ of a cubic lattice. Each edge belongs to two toric code layers, so the Hilbert space has two qubits per edge.

- Couple the layers together:

$$H = H_{\mathrm{TC}}(\mathcal{F}^{\parallel}_{\mathrm{xy}}, \mathcal{F}^{\parallel}_{\mathrm{yz}}, \mathcal{F}^{\parallel}_{\mathrm{xz}}) - K_X \sum_{\substack{\text{edges} \\ e}} \mathbf{X}^{(1)}_e \mathbf{X}^{(2)}_e$$

- When $K_X \gg 1$, the low energy effective Hamiltonian is the X-cube model. [Ma, Lake, Chen, Hermele]

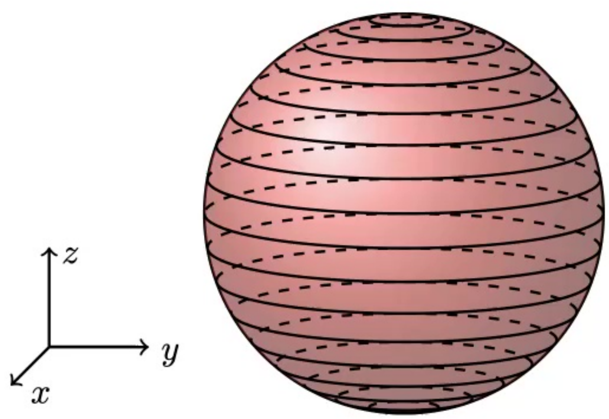
- As $K_X$ is taken from small to large, there is a phase transition from a decoupled layer phase to an X-cube phase which is driven by “p-string condensation”. Is this transition Landau?

# p-string condensation is Landau

- Note that the decoupled toric code layers have a planar one-form subsystem symmetry $\mathbb{Z}_2^{(1,1)}(\mathcal{F}^{\parallel}_{\mathrm{xy}}, \mathcal{F}^{\parallel}_{\mathrm{xz}}, \mathcal{F}^{\parallel}_{\mathrm{yz}})$ which is *relation free.* The coupling respects this symmetry.

- At small $K_X$, this entire symmetry is spontaneously broken.

- At strong $K_X$, the low energy Hilbert space coincides with the ground space of the X-cube model, which breaks a $\mathbb{Z}_2^{(1,1)}(\mathcal{F}^{\parallel}_{\mathrm{xy}}, \mathcal{F}^{\parallel}_{\mathrm{xz}}, \mathcal{F}^{\parallel}_{\mathrm{yz}})/\mathbb{Z}_2^{(1)}$ symmetry. Thus, the one-form subgroup $\mathbb{Z}_2^{(1)}$ becomes *unbroken* in this phase.



# General mechanism

- Consider a target model with a $G^{(q,k)}(\mathcal{F}_1, \mathcal{F}_2, \dots)/\mathscr{H}$ symmetry.
- Approximate this symmetry structure by placing a *(d-k)*-dimensional theory with an ordinary *q*-form symmetry on each of the leaves of the foliations $\mathcal{F}_1, \mathcal{F}_2, \dots$. This yields a theory with a $G^{(q,k)}(\mathcal{F}_1, \mathcal{F}_2, \dots)$ symmetry but without any relations imposed.
- Strongly couple the leaves together with a term which drives a Landau transition from a phase in which $\mathscr{H}$ is broken to a phase in which it is unbroken.
- Because $\mathscr{H}$ is unbroken, it acts trivially on the low energy Hilbert space, and so the low energy effective Hamiltonian looks like it has a $G^{(q,k)}(\mathcal{F}_1, \mathcal{F}_2, \dots)/\mathscr{H}$ symmetry. Phase will typically coincide with that of the target model.

# Another example: Cubic Ising model from coupled wires

- Recall that CIM has a linear subsystem symmetry group $\mathbb{Z}_2^{(0,2)}(\mathcal{F}_{\mathrm{x}}^{\parallel}, \mathcal{F}_{\mathrm{y}}^{\parallel}, \mathcal{F}_{\mathrm{z}}^{\parallel})/\mathscr{H}$. Relations described by its planar subgroup $\mathscr{H} = \mathbb{Z}_2^{(0,1)}(\mathcal{F}_{\mathrm{xy}}^{\parallel}, \mathcal{F}_{\mathrm{xz}}^{\parallel}, \mathcal{F}_{\mathrm{yz}}^{\parallel})/\mathbb{Z}_2^{(0)}$.

- Approximate this symmetry with a grid of decoupled Ising wires.

$$H_{\mathrm{Ising}}(\mathcal{F}_{\mathrm{x}}^{\parallel}, \mathcal{F}_{\mathrm{y}}^{\parallel}, \mathcal{F}_{\mathrm{z}}^{\parallel}) = \sum_{\mu=\mathrm{x,y,z}} \sum_{v} (-h\mathbf{X}_v^{\mu} - J\mathbf{Z}_v^{\mu}\mathbf{Z}_{v+\hat{\mu}}^{\mu})$$

- Couple the wires together.

$$H = H_{\mathrm{Ising}}(\mathcal{F}_{\mathrm{x}}^{\parallel}, \mathcal{F}_{\mathrm{y}}^{\parallel}, \mathcal{F}_{\mathrm{z}}^{\parallel}) - K_X \sum_{v} (\mathbf{X}_v^{\mathrm{x}}\mathbf{X}_v^{\mathrm{y}} + \mathbf{X}_v^{\mathrm{x}}\mathbf{X}_v^{\mathrm{z}} + \mathbf{X}_v^{\mathrm{y}}\mathbf{X}_v^{\mathrm{z}})$$

- As $K_X$ is taken to be large, there is a phase transition into a phase where the planar subgroup $\mathscr{H}$ becomes unbroken.

# X-cube Model from Coupled Layers: Mechanism 2

- Consider again 3 decoupled stacks of toric code/$\mathbb{Z}_2$ gauge theory layers placed on the leaves of the foliations $\mathcal{F}^{\|}_{\rm xy}, \mathcal{F}^{\|}_{\rm yz}, \mathcal{F}^{\|}_{\rm xz}$ of a cubic lattice.

- This time, couple the leaves together simply by *gauging* the desired relation:

$$H_{\rm LGT}(\mathcal{F}^{\|}_{\rm xy}, \mathcal{F}^{\|}_{\rm yz}, \mathcal{F}^{\|}_{\rm xz})/\mathbb{Z}_2^{(1)}$$

- The relation ceases to be a global symmetry of the gauged theory by fiat. The gauged theory inherits a $\mathbb{Z}_2^{(1,1)}(\mathcal{F}^{\|}_{\rm xy}, \mathcal{F}^{\|}_{\rm xz}, \mathcal{F}^{\|}_{\rm yz})/\mathbb{Z}_2^{(1)}$ global symmetry from the ungauged theory.

- However, when you gauge a discrete one-form symmetry in (3+1)D, the gauged theory gains an emergent "quantum" global one-form symmetry $\widehat{\mathbb{Z}}_2^{(1)}$. Where does this symmetry show up?

Decoupled $\mathbb{Z}_2$ gauge theory layers: $H_{\mathrm{LGT}}(\mathcal{F}^{\|}_{\mathrm{xy}}, \mathcal{F}^{\|}_{\mathrm{yz}}, \mathcal{F}^{\|}_{\mathrm{xz}})$

$\mathbb{Z}_2^{(1,1)}(\mathcal{F}^{\|}_{\mathrm{xy}}, \mathcal{F}^{\|}_{\mathrm{yz}}, \mathcal{F}^{\|}_{\mathrm{xz}})$
planar one-form
subsystem symmetry
preserving phase

$\mathbb{Z}_2^{(1,1)}(\mathcal{F}^{\|}_{\mathrm{xy}}, \mathcal{F}^{\|}_{\mathrm{yz}}, \mathcal{F}^{\|}_{\mathrm{xz}})$
planar one-form
subsystem symmetry
breaking phase

$g$

↓ gauging $\mathbb{Z}_2^{(1)}$

Gauged layers: $H_{\mathrm{LGT}}(\mathcal{F}^{\|}_{\mathrm{xy}}, \mathcal{F}^{\|}_{\mathrm{yz}}, \mathcal{F}^{\|}_{\mathrm{xz}})/\mathbb{Z}_2^{(1)}$

$\mathbb{Z}_2^{(1,1)}(\mathcal{F}^{\|}_{\mathrm{xy}}, \mathcal{F}^{\|}_{\mathrm{yz}}, \mathcal{F}^{\|}_{\mathrm{xz}})\big/\mathbb{Z}_2^{(1)}$ preserved
$\widehat{\mathbb{Z}}_2^{(1)}$ broken
((3+1)D deconfined
gauge theory phase)

$\mathbb{Z}_2^{(1,1)}(\mathcal{F}^{\|}_{\mathrm{xy}}, \mathcal{F}^{\|}_{\mathrm{yz}}, \mathcal{F}^{\|}_{\mathrm{xz}})\big/\mathbb{Z}_2^{(1)}$ broken
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(X-cube phase)

$g$

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$g$

gauging $\mathbb{Z}_2^{(1)}$

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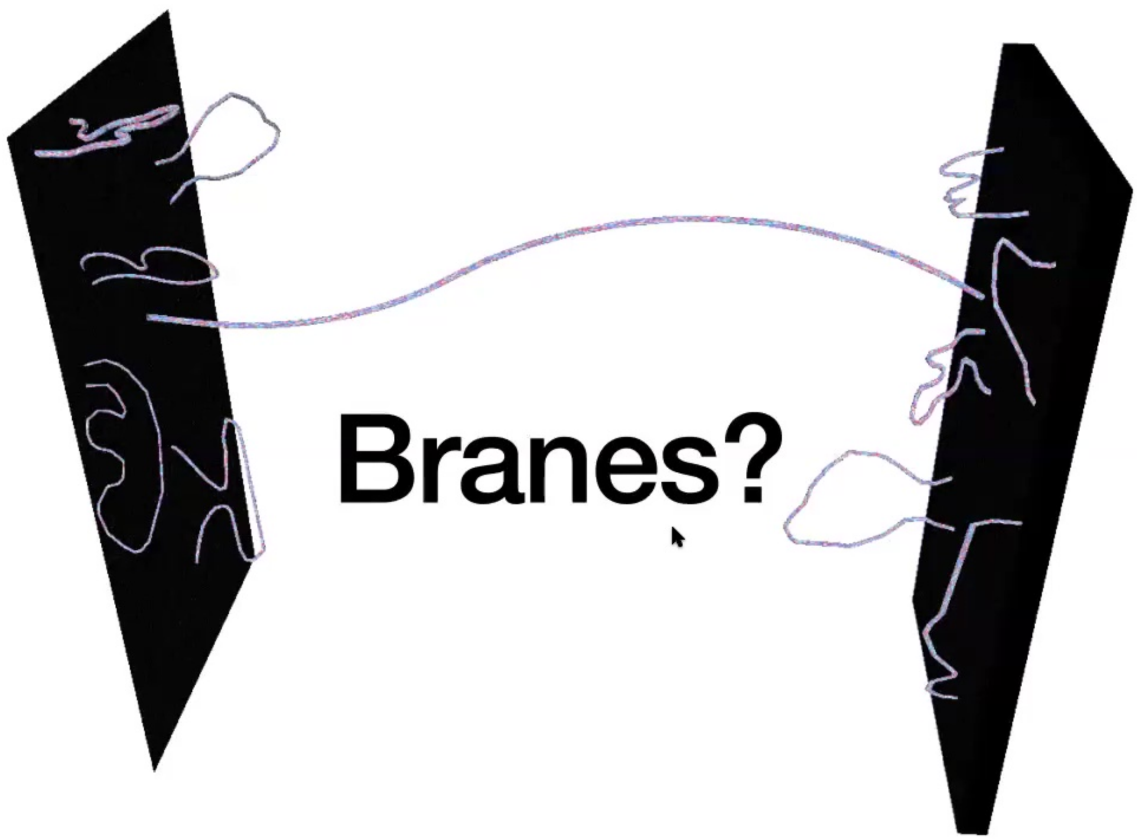
Branes?

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- Approximate this symmetry structure by placing a *(d-k)*-dimensional theory with an ordinary *q*-form symmetry on each of the leaves of the foliations $\mathcal{F}_1, \mathcal{F}_2, \dots$. This yields a theory with a $G^{(q,k)}(\mathcal{F}_1, \mathcal{F}_2, \dots)$ symmetry but without any relations imposed.

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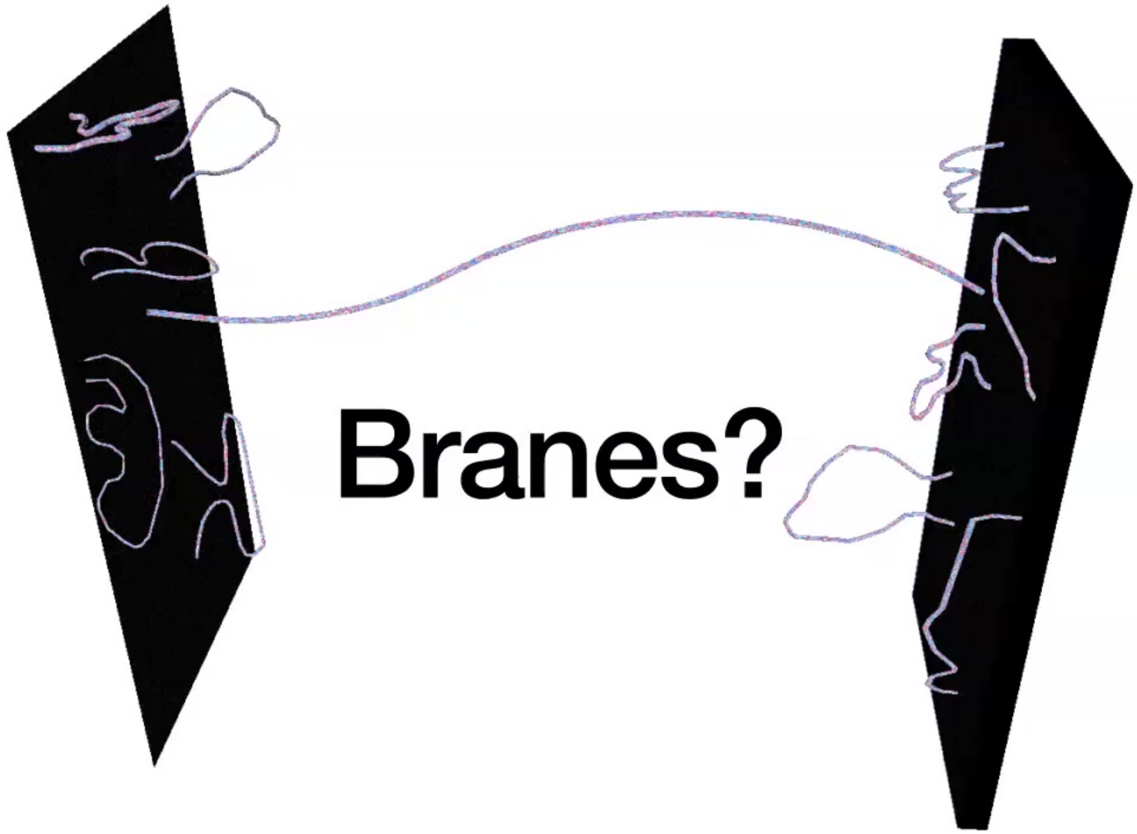
# Another example: cubic Ising model

# String theory has been historically useful for gaining insight into quantum field theories


Branes?

# String theory has been historically useful for gaining insight into quantum field theories

- In addition to strings, string theory admits extended objects called "D-branes" on which strings can end.
- In certain limits, the physics of a web of branes is described by a quantum field theory living on its world-volume.
- If you can embed a field theory into a string theory in this way, then you can leverage the structure of the latter to reason about the physics of the former.
- E.g. Hanany—Witten showed that S-duality of Type IIB string theory implies infinite families of (supersymmetric) particle-vortex dualities in (2+1)D. (The supersymmetric versions of particle-vortex dualities are known as 3d mirror symmetry.)

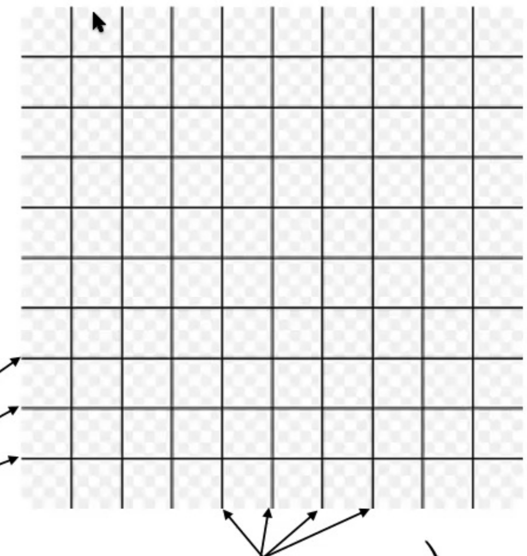
# Foliated field theories (FFTs) [Slagle, Aasen, Williamson; Slagle; Hsin-Slagle]

- Roughly obtained as the continuum limit of the kinds of coupled layer/coupled wire constructions described so far on the lattice.

- Built out of *D*-dimensional fields which resemble stacks of decoupled *d*<*D*-dimensional fields.

- FFTs can be coupled to more conventional field theories.

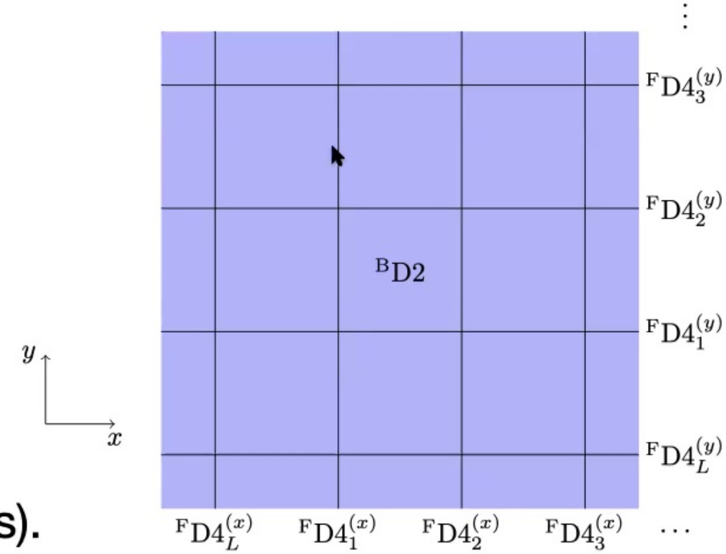
- Example: free (1+1)D compact boson wires coupled to conventional (2+1)D electromagnetism.



$$\mathcal{L} \sim -\frac{1}{2e^2} F \wedge \star F + \frac{1}{4\pi}\left(\frac{R_x^2}{2}\sum_{\mu=t,x}(\partial_\mu \varphi^{(1)} - A_\mu)(\partial^\mu \varphi^{(1)} - A^\mu) + \frac{R_y^2}{2}\sum_{\mu=t,y}(\partial_\mu \varphi^{(2)} - A_\mu)(\partial^\mu \varphi^{(2)} - A^\mu)\right)$$

# A supersymmetric foliated brane setup

| | $t$ | $x$ | $y$ | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| ${}^{\mathrm{B}}\mathrm{D2}$ | x | x | x | | | | | | | |
| ${}^{\mathrm{F}}\mathrm{D4}_{\alpha}^{(y)}$ | x | x | :: | x | x | x | | | | |
| ${}^{\mathrm{F}}\mathrm{D4}_{\alpha}^{(x)}$ | x | :: | x | x | x | | x | | | |

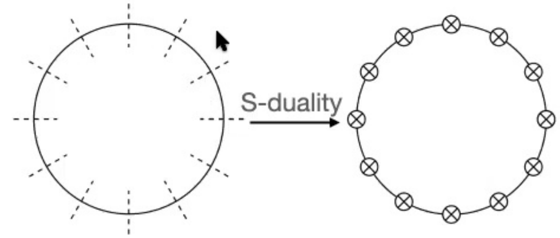



- $\mathbf{x}$ denotes a direction that the brane spans. :: indicates a direction in which the branes form an evenly spaced lattice.
- Strings which stretch from the ${}^{\mathrm{B}}\mathrm{D2}$ brane to itself contribute a (2+1)D gauge field (plus superpartners).
- Strings which stretch from the ${}^{\mathrm{B}}\mathrm{D2}$ brane to a ${}^{\mathrm{F}}\mathrm{D4}_I^{(\mu)}$ brane contribute a hypermultiplet (i.e. two oppositely charged complex scalars plus superpartners) which is localized at the intersection.
- 4-4 strings can be made to decouple. Each ${}^{\mathrm{F}}\mathrm{D4}_I^{(\mu)}$ brane supports a *U(1)* gauge field on its world-volume whose gauge coupling goes to zero. Thus in this limit, each *U(1)* becomes a global symmetry, and together they can be interpreted as a subsystem symmetry.

# A foliated Hanany—Witten setup

- Going up by one-dimension and using D3/D5/NS5-branes of Type IIB leads to foliated analogs of Hanany—Witten brane setups.

| | $t$ | $x$ | $y$ | 3 | 4 | 5 | $z$ | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| ${}^{\mathrm{B}}\mathrm{D3}$ | x | x | x | | | | x | | | |
| ${}^{\mathrm{F}}\mathrm{D5}_{\alpha}$ | x | x | x | x | x | x | :: | | | |




- ${}^{\mathrm{B}}\mathrm{D3}$-${}^{\mathrm{F}}\mathrm{D5}_{\alpha}$ system leads to (3+1)D *N=4* SYM coupled to a stack of (2+1)D hypermultiplets.

- S-duality exchanges ${}^{\mathrm{F}}\mathrm{D5}_{\alpha}$ branes with ${}^{\mathrm{F}}\mathrm{NS5}_{\alpha}$ branes. The dual brane setup is described by a long circular quiver with *L U(1)* gauge fields and hypermultiplets in bifundamentals. Taking continuum limit of both sides leads to interesting dualities of foliated field theories.

# Infinite-component Chern-Simons (iCS) theory

[Ma, Shirley, Cheng, Levin, McGreevy, Chen]

- Another approach to Abelian fracton order in (3+1)D is to consider a large *L* number of (2+1)D gauge fields $a^I$ governed by a Chern-Simons theory with a large (quasi-diagonal) *K*-matrix,

$$\mathcal{L} = \frac{1}{4\pi} \sum_{I,J=1}^{L} K_{IJ} \epsilon^{\mu\nu\rho} a_\mu^I \partial_\nu a_\rho^J$$

One thinks of the index I as labeling different layers of a stack of (2+1)D theories. Non-zero off-diagonal elements are interpreted as inter-layer couplings.

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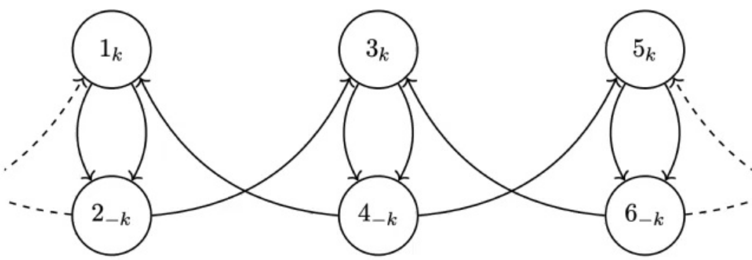
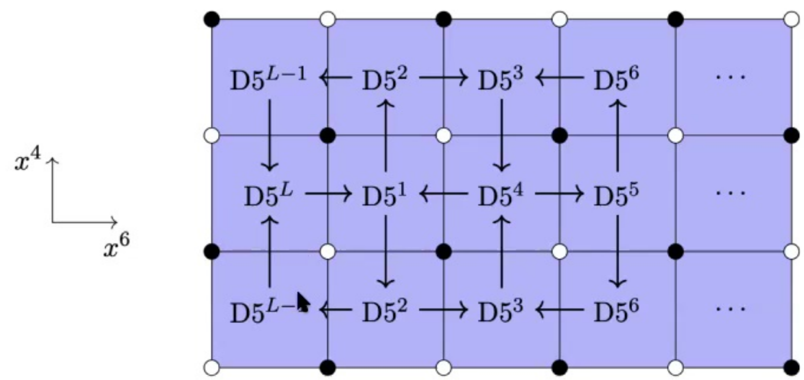
- Fractonic because e.g. ground state degeneracy on the torus is $|\det(K)|$ which can grow exponentially with *L*.

# iCS theories from brane-tilings

| | $t$ | $x$ | $y$ | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\mathrm{D4}^I$ | **x** | **x** | **x** | $\epsilon^I$ | $\lvert\mathbf{x}\rvert$ | | $\lvert\mathbf{x}\rvert$ | | | |
| NS5 | **x** | **x** | **x** | **x** | —— | $\Sigma$ | —— | | | |




- Integrating out chiral multiplets of the *N=2* quiver Chern-Simons theory leads in the IR to an Abelian CS theory with a quasi-diagonal K-matrix.

$$K(k) = \begin{pmatrix} \ddots & & & & & & & & & \\ & a & 2 & & -1 & & & & & \\ & 2 & b & 1 & & & & & & \\ & & 1 & a & 2 & & -1 & & & \\ & -1 & & 2 & b & 1 & & & & \\ & & & & 1 & a & 2 & & -1 & \\ & & & -1 & & 2 & b & 1 & & \\ & & & & & & 1 & a & 2 & \\ & & & & & -1 & & 2 & b & \\ & & & & & & & & & \ddots \end{pmatrix}$$

$$a = k - 2$$
$$b = -k - 2$$

# Conclusions

- Fractons solve long-standing problem in quantum information theory.
- They form new stable phases of quantum lattice models. They can often be understood in terms of higher-form subsystem symmetries.
- They have analogies to brane physics which we're just at the beginning of exploring.

Thank you!