Title: New Perspectives on (Quantum) Gravity

Speakers: Luca Ciambelli

Series: Quantum Gravity

Date: December 16, 2021 - 2:30 PM

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Abstract: We expect certain universal results in classical gravity to come unaltered from a quantum theory of gravity. In this talk, I will firstly discuss new ideas on this topic, and the latest understanding we have on this, in an accessible way. After that, I will focus on local symmetries in gravity, and obtain the most general off-shell algebra of diffeomorphisms that acts non-trivially at corners, where gauge charges are supported. Noether charges in Einstein gravity are then shown to generate a faithful representation of this algebra. After pausing and reviewing the covariant phase space formulation, explaining the questions and issues our community faced in successfully applying it to gravity, I will show how a careful treatment of embeddings allows us to establish a field space where Noether charges act canonically via Poisson bracket. This solves a longstanding puzzle in this field, opening doors to new promising investigations. I will conclude by mentioning them and commenting on how new proposals might shed light on old unanswered questions in quantum gravity.

Zoom Link: https://pitp.zoom.us/j/96552295909?pwd=cFZwcGxLdWlaWWhRS21RMExUd1RjQT09

New Perspectives on (Quantum) Gravity

Luca Ciambelli

Physique Mathématique des Interactions Fondamentales

Université Libre de Bruxelles &
International Solvay Institutes

Quantum Gravity Seminars
Perimeter Institute, 16-12-2021

Done

Gravity, and its quantization, remains one of the most elusive topics in physics

Approaching the problem at face value has been unsuccessful

An action, or preferred dynamics, (even a spacetime?) are classical

Difficult to disentangle classical from quantum — few guidelines

No global symmetries are expected to survive quantization
[Banks & Dixon '88, Harlow & Ooguri '18]

Local symmetries however do survive quantization, modulo anomalies

This suggests to focus on local symmetries of gravity

They provide the key to the quantum realm of gravity, as we will explain

Done

Local symmetries of gravity have support on codim 2 surfaces

[Noether 1918, Regge & Teitelboim '74, Iyer & Wald '94, Barnich & Brandt '01]

Recently denominated corners, they are the core of the corner proposal

[LC, Donnelly, Freidel, Geiller, Hopfmüller, Leigh, Oliveri, Pranzetti, Riello, Speranza, Speziale, … '16 - '21]

Quantum observables organize according to algebras at corners

[PI as one of main institutions '16 - '21]

Any insight on these algebras (and their universality) is crucial

My work has focused on corner's geometry, independent of classical features
[LC & Leigh '21]

We established one universal algebra, off-shell and metric independent
[LC & Leigh '21]

We recently included geometric tools to formulate a canonical classical phase space
[LC & Leigh & Pai '21]

Geometric pov: deep connection with celestial holography (work in progress)
[Ball, Donnay, Guevara, Kapec, Pasterski, Pate, Puhm, Raclariu, Salzer, Strominger, Zhiboedov, … '14 - '21]

Outline

Motivations ✓

Corners

Classical Phase Space

Geometry of Corners

d-dim manifold M (the bulk). $(d-k)$-dim manifold S, if $k=2 \Rightarrow$ S is a corner

$$\text{Split } TM = V \oplus H \quad \text{Rank}(H, V) = (d-k, k)$$

$H = ker(n^a) = \{\underline{\mathfrak{X}} \in TM \mid n^a(\underline{\mathfrak{X}}) = 0\}$, Using Local Coordinates on M $y^M = (u^a, x^i)$

$$n^a = du^a - a_i^a(u,x)dx^i \;\Rightarrow\; H = \text{span}\{\underline{D}_i = \underline{\partial}_i + a_i^a(u,x)\underline{\partial}_a\}$$

a_i^a is an Ehresmann connection, H=non-integrable distribution

Motivations Corners Classical Phase Space Future Perspectives

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$$[\underline{D}_i, \underline{D}_j] = f_{ij}^{a}\, \underline{\partial}_a \neq 0$$

a_i^a is an Ehresmann connection, H=non-integrable distribution

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Done

Geometry of Corners

Embedding $\phi : S \to M$, σ^α Coordinates on S, $\phi : \sigma^\alpha \mapsto y^M(\sigma)$

$\phi^* : T^*M \to T^*S$

Adapt to the split: $\phi : \sigma^\alpha \mapsto (u^a(\sigma), x^i(\sigma))$

Thus: $\phi^*(V^*) = \phi^*(n^a) = 0$, $\phi^*(H^*) = T^*S$

Example: Trivial Embedding $\phi_0 : \sigma^\alpha \mapsto (u^a(\sigma) = 0,\ x^i(\sigma) = \delta^i_\alpha \sigma^\alpha)$

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Done

Maximal Embedded Algebra

Using ϕ_0 expand vector field $\underline{\xi} = \xi^a(u,x)\underline{\partial}_a + \xi^i(u,x)\underline{\partial}_i$ in powers of $u^a \to 0$

$$\xi^b(u,x) = \xi^b_{(0)}(x) + \xi^b_{(1)a_1}(x)u^{a_1} + \frac{1}{2}\xi^b_{(2)a_1a_2}(x)u^{a_1}u^{a_2} + \ldots$$

$$\xi^i(u,x) = \xi^i_{(0)}(x) + \xi^i_{(1)a_1}(x)u^{a_1} + \frac{1}{2}\xi^i_{(2)a_1a_2}(x)u^{a_1}u^{a_2} + \ldots$$

expand then the Lie bracket of two vectors:

$$\left[\underline{\xi}_1, \underline{\xi}_2\right] = \xi^j_{(0)3}\underline{\partial}_j + \xi^b_{(0)3}\underline{\partial}_b + u^c\xi^b_{(1)3c}\underline{\partial}_b + u^c\xi^j_{(1)3c}\underline{\partial}_j + \ldots$$

Maximal Embedded Algebra

Closure of the algebra $\Leftrightarrow \underline{\xi} = \xi^k_{(0)}(x)\underline{\partial}_k + \left(\xi^a_{(0)}(x) + u^b \xi^a_{(1)b}(x)\right)\underline{\partial}_a$

$$[\underline{\xi}_1, \underline{\xi}_2] \Rightarrow (\underbrace{\mathrm{Diff}(S)}_{\xi^j_{(0)}} \ltimes \underbrace{GL(k,\mathbb{R})}_{\xi^a_{(1)b}}) \ltimes \underbrace{\mathbb{R}^k}_{\xi^a_{(0)}}$$

Called Maximal Embedded Algebra [LC & Leigh '21]

Off-shell and metric independent, no assumptions, expected to survive quantization

Motivations Corners Classical Phase Space Future Perspectives

Done

Important Sub-Cases

$$(\mathrm{Diff}(S) \ltimes GL(k,\mathbb{R})) \ltimes \mathbb{R}^k$$

k=1 $(\mathrm{diff}\, S \ltimes \mathbb{R}) \ltimes \mathbb{R}$

- AdS Holography with Weyl Symmetry: $\mathrm{Diff}(B) \ltimes \mathbb{R}$
  [LC & Leigh '19, Alessio & Barnich & LC & Mao & Ruzziconi '20]

- Generalized BMS on null structures: $\mathrm{Diff}(S) \ltimes \mathbb{R}$, and sub-classes
  [Barnich & Troessaert '10, Campiglia & Laddha, Compere & Fiorucci & Ruzziconi '14]

Done

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- Null boundaries in 2 & 3 dim: $\mathrm{Diff}(B) \ltimes \mathbb{R}$
  [Adami & Sheikh-Jabbari & Taghiloo & Yavartanoo & Zwikel '20]

Done

Important Sub-Cases

$$(\mathrm{Diff}(S) \ltimes GL(k,\mathbb{R})) \ltimes \mathbb{R}^k$$

k=2

- Weyl BMS group at null infinity: $\mathrm{Diff}(S) \ltimes \mathbb{R} \ltimes \mathbb{R}$
  [Freidel & Oliveri & Pranzetti & Speziale '21]

- Extended corner symmetry: $(\mathrm{Diff}(S) \ltimes SL(2,\mathbb{R})) \ltimes \mathbb{R}^2$
  [Donnelly, Freidel, Geiller, Oliveri, Pranzetti, Speranza, Speziale '16-'21]

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- Link with $w_{1+\infty}$

  [LC & Pate & Salzer '22]

Go Classical: Metrics

So far, all metric independent. Introduce a line element adapted to the split of TM

$$g = h_{ab}(u,x)n^a \otimes n^b + \gamma_{ij}(u,x)dx^i \otimes dx^j$$

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Metric on the normal fibres

Done

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$$g = h_{ab}(u,x)n^a \otimes n^b + \gamma_{ij}(u,x)dx^i \otimes dx^j$$

Metric on the normal fibres

Pulls back to metric on S

Expand h_{ab}, γ_{ij} and $n^a = du^a - a_i^a(u,x)dx^i$ assuming finite distance corner

use ϕ_0 and compute infinitesimal variation under a diffeo $\mathcal{L}_{\underline{\xi}} g = \delta_{\underline{\xi}} g$

$$[\underline{\xi}_1,\underline{\xi}_2] \Rightarrow \big(\underbrace{\mathrm{Diff}(S)}_{\xi^{i}} \ltimes \underbrace{GL(2,\mathbb{R})}_{\xi^a_{(1)b}}\big) \ltimes \underbrace{\mathbb{R}^2}_{\xi^a_{(0)}}$$

$$g = h_{ab}(u,x)n^a \otimes n^b + \gamma_{ij}(u,x)dx^i \otimes dx^j$$

$$\delta_{\underline{\xi}} h^{(0)}_{ab}(x) = \xi^j_{(0)}\partial_j h^{(0)}_{ab} + (h^{(0)}_{bc}\xi^c_{(1)a} + h^{(0)}_{ac}\xi^c_{(1)b}) + \xi^c_{(0)} h_{(1)ab,c}$$

$$\delta_{\underline{\xi}} a^{(0)b}_i = \left(\xi^j_{(0)}\partial_j a^{(0)b}_i + a^{(0)b}_j \partial_i \xi^j_{(0)}\right) - \xi^b_{(1)c} a^{(0)c}_i + \left(-\partial_i \xi^b_{(0)} + \xi^c_{(0)} a^{(1)b}_{ic}\right)$$

$$\delta_{\underline{\xi}} a^{(1)a}_{ib} = \left(\xi^j_{(0)}\partial_j a^{(1)a}_{ib} + a^{(1)a}_{jb}\partial_i \xi^j_{(0)}\right) + \left(-\partial_i \xi^a_{(1)b} + a^{(1)a}_{ic}\xi^c_{(1)b} - \xi^a_{(1)c} a^{(1)c}_{ib}\right) + \xi^c_{(0)} a^{(2)a}_{ibc}$$

$$\delta_{\underline{\xi}} \gamma^{(0)}_{ij} = \left(\xi^k_{(0)}\partial_k \gamma^{(0)}_{ij} + \gamma^{(0)}_{kj}\partial_i \xi^k_{(0)} + \gamma^{(0)}_{ki}\partial_j \xi^k_{(0)}\right) + \xi^c_{(0)}\gamma^{(1)}_{ij,c}$$

Done

$$[\underline{\xi}_1, \underline{\xi}_2] \;\Rightarrow\; (\underbrace{\text{Diff}(S)}_{\xi^j_{(0)}} \ltimes \underbrace{GL(2,\mathbb{R})}_{\xi^a_{(1)b}}) \ltimes \underbrace{\mathbb{R}^2}_{\xi^a_{(0)}}$$

$$\xi^a(x)\,\partial_a$$

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Motivations Corners Classical Phase Space Future Perspectives

Go Classical: Dynamics

From now on Corners: $k = 2$, Einstein gravity, charges of $\mathfrak{diff}(M)$

Compute (hard) Noether charges with our metric and trivial embedding ϕ_0

$$H_{\underline{\xi}} = \int_S \phi_0^*(Q_{\underline{\xi}}) = \int_S \phi_0^*(*dg(\underline{\xi}, \cdot))$$

The pull back is crucial

Motivations Corners Classical Phase Space Future Perspectives

Done

$$H_{\underline{\xi}} = \int_S vol_S \left(\xi^a_{(1)b} N^b{}_a + \xi^j_{(0)} b_j + \xi^a_{(0)} p_a \right)$$

[In agreement and extending: Donnelly, Freidel, Geiller, Oliveri, Pranzetti, Speranza, Speziale '16-'21]

Only $(\text{Diff}(S) \ltimes GL(2,\mathbb{R})) \ltimes \mathbb{R}^2$ contributes $\Rightarrow$ Allowed Large Surface Charges

- Translation $\quad p_d = \frac{1}{2} N^a{}_c h^{cb}_{(0)} \left(h^{(1)}_{db,a} - h^{(1)}_{da,b} \right)$

- DiffS $\quad b_j = -N^b{}_a a^{(1)a}_{jb}$

- $GL(2,\mathbb{R})$ $\quad N^b{}_a = \sqrt{-\det h^{(0)}} h^{bc}_{(0)} \varepsilon_{ca}$

$$H_{\underline{\xi}} = \int_S vol_S \left(\xi^a_{(1)b} N^b{}_a + \xi^j_{(0)} b_j + \xi^a_{(0)} p_a \right)$$

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- $GL(2,\mathbb{R}) \quad N^b{}_a = \sqrt{-\det h^{(0)}}\, h^{bc}_{(0)} \varepsilon_{ca}$

Traceless: only $SL(2,\mathbb{R})$ is dynamically realized

Evaluate the Charge Algebra $\delta_{\underline{\eta}} H_{\underline{\xi}} = H_{[\underline{\xi},\underline{\eta}]}$, later we will introduce a field space

Naively $\delta_{\underline{\eta}} H_{\underline{\xi}} = \delta_{\underline{\eta}}\Big(\int_S \phi^*(Q_{\underline{\xi}})\Big) = \int_S \phi^* \Big(\delta_{\underline{\eta}} Q_{\underline{\xi}}\Big)$

Motivations Corners Classical Phase Space Future Perspectives

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Motivations Corners Classical Phase Space Future Perspectives

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$\delta_{\underline{\eta}}$ cannot pass through ϕ^*, translations move the corner embedding

Done

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$\delta_{\underline{\eta}}$ cannot pass through ϕ^*, translations move the corner embedding

$$\delta_{\underline{\eta}} H_{\underline{\xi}} = \int_S \phi^*\left(\delta_{\underline{\eta}} Q_{\underline{\xi}}\right) + \int_S \left(\delta_{\underline{\eta}} \phi^*\right) Q_{\underline{\xi}}$$

$$\int_S (\delta_{\underline{\eta}}\phi^*)(Q_{\underline{\xi}}[g,y]) = \int_S \phi^*\left(Q_{\underline{\xi}}[g,y-\eta] - Q_{\underline{\xi}}[g,y]\right)$$

Done

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Wrong

$\delta_{\underline{\eta}}$ cannot pass through ϕ^*, translations move the corner embedding

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$$\int_S (\delta_{\underline{\eta}}\phi^*)(Q_{\underline{\xi}}[g,y]) = \int_S \phi^*\left(Q_{\underline{\xi}}[g, y-\eta] - Q_{\underline{\xi}}[g,y]\right)$$

Active vs Passive

Motivations Corners Classical Phase Space Future Perspectives

Introducing a bracket notation: $\delta_{\underline{\eta}} H_{\underline{\xi}} = H_{[\underline{\xi},\underline{\eta}]} = \{\![H_{\underline{\xi}}, H_{\underline{\eta}}]\!\}$

$$b_{\underline{\xi}} = \int_S vol_S \xi^j_{(0)}(\sigma) b_j \qquad N_{\underline{\xi}} = \int_S \mathrm{vol}_S \, \xi^a_{(1)b}(\sigma) N^b{}_a \qquad p_{\underline{\xi}} = \int_S vol_S \xi^a_{(0)}(\sigma) p_a$$

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$$\{\!\![b_{\underline{\xi}}, b_{\underline{\eta}}]\!\!\} = b_{[\underline{\xi},\underline{\eta}]} \qquad \{\!\![N_{\underline{\xi}}, N_{\underline{\eta}}]\!\!\} = N_{[\underline{\xi},\underline{\eta}]} \qquad \{\!\![p_{\underline{\xi}}, p_{\underline{\eta}}]\!\!\} = 0.$$

$$\{\!\![b_{\underline{\xi}}, N_{\underline{\eta}}]\!\!\} = N_{[\underline{\xi},\underline{\eta}]} \qquad \{\!\![N_{\underline{\xi}}, p_{\underline{\eta}}]\!\!\} = p_{[\underline{\xi},\underline{\eta}]}$$

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$$(\mathrm{Diff}(S) \ltimes SL(2,\mathbb{R})) \ltimes \mathbb{R}^2$$

Motivations Corners Classical Phase Space Future Perspectives

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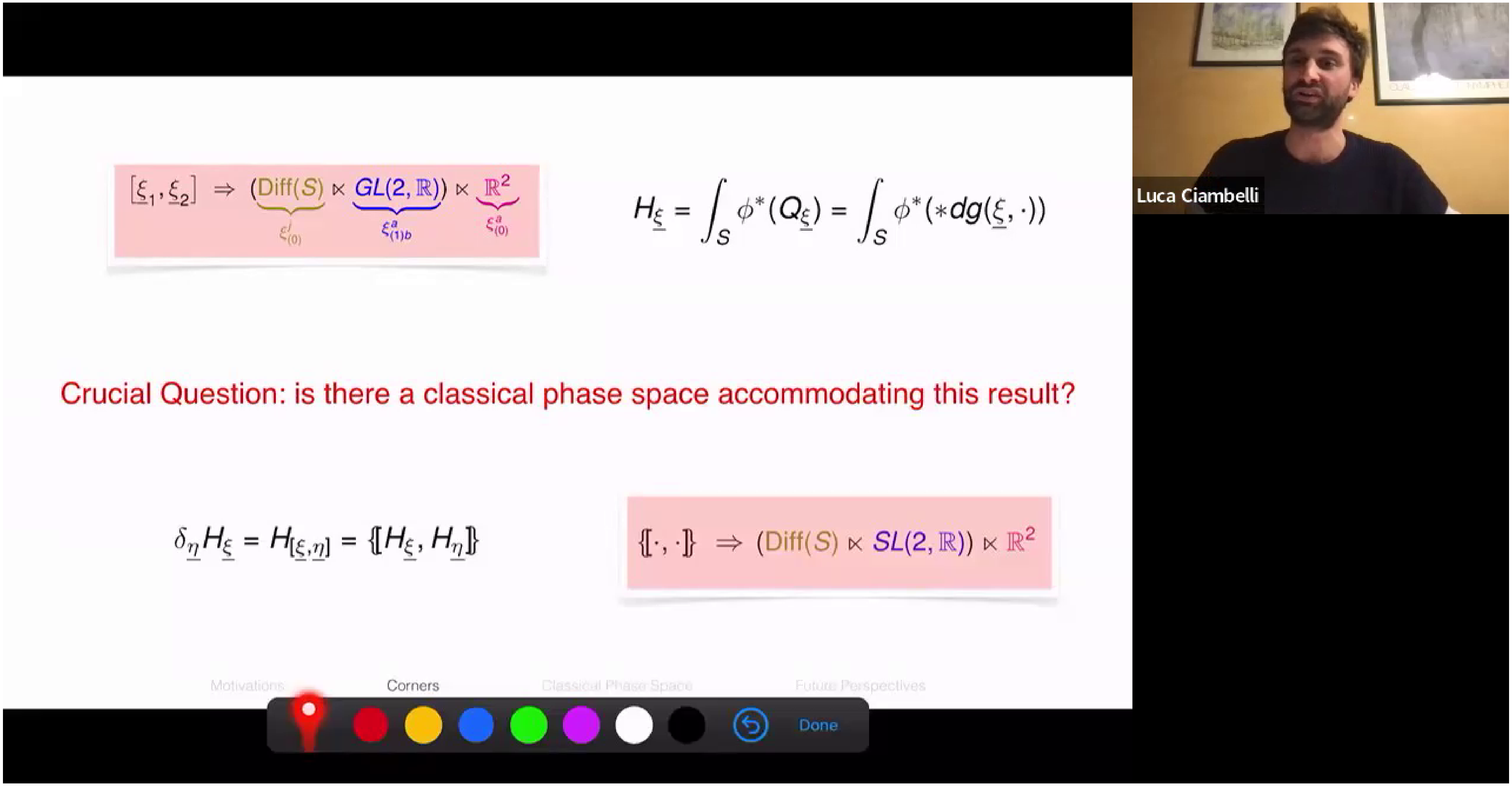
$$(\mathrm{Diff}(S) \ltimes SL(2,\mathbb{R})) \ltimes \mathbb{R}^2$$

Centerless faithful representation of the vector field algebra

Done


$[\underline{\xi}_1, \underline{\xi}_2] \Rightarrow (\underbrace{\mathrm{Diff}(S)}_{\xi^i_{(0)}} \ltimes \underbrace{GL(2,\mathbb{R})}_{\xi^a_{(1)b}}) \ltimes \underbrace{\mathbb{R}^2}_{\xi^a_{(0)}}$
$H_{\underline{\xi}} = \int_S \phi^*(Q_{\underline{\xi}}) = \int_S \phi^*(*dg(\underline{\xi}, \cdot))$
Crucial Question: is there a classical phase space accommodating this result?
$\delta_{\underline{\eta}} H_{\underline{\xi}} = H_{[\underline{\xi}, \underline{\eta}]} = \{\![H_{\underline{\xi}}, H_{\underline{\eta}}]\!\}$
$\{\![\cdot, \cdot]\!\} \Rightarrow (\mathrm{Diff}(S) \ltimes SL(2,\mathbb{R})) \ltimes \mathbb{R}^2$
Motivations
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Classical Phase Space
Future Perspectives
Done
Luca Ciambelli

$$[\underline{\xi}_1,\underline{\xi}_2] \Rightarrow (\underbrace{\mathrm{Diff}(S)}_{\xi^j_{(0)}} \ltimes \underbrace{GL(2,\mathbb{R})}_{\xi^a_{(1)b}}) \ltimes \underbrace{\mathbb{R}^2}_{\xi^a_{(0)}}$$

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Done

$$H_{\underline{\xi}} = \int_S vol_S \left(\xi^a_{(1)b} N^b{}_a + \xi^j_{(0)} b_j + \xi^a_{(0)} p_a \right)$$

[In agreement and extending: Donnelly, Freidel, Geiller, Oliveri, Pranzetti, Speranza, Speziale '16-'21]

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Motivations ✓

Corners ✓

Classical Phase Space

Future Perspectives

Classical Phase Space in a Nutshell

Idea: Geometrize the Field Space Γ, δ : Exterior Derivative on Γ

Γ is a Symplectic Manifold with Metric Ω (Symplectic 2-form)

Defining Property $\delta\Omega = 0 \;\Rightarrow\; \Omega = \delta a$ (trivial topology of Γ)

Ω Non-Degenerate: $\forall\, V_{\underline{\xi}} \mid I_{V_{\underline{\xi}}}\Omega = 0 \;\Rightarrow\; V_{\underline{\xi}} = 0$, with $I_{V_{\underline{\xi}}}$ the interior product on Γ

Symplectomorphisms of the Symplectic Manifold $\mathfrak{L}_{V_{\underline{\xi}}}\Omega = 0 \quad (\mathfrak{L}_{V_{\underline{\xi}}} \equiv I_{V_{\underline{\xi}}}\delta + \delta I_{V_{\underline{\xi}}})$

$$\text{Then: } \mathfrak{L}_{V_{\underline{\xi}}}\Omega = I_{V_{\underline{\xi}}}\underbrace{\delta\Omega}_{=0} + \delta I_{V_{\underline{\xi}}}\Omega = 0 \;\Rightarrow\; I_{V_{\underline{\xi}}}\Omega = -\delta F_{\underline{\xi}}$$

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Generator of Canonical Transformations

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Generator of Canonical Transformations

Poisson bracket: $\{F_{\underline{\xi}}, G_{\underline{\eta}}\} \equiv \mathfrak{L}_{V_{\underline{\eta}}} F_{\underline{\xi}} = I_{V_{\underline{\eta}}}\delta F_{\underline{\xi}} + \delta I_{V_{\underline{\eta}}} F_{\underline{\xi}} = -I_{V_{\underline{\eta}}} I_{V_{\underline{\xi}}}\Omega$

Motivations Corners Classical Phase Space Future Perspectives

Done

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PB algebra is projective

Motivations · Corners · Classical Phase Space · Future Perspectives

Done

Simplest Example: Classical Mechanics

$$\Omega = \delta p_i \wedge \delta q^i \quad \Rightarrow \quad \delta\Omega = 0 \;\; \& \;\; \Omega = \delta a \;\; \text{with} \;\; a = p_i \delta q^i \;\; \text{(Darboux coordinates)}$$

$$V_{\underline{\xi}} = Q^i \frac{\partial}{\partial q^i} + P_i \frac{\partial}{\partial p_i} \;\Rightarrow\; I_{V_{\underline{\xi}}}\Omega = P_i \delta q^i - Q^i \delta p_i = -\delta F_{\underline{\xi}} = -\frac{\partial F_{\underline{\xi}}}{\partial q^i}\delta q^i - \frac{\partial F_{\underline{\xi}}}{\partial p_i}\delta p_i$$

$$\Rightarrow \; V_{\underline{\xi}} = \frac{\partial F_{\underline{\xi}}}{\partial p_i}\frac{\partial}{\partial q^i} - \frac{\partial F_{\underline{\xi}}}{\partial q^i}\frac{\partial}{\partial p_i}$$

Preserves Hamilton-Jacobi eom

Then $\{F_{\underline{\xi}}, G_{\underline{\eta}}\} = -I_{V_{\underline{\eta}}} I_{V_{\underline{\xi}}}\Omega = \frac{\partial F_{\underline{\xi}}}{\partial q^i}\frac{\partial G_{\underline{\eta}}}{\partial p_i} - \frac{\partial F_{\underline{\xi}}}{\partial p_i}\frac{\partial G_{\underline{\eta}}}{\partial q^i}$ generates HJ eom

Done

What is Wrong with Gravity?

1) In gauge theories, there are zero modes in Ω (Pre-Symplectic)
Solution: quotient them out

2) Naively, for a diffeomorphism, $I_{V_{\underline{\xi}}}\Omega \neq -\delta F_{\underline{\xi}}$
This created a long debate. Our solution: be carefully with embeddings!
[Inspired by Donnelly & Freidel '16, LC & Leigh & Pai, '21]

Naively disregarding embeddings: $\delta S_R = \int_R \delta L = \int_R E^{MN} \delta g_{MN} + \int_B \theta[g, \delta g, y]$

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Done

Contract a diffeomorphism $0 = I_{V_{\underline{\xi}}}\delta S_R = \int_R I_{V_{\underline{\xi}}}\delta L \,\hat{=}\, \int_B I_{V_{\underline{\xi}}}\theta \;\Rightarrow\; J_{\underline{\xi}} = I_{V_{\underline{\xi}}}\theta - \delta_{\underline{\xi}} L \,\hat{=}\, 0$

→ Noether weakly vanishing current

Noether second theorem: $dJ_{\underline{\xi}} = 0 \;\Rightarrow\; J_{\underline{\xi}} = dQ_{\underline{\xi}}$

→ Noether Charges

Defining: $\delta\theta = \omega, \quad \Omega = \int_\Sigma \omega \quad \Rightarrow\; I_{V_{\underline{\xi}}}\Omega = \int_{\partial\Sigma}\left(-\delta Q_{\underline{\xi}} + i_{\underline{\xi}}\theta\right)$

Non-Integrable charges, Hamiltonian flux! ~~Poisson Bracket~~

Motivations Corners Classical Phase Space Future Perspectives

Done

Our Resolution

Recall the Crucial Question: classical phase space accommodating our result?

$$I_{V_\eta}(\delta \phi^*) = (\delta_\eta \phi^*) \neq 0$$

Yes, treating the embedding dynamically $\{\sigma^\alpha\} \rightarrow \left\{ y^M(\sigma^\alpha) + \chi^M(\sigma^\alpha) \right\}$

[LC & Leigh & Pai, '21]

spacetime

Enhanced Field Space: $I_{V_{\underline{\eta}}} \delta g = \mathfrak{L}_{V_{\underline{\eta}}} g = \mathcal{L}_{\underline{\eta}} g, \qquad I_{V_{\underline{\eta}}} \underline{\chi}_\phi = -\underline{\eta}\Big|_{\phi(S)}$

Then, by contruction: $(\delta\phi^*)(\alpha[g,y]) = \phi^*(\alpha[g, y+\chi] - \alpha[g,y]) = \phi^*\left(\mathcal{L}_{\underline{\chi}}\alpha[g,y]\right)$

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We obtain for the action: $\delta S_R \hat{=} \int_B \phi^* \left(\theta[g, \delta g, y] + i_{\underline{\chi}_\phi} L[g, y] \right)$

This suggests a new Symplectic Potential: $\Theta_\Sigma^{ext.} \equiv \int_\Sigma \phi^* \left(\theta[g, \delta g, y] + i_{\underline{\chi}_\phi} L[g, y] \right)$

Thus: $\Omega_\Sigma^{ext.} = \delta \Theta_\Sigma^{ext.} \hat{=} \int_\Sigma \phi^* \left(\delta\theta[g, \delta g, y] \right) + \int_{\partial\Sigma} \phi^* \left(i_{\underline{\chi}_\phi} \theta[g, \delta g, y] + \frac{1}{2} i_{\underline{\chi}_\phi} i_{\underline{\chi}_\phi} L[g, y] \right)$

(Unmodified) Noether Charges are Integrable: $I_{V_{\underline{\eta}}} \Omega_\Sigma^{ext.} \hat{=} - \delta \int_{\partial\Sigma} \phi^* \left(Q_{\underline{\eta}} \right) = -\delta H_{\underline{\eta}}$

Canonical Poisson bracket

$$I_{V_{\underline{\xi}}} I_{V_{\underline{\eta}}} \Omega_{\Sigma}^{\text{ext.}} = -I_{V_{\underline{\xi}}} \delta \int_{\partial\Sigma} \phi^* \left(Q_{\underline{\eta}} \right) = \int_{\partial\Sigma} \phi^* \left(Q_{[\underline{\xi},\underline{\eta}]} \right) = H_{[\underline{\xi},\underline{\eta}]} = -\left\{ H_{\underline{\xi}}, H_{\underline{\eta}} \right\}$$

Finally: $\{\!\![\cdot, \cdot]\!\!\} \leftrightarrow \{\cdot, \cdot\}$

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Finally: $\{\![\cdot,\cdot]\!\} \leftrightarrow \{\cdot,\cdot\}$

Maximal Embedding Algebra: non-vanishing canonical Noether Charges on the enhanced Field Space (thanks to embeddings)

Motivations ✓

Corners ✓

Classical Phase Space ✓

Future Perspectives

Done


Recap
Maximal Embedding Algebra: universal building block
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Classical Phase Space
Future Perspectives
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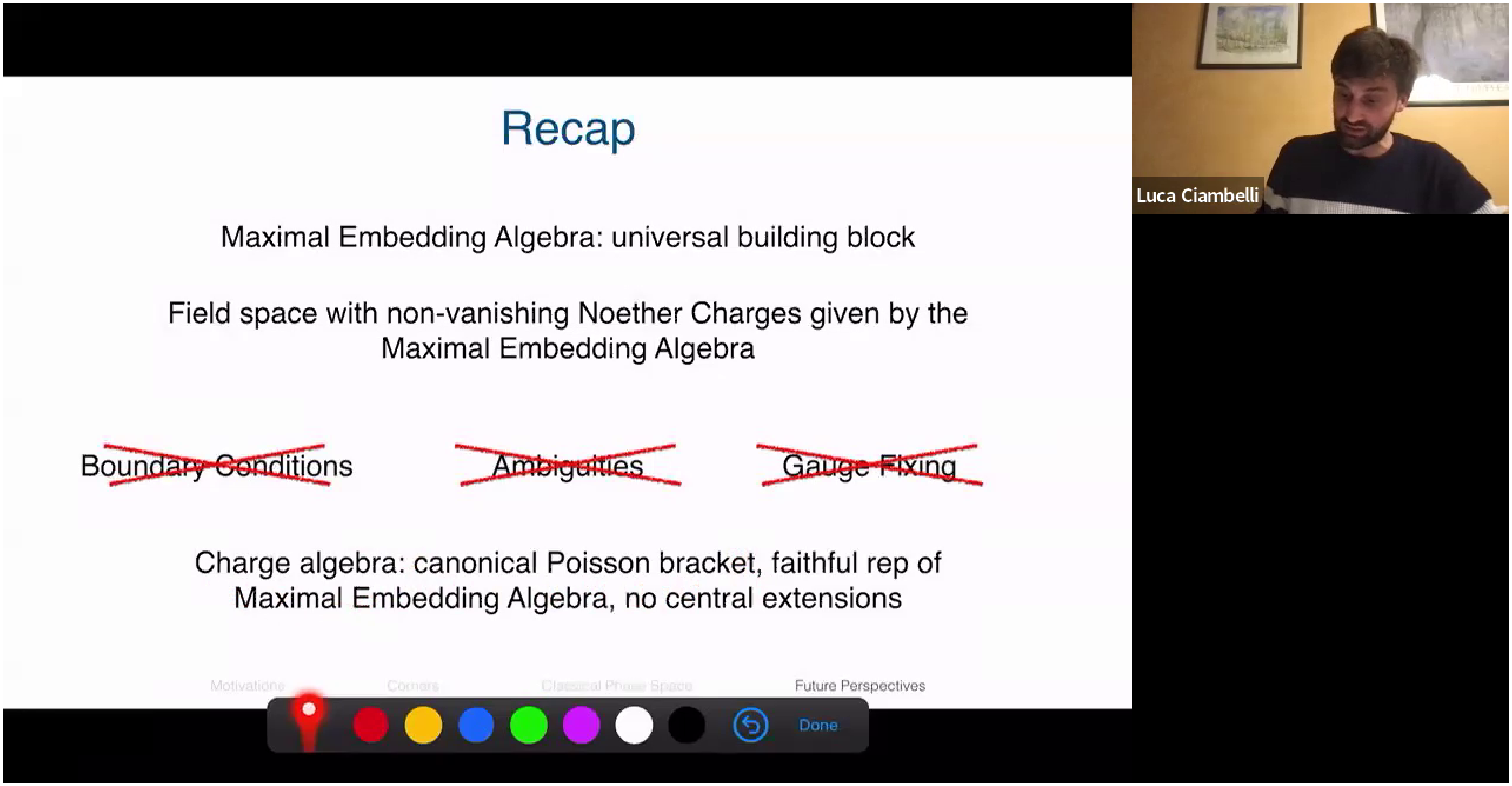
Recap
Maximal Embedding Algebra: universal building block
Field space with non-vanishing Noether Charges given by the Maximal Embedding Algebra
Boundary Conditions
Ambiguities
Gauge Fixing
Charge algebra: canonical Poisson bracket, faithful rep of Maximal Embedding Algebra, no central extensions
Future Perspectives
Done
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In the Near Future

Understand more embeddings
& physical fluxes

Application to Chern-Simons
& gauge theories

Asymptotic boundaries & corners

HPS & Information Paradox
[Hawking '15, Hawking & Perry & Strominger '16]

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Links with $w_{1+\infty}$ algebra?

Done

In the Not-So-Near Future

Understanding abstract representations of Maximal Embedding Algebra
as a probe to quantum gravity, organizing reps. of quantum observables

It is a complicated double semi-direct group, few tools

One is the coadjoint orbit method of Kirillov

[Kirillov, '61 - '04]

Understand universal isotropy groups? Quantum notion of soft and hard modes?
Topology changes and quantum phenomena?

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Geometric Structure: Principal Maximal Embedding Algebra Bundle over Corners

Done

In a Galaxy Far, Far Away

Bulk Reconstruction as a Semiclassical Approximation, Emergent Spacetime

Quantum Gravity Predictions

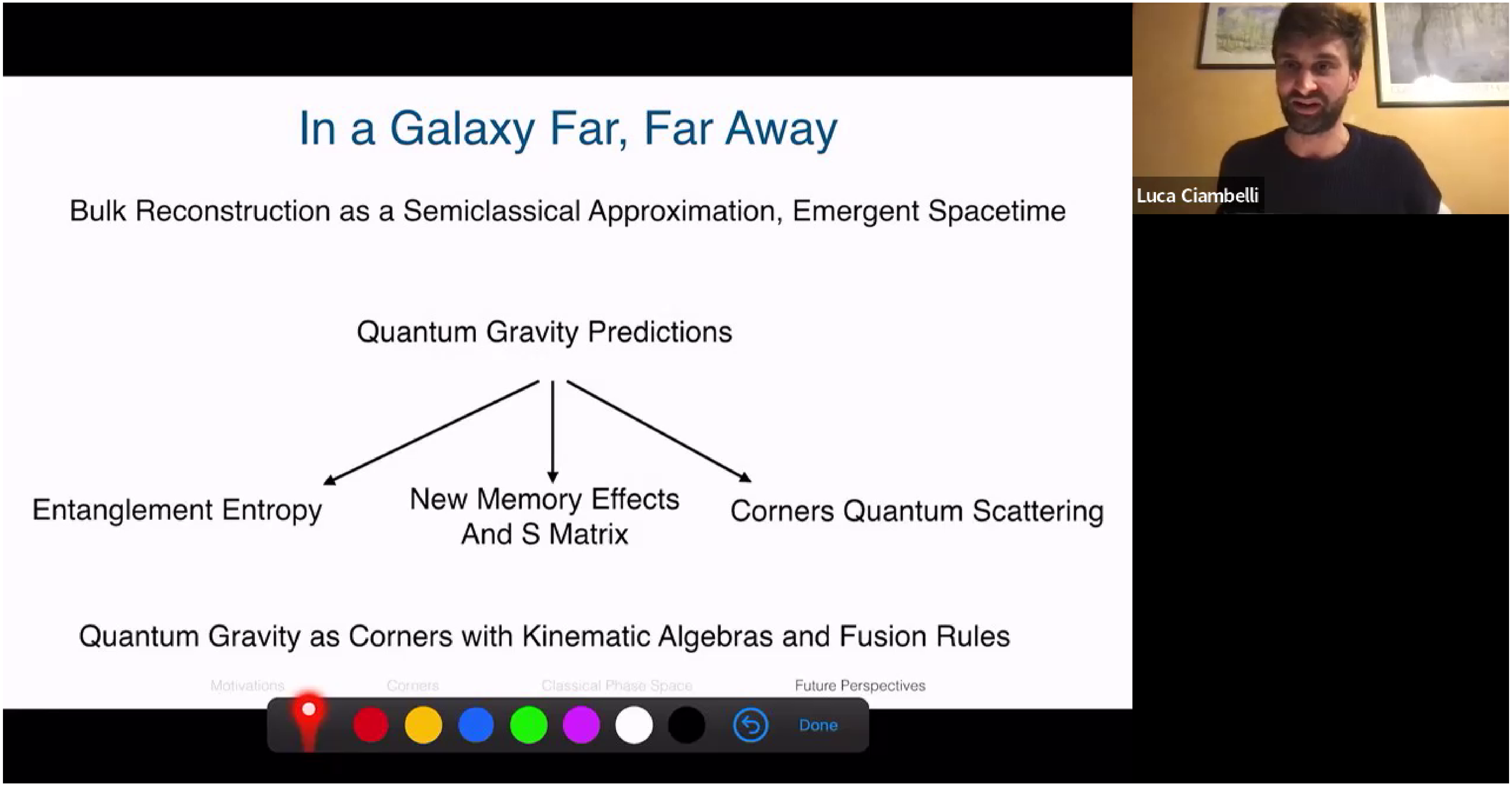
→ Entanglement Entropy

→ New Memory Effects And S Matrix

→ Corners Quantum Scattering

Motivations Corners Classical Phase Space Future Perspectives

Done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In the Near Future

Understand more embeddings
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Future Perspectives

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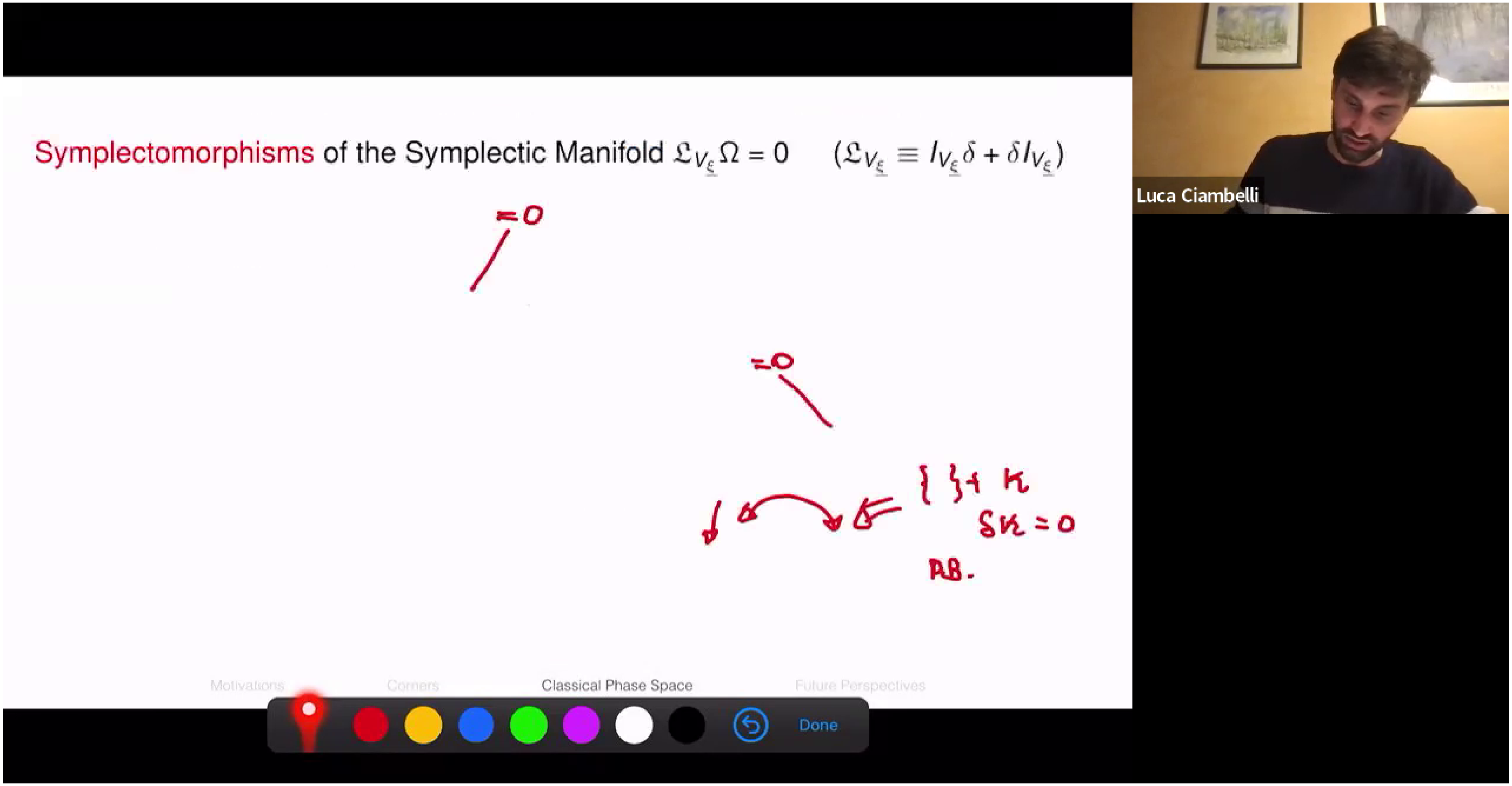
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=0
=0
{ }+ K
δK = 0
AB.
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$$[\xi_1, \xi_2] \Rightarrow (\underbrace{\mathrm{Diff}(S)}_{\xi^i_{(0)}} \ltimes \underbrace{GL(2,\mathbb{R})}_{\xi^a_{(1)b}}) \ltimes \underbrace{\mathbb{R}^2}_{\xi^a_{(0)}}$$

$\xi^a(x)\,\partial_a$

$$g = h_{ab}(u,x)\, n^a \otimes n^b + \gamma_{ij}(u,x)\, dx^i \otimes dx^j$$

$$\delta_{\underline{\xi}} h^{(0)}_{ab}(x) = \xi^j_{(0)} \partial_j h^{(0)}_{ab} + (h^{(0)}_{bc} \xi^c_{(1)a} + h^{(0)}_{ac} \xi^c_{(1)b}) + \xi^c_{(0)} h_{(1)ab,c}$$

Done

Go Classical: Metrics

So far, all metric independent. Introduce a line element adapted to the split of TM

$$g = h_{ab}(u,x)n^a \otimes n^b + \gamma_{ij}(u,x)dx^i \otimes dx^j$$

Metric on the normal fibres

Pulls back to metric on S

Expand h_{ab}, γ_{ij} and $n^a = du^a - a_i^a(u,x)dx^i$ assuming finite distance corner

use ϕ_0 and compute infinitesimal variation under a diffeo $\mathcal{L}_{\underline{\xi}} g = \delta_{\underline{\xi}} g$

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Done

$Q = \circledast\, dg(\xi, \cdot)$ (1, 2, $(d-2)$)

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Done

Go Classical: Dynamics

From now on Corners: $k = 2$, Einstein gravity, charges of $\mathfrak{diff}(M)$

Compute (hard) Noether charges with our metric and trivial embedding ϕ_0

Classical Phase Space in a Nutshell

Idea: Geometrize the Field Space Γ, δ : Exterior Derivative on Γ

We obtain for the action: $\delta S_R \hat{=} \int_B \phi^* \left(\theta[g, \delta g, y] + i_{\underline{\chi}_\phi} L[g, y]\right)$

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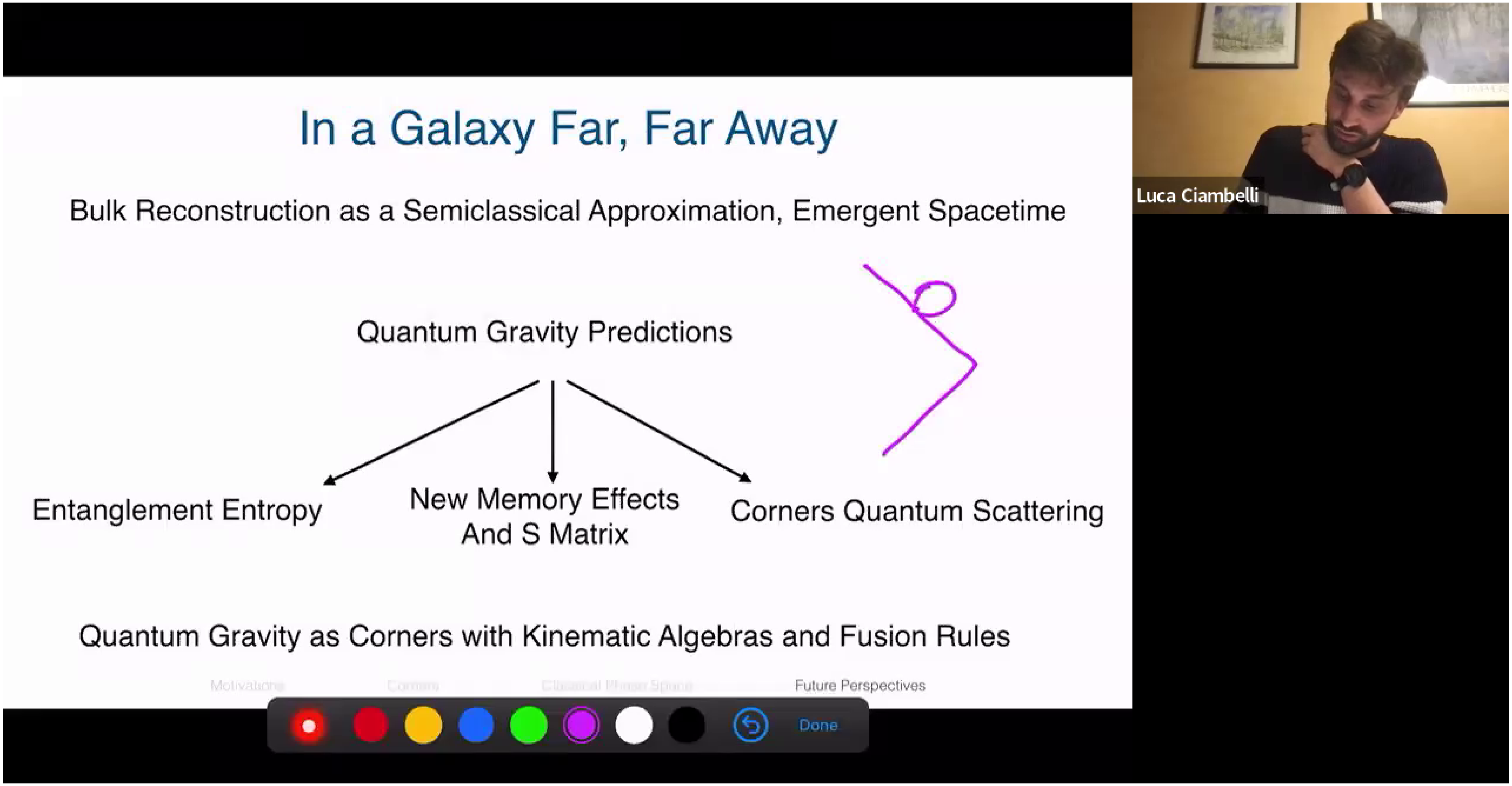
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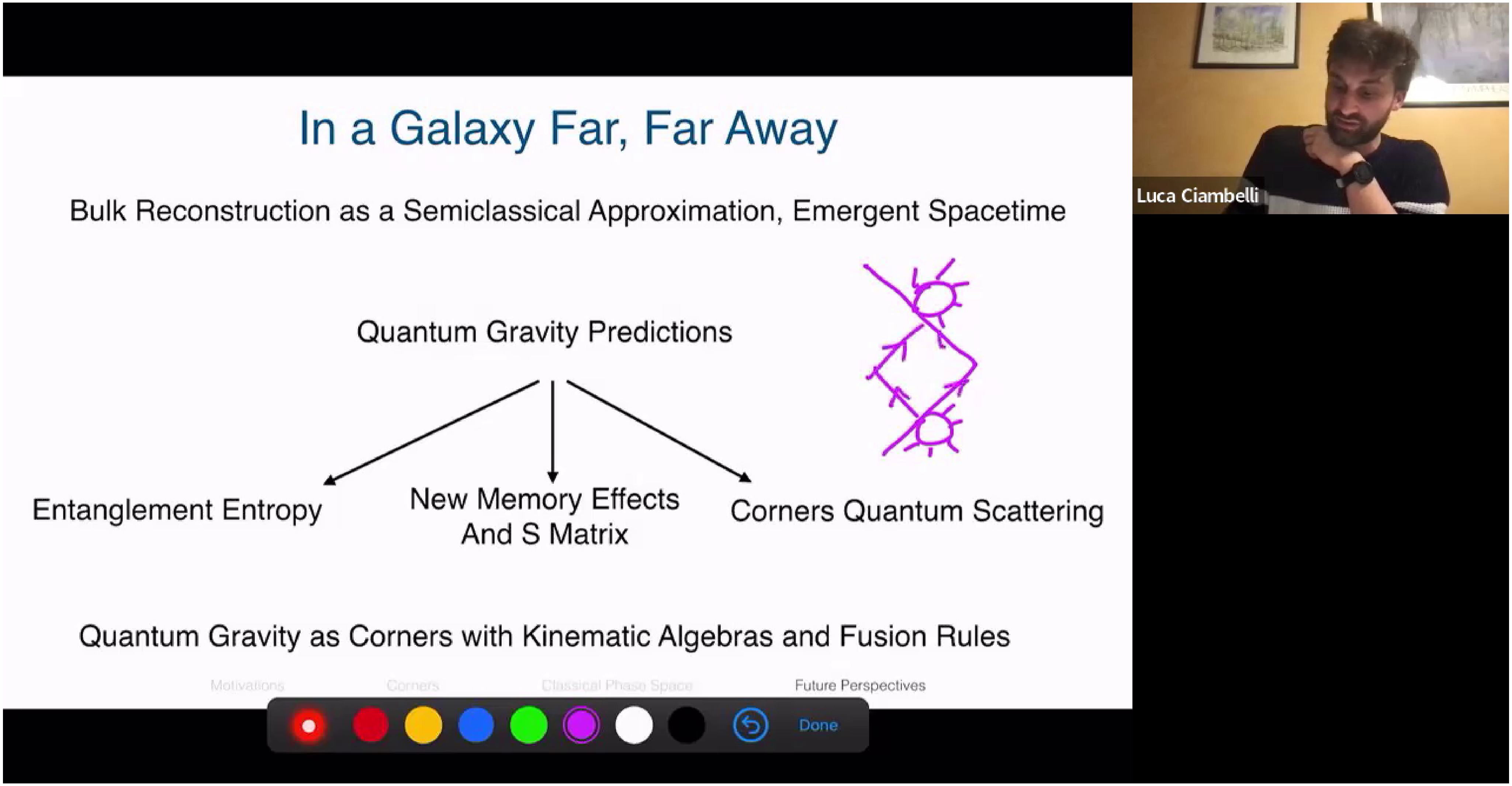
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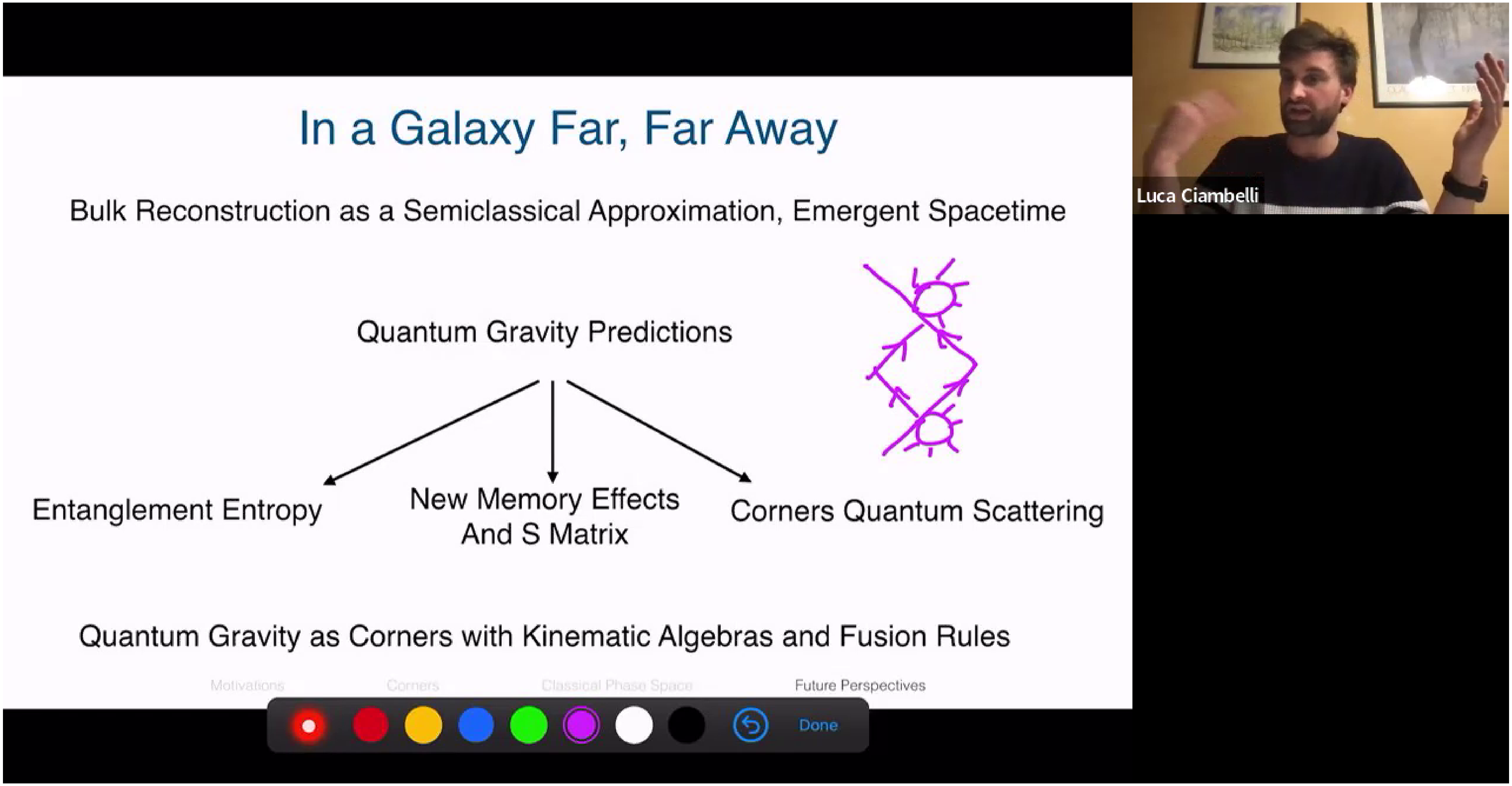
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