Title: Gyroscope memory: detecting gravitational wave memory effects with a gyroscope

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Abstract: I study the dynamics of a gyroscope far from an isolated source of gravitational waves. With respect to a local frame 'tied to the distant stars', the gyroscope precesses when gravitational waves cross its path, resulting in a net `orientation memory', carrying information on the gravitational wave profile. I show that the precession rate is given by the so-called "dual covariant mass aspect", providing a celestially local measurement protocol for this quantity. Moreover, I show that the net memory effect can be derived from the flux-balance equations for superrotation charges in the "generalized BMS" algebra. Finally, I show how the spin memory effect à la Pasterski et al. is reproduced as a special case.

Gyroscopic Gravitational Memory

Detecting gravitational memory effects with gyroscopes

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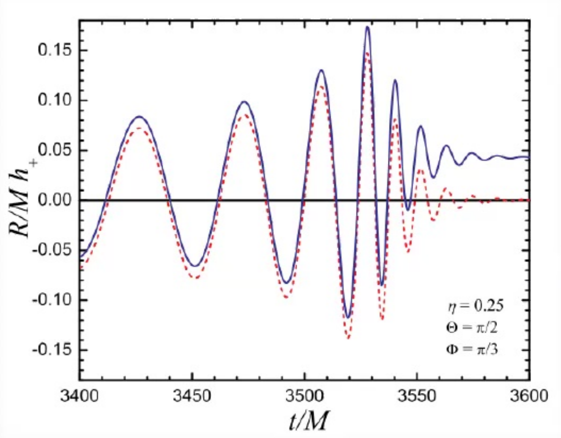
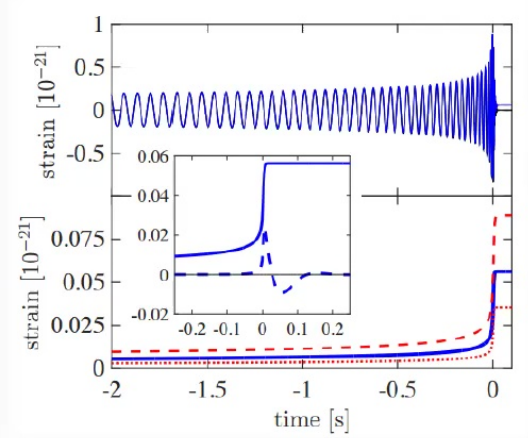
GW memory effect

Gravitational wave (GW) signal is NOT purely oscillatory



[Credit: M. Favata '09]



[Credit: Lasky et al. '16]

Memory is a low frequency effect. Recall $\mathcal{F}$(step)$= \pi\delta(\omega) + \frac{1}{i\omega}$

Poor sensitivity of detectors at low frequency

Prospects for detection: LIGO/Virgo

Not possible to detect in single event

Possible to verify it statistically, i.e. if signal with memory is preferred by data

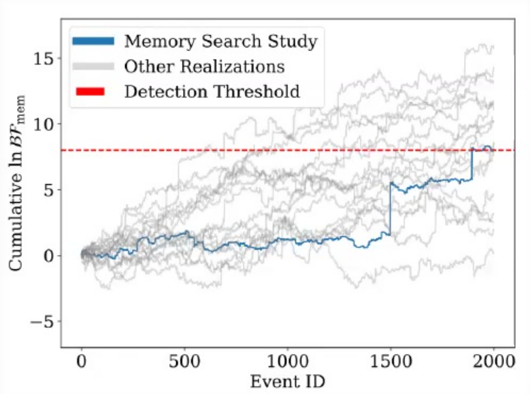
This can be done by Bayesian model selection methods, see [*Thrane et al. '19*]

Order 2000 signals for detection

[Hubner+'19]

5 years of data collection

[Boersma+'20]



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Prospects for detection with LISA

The Laser Interferometer Space Antenna (LISA) has sufficient sensitivity to observe memory in the GW signal from **supermassive black hole mergers**

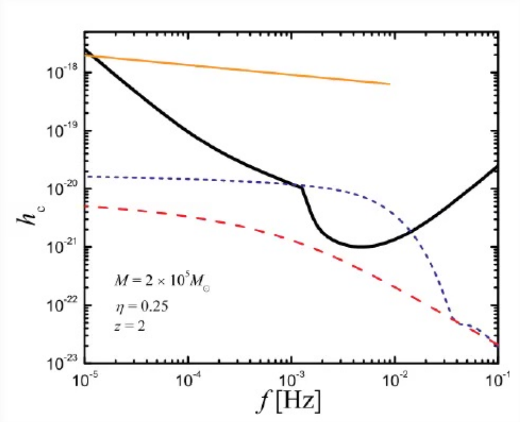
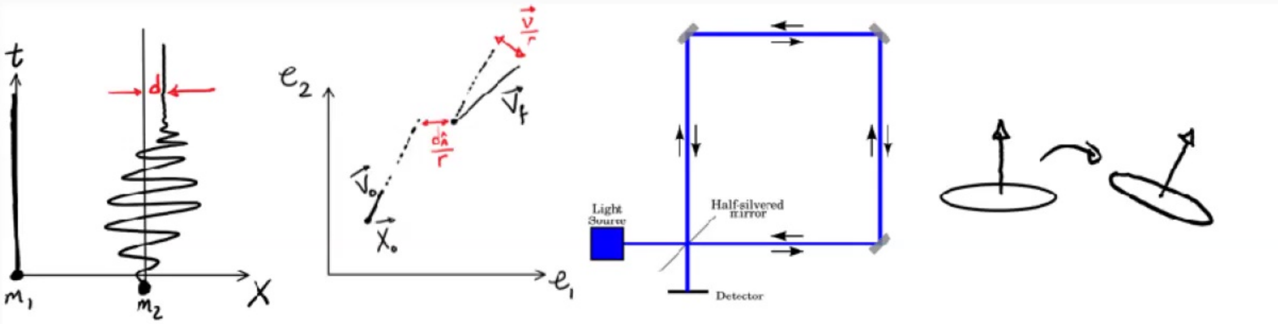


Figure 1: [*Credit: M. Favata '10*] **Black:** Sensitivity curve of LISA, Orange: inspiral signal amplitude, Blue: memory contribution

GW memory Observables

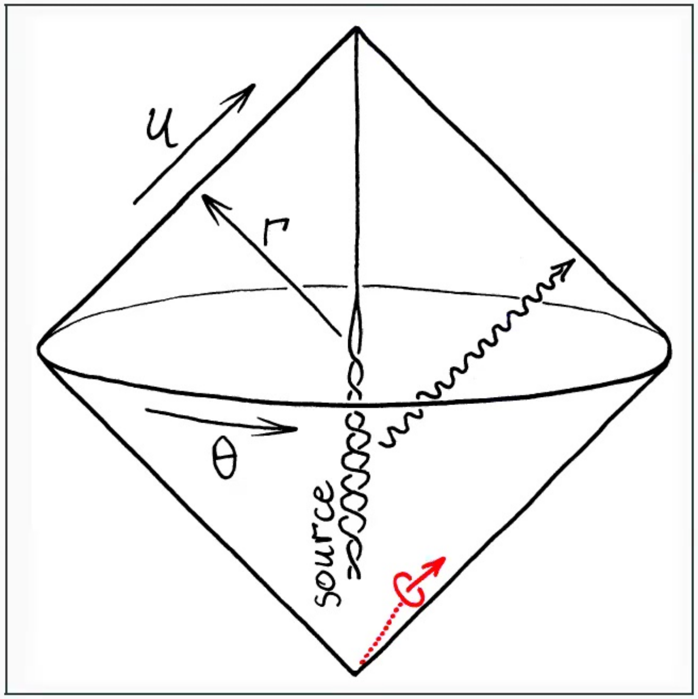
Several observables to detect memory

1. Displacement memory *[Polnarev, Zel'dovich '74]* $\Delta\vec{X} = \vec{X}(t_f) - \vec{X}(t_0)$
2. Kick (velocity) memory *[Flanagan+'19]* $\Delta\vec{V} = \vec{V}(t_f) - \vec{V}(t_0)$
3. Spin memory *[Pasterski+'16]* $\Delta T = T^+ - T^-$
4. Gyroscope memory $\Delta\vec{S} = \vec{S}(t_f) - \vec{S}(t_0)$



1,2: Gravito-electric (mass multipoles) 3,4: Gravito-magnetic (spin multipoles)

Gyroscopic memory in the action



Gyroscope dynamics

Gyroscope with velocity $\boldsymbol{u} = u^\mu \partial_\mu$ and spin $\boldsymbol{S} = S^\mu \partial_\mu$ obeys **Fermi-Walker transport** preserving $\boldsymbol{S} \cdot \boldsymbol{u} = 0$:

$$u^\nu \nabla_\nu S^\mu = (u^\mu a^\nu - u^\nu a^\mu) S_\nu$$

An observer measures spin in a **local Lorentz frame**, i.e. $\boldsymbol{S} = S^{\hat\mu} \boldsymbol{e}_{\hat\mu}$ where

$$\boldsymbol{e}_{\hat\mu} = e_{\hat\mu}{}^\nu \partial_\nu, \qquad \boldsymbol{e}_{\hat\mu} \cdot \boldsymbol{e}_{\hat\nu} = \eta_{\hat\mu\hat\nu}$$

If the observer is *comoving* with the gyroscope, i.e. $\boldsymbol{e}_{\hat 0} = \boldsymbol{u}$, then $\boldsymbol{S} = S^{\hat\imath} \boldsymbol{e}_{\hat\imath}$,

$$\frac{\mathrm{d}S^{\hat\imath}}{\mathrm{d}\tau} = \Omega^{\hat\imath}{}_{\hat\jmath} S^{\hat\jmath}, \qquad \Omega_{\hat\imath\hat\jmath} = -u^\alpha \omega_{\alpha\hat\imath\hat\jmath}$$

where the **precession rates** $\Omega^{\hat\imath}{}_{\hat\jmath}$ is given in terms of the **spin connections**

$$\omega_\mu{}^{\hat\mu\hat\nu} = e^{\hat\mu}{}_\alpha \nabla_\mu e^{\hat\nu\alpha} = e^{\hat\mu}{}_\alpha \left(\partial_\mu e^{\hat\nu\alpha} + \Gamma^\alpha_{\sigma\mu} e^{\hat\nu\sigma} \right)$$

✓ Since $\Omega_{\hat\imath\hat\jmath}$ is antisymmetric, the magnitude of the spin is conserved.

GR in Bondi gauge

Consider a source of GWs localized around the origin in a coordinate system $x^\mu = (u, r, x^a)$ subject to Bondi gauge conditions

$$g_{rr} = g_{ra} = 0, \qquad \partial_r \det\left(r^{-2} g_{ab}\right) = 0$$

The general metric takes the form

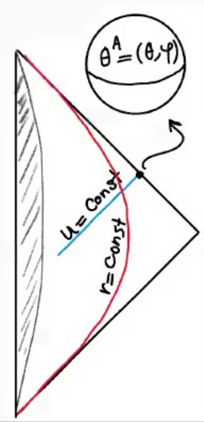
$$ds^2 = -\,e^{2\beta}(F du^2 + 2du dr) + r^2 \gamma_{ab}(dx^a - \frac{U^a}{r^2}du)(dx^b - \frac{U^a}{r^2}du)$$

For any metric in this coordinate system,

u is always lightlike

r is the areal distance

$\boldsymbol{\ell} = e^{-2\beta}\partial_r$ is tangent to affine outgoing null geodesics. It represents null rays emanating from the source.



Gyroscopes and gravitational waves

The gyroscope dynamics given by $\Omega_{\hat{i}\hat{j}} = -u^{\alpha}\omega_{\alpha\hat{i}\hat{j}}$ depends on:

Metric. Asymptotically flat spacetimes in Bondi gauge: a good setup to study GW from localized sources

Worldline. We consider two cases:
1) **Freely falling** gyroscopes,
2) **Accelerated** See paper.

Frame. Huge gauge freedom under **Local Lorentz transformations**. We fix a "**frame tied to distant stars**".

Asymptitic analysis of Einstein equations

Imposing as boundary condition $\lim_{r\to\infty}\gamma_{ab} = q_{ab}(x^a)$, the round metric on S^2,

$$\gamma_{ab} = q_{ab} + \frac{1}{r}C_{ab} + \frac{1}{4r^2}q_{ab}C^2 + O(r^{-3}) \qquad [C^2 \equiv C_{ab}C^{ab}]$$

$$\beta = -\frac{1}{32r^2}C^2 + O(r^{-3}),$$

$$F = 1 - \frac{2m}{r} + \frac{1}{r^2}\left(\frac{1}{16}C^2 + \frac{1}{3}D_aL^a + \frac{1}{4}(D_bC^{ab})^2\right) + O(r^{-3}),$$

$$U^a = -\frac{1}{2}D_bC^{ab} + \frac{1}{r}\left[-\frac{2}{3}L^a + \frac{1}{16}D^aC^2 + \frac{1}{2}C^{ab}D^cC_{bc}\right] + O(r^{-2}).$$

C_{ab}: Bondi shear, contains $h_+, h_\times$
m mass aspect, L_a angular mom. aspect, $N_{ab} = \dot{C}_{ab}$ news.

Balance equations

$$\dot{m} = +\frac{1}{4}D_aD_bN^{ab} - \mathcal{P}, , \qquad \dot{L}_a = \frac{1}{2}D^bD_{[a}D^cC_{b]c} - \mathcal{J}_a.$$

where $N_{ab} \equiv \dot{C}_{ab}$ is Bondi news and the fluxes are

$$\mathcal{P} = \frac{1}{8}N_{ab}N^{ab}, \qquad -\mathcal{J}_a = D_am + \frac{1}{4}D_b\left(N^{bc}C_{ca}\right) + \frac{1}{2}D_bN^{bc}C_{ca}$$

Gyroscope worldline

Take the velocity of the gyroscope to be $\boldsymbol{u} = \gamma(\partial_u + v^r\partial_r + v^a\partial_a)$.

Solving the geodesic equation $\boldsymbol{u}\cdot\nabla\boldsymbol{u} = 0$ asymptotically implies

Free fall gyroscope (initially at rest)

$$\gamma = 1 + \frac{m_0}{r} + \frac{\gamma_2}{r^2}\,, \qquad \gamma_2 = \int_{u_0}^{u} du'\,(m + \frac{1}{16}C^2)$$

$$v^r = \frac{m - m_0}{r} - \frac{1}{r^2}\left(\gamma_2 + \frac{1}{6}D_aL^a + \frac{1}{8}(D_bC^{ab})^2\right)$$

$$v^a = -\frac{1}{2r^2}D_bC^{ab} + \frac{1}{r^3}\left(D^a\gamma_2 - \frac{2}{3}L^a + \frac{1}{2}C^{ab}D^cC_{bc}\right)$$

Remarks.

- ✓ Static metric: $m = m_0, L^a = 0 = C_{ab} \to$ Newtonian force
- ✓ Leading angular velocity: $\mathring{v}^a = \mathring{U}^a = -\frac{1}{2}D_bC^{ab}$, dragging by GWs

Source oriented frame

Local orthonormal frame consisting of four basis vectors $\boldsymbol{e}_{\hat{\mu}} = \{\boldsymbol{e}_{\hat{0}}, \boldsymbol{e}_{\hat{r}}, \boldsymbol{e}_{\hat{a}}\}$

For an observer with $\boldsymbol{u} = \gamma(\partial_u + v^r \partial_r + v^a \partial_a)$, we choose

$$\boldsymbol{e}_{\hat{0}} = \boldsymbol{u} \quad \text{(Adapted to observer)}, \qquad \boldsymbol{e}_{\hat{r}} = \frac{1}{\gamma}\boldsymbol{\ell} - \boldsymbol{u}, \quad \text{(Source oriented)}$$

The local frame is completed by two vectors $\boldsymbol{e}_{\hat{a}}$

$$\boldsymbol{e}_{\hat{a}} = \frac{\zeta_{\hat{a}}{}^{a}}{r}\left[\partial_a + \gamma_{ab}\left(r^2 v^b - U^b\right)\boldsymbol{\ell}\right] ,$$

where $\zeta_{\hat{a}}{}^{a}(u, r, x^a)$ forms a dyad on the sphere

$$\gamma_{ab}\, \zeta_{\hat{a}}{}^{a}\, \zeta_{\hat{b}}{}^{b} = \delta_{\hat{a}\hat{b}} \, .$$

Asymptotically

$$\zeta_{\hat{a}}{}^{a} = \bar{\zeta}_{\hat{a}}{}^{b}\left(\delta_b{}^{a} - \frac{1}{2r}C_b{}^{a} + \frac{1}{16r^2}C^2\delta_b{}^{a}\right) + \mathcal{O}(1/r^3)$$

Source oriented frame: not good enough

Bad side: In Minkowski, moving gyroscope precesses in the source-oriented frame

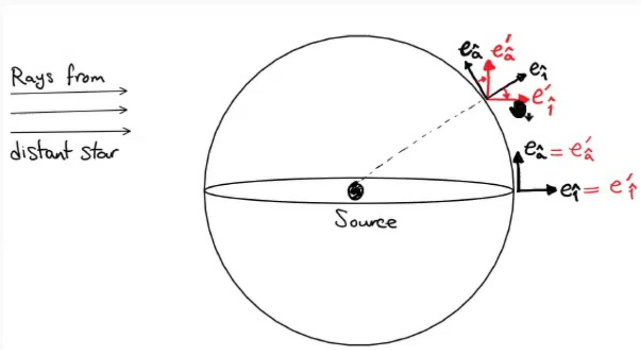
$$\overline{\omega}_{\hat{a}\hat{r}}(x^a) = \overline{\zeta}_{\hat{a}a} dx^a \,, \qquad \overline{\omega}_{\hat{a}\hat{b}}(x^a) = \overline{\zeta}_{\hat{a}}{}^{b} D_b \overline{\zeta}_{\hat{b}b} dx^a$$

Leads to a spurious precession because the frame has to adapt itself to the source.

Good side: $\overline{\omega}$ is entirely given by the asymptotic fixed structure $\overline{\zeta}_{\hat{a}}{}^{a}$

It can be cancelled by an angle dependent rotation, once for all.

In physical terms, this defines a frame **tied to the distant stars**



Frame tied to distant stars

Frame tied to distant stars $\{e'_{\hat{\mu}}\}$, is obtained by a local rotation

$$\boldsymbol{e}'_{\hat{0}} \equiv \boldsymbol{e}_{\hat{0}} = \boldsymbol{u}, \qquad \boldsymbol{e}'_{\hat{i}} \equiv R_{\hat{i}}{}^{\hat{j}}(x^a)\boldsymbol{e}_{\hat{j}},$$

under which $\omega'_{\hat{i}\hat{j}} = R_{\hat{i}}{}^{\hat{m}} R_{\hat{j}}{}^{\hat{n}} \omega_{\hat{m}\hat{n}} + R_{\hat{i}}{}^{\hat{m}} \mathrm{d} R_{\hat{j}\hat{m}}$

$R_{\hat{i}}{}^{\hat{j}}$ is found by solving $\overline{\omega}'_{\hat{i}\hat{j}} = 0$ to remove the spurious effect

$$R(x^a) = \mathcal{P} \exp \int_{x_0}^{x} \overline{\omega}\,, \qquad \omega'_{\hat{i}\hat{j}} = R_{\hat{i}}{}^{\hat{m}} R_{\hat{j}}{}^{\hat{n}} \left(\omega_{\hat{m}\hat{n}} - \overline{\omega}_{\hat{m}\hat{n}}\right)$$

Since angular displacement is small $v^a \sim \mathcal{O}(1/r^2)$, then

$$\Omega_{\hat{i}\hat{j}} = -u^{\mu} \left(\omega_{\mu\hat{i}\hat{j}} - \overline{\omega}_{\mu\hat{i}\hat{j}}\right) + \text{sub.}$$

Spin precession

Precession of the free fall gyroscope in the frame tied to distant stars is given up to $\mathcal{O}(1/r^3)$ corrections by

$$\Omega_{\hat{a}\hat{b}} = \epsilon_{\hat{a}\hat{b}}\Omega^{\mathsf{GW}}, \qquad \Omega_{\hat{r}\hat{a}} = 0$$

where

$$\Omega^{\mathsf{GW}} = \frac{1}{r^2}\epsilon^{ab}\left(\frac{1}{4}D_a D^c C_{bc} - \frac{1}{8}N_{ca}C^c{}_b\right)$$

Remarks.

- ✓ *Transverse effect*
- ✓ *Equivalence principle.* The precession is dictated by the gravitational field, irrespective of gyroscope properties (moments of inertia, spin)

Dual covariant mass aspect: measurement protocol

Defining the dual of a given tensor X_{ab} as $\widetilde{X}_{ab} \equiv \epsilon_{c(a}X_{b)}{}^{c}$ [Godazgar [2], Pope '18]

$$\Omega^{\mathrm{GW}} = \frac{\widetilde{\mathcal{M}}}{r^2} \qquad \widetilde{\mathcal{M}} \equiv \frac{1}{4}D_a D_b \widetilde{C}^{ab} - \frac{1}{8}N_{ab}\widetilde{C}^{ab}$$

$\widetilde{\mathcal{M}}$ is called dual covariant mass aspect [Freidel, Pranzetti '21] with nice properties:

- ✓ It transforms covariantly under superrotations with weight 3/2
- ✓ EM dual of covariant mass aspect $\mathcal{M} = m + \frac{1}{8}N_{ab}C^{ab}$ [Compere+ '18, Donnay+ '21]
- ✓ In the NP formalism, $\mathring{\psi}_2 = \mathcal{M} - i\widetilde{\mathcal{M}}$
- ✓ $\widetilde{Q}_\epsilon = \int_S \epsilon\widetilde{\mathcal{M}}$ defines a dual supertranslation charge

> Free fall gyroscope precession provides a measurement protocol for $\widetilde{\mathcal{M}}$

Gyroscope memory and Symmetries

GW induced precession

Parity decomposition

$$C_{ab} = D_{\langle a} D_{b\rangle} C^+ + \epsilon_{c(a} D_{b)} D^c C^-, \qquad L_a = D_a L^+ + \epsilon_{ab} D^b L^-$$

These can be inverted, e.g.

$$L^+ = D^{-2} D^a L_a, \qquad L^- = D^{-2} \epsilon^{ab} D^a L_b$$

The angular momentum balance equation implies

$$\mathcal{D} C^- = \dot{L}^- + \mathcal{J}^-, \qquad \mathcal{D} = \frac{1}{8} D^2 (D^2 + 2)$$

The precession is given by

$$\begin{aligned} \widetilde{\mathcal{M}} &= \mathcal{D} C^- - \frac{1}{8} N_{ab} \widetilde{C}^{ab} \\ &= \dot{L}^- + \mathcal{J}^- + \widetilde{\mathcal{J}}, \qquad \widetilde{\mathcal{J}} = -\frac{1}{8} N_{ab} \widetilde{C}^{ab} \end{aligned}$$

Remark. The precession in zero for non-radiative spacetimes.

Gyroscope memory

For free fall gyroscope

$$\dot{S}^{\hat{r}} = 0\,, \qquad \dot{S}^{\hat{a}} = \Omega^{\hat{a}\hat{b}} S_{\hat{b}}$$

Integrating over time, we find an *integro-differential equation*

$$S^{\hat{a}}(u) = S_0^{\hat{a}} + \int_{u_0}^{u} du'\, \Omega^{\hat{a}\hat{b}}(u')\, S_{\hat{b}}(u')$$

which can be solved by iteration.

At leading order, the net gyroscope memory is

$$\Delta S^{\hat{a}} = \frac{1}{r^2} \epsilon^{\hat{a}\hat{b}} S_{\hat{b}}(u_0) \int_{u_0}^{u_f} \widetilde{\mathcal{M}}$$

where the permanent effect of GW is given by

$$\int du \widetilde{\mathcal{M}} = \Delta L^{-} + \int du (\mathcal{J}^{-} + \widetilde{\mathcal{J}})$$

Relation to generalized BMS symmetries

Let us massage the gyroscope memory

$$\int du\Omega^{\mathrm{GW}}(x^a) = \oint_{S^2} \epsilon^{ab} D_b G(x,x') \left[\Delta L_a + \int du \mathcal{J}_a\right]_{x'} + \int du \widetilde{\mathcal{J}}$$

Therefore we find

$$\int du \widetilde{\mathcal{M}}(x) = 8\pi \left(\Delta Q_Y + \mathcal{F}_Y + \widetilde{\mathcal{F}}\right)$$

$$Y^a_x(x') = \epsilon^{ab} D_b G(x,x'), \qquad \text{superrotation parameter}$$

$$Q_Y = \oint_{S^2} \sqrt{q}\, Y^a L_a\,, \qquad \text{superrotation charge}$$

$$\mathcal{F}_Y = \int du \oint_{S^2} Y^a \mathcal{J}_a\,, \qquad \text{superrotation flux}$$

This establishes the link between gyroscope memory and superrotations.

Subleading flux and duality symmetry

What is the origin of the last flux term in the memory?

$$\widetilde{\mathcal{F}} = -\frac{1}{64\pi}\int du N_{ab}\widetilde{C}^{ab}$$

Claim: $\widetilde{\mathcal{F}}$ is the generator of duality symmetry

$$C_{ab} \to C'_{ab} = \cos\epsilon\, C_{ab} - \sin\epsilon\, \widetilde{C}_{ab},$$

where $\epsilon = \epsilon(x^a)$ is an arbitrary function on the sphere.

Proof: Start from the Asktekar-Streubel symplectic structure

$$\Omega = \frac{1}{8}\int du \oint_S \mathsf{d}_V N^{ab} \wedge \mathsf{d}_V C_{ab}$$

Contracting with the duality symmetry, $\delta C_{ab} = \epsilon\, \widetilde{C}_{ab}$, we get

$$-i_X\Omega = \mathsf{d}_V H_\epsilon\,, \qquad H_\epsilon = -\frac{1}{8}\int du \oint_S \epsilon\, N_{ab}\widetilde{C}^{ab}$$

Gyroscope memory: summary

The freely falling gyroscope memory is given by

$$\Delta S^{\hat{a}} = \frac{1}{r^2} \epsilon^{\hat{a}}{}_{\hat{b}} S_0^{\hat{b}} \int_{u_0}^{u_f} \widetilde{\mathcal{M}}$$

where

$$\boxed{\int du \widetilde{\mathcal{M}}(x) = 8\pi \big(\Delta Q_Y + \mathcal{F}_Y + H_\epsilon\big)}$$

in which

$$Q_Y = \oint_{S^2} \sqrt{q}\, Y^a L_a \,, \qquad \text{superrotation charge}$$

$$\mathcal{F}_Y = \int du \oint_{S^2} Y^a \mathcal{J}_a \,, \qquad \text{superrotation flux}$$

$$H_\epsilon = -\frac{1}{8} \int du \oint_S \epsilon\, N_{ab} \widetilde{C}^{ab} \,, \qquad \text{duality generator}$$

with superrotation and duality parameters are given by

$$Y_x^a(x') = \epsilon^{ab} D_b G(x, x'), \qquad \epsilon = \frac{1}{8\pi\sqrt{q}} \delta^2(x - x')$$

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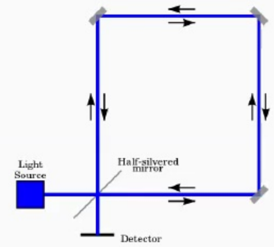
Gyroscope memory vs. spin memory [Pasterski+'15]

The Sagnac effect is given by

$$\Delta T = \frac{4}{c^2}\int_{\mathrm{Int}(\mathcal{C})}\Omega$$



where $\Omega = \Omega_{\hat{a}\hat{b}}dx^{\hat{a}} \wedge dx^{\hat{b}}$ is the precession two-form

Consider a Sagnac interferometer facing the source.

$$\Delta T = \frac{4}{c^2}\int_{\mathrm{Int}(\mathcal{C})}\left(\frac{1}{2}D_{[a}D^c C_{b]c} - \frac{1}{4}N_{c[a}C^c{}_{b]}\right)dx^a \wedge dx^b$$

If the size L of the Sagnac interferometer is small, the second term is smaller by a factor L/λ_{GW}. Therefore

$$\Omega^{GW} = -\frac{1}{2}d\mathring{U}\,, \qquad \mathring{U} = -\frac{1}{2}D^b C_{ab}dx^a$$

Therefore, we reproduce the spin memory effect of [Pasterski et al. '15]

$$\Delta T = \frac{4}{c^2}\int_{\mathrm{Int}(\mathcal{C})}\Omega^{GW} = \frac{1}{c^2}\oint_{\mathcal{C}} dx^a D^b C_{ab}$$

Conclusion

Summary

Gyroscope precession measures the dual covariant mass aspect

Gyroscope memory is related to area preserving Diff(S^2) symmetries

Plus additional flux generating electric-magnetic duality

Spin memory reproduced as a special case

Outlook

EM duality in GR and its asymptotic dynamics (with M. Henneaux)

Memory effects from Wilson loops (with T. Neogi)

Memory effects vs. Berry phases (with B. Oblak, M. Petropoulos)

Fluid dynamical effective description of the celestial holography?

Thank you for your attention!