Title: Quantum many-body topology of crystals and quasicrystals

Speakers: Dominic Else

Series: Colloquium

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Abstract: When an interacting quantum many-body system is cooled down to its ground state, there can be discrete "topological invariants" that characterize the properties of such ground states. This leads to the concept of "topological phases of matter" distinguished by these topological invariants. Experimental manifestations of these topological phases of matter include the integer and fractional quantum Hall effect, as well as topological insulators.

In this talk, after a general overview of topological phases of matter, I will explain how to define topological invariants that are specific to the ground states of regular crystals, i.e. systems that are periodic in space. I will discuss the physical manifestations of the resulting "crystalline topological phases", including implications for the properties of crystalline defects such as dislocations and disclinations. Then, I will explain how these ideas can be generalized to quasicrystals, which are a different class of materials that have long-range spatial order without exact periodicity. These ideas ultimately lead to a general classification principle for crystalline and quasicrystalline topological phases of matter.

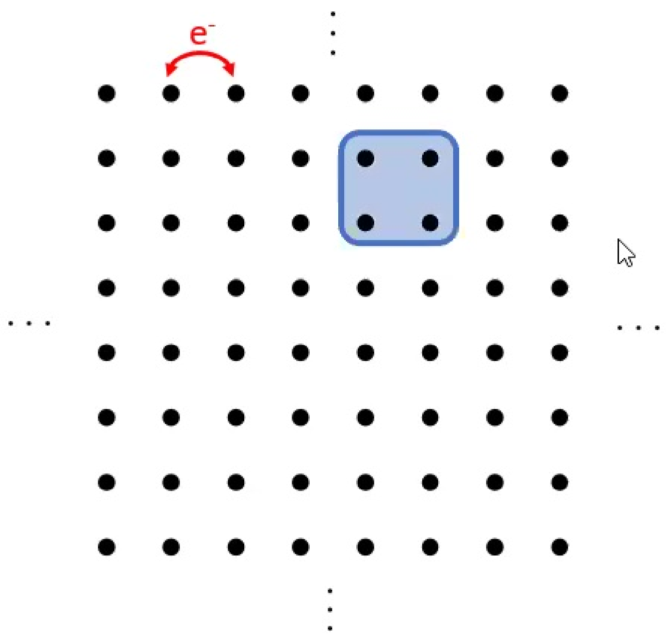
Quantum many-body topology of crystals and quasicrystals

Dominic Else (Harvard → Perimeter)

Colloquium, Perimeter Institute

December 8, 2021

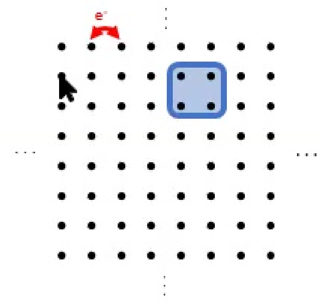
Low-temperature quantum many-body physics



Local Hamiltonian $H = \sum_X h_X$

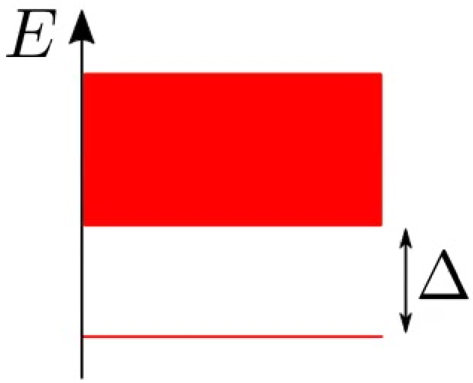
Effective field theory

UV Microscopic lattice Hamiltonian



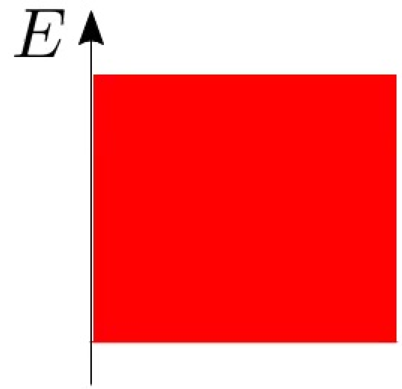
IR Low-energy, low temperature, long-wavelength physics

can be described by an "effective field theory"



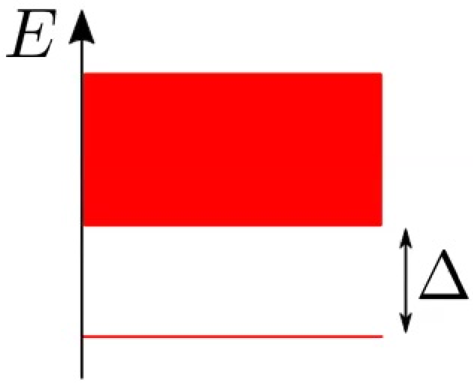
Gapped

vs.



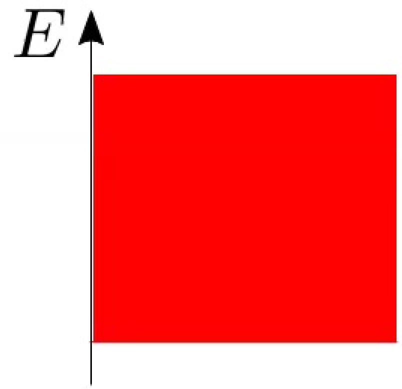
Gapless

Scale-invariant quantum field theory
(e.g. conformal field theory)



Gapped

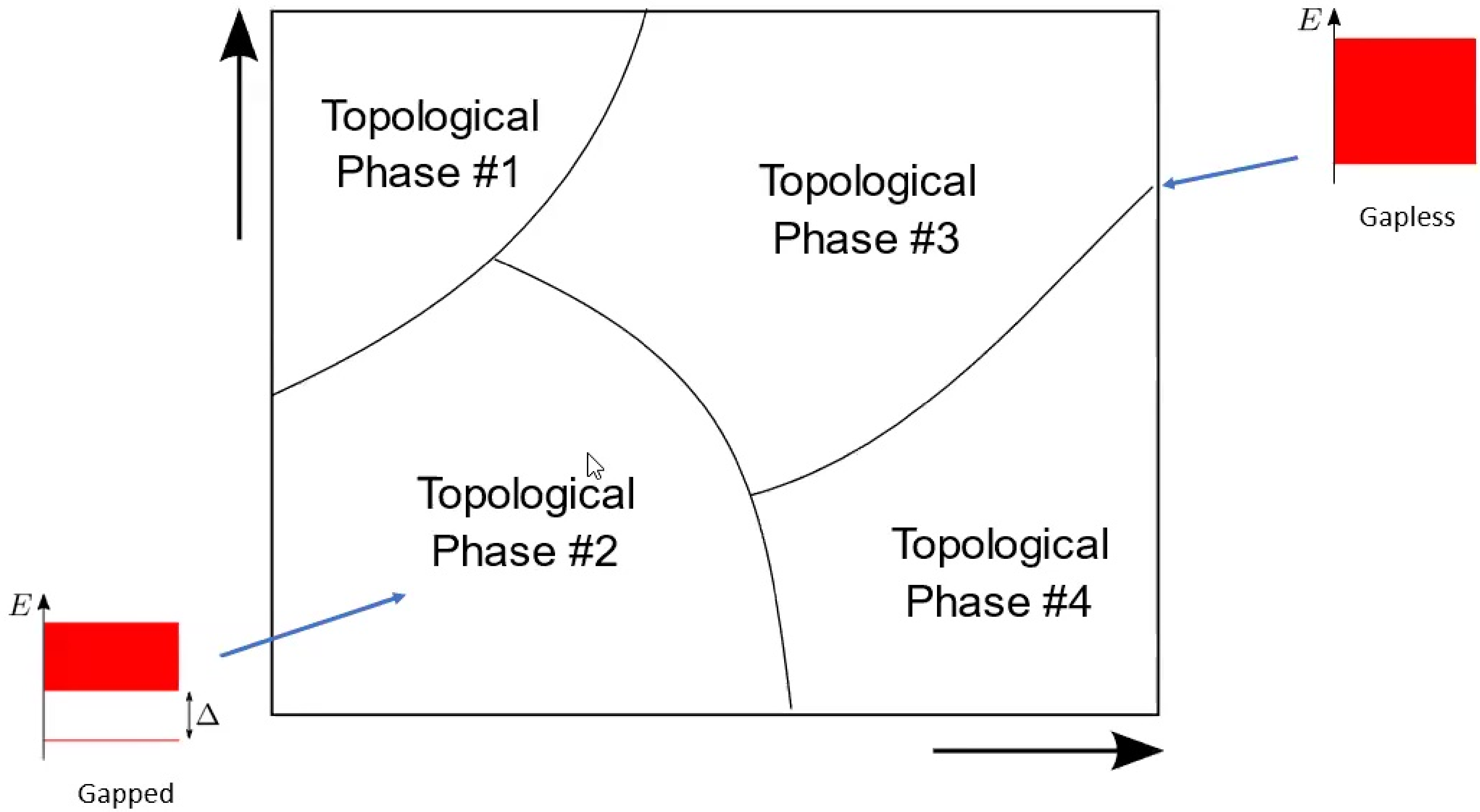
vs.



Gapless

Topological quantum field theory (TQFT)

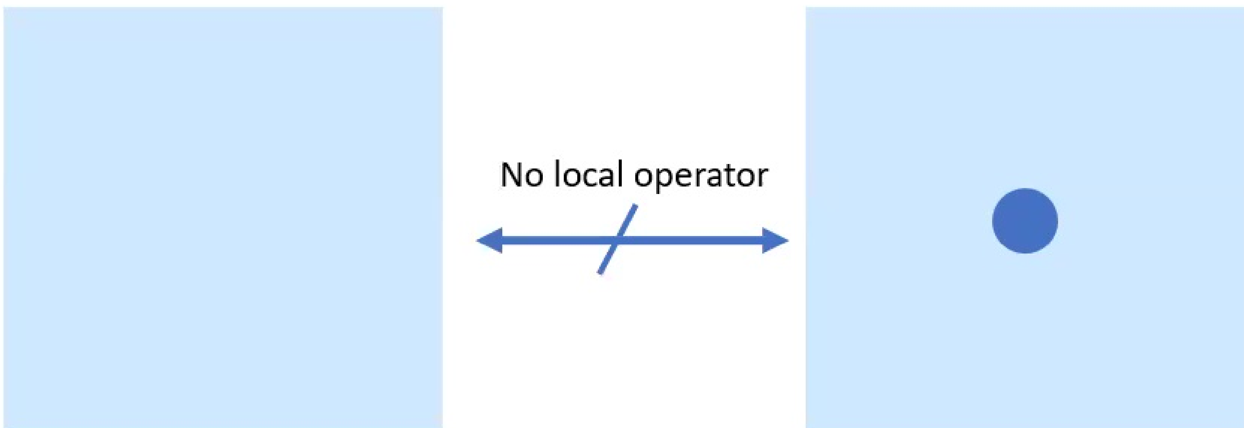
Scale-invariant quantum field theory (e.g. conformal field theory)


Topological
Phase #1
Topological
Phase #3
Topological
Phase #2
Topological
Phase #4
E
Δ
Gapped
E
Gapless

"Fractionalized" topological phase of matter

[aka "Non-invertible" or "Long-range entangled" topological phase]

Fractionalized topological phases have non-trivial *fractionalized excitations*

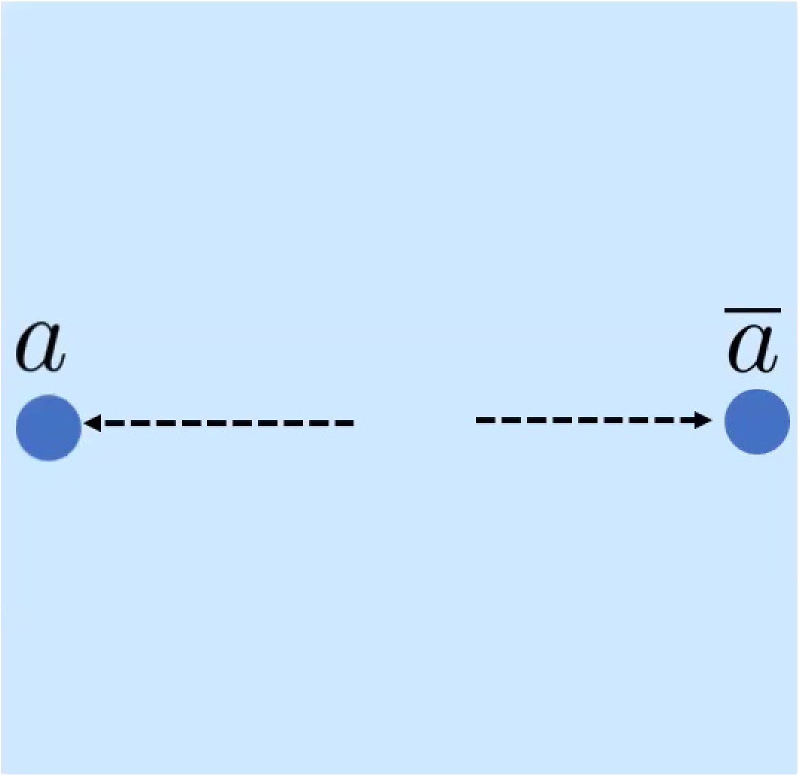


Ground state

Localized excitation on top of the ground state

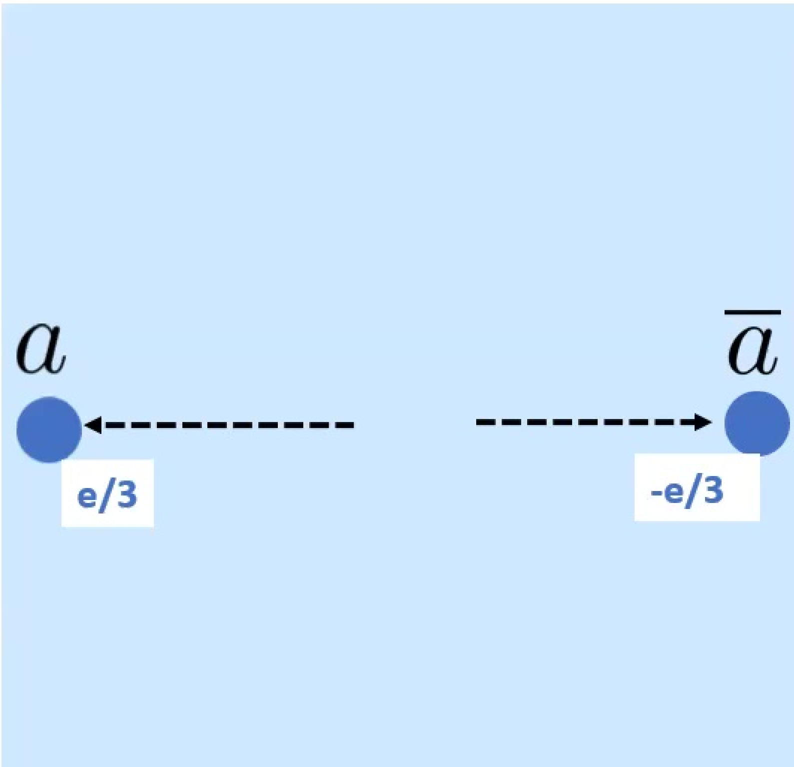
Fractionalized excitation: a *localized excitation* that *cannot be created locally*

Fractionalized excitations



- Can carry fractional charges under global symmetries

Fractionalized excitations

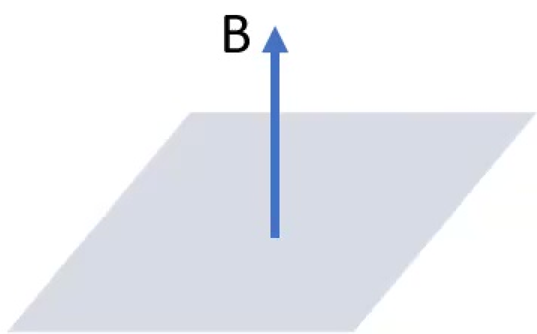


- Can carry fractional charges under global symmetries
- Can have non-trivial exchange/bradiding statistics ("anyons")
- Experimentally realized, e.g. in "fractional quantum Hall effect"

"Unfractionalized" topological phases

[aka "Invertible" or "Short-range entangled" topological phase
Also has a large overlap with "symmetry-protected topological (SPT)" phases]

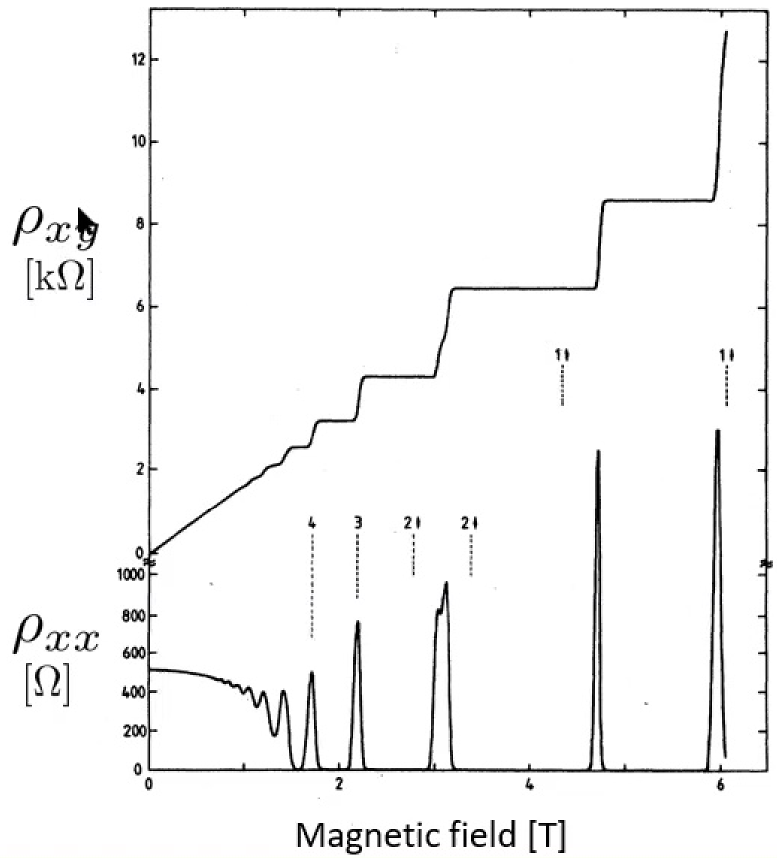
<u>Example: Integer quantum Hall effect</u>



$$\mathbf{E} = \rho \mathbf{J}$$

$$\rho = \begin{bmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{xx} \end{bmatrix}$$

[von Klitzing, Rev. Mod. Phys. 1987]



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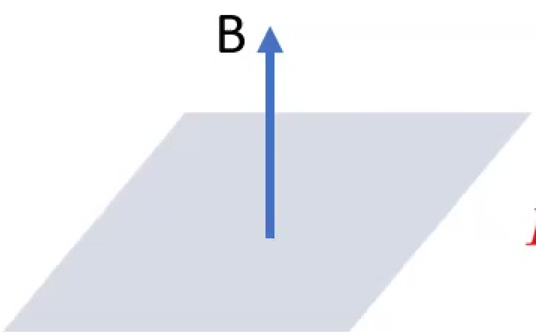
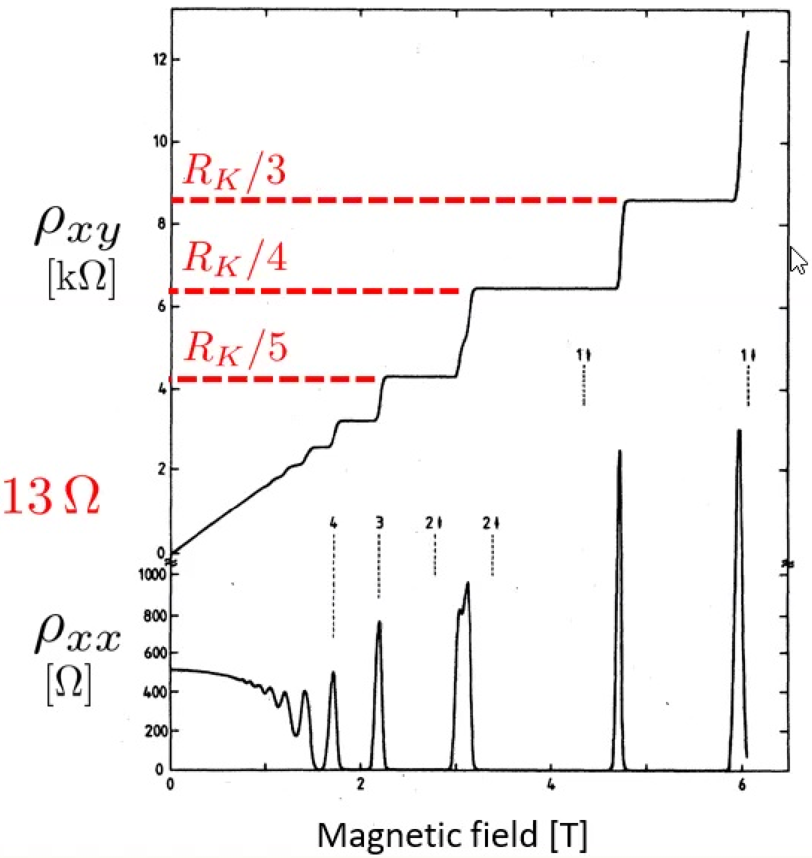


$$R_K = \frac{e^2}{2\pi\hbar} \approx 25813\,\Omega$$



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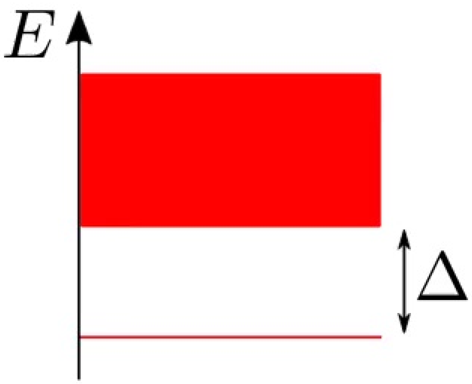
Hall conductance as a topological response

[Work in units in which $e = \hbar = 1$]



Gapped

$$\mathbf{J} = \sigma \mathbf{E}$$

Conductivity tensor

In the integer quantum hall effect:

$$\sigma = \begin{bmatrix} 0 & m/(2\pi) \\ -m/(2\pi) & 0 \end{bmatrix}$$

$$m \in \mathbb{Z}$$

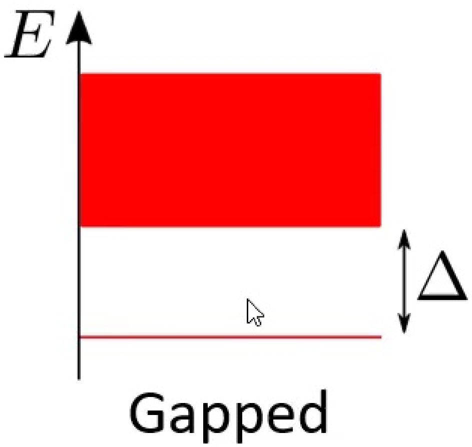
$$\sigma^{ij} = \frac{m}{2\pi}\epsilon^{ij}$$

Charge density

$$J^i = \frac{m}{2\pi}\epsilon^{ij}E_j \qquad \rho = \frac{m}{2\pi}B$$

$$\Rightarrow \quad J^\mu = \frac{m}{4\pi}\epsilon^{\mu\nu\lambda}F_{\nu\lambda}$$

Hall conductance as a topological response



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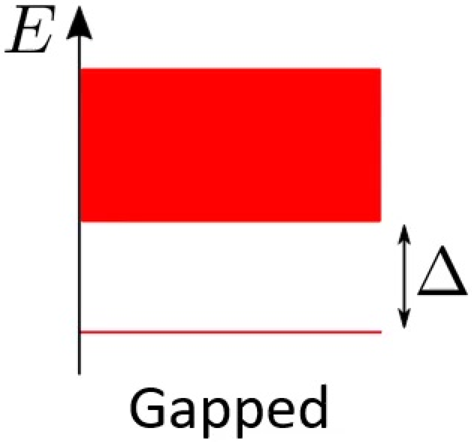
- Topological response – the above equation *does not depend on the metric* and is invariant under arbitrary diffeomorphisms of space-time

Hall conductance as a topological response



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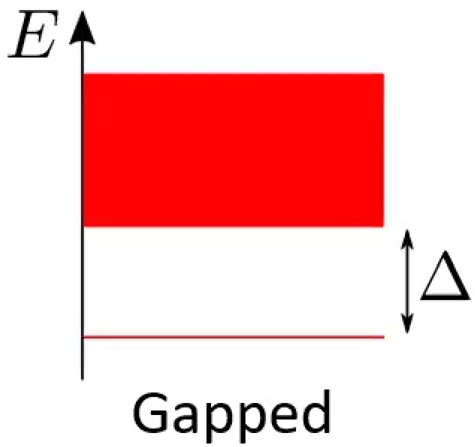
- Topological response – the above equation *does not depend on the metric* and is invariant under arbitrary diffeomorphisms of space-time
- Non-dissipative: rate of work done by the electric field is

$$J^i E_i \propto \epsilon^{ij}E_iE_j = 0$$

- Coefficient m is *quantized*

Hall conductance as a topological response



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Quantized topological responses characterize (unfractionalized) topological phases of matter

Quantization of Hall conductance

Variant of [Laughlin, 1981]

If the flux is an integer multiple of 2π , then the charge must be an integer (for an unfractionalized topological phase)

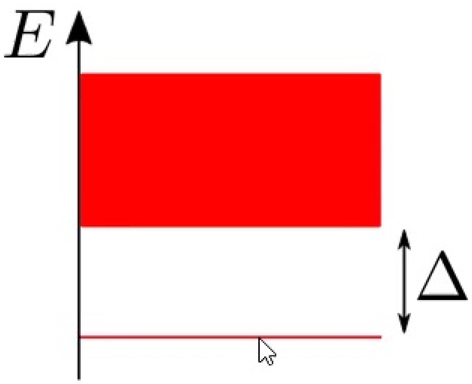
$$\rightarrow m \in \mathbb{Z}$$

$$\int \rho(\mathbf{x}) d^2\mathbf{x} = \int \frac{m}{2\pi} B(\mathbf{x}) d^2\mathbf{x}$$

Charge ↓ Flux ↓

$$Q = \frac{m}{2\pi} \Phi$$

Hall conductance as a topological response



Gapped

$$J^i = \frac{m}{2\pi}\epsilon^{ij}E_j \qquad \rho = \frac{m}{2\pi}B$$

$$J^\mu = \frac{m}{4\pi}\epsilon^{\mu\nu\lambda}F_{\nu\lambda}$$

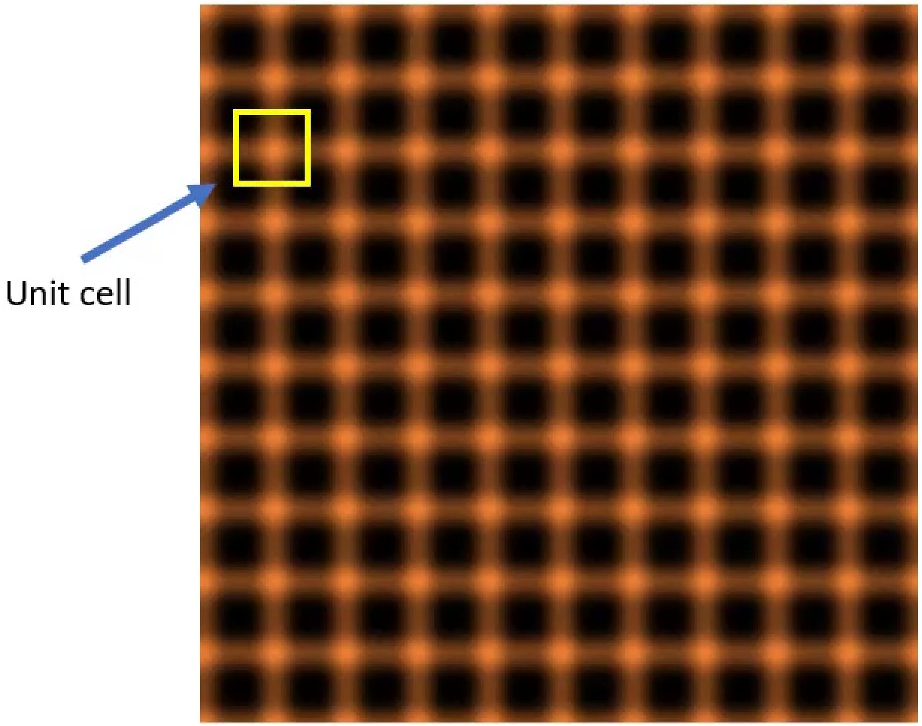
Can also think of this response as being generated by a *topological term* for the electromagnetic field (Chern-Simons term)

$$S[A] = \frac{m}{4\pi}\int d^3x\,\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda \qquad J^\mu = \frac{\delta S}{\delta A_\mu}$$

Topological responses in crystals

Presentation follows [DVE, Huang, Prem, Gromov, arXiv:2103.13393]

Crystals

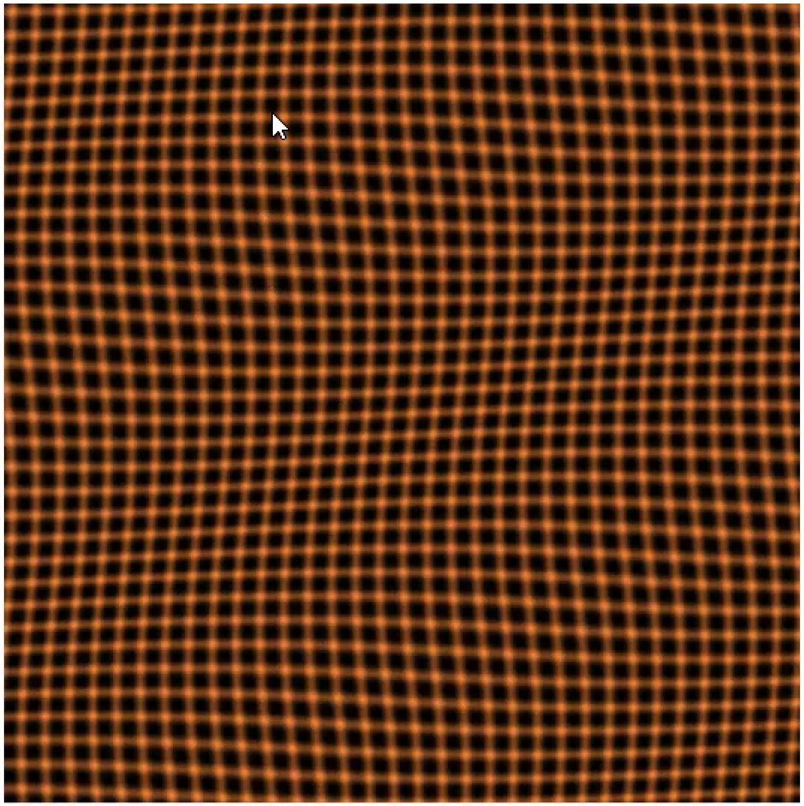


Discrete translation symmetry

Are there topological responses *specific to crystals?*

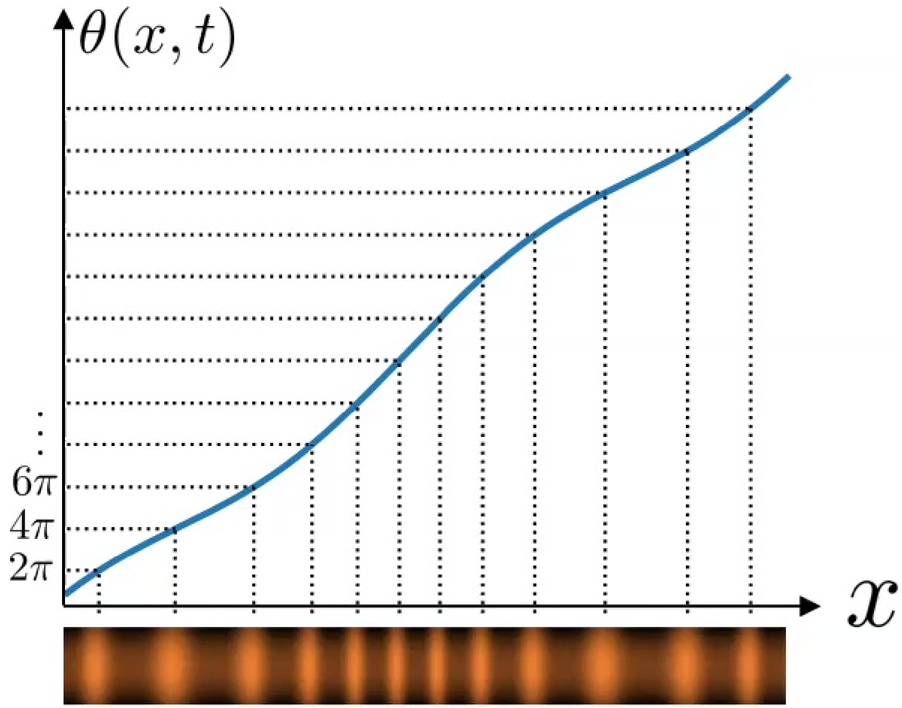
Crystalline topological phases of matter

Crystalline responses

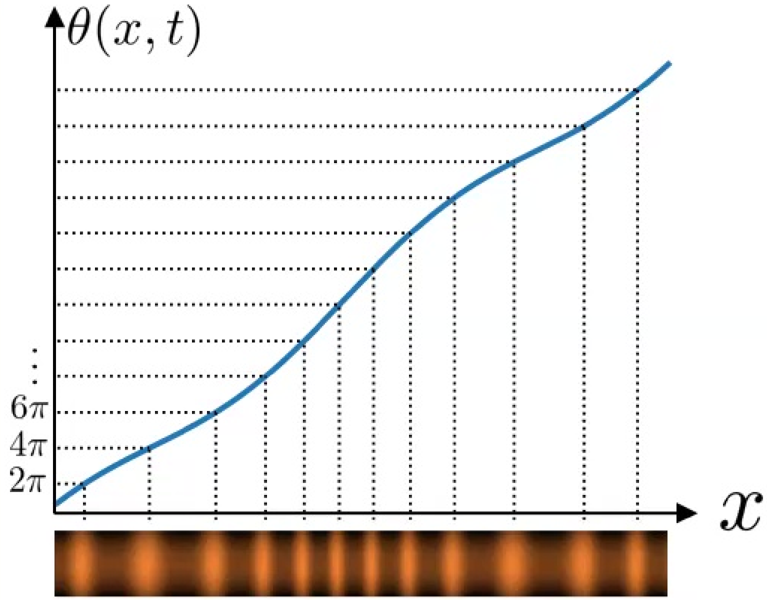


Consider responses to *elastic deformations*

Elasticity field in 1D



Topological response in 1D



Suppose we have a global U(1) symmetry.
Then there is an associated current J^μ

Then we have can have a topological response

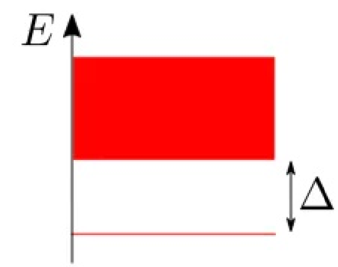
$$J^\mu = \frac{m}{2\pi}\epsilon^{\mu\nu}\partial_\nu\theta \qquad m \in \mathbb{Z}$$

A "classical" picture: point charges bound to unit cells

Charge m

$$\rho = \frac{m}{2\pi}\partial_x\theta \qquad J^x = -\frac{m}{2\pi}\partial_t\theta$$

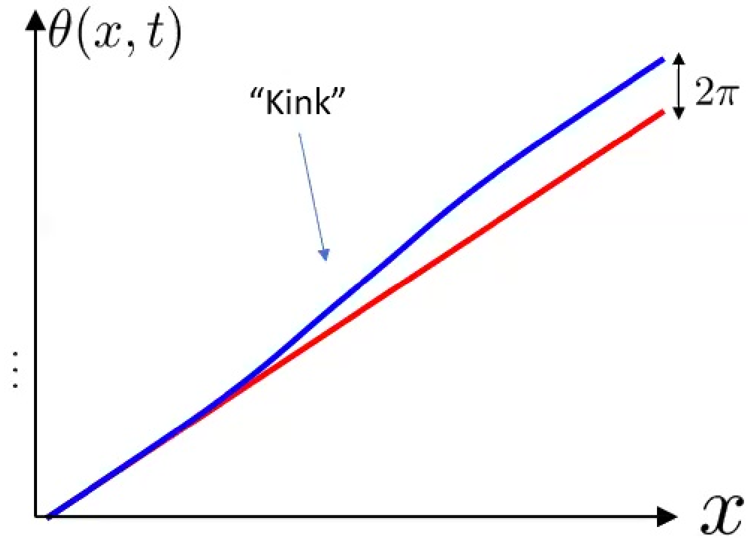
However, the topological invariant is still well-defined in *any* gapped system



Gapped

m is the "charge per unit cell"

Quantization of the topological invariant



$$\rho = \frac{m}{2\pi}\partial_x\theta$$

$$\Delta Q_{\text{kink}} = m$$

Kink defect must have integer charge (in an unfractionalized topological phase)

$$m \in \mathbb{Z}$$

Generalizing to 2D crystals

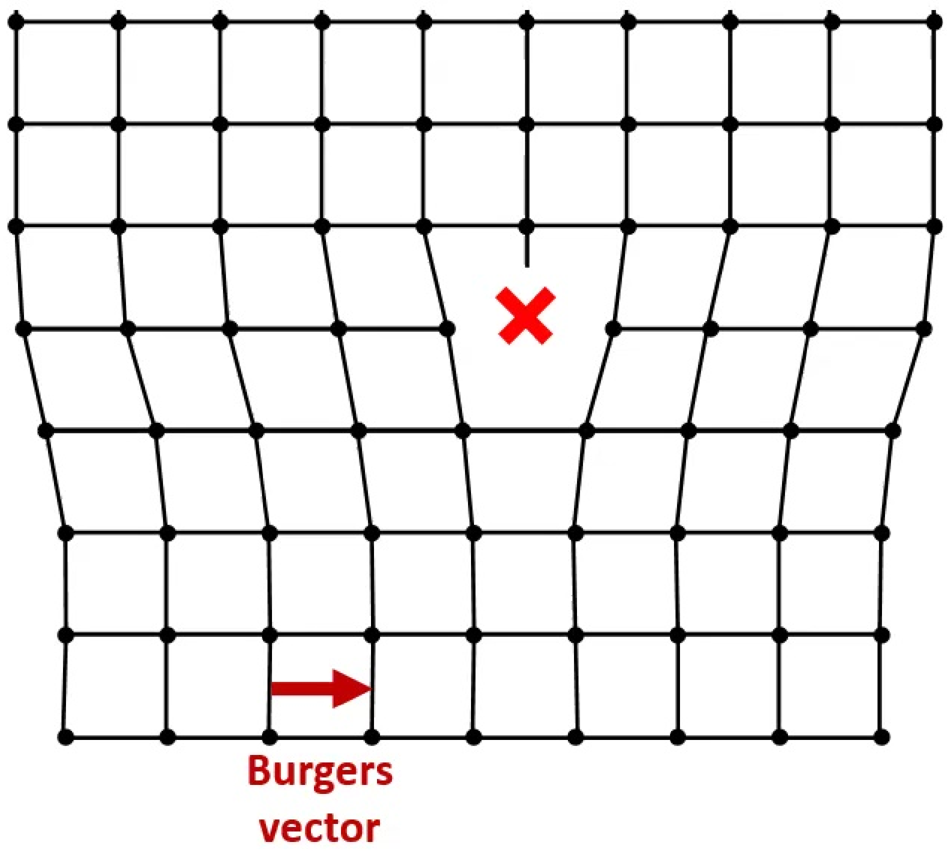
Now have two fields $\theta^1(\mathbf{x},t)$ and $\theta^2(\mathbf{x},t)$

Topological response

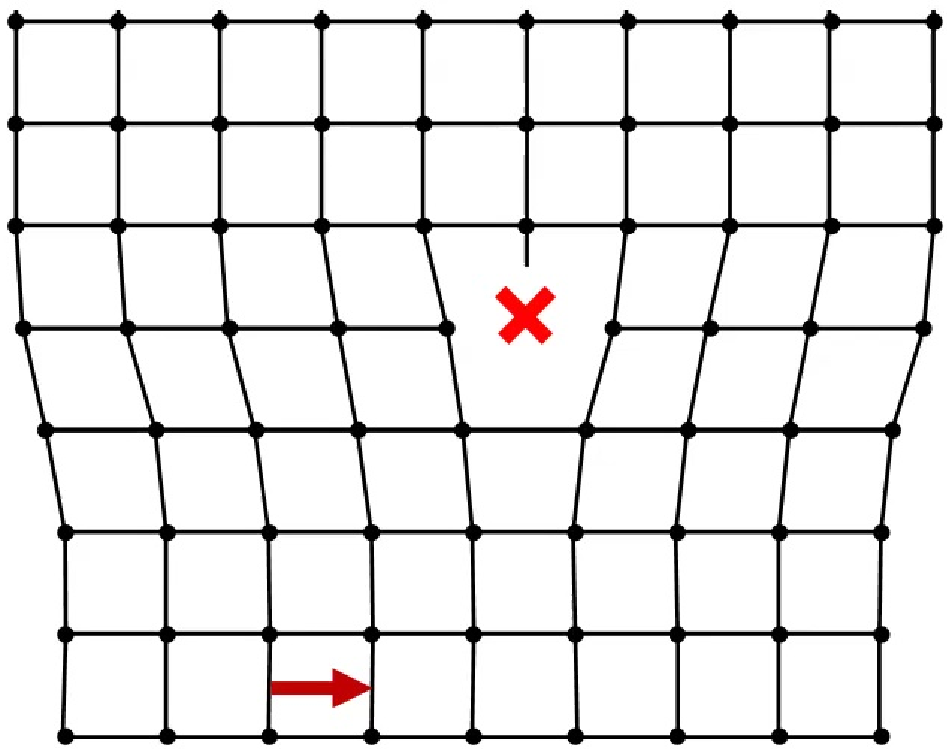
$$J^\mu = \frac{m}{(2\pi)^2}\epsilon^{\mu\nu\lambda}\partial_\nu\theta^1\partial_\lambda\theta^2$$

m is still the "charge per unit cell"

Application: mobility of dislocations2



Application: mobility of dislocations2



If the topological invariant is nonzero, then dislocations can only move in the direction of their Burgers vector without violating conservation of charge

Can derive from the topological response

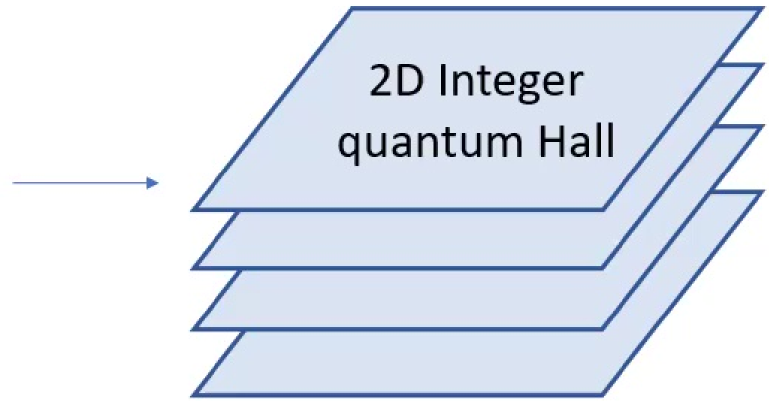
$$J^{\mu} = \frac{m}{(2\pi)^2} \epsilon^{\mu\nu\lambda} \partial_{\nu} \theta^1 \partial_{\lambda} \theta^2$$

- In 3D, two different kinds of responses with U(1) symmetry:

$$J^{\mu} = \frac{m}{(2\pi)^3}\epsilon^{\mu\nu\lambda\kappa}\partial_\nu\theta^1\partial_\lambda\theta^2\partial_\kappa\theta^3$$

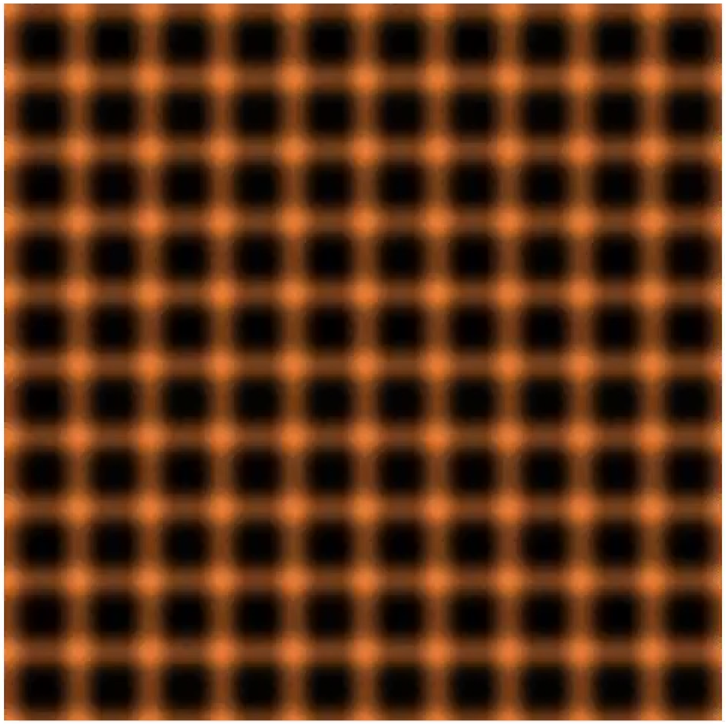
[charge per unit cell]

$$J^{\mu} = \sum_{I=3}^{3}\frac{m_I}{8\pi^2}\epsilon^{\mu\nu\lambda\kappa}\partial_\nu A_\lambda\partial_\kappa\theta^I$$



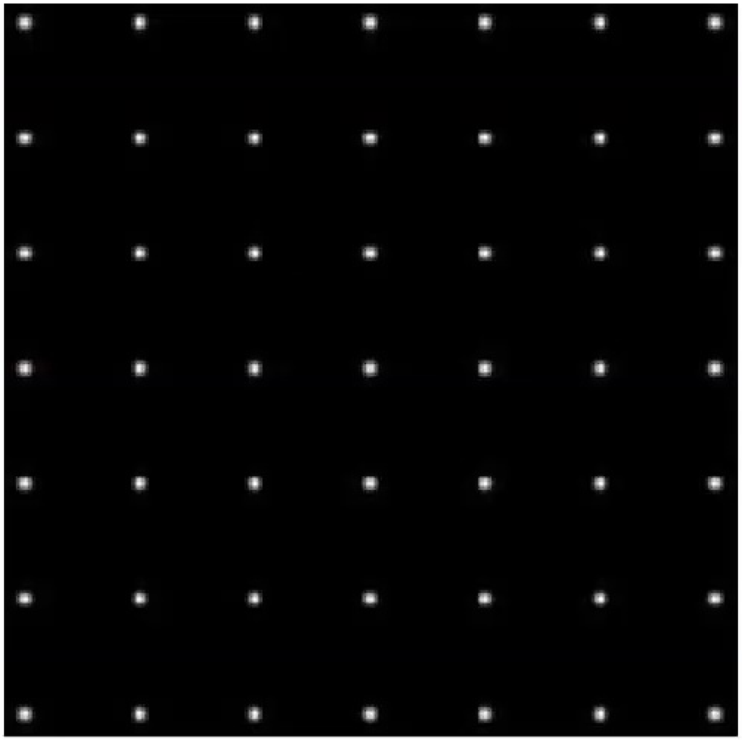
Topological responses in **quasi**crystals

Crystals



Fourier

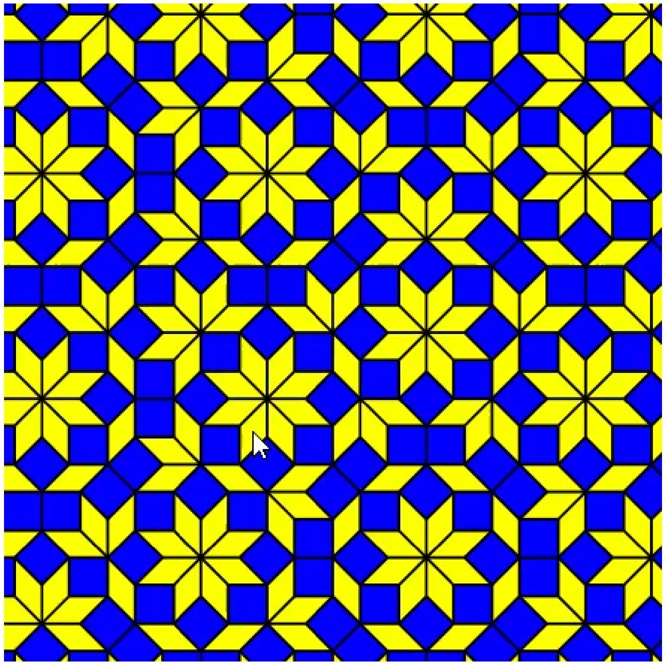
transform



Discrete translation symmetry

Quasicrystals

Ammann-Beenker tiling



Fourier

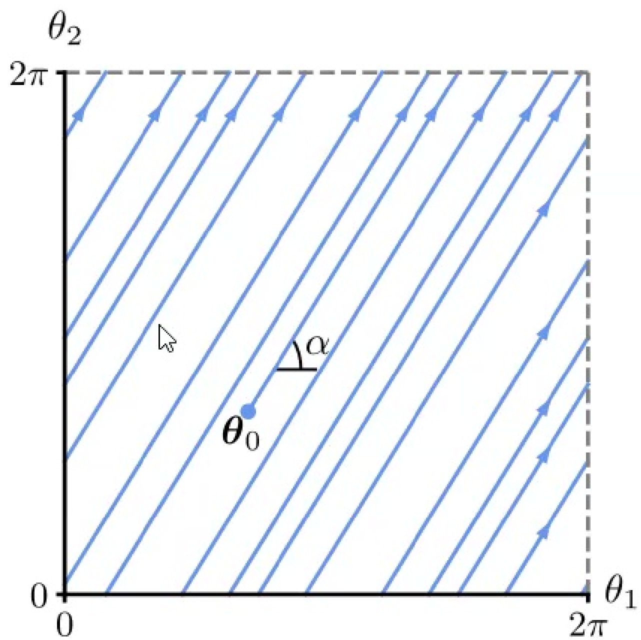
transform

Real space model: tilings

No translation symmetry

Elastic modes for crystals and quasicrystals in 1D?

- Elastic mode of 1D crystal: $\theta(x,t) \in S^1$?
- Elastic modes of 1D quasicrystal $\boldsymbol{\theta}(x,t) \in \mathbb{T}^D$ for $D > 1$



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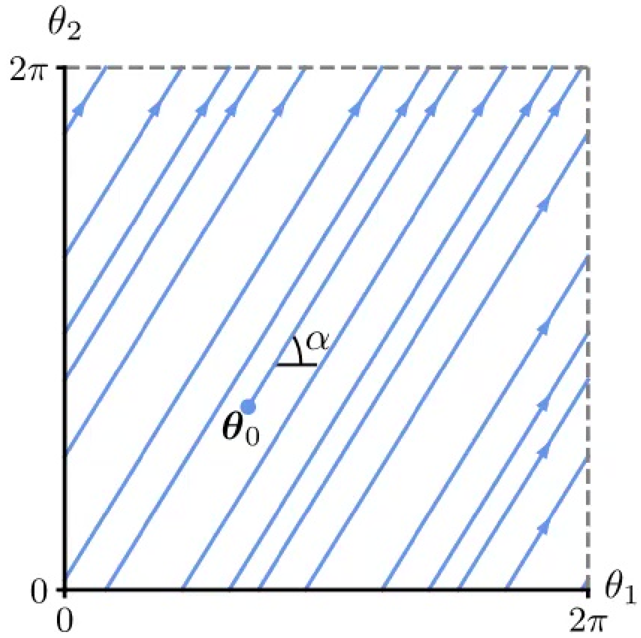


Two kinds of elastic modes: phonons and phasons

Topological response in a 1D quasicrystal

1D Crystals:

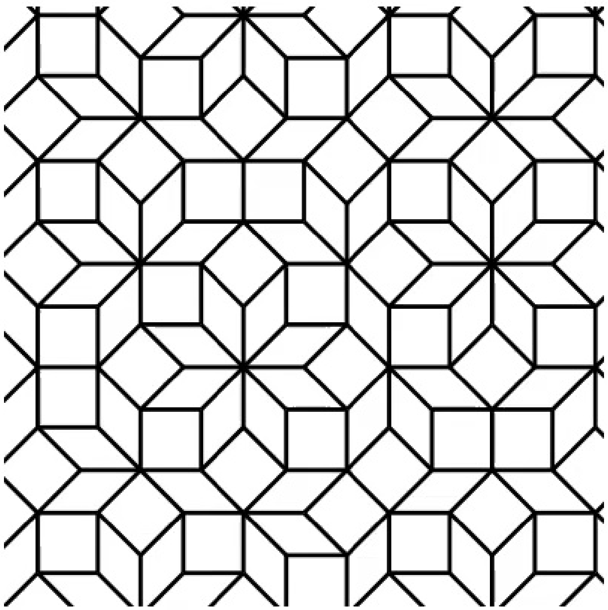
$$J^\mu = \frac{m}{2\pi}\epsilon^{\mu\nu}\partial_\nu\theta \qquad m \in \mathbb{Z}$$

1D Quasicrystals:

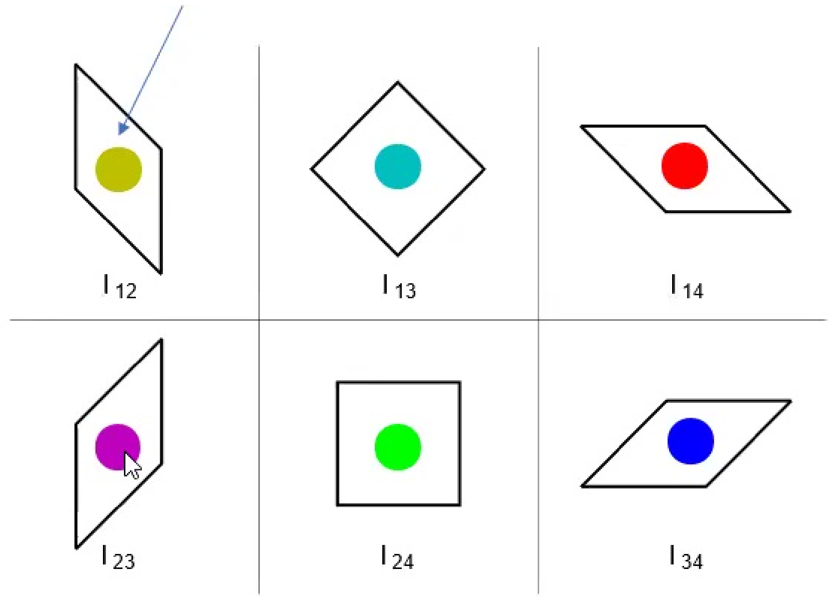
$$J^\mu = \sum_{I=1}^{D}\frac{m_I}{2\pi}\epsilon^{\mu\nu}\partial_\nu\theta^I \qquad m_1, \cdots, m_D \in \mathbb{Z}$$

$$D > 1$$

[DVE, Huang, Prem, Gromov, arXiv:2103.13393]



I pick how much charge I want to bind to each kind of tile



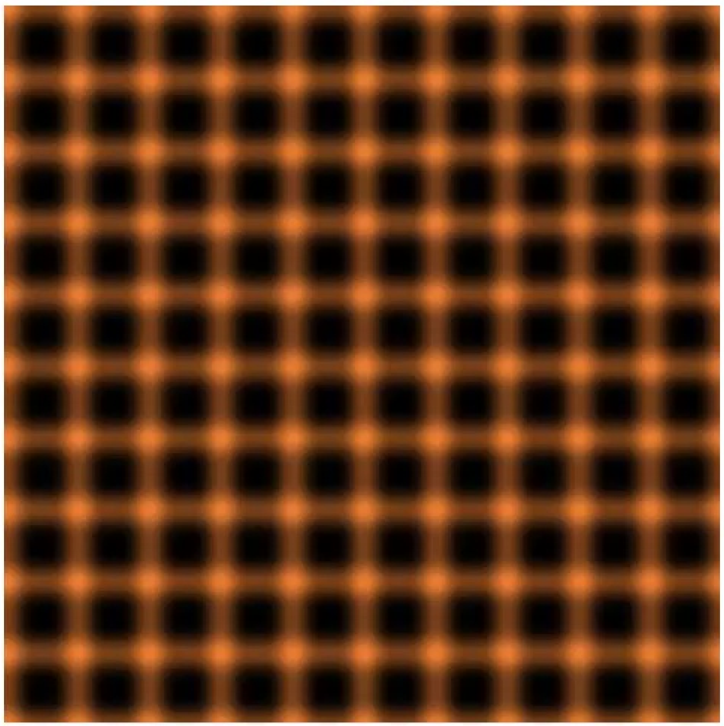
[DVE, Huang, Prem, Gromov, arXiv:2103.13393]

Dislocations in quasicrystals

The mobility constraints on dislocations are still a straightforward consequence of the topological response

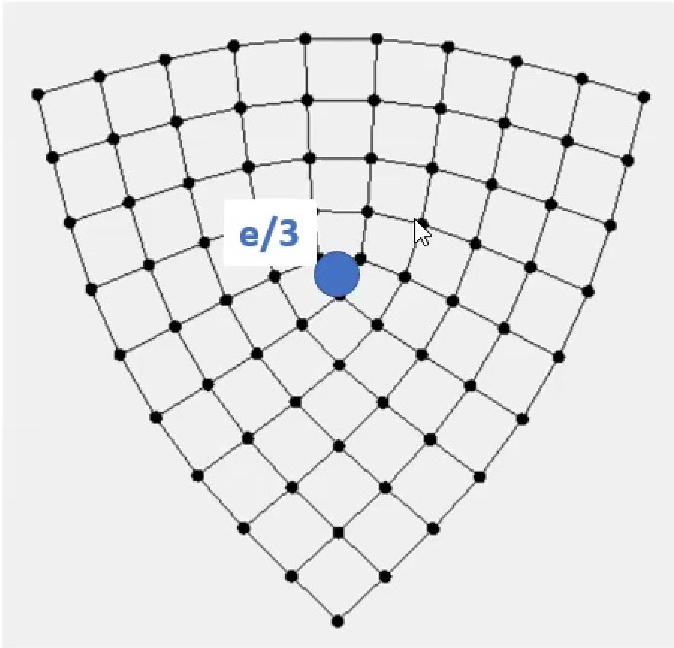
Extensions

- Crystals can also have *point-group* symmetries



Topological response affects disclination defects e.g. they can carry fractional charge

[Li, Zhu, Benalcazar, Hughes, PRB **101**, 115115]



For a general symmetry group:

[includes both translation symmetry and point-group symmetry, as well as internal symmetry groups like U(1)]

Unfractionalized topological phases are classified by
equivariant generalized cohomology

[Thorngren, DVE, PRX **8,** 011040 (2018)]
[DVE, Thorngren, PRB **99,** 115116 (2019)]

For an alternative (but, it turns out, equivalent) approach:

[Song, Huang, Fu, Hermele, PRX **7**, 01220 (2017)]
[Huang, Song, Huang, Hermele, PRB **96**, 205106 (2017)]
[Song, Huang, Qi, Fang, Hermele, Sci. Adv. **5**, eaax2007 (2019)]

Extension to quasicrystals: [DVE, Huang, Prem, Gromov, arXiv:2103.13393]

Future directions

- Experimental applications
- Metals in quasicrystals?


Thank you!