Title: Black holes beyond General Relativity: shadows, stability, and nonlinear evolution

Speakers: Aaron Held

Series: Strong Gravity

Date: December 09, 2021 - 1:00 PM

URL: https://pirsa.org/21120021

Abstract: Guided by the principles of effective field theory (EFT), I will discuss three avenues to constrain physics beyond General Relativity with black-hole observations.

1) Shadows: Without specifying any particular gravitational dynamics, I will discuss image features of black-hole shadows in general parameterizations and their relation to fundamental-physics principles like (i) regularity (no remaining curvature singularity), (ii) simplicity (a single new-physics scale), and (iii) locality (a new-physics scale set by local curvature).

2) Stability: Specifying the linearized dynamics around black-hole spacetimes determines the onset of potential instabilities and connects to the ringdown phase of gravitational waves. I will delineate how said instabilities can constrain the EFT of gravity, theories of low-scale dark energy, as well as ultralight dark matter.

3) Nonlinear evolution: The larger the probed curvature scale, the tighter the constraints on new gravitational physics. Making full use of experimental data, thus relies on predictions in the nonlinear regime of binary mergers. I will present recent progress towards achieving stable numerical evolution for the EFT of gravity up to quadratic order in curvature.

Zoom Link: https://pitp.zoom.us/j/98276687334?pwd=UnM2dElacWNtempQUHJMNVlaNHgyUT09

FRIEDRICH-SCHILLER-UNIVERSITÄT JENA

Princeton University

Princeton gravity Initiative

DAAD Deutscher Akademischer Austausch Dienst German Academic Exchange Service

Black holes beyond General Relativity: shadows, stability, and nonlinear evolution

work with
Astrid Eichhorn
Roman Gold
Philipp Johannsen
Heloise Delaporte

work with
Claudia de Rham
Sebastian Garcia-Saenz
Jun Zhang

work with
Hyun Lim

Aaron Held

DAAD PRIME Fellow, currently at **The Princeton Gravity Initiative**

09th December 2021, Strong Gravity Seminar, Perimeter Institute

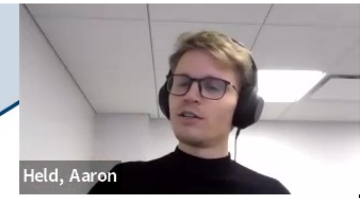
Effective Field Theory
around the free fixed point

new physics
(effective asymptotic safety)
Held, Front. in Phys. 8 (2020)
...

	classically suppressed	classically marginal
free parameters (IR-repulsive)	not predicted and unobservable	observable but not predicted
fundamental-physics predictions (IR-attractive)	predicted but (almost) unobservable	potentially observable predictions

Effective Field Theory

around the free fixed point



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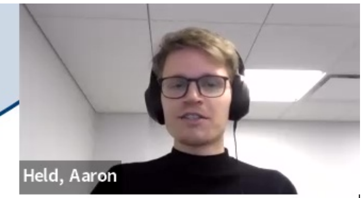
potentially observable predictions

- marginal Standard Model matter couplings

  $F_{\mu\nu}F^{\mu\nu}\,,\ \phi\,\overline{\psi}\psi\,,\ \phi^4\,,\ \phi^2\chi^2$

- Quadratic Gravity couplings

  $\alpha\,R^2\,,\ \beta\,R_{\mu\nu}R^{\mu\nu}\,,\ (\phi^2\,R\,,\ \dots)$



Effective Field Theory

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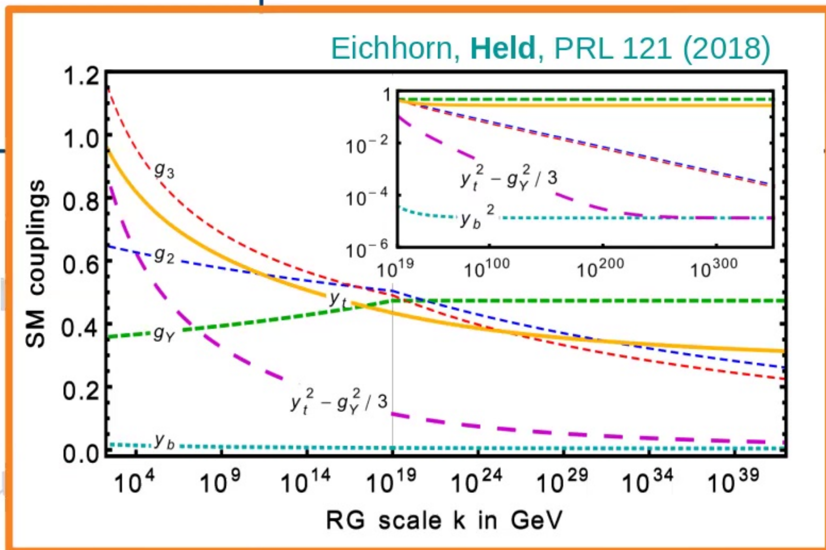

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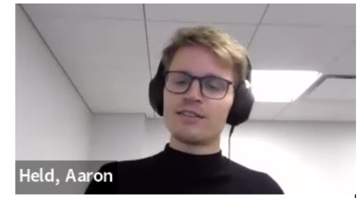
Black-hole phenomenology in EFTs beyond GR



leading-order EFT corrections

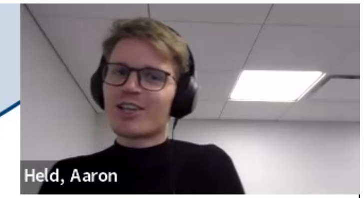
$$\mathcal{L}_{\mathrm{EFT}} = \frac{1}{16\pi G} R + \alpha R_{ab} R^{ab} - \beta R^2$$

ghost-free theories

Lovelock's theorem → field equations of GR
+ no other DOF
+ four dimensions
+ quasi-local action

Effective Field Theory

around the free fixed point



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Held, Front. in Phys. 8 (2020)

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requires **strong-gravity** (high curvature) →

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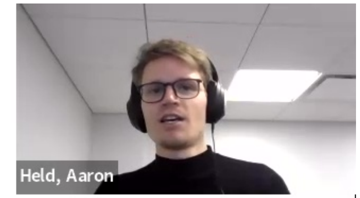

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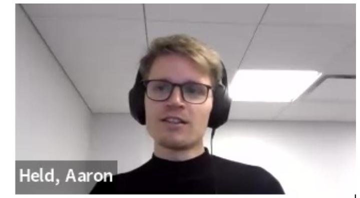
classical objections against ghosts

ghosts → ill-posedness

ghosts → runaways



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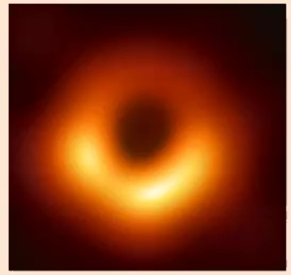
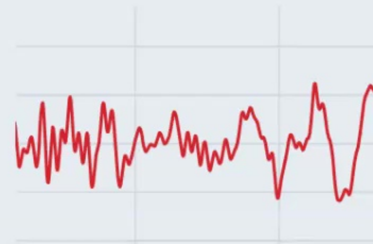
classical objections against ghosts

- ghosts $\not\Rightarrow$ ill-posedness — see Part III of today's talk
- ghosts $\not\Rightarrow$ runaways — Deffayet, Mukohyama, Vikman '21 ...

Black-hole phenomenology in EFTs beyond GR

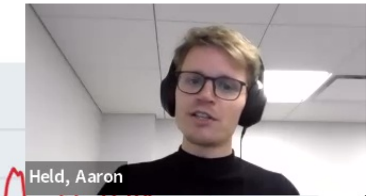


Part I: background solutions



Part II: linear dynamics

Part III: nonlinear dynamics



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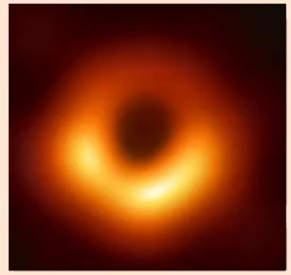

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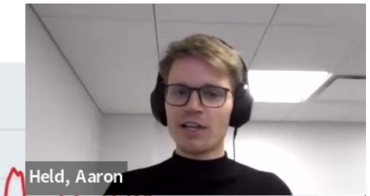

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→ field equations of GR

w/ Astrid Eichhorn
JCAP 05 (2021) 073
EPJC 81 (2021)
...

w/ Jun Zhang
(to appear)

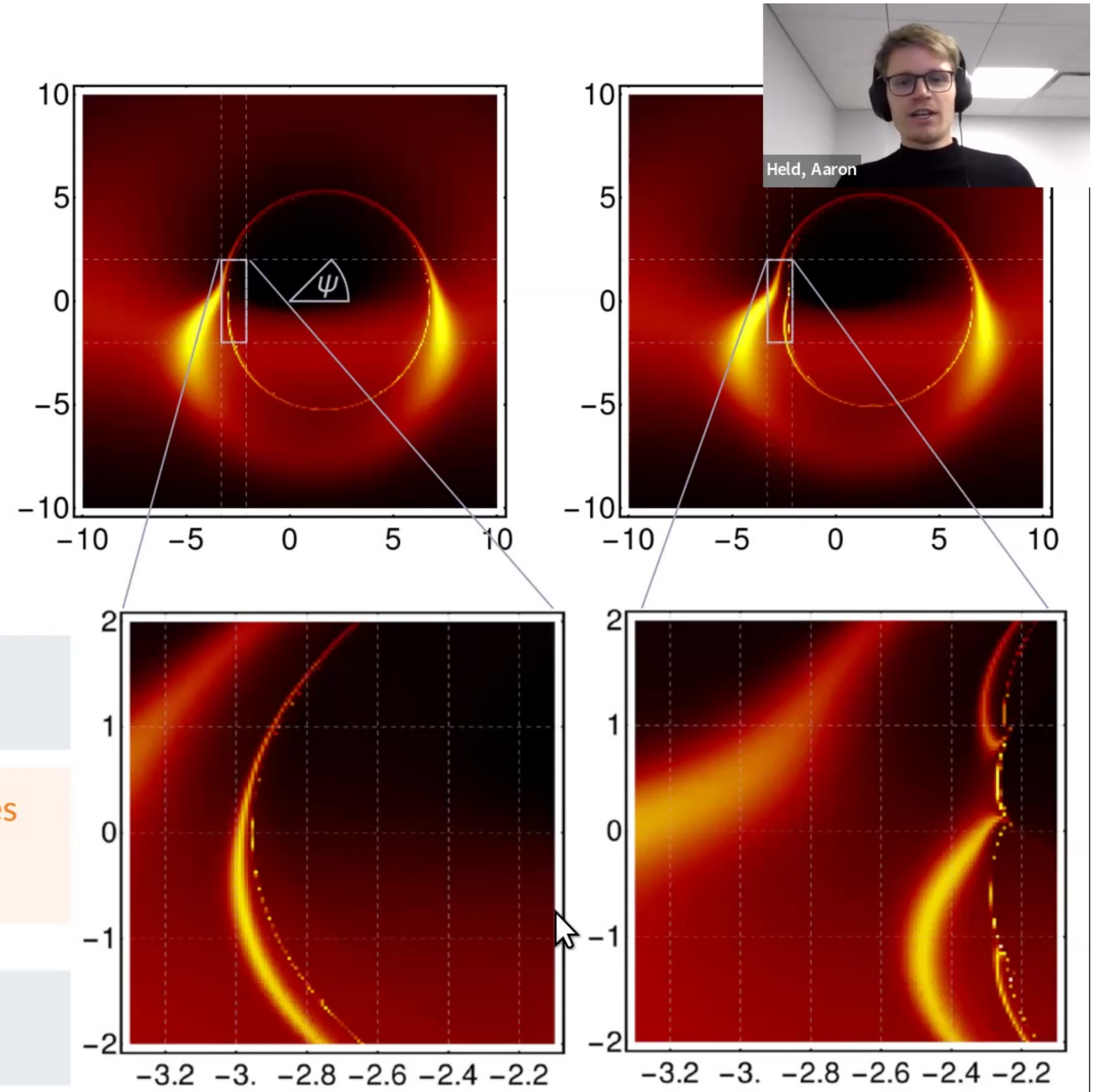
w/ Hyun Lim
PRD 104 (2021) 8
...

w/ Sebastian Garcia-Saenz & Jun Zhang
PRL 127 (2021) 13
...

w/ Hyun Lim & Claudia de Rham

Part I
Image features from fundamental principles

Eichhorn, Held, JCAP 05 (2021) 073
Eichhorn, Held, Eur.Phys.J.C 81 (2021)
Eichhorn, Held, Johannsen (to appear)
Eichhorn, Held, Delaporte (to appear)



principled approach:

"direct calculation in specific theories beyond GR"

principled-parameterized approach

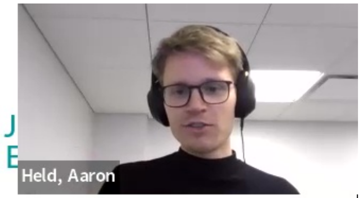
"identify characteristic image features in families of spacetimes based on fundamental principles"

parameterized approach:

"parameterize all deviations from GR"

Constructing regular spinning black holes: the principled-parameterized approach

Eichhorn, **Held**, J
Eichhorn, **Held**, E



$$ds_{\mathrm{Kerr}} = -\frac{\left(\Delta - a^2 \sin^2\theta\right)}{\Sigma} du^2 + 2\,du dr - \frac{2a \sin^2\theta \left(a^2 - \Delta + r^2\right)}{\Sigma} du d\varphi - 2a \sin^2\theta\, dr d\varphi$$
$$+ \Sigma\, d\theta^2 + \frac{\sin^2\theta \left(\left(a^2 + r^2\right)^2 - a^2 \Delta \sin^2\theta\right)}{\Sigma} d\varphi^2$$

$$\Sigma = r^2 + a^2 \cos^2\theta$$

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Kerr spacetime (metric)

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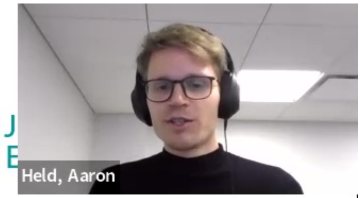

$$\left[\frac{\mathbb{I}}{2^4\,3}\right]^3 = \left[\frac{\mathbb{J}}{2^5\,3}\right]^2 = \left[\frac{\mathrm{G\,M}}{(\mathrm{r} - \mathrm{i\,a}\cos\,\theta)^3}\right]^6$$

& all other Riemann invariants vanish

Kerr spacetime **(invariants)**

Cartan '28
Karlhede '80
MacCallum, Skea, McLenaghan, McCrea
Zakhary, McIntosh '97
Carminati, McLenaghan '91
Held '21 (2105.11458)

Constructing regular spinning black holes: the principled-parameterized approach

Eichhorn, **Held**, J
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Eichhorn, **Held**, JCAP 05 (2021) 073
Eichhorn, **Held**, Eur.Phys.J.C 81 (2021)

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(all curvature invariants are regular everywhere)

ii) **Newtonian limit**

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(a single new-physics scale)

iv) **Locality**
(new physics is tied to local curvature scales)

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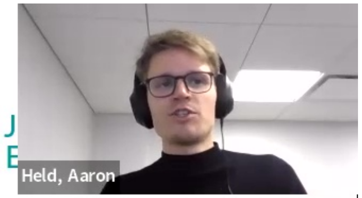

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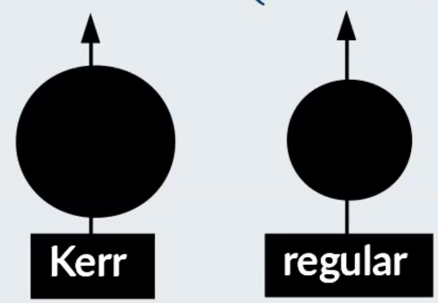
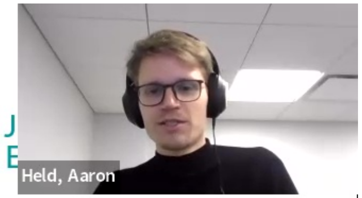
➢ **the horizon shrinks**

$$M \to M(r,\theta) \to \begin{cases} M \text{ for } r \to \infty \\ 0 \text{ for } r \to 0 \end{cases}$$



Constructing regular spinning black holes: the principled-parameterized approach

Eichhorn, Held, J
Eichhorn, Held, E



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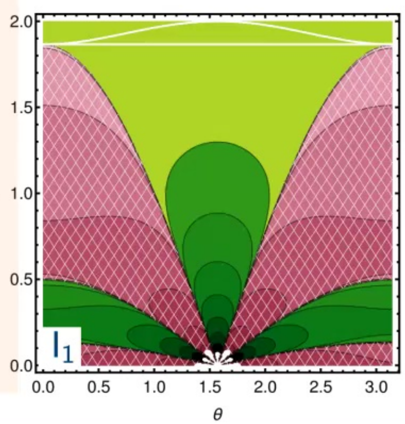
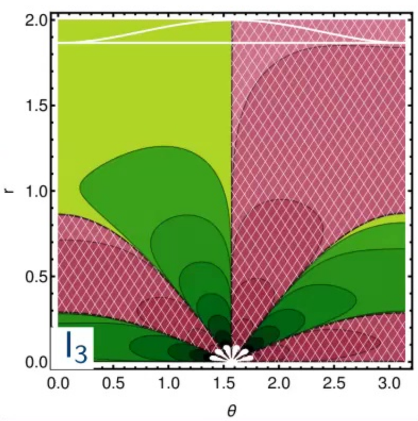
envelope to the maximum absolute-value of all curvature invariants

$$K_{GR} = \frac{48M^2}{(r^2 + a^2\cos^2(\theta))^3}$$





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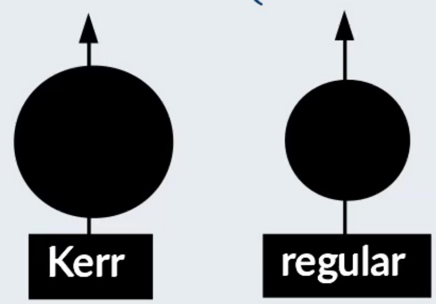

Constructing regular spinning black holes: the principled-parameterized approach

Eichhorn, Held, J
Eichhorn, Held, E

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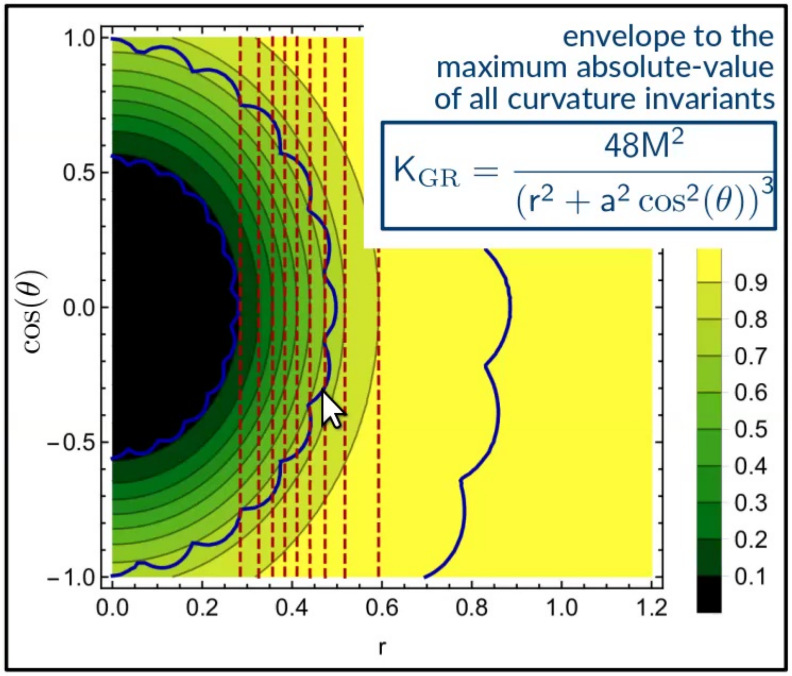

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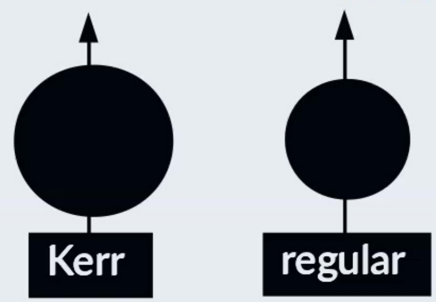

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Eichhorn, **Held**, J
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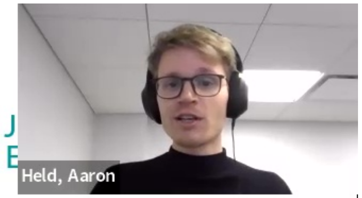

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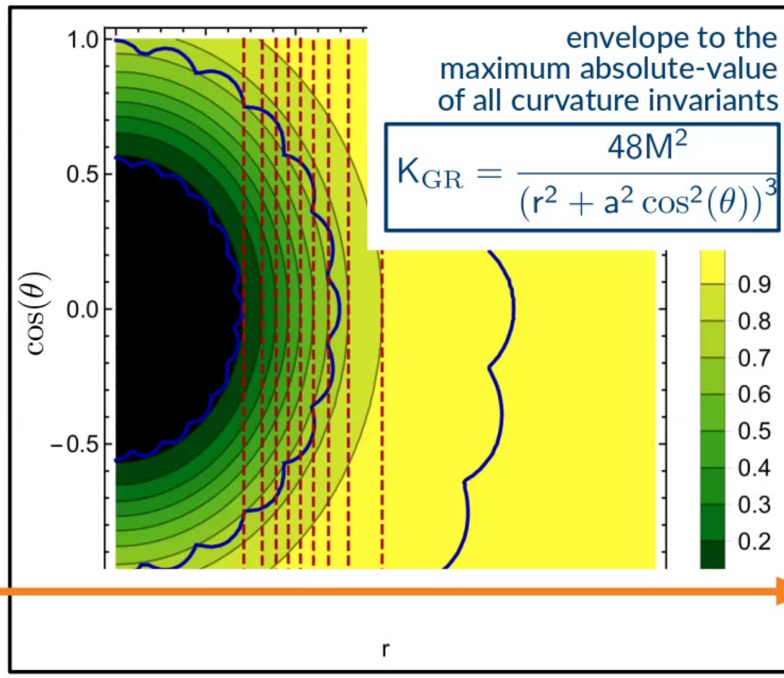

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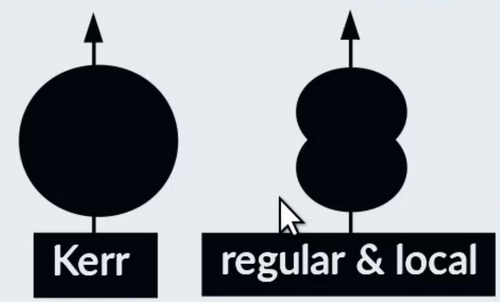
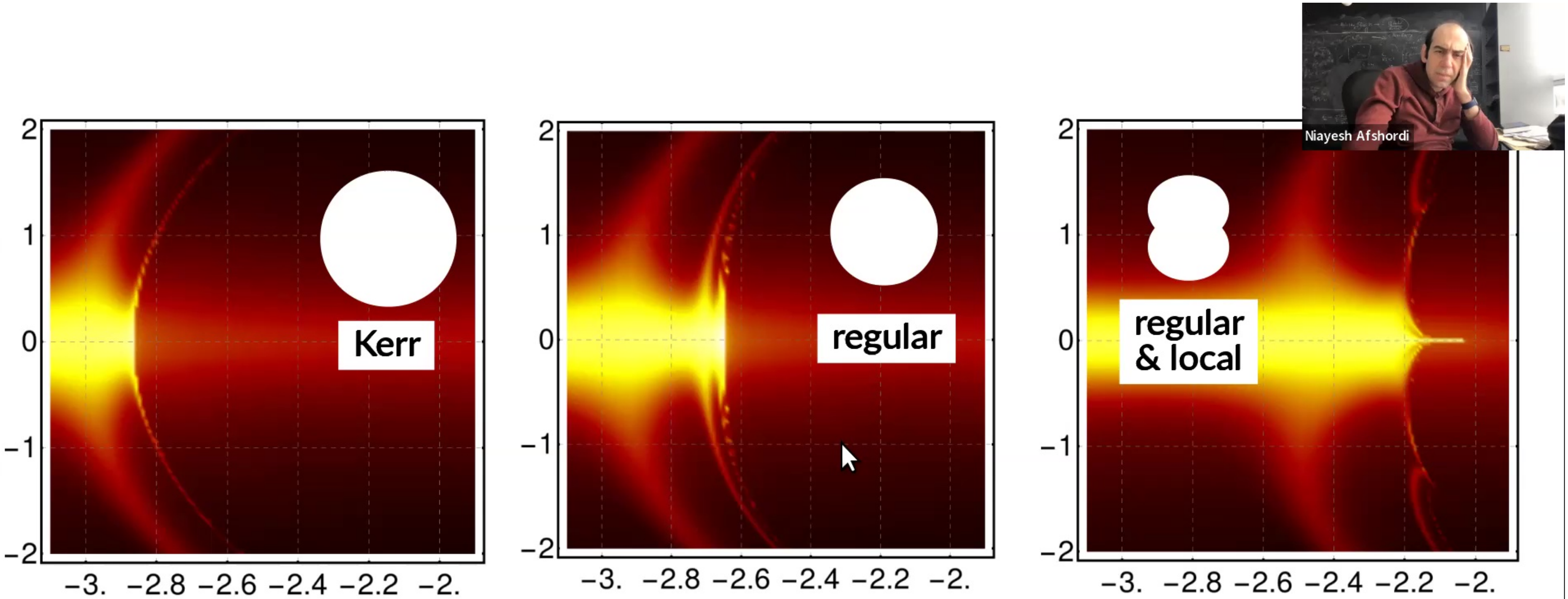
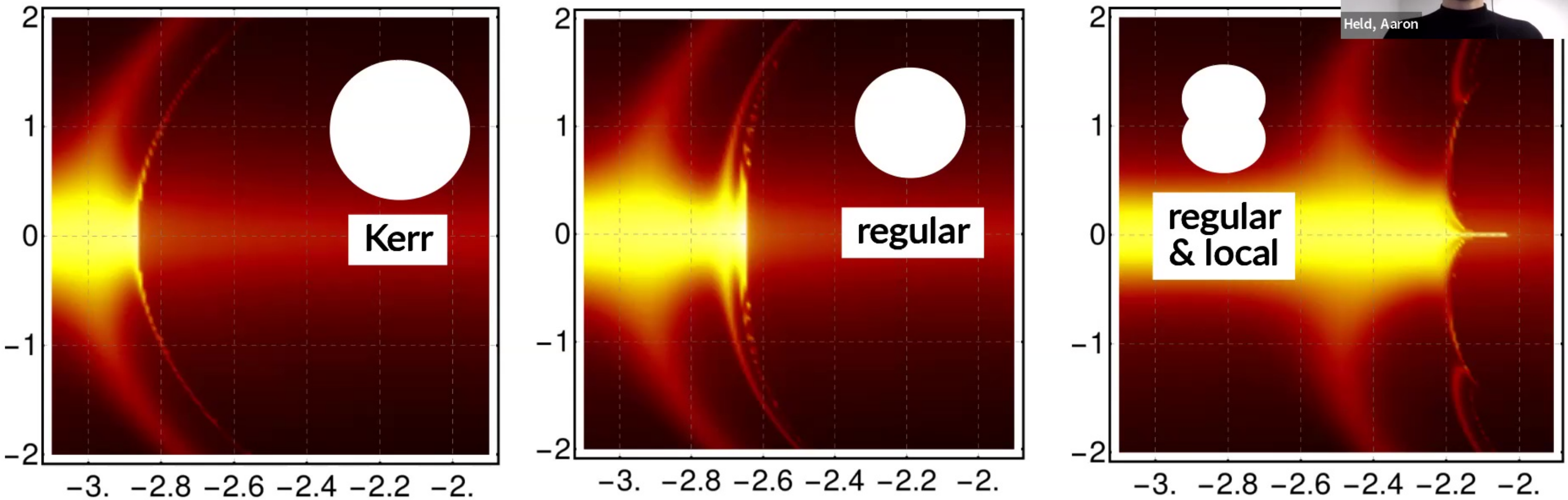


- **dependence on polar angle**


Kerr
regular
regular
& local
2
1
0
−1
−2
−3.
−2.8
−2.6
−2.4
−2.2
−2.
Niayesh Afshordi

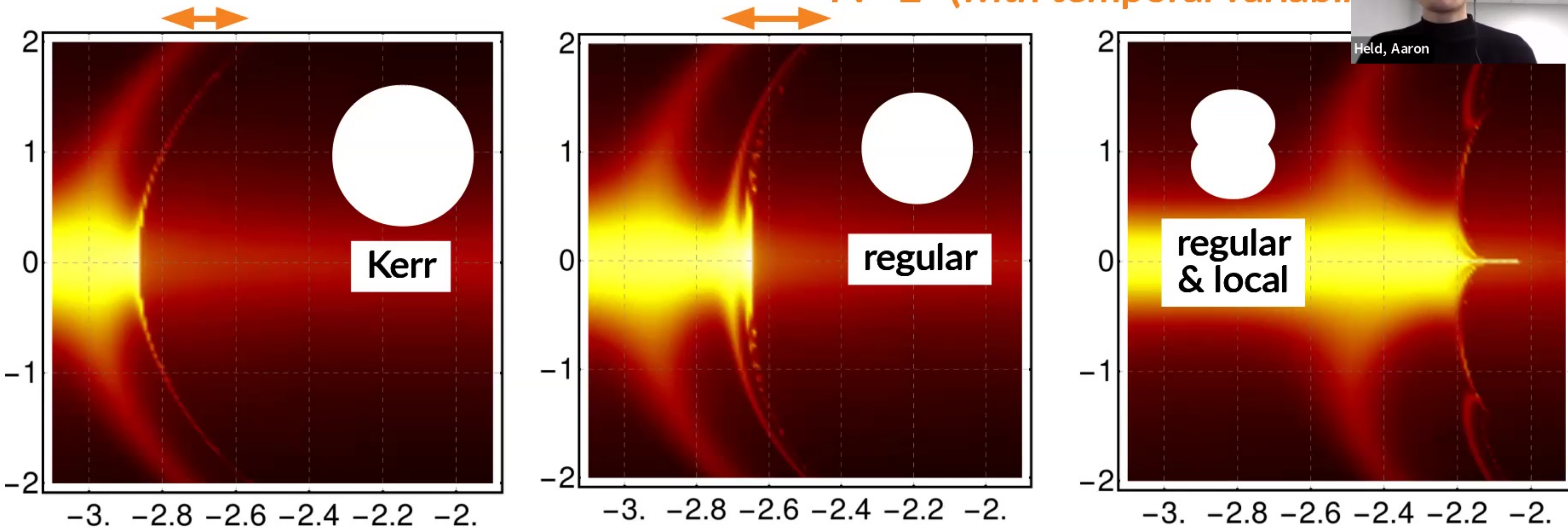


Effects have been maximized wrt ...

- Edge-on inclination
- large spin (a = 0.9)
- slow fall-off in the mass function
- $\ell_{NP} \sim M$

Broderick et. Al ‘21
Wong ‘21

N=1 (with temporal variabil



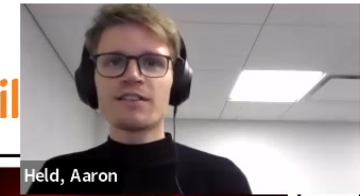


image features due to increased compactness

1) Shadow shrinks
2) Photon rings are stretched out

Broderick et. Al '21
Wong '21

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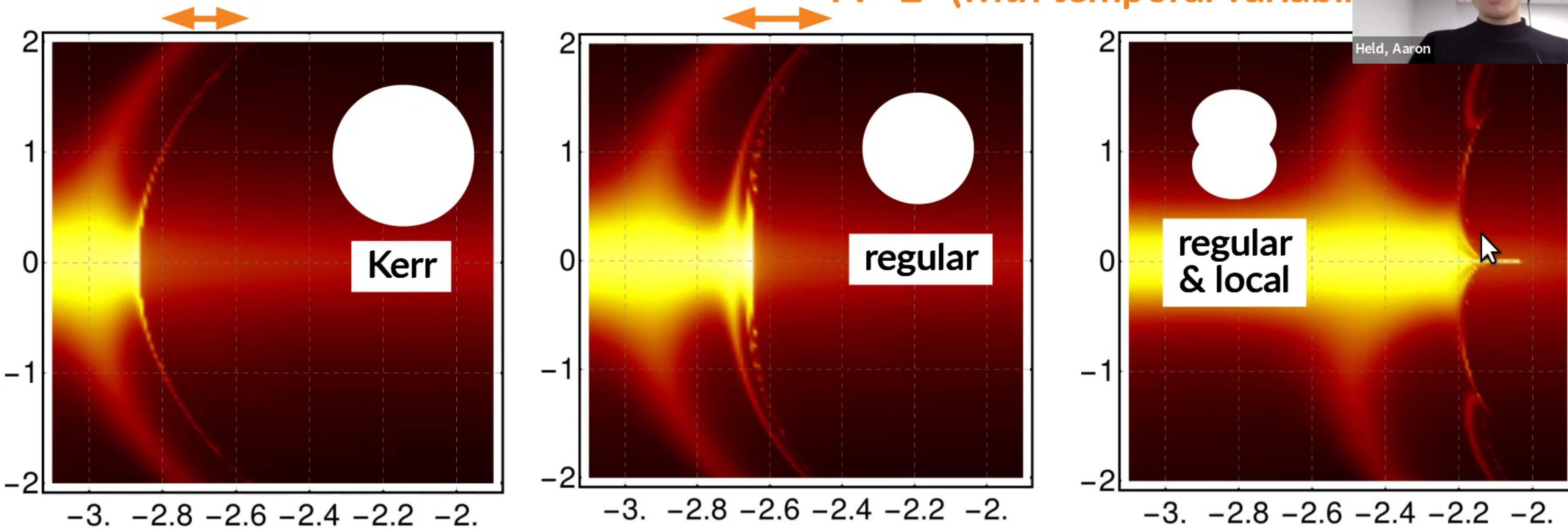
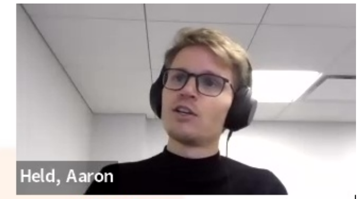
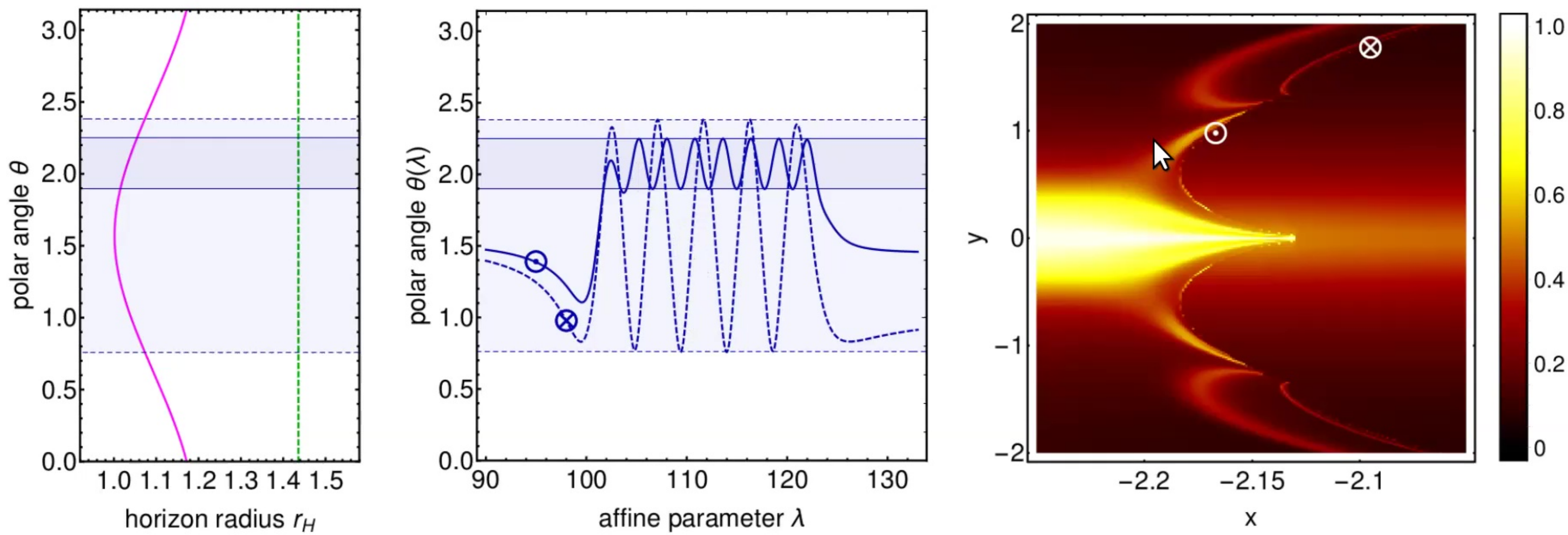


image features due to increased compactness	1) Shadow shrinks 2) Photon rings are stretched out	
prograde image features distinguishing local vs non-local models	3) No asymmetry 4) No cusps or dent	3) Asymmetry 4) **Cusps and dent**

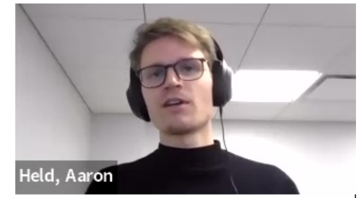


"non-spherical" horizon → **qualitatively distinct classes of photon orbits** → **cusps in the image features**

calls for GRMHD

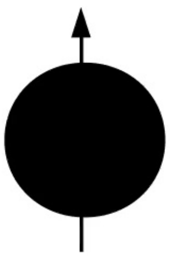


Eichhorn, **Held**, Eur.Phys.J.C 81 (2021)



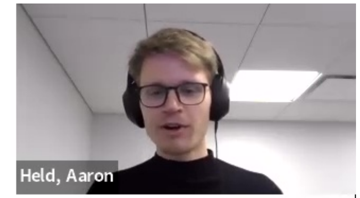
Circularity

$\xi_1^{[\mu}\xi_2^{\nu}D^{\kappa}\xi_1^{\lambda]} = 0$	at at least one point
$\xi_2^{[\mu}\xi_1^{\nu}D^{\kappa}\xi_2^{\lambda]} = 0$	at at least one point
$\xi_1^{\mu}R_{\mu}{}^{[\nu}\xi_2^{\kappa}\xi_1^{\lambda]} = 0$	everywhere
$\xi_2^{\mu}R_{\mu}{}^{[\nu}\xi_1^{\kappa}\xi_2^{\lambda]} = 0$	everywhere



vacuum
GR solutions
are **circular**

Kerr



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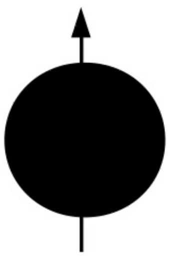

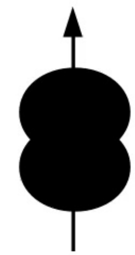

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GR solutions
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Kerr

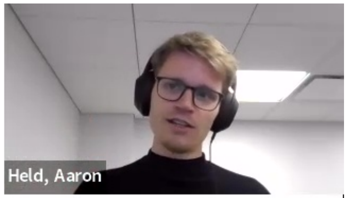
general parametrizations of stationary axisymmetric black holes are **circular**

parameterized approach

Johannsen Phys.Rev.D 88 (2013)
Konoplya et.Al., Phys.Rev.D 93 (2016)

new physics tied to local curvature scales leads to **non-circularity**

principled-parameterized approach



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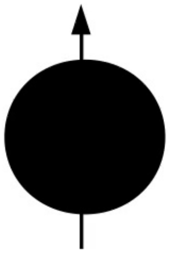

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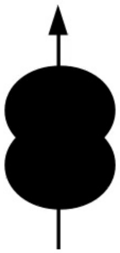

new physics tied to
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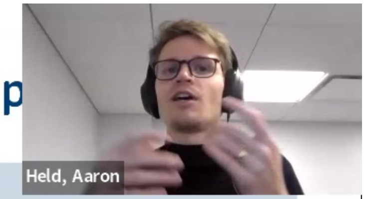
principled-parameterized approach

	Kerr	principled-parameterized approach	
axisymmetric and stationary	✔	✔	**at least 6** free metric functions Gourgoulhon, Bonazzola '93 $\begin{matrix} g_{tt} & g_{tr} & g_{t\theta} & g_{t\phi} \\ & g_{rr} & g_{r\theta} & g_{r\phi} \\ & & g_{\theta\theta} & g_{\theta\phi} \\ & & & g_{\phi\phi} \end{matrix}$
+			
circularity	✔	✗ see also Ben Achour et.Al. '20 (non-circular solutions to scalar-tensor theory)	**5** free metric functions Konoplya, Rezzolla, Zhidenko '16 $\begin{matrix} g_{tt} & 0 & 0 & g_{t\phi} \\ & g_{rr} & 0 & 0 \\ & & g_{\theta\theta} & 0 \\ & & & g_{\phi\phi} \end{matrix}$
+			
hidden symmetry	✔	✗	$g^{\mu\nu}\partial_\mu\partial_\nu = \frac{1}{S_1 + S_2}\left[\left(G_1^{AB} + G_2^{AB}\right)\partial x_A \partial x_B + \Delta_1 \partial x_1^2 + \Delta_2 \partial x_2^2 \right]$ Carter '68 Benenti, Francaviglia '79 Johannsen '15



	Kerr	principled-parameterized approach	p
axisymmetric and stationary	✔	✔	**at least 6** free metric functions Gourgoulhon, Bonazzola '93 $\begin{matrix} g_{tt} & g_{tr} & g_{t\theta} & g_{t\phi} \\ & g_{rr} & g_{r\theta} & g_{r\phi} \\ & & g_{\theta\theta} & g_{\theta\phi} \\ & & & g_{\phi\phi} \end{matrix}$
+ circularity	✔	✗ see also Ben Achour et.Al. '20 (non-circular solutions to scalar-tensor theory)	**5** free metric functions Konoplya, Rezzolla, Zhidenko '16 $\begin{matrix} g_{tt} & 0 & 0 & g_{t\phi} \\ & g_{rr} & 0 & 0 \\ & & g_{\theta\theta} & 0 \\ & & & g_{\phi\phi} \end{matrix}$
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... **stay tuned** Eichhorn, Held, Delaporte (to appear)

Part II.
Linear stability of Black Holes

Held, Zhang (to appear)
Gracia-Saenz, Held, Zhang, PRL 127 (2021) 13

Effective field theory of gravity

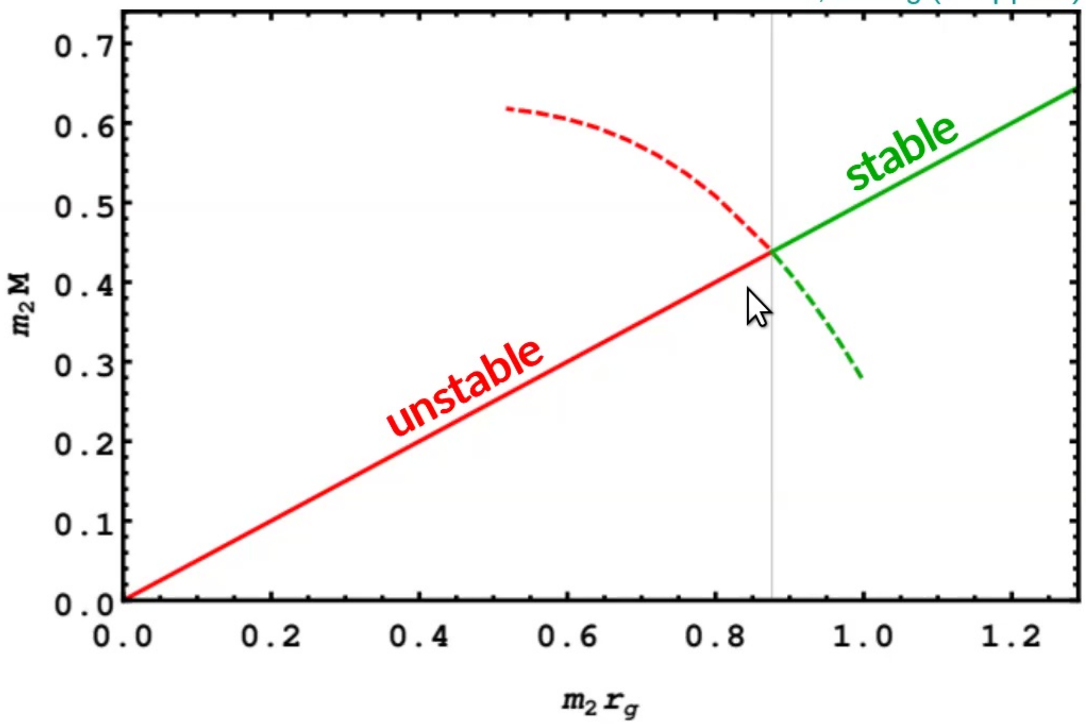
$$\mathcal{L}_{\rm EFT} = \frac{1}{16\pi\, G} R + \mathcal{O}(\text{curvature}^2)$$

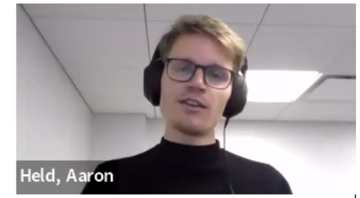
General Relativity (GR)

$$= \frac{1}{16\pi G} R + \alpha R_{ab} R^{ab} - \beta R^2 + \mathcal{O}(\text{curvature}^3)$$

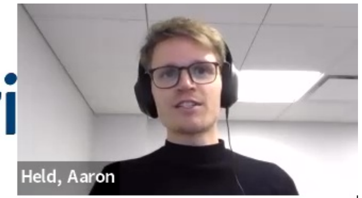
$$= \ldots$$

cf. also Lu, Perkins, Pope, Stelle '17
Held, Zhang (to appear)

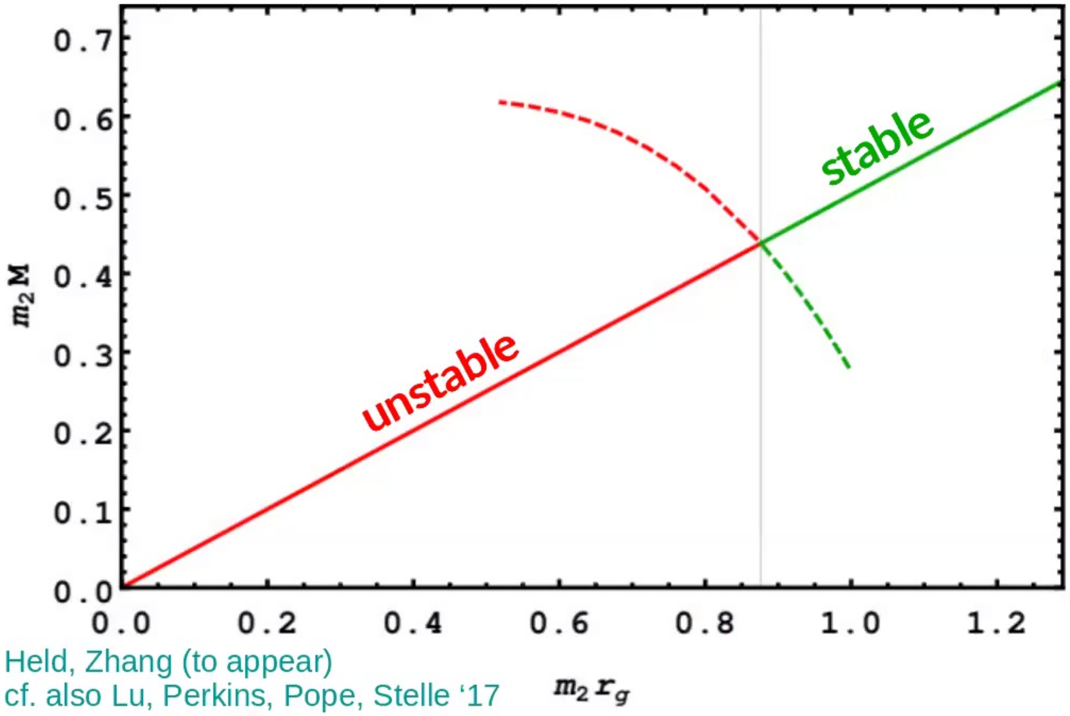




Part II. Linear stability of spherically-symmetri[c] black holes in quadratic gravity



cf. also Brito et.Al Phys. Rev. D 88, (2013)
Collingbourne Math. Phys. 62, (2021)
Held, Zhang (to appear)



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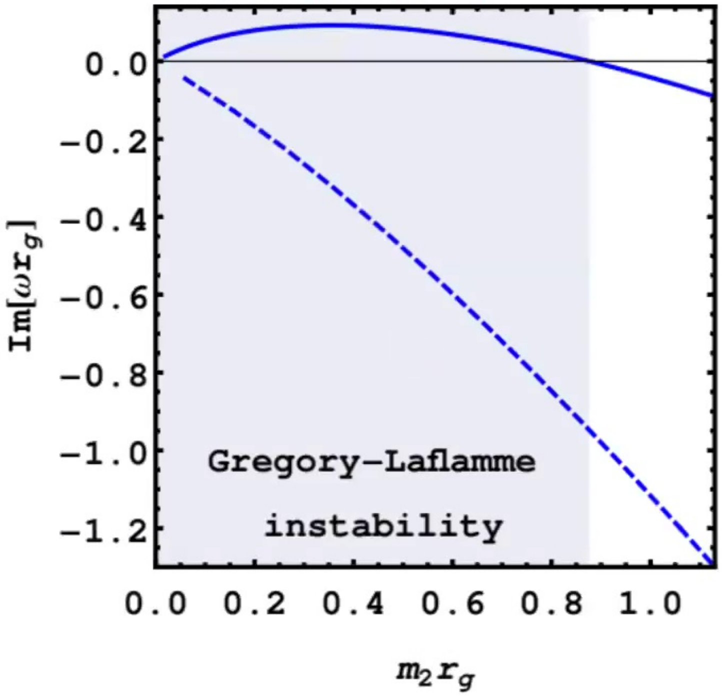

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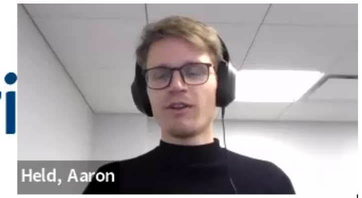

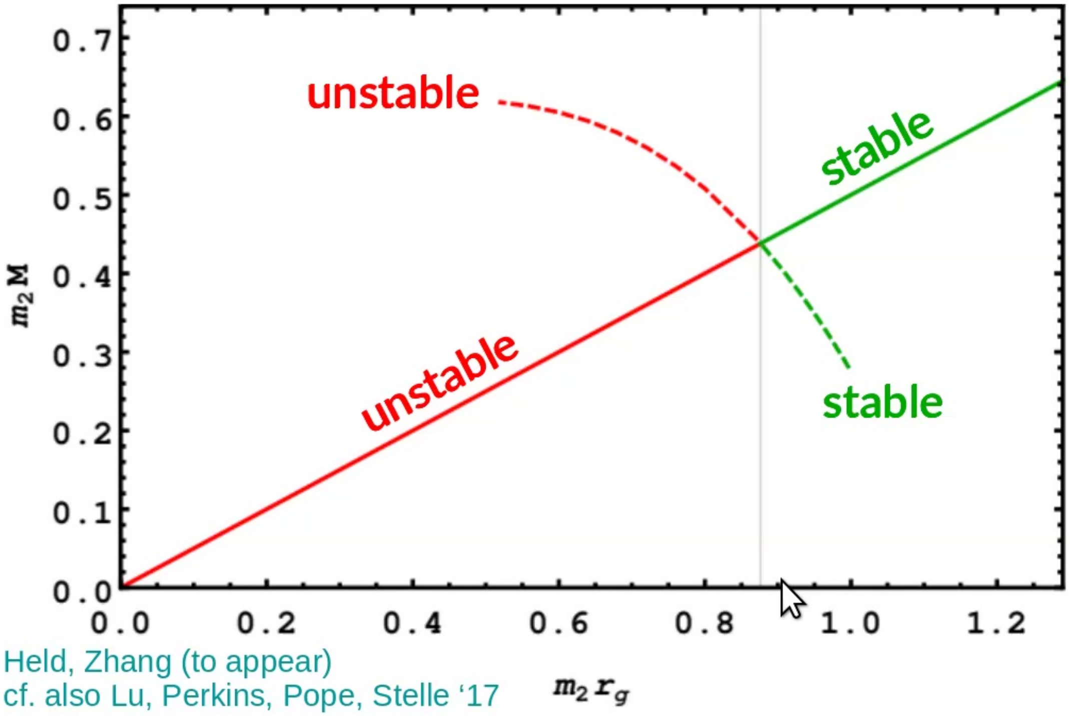

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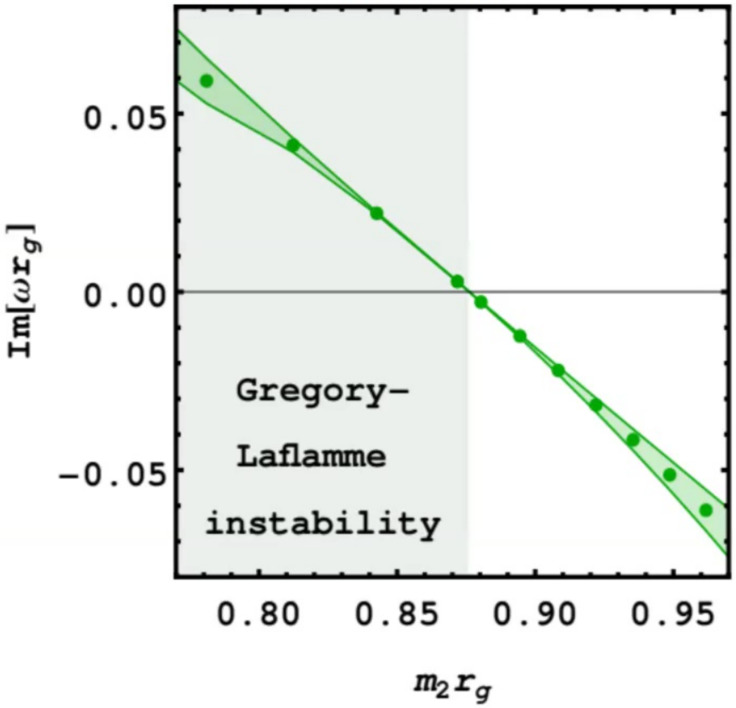
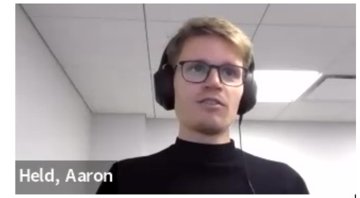


Part II.

Gracia-Saenz, **Held**, Zhang,
PRL 127 (2021) 13

Destabilization of Black Holes and Stars by Generalized Proca Fields



$$S[g, A] = \int d^4x\sqrt{-g}\Bigg[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\mu^2}{2}A^\mu A_\mu + G_{4,X}A^\mu A^\nu G_{\mu\nu} - \frac{G_6}{4}\Big(F^{\mu\nu}F_{\mu\nu}R - 4F^{\mu\rho}F^{\nu}{}_{\rho}R_{\mu\nu} + F^{\mu\nu}F^{\rho\sigma}R_{\mu\nu\rho\sigma}\Big)\Bigg]$$

see also Heisenberg '14
for motivation from generalized Proca

Part III.
Non-linear evolution in Quadratic Gravity

spherical symmetry: **Held**, Lim, PRD 104 (2021) 8
(3+1): **Held**, Lim, (to appear)

Effective field theory of gravity

$$\mathcal{L}_{\mathrm{EFT}} = \underbrace{\frac{1}{16\pi\,G}R}_{\text{General Relativity (GR)}} + \mathcal{O}(\mathrm{curvature}^2)$$

$$= \frac{1}{16\pi G}R + \alpha R_{ab}R^{ab} - \beta R^2 + \mathcal{O}(\mathrm{curvature}^3)$$

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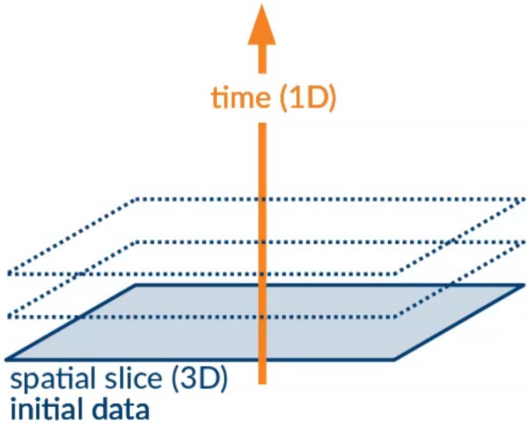
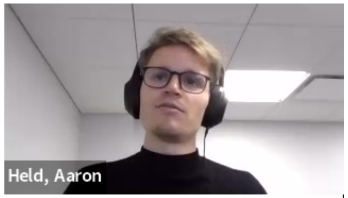
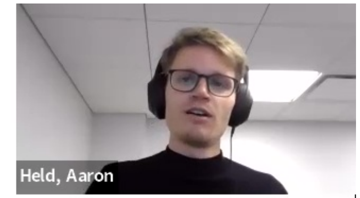
A well-posed initial value problem (IVP) ...





" An initial value problem is well-posed if a solution "

- **exists** for all future time
- is **unique**
- and depends **continuously** on the initial data



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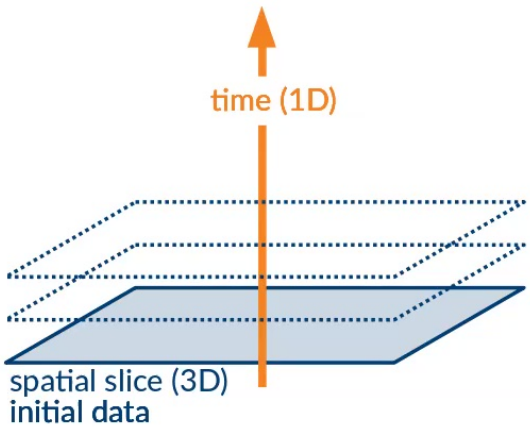

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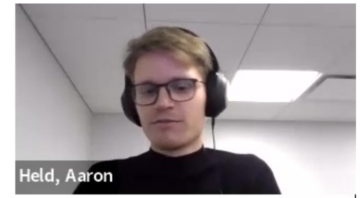
… for General Relativity

Formal proof of existence and uniqueness
Yvonne Choquet-Bruhat ‘52



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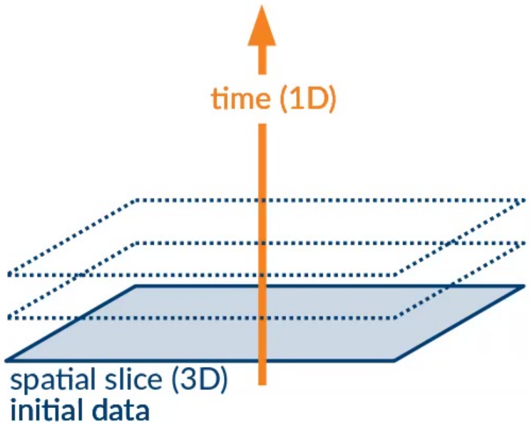

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(3+1) numerical evolution
Frans Pretorius ‘05
Baumgarte, Shapiro, Shibata, Nakamura ‘87-’99
Sarbach et.Al ‘02-’04

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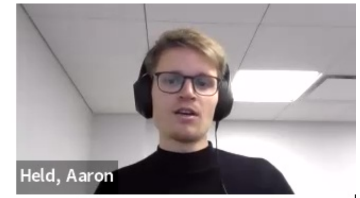

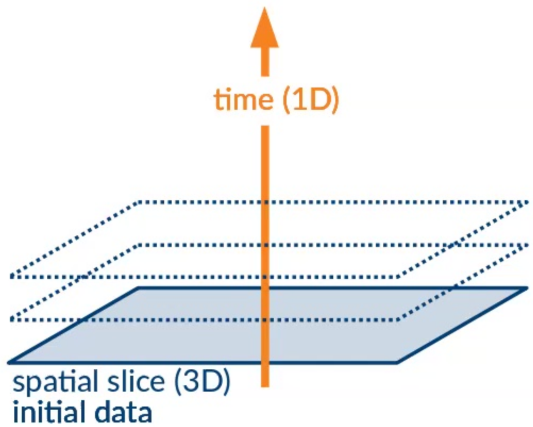

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... for Quadratic Gravity

Formal proof of existence and uniqueness
Noakes '83

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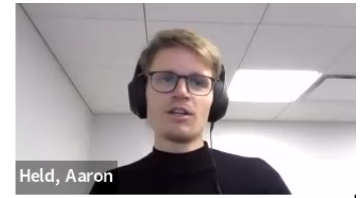

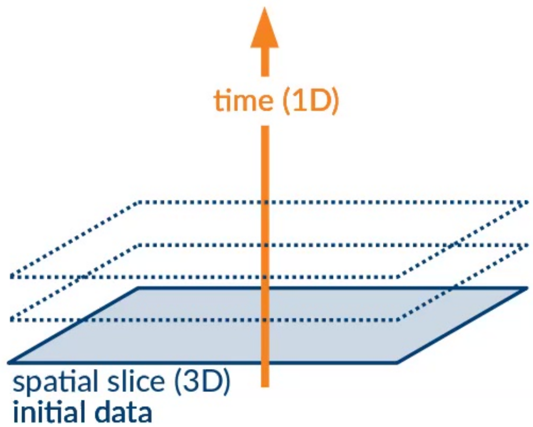

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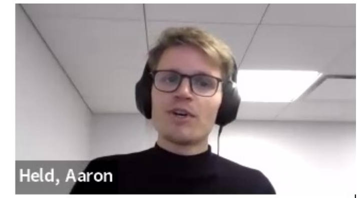
spherical symmetry: **Held**, Lim, PRD 104 (2021) 8
(3+1): **Held**, Lim, (to appear)

Well-posed IVP in Quadratic Gravity



Noakes, JMP 24, 1846 (1983)

4^{th} **order, quasilinear** equations of motion

Step 1: use linear DOFs in harmonic coords to cast to 2^{nd} order

$$g^{\alpha\beta}\partial_\alpha\partial_\beta g_{\mu\nu} = \mathcal{F}_1(g, \mathcal{R}, \widetilde{\mathcal{R}}, \partial) \quad \text{(massless spin-2)}$$
$$\Box\mathcal{R} = \mathcal{F}_2(g, \mathcal{R}, \widetilde{\mathcal{R}}, \partial) \quad \text{(massive spin-0)}$$
$$\Box\widetilde{\mathcal{R}}_{\mu\nu} = \mathcal{F}_3(g, \mathcal{R}, \widetilde{\mathcal{R}}, \partial) \quad \text{(massive spin-2)}$$

2^{nd} **order, quasilinear** + constraints

Step 2: diagonalize by adding appropriate derivatives

$$g^{\alpha\beta}\partial_\alpha\partial_\beta g_{\mu\nu} = \mathcal{F}_1(g, \mathcal{R}, \widetilde{\mathcal{R}}, h, V, \partial) \quad \text{(massless spin-2)}$$
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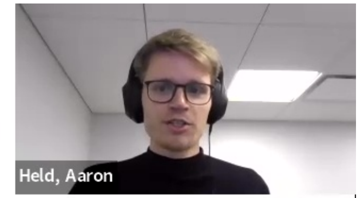
$$\Box h_{\mu\nu\gamma} = \mathcal{F}_4(g, \mathcal{R}, \widetilde{\mathcal{R}}, h, V, \partial) \quad (h_{\mu\nu\alpha} \equiv \partial_\alpha g_{\mu\nu})$$
$$\Box V_\mu = \mathcal{F}_5(g, \mathcal{R}, \widetilde{\mathcal{R}}, h, V, \partial) \quad (V_\alpha \equiv \partial_\alpha \mathcal{R})$$

2^{nd} order, quasilinear, diagonal + constraints

Well-posed IVP in Quadratic Gravity



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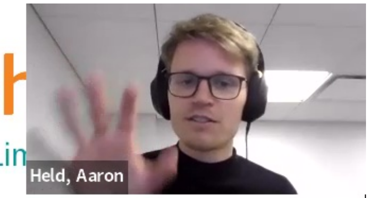
2^{nd} **order, quasilinear, diagonal** + constraints

Leray '53, Choquet-Bruhat et.Al '77

Leray's theorem guarantees well-posed IVP for $\mathcal{C}^\infty$ initial data

Numerical Evolution of Quadratic Gravity (sph

Held, Lin



Cartoon method to reduce to spherical symmetry

$$\mathbf{u} = (\mathrm{R},\, \mathrm{g_{tt}},\, \mathrm{g_{tx}},\, \mathrm{g_{xx}},\, \mathrm{g_{yy}}) \qquad \partial_t^2 \mathbf{u} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \partial_t \mathbf{u}\right)$$

$$\mathbf{v} = (\widetilde{\mathrm{R}}_{\mathrm{tt}},\, \widetilde{\mathrm{R}}_{\mathrm{tx}},\, \widetilde{\mathrm{R}}_{\mathrm{xx}}) \qquad \partial_t^2 \mathbf{v} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \partial_t \mathbf{u},\, \partial_t \mathbf{v},\, \partial_t^2 \mathbf{u}\right)$$

Diagonalization to quasi-linear 2nd-order form

$$\partial_t^2 \dot{\mathbf{u}} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \dot{\mathbf{u}},\, \partial_t \dot{\mathbf{u}},\, \partial_t \mathbf{v}\right) \qquad \partial_t \dot{\mathbf{u}} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \dot{\mathbf{u}}\right)$$

$$\partial_t^2 \mathbf{v} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \dot{\mathbf{u}},\, \partial_t \dot{\mathbf{u}},\, \partial_t \mathbf{v}\right) \qquad \partial_t \mathbf{u} \equiv \dot{\mathbf{u}}$$

Reduction to 1^{st} order in time

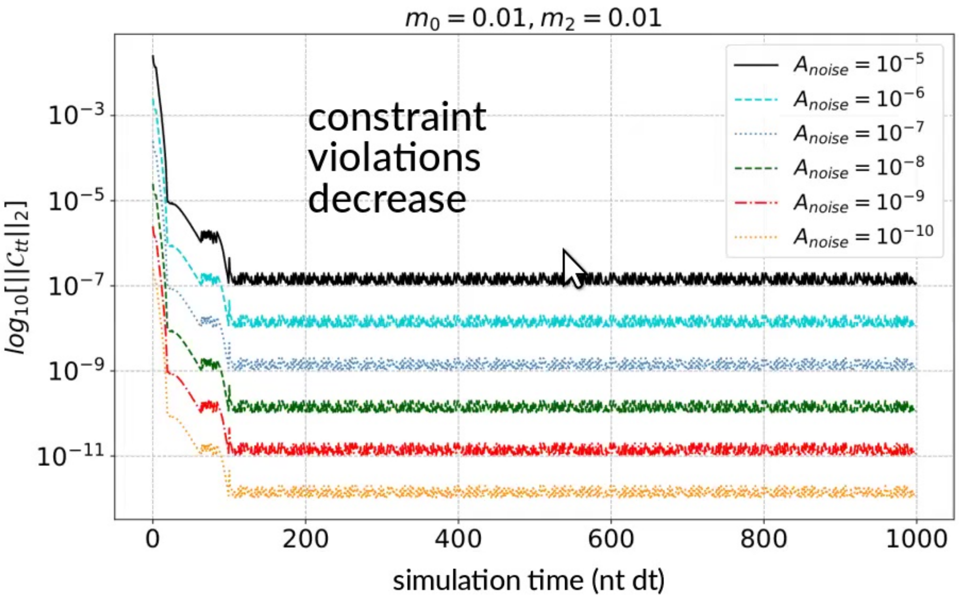
$$\partial_t \ddot{\mathbf{u}} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \dot{\mathbf{u}},\, \ddot{\mathbf{u}},\, \dot{\mathbf{v}}\right) \qquad \partial_t \dot{\mathbf{u}} \equiv \ddot{\mathbf{u}} \qquad \ddot{\mathbf{u}} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \dot{\mathbf{u}}\right)$$

$$\partial_t \dot{\mathbf{v}} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \dot{\mathbf{u}},\, \ddot{\mathbf{u}},\, \dot{\mathbf{v}}\right) \qquad \partial_t \mathbf{u} \equiv \dot{\mathbf{u}}$$

$$\partial_t \mathbf{v} \equiv \dot{\mathbf{v}}$$

Results: numerical stability …

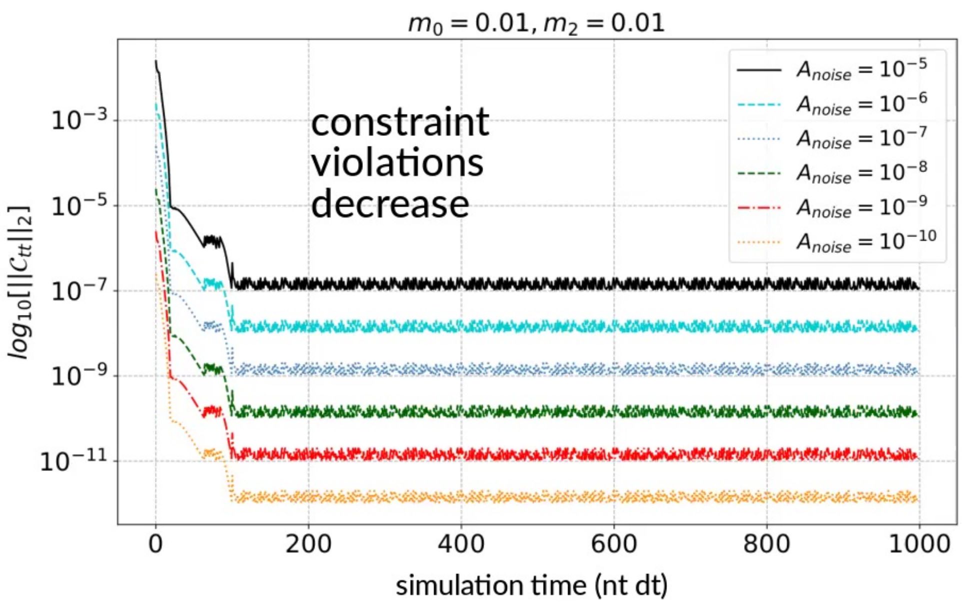
Held, Lim, PRD 104 (2021) 8

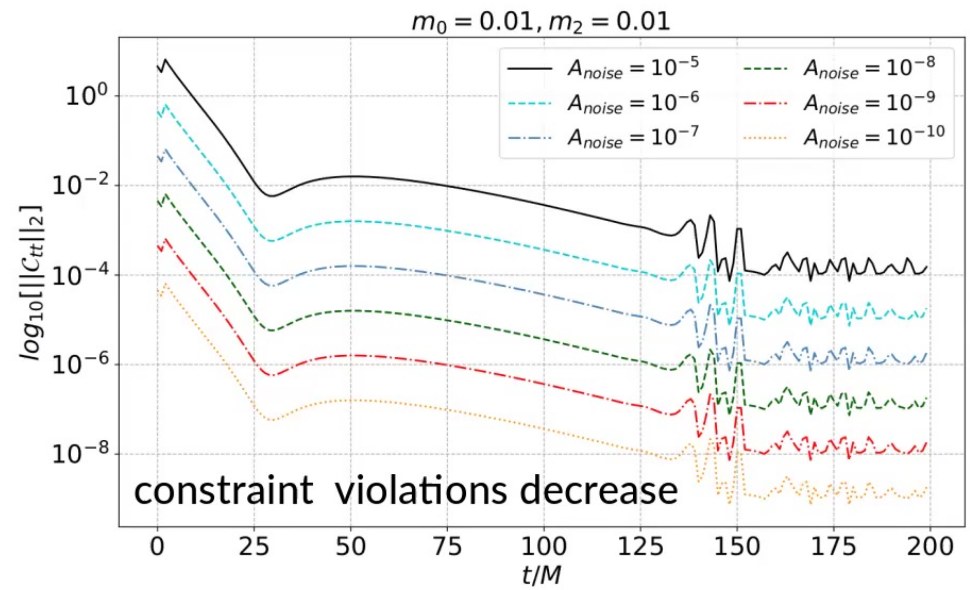


… in flat spacetime …

Results: numerical stability …

Held, Lim, PRD 104 (2021) 8

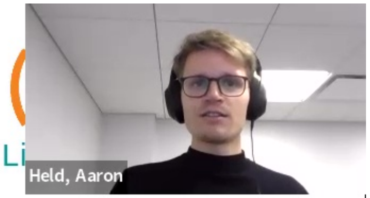




… in flat spacetime … … and about **Schwarzschild**

Numerical Evolution of Quadratic Gravity (

Held, Li



massless spin-2 $$R_{ab}(\Box g) = \widetilde{\mathcal{R}}_{ab} + \frac{1}{4} g_{ab}\mathcal{R} \equiv \widetilde{T}_{ab}$$

massive spin-0 $$\Box \mathcal{R} = m_0^2 \mathcal{R} + 2T^c{}_c$$

massive spin-2 $$\Box \widetilde{\mathcal{R}}_{ab} = -\frac{1}{3}\left(\frac{m_2^2}{m_0^2} - 1\right)\left(\nabla_a \nabla_b \mathcal{R} - \frac{1}{4} g_{ab} m_0^2 \mathcal{R}\right) + 2\widetilde{\mathcal{R}}^{cd} C_{acbd} + m_2^2 \widetilde{\mathcal{R}}_{ab} + 2T^{(TL)}_{ab} - \frac{1}{3}\left(\frac{m_2^2}{m_0^2} + 1\right)\mathcal{R}\widetilde{\mathcal{R}}_{ab} - 2\widetilde{\mathcal{R}}_a{}^c \widetilde{\mathcal{R}}_{bc} + \frac{1}{2} g_{ab} \widetilde{\mathcal{R}}^{cd} \widetilde{\mathcal{R}}_{cd}$$

1st-order variables

$$\widetilde{V}_{ab} \equiv -n^c \nabla_c \widetilde{\mathcal{R}}_{ab}$$

$$\hat{\mathcal{R}} \equiv -n^c \nabla_c \mathcal{R}$$

(3+1) decomposition

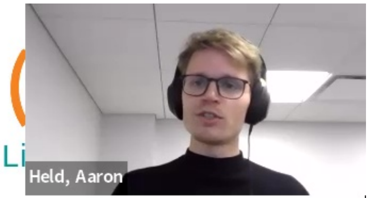
$$g_{ab} = \gamma_{ab} + n_a n_b$$

$$\widetilde{\mathcal{R}}_{ab} = \mathcal{A}_{ab} + \frac{1}{3}\gamma_{ab}\,\mathcal{A} - 2\,n_{(a}\mathcal{C}_{b)}$$

$$\widetilde{V}_{ab} = \mathcal{B}_{ab} + \frac{1}{3}\gamma_{ab}\,\mathcal{B} - 2\,n_{(a}\mathcal{E}_{b)}$$

Numerical Evolution of Quadratic Gravity (

Held, Li



ADM

massless spin-2 $$R_{ab}(\Box g) = \tilde{\mathcal{R}}_{ab} + \frac{1}{4} g_{ab}\mathcal{R} \equiv \tilde{T}_{ab}$$

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$$+ m_2^2 \tilde{\mathcal{R}}_{ab} + 2T^{(TL)}_{ab} - \frac{1}{3}\left(\frac{m_2^2}{m_0^2} + 1\right)\mathcal{R}\tilde{\mathcal{R}}_{ab} - 2\tilde{\mathcal{R}}_a{}^c \tilde{\mathcal{R}}_{bc} + \frac{1}{2} g_{ab} \tilde{\mathcal{R}}^{cd} \tilde{\mathcal{R}}_{cd}$$

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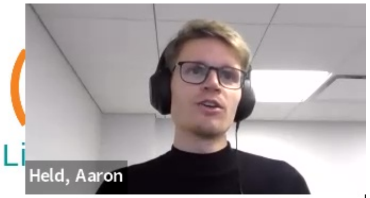
Numerical Evolution of Quadratic Gravity (

Held, Li



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massive spin-0

$$n^a\nabla_a\mathcal{R} = \mathcal{O}$$

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$$0 = D_jK^j_i - D_iK + \mathcal{C}_i$$

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constraint evolution

$$n^c\nabla_c\mathcal{C}_i = -\mathcal{E}_i + \mathcal{O}_i \qquad n^c\nabla_c\mathcal{E}_i = \ \dots$$

massive spin-2

$$n^c\nabla_c\mathcal{A} = \mathcal{O}$$

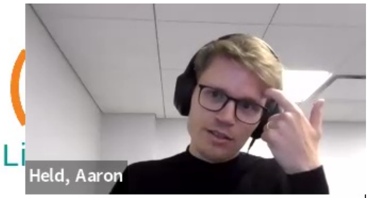
$$n^c\nabla_c\mathcal{A}_{ij} = \frac{2}{3}\mathcal{A}\,D_{(i}n_{j)} + \mathcal{O}_{ij}$$

$$n^c\nabla_c\mathcal{B} = +2\left(\mathcal{A}^{ij} + \frac{1}{3}\gamma^{ij}\mathcal{A}\right){}^{(3)}R_{ij} - \frac{1}{3}\left(\frac{m_2^2}{m_0^2}+1\right)D_iD^i\mathcal{R} - D_iD^i\mathcal{A} + 2\,a^k\mathcal{E}_k - a_iD^i\mathcal{A} + 4\,\mathcal{C}^j\left(D^iK_{ij} - D_jK\right) + \mathcal{O}$$

$$n^c\nabla_c\mathcal{B}_{ij} = +2\left(\mathcal{A}^{kl} + \frac{1}{3}\gamma^{kl}\mathcal{A}\right){}^{(3)}R_{ikjl} - \frac{1}{3}\left(\frac{m_2^2}{m_0^2}+1\right)D_iD_j\mathcal{R} - \left(D_kD^k + a_kD^k\right)\left(\mathcal{A}_{ij} + \frac{1}{3}\gamma_{ij}\mathcal{A}\right) + \frac{2}{3}\mathcal{B}\,D_{(i}n_{j)} + 2\,a^c\,\gamma_{c(i}\mathcal{E}_{j)} - \frac{1}{3}\gamma_{ij}\left(n^c\nabla_c\mathcal{B}\right) + 4\,\mathcal{C}^k\left(D_{[i}K_{k]j} + D_{[j}K_{k]i}\right) + \mathcal{O}_{ij}$$

Numerical Evolution of Quadratic Gravity (

Held, Li



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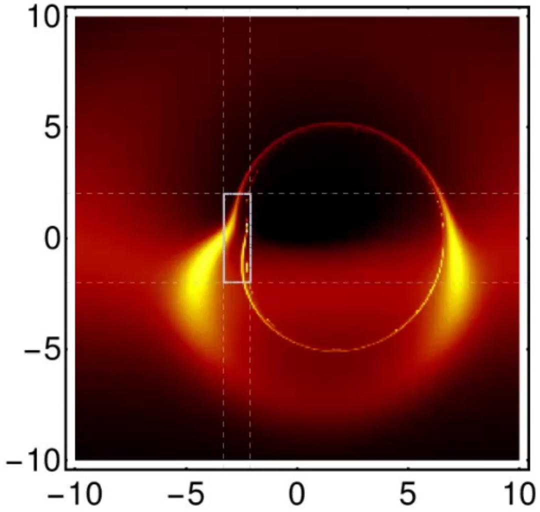
- massive spin-0/spin-2 do **not impact** massless spin-2 **principal part**
- constraint evolution **preserves BSSN**
- amenable to **1st-order strong-hyperbolicity analysis**

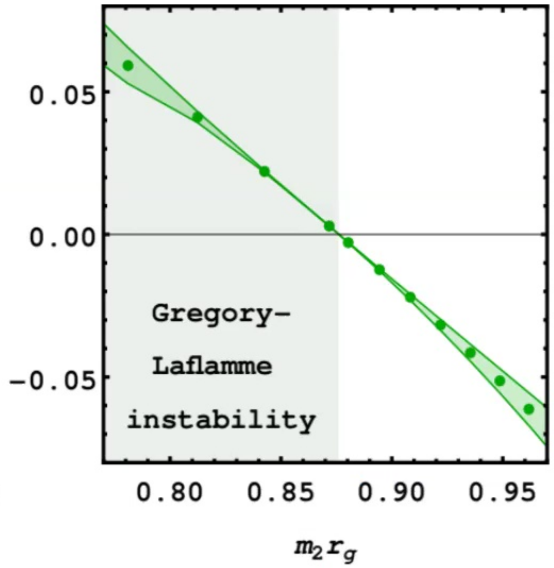
Sarbach et.Al '02-'04 (for GR)

Probing black holes …

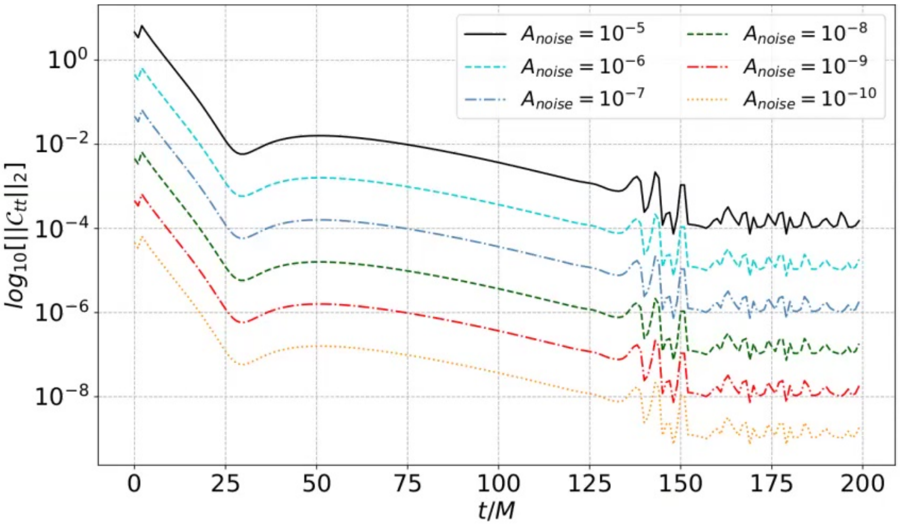
I. background spacetimes



II. linear dynamics



III. non-linear dynamics



… in the EFT of gravity

Numerical Evolution of Quadratic Gravity (3+1)

Held, Lim (ongoing)

massless spin-2

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Dynamics: linear DoF

$$\mathcal{L}_{QG} = \frac{1}{16\pi G} R + \alpha R_{ab} R^{ab} - \beta R^2$$

$$\mathcal{L}_{QG} = \frac{1}{M_{Pl}^2} \left[\frac{1}{2} R + \frac{1}{12 m_0^2} R^2 + \frac{1}{4 m_2^2} C_{abcd} C^{abcd} \right]$$

- massless spin-2 (graviton) h_{ab}
- massive spin-0 ϕ
- massive spin-2 ψ_{ab}

Decomposition (background)

- spherical harmonics $Y_{\ell m}(\theta, \phi)$
- axisymmetric perturbations $m = 0$
- focus on the monopole $\ell = 0$

$$h_{ab}^{(polar)} = e^{-i\omega t} \begin{pmatrix} A H_0 & H_1 & 0 & 0 \\ H_1 & H_2/B & 0 & 0 \\ 0 & 0 & r^2 \mathcal{K} & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta\, \mathcal{K} \end{pmatrix} Y^{\ell=0}$$

$$\psi_{ab}^{(polar)} = e^{-i\omega t} \begin{pmatrix} A F_0 & F_1 & 0 & 0 \\ F_1 & F_2/B & 0 & 0 \\ 0 & 0 & \mathcal{M} & 0 \\ 0 & 0 & 0 & \sin^2\theta\, \mathcal{M} \end{pmatrix} Y^{\ell=0}$$

$$\frac{d^2}{dr_*^2} \psi(r) + \psi(r) \left[\omega^2 - V(r) \right] = 0$$

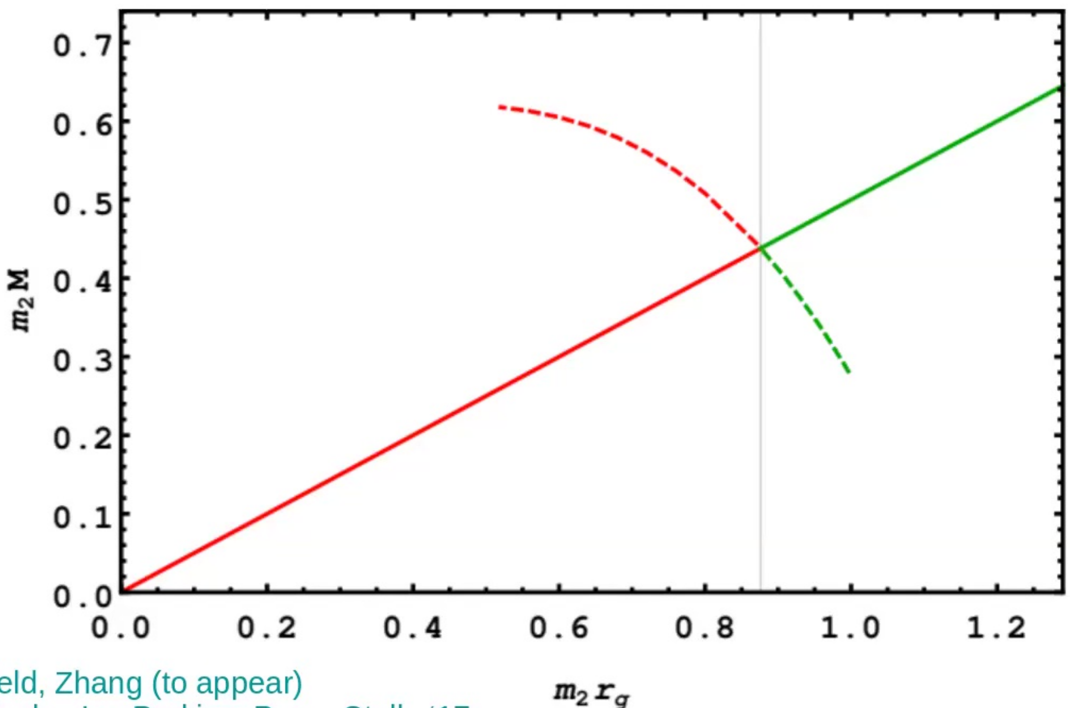
GR-background: Brito, Cardoso, Pani '13
non-GR: Held, Zhang (to appear)

Boundary conditions:

- purely ingoing waves at the horizon
- outgoing waves at asymptotic infinity define QNMs
- ingoing waves at asymptotic infinity define bound states

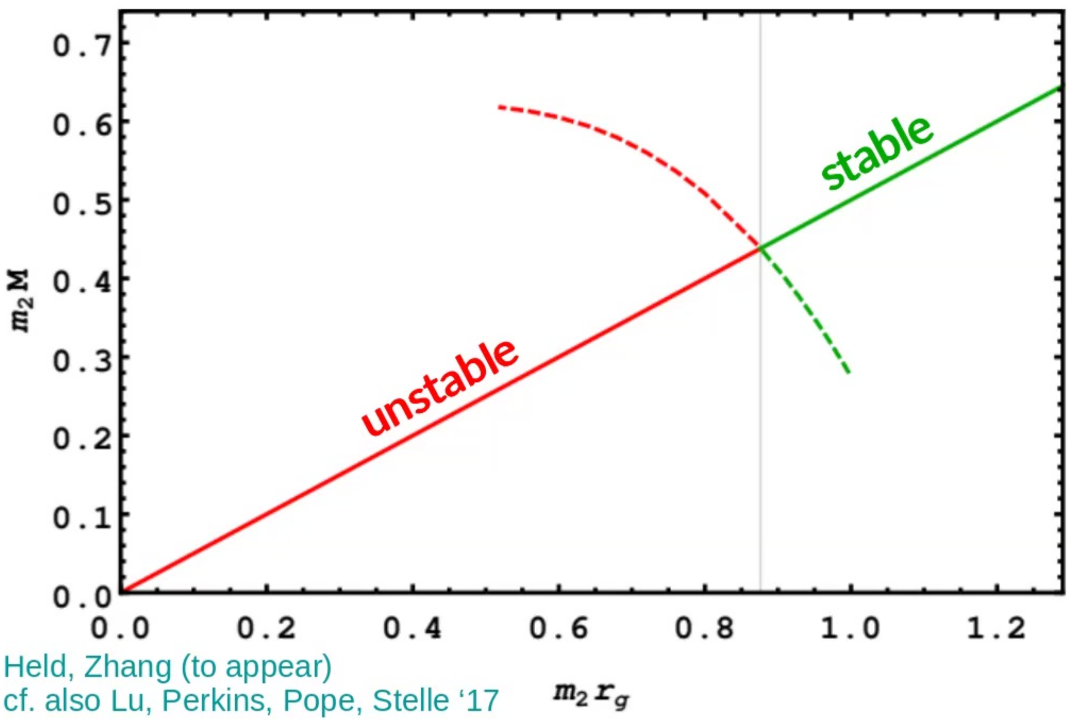
- positive imaginary part signals instability
- negative imaginary part signals stability

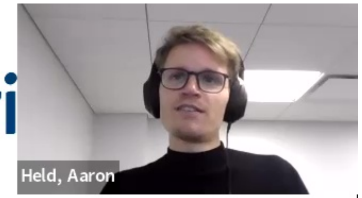
Part II. Linear stability of spherically-symmetric black holes in quadratic gravity



Held, Zhang (to appear)
cf. also Lu, Perkins, Pope, Stelle ‘17

Part II. Linear stability of spherically-symmetri[c] black holes in quadratic gravity





cf. also Brito et.Al Phys. Rev. D 88, (2013)
Collingbourne Math. Phys. 62, (2021)
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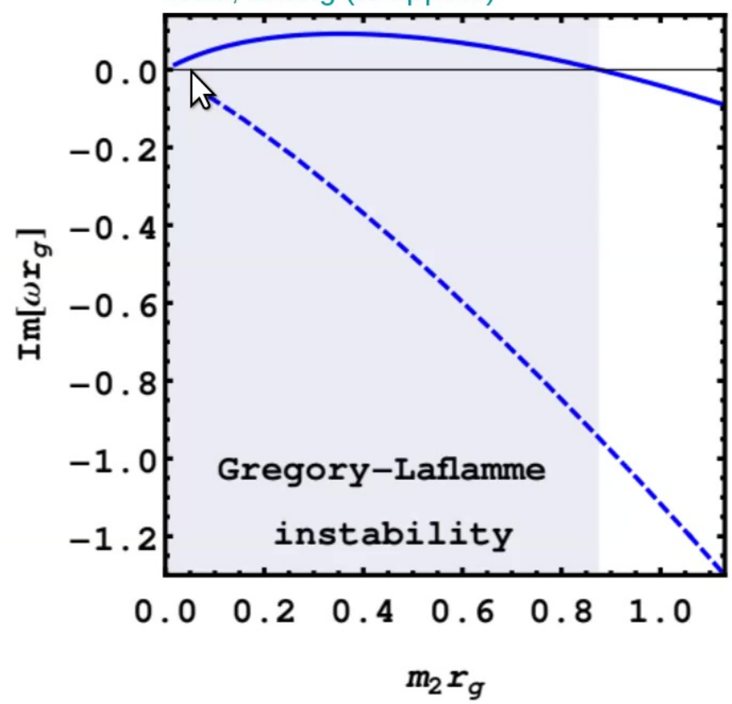

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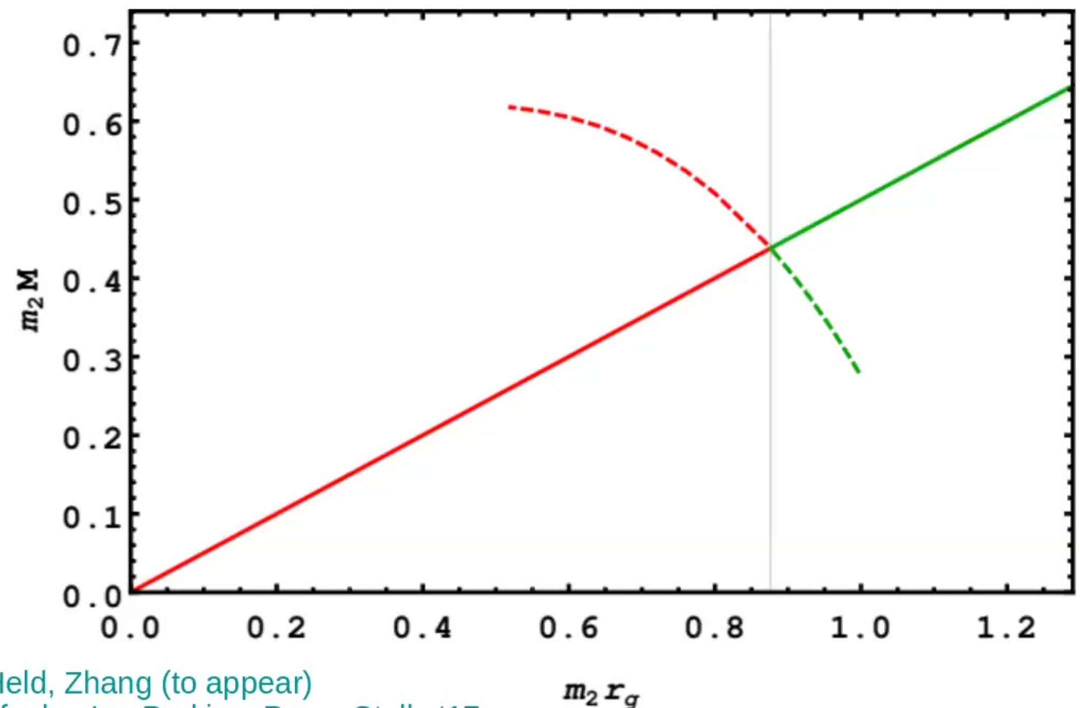

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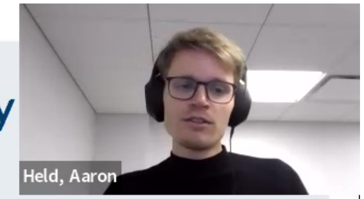

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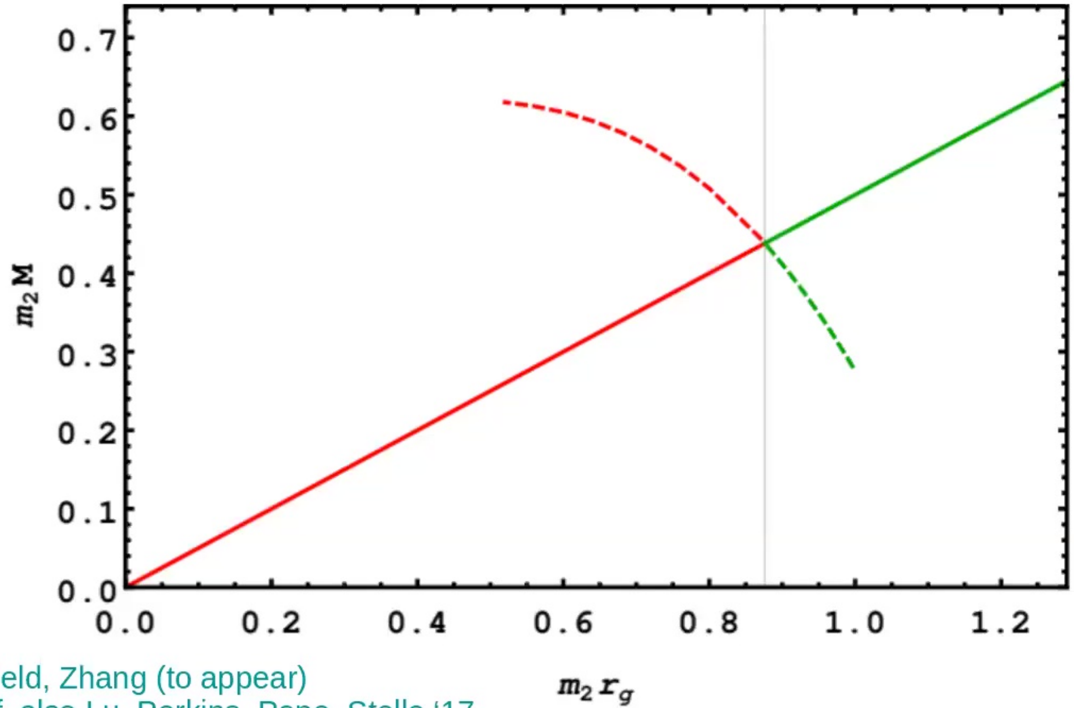
Part II. Linear stability of spherically-symmetri black holes in quadratic gravity



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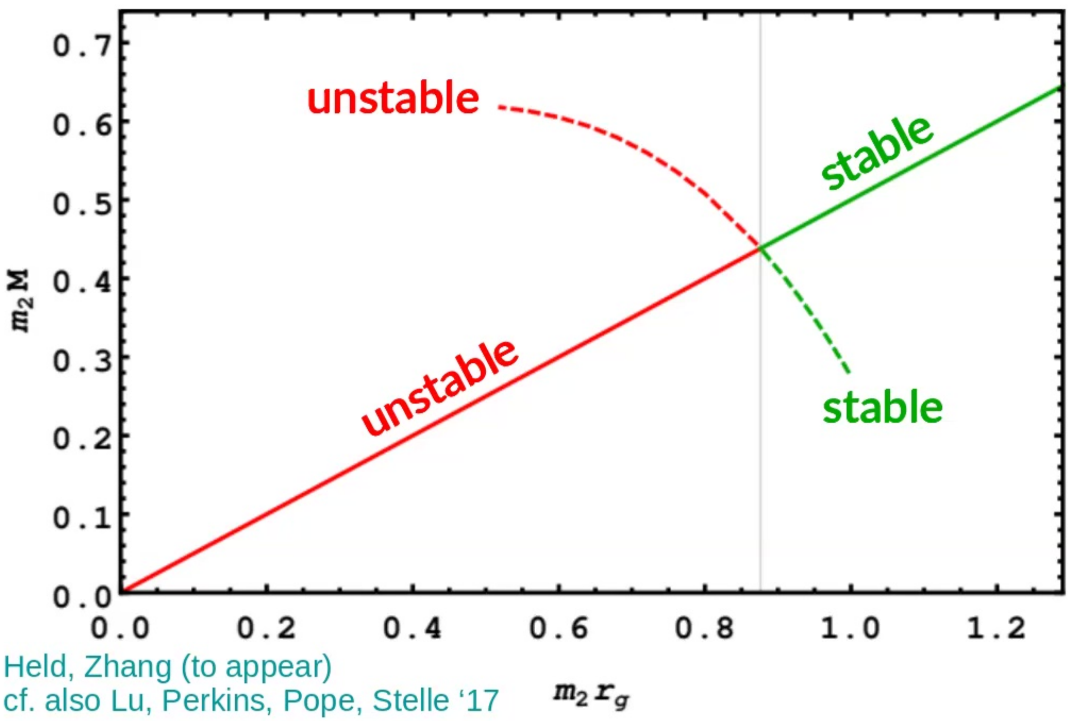

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