Title: Realizing a dynamical topological phase without symmetry protection in trapped ions

Speakers: Andrew Potter

Series: Quantum Matter

Date: December 06, 2021 - 12:00 PM

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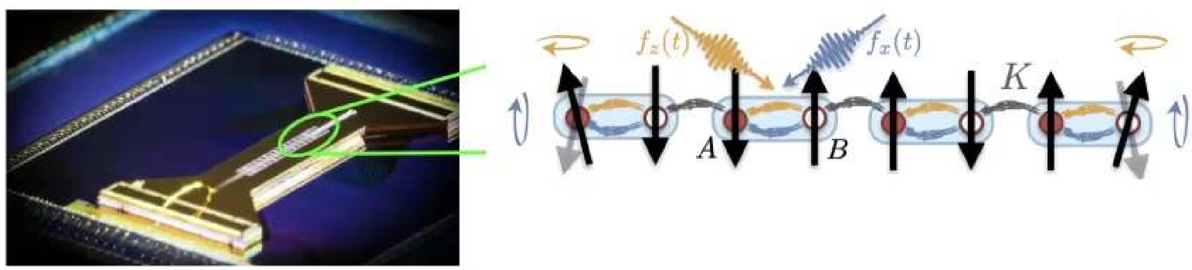
Abstract: In thermal equilibrium, 1d bosonic systems (e.g. spin- or qubit- chains) cannot support intrinsically topological phases without symmetry protection. For example, the edge states of the Haldane spin chain are fragile to magnetic fields, in contrast to the absolutely stable Majorana edge states of a topological superconducting wire of fermionic electrons. This fragility is a serious drawback to harnessing topological edge states as protected quantum memories in existing AMO and qubit platforms for quantum simulation and information processing. In this talk, I will present evidence for a non-equilibrium topological phase of quasiperiodically-driven trapped ion chains, that exhibits topological edge states that are protected purely by emergent dynamical symmetries that cannot be broken by microscopic perturbations. This represents both the first experimental realization of a non-equilibrium quantum phase, and the first example of a 1d bosonic topological phase that does not rely on symmetry-protection.

# Realizing a dynamical topological phase in a trapped ion chain

Andrew C. Potter (UBC)

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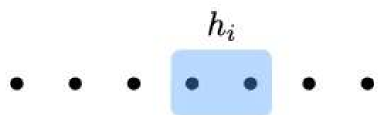
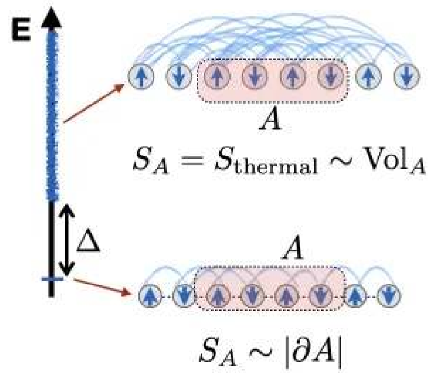
# Phases of Gapped Ground-States

Local, Gapped Hamiltonians:

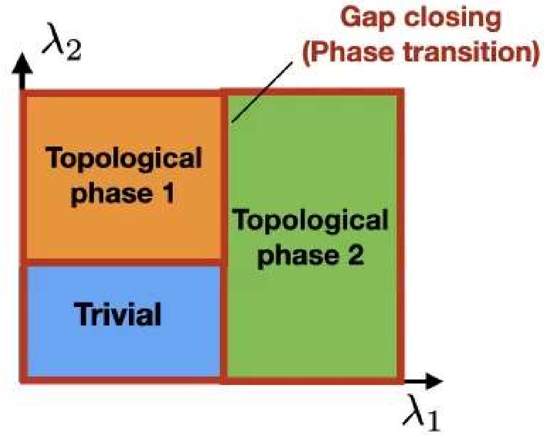
$$H(t) = \sum_i \lambda_i(t) h_i$$




$$H = \sum_i \lambda_i h_i$$

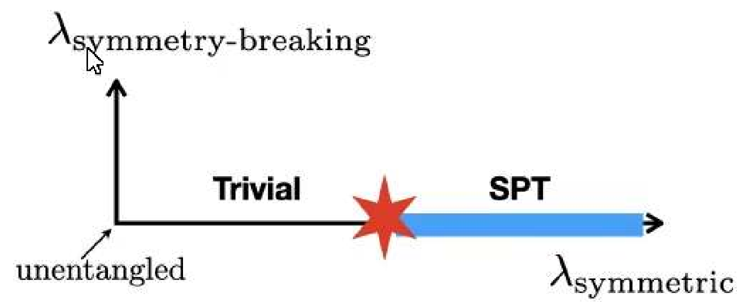



Topological Phases:

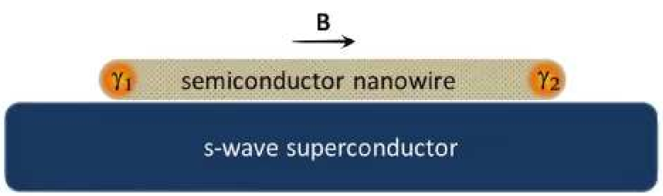



Symmetry-protected topological (SPT) phases:

**(Only possible phases for 1d bosons/spins/qubits)**

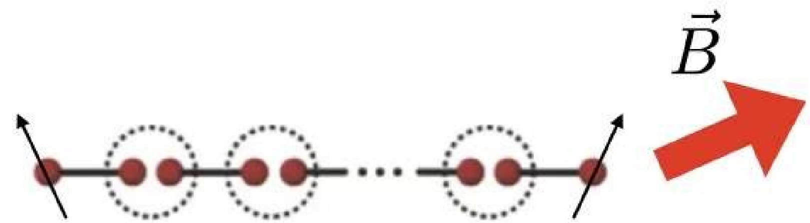


# Topological quantum memories



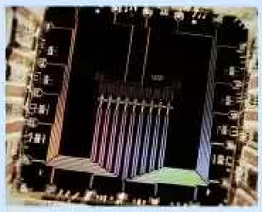

**Intrinsic topological (not-SPT)
superconducting wire
of electrons/fermions**

**Majorana edge states are immune to
local perturbations**



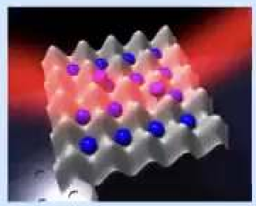

**SPT edge states are poor memories,
killed by any stray field!**

# Quantum Dynamics Toolbox

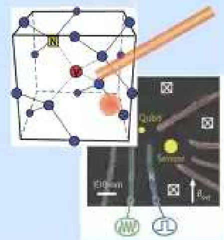


Superconducting Qubits

Trapped Ions

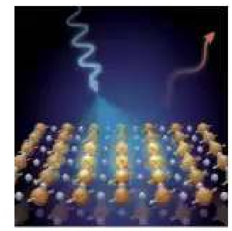


Neutral atoms & molecules



Spin Qubits & Vacancy Centers

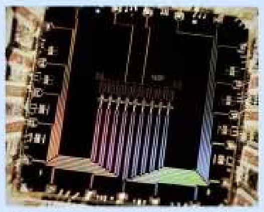
**Closed/Isolated (this talk)**



Ultrafast pump-probe methods

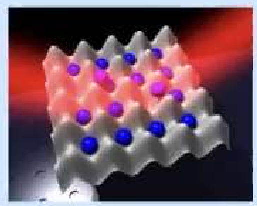
**Coherent Spatio-temporal control:** $H(t) = \sum_i \lambda_i(t) h_i$

# Quantum Dynamics Toolbox



**Superconducting Qubits**

**Trapped Ions**



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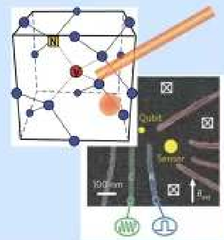

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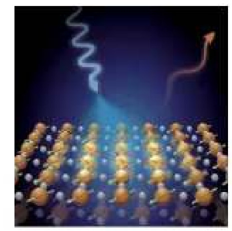


**Ultrafast pump-probe methods**

**Coherent Spatio-temporal control:** $H(t) = \sum_i \lambda_i(t) h_i$

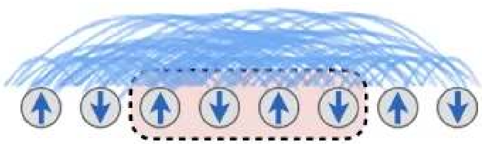
- **Can we find stable methods to create/protect/manipulate entanglement out-of-equilibrium?**
- **Are new dynamical phases possible?**

# What is a "phase" without a ground-state?

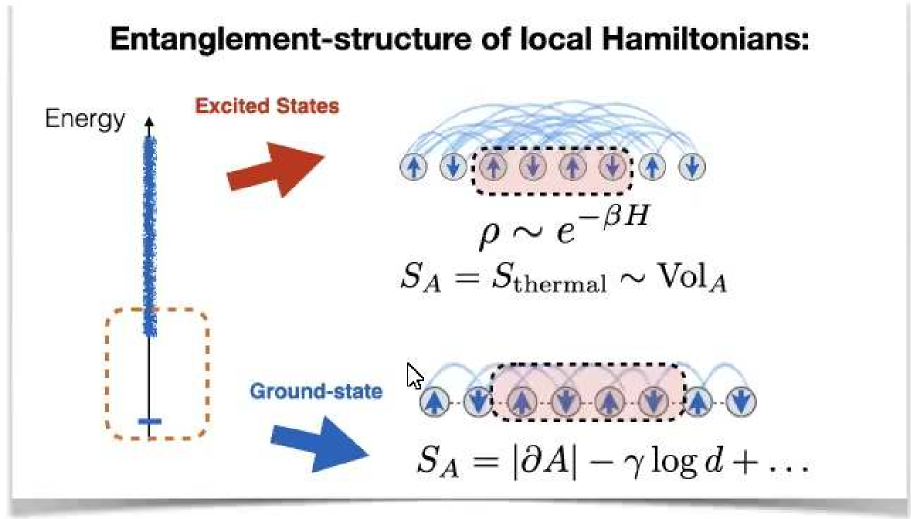
- Energy is not conserved => **No ground-state!**

$$H(t) = \sum_{\vec{n}} e^{-i\vec{\Omega}\cdot\vec{n}t} h_{\vec{n}} \qquad \varepsilon \simeq \varepsilon + 2\pi\vec{n}\cdot\vec{\Omega}$$

- W/ Generic interactions expect:
  Maximize entropy, heat to infinite temperature

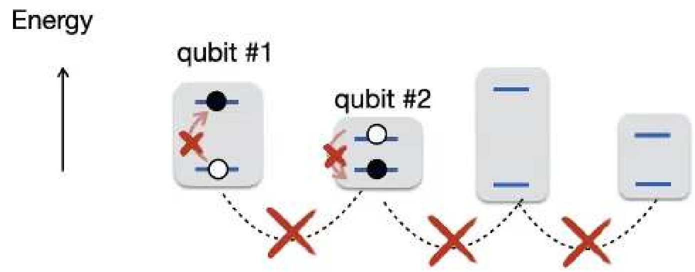


$$\rho \sim 1$$



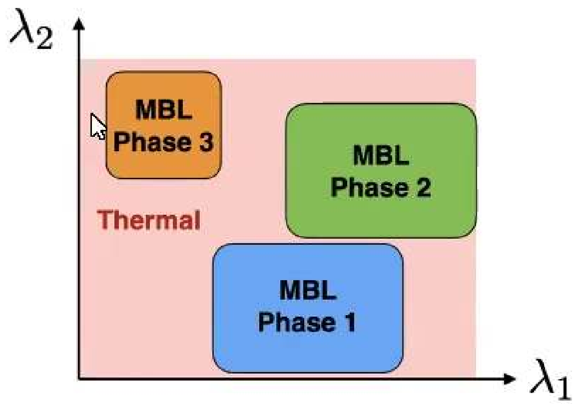
# Many-body localization (MBL)

**Strong Disorder + Isolation from environment => MBL**




- All eigenstates behave like ground-states <u>in the same phase</u>
- Properties of phase evident in quenches from generic initial states
  **(No need to cool or prepare an eigenstate)**

**Phases distinguished by delocalizing/ thermalizing transitions**




See e.g. arXiv:1804.11065 for review

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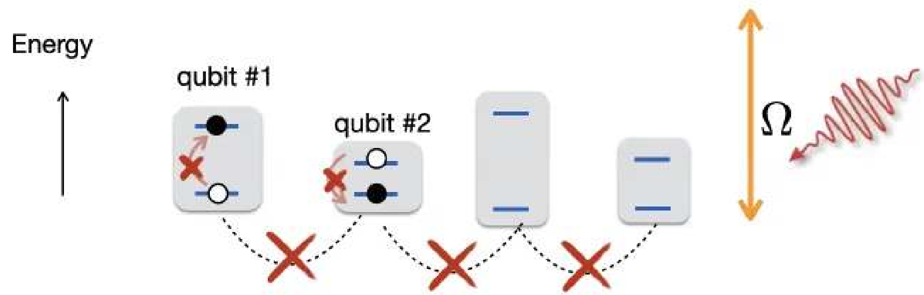


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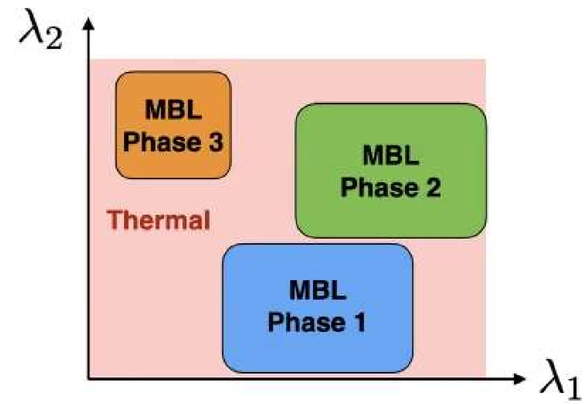



Driven MBL appears stable for:

- Periodic (1-tone, Floquet)
- Quasiperiodic (up to 2-tones) w/ smooth t-dependence

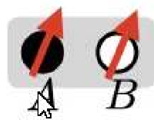
See e.g. arXiv:1804.11065 for review

Long, Crowley, Chandran

# SPT +MBL: Disordered AKLT Chain

Nandkishore, Sondhi, Huse; Bahri, Vishwanath, Altman; Bauer, Nayak, …

**Single "Site":**



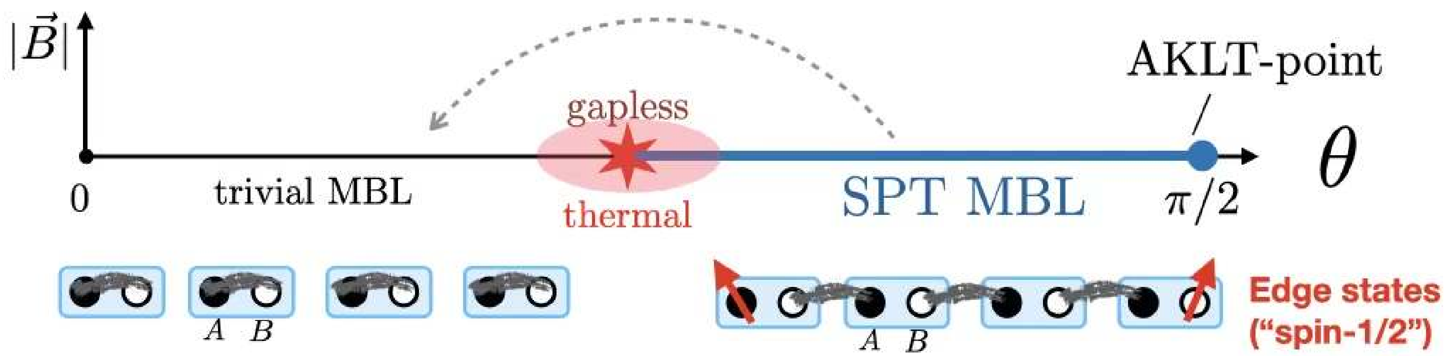


Symmetries: $R_x(\pi), R_z(\pi)$

group: $\mathbb{Z}_2^{(x)} \times \mathbb{Z}_2^{(z)}$

$$H = \sum_{i;\mu\in\{x,z\}} J_{\mu,i} \left[\cos\theta\sigma^{\mu}_{A,i}\sigma^{\mu}_{B,i} + \sin\theta\sigma^{\mu}_{B,i}\sigma^{\mu}_{A,i+1}\right]$$

$J_{\mu,i} \overset{\text{i.i.d.}}{\sim} [0, J]$



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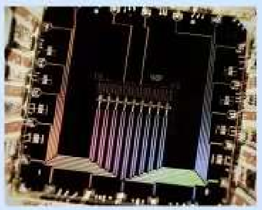

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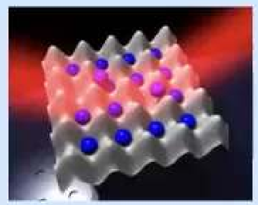

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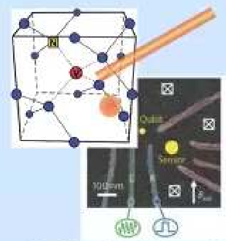

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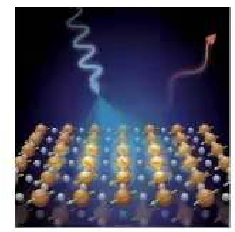
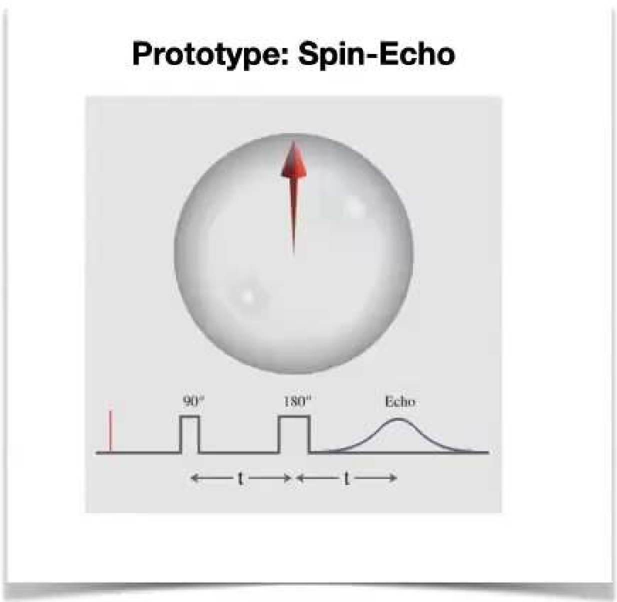
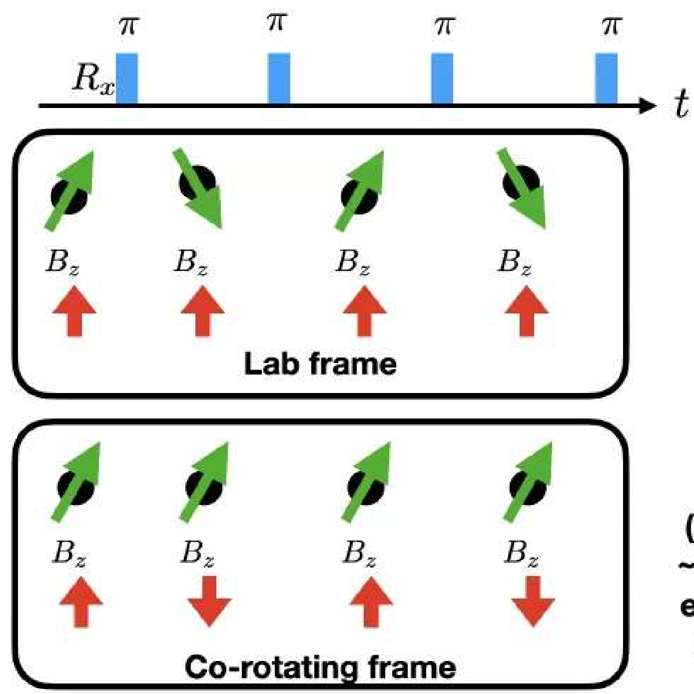


Ultrafast pump-probe methods

**Can we use MBL + time-dependent driving to remove reliance on symmetry protection for qubit chains?**

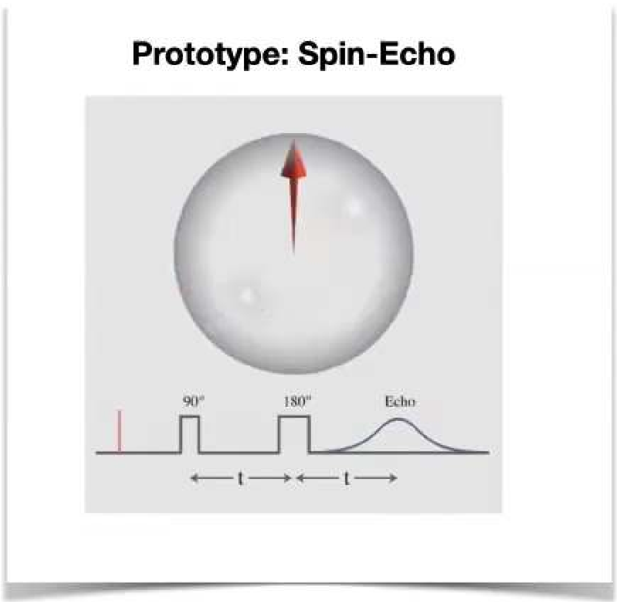
# Dynamical Decoupling

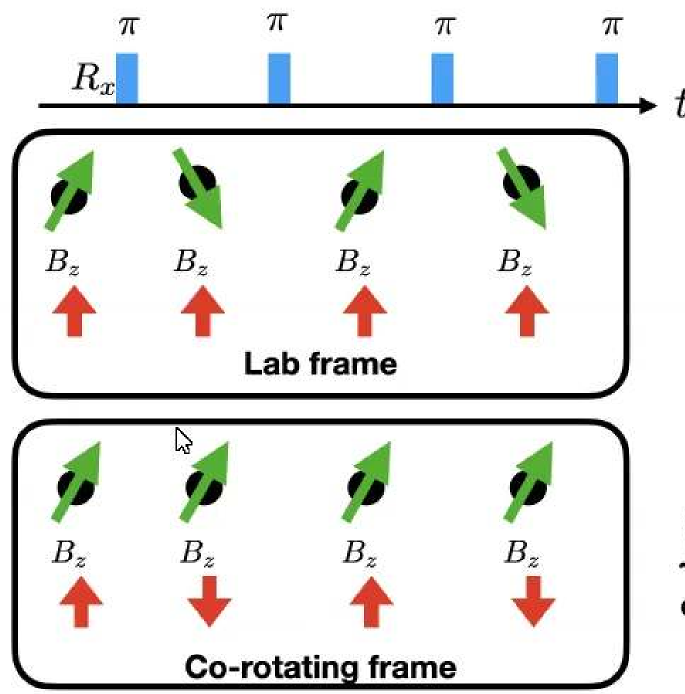



**(Static fields ~ cancel over even number of periods)**

# Dynamical Decoupling

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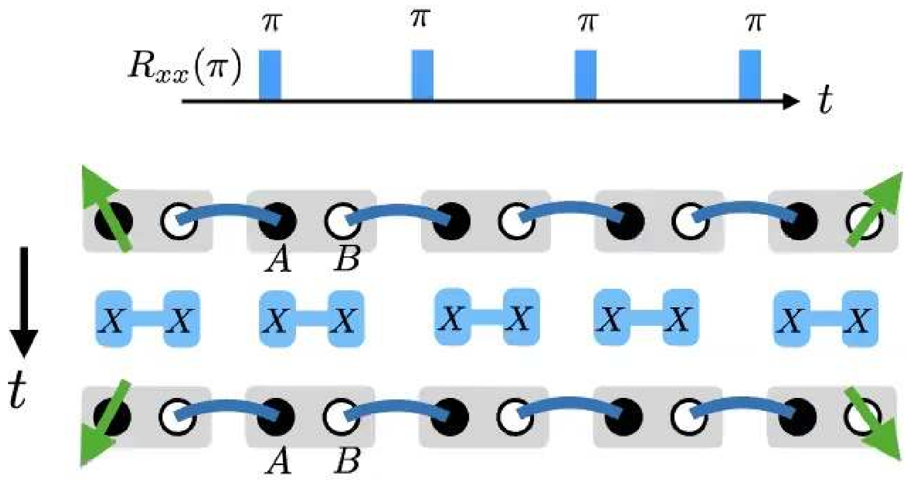
# Many-Body Dynamical Decoupling




$$R_{xx}(\theta) = \exp\left[-\frac{i}{2}\theta\sum_{i=1}^{L}\sigma^x_{A,i}\sigma^x_{B,i}\right]$$

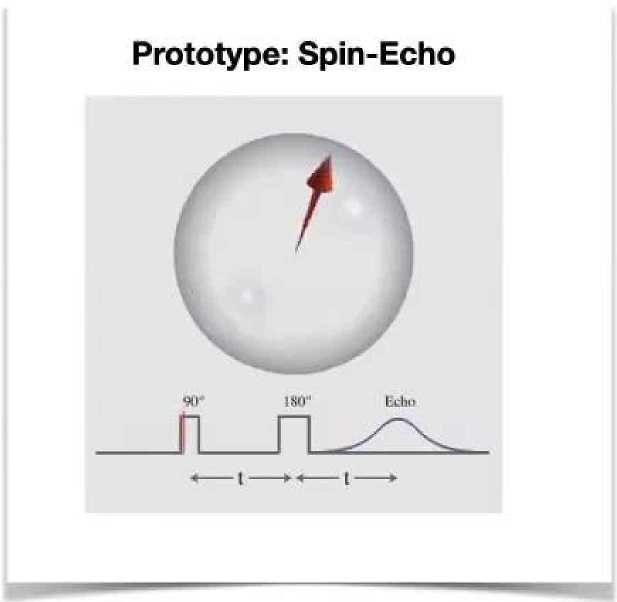
$$R_{xx}(\pi) = R_x(\pi)$$

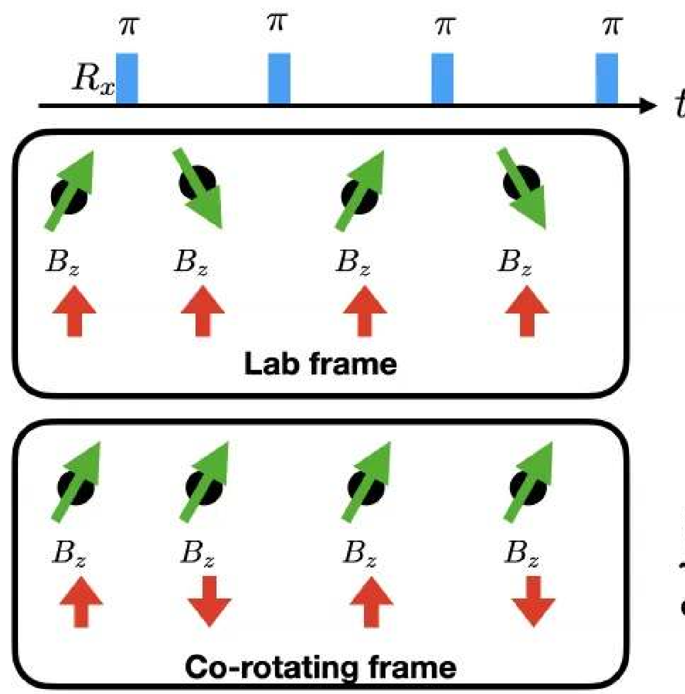
**Implements dynamical decoupling (DD) pulse**

$$[R_{xx}(\theta), R_z(\pi)] = 0\ \forall\theta$$

**Two-qubit DD Pulses required to preserve Rz symmetry**

# Dynamical Decoupling

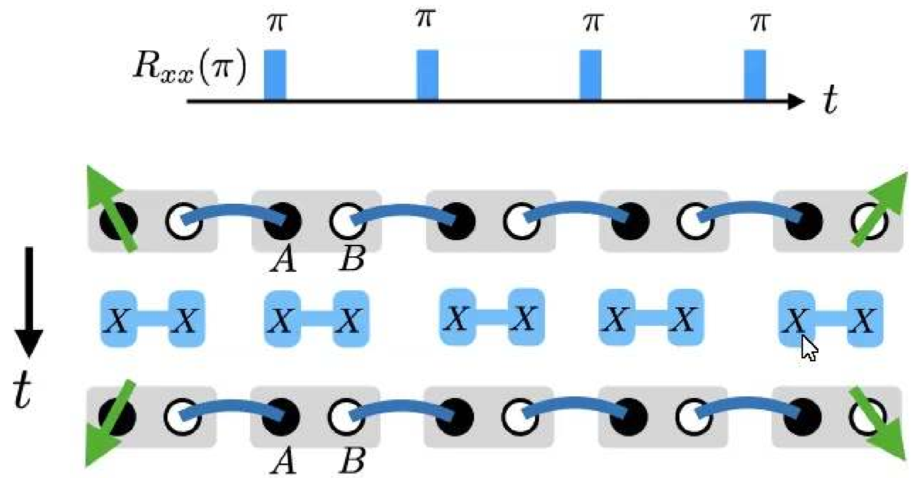



**(Static fields ~ cancel over even number of periods)**

1. **Fine-Tuned: Small errors in pulse weight will accumulate with time** $R_x(\pi) \to R_x(\pi - \epsilon)$
2. **Only cancels Bz,y (not Bx)**

# Many-Body Dynamical Decoupling

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$$R_{xx}(\pi) = R_x(\pi)$$

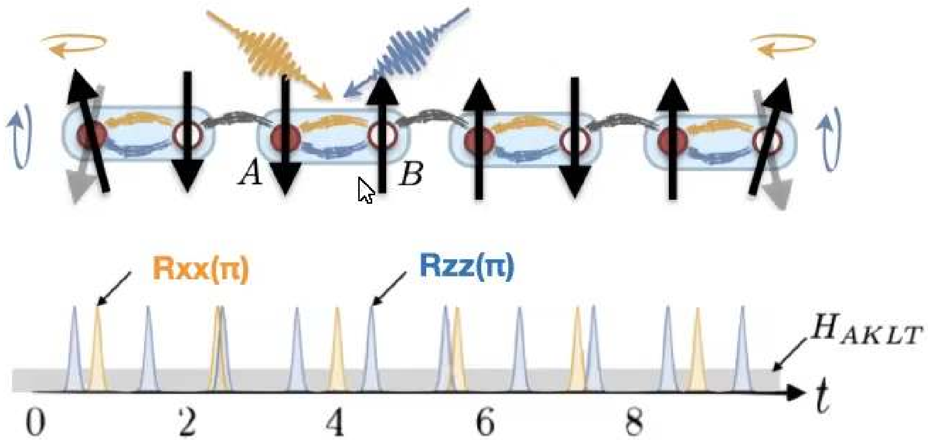
**Implements dynamical decoupling (DD) pulse**

$$[R_{xx}(\theta), R_z(\pi)] = 0 \;\forall\theta$$

**Two-qubit DD Pulses required to preserve Rz symmetry**

- Main idea: Imperfect pulses simply shift the AKLT couplings (in a symmetric way)
- As long as we **stay in the AKLT phase** there will still be edge states **(but now dynamically decoupled from Z,Y-fields!)**
- **"Emergent $Z_2$ Symmetry!"** Inherited from the periodicity (time-translation symmetry) of the pulses
- **Can we get rid of all symmetry protection in this way?**

# Dynamically Enforcing Both Symmetries



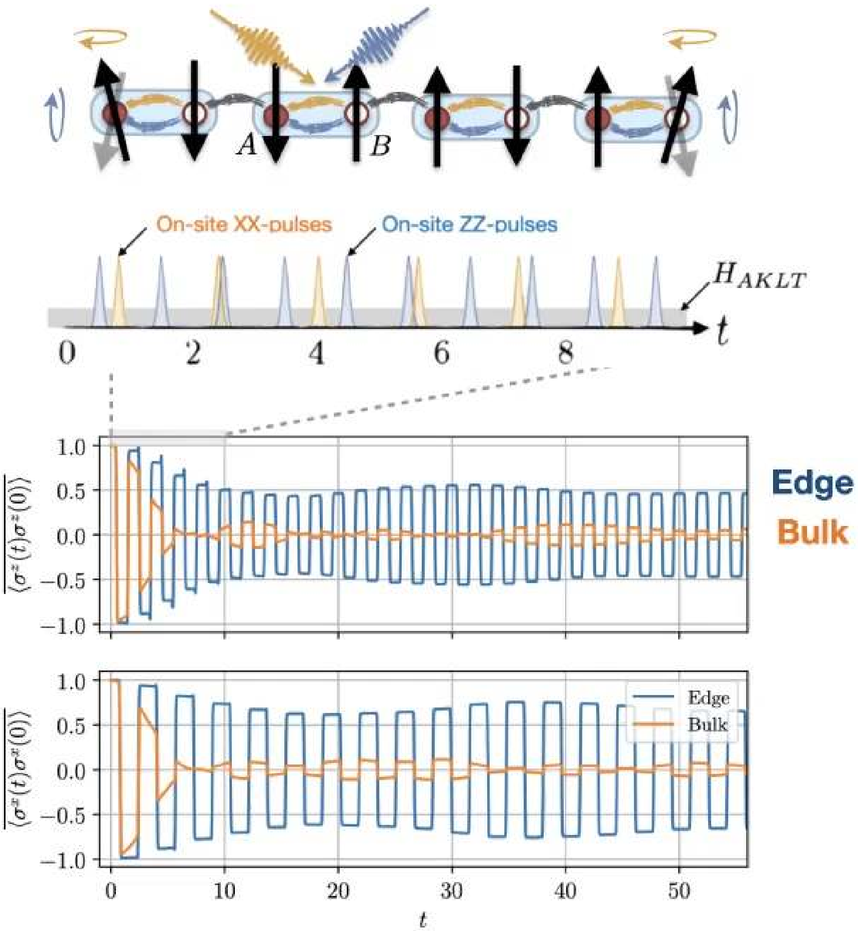

**Incommensurate frequencies:** $\Omega_z/\Omega_x = \varphi = \frac{1+\sqrt{5}}{2}$

**Else, if XX and ZZ pulses always came together, B~y would not get canceled**

- Both $R_x$ and $R_z$ symmetries are emergent
- Emergent symmetries will survive even if we perturb with symmetry breaking fields to H(t)
- If $H_F$ stays in the AKLT phase, this protocol would be stable to generic perturbations!
- **Disordered AKLT couplings => MBL, prevents heating by drive**

Friedman, Ware, Vasseur, ACP arXiv:2009.03314

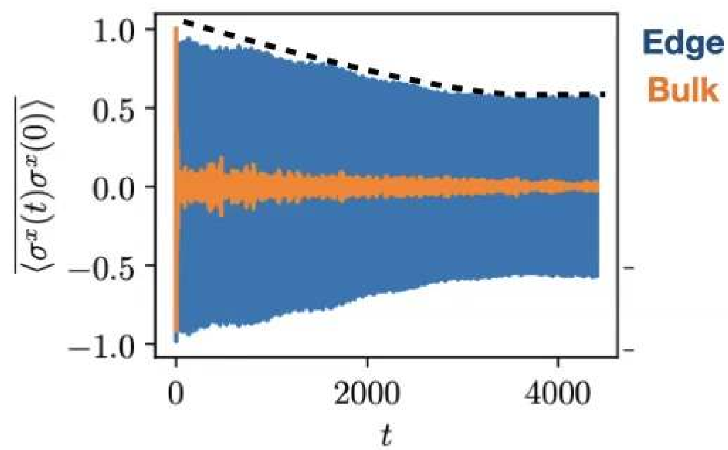
# Coherent Topological Edge states




**50-spins, TEBD, Short Times**

- Imperfect pulses (~95% of pi-pulses)
- ~5% Random direction & strength onsite fields (all symmetries broken)

**14-spins, Longer Times (ED)**




Friedman, Ware, Vasseur, ACP arXiv:2009.03314

# Some formalism

**Multi-tone drive:**

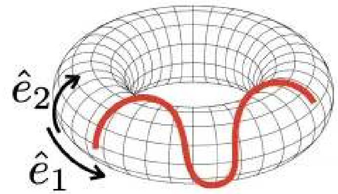
$$H(t) = \sum_{\vec{n}} e^{-i\vec{\Omega}\cdot\vec{n}t} h_{\vec{n}}$$

$$\vec{\Omega} = (\Omega_1, \Omega_2, \ldots, \Omega_p)^T$$

$$\vec{n} \in \mathbb{Z}^p \qquad \Omega_i/\Omega_j \notin \mathbb{Q}$$

p=0: static, time-independent
p=1: periodic "Floquet", H(t+T)=H(t)
p≥2: quasiperiodic/non-repeating
~~p→∞: continuous spectrum~~

**"Cut and project"**




$(p = 2)$

$$H[\vec{\theta} + 2\pi\hat{e}_\mu] = H[\vec{\theta}]$$

$\mu \in \{1 \ldots p\}$

*(multi-periodic time-dependence)*

$$H(t) = H[\vec{\theta}(t)]$$

$$\vec{\theta}(t) = \vec{\Omega}t$$

# Floquet Theorem

$$H(t) = H[\vec{\theta}(t)]$$

$$U(t) = \mathcal{T}e^{\int_0^t H(s)ds} = W^\dagger Q[\vec{\theta}(t)]e^{-iH_F t}W$$

**Finite-depth unitary "frame transformation"** ($W$)

**(Quasi)-periodic "micro-motion" Q(0)=1**

**Effective time-independent Hamiltonian** ($H_F$)

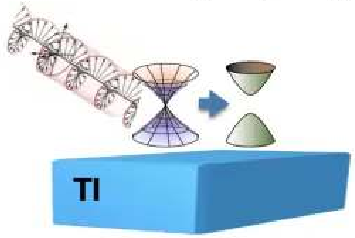
**Def. 1: "Floquet engineering"**

- $H_F$ may be more interesting than H(t)
- But could deform H(t)→$H_F$ w/out phase transition
- Corollary: Q(t) has same (quasi)periodicity as drive

$$Q[\vec{\theta} + 2\pi\hat{e}_\mu] = Q[\vec{\theta}]$$

**Examples:** Photo-induced superconductivity, topology…

Fausti, Daniele, et al. (Cavalleri group) Science '11




Wang et al. (Gedik Group) Science '13

# Floquet Theorem

$$H(t) = H[\vec{\theta}(t)]$$

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Finite-depth unitary "frame transformation"

(Quasi)-periodic "micro-motion"

Effective time-independent Hamiltonian

**Def. 2: "Intrinsically dynamical" phases**

- Dynamics is essential **[Topology comes from Q]**
- H(t) →$H_F$ requires a <u>phase transition</u>
- Q has "enlarged" (quasi)periodicity

$$Q(\vec{\theta} + 2\pi\hat{e}_\mu) = Q(\vec{\theta})\mathcal{X}_\mu \quad (\mathcal{X}_\mu)^{N_\mu} = \mathbb{1}$$

**"Emergent dynamical symmetry(s)"**

$[\mathcal{X}_\mu, H_F] = 0$ even if $[\mathcal{X}_\mu, H(t)] \neq 0$

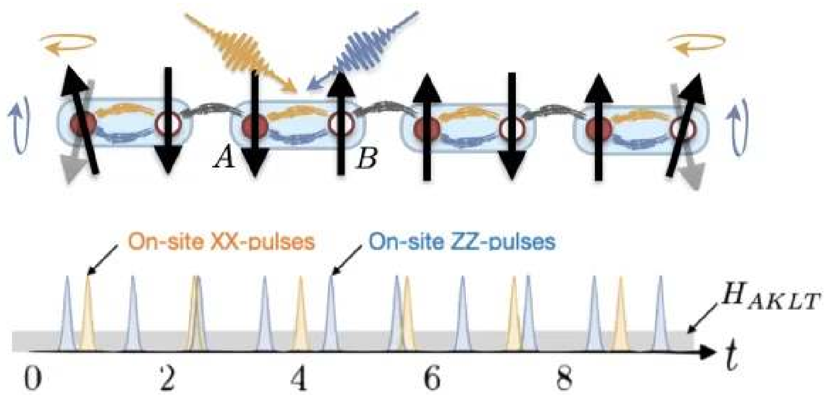
# Driven AKLT Model

**Usual AKLT:**


Symmetries : $R_x(\pi), R_z(\pi)$

Symmetry group: $\mathbb{Z}_2^{(x)} \times \mathbb{Z}_2^{(z)}$

**Qpdc-drive AKLT:**




$$U(t) = W^{\dagger} Q[\vec{\theta}(t)] e^{-iH_{\text{AKLT}}} W$$

$$W = 1$$

$$Q(\vec{\theta} + 2\pi \hat{e}_1) = Q(\vec{\theta}) R_x(\pi)$$

$$Q(\vec{\theta} + 2\pi \hat{e}_2) = Q(\vec{\theta}) R_z(\pi)$$

**Both protecting symmetries are emergent**

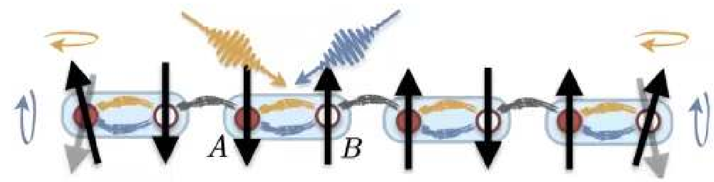
**(Cannot be broken by generic perturbations!)**

- Can Compute W, Q, HF, perturbatively in deviation, g, from ideal drive
- Asymptotic series: accurate to ~ $\exp[1/g^{2/3}]$

Friedman, Ware, Vasseur, ACP

**using formalism of: Ho, Dumitrescu, Else**

# Topological Edge Dynamics


**SPT invariant of usual AKLT chain:**

$$[R_x(\pi)R_z(\pi)]_{\text{edge}} = \underbrace{(-\mathbf{1})}_{\omega}[R_z(\pi)R_x(\pi)]_{\text{edge}}$$

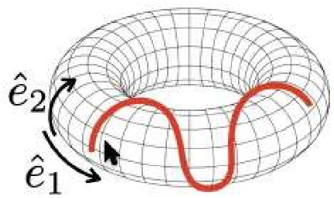
$$[\omega] \in \mathcal{H}^2(Z_2 \times Z_2, U(1))$$

**Dynamical Topological Invariant?**

$$U(t) = W^\dagger Q[\vec{\theta}(t)]e^{-iH_{\text{AKLT}}}W$$

$$Q(\vec{\theta} + 2\pi\hat{e}_\mu) = Q(\vec{\theta})\mathcal{X}_\mu$$

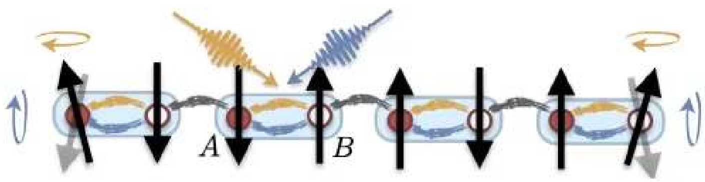
$$[\mathcal{X}_1\mathcal{X}_2]_{\text{edge}} = (-\mathbf{1})[\mathcal{X}_2\mathcal{X}_1]_{\text{edge}}$$




**Issue: Physically we only have access to a single time direction:** $\vec{\theta}(t) = \vec{\Omega}t$

**Dynamics of edge "buried" in non-repeating micromotion**

# Topological Edge Dynamics




$$U(t) = W^\dagger Q[\vec{\theta}(t)] e^{-iH_{\mathrm{AKLT}}} W$$

**Golden ratio drive:** $\Omega_z / \Omega_x = \varphi = \frac{1+\sqrt{5}}{2}$

**"Fibonacci times":** $t_n = \frac{2\pi}{\Omega} F_n$

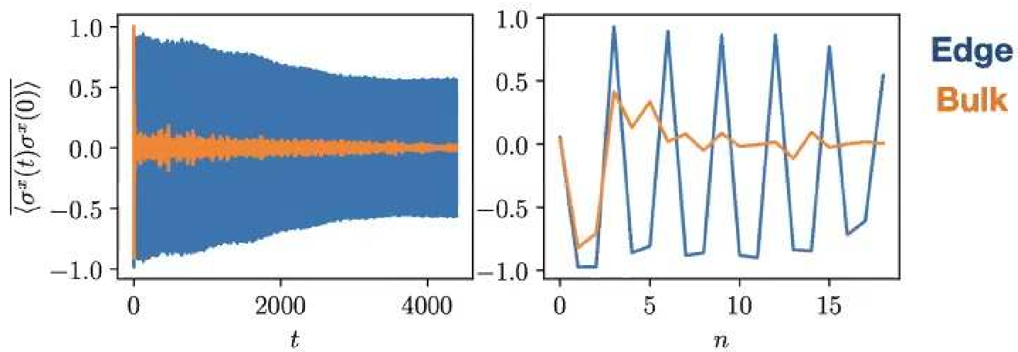
**Near-Recurrences:** $\Omega_x t_n = \Omega_z t_n + 2\pi N + \mathcal{O}(\varphi^{-n})$

**Edge evolution in Fibonacci time:** $Q[\vec{\Omega} t_n] \approx Q[0] \mathcal{X}_n$

Fibonacci sequence of X and Z rotations: $\mathcal{X}_{n+1} = \mathcal{X}_{n-1} \mathcal{X}_n$ $\quad \mathcal{X}_2 = \sigma^z \sigma^x \quad \mathcal{X}_1 = \sigma^x$

$$\mathcal{X}_n \sim \pm (\sigma^x)^{F_{n-1}} (\sigma^z)^{F_{n-2}}$$

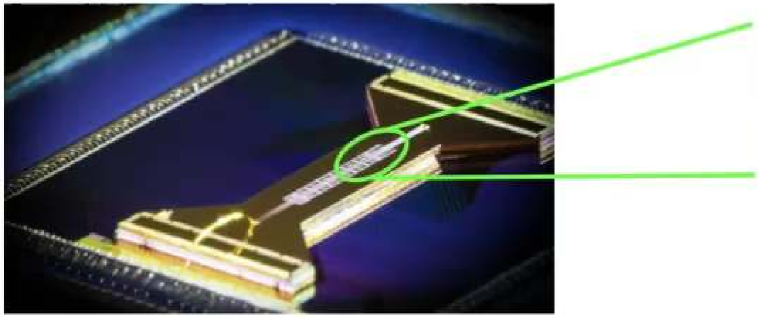
Exhibits 3-Fold periodicity: $\mathcal{X}_{n+3} = \mathcal{X}_n$

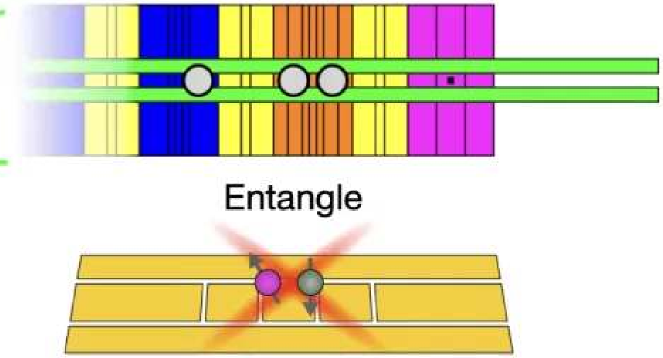



**3-fold periodicity in Fibonacci-Time = signature of edge topology**

Friedman, Ware, Vasseur, ACP

# Experimental Realization (Trapped Ions)





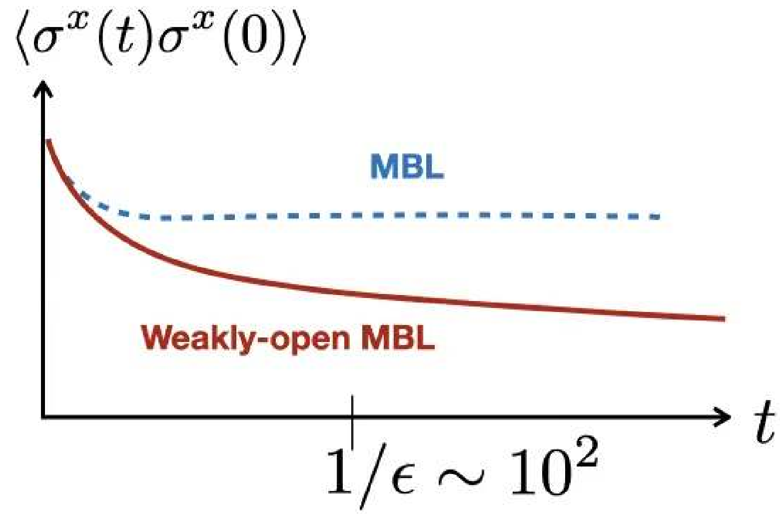

Honeywell QCCD Architecture

- 10x qubits ($^{171}Yb^{+}$ HF “clocks”)
- Linear Array (Can shuffle positions)
- Arbitrary 1q gates: $\epsilon_{1q}$ ~ 0.01%
- 1d NN 2q gates: $\epsilon_{2q}$ ~ 0.5%

# What do we expect?

**"Weakly-open MBL":**

- MBL + Slowly decaying amplitude from incoherent (non-unitary) errors
- Ex: Depolarizing errors
  [$\epsilon_{2q}$ ~ 0.5% (RBN)] $\rho \to (1-\epsilon)\rho + \epsilon \mathbb{1}$




Dumitrescu, Kumar, et al. (Honeywell+ACP) 2107.09676

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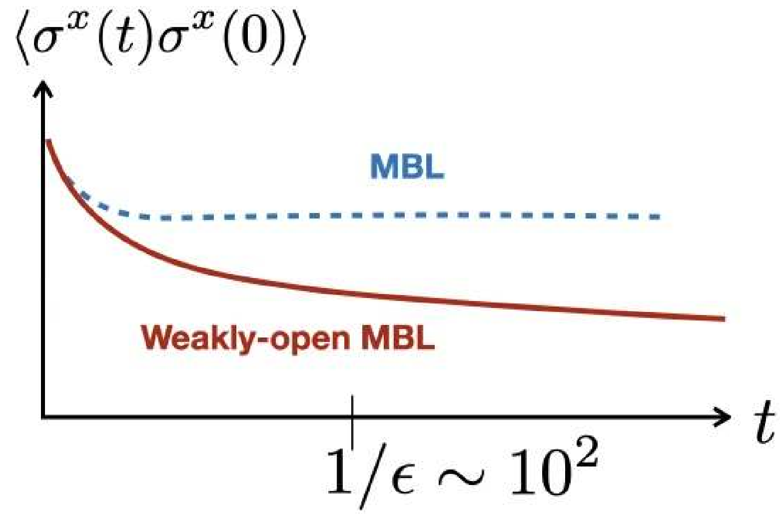


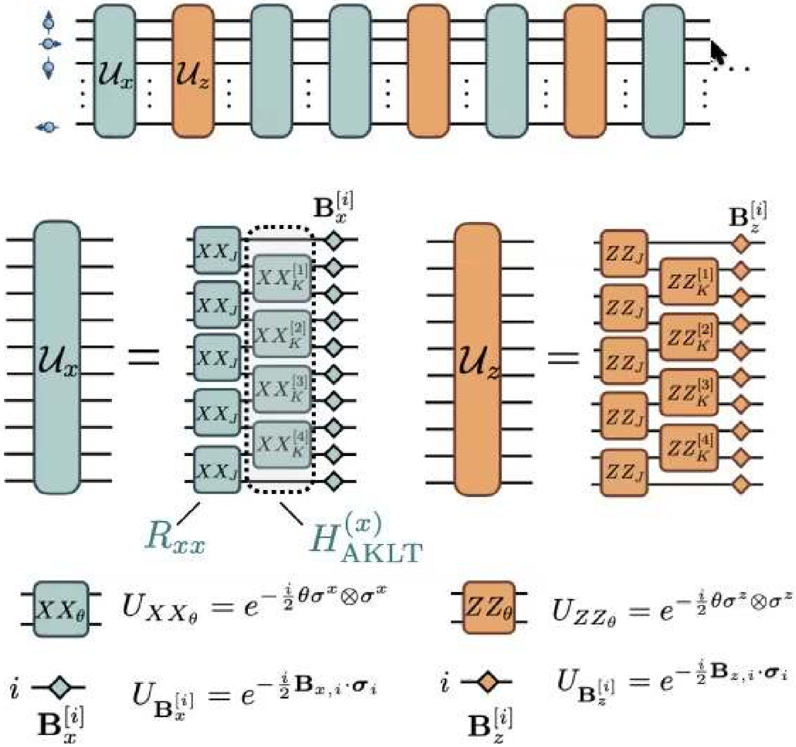

**Coherent (unitary) errors**
(Stray fields, cross-talk, pulse amplitude/phase errors,….)

- Accumulate as ~ $t^2\epsilon$
- MBL is stable to these *(may break protecting symmetries)*

Dumitrescu, Kumar, et al. (Honeywell+ACP) 2107.09676

# Recursive Fibonacci Drive

**Continuous time-dependence is hard to implement!?**



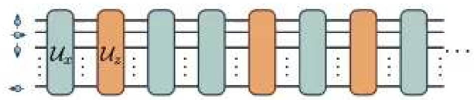

Recursive/Pulsed Version: $U_{n+1} = U_{n-1}U_n$

$$U_1 = U_x \quad U_2 = U_z U_x$$

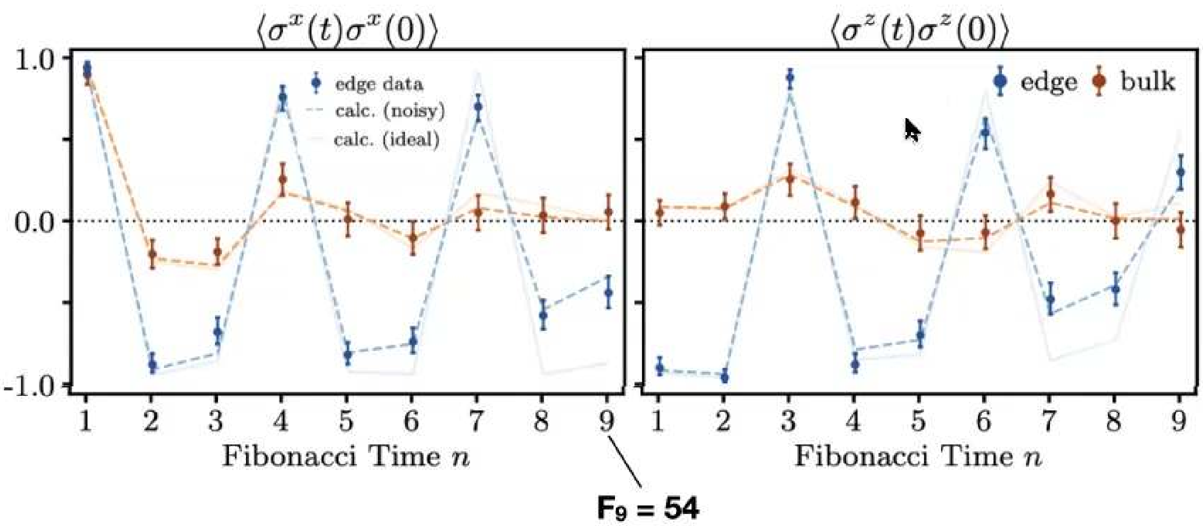
- Fibonacci sequence of X and Z pulses
- Relative freq. of Z to X is still: $\sim \varphi$
- MBL is (meta)stable: $\tau_{\text{heating}} \sim e^{-1/\epsilon}$ Dumitrescu, Vasseur, ACP
- Signature: coherent edge states w/ 3-fold periodic dynamics in Fibonacci time

Dumitrescu, Kumar, et al. (Honeywell+ACP) 2107.09676

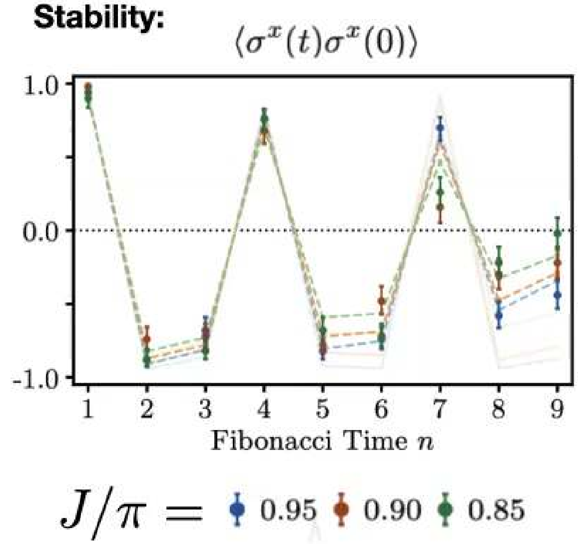
# Experimental results



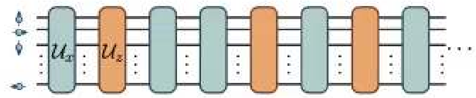

**W/ 5% "Errors" in all couplings**




**Stability:**



# Experimental results

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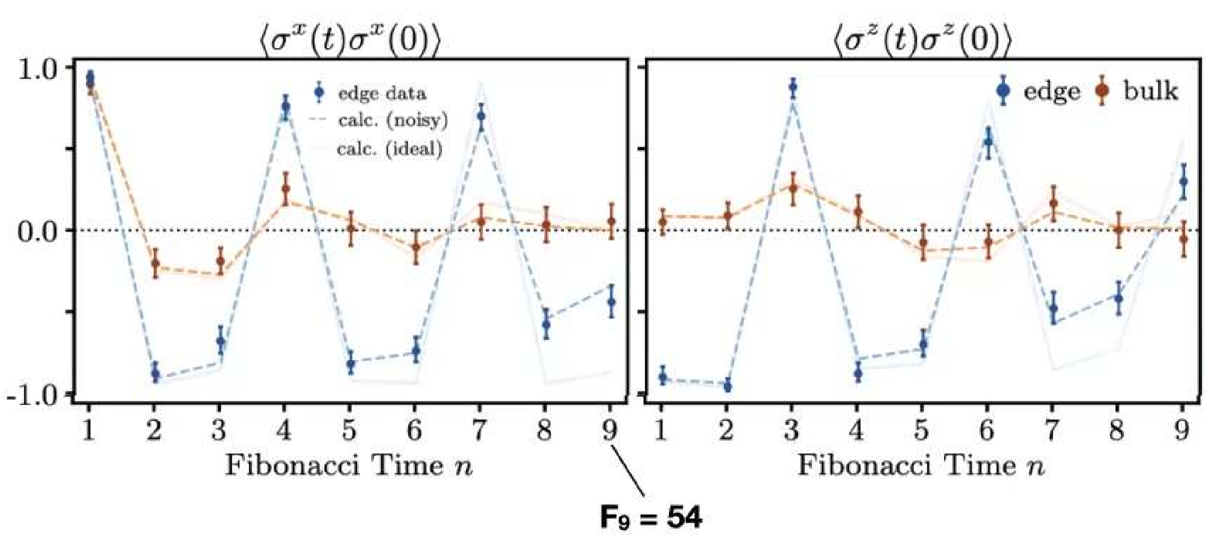
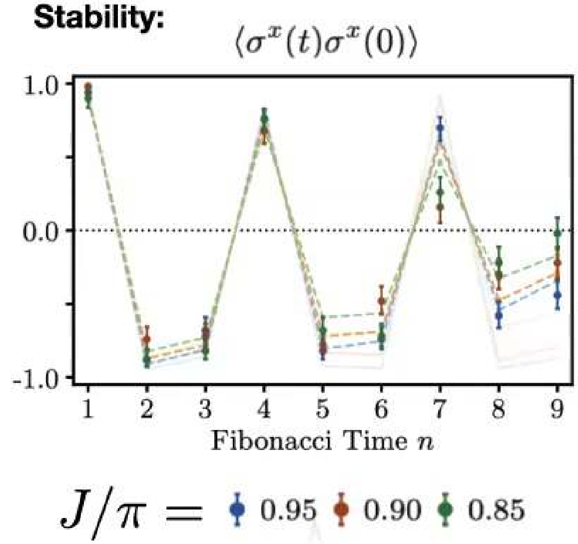
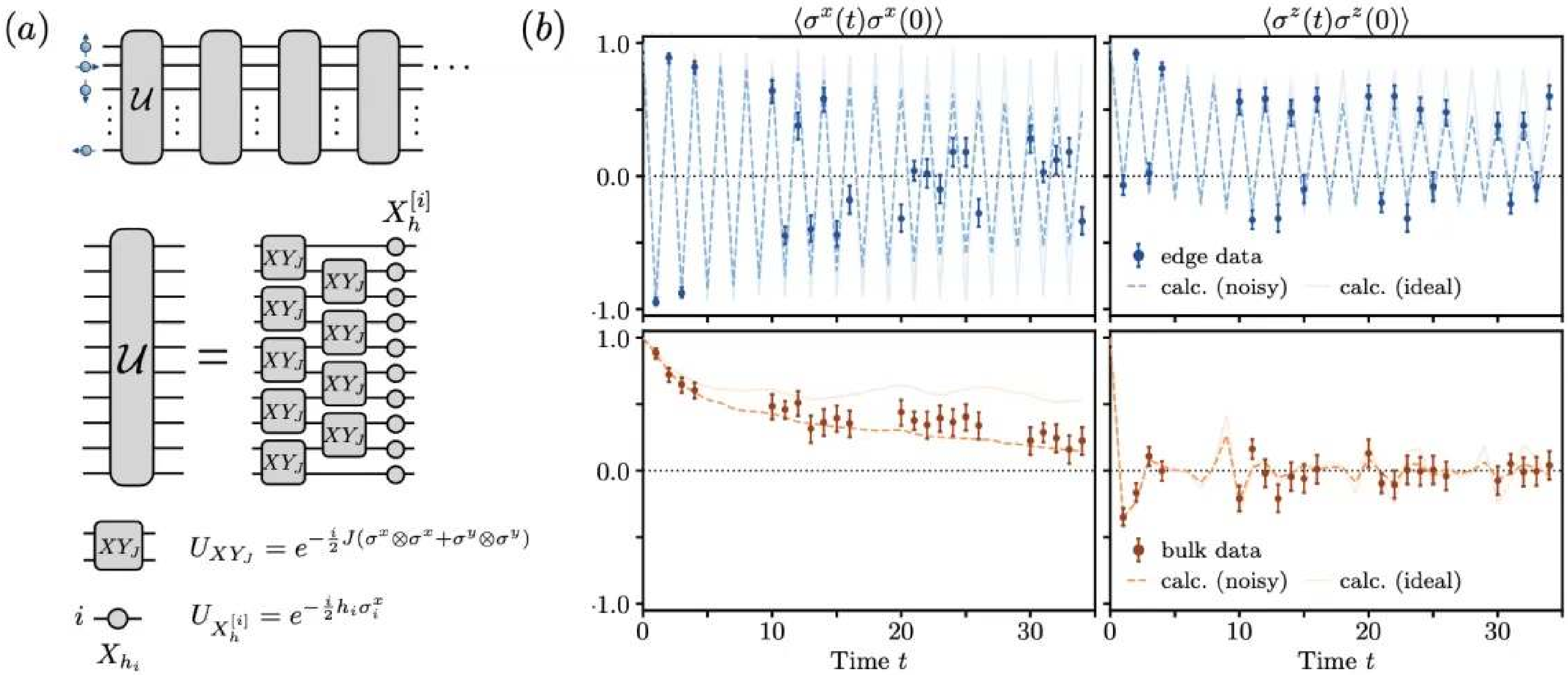



**Stability:**




**Edge physics stable despite that we've explicitly broken all symmetries and artificially amplified control "imperfections"**

# Contrast: An Ersatz Floquet SPT

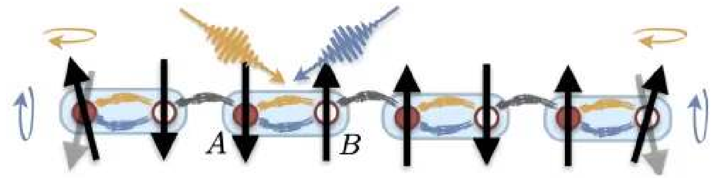



**X-only, Floquet Version**
**+**
Small Integrability-Breaking Terms
*(gives a chance to thermalize)*

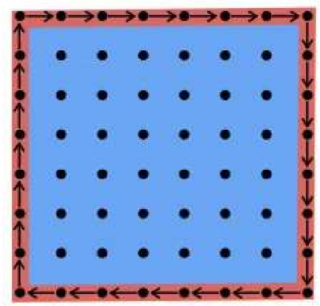
- Bulk shows weakly-open MBL behavior
- Edge dephases much faster than 1/ϵ ~ 100 Floquet periods
- Consistent with: Coherent 2-3% symmetry-breaking (Z “phase”) errors
- Likely culprit: B-field inhomogeneity **[FSPT/MBL = selective noise probe!]**

# Outlook

- **1st experimental realization of:**
  1. Dynamical *quantum* phase;
  2. 1d Boson SPT

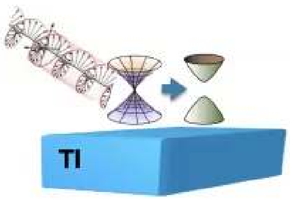



- **Other dynamical topological phases (2d):**
  - Chiral + quantized quantum information channels
  - Robust methods to manipulate topological qubits (boundary defects of surface code etc…)
  - Applications for error-resilient manipulation of QI?



Rudner, Lindner, Berg, Levin; Po, ACP, Morimoto, Fidkowski, Vishwanath

- **Solid-state realizations?**
  - No MBL, stabilize with dissipation?



# Thanks for your attention

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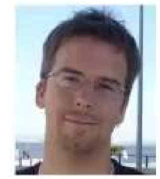

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Honeywell

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