Title: Predicting many properties of quantum systems with chaotic dynamics

Speakers: Hong-Ye Hu

Series: Perimeter Institute Quantum Discussions

Date: December 08, 2021 - 11:00 AM

URL: https://pirsa.org/21120015

Abstract: Classical shadow tomography provides an efficient method for predicting functions of an unknown quantum state from a few measurements of the state. It relies on a unitary channel that efficiently scrambles the quantum information of the state to the measurement basis. However, it is quite challenging to realize deep unitary circuits on near-term quantum devices, and an unbiased reconstruction map is non-trivial to find for arbitrary random unitary ensembles. In this talk, I will discuss our recent progress on combining classical shadow tomography with quantum chaotic dynamics. Particularly, I will introduce two new families of shadow tomography schemes: 1) Hamiltonian-driven shadow tomography and 2) Classical shadow tomography with locally scrambled quantum dynamics. In both works, I'll derive the unbiased reconstruction map, and analyze the sample complexity. In the Hamiltonian-driven scheme, I will illustrate how to use proper time windows to achieve a more efficient tomography. In the second work, I will demonstrate advantages of shadow tomography in the shallow circuit region. Then I'll conclude by discussing approximate shadow tomography with local Hamiltonian dynamics, and demonstrate that a single quench-disordered quantum spin chain can be used for approximate shadow tomography.

References:
[1] Hong-Ye Hu, Yi-Zhuang You. "Hamiltonian-Driven Shadow Tomography of Quantum States". arXiv:2102.10132 (2021)
[2] Hong-Ye Hu, Soonwon Choi, Yi-Zhuang You. "Classical Shadow Tomography with Locally Scrambled Quantum Dynamics". arXiv: 2107.04817 (2021)

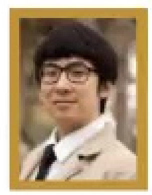
Zoom Link: https://pitp.zoom.us/j/99011187936?pwd=OVU3VkpyZ21YcXRCOW5DOHlnSWlVQT09

Dec. 8th, 2021

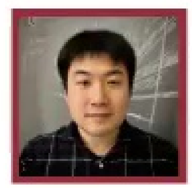
Seminar @ Perimeter Institute

Predicting many properties of quantum systems with chaotic dynamics

Hong-Ye Hu



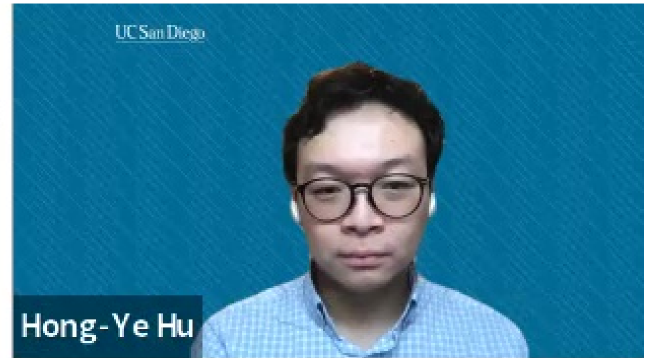
Yi-Zhuang You



Soonwon Choi

[1] **Hong-Ye Hu**, Yi-Zhuang You. arXiv:2102.10132

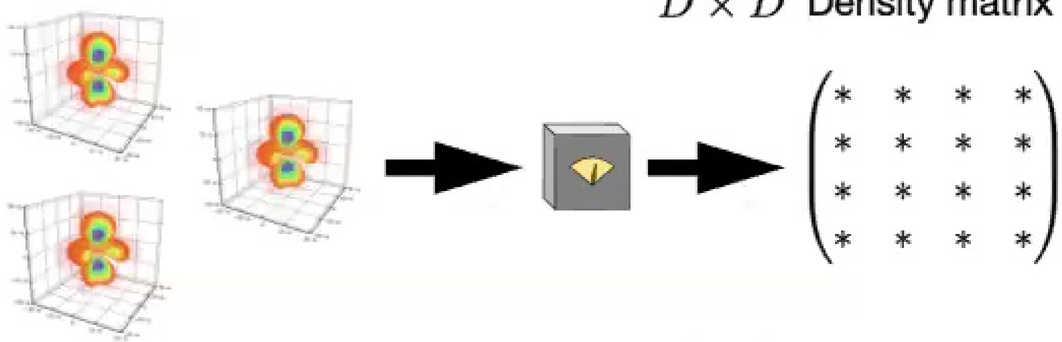
[2] **Hong-Ye Hu**, Soonwon Choi, Yi-Zhuang You. arXiv:2107.04817



Quantum State Tomography

- Full tomography of quantum states: A complete classical representation of quantum states

$D \times D$ Density matrix $(D = d^N)$



- Quantum state tomography is an essential task in many quantum technology applications:

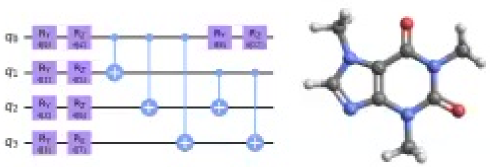
  Quantum devices benchmarking,
  discrimination of quantum states, …

- Full tomography is HARD! J. Haah, A. Harrow, Z. Ji, X. Wu, N. Yu (2017)

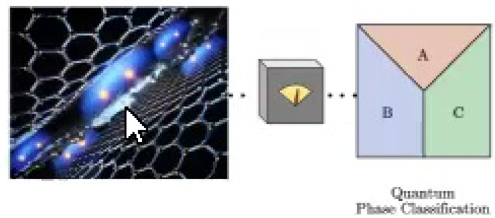
Predicting quantum properties

- Most of the cases, we want to predict properties of the underlying quantum states

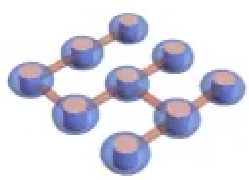
Quantum-Classical Hybrid Optimization (VQE,QAOA)



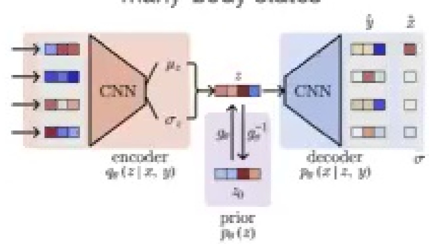
Quantum Phase of Matter



(Variational) State Preparation

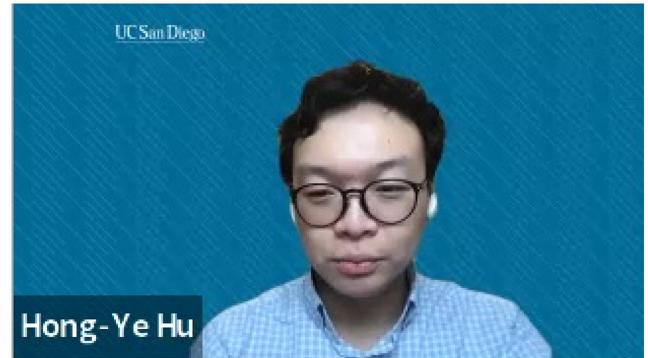


Efficient machine learning of quantum many-body states



Perruzzo, McClean, Shadbolt, Yung, Zhou, Love, Aspuru-Guzik, O'Brien(2014) PW Anderson(1972) Carrasquila, Melko(2016)

Farhi, Goldstone, Gutmann(2014) Sagastizabal, Premaratne, et al (2021)

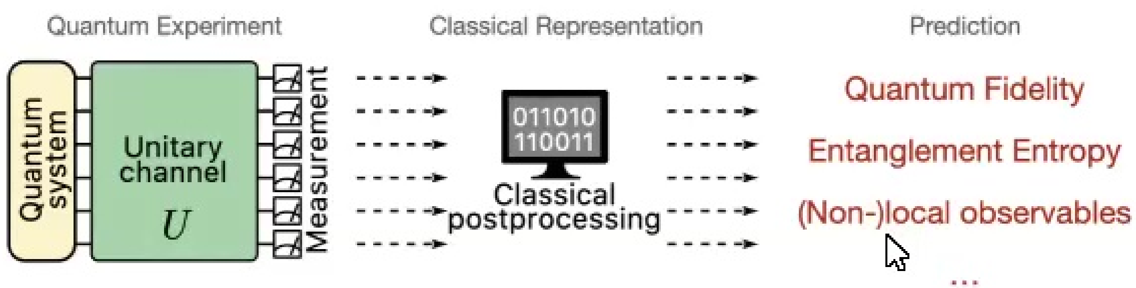


Classical Shadow Tomography

- If we are not interested in the full tomography, but **predicting** some "relevant" types of observables, the tomography scheme can be more efficient.

- Classical Shadow Tomography. Huang, Kueng, Preskill (2020)

Overview of the process



$U \in \mathcal{E}_U = \{U | P(U)\}$

Classical Shadow Tomography

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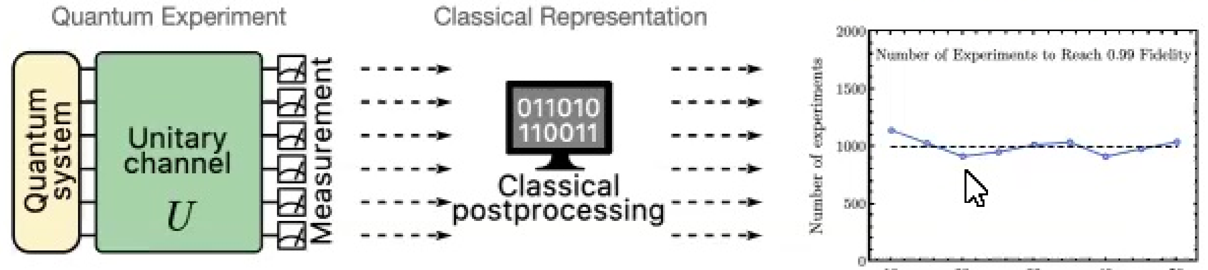
Overview of the process

Provable efficiency!

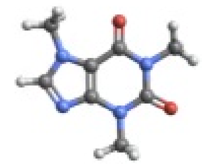


$$U \in \mathcal{E}_U = \{U|P(U)\}$$

Why using randomized measurements

- Predicting many properties simultaneously

  M linear properties O_i requires O(***log M)*** data

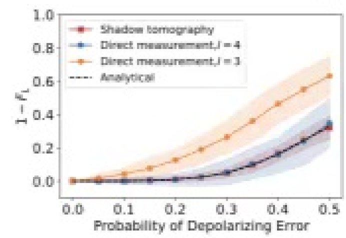


- During classical post-processing : do operation that is not easy to perform in experiments

  Error mitigation = Error correction code + shadow tomography

$$\langle \Gamma \rangle_{\mathrm{QEM}} = \frac{\mathrm{Tr}[\Pi \mathcal{E}(\rho) \Pi^\dagger \Gamma]}{\mathrm{Tr}[\Pi \mathcal{E}(\rho) \Pi^\dagger]}$$

Project out non-logical errors



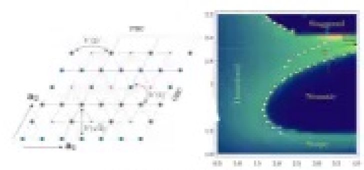
with

HYH, RL, YZY, ER, ZW (In preparation)

- Unsupervised machine learning

  1. Unsupervised learning: no knowledge about order parameter

  2. Propose better order parameter



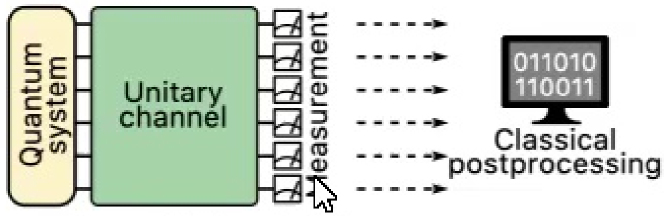
HYH (on going work)

Giulia Semeghini, Harry Levine, Alexander Keesling, Sepehr Ebadi, Tout T. Wang, Dolev Bluvstein, Ruben Verresen, Hannes Pichler, Marcin Kalinowski, Rhine Samajdar, Ahmed Omran, Subir Sachdev, Ashvin Vishwanath, Markus Greiner, Vladan Vuletic, Mikhail D. Lukin

Rhine Samajdar, Wen Wei Ho, Hannes Pichler, Mikhail D. Lukin, and Subir Sachdev

Classical Shadow Tomography

- Data Acquisition Protocol (Randomized Measurements)

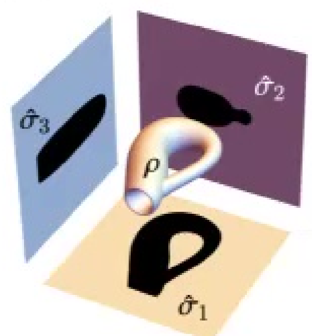


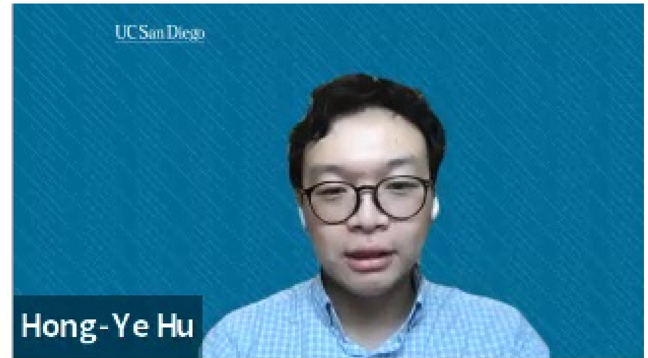
$U \in \mathcal{E}_U = \{U|P(U)\}$

- Step 1 (Quantum evolution): $\rho \to \rho' = U\rho U^\dagger$
- Step 2 (Measure in Z-basis): $\rho' \to |b\rangle\langle b|$
- Step3 (Classical computation):

$$|b\rangle\langle b| \to \hat{\sigma}_{U,b} = U^\dagger |b\rangle\langle b| U$$

Classical Shadows





Classical Shadow Tomography

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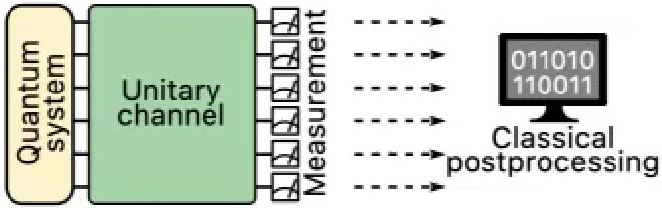

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Associated Probability

$P(U)$

- Step 1 (Quantum evolution): $\rho \rightarrow \rho' = U\rho U^\dagger$
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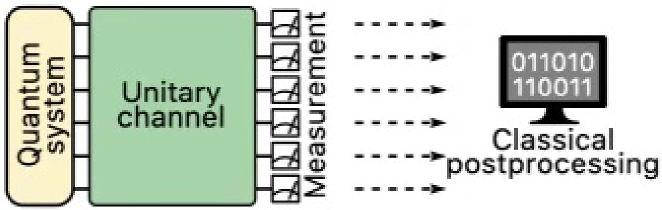


$U \in \mathcal{E}_U = \{U|P(U)\}$

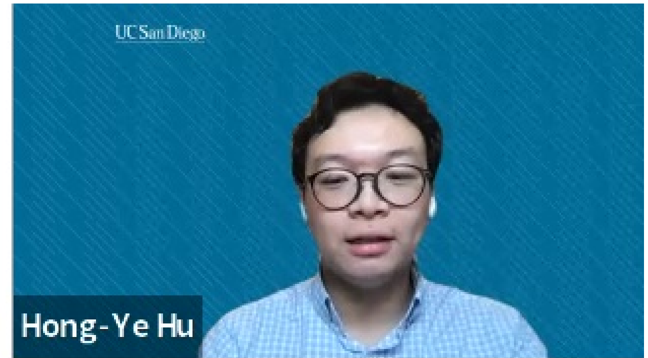
Associated Probability

- Step 1 (Quantum evolution): $\rho \to \rho' = U\rho U^\dagger$ $\quad P(U)$
- Step 2 (Measure in Z-basis): $\rho' \to |b\rangle\langle b|$ $\quad \mathrm{Tr}(|b\rangle\langle b|\rho') = \mathrm{Tr}(|b\rangle\langle b|U\rho U^\dagger)$
  $= \mathrm{Tr}(U^\dagger|b\rangle\langle b|U\rho) = \mathrm{Tr}(\hat{\sigma}_{U,b}\rho)$
- Step3 (Classical computation):

$$|b\rangle\langle b| \to \hat{\sigma}_{U,b} = U^\dagger|b\rangle\langle b|U \xrightarrow{\text{Total probability}} P(\hat{\sigma}_{U,b}|\rho) = P(U)\mathrm{Tr}(\hat{\sigma}_{U,b}\rho)$$

Classical Shadows

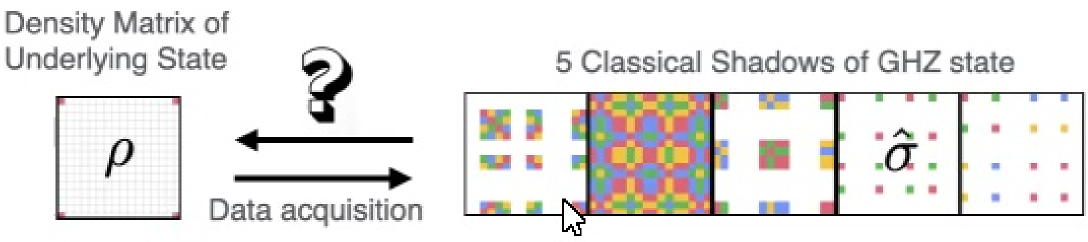
- Dataset: $\{\hat{\sigma}_{U,b}|P(\hat{\sigma}_{U,b}|\rho) = P(U)\mathrm{Tr}(\hat{\sigma}_{U,b}\rho)\}$



Reconstruction Map

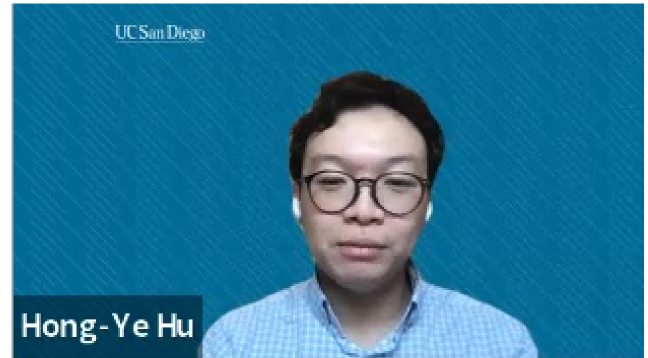
- Dataset:

$$\mathcal{E}_{\sigma|\rho} = \{\hat{\sigma}_{U,b} | P(\hat{\sigma}_{U,b}|\rho) = P(U)\mathrm{Tr}(\hat{\sigma}_{U,b}\rho)\}$$



- Take average of the classical shadows:

$$\sigma = \mathop{\mathbb{E}}_{\hat{\sigma} \in \mathcal{E}_{\sigma|\rho}} \hat{\sigma} = \sum_{U,\hat{\sigma}} \hat{\sigma} P(U)\mathrm{Tr}(\hat{\sigma}\rho)$$



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- Take average of the classical shadows:

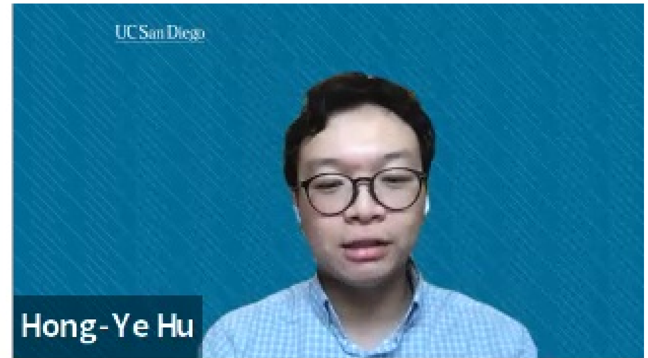
$$\sigma = \mathop{\mathbb{E}}_{\hat{\sigma}\in\mathcal{E}_{\sigma|\rho}} \hat{\sigma} = \sum_{U,\hat{\sigma}} \hat{\sigma} P(U)\mathrm{Tr}(\hat{\sigma}\rho) = \mathcal{M}[\rho]$$

Linear map: measurement channel

- If the unitary ensemble $P(U)$ is tomography complete, the channel is invertible.

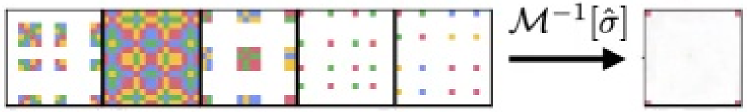
$$\rho = \mathcal{M}^{-1}[\sigma] = \mathop{\mathbb{E}}_{\hat{\sigma}\in\mathcal{E}_{\sigma|\rho}} \mathcal{M}^{-1}[\hat{\sigma}]$$

Linear map: reconstruction map



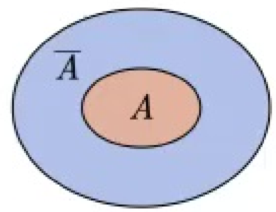
Reconstruction Map is Foundation

- $\mathcal{M}^{-1}$—reconstruction map is the foundation in the prediction:
  - Reconstructing density matrix $\rho = \underset{\hat{\sigma}\in\mathcal{E}_{\sigma|\rho}}{\mathbb{E}} \mathcal{M}^{-1}[\hat{\sigma}]$



  - Linear property $\langle O \rangle = \mathrm{Tr}(O\rho)$

$$\hat{o} = \mathrm{Tr}(O\mathcal{M}^{-1}[\hat{\sigma}]) \longrightarrow \mathbb{E}\hat{o} = \langle O \rangle$$



  - Nonlinear property

$$e^{-S_\rho(A)} = \mathrm{Tr}_A(\mathrm{Tr}_{\bar{A}}\,\rho)^2$$
$$= \underset{\hat{\sigma},\hat{\sigma}'\in\mathcal{E}_{\sigma|\rho}}{\mathbb{E}} \mathrm{Tr}_A\left(\mathrm{Tr}_{\bar{A}}\,\mathcal{M}^{-1}[\hat{\sigma}]\,\mathrm{Tr}_{\bar{A}}\,\mathcal{M}^{-1}[\hat{\sigma}']\right)$$

Elben, Vermersch, Dalmonte, Cirac, Zollar (2019)

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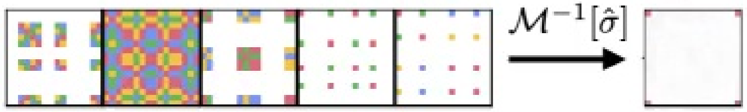

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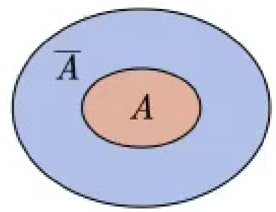

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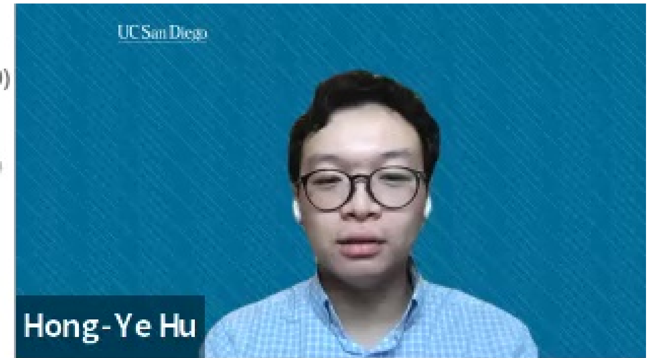
Elben, Vermersch, Dalmonte, Cirac, Zollar (2019)

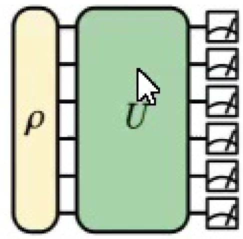
- Yet, the reconstruction map $\mathcal{M}^{-1}$ is not easy to find!

Examples of CST

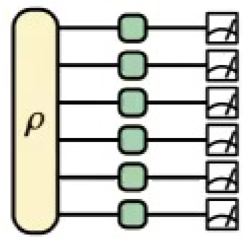
Ohliger, Nesme, Eisert (2012)
Elben, Vermersch, Dalmonte, Cirac, Zollar (2019)
Huang, Kueng, Preskill (2020)
Brandao, Harrow, Horodecki (2012)



- Global Haar (3-design) Unitary
  - $\mathcal{M}[\rho] = \frac{1}{D+1}(\rho + \mathbb{1})$
  - $\mathcal{M}^{-1}[\sigma] = (D+1)\sigma - \mathbb{1}$ $(D = 2^N)$
  - ✓ Low rank observable: Fidelity
  - ✗ Deep random circuits



- Local Haar (3-design) Unitary
  - $\mathcal{M}[\rho] = \bigotimes_i \frac{1}{d+1}(\rho_i + \mathbb{1}_i)$
  - $\mathcal{M}^{-1}[\sigma] = \bigotimes_i ((d+1)\sigma_i - \mathbb{1}_i)$ $(d = 2)$
  - ✓ Local observable: local correlation functions
  - ✗ Non-local observable, Fidelity



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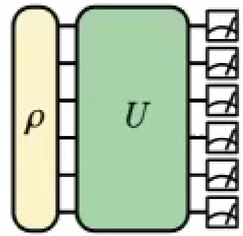

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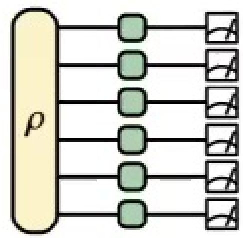
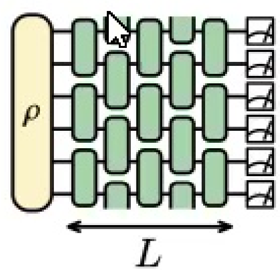


- What about shallow circuit?

  ✓ Nice interpolation between two limits

  ✓ Shallow circuits, Hamiltonian dynamics without time dependent control



Flexible Shallow Unitary

- **Desire:**
  - ✓ Adaptive: optimize sample efficiency in adaptation to target set of physical observables
  - ✓ Easy to implement: Shallow circuits, Hamiltonian dynamics without time dependent control

- **Challenges:** Highly non-trivial to compute reconstruction map for arbitrary unitary ensembles.

- New progresses: CST + Chaotic Quantum Dynamics
  - Global GUE Hamiltonian dynamics. Hu, You (2021)
  - Locally scrambled quantum dynamics. Hu, Choi, You (2021)

Hamiltonian-Driven Shadow Tomography

- Unitary channel: time evolution generated by generic quantum chaotic Hamiltonian H , modeled by (global) GUE random matrix
  - $t \to 0$: No scrambling: $U = \mathbb{1}$

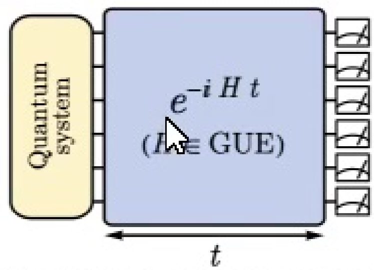
Diagonal-traceless

$$\mathcal{M}[\rho] \to \frac{\mathbb{1}}{D} + \rho_d$$

  - $t \to \infty$: Maximal scrambling: Global Haar

Off-diagonal Diagonal-traceless

$$\mathcal{M}[\rho] \to \frac{\mathbb{1}}{D} + \frac{\rho_o}{D+1} + \frac{\rho_d}{D+1}$$



D : Total Hilbert space dimension

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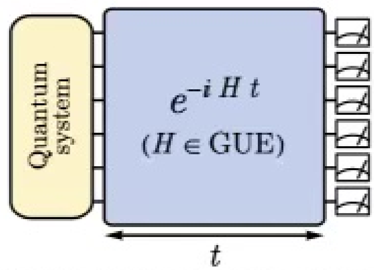

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  - Intermediate t:

$$\mathcal{M}[\rho] = \frac{\mathbb{1}}{D} + \frac{\rho_o}{\alpha_D(t)} + \frac{\rho_d}{\beta_D(t)}$$

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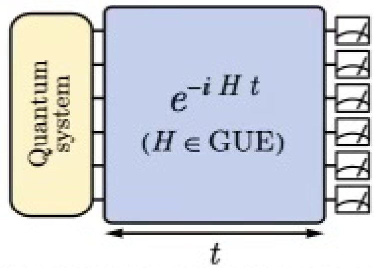

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- Intermediate t:

$$\mathcal{M}[\rho] = \frac{\mathbb{1}}{D} + \frac{\rho_o}{\alpha_D(t)} + \frac{\rho_d}{\beta_D(t)}$$

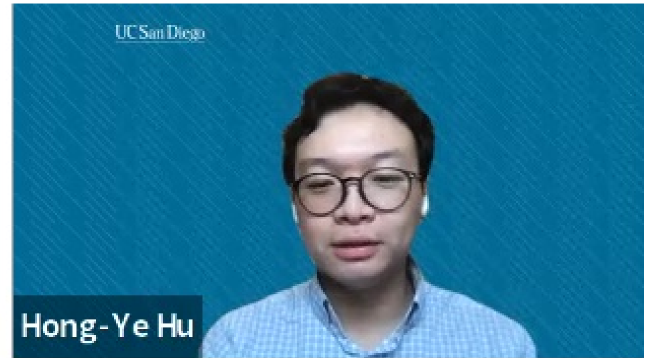
$$\alpha_D(t) = \left(\frac{1}{D+1} - \lambda_D(t)\right)^{-1},$$

$$\beta_D(t) = \left(\frac{1}{D+1} + D\lambda_D(t)\right)^{-1},$$

$$\lambda_D(t) = \frac{(Dr^2(t) + r(2t))^2 - 4r^2(t)}{(D+3)(D^2-1)},$$

$$r(t) = J_1(2t)/t$$

Spectral form factor

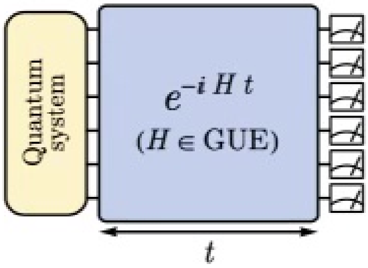


Prediction & Sample Complexity

- $\mathcal{M}^{-1}$ — reconstruction map:

$$\mathcal{M}[\rho] = \mathbb{1}/D + \frac{\rho_o}{\alpha_D(t)} + \frac{\rho_d}{\beta_D(t)}$$

$$\mathcal{M}^{-1}[\rho] = \mathbb{1}/D + \alpha_D(t)\,\rho_o + \beta_D(t)\,\rho_d$$



- Predicting properties:

Single shot estimation

$$\hat{o} = \mathrm{Tr}(O\mathcal{M}^{-1}[\hat{\sigma}]) \longrightarrow \mathbb{E}\hat{o} = \langle O \rangle$$

Mean estimation

$$\bar{o} = \frac{1}{M}\sum_{k=1}^{M} \hat{o}(\hat{\sigma}_k) \longrightarrow \Pr(|\bar{o} - \langle O \rangle| \geq \epsilon) \leq \frac{\mathrm{Var}\hat{o}}{\epsilon^2 M} \leq \delta$$

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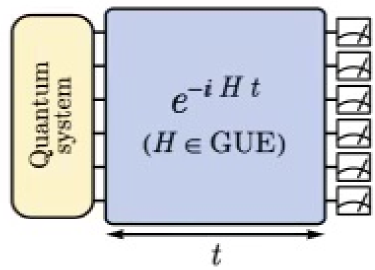

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- Shadow Norm: $\mathrm{Var}\hat{o} \leq \max_{\rho} \mathrm{Var}\hat{o} = ||O||^2_{\mathrm{shadow}}$
  - Off-diagonal observable
  - Diagonal observable

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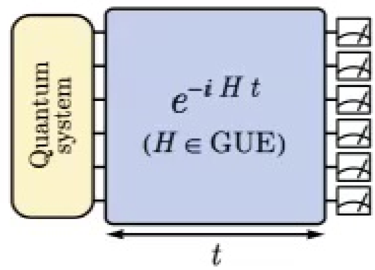

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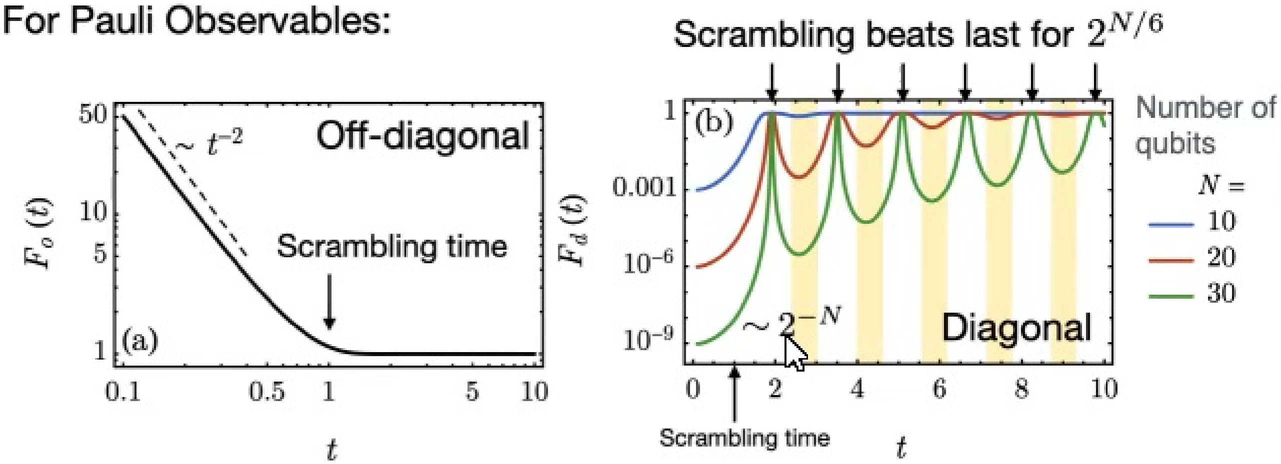
  - Off-diagonal observable $||O_o||^2_{\mathrm{shadow}} = \mathrm{Tr}(O_o^2)F_o(t)$
  - Diagonal observable $||O_d||^2_{\mathrm{shadow}} = \mathrm{Tr}(O_d^2)F_d(t)$

Dynamic Form Factors

Hamiltonian-Driven Shadow Tomography

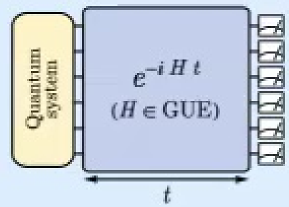
- For Pauli Observables:



- $t \to 0$, No scrambling: computational basis measurements can only probe diagonal observables.
- $t \to \infty$, U approaches Haar (leading D): all observables are equally scrambled. No difference between d/o observables.
- Intermediate t, a sequence of scrambling beats.

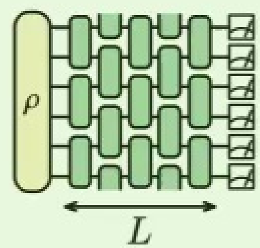
Hamiltonian-Driven Shadow Tomography

- Lessons Learned:
  - Time is our friend. (Tunable unitary channel)
  - Only requires Hamiltonian dynamics without time dependent control



Hong-Ye Hu, Yi-Zhuang You. arXiv:2102.10132

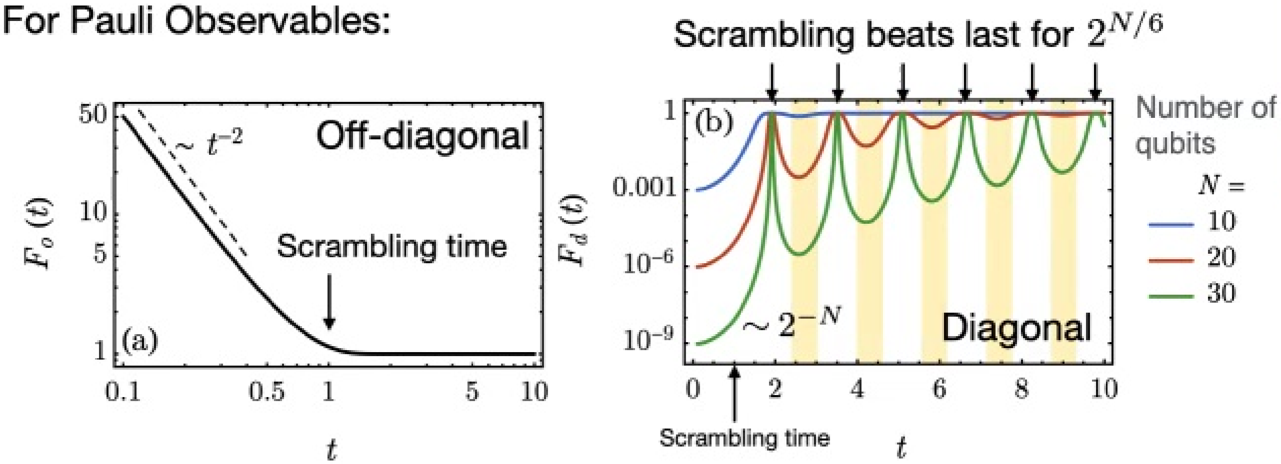
- Next step:
  - Locally scrambled quantum dynamics (finite-depth & local random circuit, local Hamiltonian)



Hong-Ye Hu, Soonwon Choi, Yi-Zhuang You. arXiv:2107.04817

Hamiltonian-Driven Shadow Tomography

- For Pauli Observables:



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  - Intermediate t, a sequence of scrambling beats.

Hamiltonian-Driven Shadow Tomography

- Unitary channel: time evolution generated by generic quantum chaotic Hamiltonian H , modeled by (global) GUE random matrix
  - $t \to 0$: No scrambling: $U = \mathbb{1}$

    Diagonal-traceless

    $$\mathcal{M}[\rho] \to \frac{\mathbb{1}}{D} + \rho_d$$

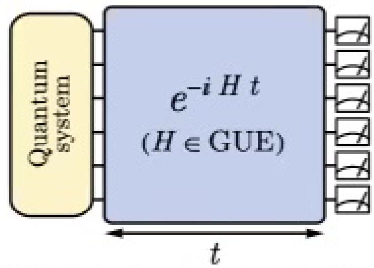
  - $t \to \infty$: Maximal scrambling: Global Haar

    Off-diagonal Diagonal-traceless

    $$\mathcal{M}[\rho] \to \frac{\mathbb{1}}{D} + \frac{\rho_o}{D+1} + \frac{\rho_d}{D+1}$$

  - Intermediate t:

    $$\mathcal{M}[\rho] = \frac{\mathbb{1}}{D} + \frac{\rho_o}{\alpha_D(t)} + \frac{\rho_d}{\beta_D(t)}$$



D : Total Hilbert space dimension

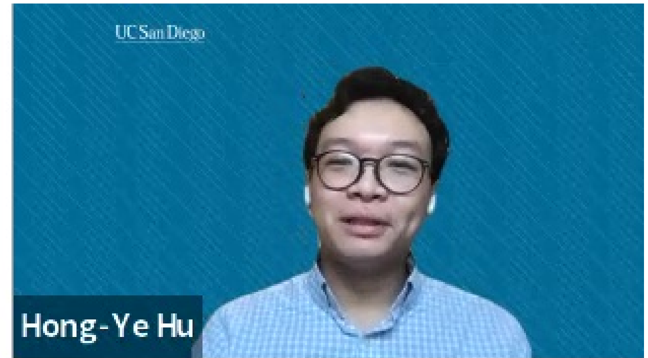
$$\alpha_D(t) = \Big(\frac{1}{D+1} - \lambda_D(t)\Big)^{-1},$$

$$\beta_D(t) = \Big(\frac{1}{D+1} + D\lambda_D(t)\Big)^{-1},$$

$$\lambda_D(t) = \frac{(Dr^2(t) + r(2t))^2 - 4r^2(t)}{(D+3)(D^2-1)},$$

$$r(t) = J_1(2t)/t$$

Spectral form factor



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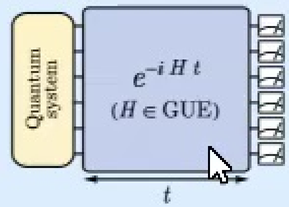

Hong-Ye Hu, Yi-Zhuang You. arXiv:2102.10132

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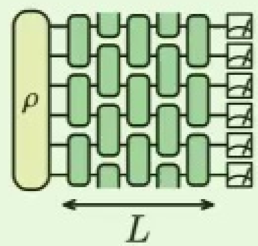
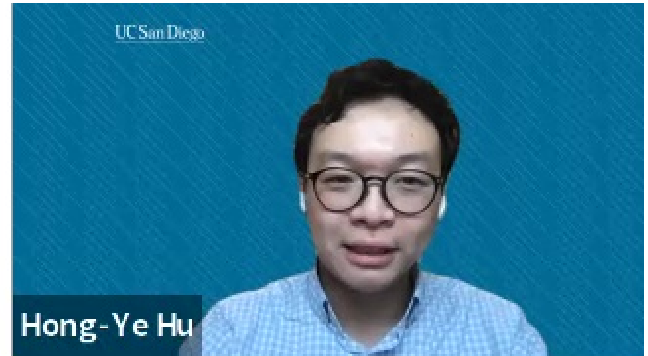


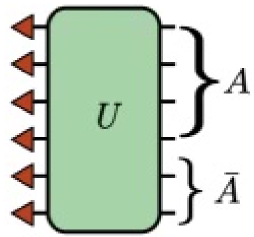
Hong-Ye Hu, Soonwon Choi, Yi-Zhuang You. arXiv:2107.04817



Examples of CST

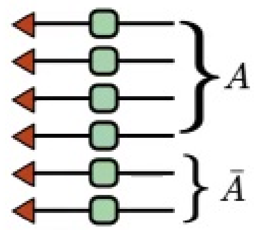
- The entanglement created by circuits are different:
  - Global Haar (3-design) Unitary

    Page states, *volume law entanglement*



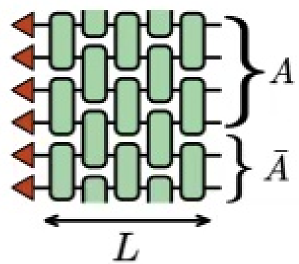
  - Local Haar (3-design) Unitary

    Product states, *no entanglement*



- What about shallow circuit? [**Our work**]

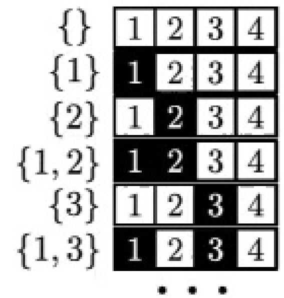
  Entanglement created by shallow circuit
  will be between those limits.



Locally Scrambled Shadow Tomography

- Reconstruction map:
  1. Linear map
  2. Contain basis independent information (Entanglement Entropy)

$A \in 2^{\Omega_N}$

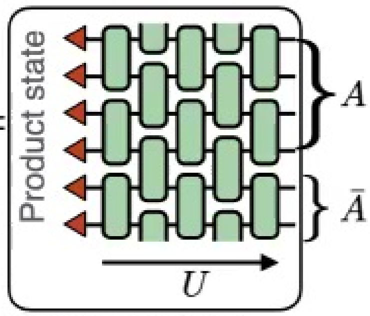


$$\rho = \mathcal{M}^{-1}[\sigma] = d^N \sum_{A \in 2^{\Omega_N}} r_A \sigma_A$$

Only depends on entanglement information

- Entanglement Feature: A vector of all entanglement patterns

$\sigma =$



Quantum evolution

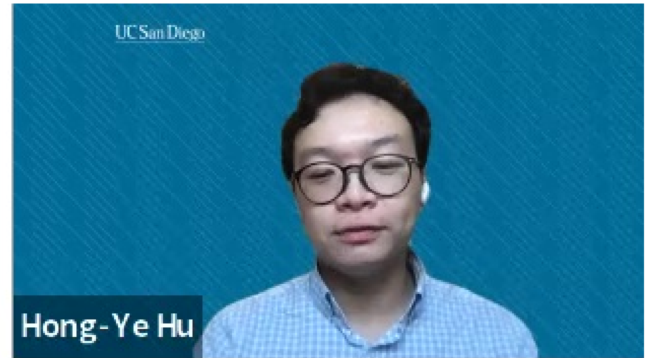
Create entanglement!

$$W^{(2)}[\sigma] = \mathbb{E}_U \left(e^{-S^{(2)}_{\{\}}}, e^{-S^{(2)}_{\{1\}}}, e^{-S^{(2)}_{\{2\}}}, e^{-S^{(2)}_{\{1,2\}}}, \cdots, e^{-S^{(2)}_{A}}, \cdot\right)$$

Solve linear constraints equation:

$$\sum_{A,C} r_A F_{A,B,C} W^{(2)}_C[\sigma] = \delta_{B,(11\cdots 1)}$$

where $F_{A,B,C}$ is a constant fusion factor depending local Hilbert space dimension d.



Locally Scrambled Shadow Tomography

- **A new formulation**: Entanglement pattern created by the unitary ensemble will determine the reconstruction map!

1. Calculate entanglement feature of unitary ensemble.

$$W^{(2)}[\sigma] = \mathop{\mathbb{E}}_{U} \left(e^{-S^{(2)}_{\{\}}}, e^{-S^{(2)}_{\{1\}}}, e^{-S^{(2)}_{\{2\}}}, e^{-S^{(2)}_{\{1,2\}}}, \cdots\right)$$

2. Solve linear constraints equation:

$$\sum_{A,C} r_A F_{A,B,C} W^{(2)}_C[\sigma] = \delta_{B,(11\cdots 1)}$$

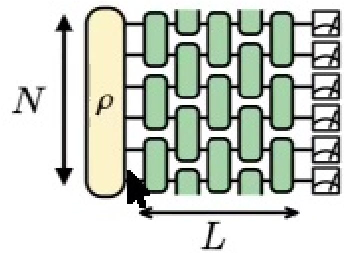
3. Then reconstruction map is given by:

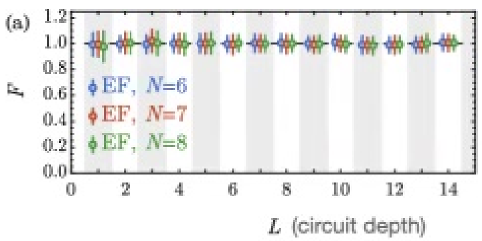
$$\rho = \mathcal{M}^{-1}[\sigma] = d^N \sum_{A \in 2^{\Omega_N}} r_A \sigma_A$$

- The new formulation bridges **quantum dynamics** and **quantum state tomography** (quantum information)

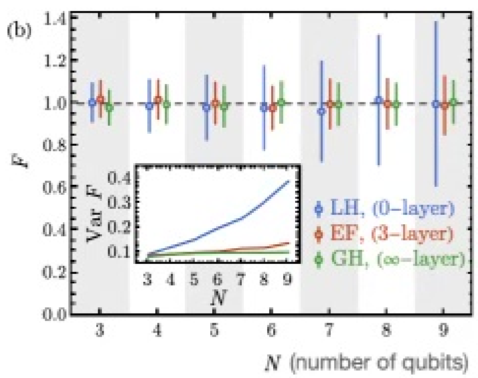
Numerical results

- Random unitary (Clifford) circuit
- Fidelity (low rank operator): $\mathcal{F}(\rho, \rho_{\text{construct}})$





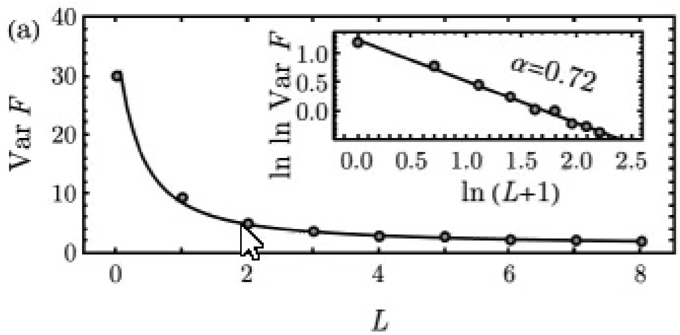
Unbiased reconstruction for different depth quantum circuits



Great advantage in reducing sample complexity even for a few layers!

Sample complexity of low-rank operator

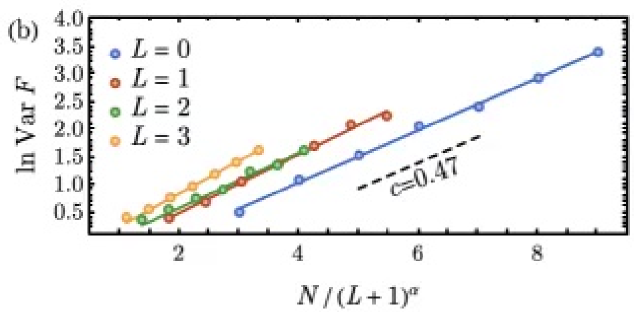
- Empirical formula



$$\mathrm{Var}F \propto \exp\left(0.47\frac{N}{(L+1)^{0.72}}\right)$$

Sample complexity decreases **faster than exponential** in a few layers.

- Data collapse of small system size simulation



Great advantage in shallow circuit regions!

Sample complexity of full-rank operator

- Pauli operators

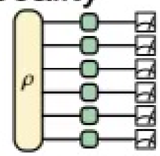
$$Z^{(k)} = Z^{\otimes k} I^{\otimes (N-k)}$$

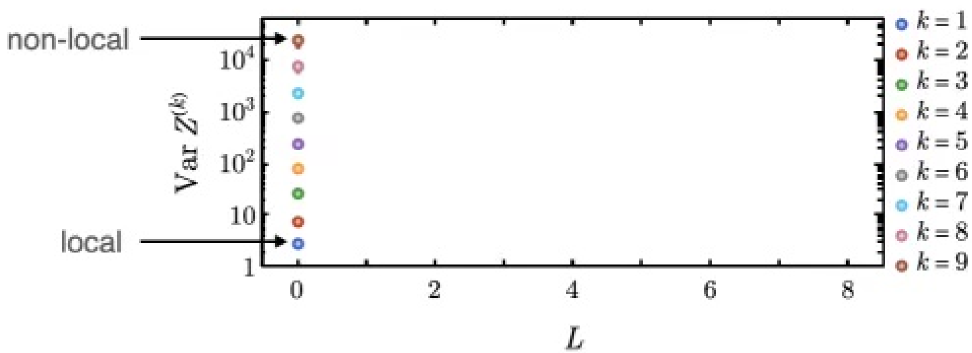
  - In the shallow circuit limit $L \to 0$, complexity depends on locality

$$\mathrm{Var} Z^{(k)} \propto 4^k$$

local operator ~ low sample complexity

non-local operator ~ high sample complexity





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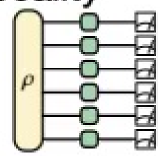
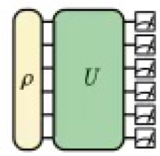
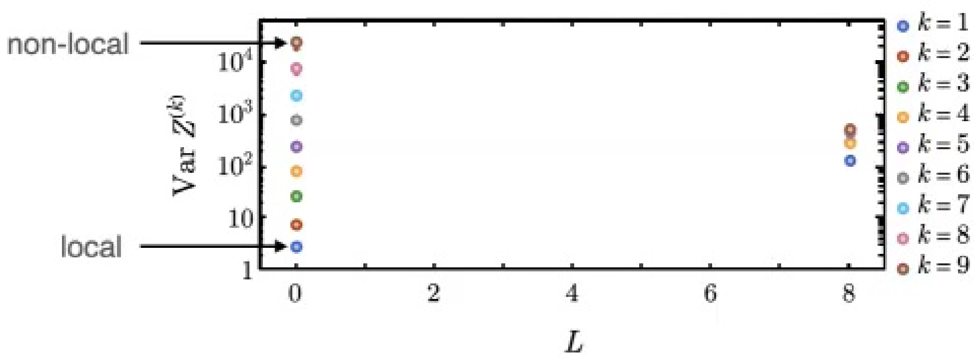


  - In the deep circuit limit $L \to \infty$, there is no difference between local and non-local operator $\mathrm{Var} Z^{(k)} \propto D = 2^N$





Approximate Tomography with Local Hamiltonian Dynamics

- Realizing 2-local random Clifford gates may not be easy.
- Can realistic local Hamiltonian generate unitary dynamics that approximate locally scrambling assumption?

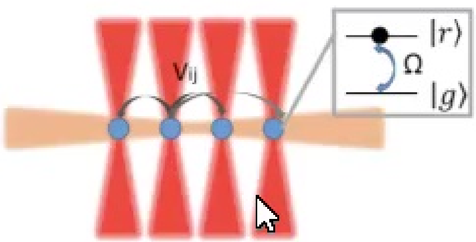


Image from *M. Wu@Stanford*

$H_{\text{Rydberg atoms}}$

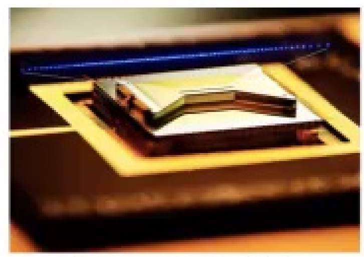
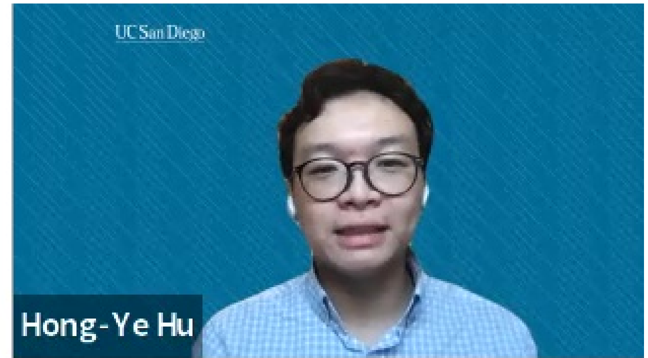


Image from *Physics World*

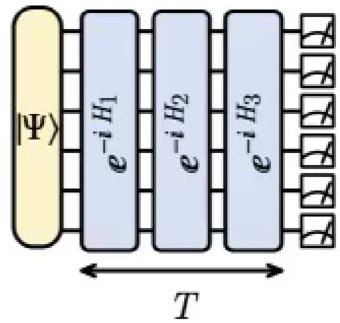
$H_{\text{Trapped ions}}$



Approximate Tomography with Local Hamiltonian Dynamics

- Unitary evolutions $U = e^{-iHT}$ generated by a local Hamiltonian for finite amount of time T.

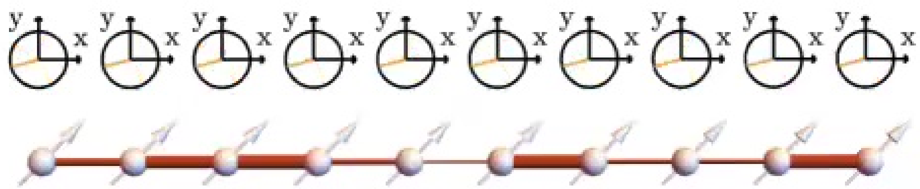
- Disordered Quantum Spin Model:



Quench disorder

$$H_t = \sum_{\langle i,j \rangle} J_{ij} X_i X_j + h \sum_i (\cos\theta_t X_i + \sin\theta_t Y_i)$$

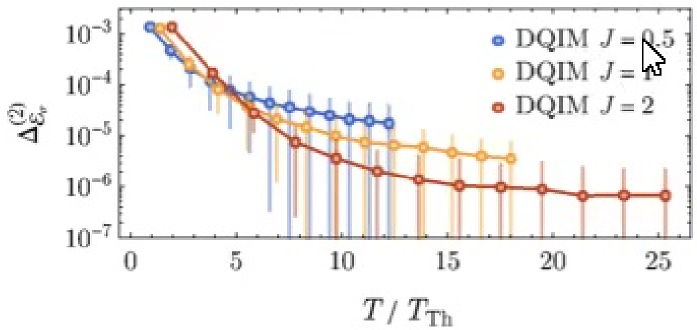
$$U = \prod_{t=1}^{T} e^{-iH_t} \ , \ J_{ij} \sim \mathrm{Uni}[J - \tfrac{J}{2}, J + \tfrac{J}{2}]$$



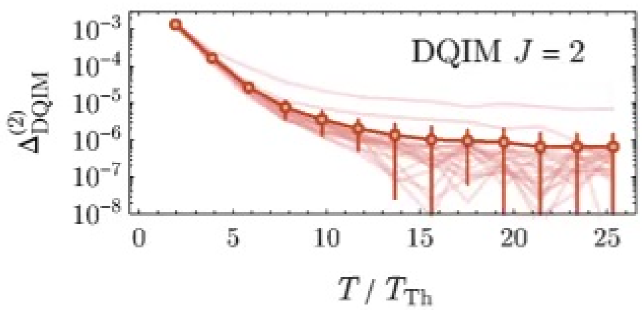
- Question: Does this practical unitary ensemble satisfy *locally scrambling* assumption? How to characterize it?

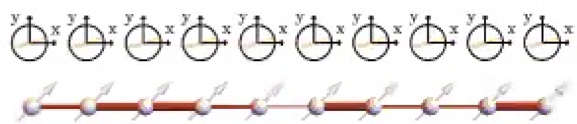
Local Frame Potential

- Local frame potential as a distance



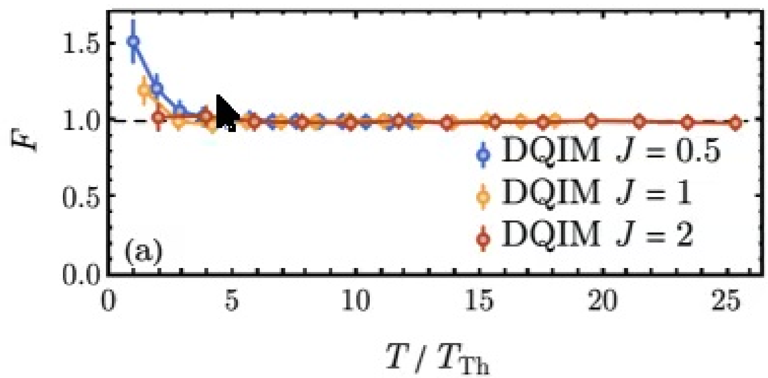
- One quench disorder Hamiltonian $\approx$ ensemble average





Approximate shadow tomography

- Fidelity prediction



- Before locally scrambling condition is reached, prediction is biased

- After locally scrambling condition is reached, prediction is unbiased

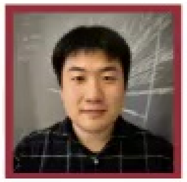
Summary

- Shadow tomography is a great tool, but finding $\mathcal{M}^{-1}$ for shallow circuit is hard.
- **Solution**: CST + Chaotic quantum dynamics
- Merits:
  - New formulation naturally bridges the study of Quantum Dynamics and Quantum State Tomography.
  - Nice interpolation between global Haar and local Haar. Information scrambling assists quantum measurements.
  - Experiment friendly: shallow circuits, and local-Hamiltonian dynamics without time dependent fine tuning.
  - Flexible design of unitary circuits. Adaptation to target observables

Acknowledgement

- My fantastic collaborators:

Dr.Yi-Zhuang You



Dr. Soonwon Choi

- Thanks for discussions and comments

Daniel Mark

Hsin-Yuan Huang

Dr. Richard Keung

Dr. Andreas Elben

Dr. Jens Eisert

Dr. Xun Gao

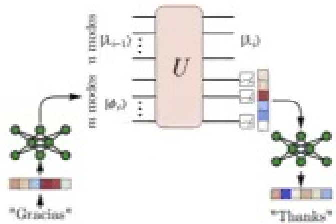
Dr. Junyu Liu

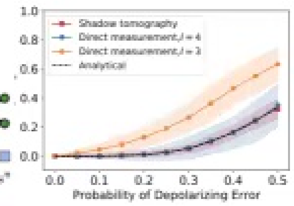
My Other Researches

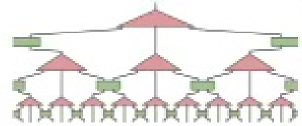
Open source software

Quantum Information/Computation

- Provable quantum advantage in neural sequence modeling (In preparation) EA, **HYH**, JH,XG
- Classical shadow tomography+error mitigation (In preparation) **HYH**, RL, YZY, RE, ZW
- Differential programming of isometric tensor network CG, **HYH**, YZ
- Resonant quantum search with monitor qubits FW, **HYH**, BW

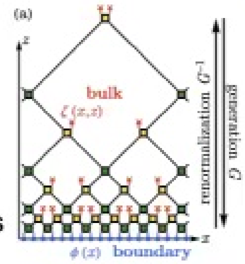


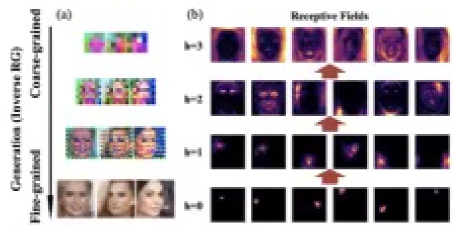




Machine Learning Holography

- Neural Network Renormalization Group and Holographic Duality **HYH**, SL, LW, YZY
- Holographic QCD and Neural ODE KH, **HYH**, YZY
- Hierarchical Representation of Human Faces **HYH**, DW, YZY, BO, YC





Quantum many-body physics

- Strong Disordered Many-body Physics and SBRG **HYH**, YZY
- Gapless SPT + Strong Disorder **HYH***, CD*, VK, YZY, RV, RV