Title: Probing topological invariants from a ground state wave function

Speakers: Ze-Pei Cian

Series: Perimeter Institute Quantum Discussions

Date: December 06, 2021 - 11:00 AM

URL: https://pirsa.org/21120013

Abstract: With the rapid development of programmable quantum simulators, the quantum states can be controlled with unprecedented precision. Thus, it opens a new opportunity to explore the strongly correlated phase of matter with new quantum technology platforms. In quantum simulators, one can engineer interactions between the microscopic degree of freedom and create exotic phases of matter that presumably are beyond the reach of natural materials. Moreover, quantum states can be directly measured instead of probing physical properties indirectly via optical and electrical responses of material as done in traditional condensed matter. Therefore, it is pressing to develop new approaches to efficiently prepare and characterize desired quantum states in the novel quantum technology platforms.

In this talk, I will introduce our recent works on the characterization of the topological invariants from a ground state wave function of the topological order phase and the implementation in noisy intermediate quantum devices. First, using topological field theory and tensor network simulations, we demonstrate how to extract the many-body Chern number (MBCN) given a bulk of a fractional quantum Hall wave function [1]. We then propose an ancilla-free experimental scheme for measuring the MBCN without requiring any knowledge of the Hamiltonian. Specifically, we use the statistical correlations of randomized measurements to infer the MBCN of a wave function [2]. Finally, I will present an unbiased numerical optimization scheme to systematically find the Wilson loop operators given a ground state wave function of a gapped, translationally invariant Hamiltonian on a disk. We then show how these Wilson loop operators can be cut and glued through further optimization to give operators that can create, move, and annihilate anyon excitations. We then use these operators to determine the braiding statistics and topological twists of the anyons, yielding a way to fully characterize topological order from the bulk of a ground state wave function [3].

[1] H. Dehghani, Z.P. Cian, M. Hafezi, and M. Barkeshl, Phys. Rev. B 103, 075102
[2] Z.P. Cian, H. Dehghani, A. Elben, B. Vermersch, G. Zhu, M. Barkeshli, P. Zoller, and M. Hafezi, Phys. Rev. Lett. 126, 050501
[3] Z.P. Cian, M. Hafezi, and M. Barkeshl, Manuscript in preparation.

Probing topological properties from the ground state wave function

Ze-Pei Cian

Department of physics, University of Maryland, College Park

Joint Quantum Institute

Topological ordered phase:

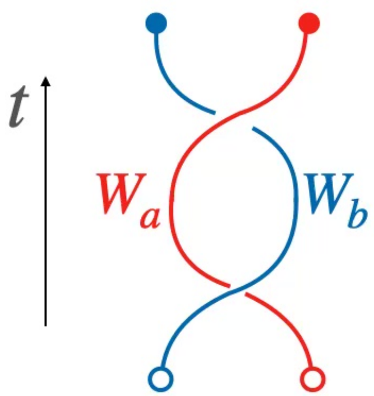
-The phase can not be characterized by local order parameter and SSB. $\langle \phi \rangle \neq 0$

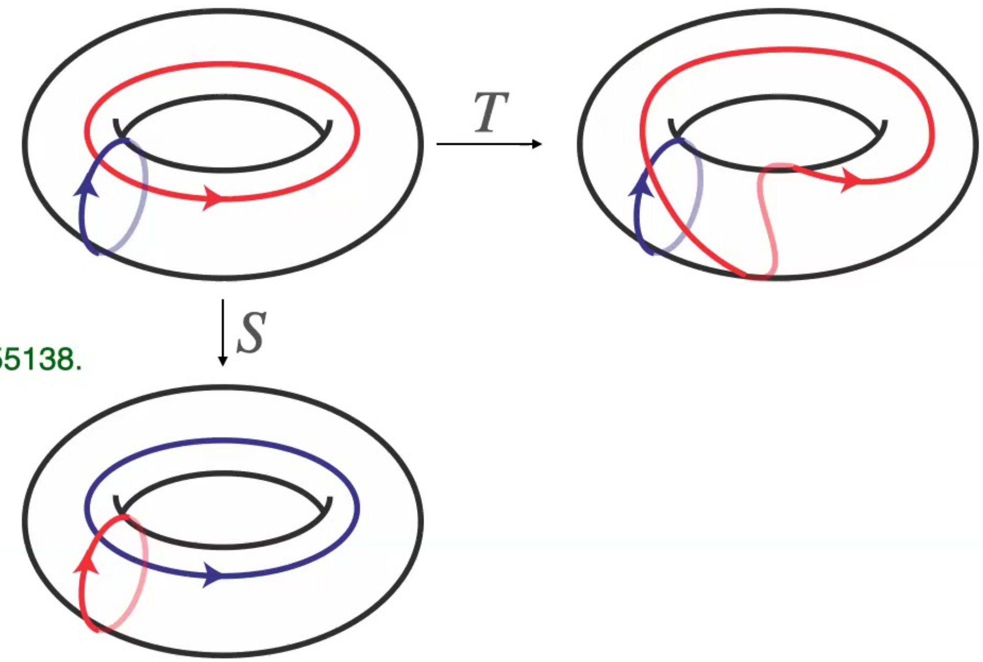
- Macroscopically :
  - Robust ground state degeneracy
  - Modular S and T matrix

Wen, Niu, Phys. Rev. B **41**, 9377

- Microscopically : Wen, Int. J. Mod. Phys. B. **4** (2): 239
  - Long-range Entanglement
  - Quasi-particles carry fractional statistic

Chen, Gu, Wen, Phys. Rev. B. **82** (15): 155138.





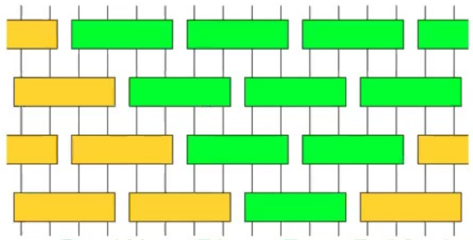
Probing topological properties from the ground state wave function

Motivation No. 1 :

- From a fundamental perspective :

  -Two gapped ground state $|\psi_{1(2)}\rangle$ are in the same phase $\leftrightarrow |\psi_1\rangle = U|\psi_2\rangle$

  constant depth circuit



Chen, Gu, Wen, Phys. Rev. B ***82****, 155138 (2010)*

Information from H?

Linear response paradigm $H + \delta H$?

  - $|\psi\rangle$ contains enough topological informations to characterize the phase.

  - How do we probe topological invariants that allows the full characterization of a topological order phase from $|\psi\rangle$ only?

    i.e. total quantum dimension $(\mathscr{D})$, $S = \alpha L - \log \mathscr{D}$

Kitaev, Preskill, Phys. Rev. Lett. ***96****, 110404*

Levin, Wen, Phys. Rev. Lett. ***96****, 110405*

Probing topological properties from the ground state wave function

Motivation No. 2 :

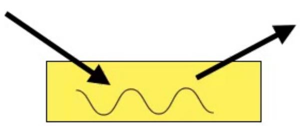
- From a quantum simulation perspective :

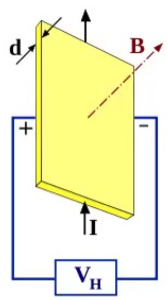
  (1) Directly access to the wave function ✅ (Equal-time Multipartite Correlation function)

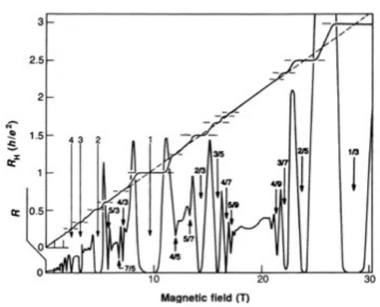
  i.e. $\langle \psi | Z_1 Z_2 X_3 Y_4 \ldots Z_N | \psi \rangle$

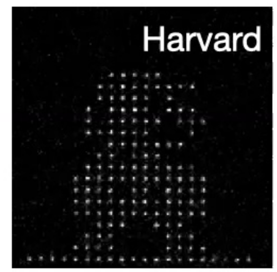
  (2) Linear response might not be applicable ⛔

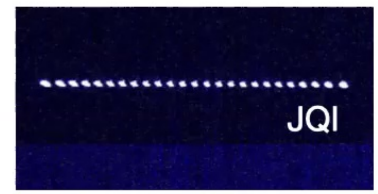
Linear response paradigm:

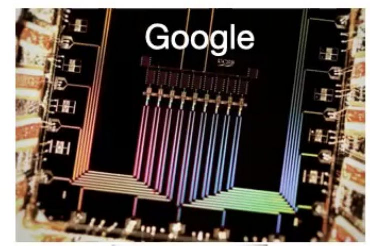


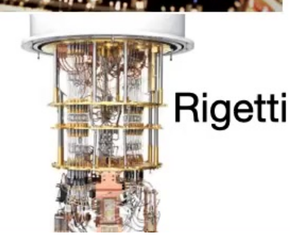












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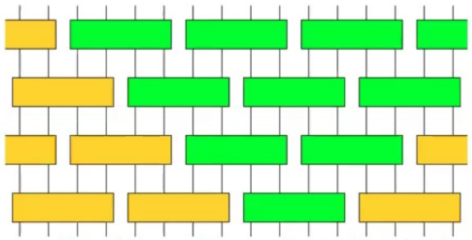

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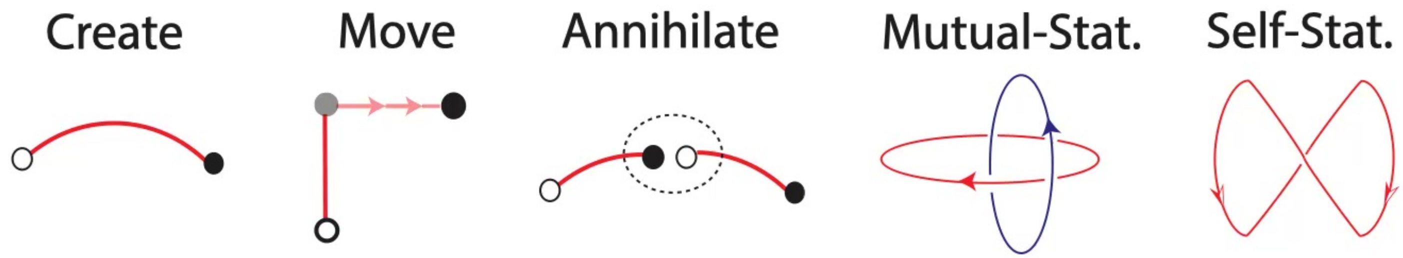
    Kitaev, Preskill, Phys. Rev. Lett. ***96****, 110404*

    Levin, Wen, Phys. Rev. Lett. ***96****, 110405*

Outline :

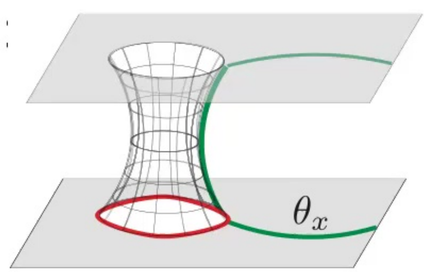
(1) Extracting Wilson loops and fractional statistics from a ground state wave function on a disk   Manuscript in preparation

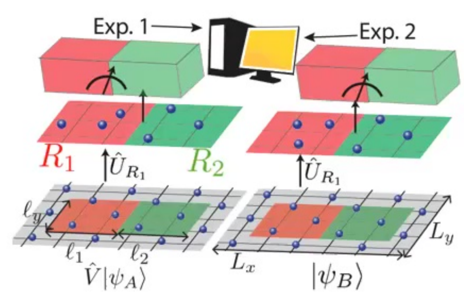


(2) Extracting many-body Chern number in a quantum simulator

Phys. Rev. B, 103, 075102 (2021)

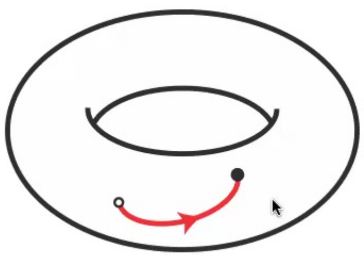
Phys. Rev. Lett., 126, 050501 (2021)



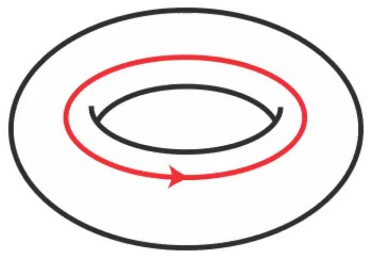


Wilson Loop operator refresher

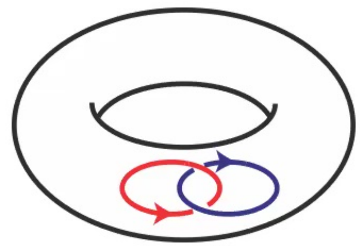
$W_{open}|G\rangle = |E\rangle$



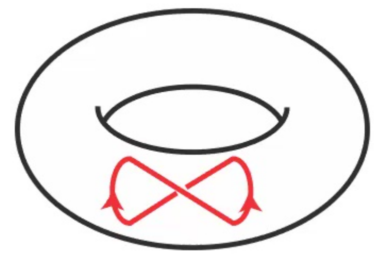
$W_{close}|G_1\rangle = |G_2\rangle$



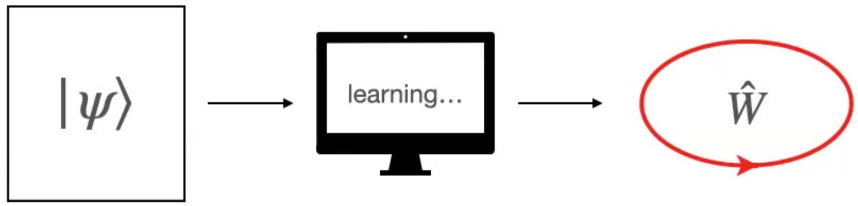
$W_1 \infty W_2|G\rangle = e^{i\phi_{12}}|G\rangle$



$W_{twist}|G\rangle = e^{i\theta}|G\rangle$

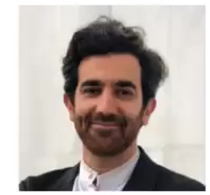


Given a wave function with topological order, we want to extract the Wilson loop operators from it.



Maissam Barkeshkli

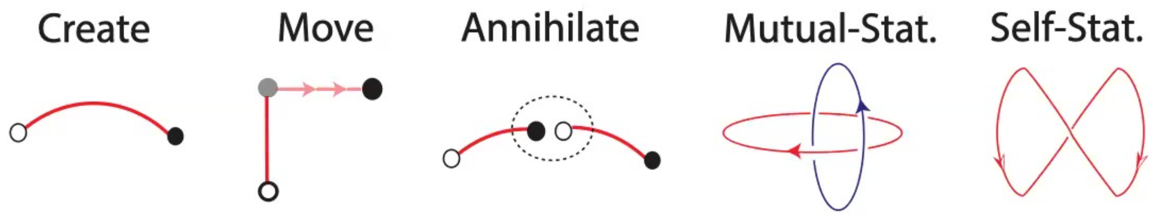
(U. Maryland)



Mohammad Hafezi

(U. Maryland)

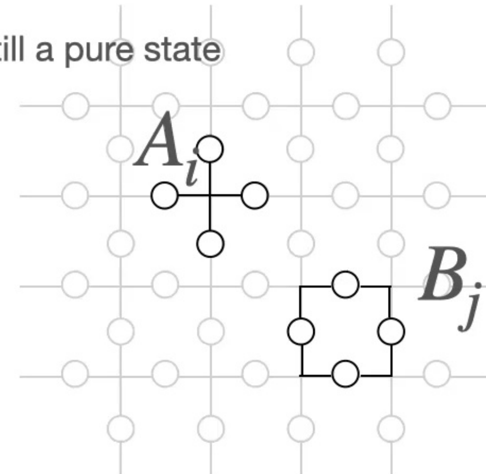
After the extraction of Wilson loop operators …



Fundamental : characterize topological order from wave function.

Quantum simulation : real world is not perfect.
After preparing a topological ordered state in a noisy device, how do we manipulate anyons?

In this work, we focus on the coherent error —> non-ideal wave function but still a pure state



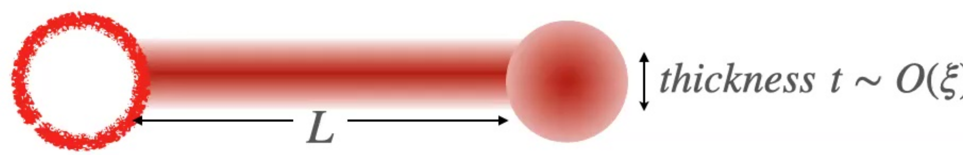
Example : Toric Code model in a magnetic field.

$$H = -\sum_{i\in +} A_i - \sum_{j\in \square} B_j + h_z \sum_k Z_k + h_x \sum_l X_l$$

$$A_i = \prod_{s\in i} Z_s \quad B_j = \prod_{p\in j} X_p$$

Exactly solvable point $h_x = h_z = 0$: WLOs $\prod_{i\in\gamma} Z_i, \prod_{j\in\gamma'} X_j$

Non-zero h_x, h_z $\rightarrow$Non-zero correlation length ξ

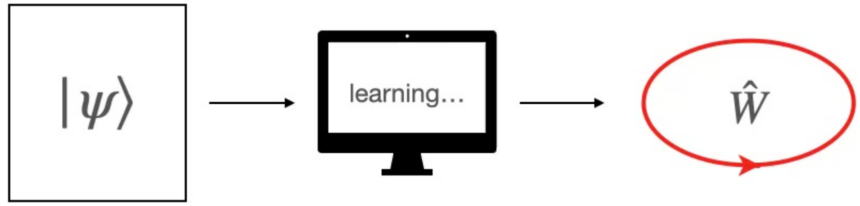


Truncation error : $\epsilon \sim O(N_s e^{-\frac{t}{\xi}})$

Hasting, Wen, Phys. Rev. B ***72****, 045141 (2005)*

, where N_s is the number of site the WLO acts on.

Numerical Scheme :

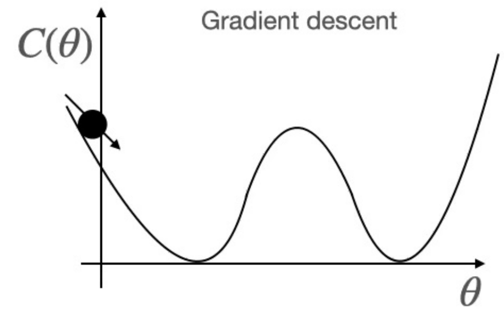


Input : Wave function $|\psi\rangle$, Wilson loop ansatz $W(\theta)$.

Output : Wilson loop operator $\{W_i \,|\, 0 \leq i < N_W\}$.

Scheme : Optimize the following cost function :

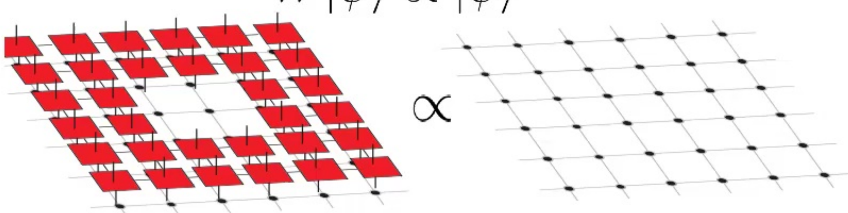
$$C(\theta) = [\langle\psi|\,W(\theta)\,|\psi\rangle - 1]^2 + [\langle\psi|\,W^\dagger(\theta)W(\theta)\,|\psi\rangle - 1]^2.$$



-$C(\theta) \geq 0$, when $C(\theta = \theta^*) = 0 \leftrightarrow W(\theta^*)\,|\psi\rangle = |\psi\rangle$

-We parametrize the Wilson loop ansatz by tensor network with bond dimension χ and thickness t.

$W|\psi\rangle \propto |\psi\rangle$



Evaluation of $\langle\psi|\,W(\theta)\,|\psi\rangle$ and $\langle\psi|\,W^\dagger(\theta)W(\theta)\,|\psi\rangle$ is efficient!

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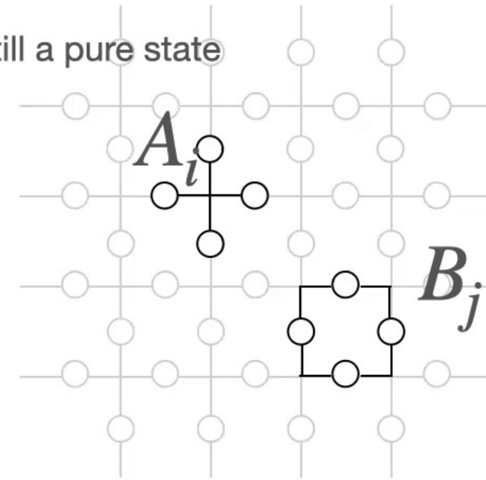

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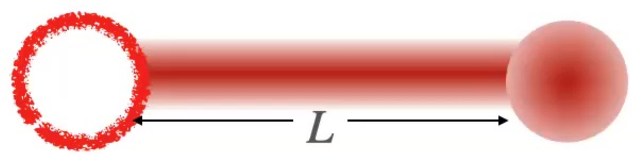

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*Hasting, Wen, Phys. Rev. B **72**, 045141 (2005)*

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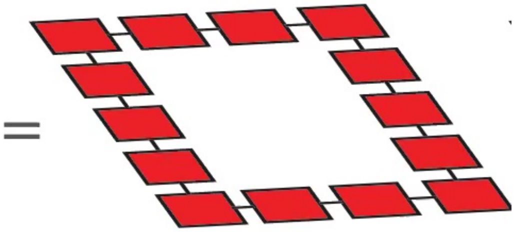
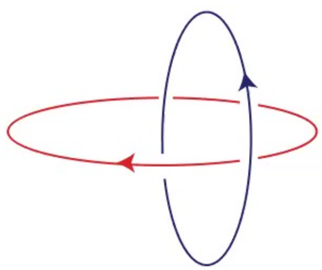
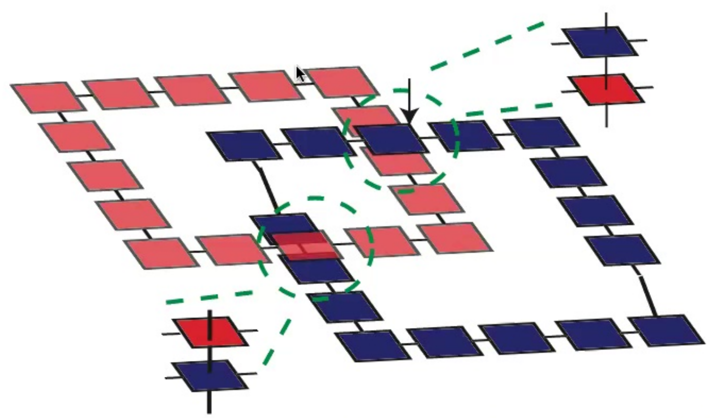
After numerical optimization :

$W_a =$ 

Mutual-Braiding Statistic





Haah, J., *Communications in Mathematical Physics*, *342*(3), pp.771-801. (2016)

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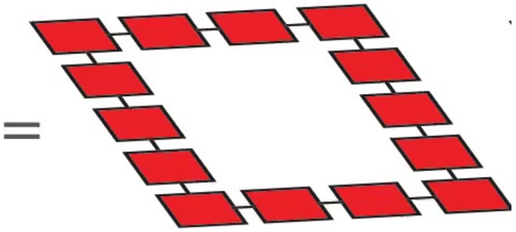
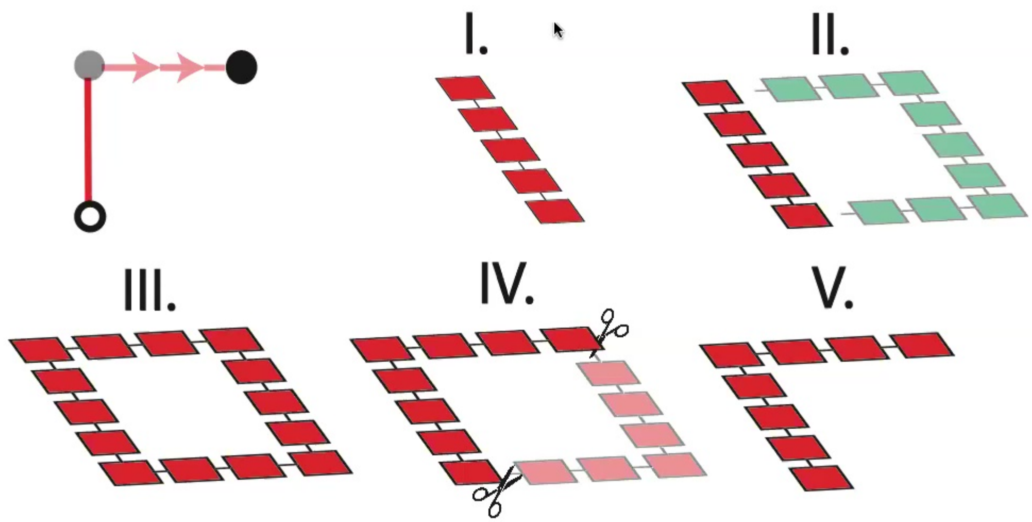
$W_a =$ 

Move anyons:



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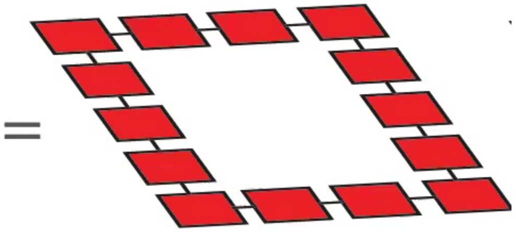
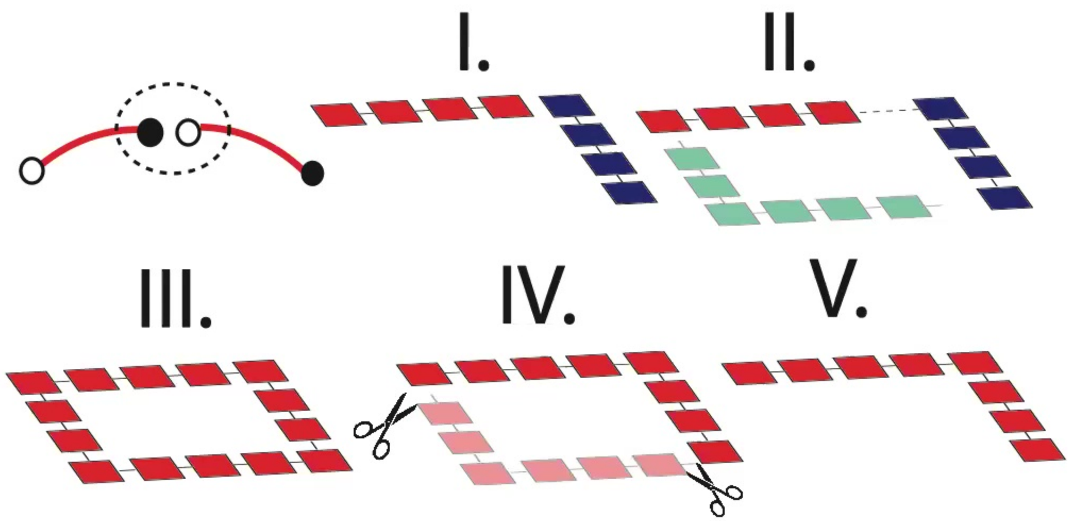
$W_a =$ 

Merge anyons:



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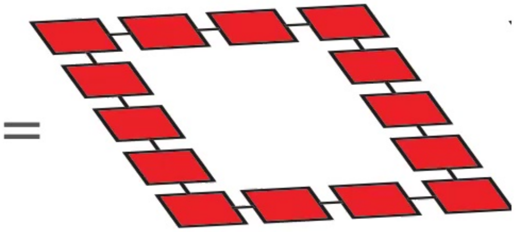
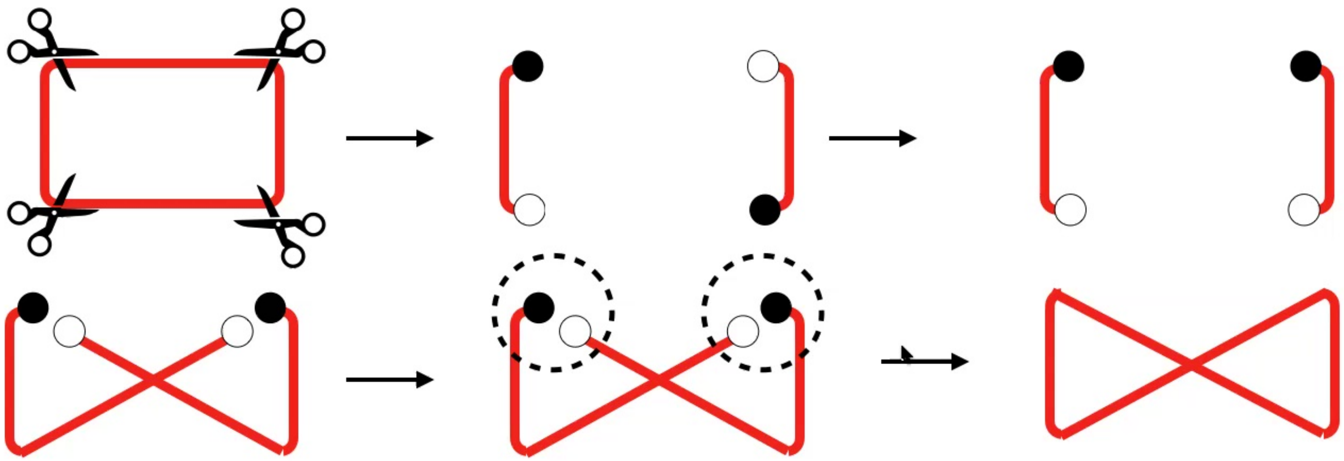
$W_a =$ 

(Spin-Statistic theorem)

Self-braiding statistic :



How to find the ground state numerically :
Infinite projected entangled pair states,
Corner transfer matrix renormalization group,
Differential programming

Jordan, Orus, Vidal, Verstraete, Cirac, Phys. Rev. Lett. 101, 250602 (2008)
Nishino,Okunishi, J. Phys. Soc. Jpn. 65, 891 (1996).
Liao, Liu, Wang, Xiang, Phys. Rev. X ***9****, 031041(2020)*

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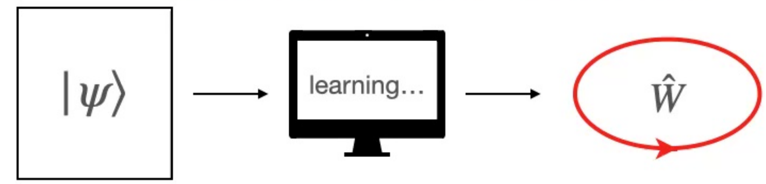
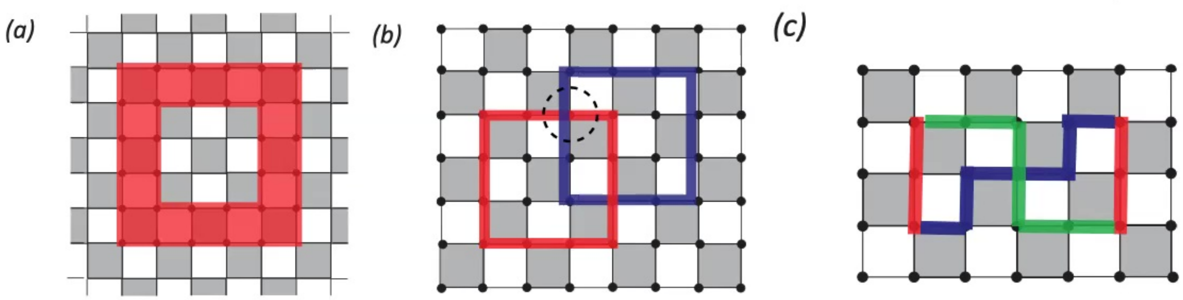


** Simulation on dual lattice*



$$\begin{array}{cc} & \begin{array}{cccc} I & e & m & em \end{array} \\ \tilde{S} = & \begin{pmatrix} 1.0 & 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & -1.00001 & -0.99996 \\ 1.0 & -0.99999 & 1.0 & -0.99977 \\ 1.0 & -1.00004 & -1.00024 & 1.0 \end{pmatrix} \begin{array}{c} I \\ e \\ m \\ em \end{array} \end{array}$$

$$\begin{array}{c} \qquad\qquad I \quad e \quad m \quad em \\ \tilde{T} = diag(1.0, 0.981, 0.978, -0.969). \end{array}$$

$$h_x = 0.1, h_z = 0.05, t = 2, L = 24, \chi = 1$$

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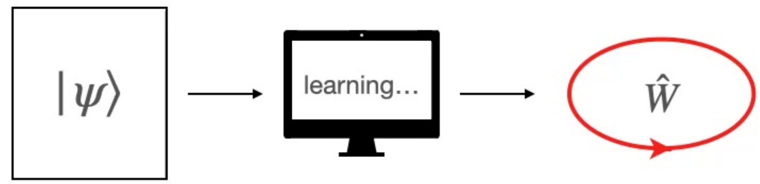

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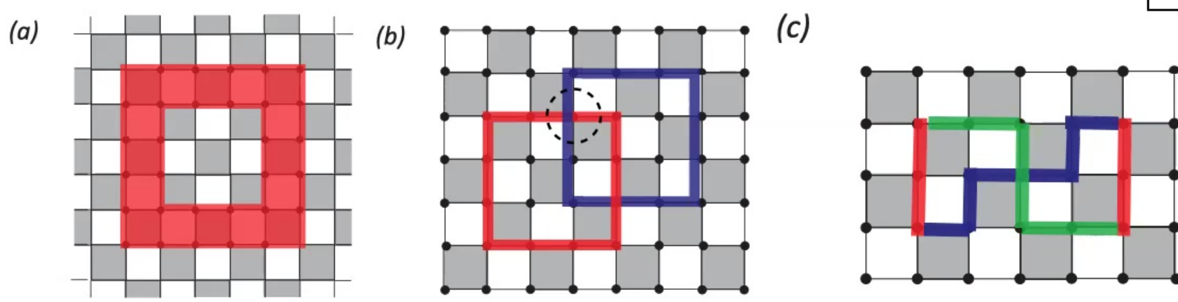

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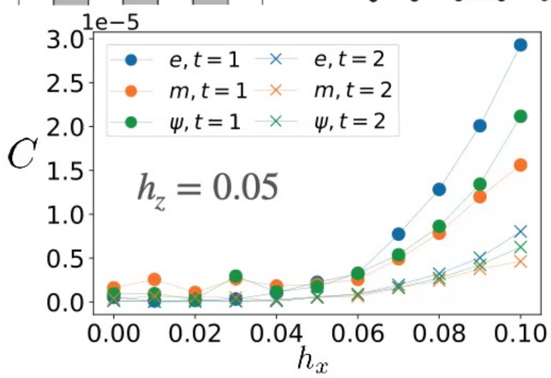
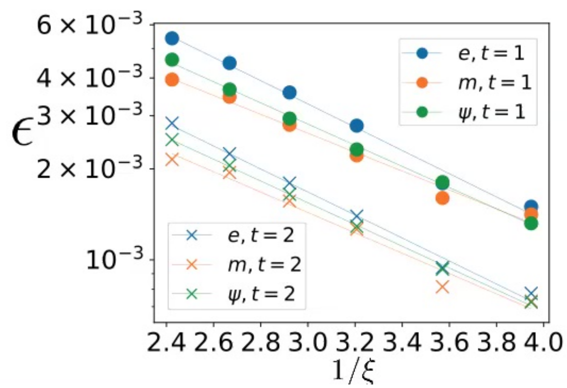
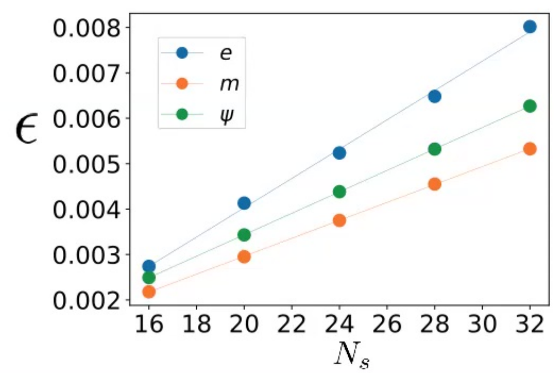
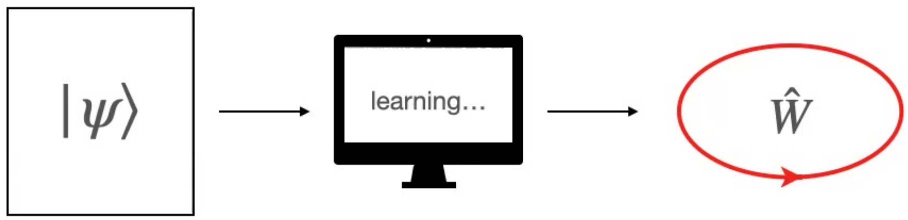




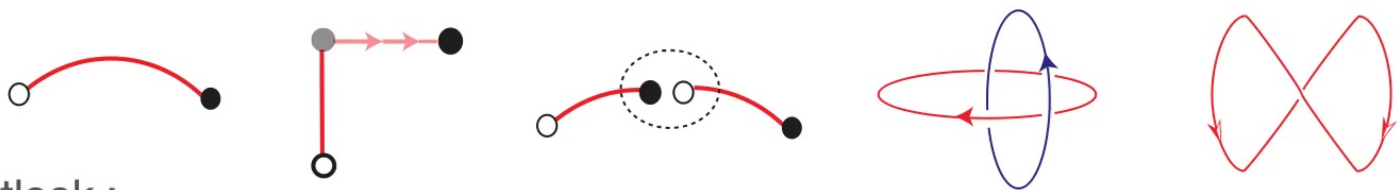


Summary and Outlook (for part I)

Summary :



Create　　Move　　Annihilate　　Mutual-Stat.　　Self-Stat.

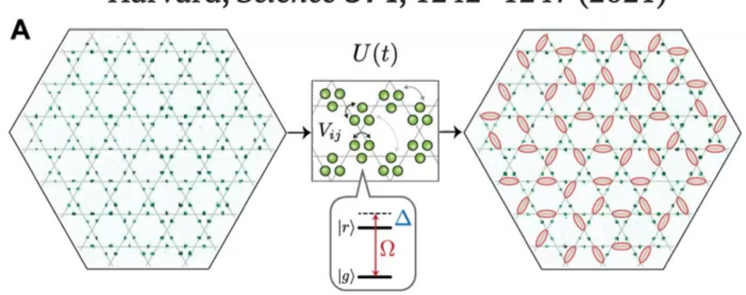


Outlook :

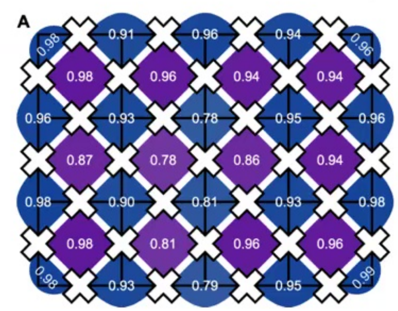
Measurement protocol with quantum simulator?

Variational Quantum Wilson Loop optimizer?

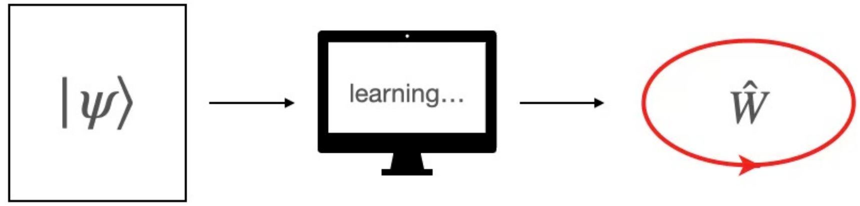
Harvard, *Science* **374**, 1242–1247 (2021)



Google, *Science* **374**, 1237–1241 (2021)



Numerical Scheme :

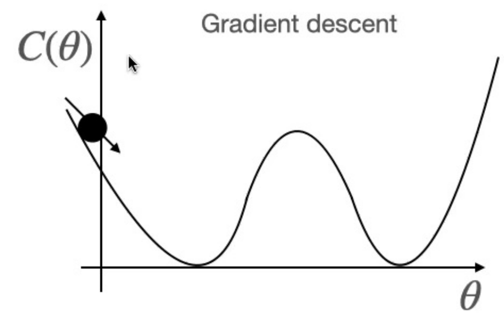


Input : Wave function $|\psi\rangle$, Wilson loop ansatz $W(\theta)$.

Output : Wilson loop operator $\{W_i \,|\, 0 \le i < N_W\}$.

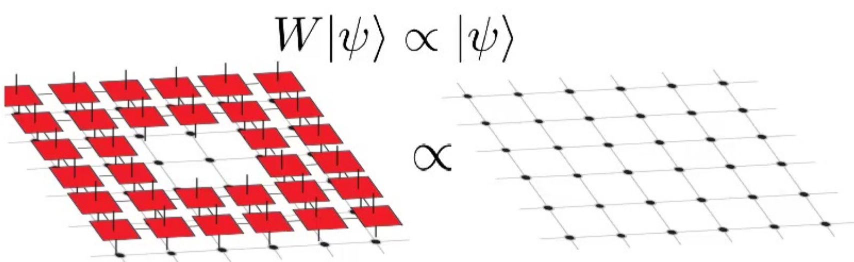
Scheme : Optimize the following cost function :

$$C(\theta) = [\langle\psi|\,W(\theta)\,|\psi\rangle - 1]^2 + [\langle\psi|\,W^\dagger(\theta)W(\theta)\,|\psi\rangle - 1]^2.$$



-$C(\theta) \ge 0$, when $C(\theta = \theta^*) = 0 \leftrightarrow W(\theta^*)\,|\psi\rangle = |\psi\rangle$

-We parametrize the Wilson loop ansatz by tensor network with bond dimension χ and thickness t.



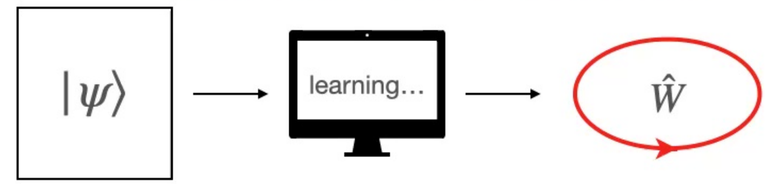
Evaluation of $\langle\psi|\,W(\theta)\,|\psi\rangle$ and $\langle\psi|\,W^\dagger(\theta)W(\theta)\,|\psi\rangle$ is efficient!

How to find the ground state numerically :
Infinite projected entangled pair states,
Corner transfer matrix renormalization group,
Differential programming

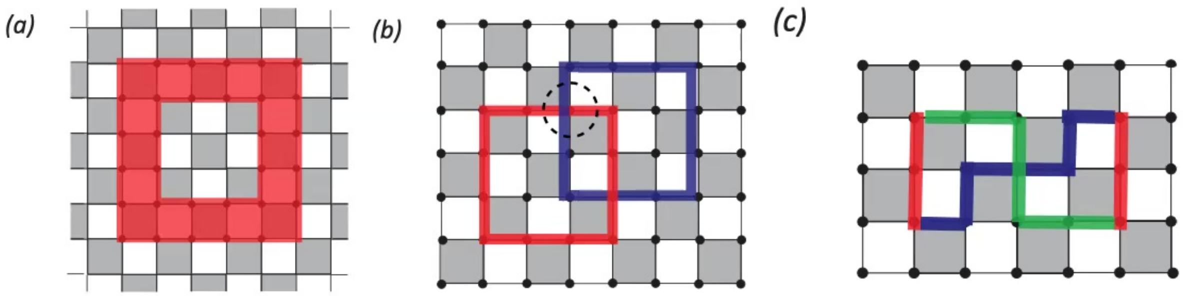
Jordan, Orus, Vidal, Verstraete, Cirac, Phys. Rev. Lett. 101, 250602 (2008)
Nishino,Okunishi, J. Phys. Soc. Jpn. 65, 891 (1996).
*Liao, Liu, Wang, Xiang, Phys. Rev. X **9**, 031041(2020)*

Example : Toric Code model in a magnetic field.

$$H = -\sum_{i\in +} A_i - \sum_{j\in \square} B_j + h_z \sum_k Z_k + h_x \sum_l X_l$$



** Simulation on dual lattice*



$$\tilde{S} = \begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{array}{cccc} I & e & m & em \end{array}$$

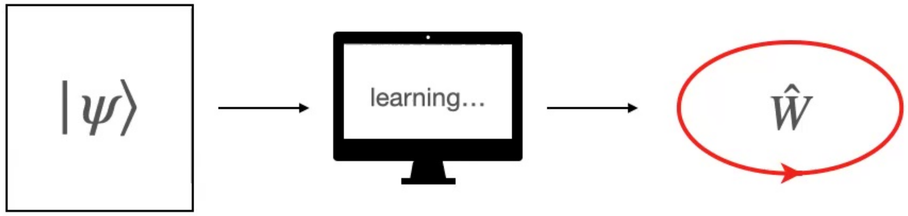
$$\tilde{S} = \begin{pmatrix} 1.0 & 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & -1.00001 & -0.99996 \\ 1.0 & -0.99999 & 1.0 & -0.99977 \\ 1.0 & -1.00004 & -1.00024 & 1.0 \end{pmatrix} \begin{array}{c} I \\ e \\ m \\ em \end{array}$$

$$\begin{array}{cccc} I & e & m & em \end{array}$$
$$\tilde{T} = diag(1.0, 0.981, 0.978, -0.969).$$

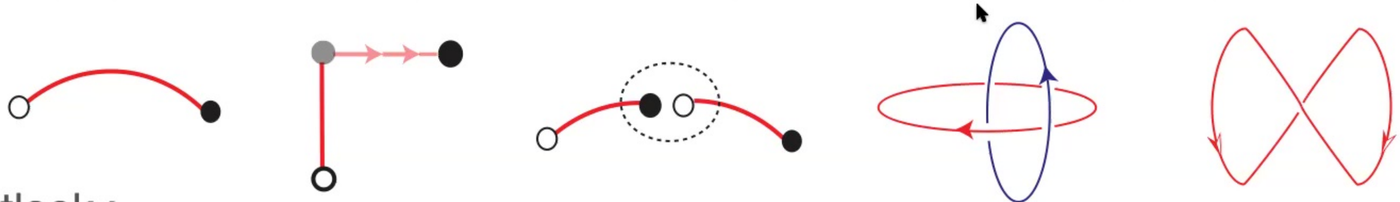
$$h_x = 0.1, h_z = 0.05, t = 2, L = 24, \chi = 1$$

Summary and Outlook (for part I)

Summary :



Create | Move | Annihilate | Mutual-Stat. | Self-Stat.

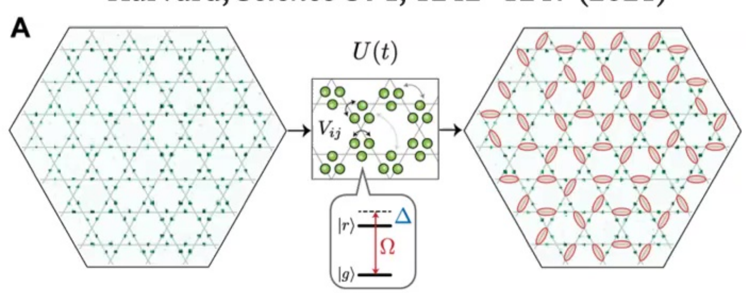


Outlook :

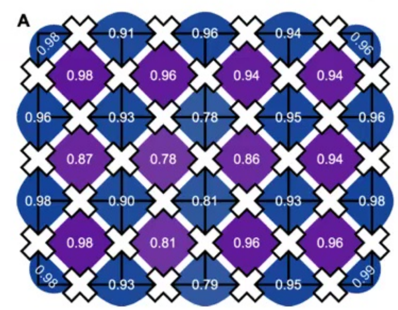
Measurement protocol with quantum simulator?

Variational Quantum Wilson Loop optimizer?

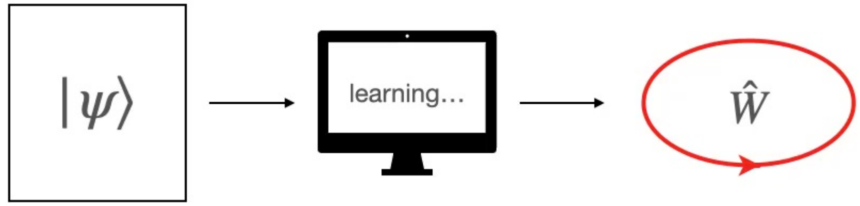
Harvard, *Science* **374**, 1242–1247 (2021)



Google, *Science* **374**, 1237–1241 (2021)



Numerical Scheme :



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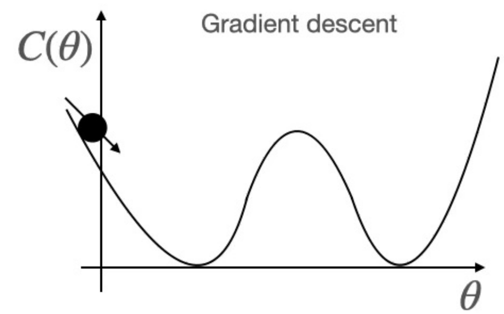

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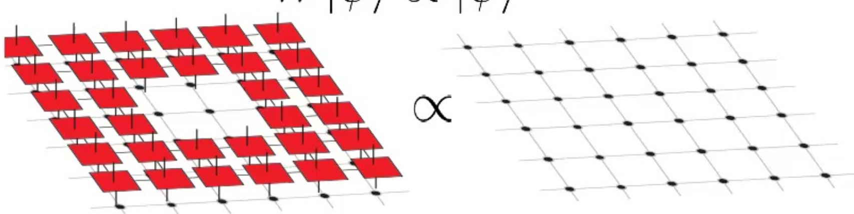
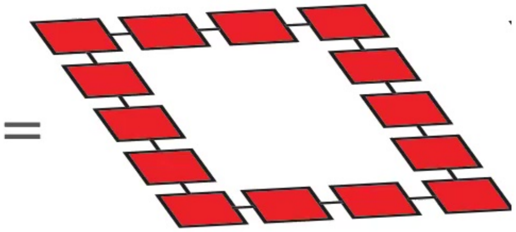
$$W|\psi\rangle \propto |\psi\rangle$$

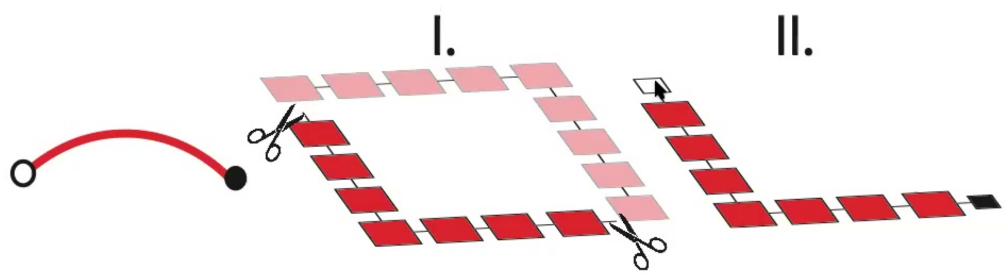


Evaluation of $\langle\psi|\, W(\theta)\,|\psi\rangle$ and $\langle\psi|\, W^\dagger(\theta)W(\theta)\,|\psi\rangle$ is efficient!

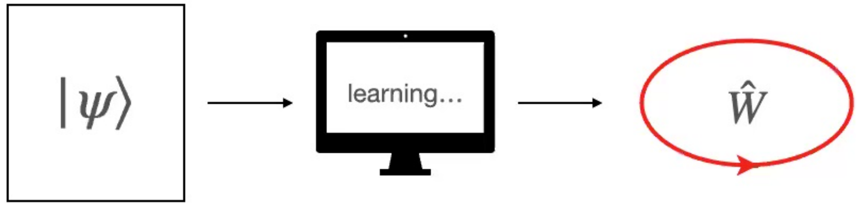
After numerical optimization :

$W_a =$ 

Create a pair of anyons:



Numerical Scheme :



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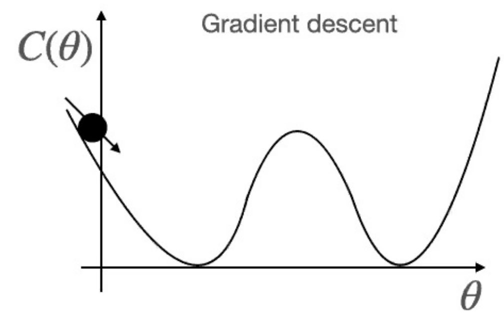

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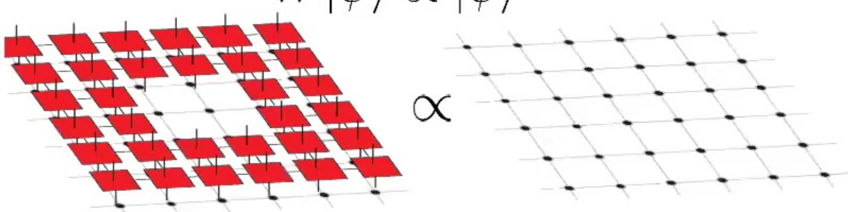

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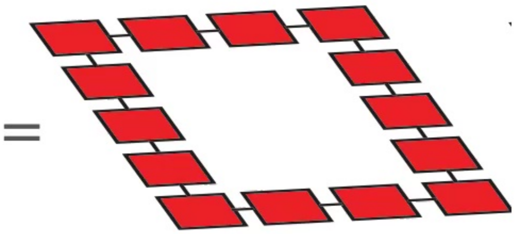
$W_a =$ 

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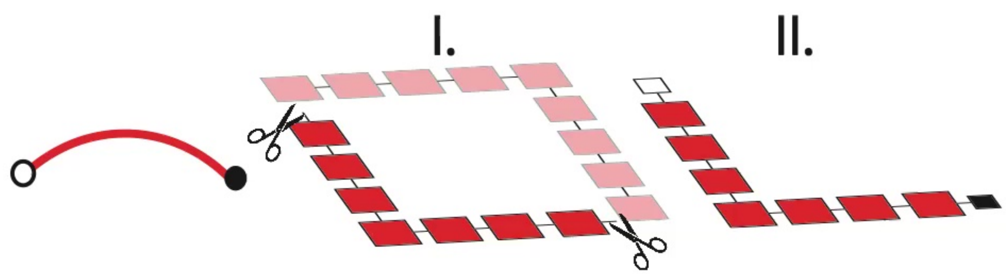
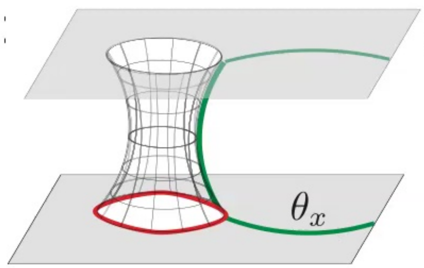


How can we measure many-body Chern Number?

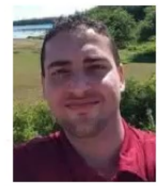
1. Topological quantum field theory

→Measure Many-Body Chern Number through a set of equal-time correlation function.

NO LINEAR RESPONSE! NO PBC! NO TWIST BOUNDARY CONDITION!

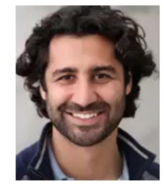


Hossein Dehghani, **Ze-Pei Cian**, Mohammad Hafezi, Maissam Barkeshli,
Phys. Rev. B, 103, 075102 (2021)



Hossein Dehghani

(U. Maryland)



Maissam Barkeshkli

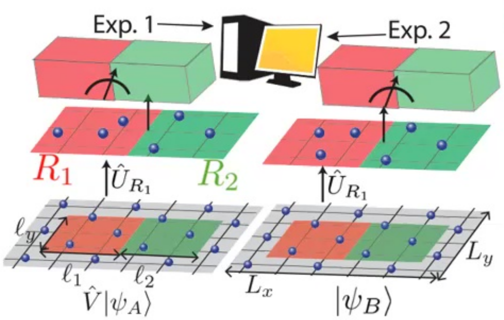
(U. Maryland)

Mohammad Hafez

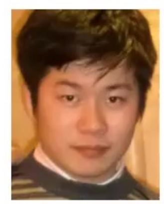
(U. Maryland)

2. Equal-time correlation function can be measured through Randomized Measurement :

(à la measurement of 2nd order Renyi $tr(\rho^2)$)



Ze-Pei Cian, Hossein Dehghani, Andreas Elben, Benoît Vermersch, Guanyu Zhu, Maissam Barkeshli, Peter Zoller, Mohammad Hafezi
Phys. Rev. Lett., 126, 050501 (2021)



Guanyu Zhu

(IBM)

Andreas Elben

(Caltech)

Benoît Vermersch

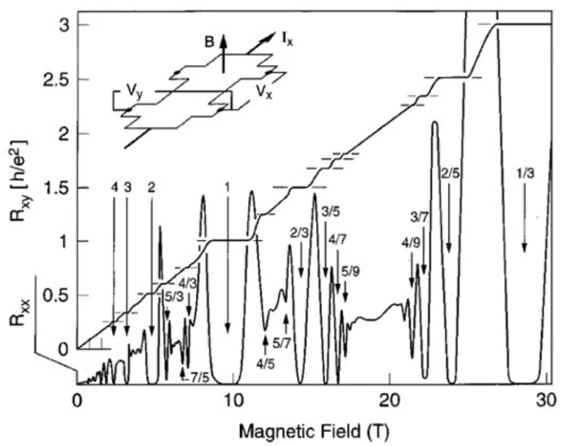
(U. Grenoble-Alpes)

Peter Zoller

(Innsbruck)

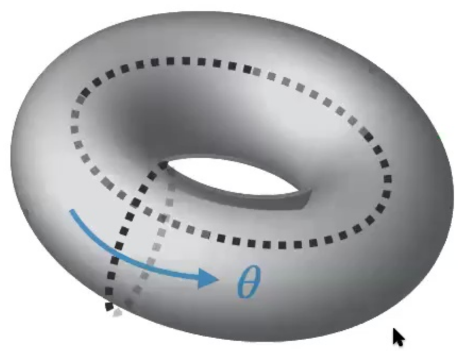
Crash course of Quantum Hall effect, Chern Number, Laughlin's Argument:

Hall conductance -> Chern number



Stormer, Tsui, Gossard, Rev. Mod. Phys. **71**, S298

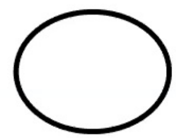
1. Flux insertion
2. Quantized Charge Transport (Polarization)



$y_{OBC} = \langle \hat{y} \rangle$

$\downarrow$

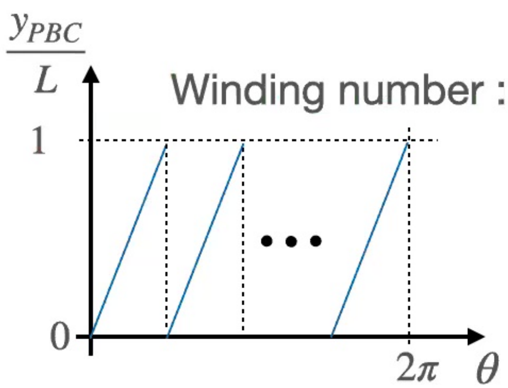
$\hat{y} \sim \hat{y} + L$



$$y_{PBC} = \frac{L}{2\pi}\angle[\langle \exp(i\frac{2\pi}{L}\hat{y})\rangle]$$

Resta, Phys. Rev. Lett. **80**, 1800 (1988)

$\angle$: Phase angle of a complex number

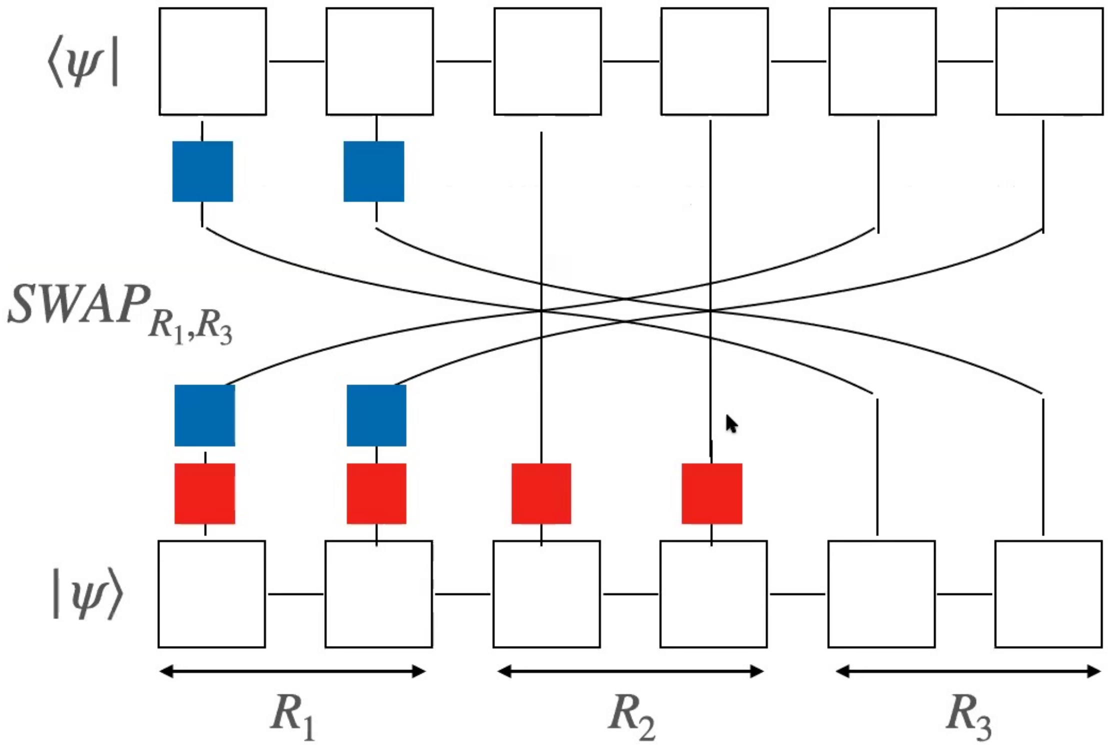


1. Periodic boundary condition(PBC)? ⛔
2. Flux insertion? ⛔

SWAP : space-time surgery

Matrix Product State point of view:

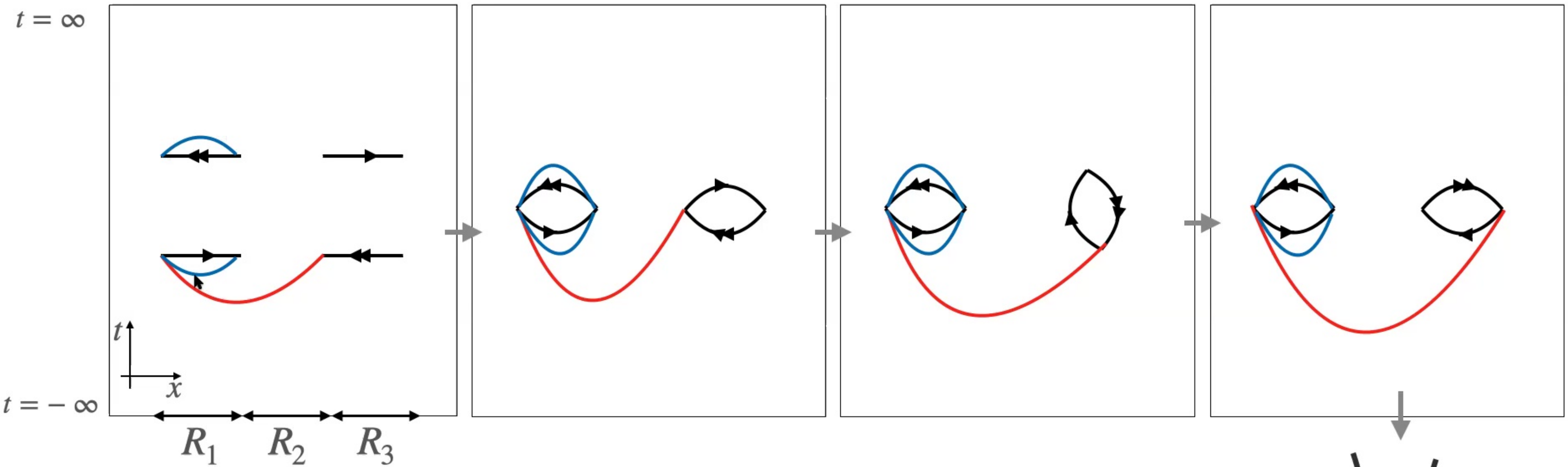
$$\langle\psi|\, W^\dagger SWAP_{R_1,R_3} W \; V \,|\psi\rangle$$

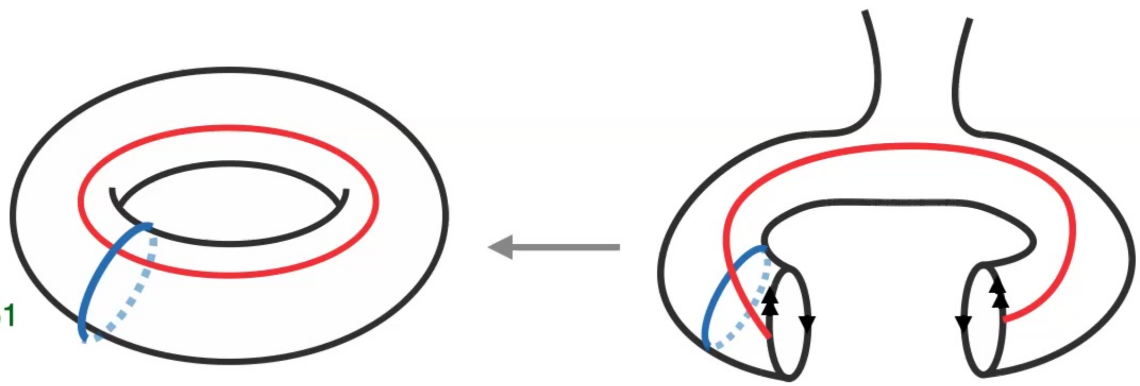


Pollmann, Turner, Phys. Rev. B **86**, 125441

SWAP : space-time surgery

Topological Field Theory point of view:





$$\langle \psi | \, W^{\dagger} SWAP_{R_1,R_3} W \;\; V \; | \psi \rangle$$

Shiozaki, Shapourian, Gomi, Ryu, Phys. Rev. B **98**, 035151
Shiozaki, Ryu, JHEP 2017(4), 1-47.

SWAP : space-time surgery

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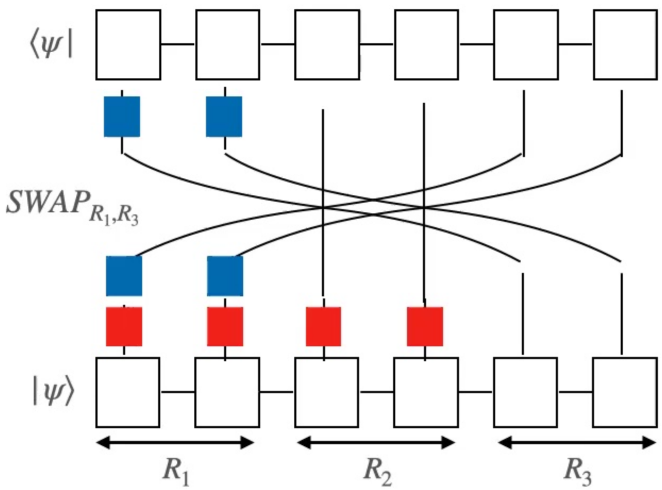
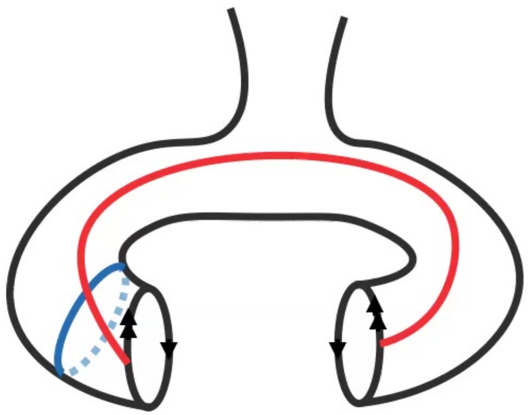


Pollmann, Turner, Phys. Rev. B **86**, 125441

Topological Field Theory point of view:

Synthesis spacetime torus with equal-time correlation function

We don't need periodic boundary condition anymore!

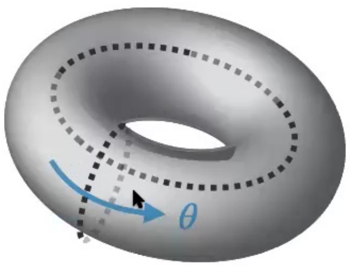


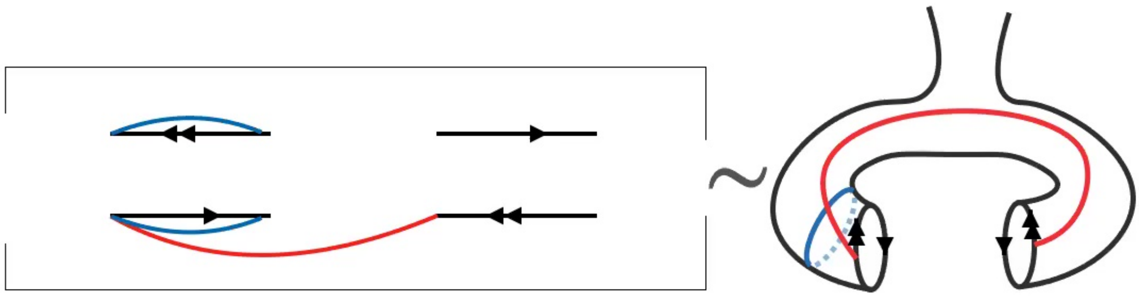
Shiozaki, Shapourian, Gomi, Ryu, Phys. Rev. B **98**, 035151

Shiozaki, Ryu, JHEP 2017(4), 1-47.

Recap : Measuring MBCN without varying boundary condition

Polarization operator : $V = exp(i\frac{2\pi}{L}\hat{y})$ Flux insertion operator : $W(\theta) = exp[i\theta\hat{n}(x,y)]$





$P(\theta) = \angle[\langle\psi_{PBC}(\theta)\,|\,V|\,\psi_{PBC}(\theta)\rangle]$ $\xrightarrow{\text{Spacetime surgery}}$ $P(\theta) = \angle\langle\psi|\,W^{\dagger}(\theta)SWAP_{R_1,R_3}W(\theta)\;V\,|\,\psi\rangle$

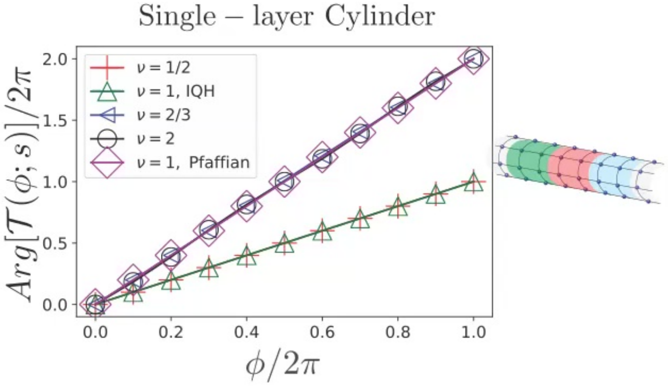
Periodic Boundary Condition $\longrightarrow$ $SWAP_{R_1,R_3}$

Flux insertion via twist boundary condition $\longrightarrow$ Flux insertion operator $W(\theta) = exp[i\theta\hat{n}(x,y)]$

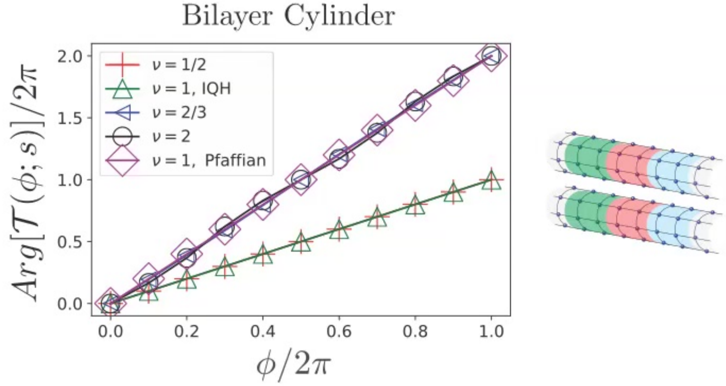
PBC, flux insertion ✗ $\longrightarrow$ Equal-time correlation function ✓

Numerical results

$$\langle\Psi|W_1^\dagger(\phi)\mathbb{S}_{1,3}W_1(\phi)V^s_{1\cup 2}|\Psi\rangle$$



$$\frac{\langle\Psi_B|\langle\Psi_A|W^\dagger_{A1}(\phi)V^{s\dagger}_{B2}\mathbb{S}_{A1,B1}\mathbb{S}_{A3,B3}W_{A1}(\phi)V^s_{A2}|\Psi_A\rangle|\Psi_B\rangle}{\langle\Psi_B|\langle\Psi_A|W^\dagger_{A1}(\phi)\mathbb{S}_{A1,B1}\mathbb{S}_{A3,B3}W_{A1}(\phi)|\Psi_A\rangle|\Psi_B\rangle}$$



Model	s	C
QH $\nu = 1/2$ B	2	1
QH $\nu = 2/3$ B	3	2
QH $\nu = 2$ B	1	2
QH $\nu = 1$ F	1	2
QH $\nu = 1$ B MR	2	2
QH $\nu = 1/2$ B	2	1
QH $\nu = 1$ F	1	1
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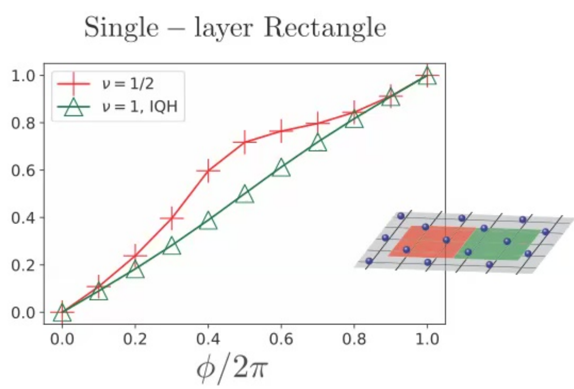
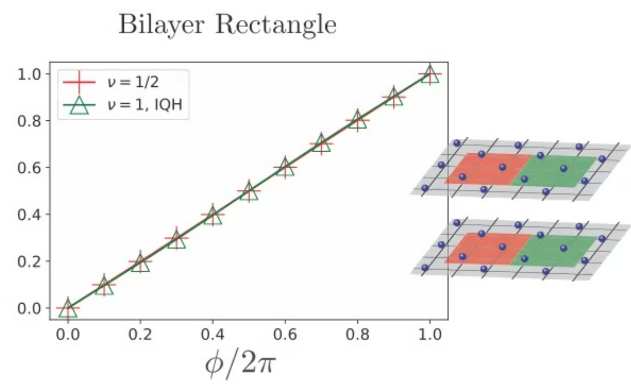
QH $\nu = 1$ B MR	2	2
QH $\nu = 1/2$ B	2	1
QH $\nu = 1$ F	1	1
Chern insulator	1	1





DMRG simulations with MPS.

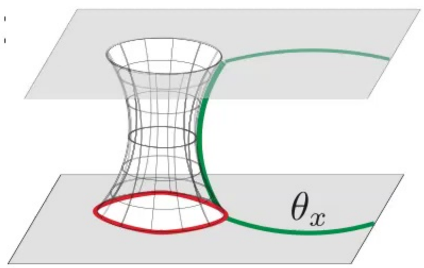
Our formula holds valid for open sheets

Summary (for part II)

1. Topological quantum field theory

→Measure Many-Body Chern Number through a set of <u>equal-time correlation function</u>.

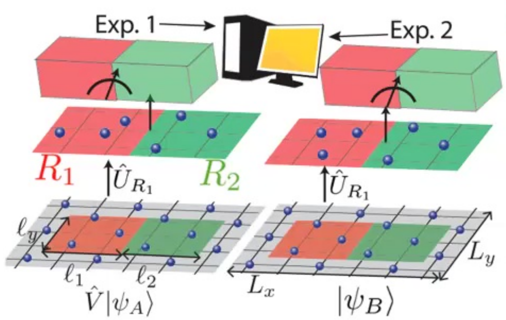
NO LINEAR RESPONSE! NO PBC! NO TWIST BOUNDARY CONDITION!



Hossein Dehghani, **Ze-Pei Cian**, Mohammad Hafezi, Maissam Barkeshli,
Phys. Rev. B, 103, 075102 (2021)

2. <u>Equal-time correlation function</u> can be measured through Randomized Measurement :

(à la measurement of 2nd order Renyi $tr(\rho^2)$)

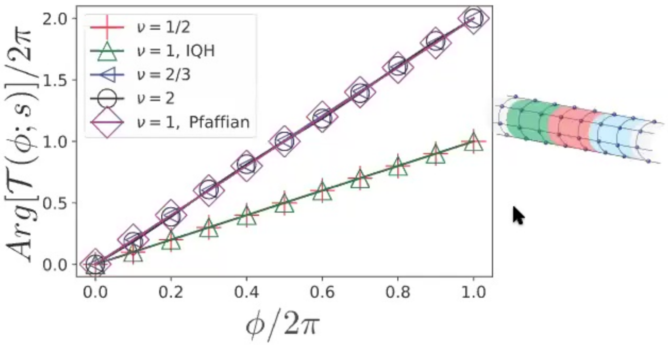


Ze-Pei Cian, Hossein Dehghani, Andreas Elben, Benoît Vermersch, Guanyu Zhu, Maissam Barkeshli, Peter Zoller, Mohammad Hafezi
Phys. Rev. Lett., 126, 050501 (2021)

Numerical results

$$\langle\Psi|W_1^\dagger(\phi)\mathbb{S}_{1,3}W_1(\phi)V^s_{1\cup 2}|\Psi\rangle$$

Single – layer Cylinder



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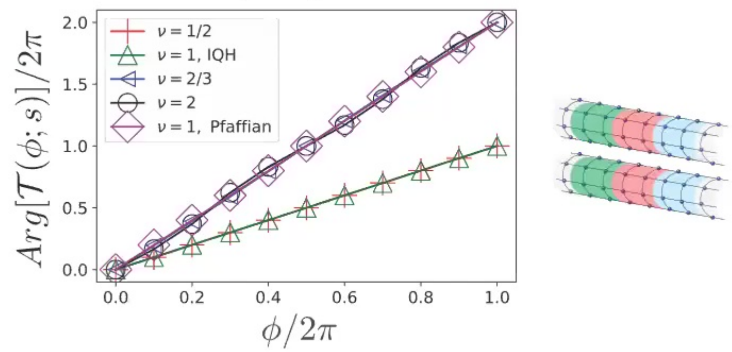
Bilayer Cylinder



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QH $\nu = 1$ F	1	1
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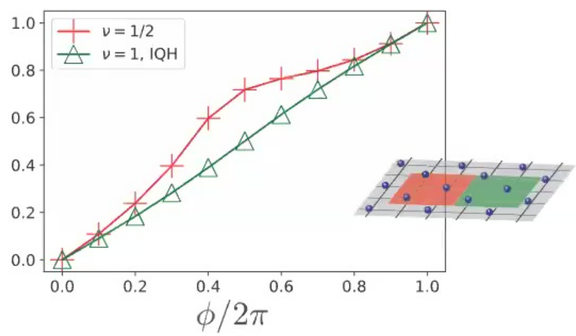
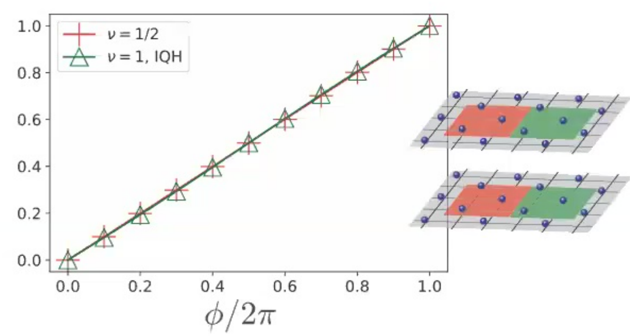
QH $\nu = 2$ B	1	2
QH $\nu = 1$ F	1	1
QH $\nu = 1$ B MR	2	2
QH $\nu = 1/2$ B	2	1
QH $\nu = 1$ F	1	1
Chern insulator	1	1

Single – layer Rectangle



Bilayer Rectangle



DMRG simulations with MPS.

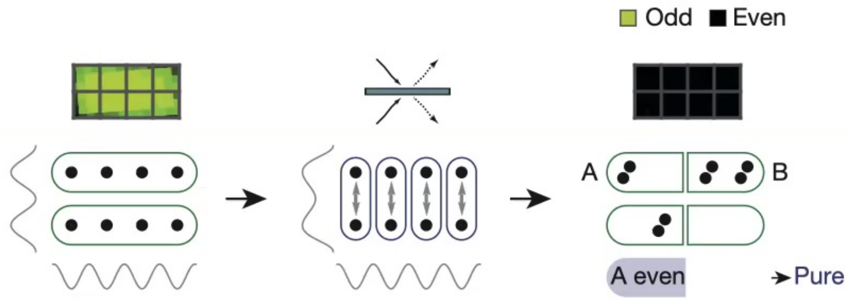
Our formula holds valid for open sheets

How to measure $\langle \psi_A | \langle \psi_B | SWAP_{A_1,B_1} | \psi_A \rangle | \psi_B \rangle$?

In principle, one can measure the SWAP expectation value.

Two identical copies + tunneling + Parity Measurement

Daley et al. PRL (2012)
Islam et al. Nature (2015)



Harvard

But what if we don't have access to two copies at the same time?

- The $SWAP$ operation can be implemented via randomized measurement.

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$$\mathrm{Tr}\left[\rho_A^2\right] = \mathcal{D}_A \sum_{\mathbf{s}_A, \mathbf{s}'_A} (-\mathcal{D}_A)^{-D_G[\mathbf{s}_A, \mathbf{s}'_A]} \overline{P_U(\mathbf{s}_A) P_U(\mathbf{s}'_A)}$$

Theory : A. Elben, B. Vermersch, M. Dalmonte, J.I. Cirac, and P. Zoller PRL (2018)
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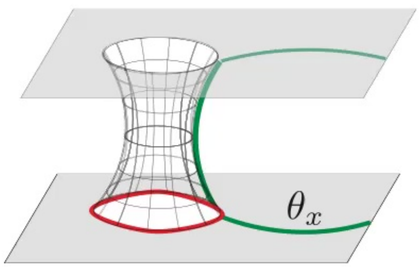

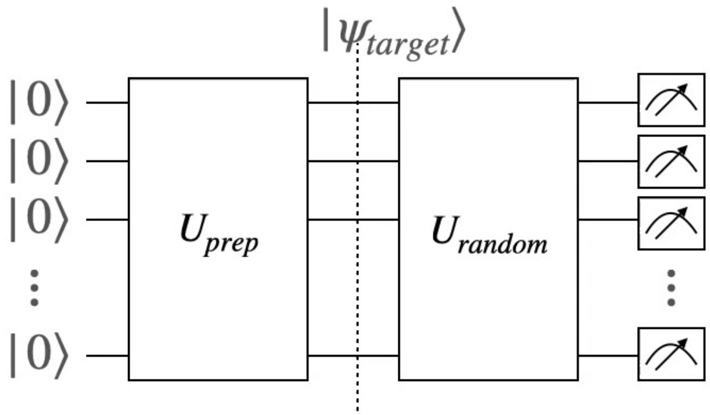

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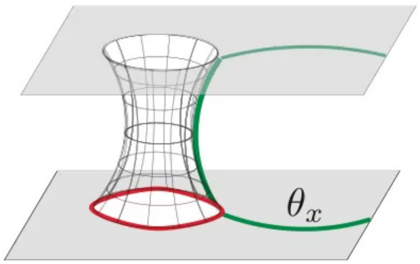

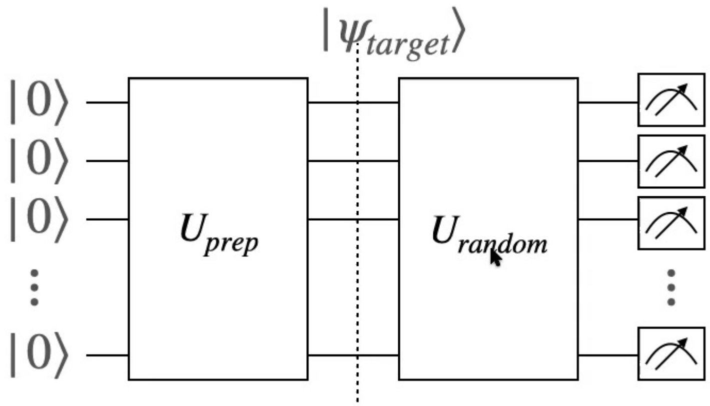

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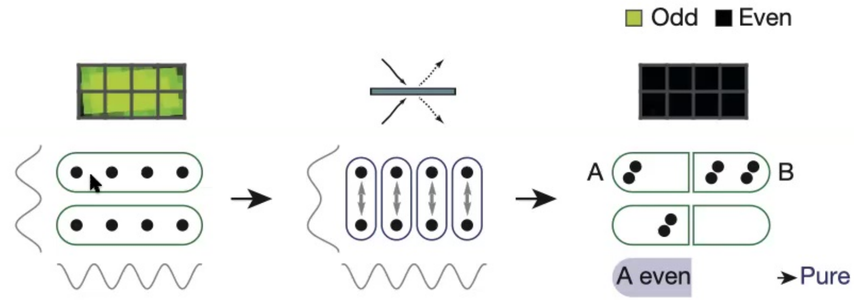

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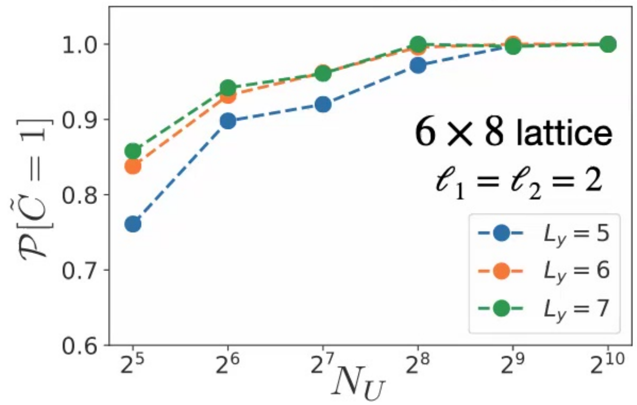
Numerical results for $\nu = 1/2$ bosonic fractional Chern insulator

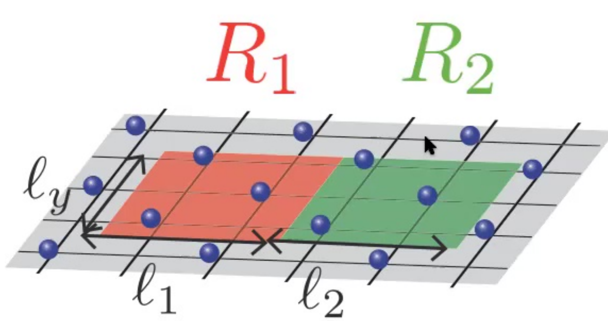
$$H = -\sum_{x,y} e^{i2\pi\alpha y} a^\dagger_{x,y} a_{x+1,y} + a^\dagger_{x,y} a_{x,y+1} + h.c. + U \sum_r n_r(n_r - 1)$$

$$(U \to \infty)$$

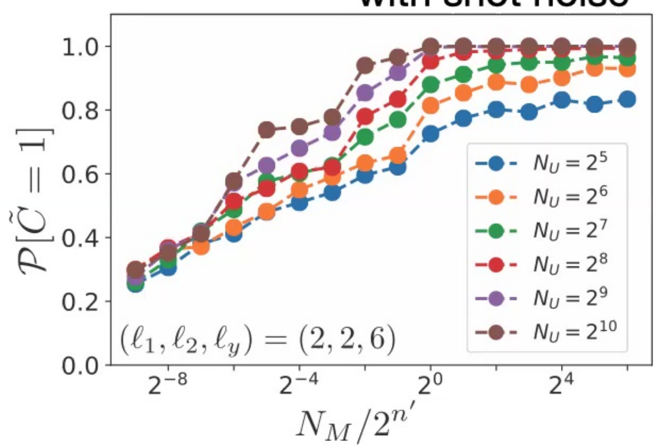
No shot noise



Random unitaries could be implemented by quench dynamics: tunneling + random onsite potential



with shot noise



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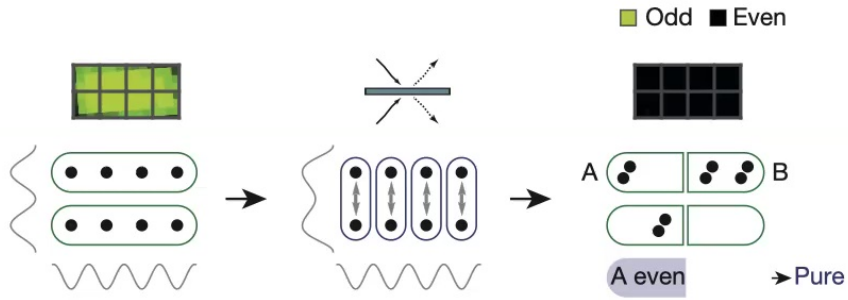

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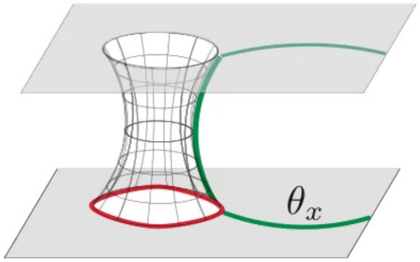
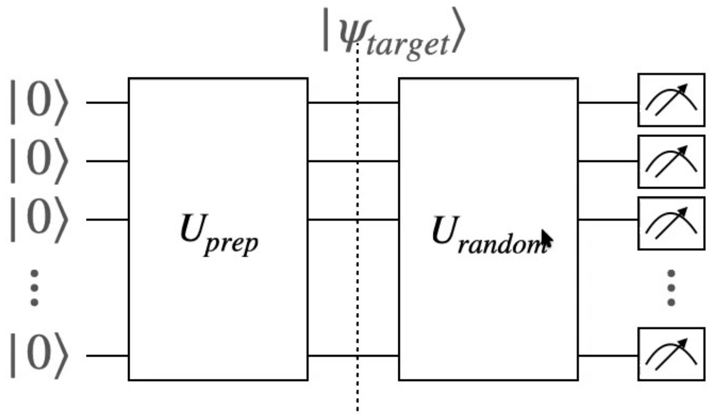


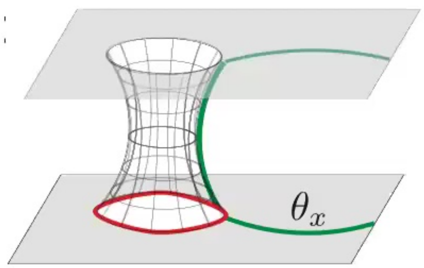


Summary (for part II)

1. Topological quantum field theory

→Measure Many-Body Chern Number through a set of equal-time correlation function.

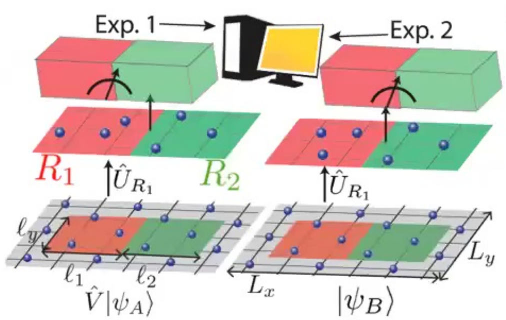
NO LINEAR RESPONSE! NO PBC! NO TWIST BOUNDARY CONDITION!



Hossein Dehghani, **Ze-Pei Cian**, Mohammad Hafezi, Maissam Barkeshli, Phys. Rev. B, 103, 075102 (2021)

2. Equal-time correlation function can be measured through Randomized Measurement :

(à la measurement of 2nd order Renyi $tr(\rho^2)$)



Ze-Pei Cian, Hossein Dehghani, Andreas Elben, Benoît Vermersch, Guanyu Zhu, Maissam Barkeshli, Peter Zoller, Mohammad Hafezi Phys. Rev. Lett., 126, 050501 (2021)