Title: Holographic entanglement in spin network states: bulk-to-boundary isometries and horizon-like regions from volume correlations

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Abstract: For quantum gravity states associated to open spin network graphs, we study how the boundary (the set of open edges, which carries spin degrees of freedom) is affected by the bulk, specifically by its combinatorial structure and by the quantum correlations among the intertwiners. In particular, we determine under which conditions certain classes of quantum gravity states map bulk degrees of freedom into boundary ones isometrically (which is a necessary condition for holography). We then look at the entanglement entropy of the boundary and recover, for slightly entangled intertwiners, the Ryu-Takayanagi formula with corrections induced by the entanglement entropy of the bulk state. We also show that the presence of a region with highly entangled intertwiners deforms the minimal-area surface, which is prevented from entering that region when the entanglement entropy of the latter exceeds a certain bound, a mechanism which thus leads to the rise of a black hole-like region in the bulk.

Zoom Link: https://pitp.zoom.us/j/96356007543?pwd=U2VrRlhyOThMODdMYllDMnB6VjlZQT09

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HOLOGRAPHIC ENTANGLEMENT IN SPIN NETWORK STATES:

BULK-TO-BOUNDARY ISOMETRIES AND HORIZON-LIKE REGIONS FROM VOLUME CORRELATIONS

Based on work with G. Chirco and D. Oriti

2110.15166, 2105.06454

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Perimeter Institute

2 December 2021

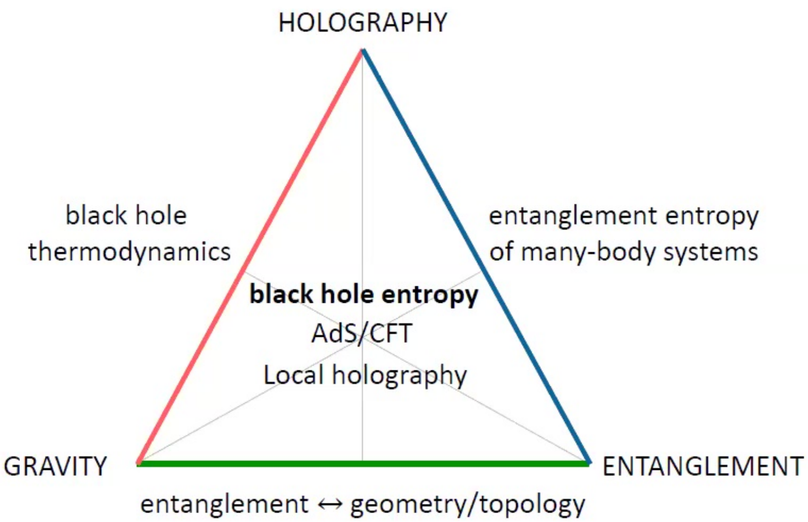
OVERVIEW

1. Introduction
2. Framework: spin network formalism for quantum spacetime
3. Holographic entanglement in spin network states
   - Bulk-to-boundary isometries
   - Horizon-like regions from volume correlations

INTRODUCTION

A deep threefold connection between **gravity**, **holography**, and **entanglement** emerges from results at th

between quantum gravity and quantum information:



I. (Quantum) gravity exhibits holographic features
   - **Bekenstein-Hawking area law** for black hole entropy;
   - **AdS/CFT** correspondence and **Ryu-Takayanagi formula**;
   - **Local holography**: corner symmetry charges encoding bulk quantum geometry [Freidel, Donnelly…]
   - …

II. **Entanglement** plays a crucial role in the emergence of **space(time) geometry/topology**
   - **Entanglement** as necessary and sufficient condition for **spacetime connectivity** [Van Raamsdonk, Maldacena, Susskind];
   - **spatial geometries** from **entanglement** structure of **abstract quantum states** [Cao, Carroll, Michalakis];
   - **Einstein's equations** linearized about AdS from **entanglement "first law"** in CFT [Lashkari, McDermott…]
   - …

III. **Entanglement entropy** of (classes of) quantum many-body systems satisfies **area laws**

4

Which is the origin of the gravity/holography/entanglement threefold connection,
and what does it tell us about quantum gravity?

Suitable formalism: **spin networks** for quantum geometries arising as **patterns of entanglement** among space quanta

RESEARCH DIRECTIONS

GEOMETRY/TOPOLOGY FROM QUANTUM INFORMATION

Characterize spin network states from a quantum-information perspective, to facilitate the extraction of geometric and topological features

QUANTUM ORIGIN OF HOLOGRAPHY

Identify properties of the entanglement structure of a spin network which make bulk and boundary holographically related

⟶ BULK-TO-BOUNDARY ISOMETRIES

TOWARDS MODELLING OF BLACK HOLES MICROSTATES

Investigate how entanglement of bulk degrees of freedom affects the boundary state and its entropy

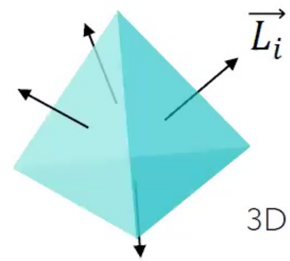
⟶ HORIZON-LIKE REGIONS FROM VOLUME CORRELATIONS

Remark: we look at finite bounded regions; "**local**" notion of **holography** for spin networks

SPIN NETWORK FORMALISM FOR QUANTUM SPACETIME

Building block of quantum space as a **spin network vertex**:

phase space of classical geometries of a simplex



$\Sigma_i \overrightarrow{L_i} = 0$ closure relation

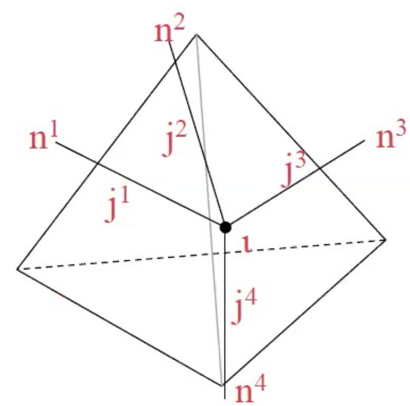
3D Euclidian space

quantization

$$\mathcal{H}_v = \bigoplus_{\vec{j}_v} \left(\bigotimes_{i=1}^{d} \underbrace{V^{j^i_v}}_{\text{repr. space}} \otimes \underbrace{\mathcal{I}^{\vec{j}_v}}_{\text{intertwiner space}} \right)$$

j = spin labelling irrep of **SU(2)**

spin network vertex



$|j^i n^i\rangle \in V^{j^i}$ diagonalises **area operator**

$|\iota^{\vec{j}}\rangle \in \mathcal{I}^{\vec{j}} = \mathrm{Inv}_G \left[V^{j^1} \otimes \ldots \otimes V^{j^d} \right]$ diagonalises **volume operator**

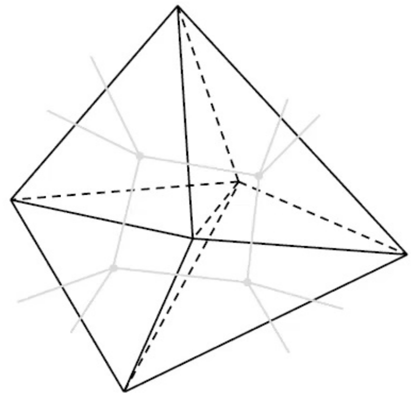
(different perspective: spin networks from canonical quantization of GR in first order variables)

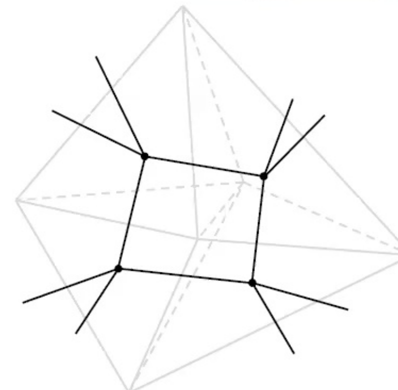
7

SPIN NETWORK FORMALISM FOR QUANTUM SPACETIME

Spin networks = graphs dual to **simplicial complexes**

- **edges** carrying SU(2) spins
- **nodes** carrying intertwiners (gauge invariant tensors)





As kinematical states, spin networks enter* various related QG approaches:

- **Loop quantum gravity** (canonical quantization of general relativity)
- **Spin foam models** (covariant LQG or gravity as generalized lattice gauge theory)
- **Group field theory** (quantum field theory for simplicial geometry)

*with different Hilbert space structures for graph superposition!

Spin network formalism for quantum spacetime - Group field theory perspective

Spin network states arising from the entanglement of individual vertices (fundamental excitations of the

Hilbert space for V open vertices $\mathcal{H}_V = \bigotimes_v \mathcal{H}_v \longrightarrow \mathcal{H}_\gamma = \bigoplus_J \left(\bigotimes_v \mathcal{I}^{\vec{j}_v} \otimes \bigotimes_{e \in \partial\gamma}^{d} V^{j_e} \right)$ Hilbert space for graph γ

$$J = \{ j_e | e \in \gamma \}$$

$$|\psi\rangle \xrightarrow[\text{(entangling edge dof)}]{\text{gluing}} |\psi_\gamma\rangle = \left(\bigotimes_{e \in L} \langle e| \right) |\psi\rangle$$

internal links of combinatorial pattern γ

link state

$$|e\rangle = \bigoplus_j \frac{1}{\sqrt{d_j}} \sum_n |jn\rangle \otimes |jn\rangle$$

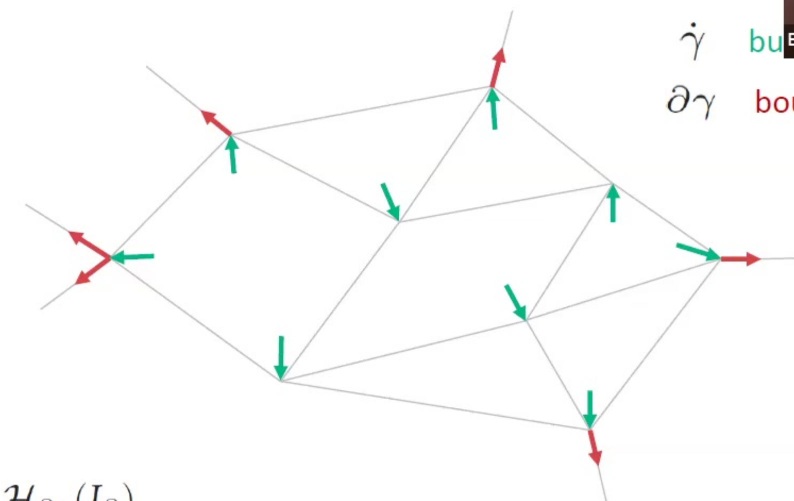
maximally entangled state of edge degrees of freedom

9

Open spin network states as bulk-to-boundary maps

$$\text{State}\quad |\psi_\gamma\rangle \in \mathcal{H}_\gamma(J) = \underbrace{\overset{\text{bulk}}{\bigotimes_v \mathcal{I}^{\vec{j}_v}}}_{\mathcal{H}_{\dot\gamma}(J)} \otimes \underbrace{\overset{\text{boundary}}{\bigotimes_{e\in\partial\gamma} V^{j_e}}}_{\mathcal{H}_{\partial\gamma}(J_\partial)}$$

input space output space



$\dot{\gamma}$ bu

$\partial\gamma$ boundary edges

Bulk-to-boundary map $\quad M[\psi_\gamma]: \quad \mathcal{H}_{\dot\gamma}(J) \rightarrow \mathcal{H}_{\partial\gamma}(J_\partial)$

$$|\zeta\rangle \qquad \langle\zeta|\psi_\gamma\rangle = |\psi_{\partial\gamma}(\zeta)\rangle$$

bulk input state $\displaystyle\sum_{\iota_1,\dots,\iota_N} \zeta_{\iota_1,\dots,\iota_N} \bigotimes_v |\iota_v\rangle$

$\displaystyle\sum_{\{n_{e\in\partial\gamma}\}} (\psi_{\partial\gamma}(\zeta))_{\{n_{e\in\partial\gamma}\}} \bigotimes_{e\in\partial\gamma} |j_e n_e\rangle$ boundary output state

Isometry condition

Map-state duality:

bulk basis state

$$\underbrace{\sum_\alpha |\alpha\rangle \otimes |\alpha\rangle}_{\text{maximally entangled state of two copies of the bulk}}$$

$$|\psi_\gamma\rangle = \sum_\alpha (M \otimes \mathbb{I})\, |\alpha\rangle \otimes |\alpha\rangle$$

Reduced bulk state:

$$\rho_{\dot\gamma} = \mathrm{Tr}_{\partial\gamma}\left(\frac{|\psi_\gamma\rangle\langle\psi_\gamma|}{D_{\dot\gamma}}\right) = \frac{1}{D_{\dot\gamma}} \sum_{\alpha\alpha'} \left(M^\dagger M\right)^*_{\alpha\alpha'} |\alpha\rangle\langle\alpha'|$$

bulk dimension

isometry condition $\quad M^\dagger M = \mathbb{I} \iff \rho_{\dot\gamma} = \dfrac{\mathbb{I}}{D_{\dot\gamma}}$ reduced bulk state **maximally mixed**

$\downarrow$

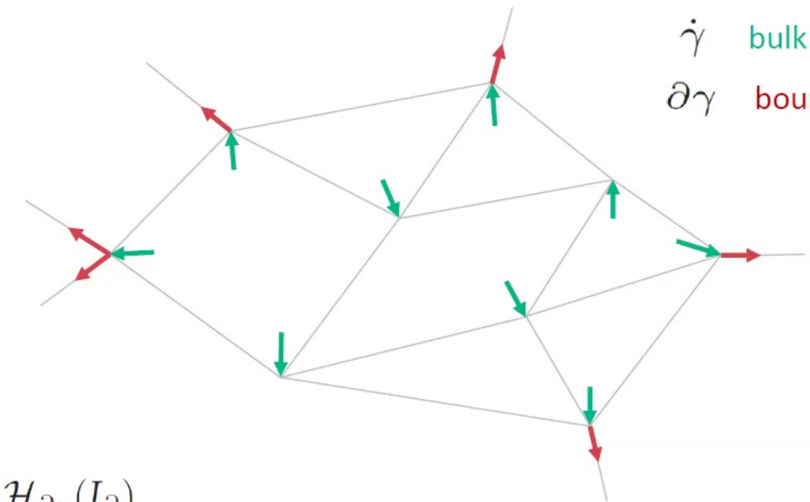
checking the isometry condition by computing the bulk entropy

Open spin network states as bulk-to-boundary maps

State $|\psi_\gamma\rangle \in \mathcal{H}_\gamma(J) = \underbrace{\bigotimes_v \mathcal{I}^{\vec{j}_v}}_{\mathcal{H}_{\dot\gamma}(J)} \otimes \underbrace{\bigotimes_{e\in\partial\gamma} V^{j_e}}_{\mathcal{H}_{\partial\gamma}(J_\partial)}$

bulk — boundary

input space — output space



$\dot\gamma$ bulk intertwiners

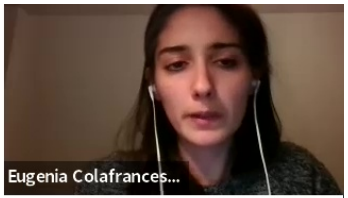
$\partial\gamma$ boundary edges

Bulk-to-boundary map $M[\psi_\gamma]: \quad \mathcal{H}_{\dot\gamma}(J) \rightarrow \mathcal{H}_{\partial\gamma}(J_\partial)$

$$|\zeta\rangle \qquad \langle\zeta|\psi_\gamma\rangle = |\psi_{\partial\gamma}(\zeta)\rangle$$

bulk input state $\sum_{\iota_1,\ldots,\iota_N} \zeta_{\iota_1,\ldots,\iota_N} \bigotimes_v |\iota_v\rangle$

$\sum_{\{n_{e\in\partial\gamma}\}} (\psi_{\partial\gamma}(\zeta))_{\{n_{e\in\partial\gamma}\}} \bigotimes_{e\in\partial\gamma} |j_e n_e\rangle$ boundary output state



ISOMETRY CONDITION

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bulk dimension

isometry condition $\quad M^\dagger M = \mathbb{I} \quad \Longleftrightarrow \quad \rho_{\dot\gamma} = \dfrac{\mathbb{I}}{D_{\dot\gamma}}$ reduced bulk state **maximally mixed**

↓

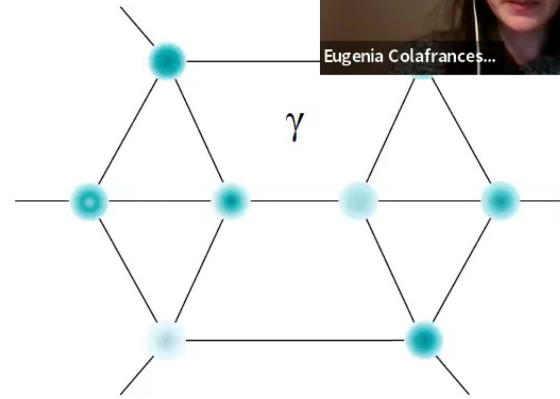
checking the isometry condition by computing the **bulk entropy**

Special class of spin network states: random tensor networks

$$|\psi_\gamma\rangle = \left(\bigotimes_{e\in L}\langle e|\right)\bigotimes_v |f_v\rangle$$

random vertex wavefunctions

$$O(|f_v\rangle) \quad \xrightarrow{\text{randomization}} \quad \underbrace{\int \mathrm{d}U}_{\text{Haar measure}}\, O\left(U|f_v^0\rangle\right)$$



Rényi-2 entropy of a portion P of quantum system described by the density matrix ρ:

$$S_2(\rho_P) = -\log \mathrm{Tr}\left(\rho_P^2\right) \qquad \text{replica trick:} \quad S_2(\rho_P) = -\log\left(\frac{Z_1}{Z_0}\right) \qquad \begin{aligned} Z_1 &= \mathrm{Tr}\left[(\rho\otimes\rho)\,\mathcal{S}_P\right] \\ Z_0 &= \mathrm{Tr}\left[\rho\otimes\rho\right] \end{aligned}$$

operator swapping the two copies of subsystem P

large spins regime

$$\overline{S_2(\rho_{A\cup\Omega})} \approx -\log\left(\frac{\overline{Z_1}}{\overline{Z_0}}\right) \qquad \text{with} \qquad \overline{Z_{1/0}} = \mathrm{Tr}\left[\bigotimes_{e\in L}\left(|e\rangle\langle e|\right)^{\otimes 2}\bigotimes_v \overline{\left(|f_v\rangle\langle f_v|\right)^{\otimes 2}}\left(\mathcal{S}_{A\cup\Omega}/\mathbb{I}\right)\right]$$

boundary portion

bulk portion

13

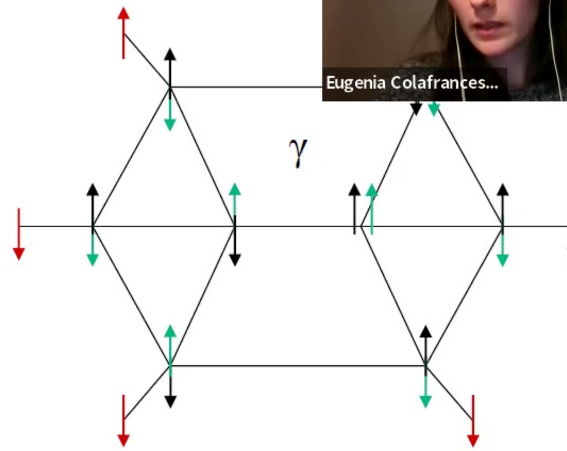
Rényi entropy from Ising partition functions

$$\overline{Z_{1/0}} = \mathrm{Tr}\left[\bigotimes_{e\in L}(|e\rangle\langle e|)^{\otimes 2}\bigotimes_{v}\underbrace{\overline{(|f_v\rangle\langle f_v|)^{\otimes 2}}}_{\mathcal{S}_v \;+\; \mathbb{I}}\underbrace{(\mathcal{S}_{A\cup\Omega}/\mathbb{I})}\right] = \sum_{\vec{\sigma}} e^{-\mathcal{A}_{1/0}(\vec{\sigma})}$$

$$\underbrace{\sigma_v = -1 \quad \sigma_v = +1}_{\text{Ising spins}}$$

$$\left.\begin{matrix}\mu_{e\in A} = -1\\ \mu_{e\notin A} = +1\end{matrix}\right\}\text{ boundary fields}$$

$$\left.\begin{matrix}\nu_{v\in\Omega} = -1\\ \nu_{v\notin\Omega} = +1\end{matrix}\right\}\text{ bulk fields}$$



Ising-like action:

$$\mathcal{A}_{1/0}(\vec{\sigma}) = -\frac{1}{2}\left[\sum_{e\in L}(\sigma_v\sigma_w - 1)\underbrace{\log d_{j_e}}_{\text{link entropy}} + \sum_{e\in\partial\gamma}(\sigma_v\mu_e - 1)\underbrace{\log d_{j_e}}_{\text{link entropy}} + \sum_{v}(\sigma_v\nu_v - 1)\underbrace{\log D_{\vec{j}_v}}_{\text{intertwiner entropy}}\right]$$

$\mathcal{A}_1 \to$ $\vec{\mu}_A = \downarrow$, $\vec{\nu}_\Omega = \downarrow$

$\mathcal{A}_0 \to$ all fields $\uparrow$

Free energy:

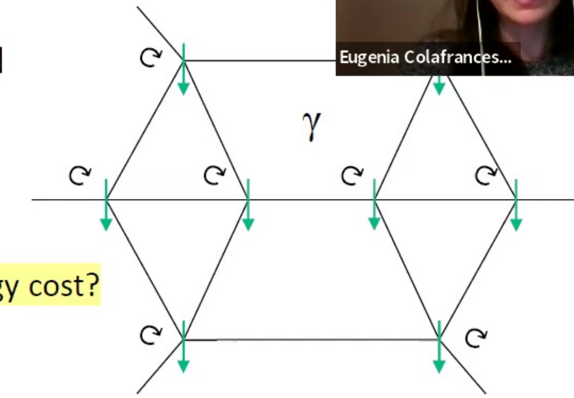
$$F = -\log\overline{Z} \quad\Rightarrow\quad \overline{S_2(\rho_{A\cup\Omega})} \approx -\log\left(\frac{\overline{Z_1}}{\overline{Z_0}}\right) = F_1 - F_0$$

energy cost of flipping down fields in $A\cup\Omega$

CHECKING THE ISOMETRIC CHARACTER OF BULK-TO-BOUNDARY MAPS – HOMOGENEOUS CASE

The map is an isometry if the corresponding **reduced bulk state** is **maximally mixed**

$\overline{S_2(\rho_{\dot\gamma})} = \ ?$ boundary fields ↑ frozen bulk fields flipped down



free-energy cost?

Homogeneous case

$\beta = \log d_j$ inverse Ising temperature

$$\overline{Z} = \sum_{\vec\sigma} e^{-\beta H(\vec\sigma)} \qquad \text{where} \qquad H(\vec\sigma) = -\frac{1}{2}\left[\sum_{e\in L}(\sigma_v\sigma_w - 1) + \sum_{e\in\partial\gamma}(\sigma_v - 1) + \frac{\log D_j}{\beta}\sum_v(\sigma_v\nu_v - 1)\right]$$

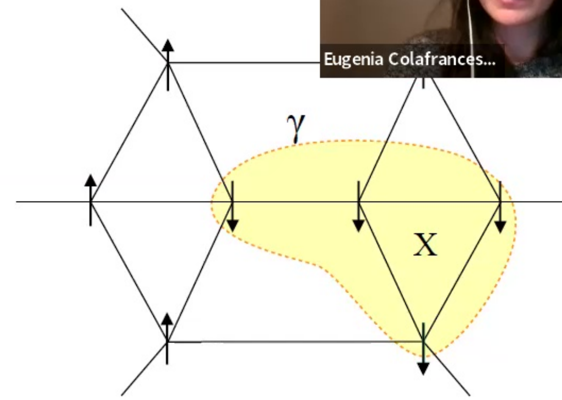
large spins regime: $\beta \gg 1 : \ F = -\log \overline{Z} \approx \beta \min_{\vec\sigma} H(\vec\sigma)$

observation: $H(\vec\sigma = \uparrow) = H_{\text{min}} \quad \Rightarrow \quad F = N\log D_j = \overline{S_2(\rho_{\dot\gamma})}_{\text{max}}$

The isometry condition translates into the requirement of **stability of the all-up configuration:** $\beta H(\vec\sigma) > N\log D_j \quad \forall\vec\sigma$

Checking the isometric character of bulk-to-boundary maps – homogeneous case

$\mathbf{X}$ = region in which the Ising spins point down



$$|\partial X| \log d_j > |X| \log D_j \quad \forall X$$ **stability of the all-up configuration**

graphs made of **four-valent vertices** $|\partial X| > |X| \quad \forall X$ violated

The bulk-to-boundary map of homogeneous graphs made of four-valent vertices (with at most one boundary link for each vertex) cannot be an isometry

Inhomogeneous case

$$\beta = \log d$$

average link dimension

Ising-like Hamiltonian:

$$H(\vec{\sigma}) = -\frac{1}{2}\left[\sum_{e\in L}(\sigma_v\sigma_w - 1)\frac{\log d_{j_e}}{\beta} + \sum_{e\in\partial\gamma}(\sigma_v - 1)\frac{\log d_{j_e}}{\beta} + \sum_{v}(\sigma_v\nu_v - 1)\frac{\log D_{\vec{j}_v}}{\beta}\right]$$

CHECKING THE ISOMETRIC CHARACTER OF BULK-TO-BOUNDARY MAPS – INHOMOGENEOUS CAS

Isometry condition as **stability of the all-up configuration:** $\beta H(\vec{\sigma}) > \log\left(\prod_v D_{\vec{j}_v}\right) \quad \forall \vec{\sigma}$

4-valent vertices $\quad j_a \leq j_b \leq j_c \leq j_d$

intertwiner dimension $\quad D_{\vec{j}} = \min\{j_a + j_d, j_b + j_c\} - \Delta + 1 \qquad$ with $\quad \Delta = j_d - j_a = j_{\text{max}} - j_{\text{min}}$

inhomogeneity of vertex structure

stability of the all-up configuration: $\displaystyle\sum_{e \in \partial X} \log d_{j_e} > \sum_{v \in X} \log\left(\min\{j_v^a + j_v^d, j_v^b + j_v^c\} - \Delta_v + 1\right) \quad \forall X$

- $\Delta_v = 0 \ \forall v \quad$ homogeneous case, the bulk-to-boundary map is not an isometry
- $\Delta_v = \Delta_{v_{\text{max}}} - 1 \ \forall v \quad$ the stability condition becomes

$$\sum_{e \in \partial X} \log d_{j_e} > |X| \log 2 \quad \forall X$$

increasing the **inhomogeneity of the spin assignment** increases the **holographic character** of the map!

HOLOGRAPHIC ENTANGLEMENT IN SPIN NETWORK STATES:
HORIZON-LIKE REGIONS FROM VOLUME CORRELATIONS

BOUNDARY ENTROPY FOR RANDOM SPIN NETWORK

Consider the **boundary state** resulting from the application of **M** to a bulk state $|\zeta\rangle$:

$$|\psi_{\partial\gamma}(\zeta)\rangle = \underbrace{\langle\zeta|}_{\text{bulk state}} \left(\bigotimes_{e\in L}\langle e|\right) \bigotimes_{v} \underbrace{|f_v\rangle}_{\text{random}}$$

What is its boundary entropy and how the bulk state affects it?

$$\overline{S_2(\rho_A)} \approx -\log\left(\frac{\overline{Z_1}}{\overline{Z_0}}\right)$$

$$\overline{Z_{1/0}} = \mathrm{Tr}\left[\underbrace{(|\zeta\rangle\langle\zeta|)^{\otimes 2}}_{\text{bulk contribution}} \bigotimes_{e\in L} (|e\rangle\langle e|)^{\otimes 2} \bigotimes_{v} \underbrace{\overline{(|f_v\rangle\langle f_v|)^{\otimes 2}}}_{\text{Ising spins } \vec{\sigma}} \underbrace{(\mathcal{S}_A/\mathbb{I})}_{\text{boundary fields } \vec{\mu}}\right] = \sum_{\vec{\sigma}} e^{-\mathcal{A}_{1/0}(\vec{\sigma})}$$

$$\mathcal{A}_{1/0}(\vec{\sigma}) = -\frac{1}{2}\sum_{e\in L}(\sigma_v\sigma_w - 1)\log d_{j_e} - \frac{1}{2}\sum_{e\in\partial\gamma}(\sigma_v\mu_e - 1)\log d_{j_e} + \underbrace{S_2(\zeta;\downarrow)}_{\text{bulk entropy in }\downarrow\text{-region}}$$

$$\overline{S_2(\rho_A)} \approx -\log\left(\frac{\overline{Z_1}}{\overline{Z_0}}\right) = F_1 - F_0$$

19

Boundary entropy for random spin network

Consider the **boundary state** resulting from the application of **M** to a bulk state $|\zeta\rangle$:

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$$\overline{S_2(\rho_A)} \approx -\log\left(\frac{\overline{Z_1}}{\overline{Z_0}}\right) = F_1 - F_0$$

HOLOGRAPHIC ENTANGLEMENT IN SPIN NETWORK STATES:

HORIZON-LIKE REGIONS FROM VOLUME CORRELATIONS

BOUNDARY ENTROPY FOR RANDOM SPIN NETWORK – HOMOGENEOUS CASE

$\beta = \log d_j$ inverse Ising temperature

$$\overline{Z} = \sum_{\vec{\sigma}} e^{-\beta H(\vec{\sigma})} \qquad H(\vec{\sigma}) = \underbrace{-\frac{1}{2}\left[\sum_{e\in L}(\sigma_v\sigma_w - 1) + \sum_{e\in\partial\gamma}(\sigma_v\mu_e - 1)\right]}_{\substack{\text{area of domain wall} \\ |\Sigma(\vec{\sigma})|}} + \beta^{-1}S_2(\zeta;\downarrow)$$

$$\beta \gg 1: \; F = -\log \overline{Z} \approx \beta \min_{\vec{\sigma}} H(\vec{\sigma})$$

Small bulk entropy contribution:

$$\overline{S_2(\rho_A)} \approx \underbrace{\log d_j \min_{\vec{\sigma}} |\Sigma(\vec{\sigma})|}_{\text{area law}} + \underbrace{S_2(\zeta;\downarrow)}_{\text{bulk correction}}$$

bulk area law for boundary entropy (with bulk induced corrections)

BOUNDARY ENTROPY FOR RANDOM SPIN NETWORK – HOMOGENEOUS CASE

Large bulk entropy contribution:

$$\overline{S_2(\rho_A)} \approx \min_{\vec{\sigma}} \left\{ -\beta \frac{1}{2} \left[\sum_{e \in L} (\sigma_v \sigma_w - 1) + \sum_{e \in \partial\gamma} (\sigma_v \mu_e - 1) \right] + S_2(\zeta; \downarrow) \right\}$$

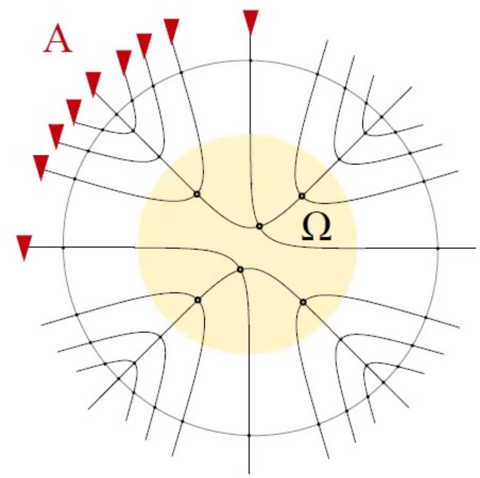
EXAMPLE

Bulk entanglement only in a region $\mathbf{\Omega}$:

$$|\zeta\rangle = |\zeta_\Omega\rangle \otimes \bigotimes_{v \notin \Omega} |\eta_v\rangle$$

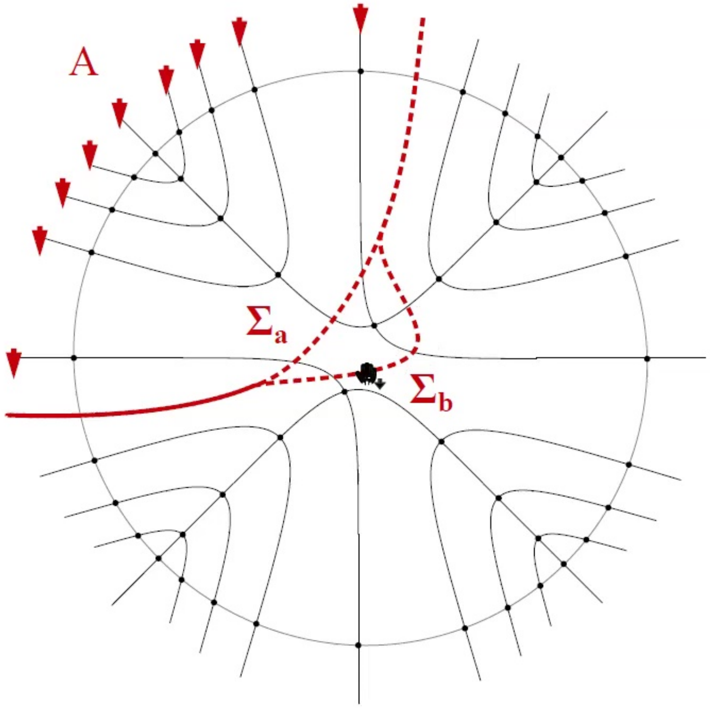
random pure state

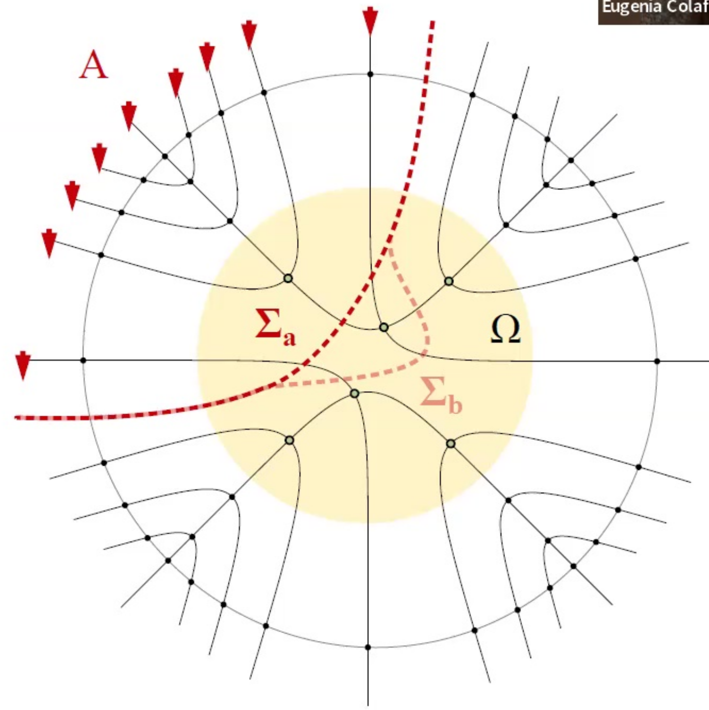
$$S_2(\zeta; \downarrow) = \log \frac{d_j^{|\Omega|} + 1}{d_j^{|\Omega_\downarrow|} + d_j^{|\Omega_\uparrow|}}$$



$$\overline{S_2(\rho_A)} \approx \beta \min_{\vec{\sigma}} \left\{ -\frac{1}{2} \left[\sum_{e \in L} (\sigma_v \sigma_w - 1) + \sum_{e \in \partial\gamma} (\sigma_v \mu_e - 1) \right] + \min\{|\Omega_\uparrow|, |\Omega_\downarrow|\} \right\}$$

Boundary entropy for random spin network – homogeneous case





The minimum of H is degenerate, the corresponding surfaces have area

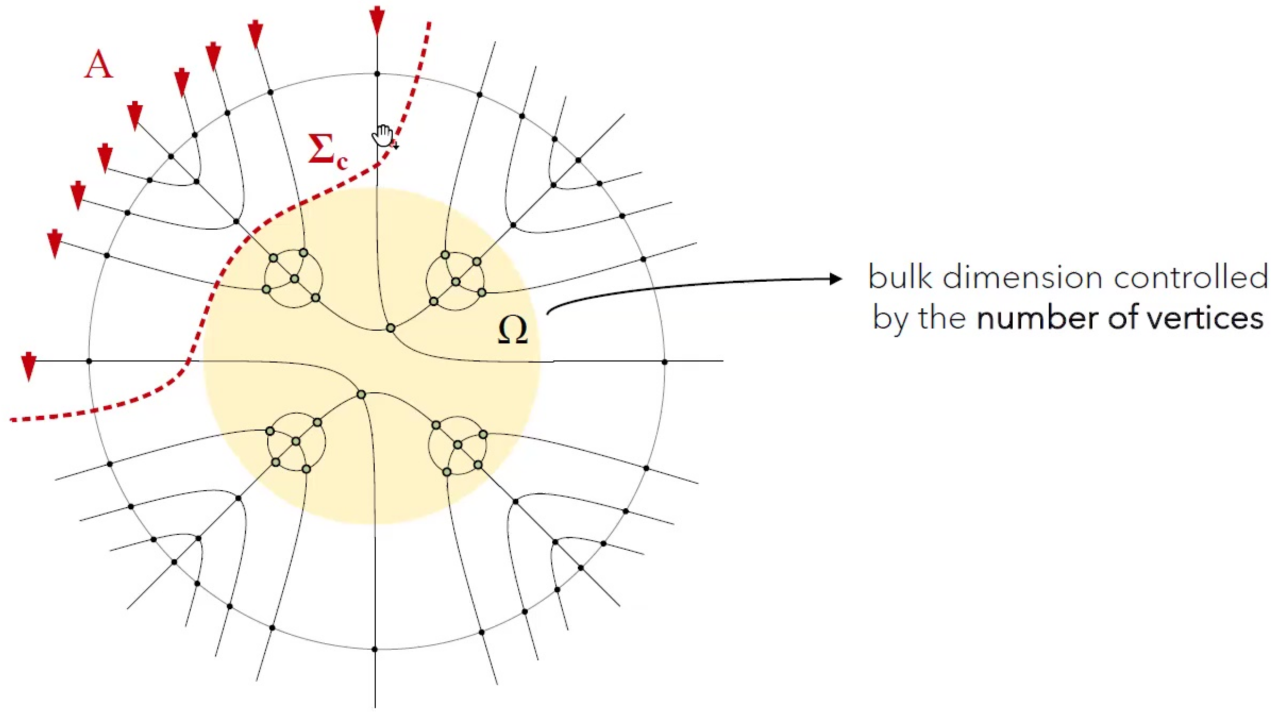
$$|\Sigma_a| = |\Sigma_b| = 5$$

If intertwiner entanglement is present in a region $\boldsymbol{\Omega}$, the degeneracy of the minimal energy is removed:

$$|\Sigma_a| = 6,\ |\Sigma_b| = 7$$

Boundary entropy for random spin network – homogeneous case

By increasing the dimension of the bulk disk Ω via refinement of vertices, the minimal-energy surface is prevented from entering it:



emergence of a **horizon-like region** in the bulk

Boundary entropy for random spin network – inhomogeneous case

$\beta = \log d$ → average link dimension

$$\overline{Z} = \sum_{\vec{\sigma}} e^{-\beta H(\vec{\sigma})} \qquad H(\vec{\sigma}) = \underbrace{-\frac{1}{2}\left[\sum_{e\in L}(\sigma_v\sigma_w - 1)J_e + \sum_{e\in\partial\gamma}(\sigma_v\mu_e - 1)J_e\right]}_{\text{area of domain wall}} + \beta^{-1}S_2(\zeta;\downarrow) \qquad J_e = \frac{\log d_{j_e}}{\beta}$$

$$|\Sigma(\vec{\sigma})|$$

$$\beta \gg 1: \; F = -\log\overline{Z} \approx \beta \min_{\vec{\sigma}} H(\vec{\sigma})$$

Small bulk entropy contribution:

$$\overline{S_2(\rho_A)} \approx \underbrace{\log d_j \min_{\vec{\sigma}} |\Sigma(\vec{\sigma})|}_{\text{area law}} + \underbrace{S_2(\zeta;\downarrow)}_{\text{bulk correction}}$$

bulk area law for boundary entropy (with bulk induced corrections)

BOUNDARY ENTROPY FOR RANDOM SPIN NETWORK – HOMOGENEOUS CASE

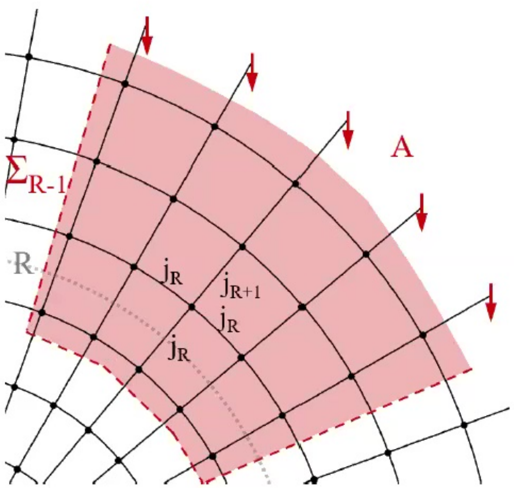
$H(r)$ = Ising-like Hamiltonian of a configuration whose domain wall Σ_r lies between shell r and shell r - 1

bulk entanglement not present

condition for the minimal-energy surface to drop from shell $r + 1$ to shell r :

$$H(r+1) > H(r) \iff d_{j_{r+1}} > d_{j_r}^{\frac{|A|+2}{A}}$$

satisfied for $|A| \gg 1$

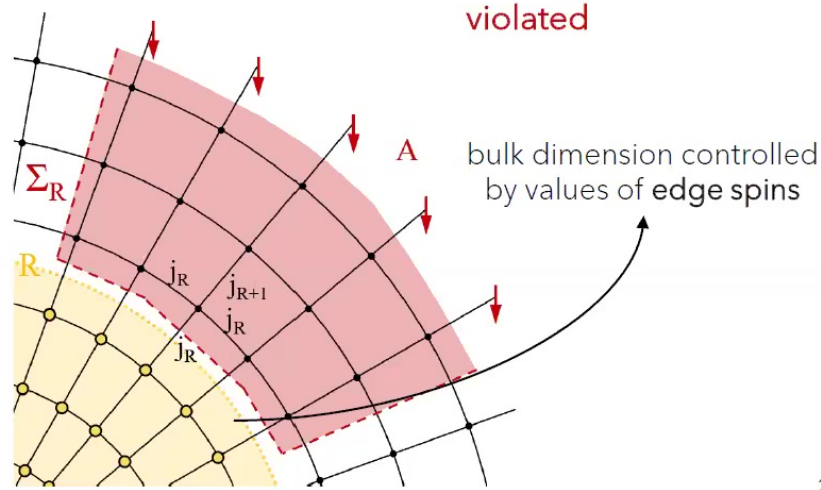


bulk entanglement present for $r \leq R$

condition for the minimal-energy surface to enter the disk Ω of radius R:

$$H(R+1) - H(R) > 0 \iff \underbrace{2 - d_{j_R}^{\frac{|A|+2}{A}}}_{\text{negative}} > \frac{d_{j_R}^{1+\frac{|A|+2}{A}}}{d_{j_{R+1}}}$$

violated



CONCLUSIONS

Tool: for spin network states with **random vertex weights**, entropy calculation translates into the evaluation of the free energy of a dual **Ising model**

BULK-TO-BOUNDARY ISOMETRIES

- ✓ Quantum gravity states associated to open graphs define **bulk/boundary maps** in the language of tensor networks
- ✓ Bulk-to-boundary maps of **homogeneous** graph made of four-valent vertices, each of them with at most one boundary link, **cannot be isometric**
- ✓ For generic graphs made of four-valent vertices, **increasing the inhomogeneity** of the spin assignment **increases the holographic character** of the map

HORIZON-LIKE REGIONS FROM VOLUME CORRELATIONS

- ✓ **Bulk area law for boundary entropy** for null (or small) values of the bulk entropy (with bulk-induced corrections)
- ✓ Emergence of a **horizon-like surface**: when a bulk region with high entanglement entropy is present, the Ising domain wall cannot enter it

FUTURE STEPS

- Generalization of the analysis to
  - **spin superposition** (entropy from a class of combinatorially equivalent Ising models)
  - **graph superposition:** by enriching the spin network with data encoding the amount of **link-entanglement** between vertices (entropy calculation mapped to random Ising model)
  - **GFT condensate states**
- Information-theoretic characterization of black hole horizons
  - derivation of a **"threshold condition"** for the emergence of horizon-like surfaces in the bulk, analogously to the one obtained from the **typicality approach** to the study of the local behavior of spin networks [Chirco, Anzà 2017]
  - **Numerical simulation** of boundary entropy and its phase transition through random tensor network techniques
- Implementation of the **dynamics**
  Several possibilities dictated by the QG approach (Loop quantum gravity, Spin foams, Group field theories…).
  - **Quasi-local holographic dualities in 3d quantum gravity** [Dittrich, Goeller, Livine, Riello 2017]
    bulk quantum geometrodynamics (given by Ponzano-Regge state-sum model) ↔ 2d statistical models
  - **LQG boundary dynamics as a 2+1-dimensional SL(2,C) gauge theory** [Livine 2021]