Title: Algebraically closed higher categories

Speakers: Theo Johnson-Freyd

Series: Mathematical Physics

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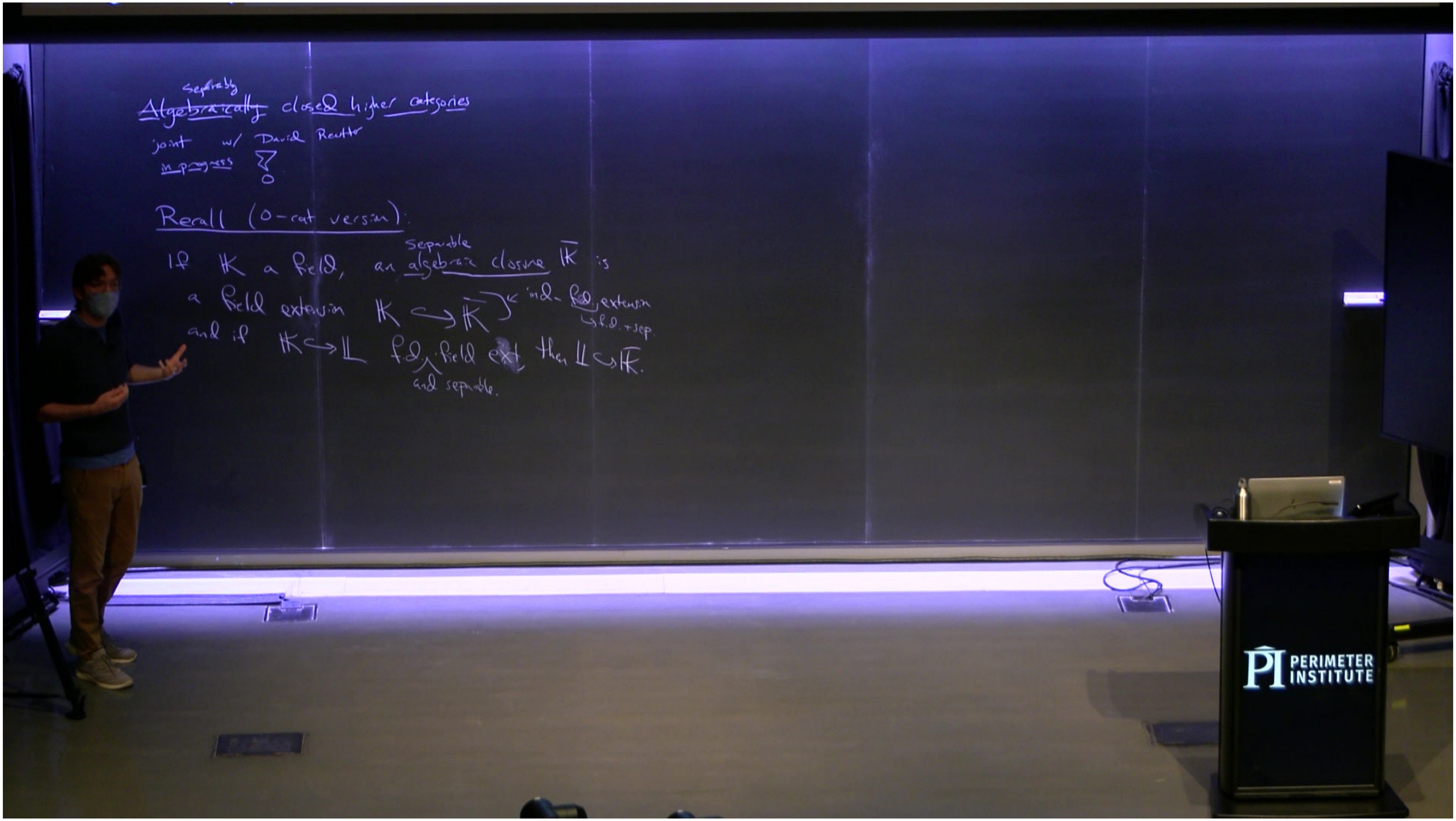
Abstract: I will report on my progress, joint with David Reutter, to construct and analyze the algebraic closure of nVec --- in other words, the universal n-category of framed nD TQFTs. The invertibles are Pontryagin dual to the stable homotopy groups of spheres. The Galois group is almost, but not quite, the stable orthogonal group. And an invertible TQFT can be condensed from the vacuum if and only if it is in the Pontryagin dual to the cokernel of the J-homomorphism.

Zoom Link: https://pitp.zoom.us/j/97015505587?pwd=MTJCYzd3djk2SkZENmVNVm01S0JQdz09

Algebraically closed higher categories

joint w/ David Reutter

in progress


Separably
~~Algebraically~~ closed higher categories
joint w/ David Reutter
in progress
Recall (0-cat version):
If $\mathbb{K}$ a field, an separable ~~algebraic~~ closure $\overline{\mathbb{K}}$ is
a field extension $\mathbb{K} \hookrightarrow \overline{\mathbb{K}}$ ← ind-f.d. extension
↳ f.d. + sep.
and if $\mathbb{K} \hookrightarrow \mathbb{L}$ f.d. and separable field ext then $\mathbb{L} \hookrightarrow \overline{\mathbb{K}}$.
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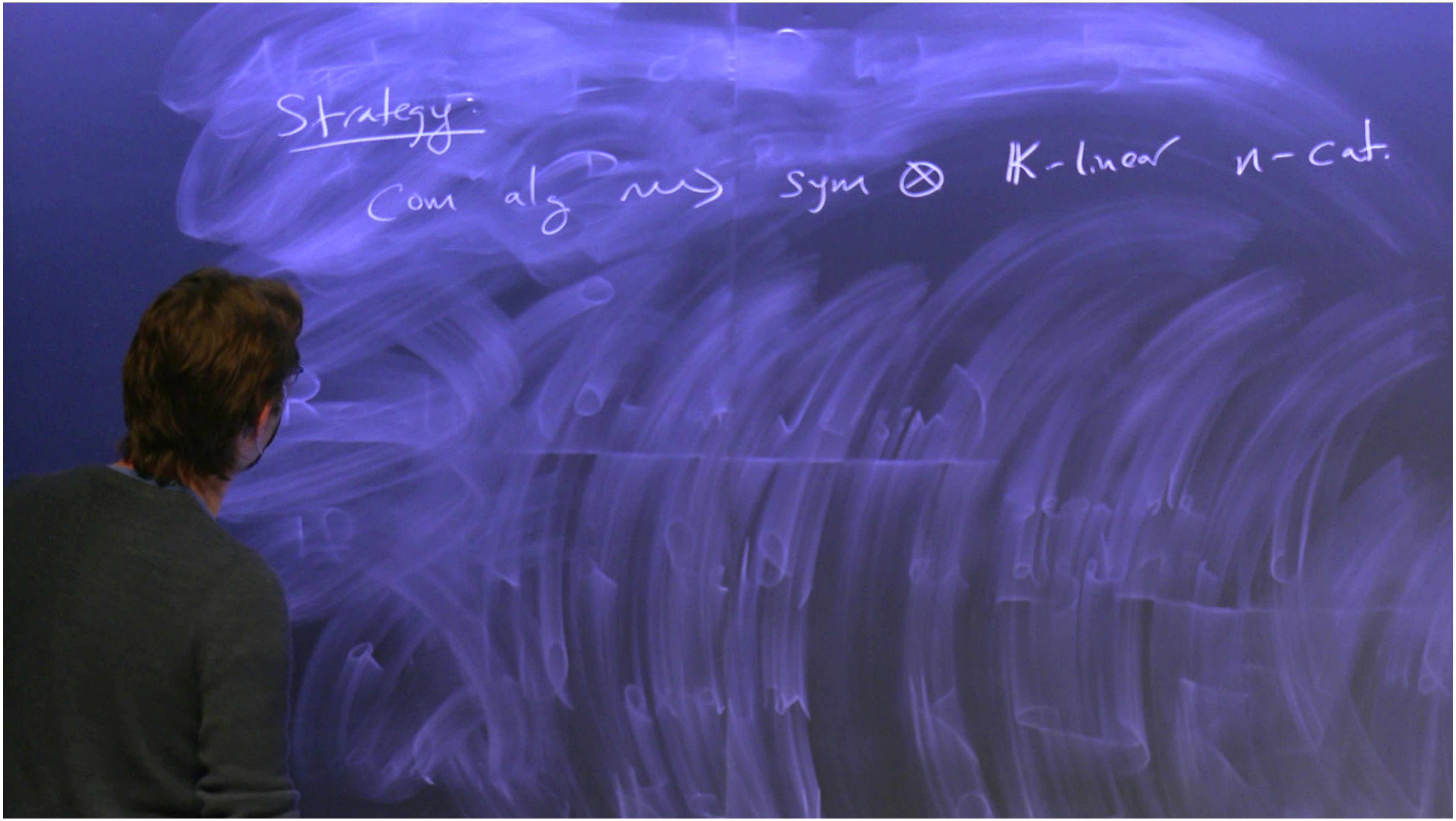
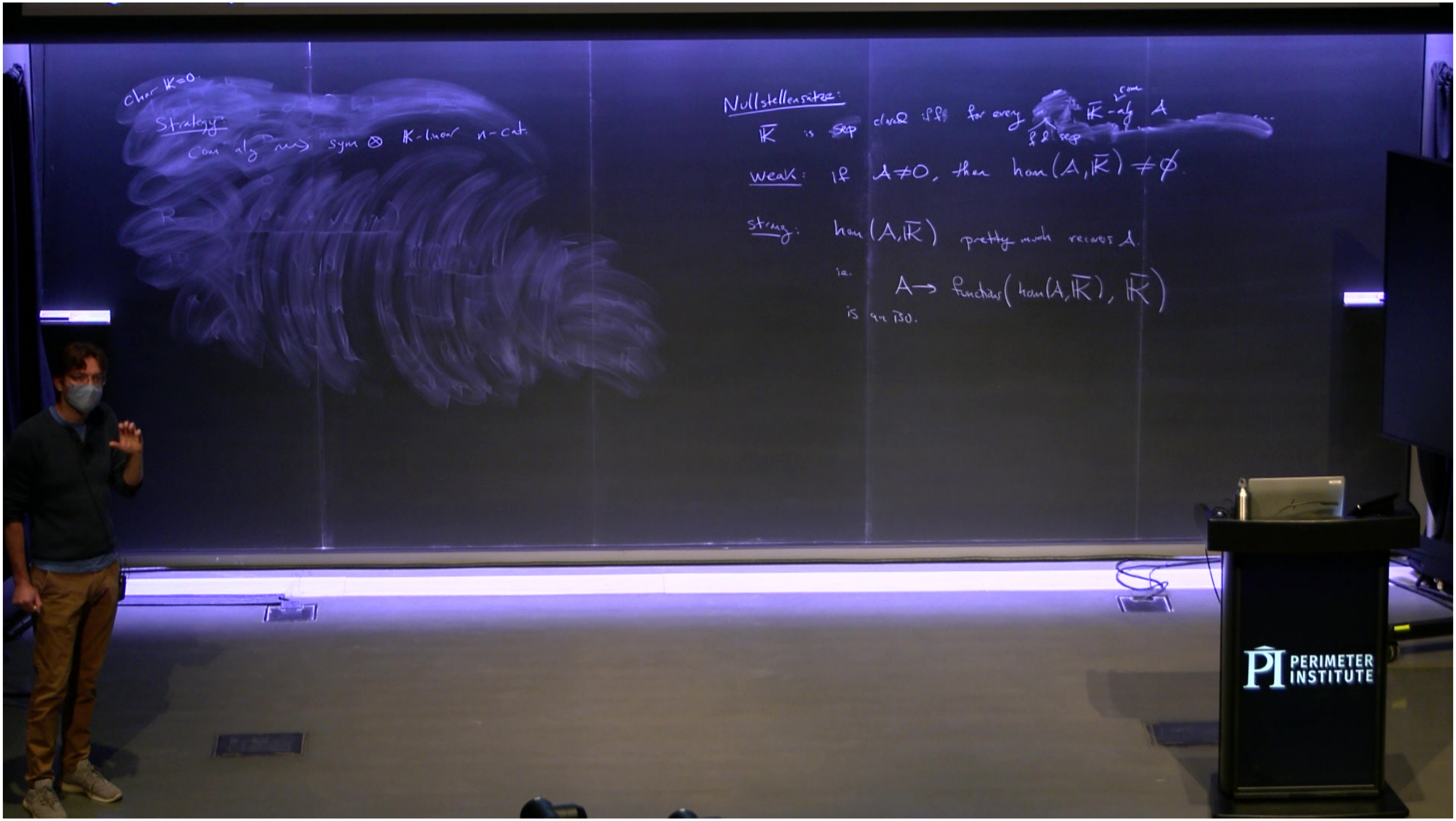

Strategy:
Com alg^D ⇝ sym ⊗ 𝕂-linear n-cat.


char K=0.
Strategy:
Com alg ~> sym ⊗ K-linear n-cat.
Nullstellensatz:
K̄ is sep closed iff for every f.g. sep K̄-alg A
weak: if A≠0, then hom(A,K̄) ≠ ∅.
strong: hom(A,K̄) pretty much recovers A.
i.e. A → functions(hom(A,K̄), K̄)
is an iso.
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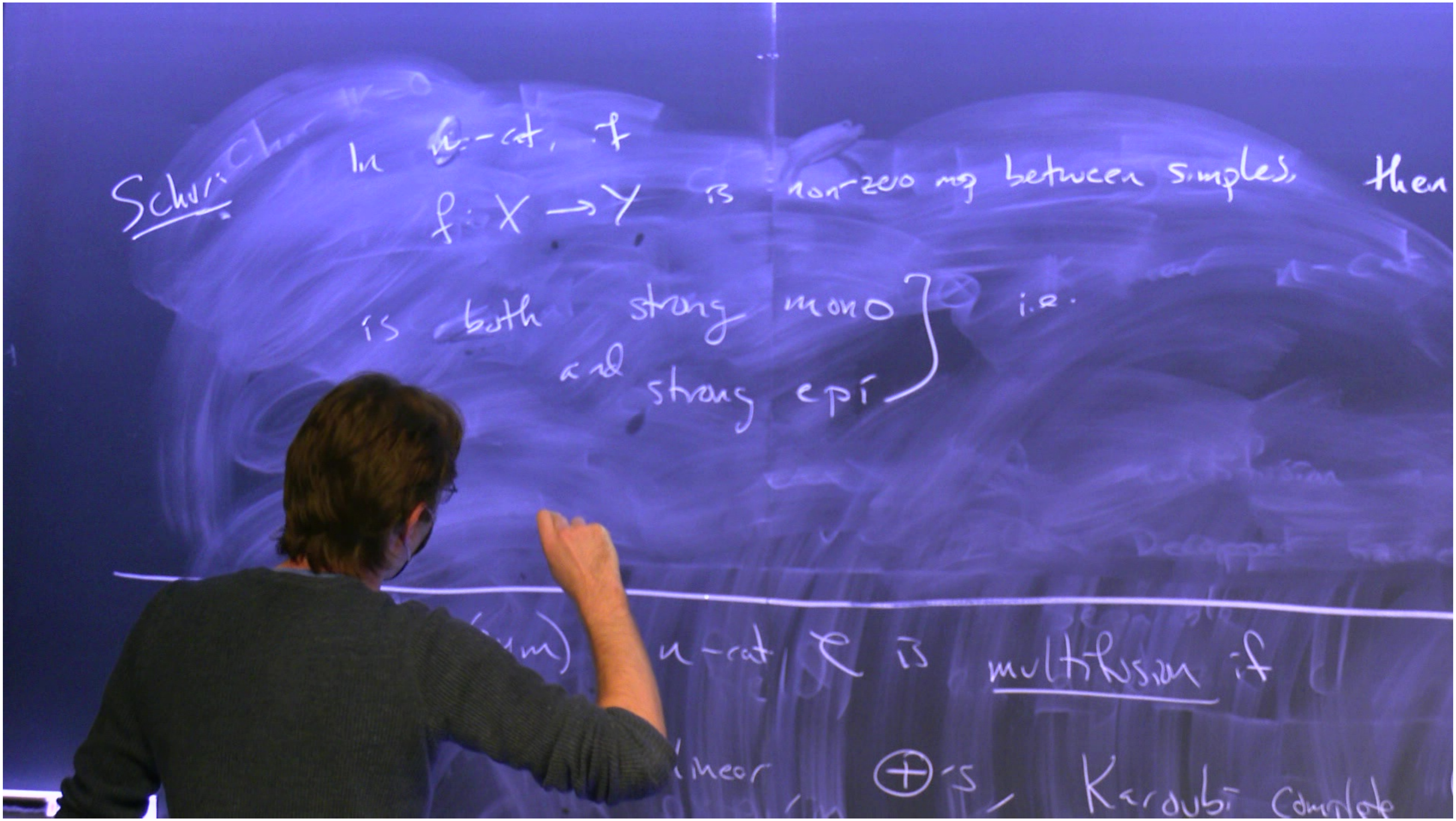
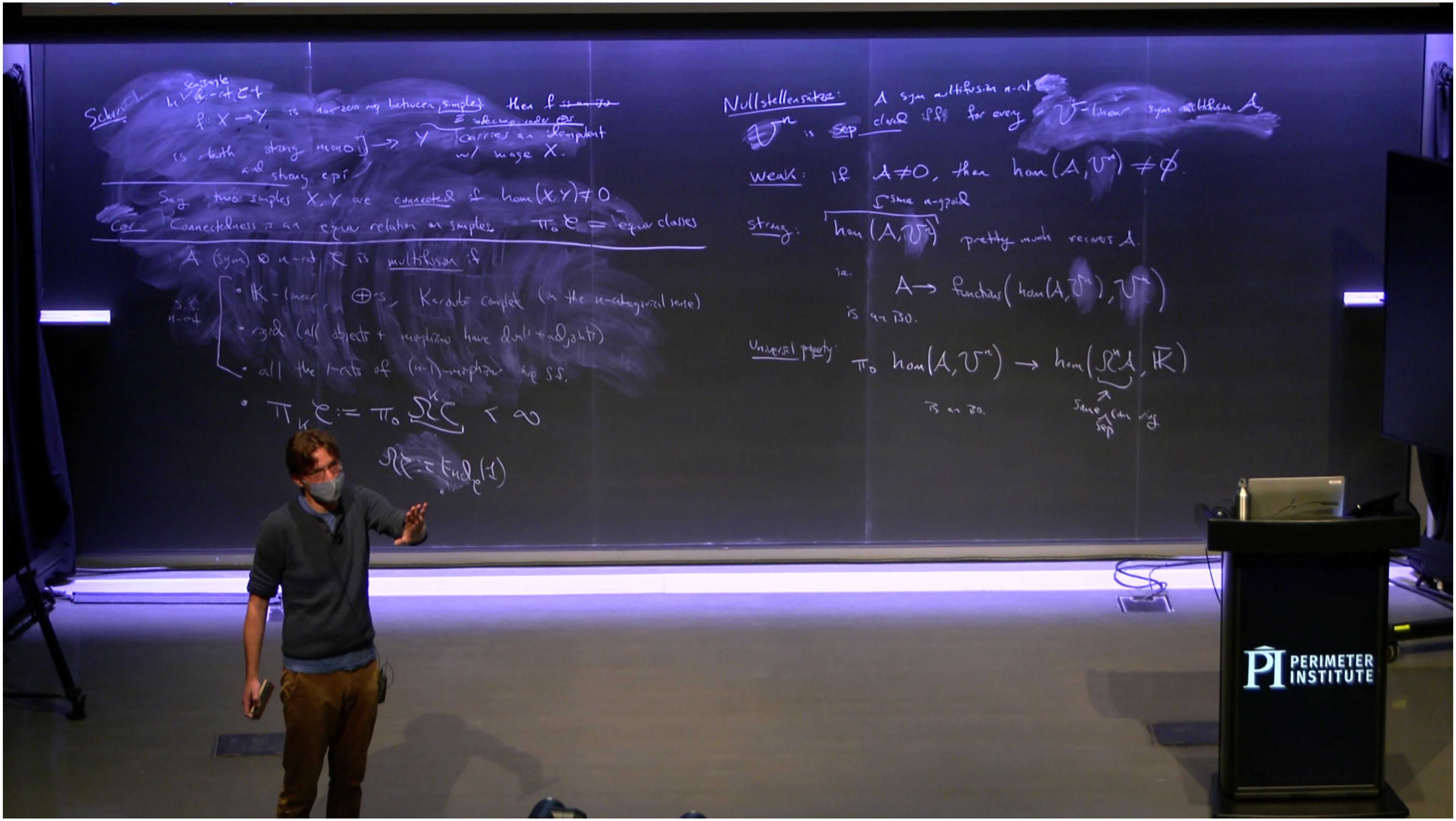
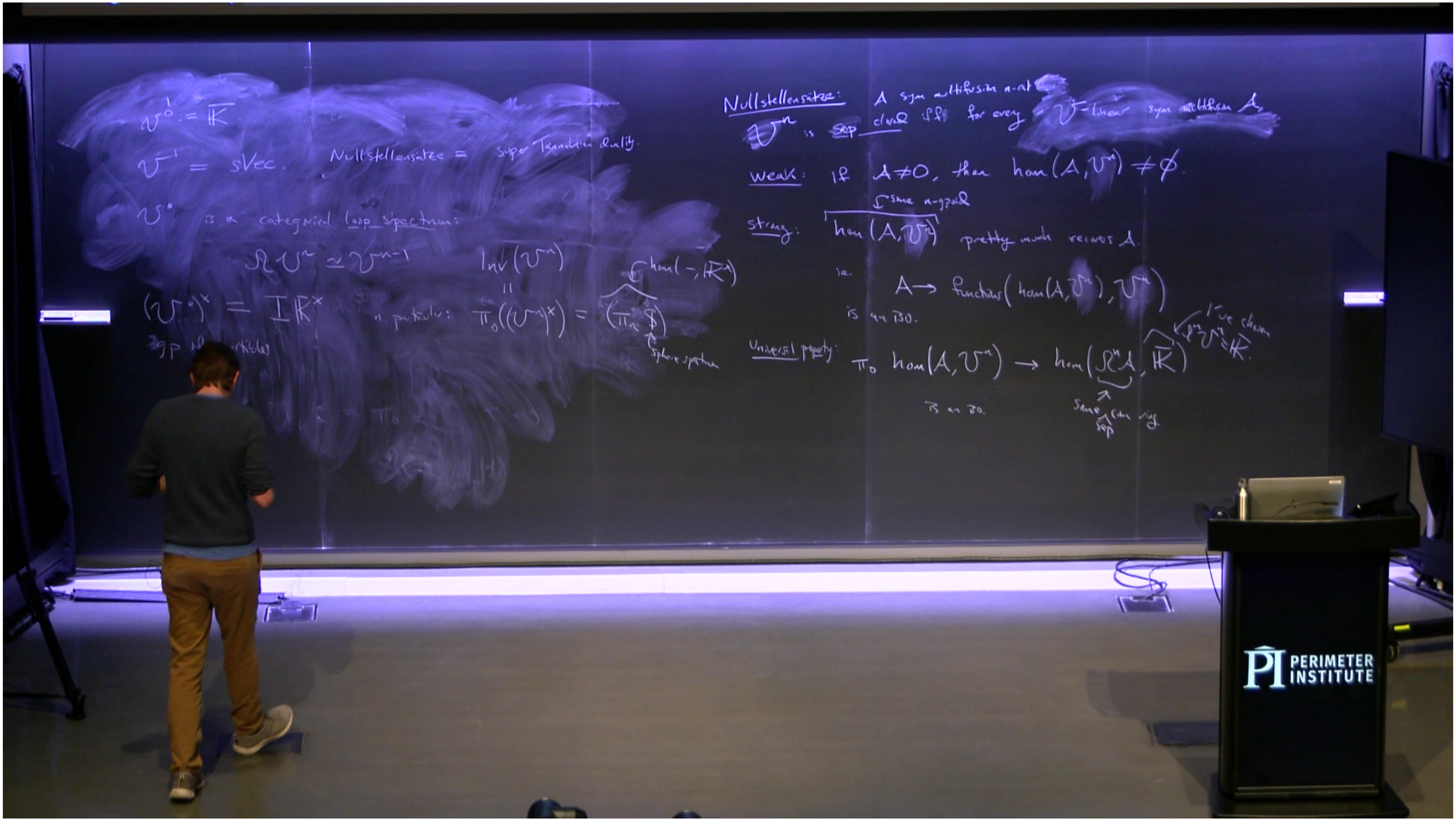
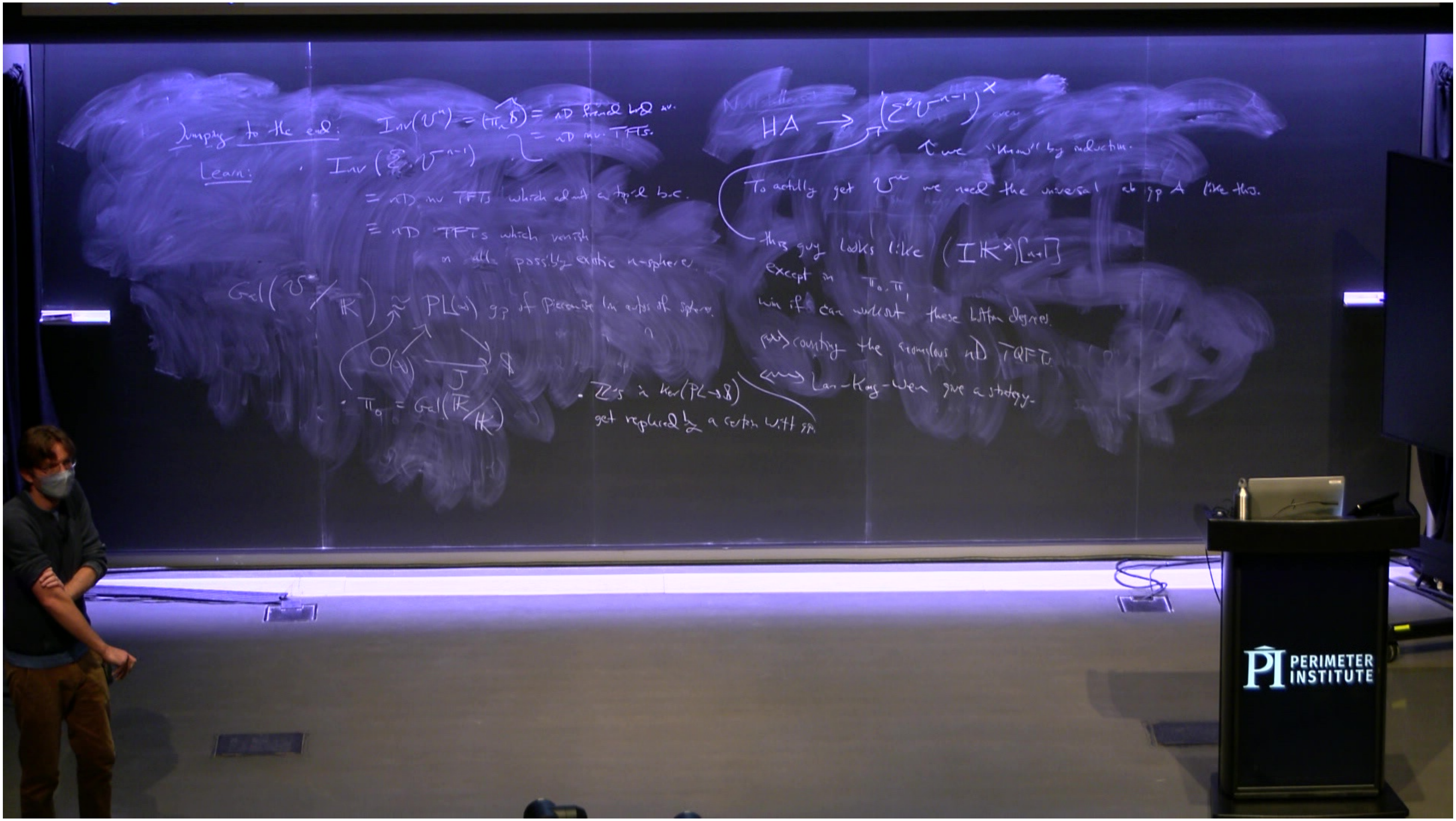

Schur: In n-cat, if
f: X → Y is non-zero map between simples, then
is both strong mono
and strong epi
i.e.
n-cat 𝒞 is multifusion if
linear
⊕'s
Karoubi complete



Schur:
f : X → Y is non-zero mor between simples then f
is both strong mono and strong epi
carries an idempotent w/ image X.
Say two simples X, Y are connected if hom(X, Y) ≠ 0
Cor: Connectedness is an equiv relation on simples. π₀ 𝒞 = equiv classes
A (sym) ⊗ n-cat 𝒞 is multifusion if
s.s. n-cat
• 𝕂-linear, ⊕-s, Karoubi complete (in the n-categorical sense)
• rigid (all objects + morphisms have duals + adjoints)
• all the 1-cats of (n-1)-morphisms are s.s.
• π_k 𝒞 := π₀ Ω^k 𝒞 < ∞
Ω𝒞 := End_𝒞(1)
Nullstellensätze: A sym multifusion n-cat
is sep closed iff for every -linear sym multifusion A,
weak: If A ≠ 0, then hom(A,) ≠ ∅.
some n-gpoid
strong: hom(A,) pretty much recovers A.
i.e. A → functions(hom(A,),)
is an iso.
Universal property: π₀ hom(A,) → hom(Ωⁿ A, 𝕂̄)
is an iso.



Nullstellensätze:
A sym multifusion n-cat
is sep closed iff for every
-linear sym multifusion A,
weak: if A ≠ 0, then hom(A,) ≠ ∅.
some n-groupoid
strong: hom(A,) pretty much recovers A.
i.e.
A → functors(hom(A,),)
is an iso.
Universal property:
is an iso.
I've chosen
Sphere spectrum
sVec.
Nullstellensatze = super Tannakian duality.
is a categorical loop spectrum:
in particular:


Jumping to the end:
Learn:
which admit a top'l b.c.
which vanish
on all possibly exotic n-sphere
gp of piecewise lin autos of sphere
get replaced by a certain Witt gp.
HA
we "know" by induction.
To actually get we need the universal ab gp A like this.
this guy looks like
except in
win if can work out these bottom degrees.
counting the anomalous nD TQFTs
Lan-Kong-Wen give a strategy.
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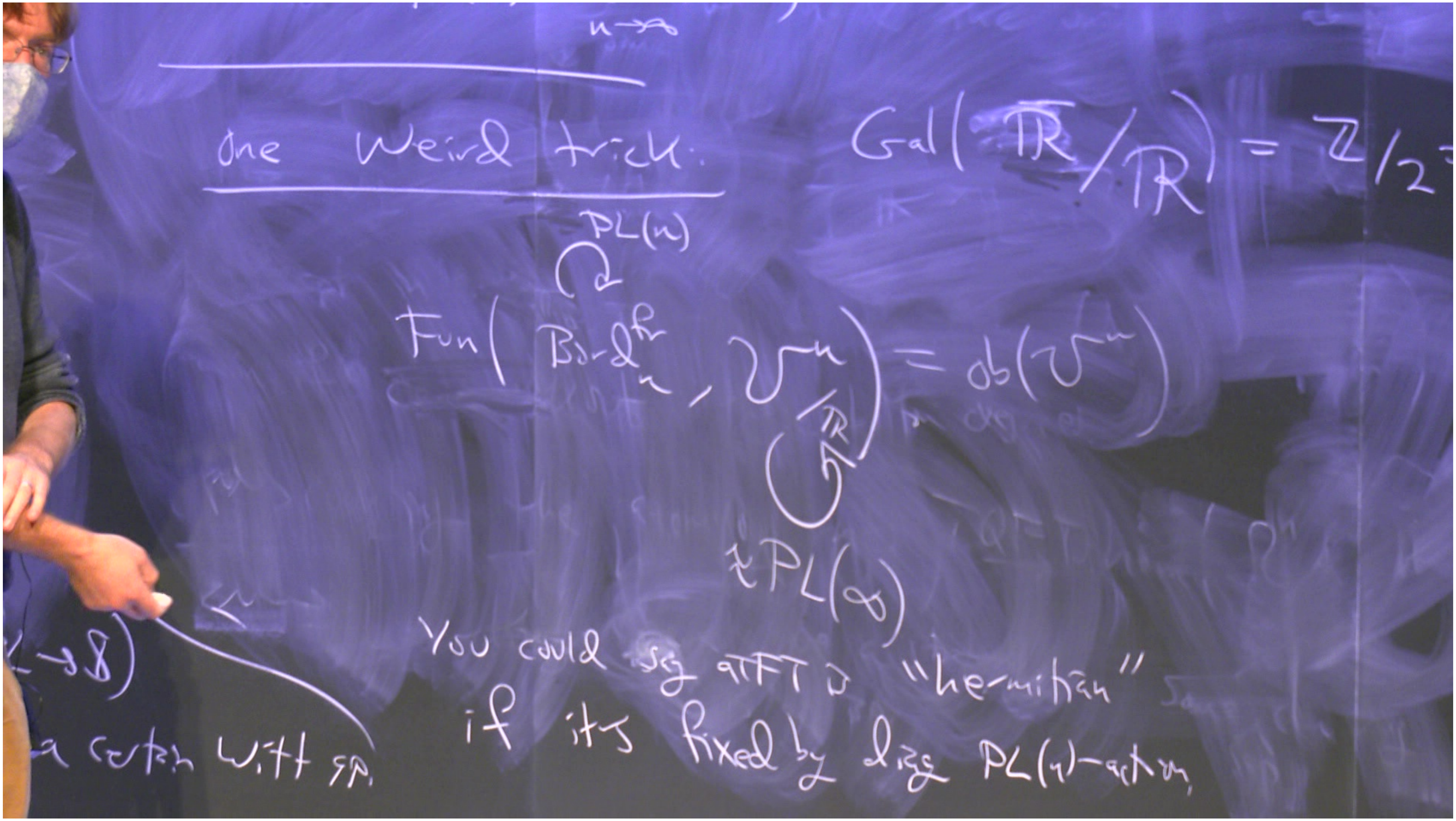
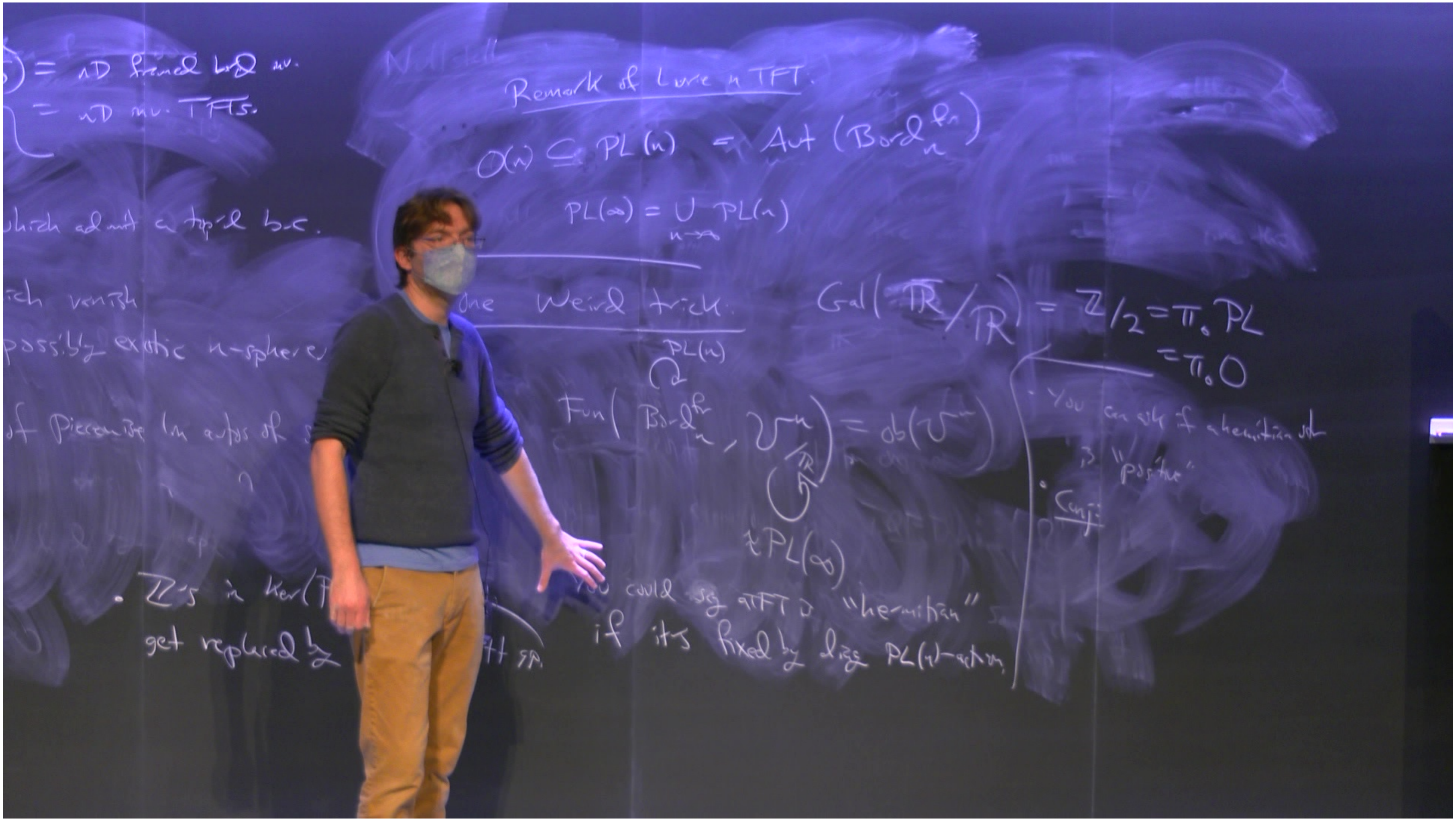
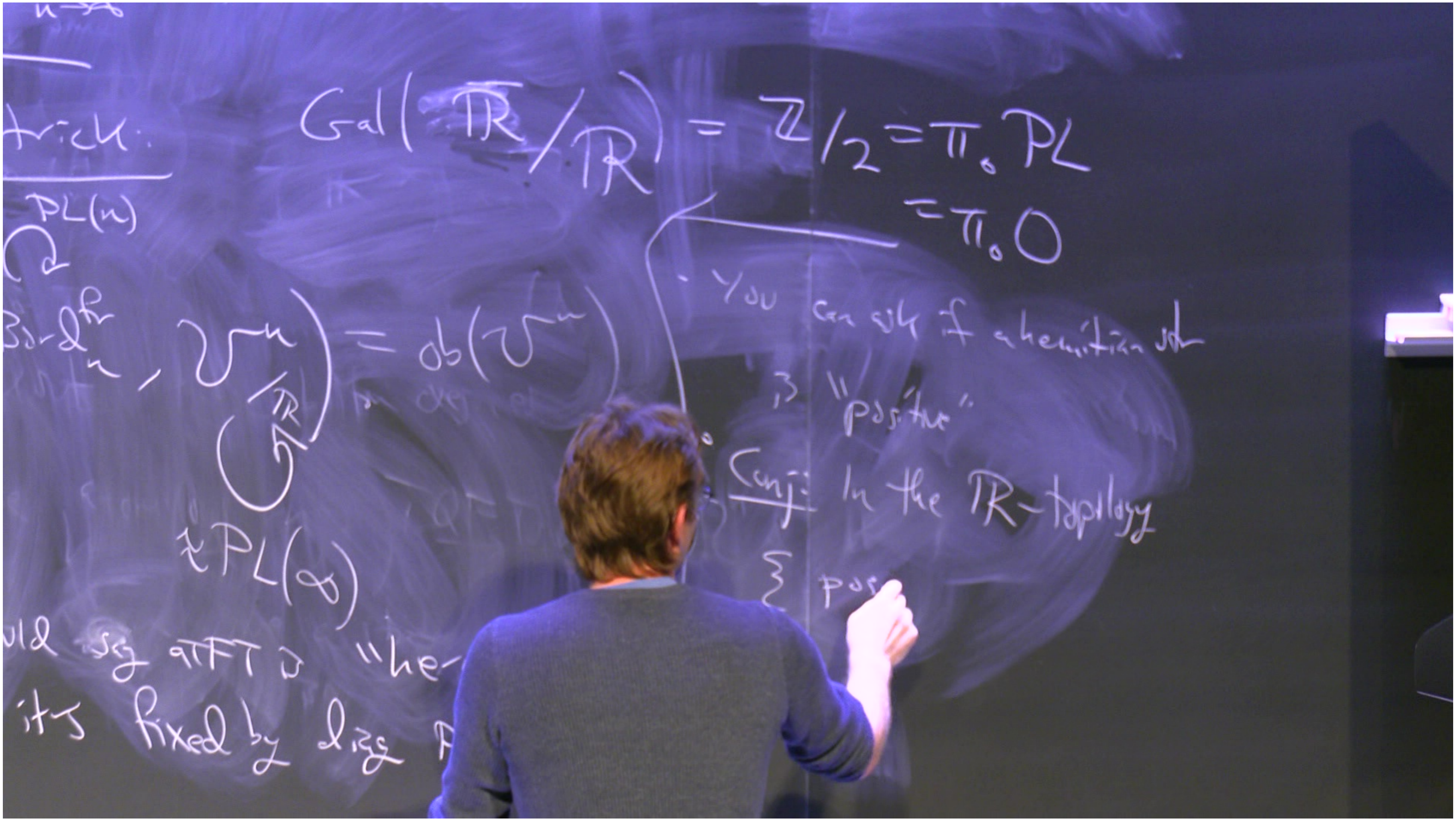

One Weird trick:
$\mathrm{Gal}(\overline{\mathbb{R}}/\mathbb{R}) = \mathbb{Z}/2$
$PL(n)$
$\mathrm{Fun}(\mathrm{Bord}_n^{fr}, \mathcal{V}^n_{/\mathbb{R}}) = \mathrm{ob}(\mathcal{V}^n)$
$\mathbb{Z}PL(\infty)$
You could say a TFT is "hermitian" if it's fixed by diag $PL(n)$-action



O(n) ⊆ PL(n) = Aut(Bord_n^fr)
PL(∞) = ∪ PL(n)
One Weird trick
Gal(ℝ̄/ℝ) = ℤ/2 = π0 PL = π0 O
Fun(Bord_n^fr, V^n)
τPL(∞)
You could say a TFT is "hermitian" if it's fixed by diag PL(n)-action
possibly exotic n-spheres
get replaced by


trick:
Gal(R̄/R) = Z/2 = π₀ PL
= π₀ O
PL(n)
You can ask if a hermitian str
is "positive"
Conj: In the R-topology
PL(∞)
fixed by

$) =$ nD framed bord mv.

$=$ nD mv. TFTs.

which admit a top'l b.c.

...ich vanish

possibly exotic n-sphere

of piecewise lin autos of sphere

• $\mathbb{Z}$'s in $\ker(PL \to \mathbb{S})$ get replaced by a certain Witt gp.

Remark of Lurie n TFT.

$$O(n) \subseteq PL(n) = \mathrm{Aut}(\mathrm{Bord}_n^{fr})$$

$$PL(\infty) = \bigcup_{n \to \infty} PL(n)$$

One Weird trick.

$$\mathrm{Gal}(\overline{\mathbb{R}}/\mathbb{R}) = \mathbb{Z}/2 = \pi_0 PL = \pi_0 O$$

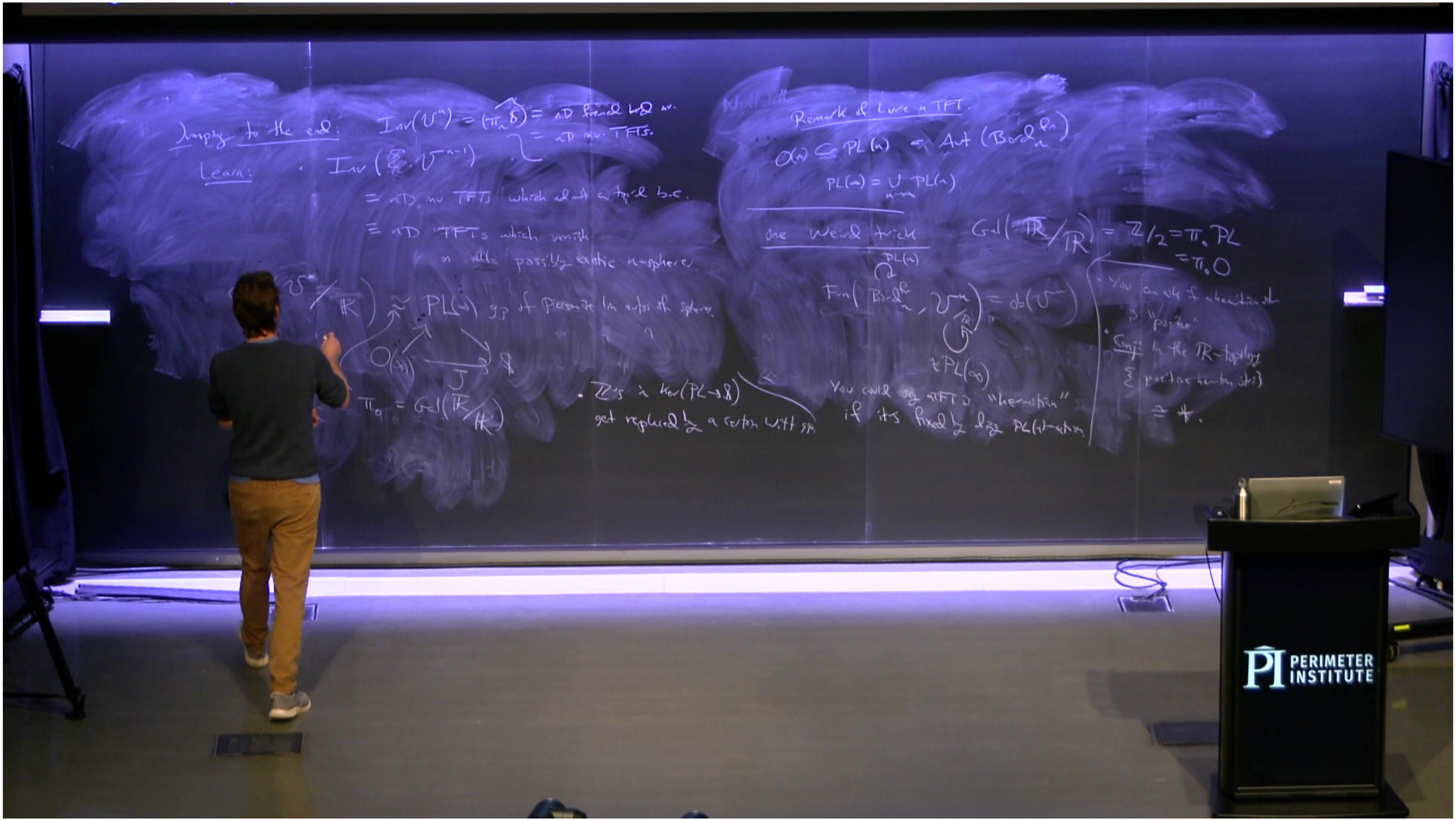
$$\mathrm{Fun}(\mathrm{Bord}_n^{fr}, \mathcal{V}^n_{\overline{\mathbb{R}}}) = \mathrm{ob}(\mathcal{V}^n)$$

with $PL(n)$ acting on Bord_n^{fr}, and $PL(\infty)$ acting on $\mathcal{V}^n_{\overline{\mathbb{R}}}$.

You could say a TFT is "hermitian" if it's fixed by diag $PL(n)$-action.

• You can ask if a hermitian str is "positive"

• Conj: In the $\mathbb{R}$-topology $\{$ positive hermitian strs $\} \cong *$.


Jumping to the end:
Learn:
One Weird trick
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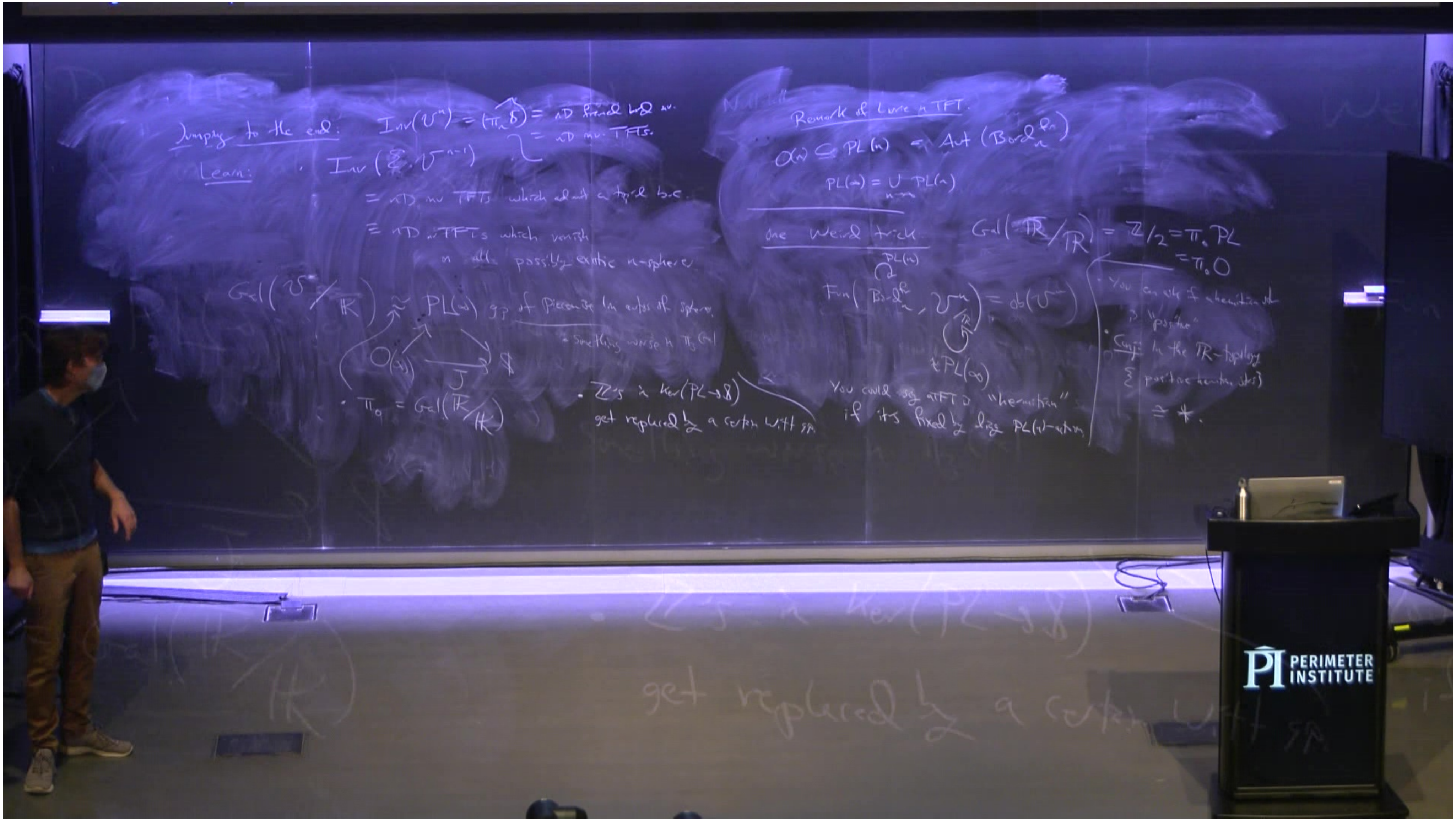
Jumping to the end:
Learn:
Remark of Lurie n TFT.
One Weird trick
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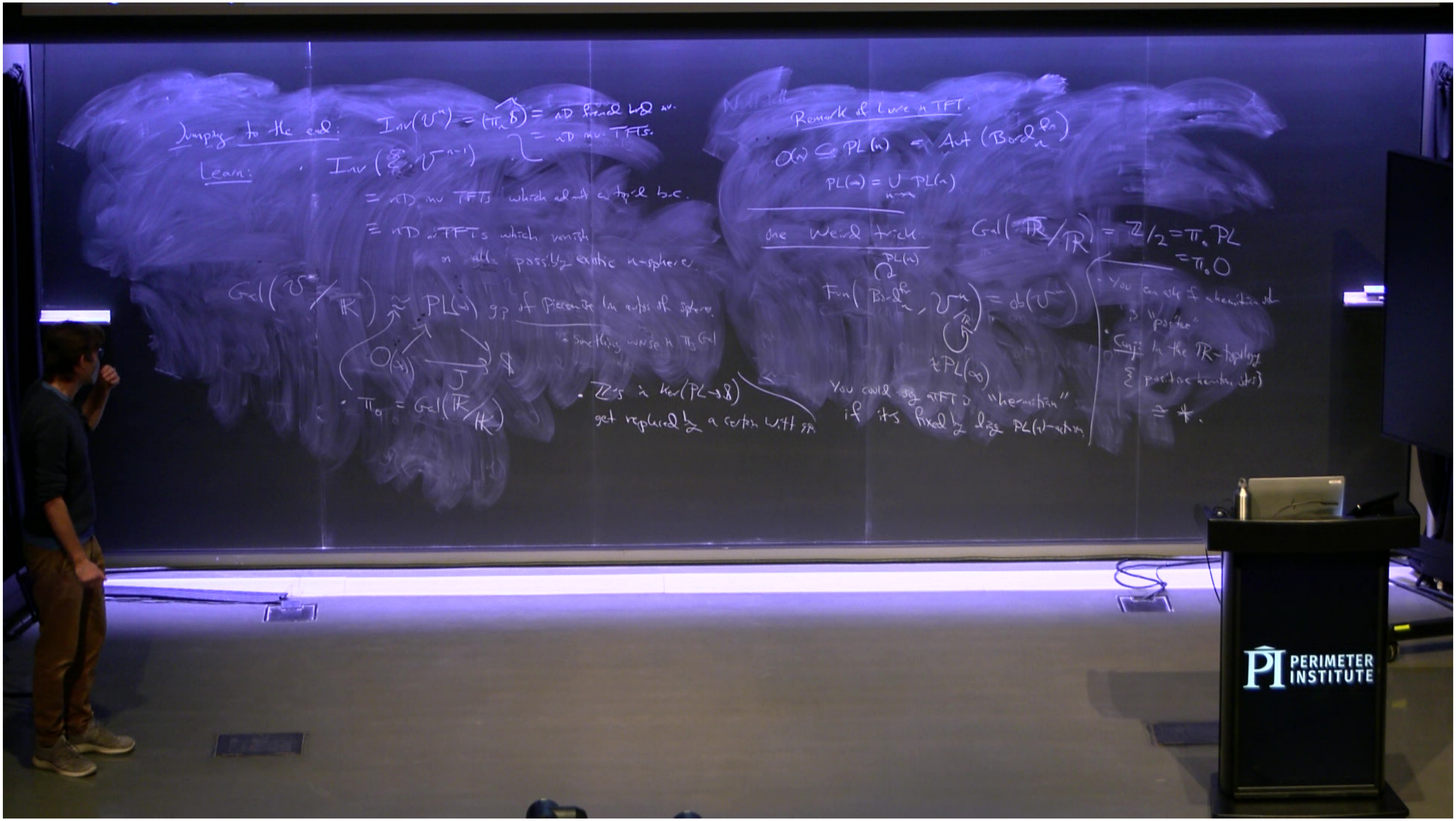
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