

Title: Subdiffusion and ergodicity breaking in systems with emergent or microscopic dipole-moment conservation

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Series: Quantum Matter

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Abstract: I will first give a brief overview of my research in the field of out-of-equilibrium quantum many-body physics, ranging from the theory of many-body localization, to the recent application of Tensor

Processing Units for accelerating simulations of quantum dynamics. I'll then focus on (1) the

experimental observation and theoretical explanation of subdiffusive dynamics in a "tilted" Fermi-Hubbard system [PRX 10, 011042 (2020)], and (2) a "freezing" phase transition between weak and

strong ergodicity breaking in systems with particles that are immobile by themselves, but undergo coordinated pair hopping [PRB 101, 214205 (2020)]. These topics contain the common thread of either an emergent or microscopic conservation of the dipole moment (center of mass of the particle distribution), and I will provide simple pictures for how this leads to the subdiffusion and ergodicity breaking.

Subdiffusion and ergodicity breaking from emergent or microscopic dipole- moment conservation

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PRX 10, 011042 (2020)
PRB 101, 214205 (2020)



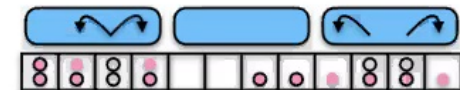
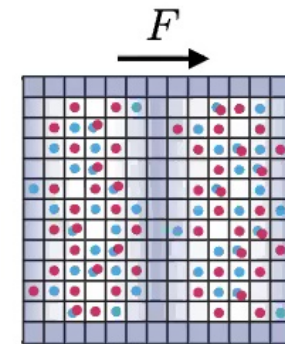
Research overview

- dynamics of *highly-excited* quantum many-body systems
- universal aspects of quantum thermalization
- many-body localization
- phase transitions in quantum dynamics

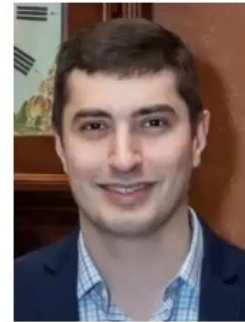


Outline

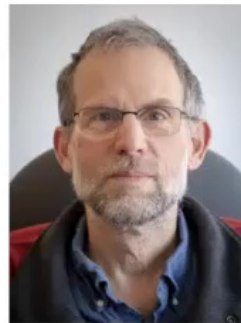
- Part 1: Tilted Fermi-Hubbard system
 - novel hydrodynamics of charge and energy
 - a mechanism for emergent dipole-moment conservation and subdiffusion
- Part 2: Kinetically constrained circuit model
 - subdiffusion and freezing phase transition due to microscopic dipole-moment conservation



Collaborators



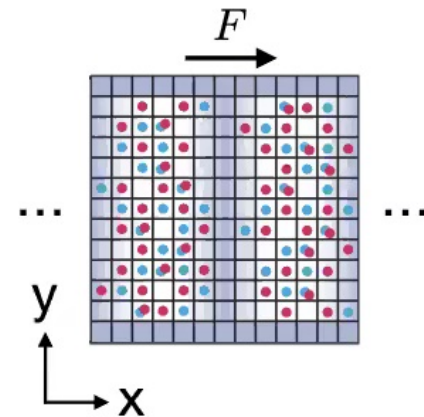
Elmer Guardado-Sanchez | Ben Spar | Peter Brown | Waseem Bakr
(Princeton)



David Huse (Princeton)

System

- isolated 2D cold-atom Fermi-Hubbard system
- atoms remain in first band of the lattice
- strong external linear potential (“tilt”) along x direction
- density comparable to half filling
- spin balanced



$$H = \sum_{\langle ij \rangle, \sigma} -t_h (c_{i, \sigma}^\dagger c_{j, \sigma} + \text{h.c.}) + \sum_i U n_{i, \uparrow} n_{i, \downarrow} - F x_i (n_{i, \uparrow} + n_{i, \downarrow}) \quad F a_{\text{latt}} \sim 4t_h \sim U$$

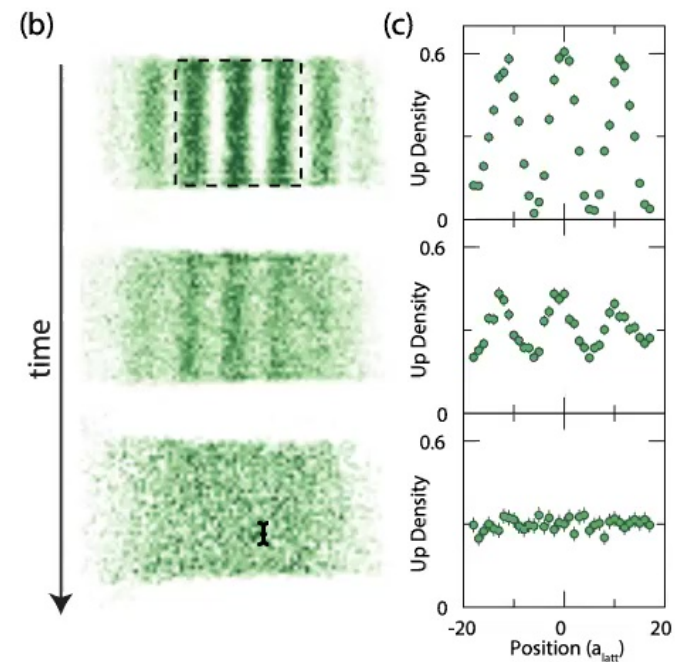
Part 1: Tilted Fermi-Hubbard system

Motivation

- initially motivated by an interest in MBL due to a strong linear potential and connection to fractons
 - tilt strong enough to generate microscopic dipole-moment conservation, particles mobile via pair hopping
 - but this is not what we found!
- I
- explore new regime of strong tilts and strong interactions ($F \sim t_h \sim U$) available in *isolated* cold-atoms systems

Experiment

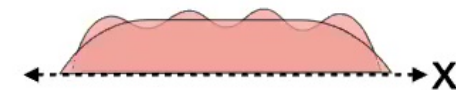
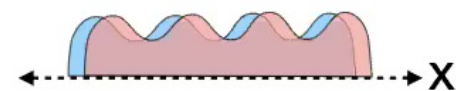
- initialize charge density wave in x direction
- measure occupancies of lattice sites as a function of position and time, average over y and samples
- also probe the local energy density if/when possible



Early-time heating and local equilibrium

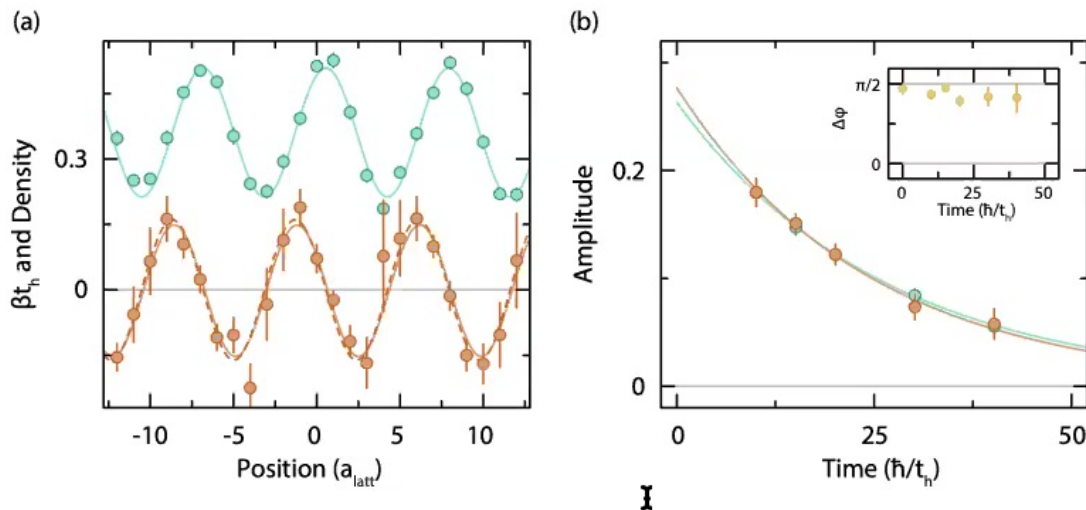
- the cloud “falls down” the tilt and heats up, achieving *local* equilibrium at near-infinite *local* temperatures
- final global equilibrium is at infinite temperature and uniform density
- at $\beta=T^{-1}$ near 0 local equilibrium implies

$$\frac{dn(x, t)}{dx} \sim F\beta(x, t)$$



Early-time heating and local equilibrium

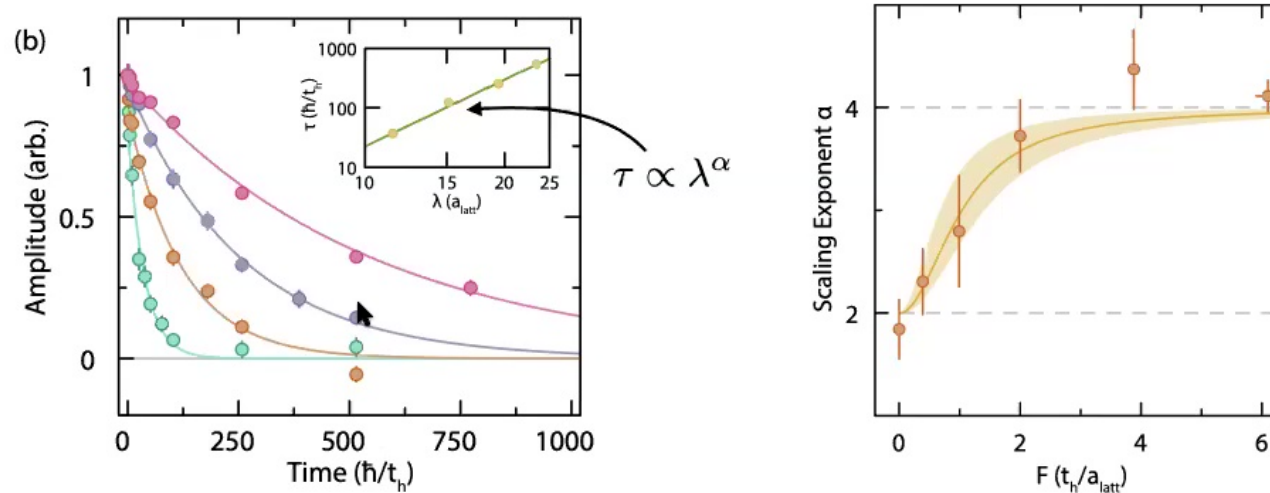
- estimate local temperature in “atomic limit” and test prediction of local equilibrium in tilted system



- late-time hydrodynamic regime is after this local equilibrium is quickly achieved

Late-time relaxation of density waves

$$n(x, t) = \bar{n} + A_0 e^{-t/\tau} \cos(kx + \phi(t))$$



- density waves relax diffusively at weak tilts
- crossover to subdiffusion at strong tilts

Hydrodynamic model

- linear theory around the uniform infinite-temperature global equilibrium
- density $n(x,t)$, non-tilt energy density $e(x,t)$, entropy density $s(e,n)$

$$\dot{n} + \nabla \cdot j_n = 0 \quad \dot{e} + \nabla \cdot j_e = F j_n$$

$$j_n = M_n \chi_n + M_{en} \chi_e \quad j_e = M_e \chi_e + M_{ne} \chi_n$$

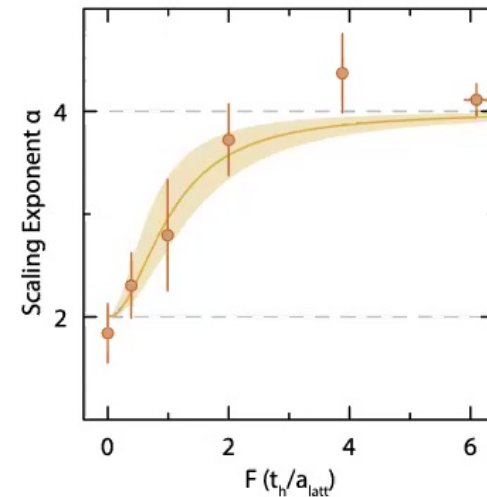
$$\chi_n = \nabla \left(\frac{\partial s}{\partial n} \right) + F \left(\frac{\partial s}{\partial e} \right) \quad \chi_e = \nabla \left(\frac{\partial s}{\partial e} \right)$$

Hydrodynamic model

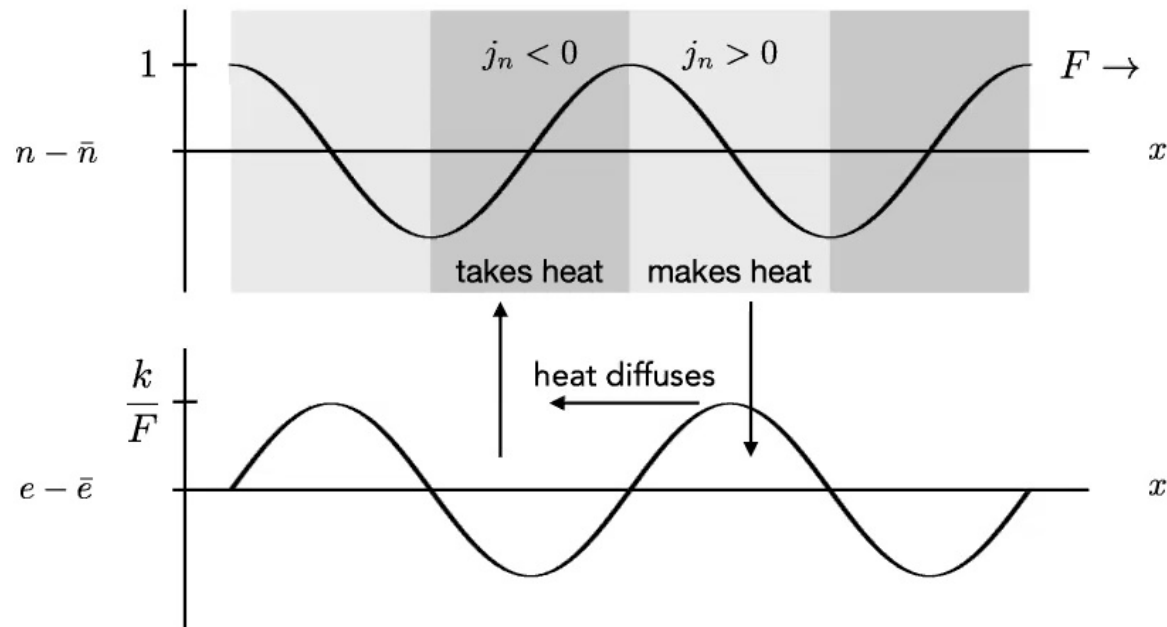
- linear system of equations solved for decay time $\tau(F, \lambda)$
- can extract prediction for $\alpha(F)$ in $\tau \propto \lambda^\alpha$

$$\alpha = \frac{d \log \tau}{d \log \lambda} = 2 + \frac{2}{1 + \frac{s_{nn}}{s_{ee}} \frac{k^2}{F^2}}$$

- so tilted system will cross over from diffusive ($\tau \propto \lambda^2$) to subdiffusive with $\tau \propto \lambda^4$ at strong tilts (or long wavelengths)



Subdiffusion at strong tilts



- need to convert $\sim F\lambda^2$ tilt energy into heat in chunks of $\sim 1/F$
- so $\sim F^2\lambda^2$ chunks, one after the other, each diffuse in a time $\sim \lambda^2/D_{th}$
- thus the density wave decays in time $\sim F^2\lambda^4/D_{th}$
- this mechanism generates an emergent dipole-moment conservation

Summary of Part 1

I

- discovered a new universal hydrodynamic regime of subdiffusion in “tilted band fluids”
- comes from interplay of energy conservation and external tilted potential which couples charge transport to energy production
- an example of quantum “inputs” into a classical framework yielding interesting new physics!

PRX 10, 011042 (2020)

Part 2: Kinetically constrained circuit model

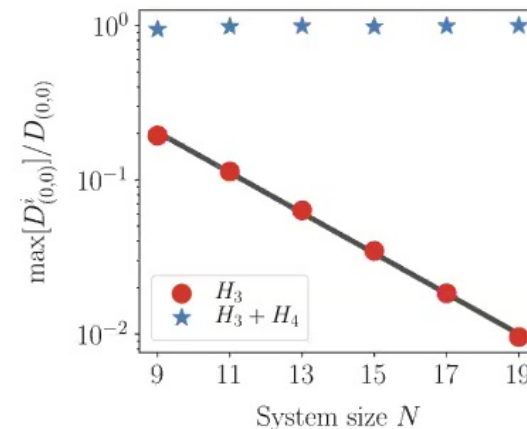
Background

- “Hilbert space fragmentation” in dipole-conserving Hamiltonians
- constraints cause dynamically disconnected sectors of Fock states
- weak and strong versions
- problem is essentially classical
- transition between weak and strong possible?

spin $\frac{1}{2}$

$$H_3 = -\sum_n [S_n^+ (S_{n+1}^-)^2 S_{n+2}^+ + \text{H.c.}]$$

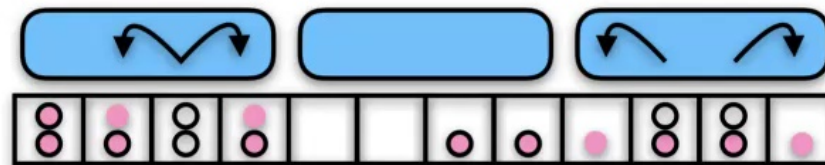
$$H_4 = -\sum_n [S_n^+ S_{n+1}^- S_{n+2}^- S_{n+3}^+ + \text{H.c.}],$$



Sala et al. 2020
 Khemani et al. 2020
 Moudgalya et al. 2020, 2021

Model

- classical stochastic lattice gas in 1D \mathbb{Z}
- $n_i \in \{0,1,2\}$ particles on site i
- at each time step, a layer of 4-site gates acts
- gates are totally random but conserve the charge and dipole moment of the particle distribution
- same fragmentation as H_3+H_4



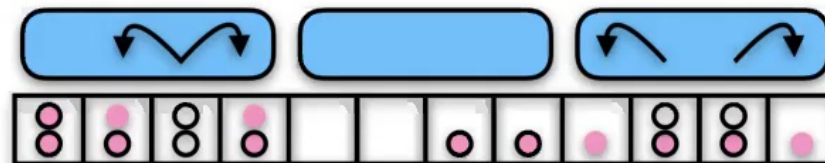
Model

- kinetic constraints lead to “pair hopping” on 3 or 4 sites as the only allowed local transitions
- single particles (or holes) are immobile (“fractons”)
- expect an “active” phase around half filling, and a “frozen” phase at high/low densities
- interested in dynamics of typical states at a given density and no dipole moment

allowed transitions

0120	0201	1011
0121	0202	1012
0210	1020	1101
0211	1021	1102
0220	1111	2002
1120	1201	2011
1121	1202	2012
1210	2020	2101
1211	2021	2102

0020	0101
0021	0102
0110	1001
0111	1002
0200	1010
0212	1022
0221	1112
1110	2001
1200	2010
1212	2022
1220	2111
1221	2112
2120	2201
2121	2202

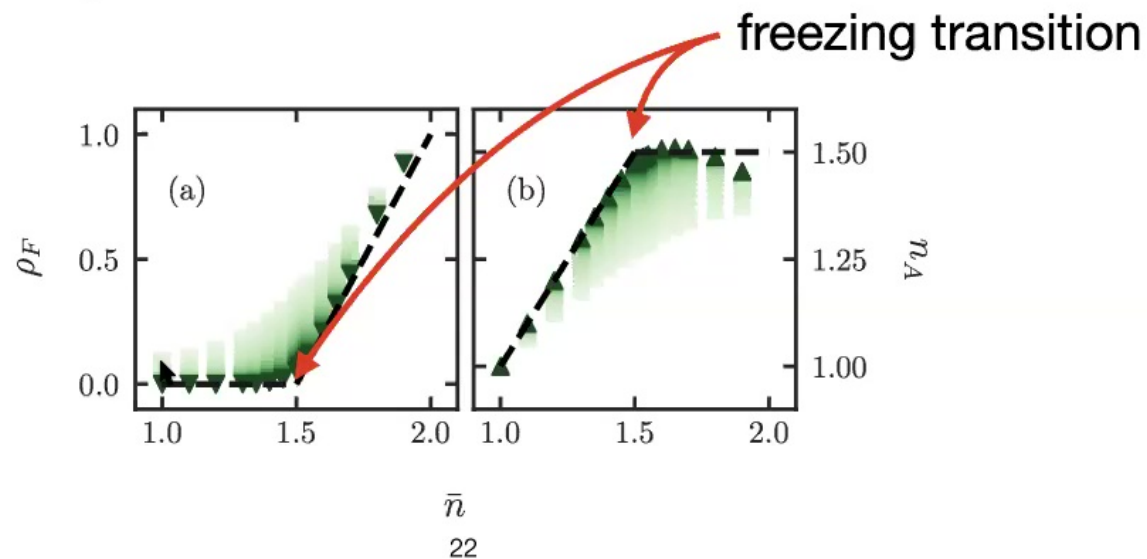


Active and frozen regions

- Active site: a site which has, at any point during the dynamics, changed its occupancy n_i
- Frozen site: a site which has never changed its occupancy
- active vs. frozen depends on the initial state, the time, and the site i
- key quantity: density of frozen sites ρ_F

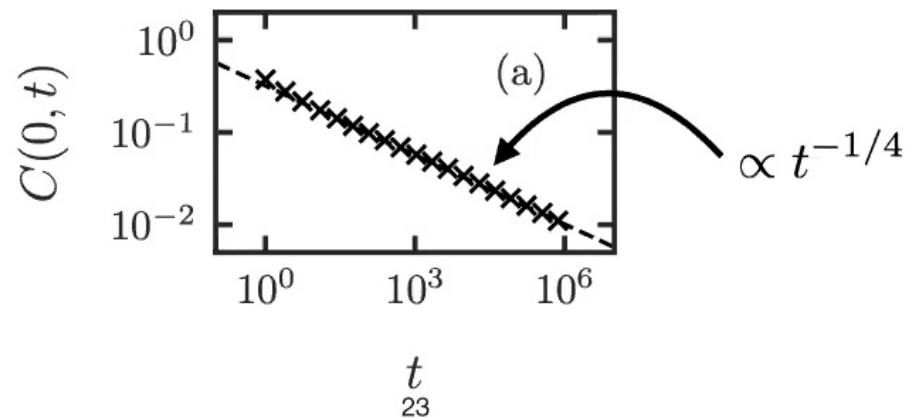
Simulations

- simulate dynamics of stochastic circuit model and track the number density of frozen sites and charge density of active regions
- $L=10^4$ sites, out to times $t=10^8$



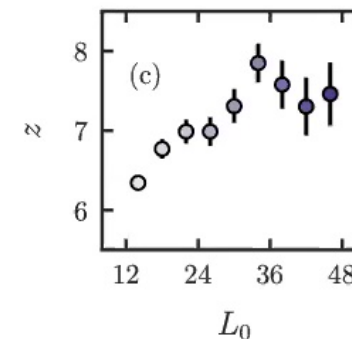
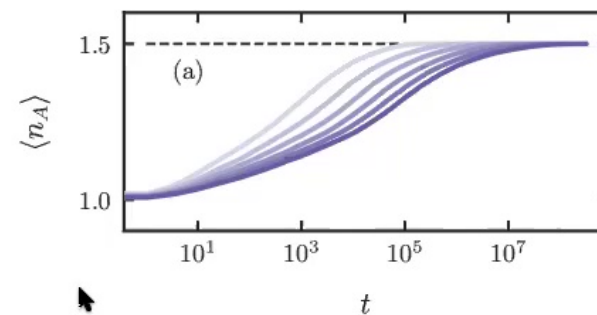
Active phase

- simulate dynamics of stochastic circuit model and compute charge autocorrelation function $\langle n_i(t)n_i(0) \rangle$
- half filling $\langle n_i \rangle = 1$, 10^3 random initial states, 10^3 sites
- subdiffusion $\tau \propto \lambda^4$



Isolated active bubble

- time evolve
....22221111....11112222....
- isolated active bubble acts as a bath to “activate” its frozen surroundings
- bubbles grow and merge, but halt when their density hits critical value
- time it takes to reach this critical value gives dynamical scaling exponent $z \sim 7$

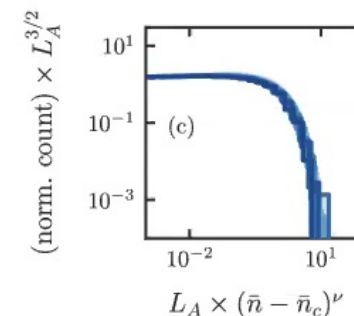
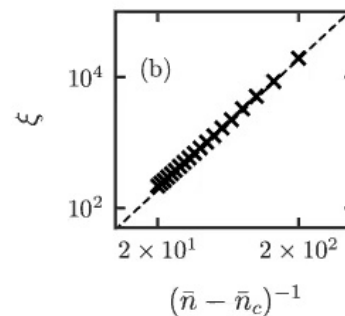
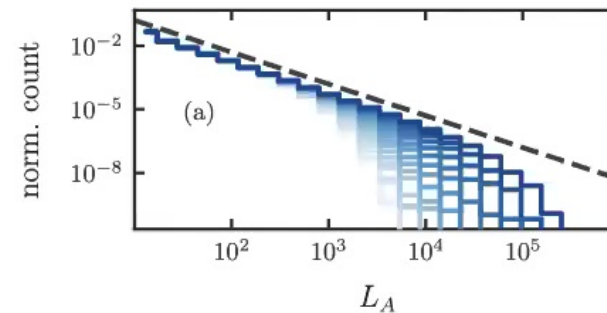


Approximate model

- low density active regions grow and “unfreeze” high density frozen regions, but lose power as a bath as they approach critical density
- approximate model:
 - sample an initial charge configuration (this does not explicitly evolve)
 - identify initially active regions and their densities
 - move active-frozen boundaries as long as charge density of active region remains subcritical
 - yields a final $t \rightarrow \infty$ pattern of frozen and critical active regions

Critical properties from approximate model

- distribution of active block lengths approaches $p(L) \sim L^{-3/2}$
- correlation length critical exponent $\nu=2$
- “order parameter” exponent $\beta=1$
- analytic derivations of these exponents



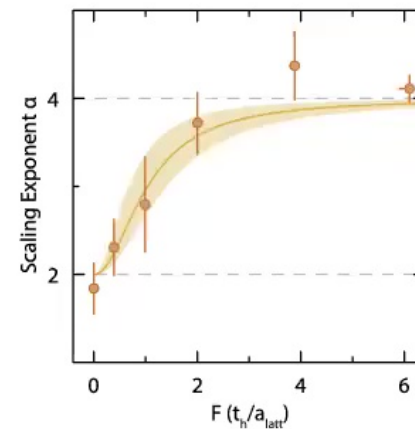
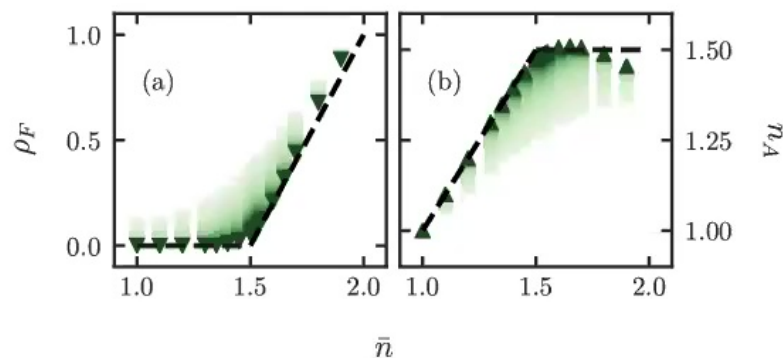
Summary of Part 2

- microscopic charge and dipole-moment conservation leads to universal subdiffusion in the active phase, and a freezing phase transition as a function of density
- simple understanding of this transition is possible through considering basic elements like isolated active bubbles and approximate models of their growth and interaction

I

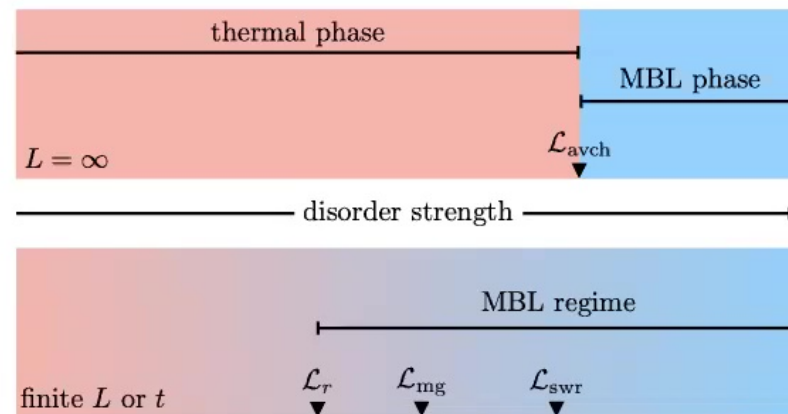
Summary

- emergent and microscopic conservation of higher moments of the charge distribution can lead to interesting dynamical crossovers, phases, and critical points



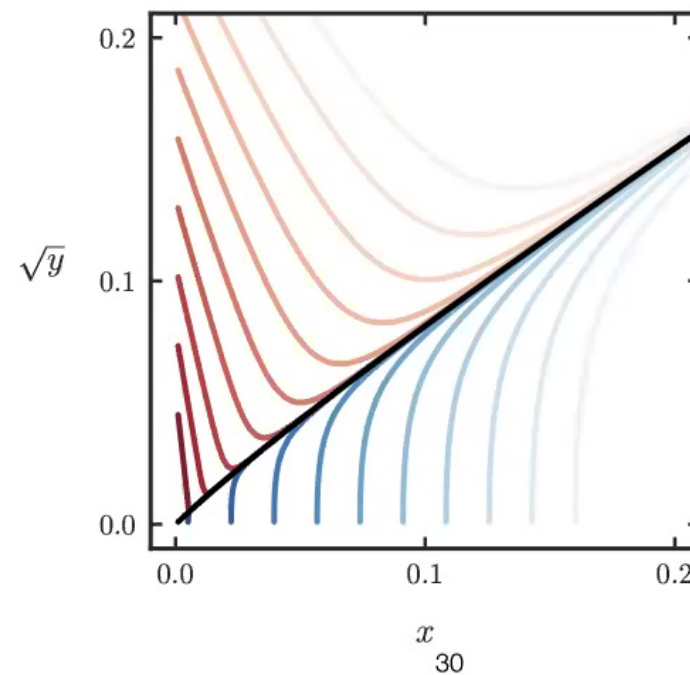
Other work

- “avalanches” and rare many-body resonances in small many-body localized systems



Other work

- renormalization group approaches for understanding the many-body localization phase transition



Other work

- repurposing Google's Tensor Processing Units for simulations of quantum many-body dynamics

