Title: A construction of exotic metal and metal-insulator transition

Speakers: Xiaochuan Wu

Series: Quantum Matter

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Abstract: The charge resistivity/conductivity can take universal values in various scenarios of two-dimensional condensed matter systems. Well-known examples of universal resistivity include 2+1d quantum critical points, (fractional) quantum Hall effects, the criterion of two-dimensional "bad metal", and the universal resistivity jump predicted at the interaction-driven metal-insulator transition. We construct examples of two-dimensional metallic states with the following exotic behaviors: (1) at low temperature this state is a "bad metal" whose resistivity can be much larger than the Mott-Ioffe-Regel limit; (2) while increasing temperature T the resistivity ?(T) crosses over from a bad metal at low T to a good metal at intermediate T; (3) at low temperature the metallic state has a large Lorenz number, which strongly violates the Wiedemann-Franz law; (4) the state also has a large thermopower (Seebeck coefficient). Motivated by the recent experiment in transition metal dichalcogenides, an exotic interaction-driven metal-insulator transition will also be constructed. The universal resistivity jump at this transition far exceeds what was proposed in previous theory.

Zoom Link: https://pitp.zoom.us/j/93625929658?pwd=Y284ZTZpWFM1RnduSmhXdDZBRjgyQT09

Construction of Exotic Metal and Metal-Insulator Transition

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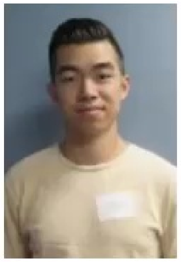
Collaborators

UCSB



Yichen Xu

Mengxing Ye

Zhu-Xi Luo

Cenke Xu

Cornell

Chao-Ming Jian

- **XW, M. Ye, Z. Luo, C. Xu, in preparation**
- **Y. Xu, XW, Z. Luo, M. Ye, C.-M. Jian, C. Xu, arXiv:2106.14910**

Quantum states of matter

Gapped quantum matter

Excited states

Ground states

Fully gapped states (bulk and boundary) have infinite resistivity

$\rho = \infty$ at $T = 0$

Gapless quantum matter

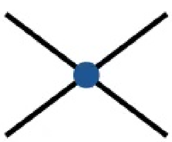
Excited states

Ground states

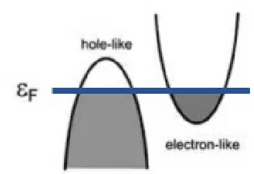
Gapless states can have finite resistivity

$\rho < \infty$ at $T = 0$

Quantum critical point (QCP)



Metal



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Universal values of electrical resistivity in $d = 2$

$$\rho \sim \frac{h}{e^2} a^{d-2} \quad \text{(universal number in } d = 2\text{)}$$

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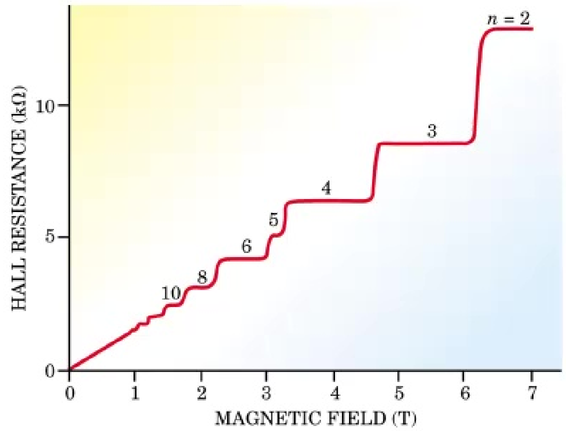


- **Quantum Hall effect**: precisely quantized plateaus $\rho_{xy} = \frac{1}{n}\frac{h}{e^2}, n \in \mathbb{Z}$.
- Different samples have different impurities, different geometries, etc.

Universal values of electrical resistivity in $d = 2$ (metals and QCPs)

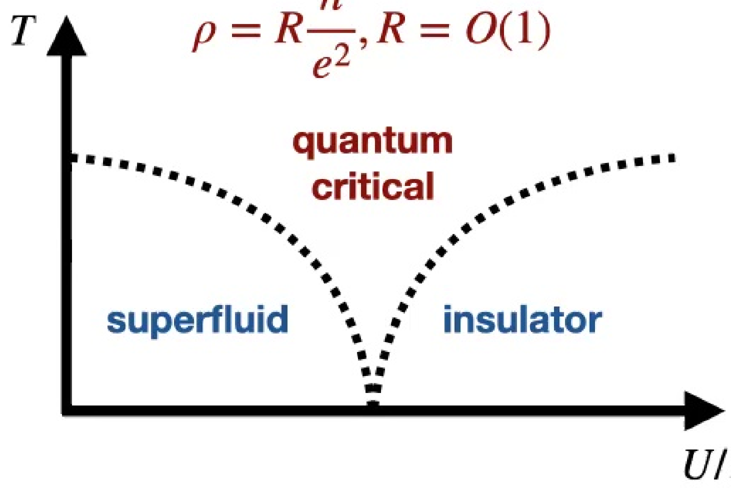
$$\rho \sim \frac{h}{e^2} a^{d-2} \quad \text{(universal number in } d = 2\text{)}$$

(1) **Upper bound of "good metal"** $\rho \leq \dfrac{h}{e^2}$:

to have well-defined wave packet for mobile electrons (Boltzmann transport is valid)

(2) **Universal resistivity at QCP**

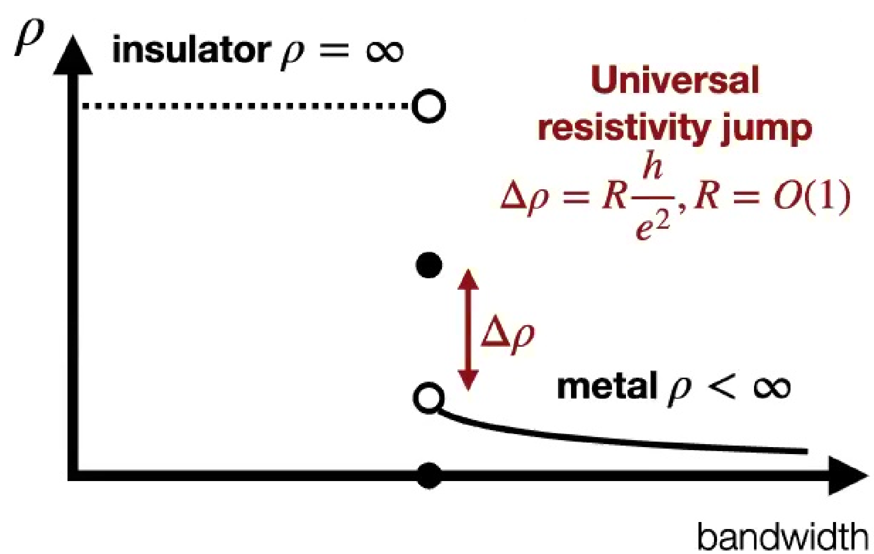
e.g., superfluid-insulator transition of charged bosons



Universal values of electrical resistivity in $d = 2$ (metals and QCPs)

$$\rho \sim \frac{h}{e^2} a^{d-2} \quad \text{(universal number in } d = 2\text{)}$$

Interaction-driven **continuous metal-insulator transition** at half-filling (spin-charge separation $c_{r,\alpha} = b_r f_{r,\alpha}$, $\alpha = \uparrow \downarrow$) in Lee-Lee, PRL (2005), Senthil, PRB (2008), etc.



Universal values of electrical resistivity in $d = 2$ (metals and QCPs)

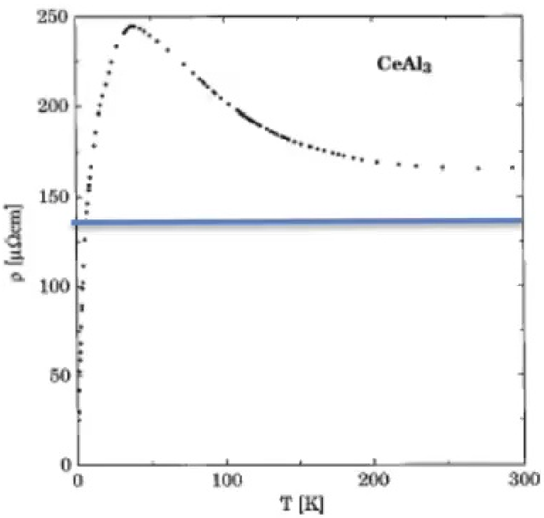
$$\rho \sim \frac{h}{e^2} a^{d-2} \quad \text{(universal number in } d = 2\text{)}$$

What about **metal** + **quantum critical point** ?

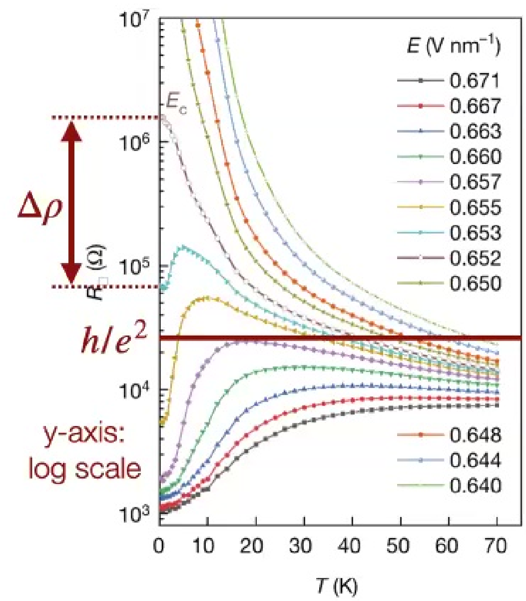
"Bad conductor" behaviors

Our expectations: "good metal" $\rho < h/e^2$ metal-insulator transition $\Delta\rho = Rh/e^2, R = O(1)$

But… "bad metal" $\rho > h/e^2$ big resistivity jump $\Delta\rho = Rh/e^2, R \gg 1$



Andres et al., PRL (1975)



(**TMD Moiré**) Shan, Mak et al. Nature (2021)

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Is electron interaction doing something dramatic?

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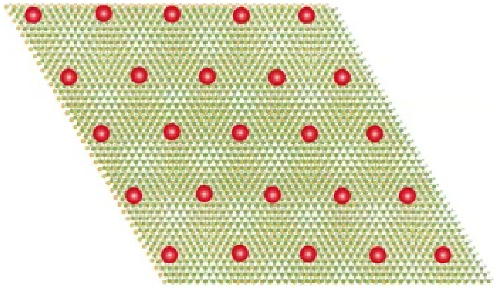
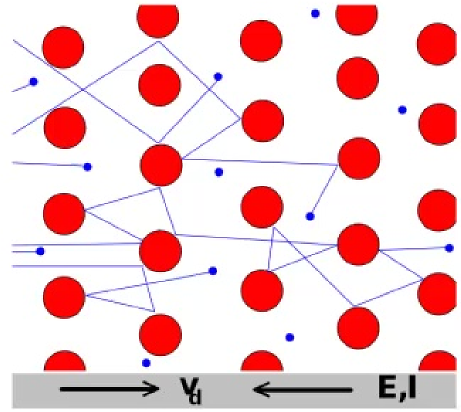
Content - Metal and MIT with Charge Fractionalization

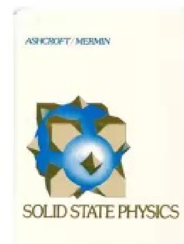
- **I. Bad metal** constructed by (1) nearly-free fermionic parton gas with Z_N gauge field; (2) parton non-fermi liquid with $SU(N)$ gauge field.
- **II. Big universal resistivity jump** at interaction-driven continuous **metal-insulator transition (MIT)** at half-filling, and possible connections to the recent experiment in transition metal dichalcogenides Moiré superlattice.



Background: criterion of "bad metal" in $d = 2$

- **Quasiparticle transport** (simplest level): Drude formula of metal





- $$\rho = \frac{m}{ne^2\tau} = \frac{h}{e^2}\frac{1}{k_F l_m}$$

- To have well-defined wave packet for mobile electrons (Boltzmann transport is valid), we need $k_F l_m \geq 1$.

- The upper bound for "good metal" (Mott-Ioffe-Regel limit): $\rho = \frac{h}{e^2}\frac{1}{k_F l_m} \leq \frac{h}{e^2}$.

- Beyond Mott-Ioffe-Regel limit, the theoretical understanding is challenging.

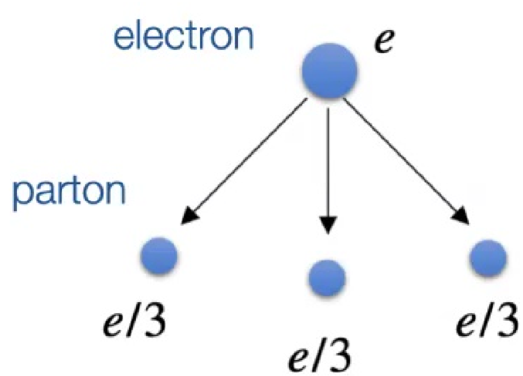
Emery-Kivelson PRL (1995)

XW, M. Ye, Z. Luo, C. Xu, in preparation

A simple construction: fermionic parton gas with $e_* = e/N$

- Each parton is a spin-$1/2$ fermion with an electric charge $e_* = e/N$, and a Z_N gauge charge (N is odd).

- **Electron Hilbert space**: the gauge-invariant states are fermions (with odd e and half-integer spin) and bosons (with even e and integer spin).



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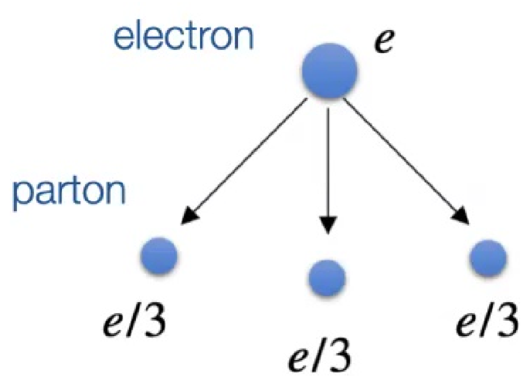

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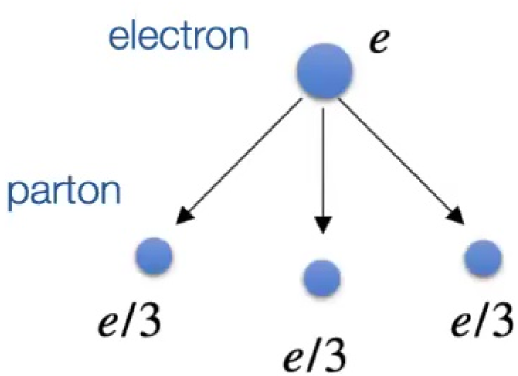
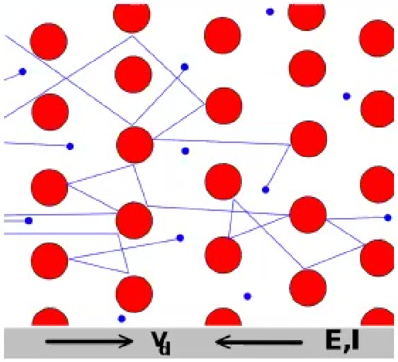


Parton transport (simplest level)

Drude formula $\rho = \frac{h}{e_*^2} \frac{1}{k_F^* l_m}$

Impurity scattering: hard-sphere like potential (insensitive to T)



This is the simplest toy model. In TMD Moiré, there is a natural reason for charge fractionalization (anther construction later).

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A simple construction: fermionic parton gas with $e_* = e/N$

- Z_N gauge field is gapped
- Partons are nearly free at low T
- Parton wave packet is well-defined when

$$k_F^* l_m \geq 1 \;\Rightarrow\; \rho \leq \frac{h}{e_*^2} = N^2 \frac{h}{e^2}$$

- The "good metal of partons" (with semiclassical quasiparticle picture) can be a very bad metal in the conventional sense.

Parton transport (simplest level)

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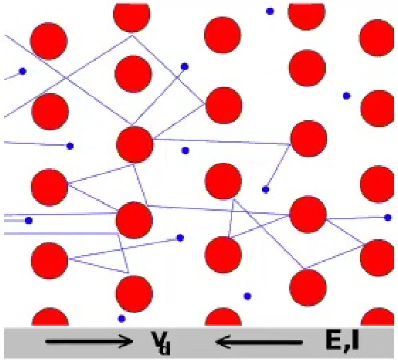

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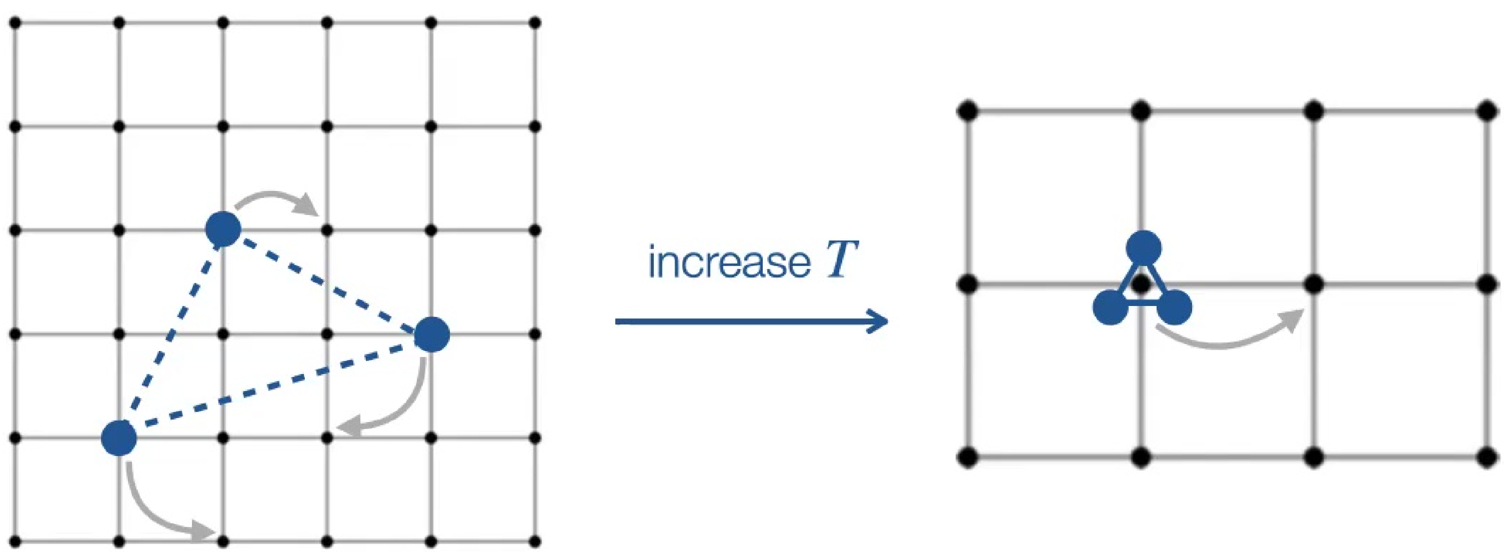
Finite-T confinement of Z_N gauge field

Thermally activated Z_N gauge fluxes $\Rightarrow$ confinement.
The confinement length is $\xi_c^2 \sim e^{\Delta/T}$, where Δ is the gap of Z_N gauge flux.

deconfinement-confinement crossover

when $\xi_c \sim a$, fully confined $\rightarrow$ electron



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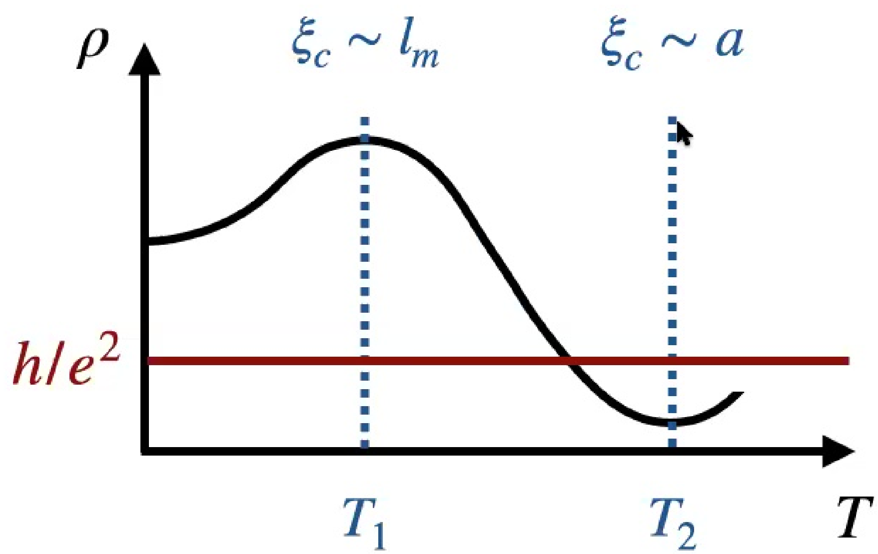
(Expected) T-dependence of electrical resistivity

Three length scales: confinement length $\xi_c(T)$, scattering mean free path l_m, lattice constant a.

(Assuming the impurity satisfies $1 < k_F l_m < N^2$ such that electrons have semiclassical picture.)



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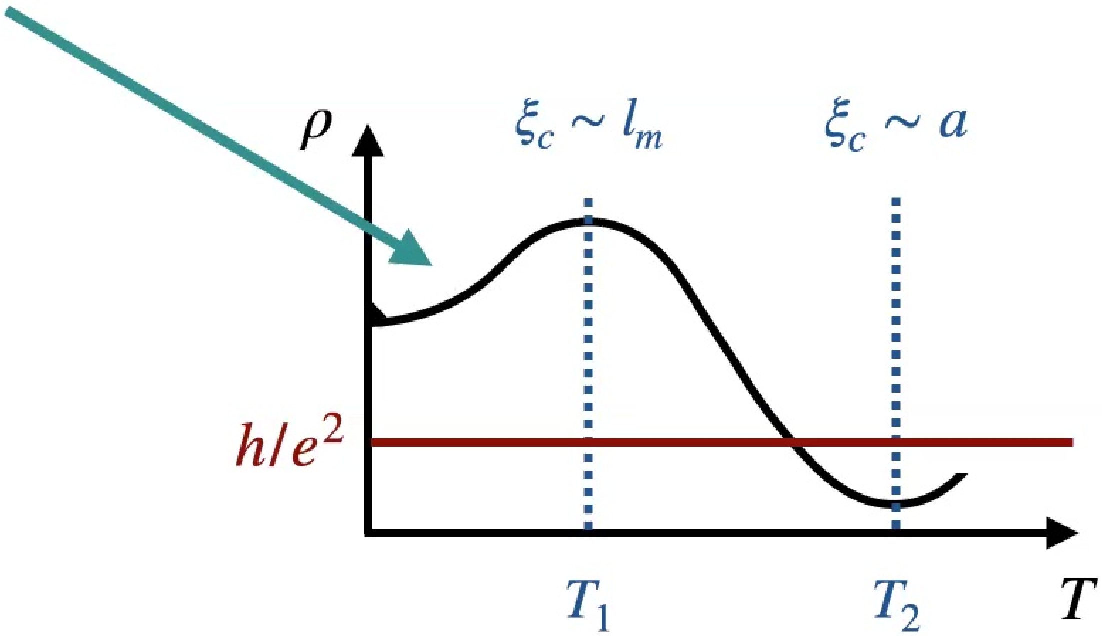
Parton fermi-liquid behavior $\sim T^2$, parton-parton interaction is suppressed by $e_*^2 \sim e^2/N^2$



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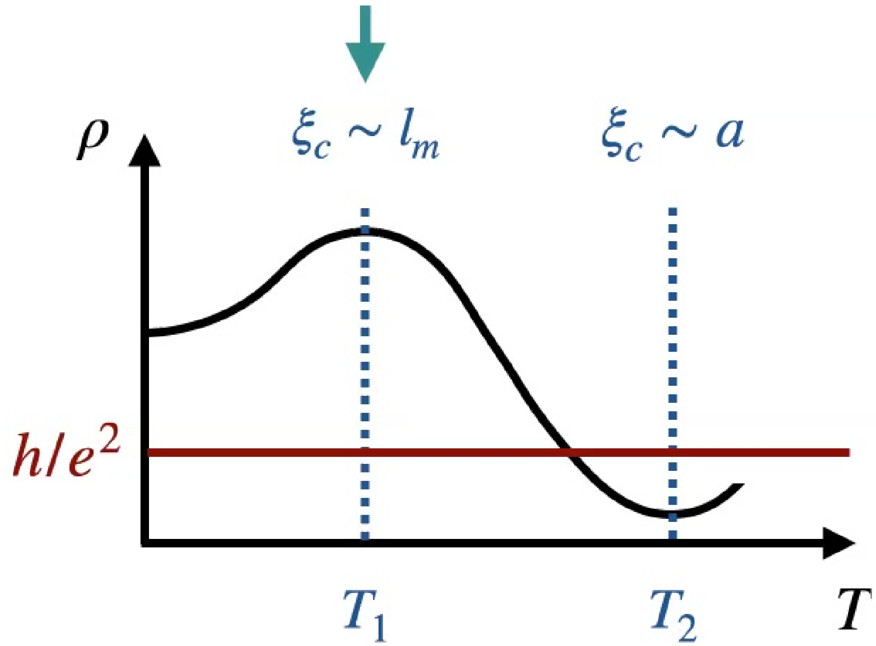
The semiclassical picture of partons starts to break down



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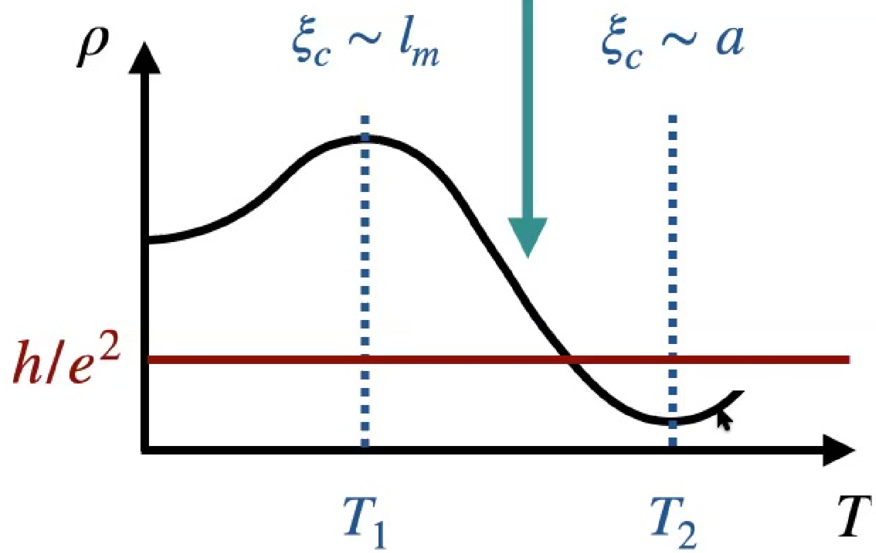
The deconfinement-to-confinement crossover $\Rightarrow$ bad metal to good metal crossover



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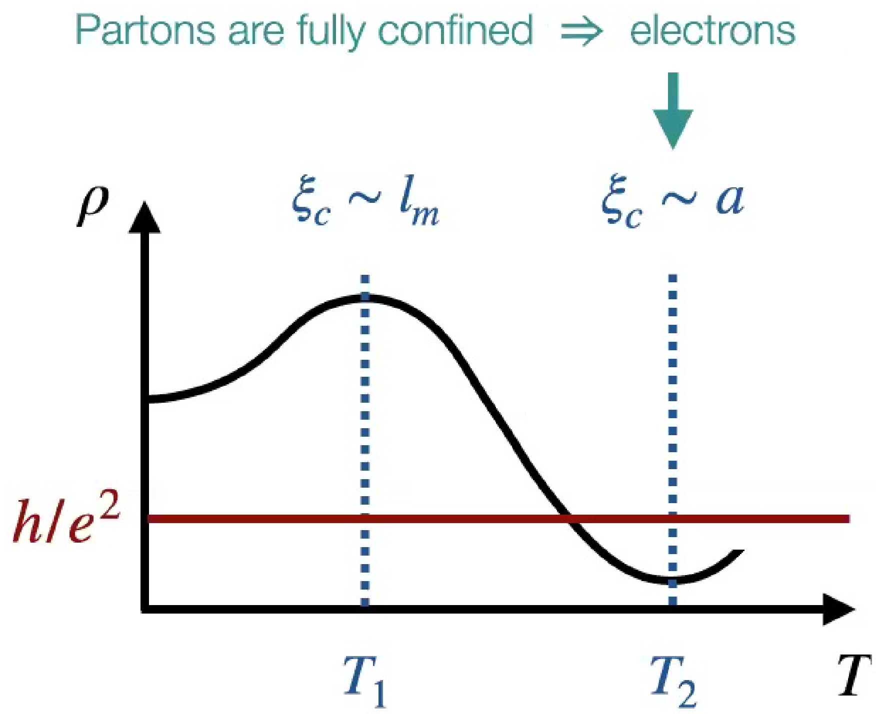

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(Expected) T-dependence of electrical resistivity

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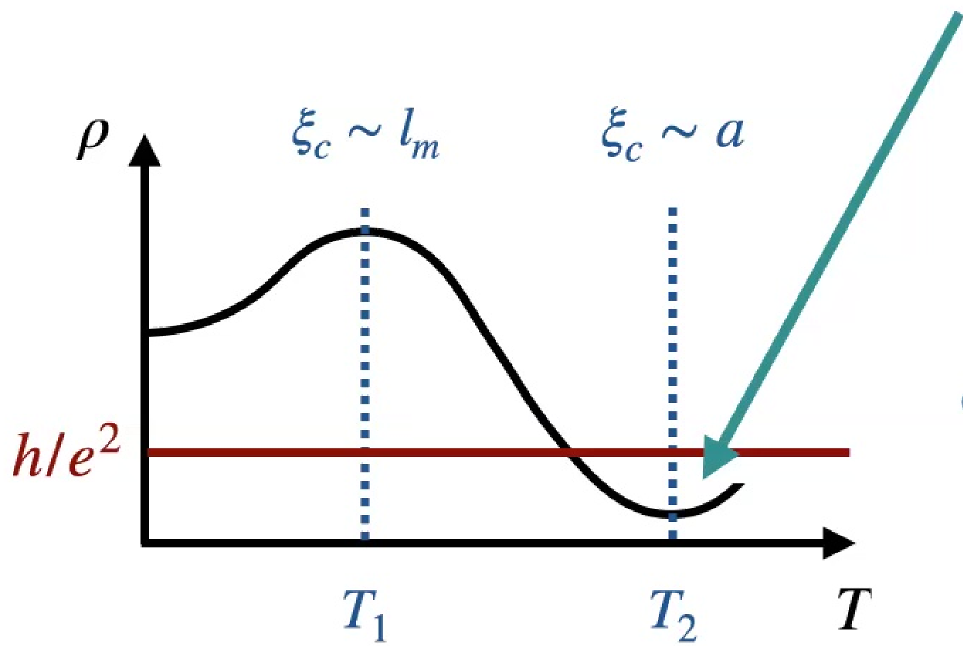
Electron fermi-liquid behavior $\sim T^2$



(We are still looking for experimental realizations for this simple theory.)

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Thermoelectric properties of fermionic partons with $e_* = e/N$

- **Thermoelectric transport** $$\begin{pmatrix} \boldsymbol{J}^e \\ \boldsymbol{J}^h \end{pmatrix} = \begin{pmatrix} \sigma & \alpha \\ T\alpha & \bar{\kappa} \end{pmatrix} \begin{pmatrix} \boldsymbol{E} \\ -\nabla T \end{pmatrix}$$

- Low-T semiclassical dynamics (parton band $\epsilon_n(\boldsymbol{k})$ and Berry curvature $\boldsymbol{\Omega}_n(\boldsymbol{k})$)

- $$\dot{\boldsymbol{x}} \equiv \boldsymbol{v}_n(\boldsymbol{k}) = \frac{1}{\hbar}\frac{\partial \epsilon_n(\boldsymbol{k})}{\partial \boldsymbol{k}} - \dot{\boldsymbol{k}} \times \boldsymbol{\Omega}_n(\boldsymbol{k}), \quad \hbar\dot{\boldsymbol{k}} = -\frac{e}{N}\boldsymbol{E}(\boldsymbol{x}) - \frac{e}{N}\dot{\boldsymbol{x}} \times \boldsymbol{B}(\boldsymbol{x}) .$$

- Semiclassical Boltzmann transport equation (relaxation-time approximation)

- $$\frac{\partial g}{\partial t} + \dot{\boldsymbol{x}} \cdot \frac{\partial g}{\partial \boldsymbol{x}} + \dot{\boldsymbol{k}} \cdot \frac{\partial g}{\partial \boldsymbol{k}} = \left(\frac{\partial g}{\partial t}\right)_{\text{coll}} \approx -\frac{\delta g}{\tau} .$$

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Thermoelectric properties of fermionic partons with $e_* = e/N$

- Thermopower (Seebeck coefficient) $Q(T_1) = \frac{\alpha_{xx}}{\sigma_{xx}} = -N\frac{\pi^2}{3}\frac{k_B^2 T}{e}\frac{\sigma}{\sigma'}.$
- Lorenz number (Wiedemann-Franz law) $L(T_1) = \frac{\kappa_{xx}}{T\sigma_{xx}} = N^2\frac{\pi^2}{3}\frac{k_B^2}{e^2}.$
- The deconfinement-to-confinement crossover: $\frac{Q(T_2)}{Q(T_1)} \sim \frac{1}{N}, \quad \frac{L(T_2)}{L(T_1)} \sim \frac{1}{N^2}.$
- **The electrical transport is suppressed, while the thermal transport is enhanced.**
- The ratios are independent of the scattering mean free path l_m (good metal or bad metal).

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- **I. Bad metal** constructed by (1) nearly-free fermionic parton gas with Z_N gauge field; (2) parton non-fermi liquid with $SU(N)$ gauge field.
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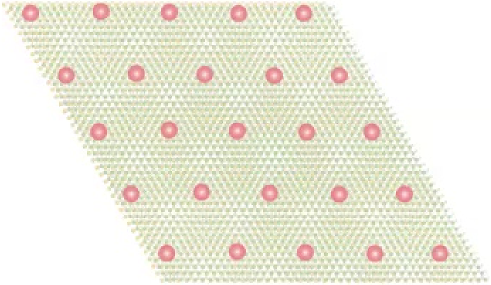

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Parton fermi surface + $SU(N)$ gauge field

- Electron $c = \psi_1\psi_2\ldots\psi_N = (1/N!)\varepsilon_{IJ\ldots K}\psi_I\psi_J\ldots\psi_K$ by N (= odd) fermionic partons ψ
- Electron c is invariant under $\psi_I \to W_{IJ}\psi_J, W \in SU(N)$ under fundamental representation
- $SU(N)$ gauge redundancy $\Rightarrow$ emergent dynamical $SU(N)$ gauge field
- $SU(N)$ can be Higgsed down to Z_N (the previous example)

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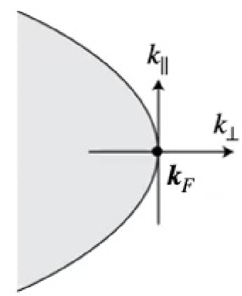
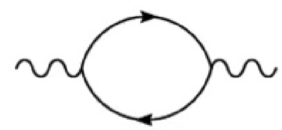
- Electron c is invariant under $\psi_I \to W_{IJ}\psi_J$, $W \in SU(N)$ under fundamental representation
- $SU(N)$ gauge redundancy $\Rightarrow$ emergent dynamical $SU(N)$ gauge field
- **Fermi-surface (FS) state is strongly renormalized by gapless gauge fluctuations** (e.g., **FS + U(1)**: Nayak-Wilczek 1994, Lee 2008, 2009, Mross, et.al. 2010, Metlitski-Sachdev 2010, etc)
- **Generalization to FS + SU(*N*)**: gluon self-interactions, gluon-ghost interactions, fermion-gluon interactions (other than $\psi^\dagger a_\perp \psi$, transverse gauge field $a_\perp$) are all irrelevant under RG (similar to **FS + U(2)** in Zou-Chowdhury 2020)
- Fermion dynamical exponent $z_\psi = 1 + \epsilon/2 \Rightarrow$ **well-defined FS without quasiparticles**

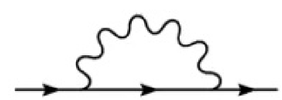
Background: fermi surface + $U(1)$ gauge field

FS + U(1): Nayak-Wilczek 1994, Lee 2009, Mross, et.al. 2010, Metlitski-Sachdev 2010
FS + U(2): Zou-Chowdhury 2020

- The Landau-damped gauge field $a_\perp$
- $$\mathscr{L}[a_\perp] \sim \frac{1}{2g^2}\left(\gamma\frac{|\omega|}{|k_\parallel|} + |k_\parallel|^{z-1}\right)|a_\perp(\omega,\boldsymbol{k})|^2$$
- Perturbative RG flow using ϵ expansion ($\epsilon = z - 2$)
- $\Rightarrow$ fermion dynamical exponent $\mathtt{i}\omega \to \mathtt{i}\mathrm{sgn}(\omega)|\omega|^{\frac{2}{2+\epsilon}}$
- $\Rightarrow$ well-defined fermi surface without quasiparticles







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Generalization: parton fermi surface + $SU(N)$ gauge field

- FS + $SU(N)$: gluon self-interactions, gluon-ghost interactions, fermion-gluon interactions (other than $\psi^\dagger a_\perp \psi$) are all irrelevant.

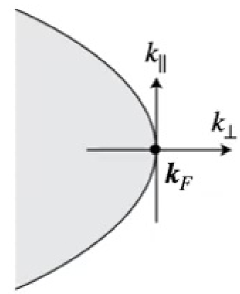
  similar to **FS + U(2)** in Zou-Chowdhury 2020

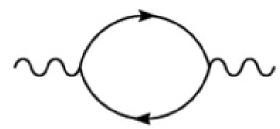
- $G = SU(N)$ looks "abelianized" ($dimG = N^2 - 1$ copies):

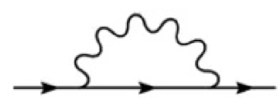
- $$\mathscr{L}[a_\perp] \sim \frac{1}{2g^2}\left(\gamma \frac{|\omega|}{|k_\parallel|} + |k_\parallel|^{z-1}\right) \sum_{a=1}^{dimG} |a_\perp^a(\omega, k)|^2$$

- ϵ-expansion ($\epsilon = z - 2$) $\Rightarrow$ **parton non-fermi liquid**

- (pairing temperature $< T \ll$ confinement temperature)







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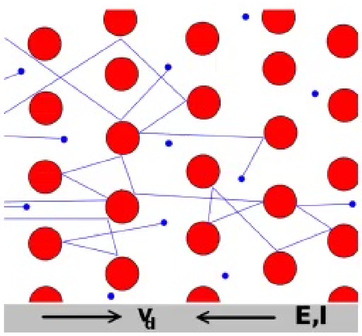
Transport properties of parton fermi surface + $SU(N)$ gauge field

	Electrical resistivity ρ	Thermal conductivity κ
Partons (deconfinement)	$\rho(T) \sim N\rho_{imp,0} + (N^2-1)T^{4/3}$	$T\kappa(T)^{-1} \sim \frac{1}{N}W_{imp,0} + \frac{N^2-1}{N^2}T^{2/3}$
Electrons (confinement)	$\rho(T_2) \sim \rho_{imp,0}$	$T\kappa(T_2)^{-1} \sim W_{imp,0}$

Impurity scattering: hard-sphere like potential (insensitive to T)

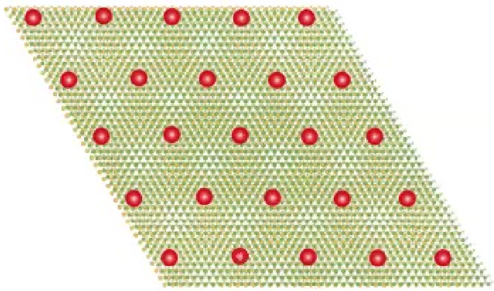


- Deconfinement (T_1) to confinement (T_2) crossover (leading-term from impurity scattering):
- Thermopower (Seebeck coefficient) $\dfrac{Q(T_2)}{Q(T_1)} \sim \dfrac{1}{N}$,
- Lorenz number (Wiedemann-Franz) $\dfrac{L(T_2)}{L(T_1)} \sim \dfrac{1}{N^2}$.

Content - Metal and MIT with Charge Fractionalization

- **I. Bad metal** constructed by (1) nearly-free fermionic parton gas with Z_N gauge field; (2) parton non-fermi liquid with $SU(N)$ gauge field.

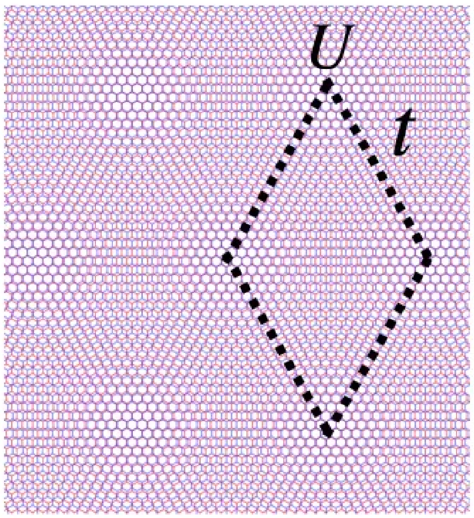
- **II. Big universal resistivity jump** at interaction-driven continuous **metal-insulator transition (MIT)** at half-filling, and possible connections to the recent experiment in transition metal dichalcogenides Moiré superlattice.



Shan, Mak et al. Nature (2021)

Motivations from big resistivity jump in $MoTe_2/WSe_2$

$MoTe_2/WSe_2$ bilayers (0-degree)



7% lattice mismatch

⇓

Moiré superlattice

- TMD Moiré systems: t ~ 1-10 meV $\ll$ U ~ 50-100 meV.
- Topologically trivial bands ⇒ No Wannier obstruction
- $$H = \sum_{r,r',\alpha} (-t_{r,r'} c^{\dagger}_{r,\alpha} c_{r',\alpha} + \text{h.c.}) + \sum_{r} U n_{r,\uparrow} n_{r,\downarrow} + \dots$$
- Spin-orbital coupling ⇒ spin-valley locking ($\alpha = \uparrow \downarrow$)
- In experiment, tunable bandwidth at half-filling ⇒ (interaction driven?) continuous metal-insulator transition
- Half-band filling density (two orders) $\gg$ disorder density

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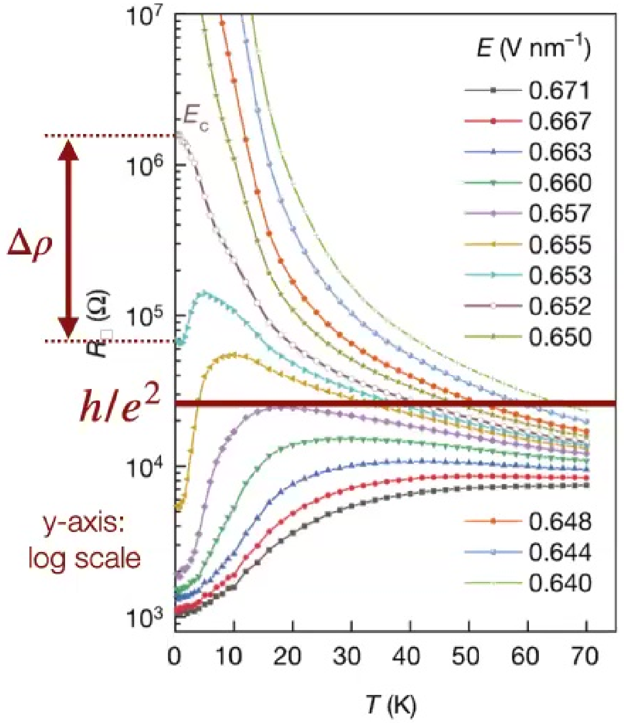
Motivations from big resistivity jump in $MoTe_2/WSe_2$

Metal-insulator transition at half-filling:



Shan, Mak et al. Nature (2021)

If interaction driven continuous transition

$\Delta\rho$ is much larger than the known universal value within the current theoretical understanding.

Universal resistivity jump at interaction-driven metal-insulator transition

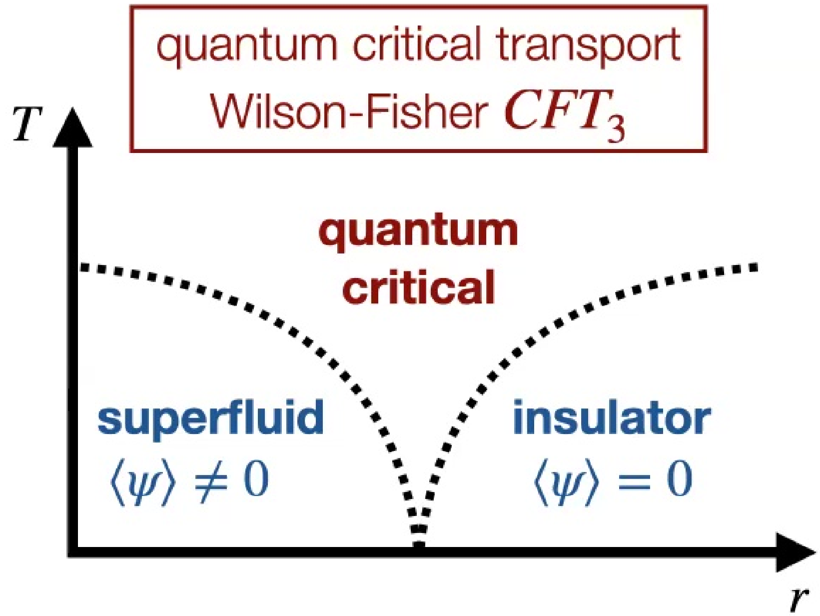
Universal resistivity at superfluid-insulator transition of charged bosons

Try to understand big resistivity jump in transition metal dichalcogenides

Background: universal conductivity at superfluid-insulator transition

Landau symmetry-breaking transition in $2+1d$

$$S = \int d\tau d^2\boldsymbol{x}\, |\partial_\tau \psi|^2 + |\nabla \psi|^2 + r|\psi|^2 + u|\psi|^4 + \dots$$

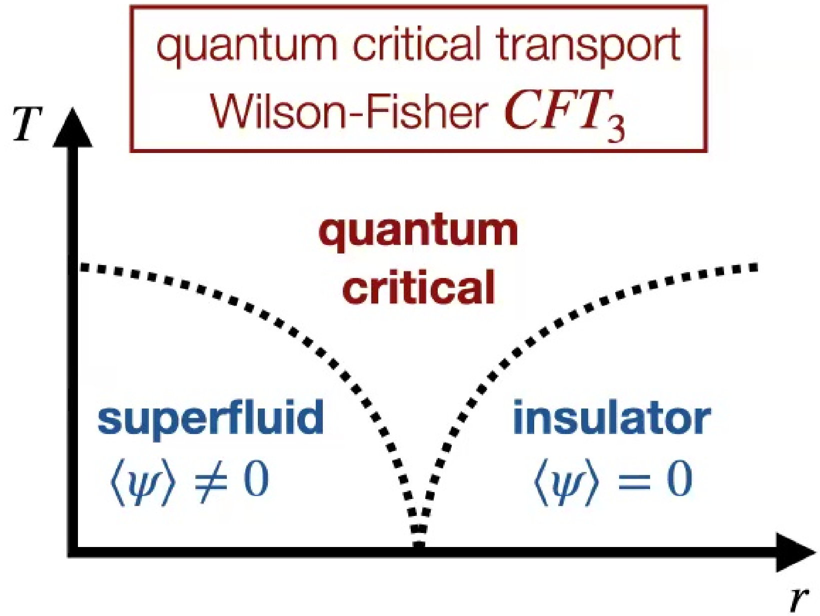


Haviland et al. PRL 62, 2180 (1989)
Fisher et al. PRL 64, 587 (1990)
Cha et al. PRB 44, 6883 (1991)
Fazio-Zappalà PRB(R) 8883 (1996)
Damle-Sachdev PRB 56, 8714 (1997)
Šmakov-Sørensen PRL 95, 180603 (2005)
Witczak-Krempa et al. PRB 86, 245102 (2012)
Chen et al. PRL 112, 030402 (2014)
Chester et al. JHEP 2020, 142 (2020)

Background: universal conductivity at superfluid-insulator transition

Universal conductivity: $\Sigma(\tilde{\omega})$ is a dimensionless universal scaling function in $2+1d$

$$\sigma(\omega/T) = \frac{e^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right)$$



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Ordinary transport: finite conductivity always needs impurity or Umklapp scattering (for momentum relaxation)

Quantum critical transport (with particle-hole symmetry): conductivity is finite **without disorder and Umklapp**

	Particles	Holes
Momentum	→	←
Electrical current	→	→

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Background: universal conductivity at superfluid-insulator transition

Universal conductivity: $\Sigma(\tilde{\omega})$ is a dimensionless universal scaling function in $2+1d$

$$\sigma(\omega/T) = \frac{e^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right)$$

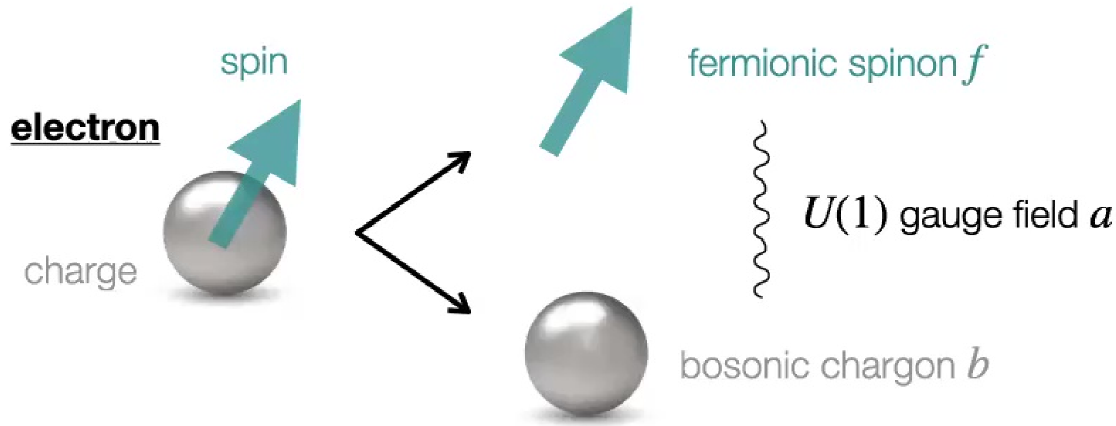
$\Sigma(0)$	$\Sigma(\infty)$		$\Sigma(0)$	$\Sigma(\infty)$	
≈ 1		experiment in PRL 62, 2180 (1989)	1.068		large-N in PRB 86, 245102 (2012)
	0.315	ϵ-expansion in PRB 8883 (1996)		0.359(4)	Monte Carlo in PRL 112, 030402 (2014)
1.037	0.3927	ϵ-expansion in PRB 56, 8714 (1997)		0.355155(11)	conformal bootstrap in JHEP 2020, 142 (2020)

- Maybe we can trust $\Sigma(\infty) \approx 0.36$ (conformal bootstrap) and $\Sigma(0) \approx 1$ (experiment).
- Universal resistivity $\rho = \sigma^{-1}$ (two values the same order): $\rho(\infty) \approx 3\rho(0) \approx 3\frac{h}{e^2}$.

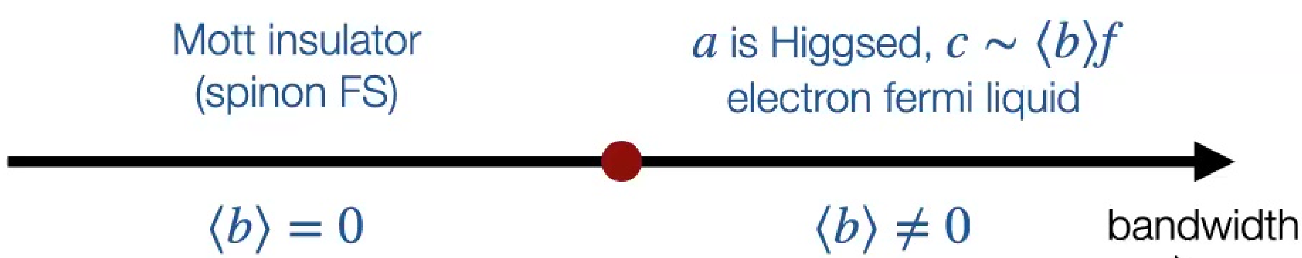
Lee-Lee, PRL (2005), Senthil, PRB (2008), etc

Background: universal resistivity jump at metal-insulator transition

Interaction-driven transition at half-filling



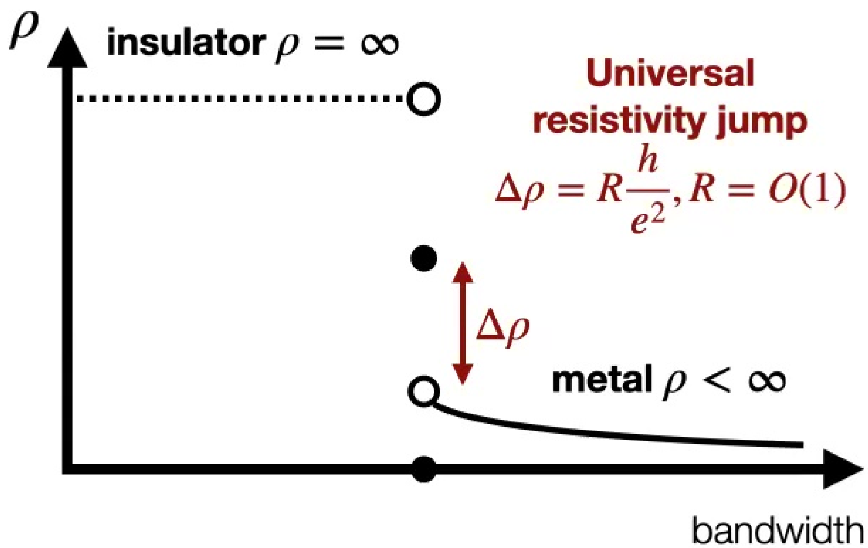
b (dynamically decoupled from f, a): **superfluid-insulator transition**



Background: universal resistivity jump at metal-insulator transition

- Ioffe-Larkin rule $\rho = \rho_f + \rho_b$; Insulator $\rho_b = \infty \Rightarrow \rho = \infty$; Metal $\rho_b = 0 \Rightarrow \rho = \rho_f$.

- Critical point: $\rho_b = R\frac{h}{e^2} \;\Rightarrow\; \rho = \rho_f + R\frac{h}{e^2}$, where R is of the order $1 < R < 10$.



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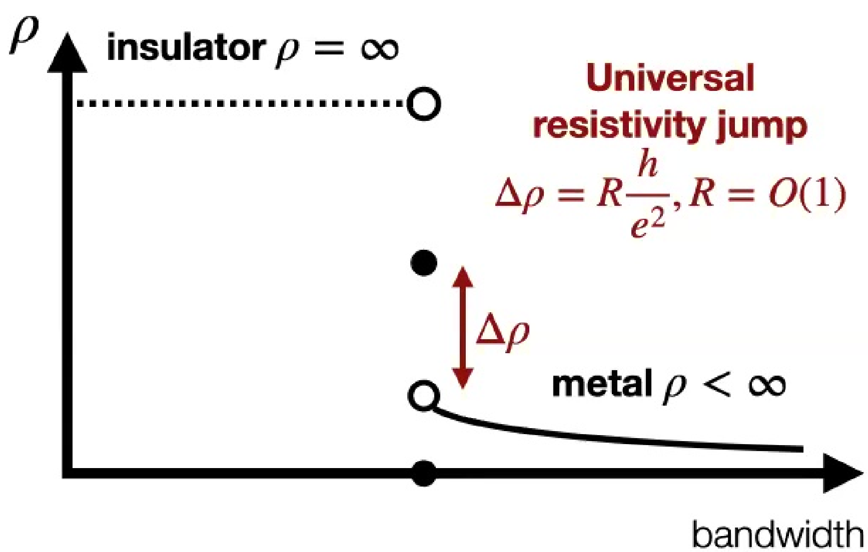


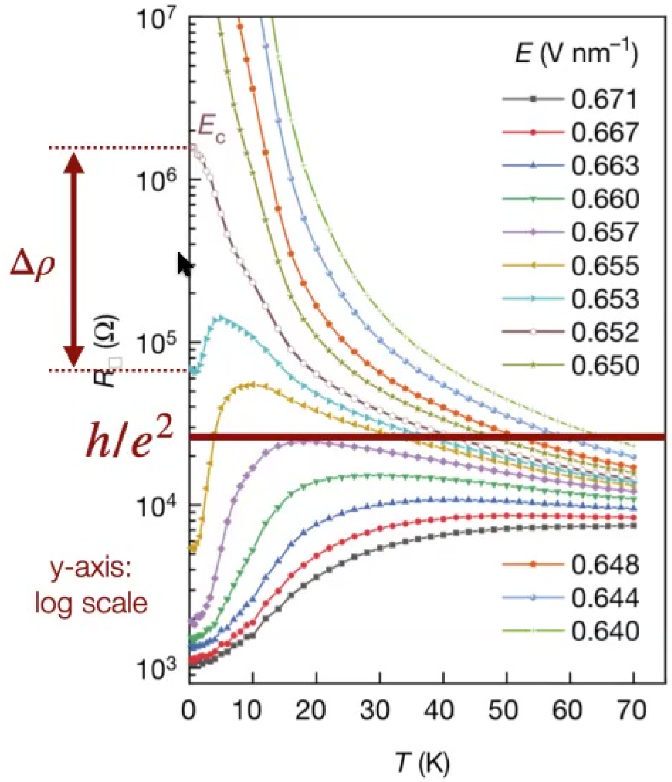
Universal resistivity jump $\rho(\omega/T)$ (large-N, MC results in Witczak-Krempa et al. PRB (2012)):

$$\Delta\rho(\infty) = 3.51\frac{h}{e^2} \text{ (Wilson-Fisher CFT)}$$

$$\Delta\rho(0) = 7.93\frac{h}{e^2} \text{ (WF CFT + damped gauge)}$$

$\Delta\rho$ is NOT significantly larger than $\frac{h}{e^2}$

Motivations from big resistivity jump in $MoTe_2/WSe_2$



Shan, Mak et al. Nature (2021)

$\Delta\rho$ is much larger than the known universal value

$MoTe_2/WSe_2$ seems beyond the "convention construction" of interaction-driven continuous metal insulator transition

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Y. Xu, XW, Z. Luo, M. Ye, C.-M. Jian, C. Xu, arXiv:2106.14910

Metal-insulator transition (half-filling) with large resistivity jump

- "Standard construction" in Senthil 2008
- $c_{\boldsymbol{r},\alpha} = b_{\boldsymbol{r}} f_{\boldsymbol{r},\alpha},\ \alpha = \uparrow\downarrow$
- One emergent $U(1)$ gauge field a
- f: spinon fermi surface
- b: superfluid-insulator transition

- New construction: $c_{\boldsymbol{r},\alpha} = b_{\alpha,\boldsymbol{r}} f_{\boldsymbol{r},\alpha},\ \alpha = \uparrow\downarrow$
- (time-reversal symmetry, no $SU(2)$ rotation)
- Two emergent $U(1)$ gauge fields $a_\uparrow, a_\downarrow$
- f: spinon fermi surface
- $b_\uparrow, b_\downarrow$: superfluid-insulator transitions simultaneously (time-reversal symmetry)
- **$\Rightarrow$ charge fractionalization $\Rightarrow$ Large resistivity jump at critical point**

Y. Xu, XW, Z. Luo, M. Ye, C.-M. Jian, C. Xu, arXiv:2106.14910

Metal-insulator transition (half-filling) with large resistivity jump

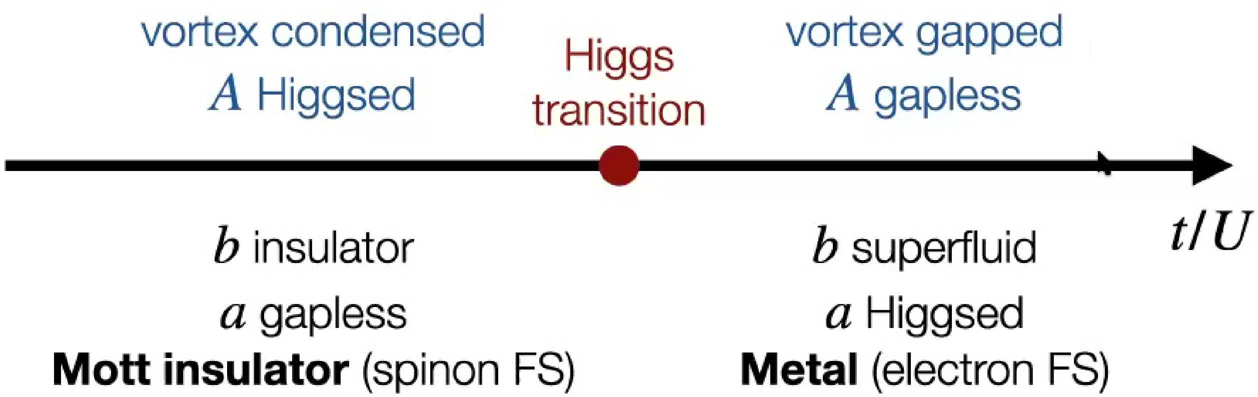
- "Standard construction" in Senthil 2008
- $c_{\boldsymbol{r},\alpha} = b_{\boldsymbol{r}} f_{\boldsymbol{r},\alpha}$, $\alpha = \uparrow\downarrow$
- Electron c at half-filling
- $\Rightarrow$ b at integer filling
- the Mott insulator of b trivially gapped

- New construction: $c_{\boldsymbol{r},\alpha} = b_{\alpha,\boldsymbol{r}} f_{\boldsymbol{r},\alpha}$, $\alpha = \uparrow\downarrow$
- Electron c at half-filling
- $\Rightarrow$ $b_\uparrow$ at half-filling, and $b_\downarrow$ at half-filling
- Lieb-Schultz-Mattis (LSM) theorem $\Rightarrow$ the Mott insulator of $b_\uparrow, b_\downarrow$ can NOT be trivially gapped.
- (1) topological order;
- (2) commensurate density wave (spontaneously breaks translation symmetry).

Y. Xu, XW, Z. Luo, M. Ye, C.-M. Jian, C. Xu, arXiv:2106.14910

Critical theory of $b_\uparrow$ (or $b_\downarrow$) at fractional filling: dual vortex theory

Dual theory: vortex of b + dynamical $U(1)$ gauge field A (dual to Goldstone of superfluid of b)



- Case 1: the condensation of N-vortex (bound state) at $\boldsymbol{k} = 0$ gives Z_N topological order.
- Case 2: the condensation of vortex at finite momentum $\boldsymbol{k} \neq 0$ breaks translation symmetry.

Case 2 is similar to the superfluid-insulator transition in Burkov-Balents-PRB 2005.

Y. Xu, XW, Z. Luo, M. Ye, C.-M. Jian, C. Xu, arXiv:2106.14910

Case 1: Z_N topological order

- The critical theory of N-vortex (bound state) condensation for $b_\uparrow$ (or $b_\downarrow$)
- $\mathscr{L} = |(\partial_\mu - \mathrm{i}NA_\mu)\psi|^2 + r|\psi|^2 + u|\psi|^4 + \frac{\mathrm{i}}{2\pi}A \wedge d(a + eA_{ext}) + \ldots$
- The chargon sector (3D XY* universality) is dynamically decoupled from the spinon fermi-surface.

Y. Xu, XW, Z. Luo, M. Ye, C.-M. Jian, C. Xu, arXiv:2106.14910

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- The chargon sector (3D XY* universality) is dynamically decoupled from the spinon fermi-surface.
- **Charge fractionalization** (both critical point and Mott insulator): the charge carrier is the anyon $\tilde{\psi}$ of the Z_N topological order, with $e_* = e/N$. We find $\tilde{\psi}$ has universal DC resistivity $\tilde{\rho} = 7.93h/e_*^2$. The total species of $\tilde{\psi}$ is 2 ($b_\uparrow$ and $b_\downarrow$).
- **Large universal resistivity jump** $\Delta\rho = \frac{1}{2}\tilde{\rho} = 3.96N^2\frac{h}{e^2} \sim N^2\frac{h}{e^2}$.

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Y. Xu, XW, Z. Luo, M. Ye, C.-M. Jian, C. Xu, arXiv:2106.14910

Case 2: commensurate density wave

- The vortex band structure: N minima in Brillouin zone ~ $\sum_{I=1}^{N} \psi_I e^{\mathrm{i}\boldsymbol{Q}_I \cdot \boldsymbol{r}}$, where low-energy fields ψ_I
- $\mathscr{L} = \sum_{I=1}^{N} (|(\partial_\mu - \mathrm{i}A_\mu)\psi_I|^2 + r|\psi_I|^2) + u(\sum_{I=1}^{N} |\psi_I|^2)^2 + \frac{\mathrm{i}}{2\pi} A \wedge d(a + eA_{ext}) + \ldots$

Y. Xu, XW, Z. Luo, M. Ye, C.-M. Jian, C. Xu, arXiv:2106.14910

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- **Charge fractionalization** at critical point (Landau-forbidden transition in chargon sector): the charge carrier is the vortex $\tilde{\psi}_I$ of each ψ_I with $e_* = e/N$. Each $\tilde{\psi}_I$ has universal DC resistivity $\tilde{\rho} = \tilde{R}h/e_*^2$, where $\tilde{R} > 7.93$ does not scale with N. The total species of $\tilde{\psi}$ is $2N$ ($b_\uparrow$ and $b_\downarrow$).
- **Large universal resistivity jump** $\Delta\rho = \frac{1}{2N}\tilde{\rho} \sim N\frac{h}{e^2}$.

Y. Xu, XW, Z. Luo, M. Ye, C.-M. Jian, C. Xu, arXiv:2106.14910

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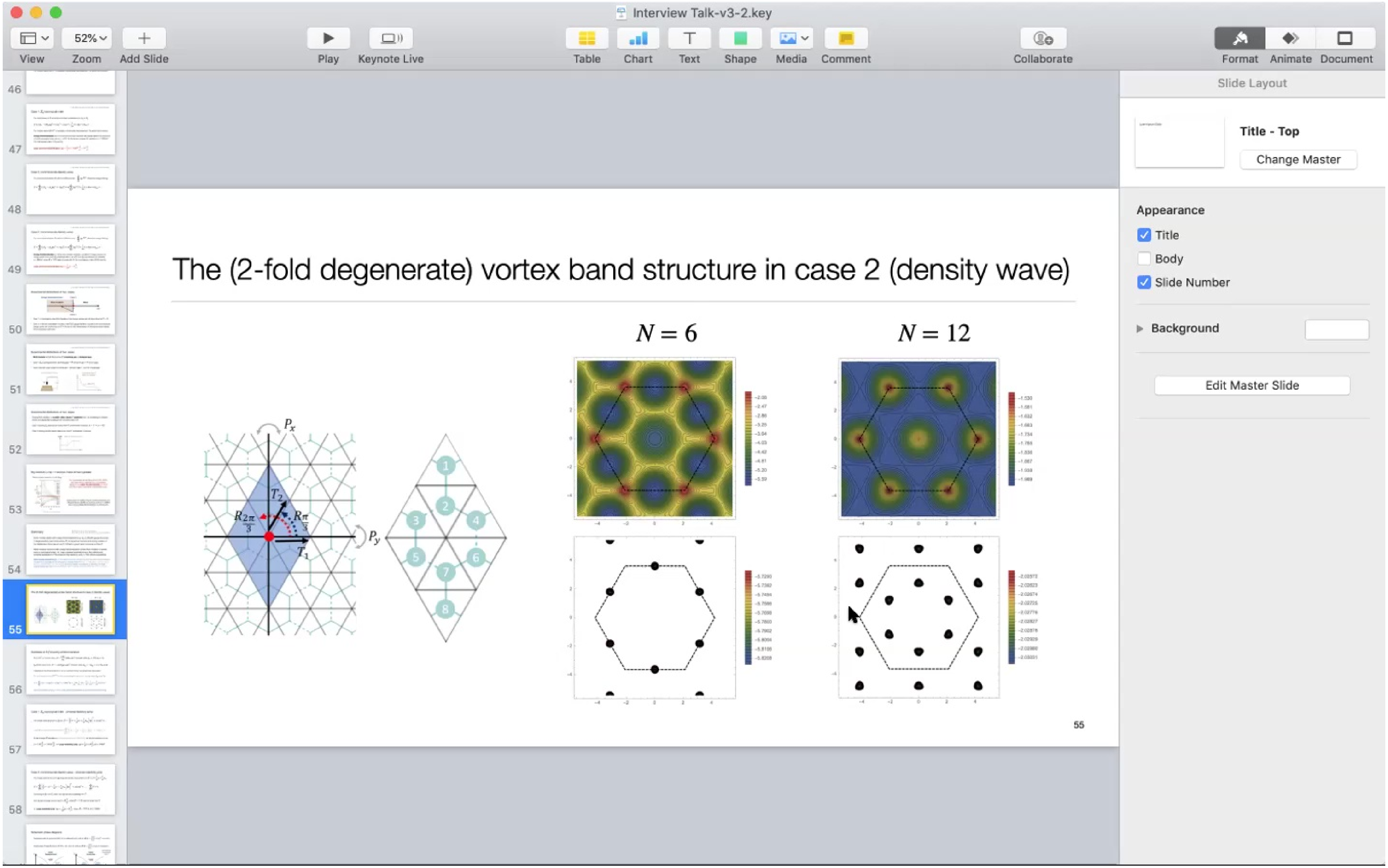
- **Large universal resistivity jump** $\Delta\rho = \frac{1}{2N}\tilde{\rho} \sim N\frac{h}{e^2}$.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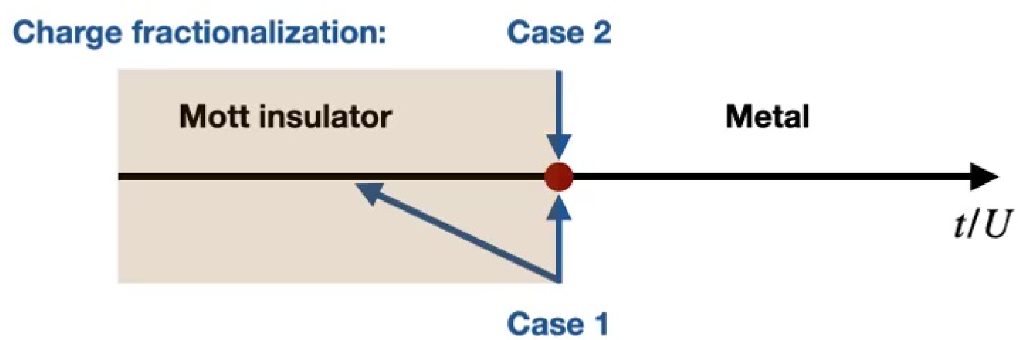
Slide Layout
Title - Top
Change Master
Appearance
Title
Body
Slide Number
Background
Edit Master Slide
The (2-fold degenerate) vortex band structure in case 2 (density wave)
N = 6
N = 12
55

Y. Xu, XW, Z. Luo, M. Ye, C.-M. Jian, C. Xu, arXiv:2106.14910

Experimental distinctions of two cases



- Case 1: in topological order (Mott insulator), the charge carriers are still deconfined at $T = 0$.
- Case 2: in density wave (Mott insulator), the $U(1)$ gauge field that couples to the fractionalized charge carrier will confine even at $T = 0$, due to the condensation of monopole which carries lattice translation symmetry.

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Subtleties of b-f coupling at Mott transition

- $O = |b|^2$ or nematic order, $\mathscr{L} \sim \frac{|\omega|}{|q|}|O(\omega, q)|^2$ irrelevant when $\Delta_O > 3/2$ ($\eta_O > 2$).
- S_Q density wave order, $\mathscr{L} \sim |\omega||S_Q(\omega, q)|^2$ irrelevant when $\Delta_{S_Q} > 1$ ($\eta_{S_Q} > 1$) or $2k_F \neq Q$.
- If satisfied at the critical point of b, the two sectors b and f are dynamically decoupled.
- The dual vortex theory (CP^{N-1} model) considering hot spots from density-wave $S_Q = \psi^\dagger T \psi$
- $\mathscr{L} = \sum_{I=1}^{N} (|(\partial - \mathrm{i}A)\psi_I|^2 + \mathrm{i}\lambda|\psi_I|^2) + \mathrm{i}\Phi\psi^\dagger T\psi + \frac{1}{2g}\Phi\frac{1}{|\partial_\tau|}\Phi + \frac{N}{16}\lambda\frac{1}{|\partial|}\lambda + \frac{N}{32}A|\partial|A + \ldots$
- which is controlled when $g \sim 1/N$, check relevance/irrelevance of all PSG-allowed terms of ψ.

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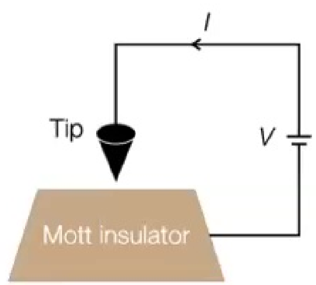
Y. Xu, XW, Z. Luo, M. Ye, C.-M. Jian, C. Xu, arXiv:2106.14910

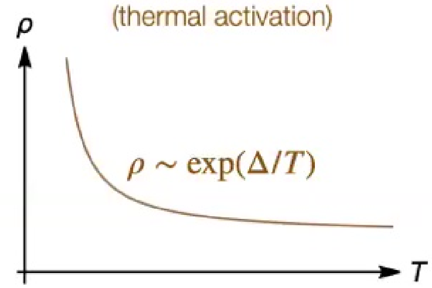
Experimental distinctions of two cases

- **Mott insulator** at half-filling at low-T: **tunneling gap** vs **transport gap**
- Case 1 (Z_N topological order): tunneling gap = N transport gap (= N anyon gap)
- Case 2 (density wave state): tunneling gap = transport gap (= electron charge gap)

Tunneling spectroscopy



Resistivity at finite-T (thermal activation)



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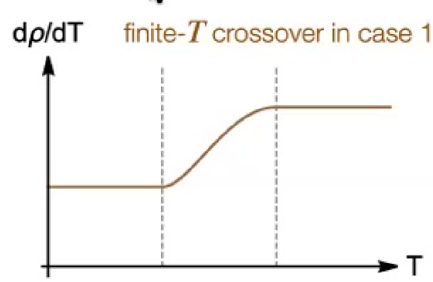
Y. Xu, XW, Z. Luo, M. Ye, C.-M. Jian, C. Xu, arXiv:2106.14910

Experimental distinctions of two cases

- Doping Mott insulator → **metallic state**: **linear-T resistivity** from the scattering of charged boson and gauge field a (shared with fermionic spinon f).
- Case 1 (doping Z_N topological order): finite-T confinement crossover $\rho \sim T \rightarrow \rho \sim NT$.
- Case 2 (doping density wave): there is no finite-T confinement crossover.



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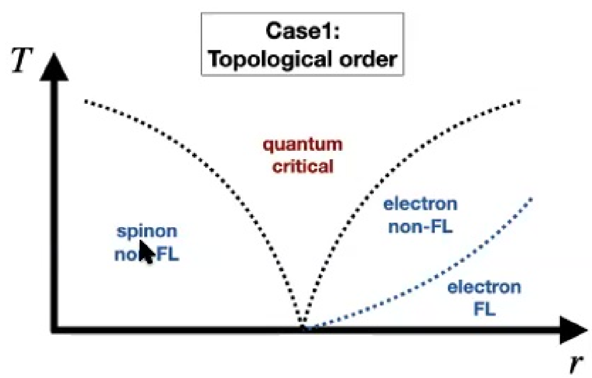
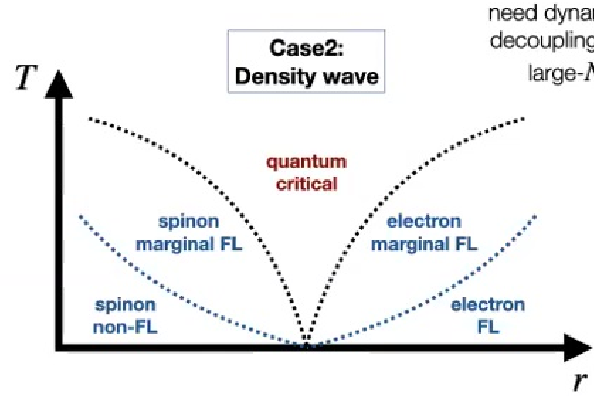
Schematic phase diagrams

- Topological order: A gapped at QCP $\Rightarrow a$ unaffected by $A \wedge da \Rightarrow \mathscr{L}[a] \sim \frac{|\omega|}{|q|} + |q|^2 \Rightarrow$ non-FL

- Density wave: if large-N of $\psi \Rightarrow \mathscr{L}[A] \sim |k|$, then $A \wedge da \Rightarrow \mathscr{L}[a] \sim \frac{|\omega|}{|q|} + |q| \Rightarrow$ marginal FL





need dynamical decoupling and large-N

Summary

XW, M. Ye, Z. Luo, C. Xu, in preparation
Y. Xu, XW, Z. Luo, M. Ye, C.-M. Jian, C. Xu, arXiv:2106.14910

- Exotic metallic states with charge fractionalization (e.g. Z_N or $SU(N)$ gauge structure): (1) large resistivity (bad metal) at low-T; (2) large thermopower and strong violation of the Wiedemann-Franz law at low-T; (3) bad-to-good metal crossover at finite-T.

- Metal-insulator transition with charge fractionalization (where Mott insulator is density wave or topological order) $\Rightarrow$ large universal resistivity jump at the critical point (potential explanation of the observed big resistivity jump in TMD Moiré superlattice).

- **Other loosely related topics**: (1) The relation between universal conductivity and universal features of higher-form symmetry at $2+1d$ quantum critical points (XW et al., J. Stat. Mech. 073101 (2021), XW et al., SciPost Phys. 11, 033 (2021)); (2) Exotic metallic states based on Sachdev-Ye-Kitaev physics without disorder (XW et al., PRB 98, 165117 (2018), XW et al., PRB 100, 075101 (2019)).

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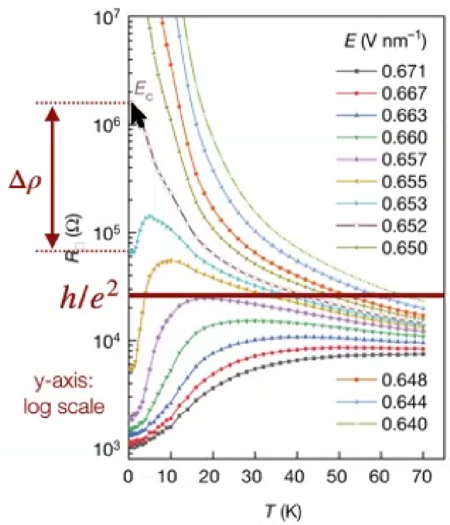
Big resistivity jump in transition metal dichalcogenides

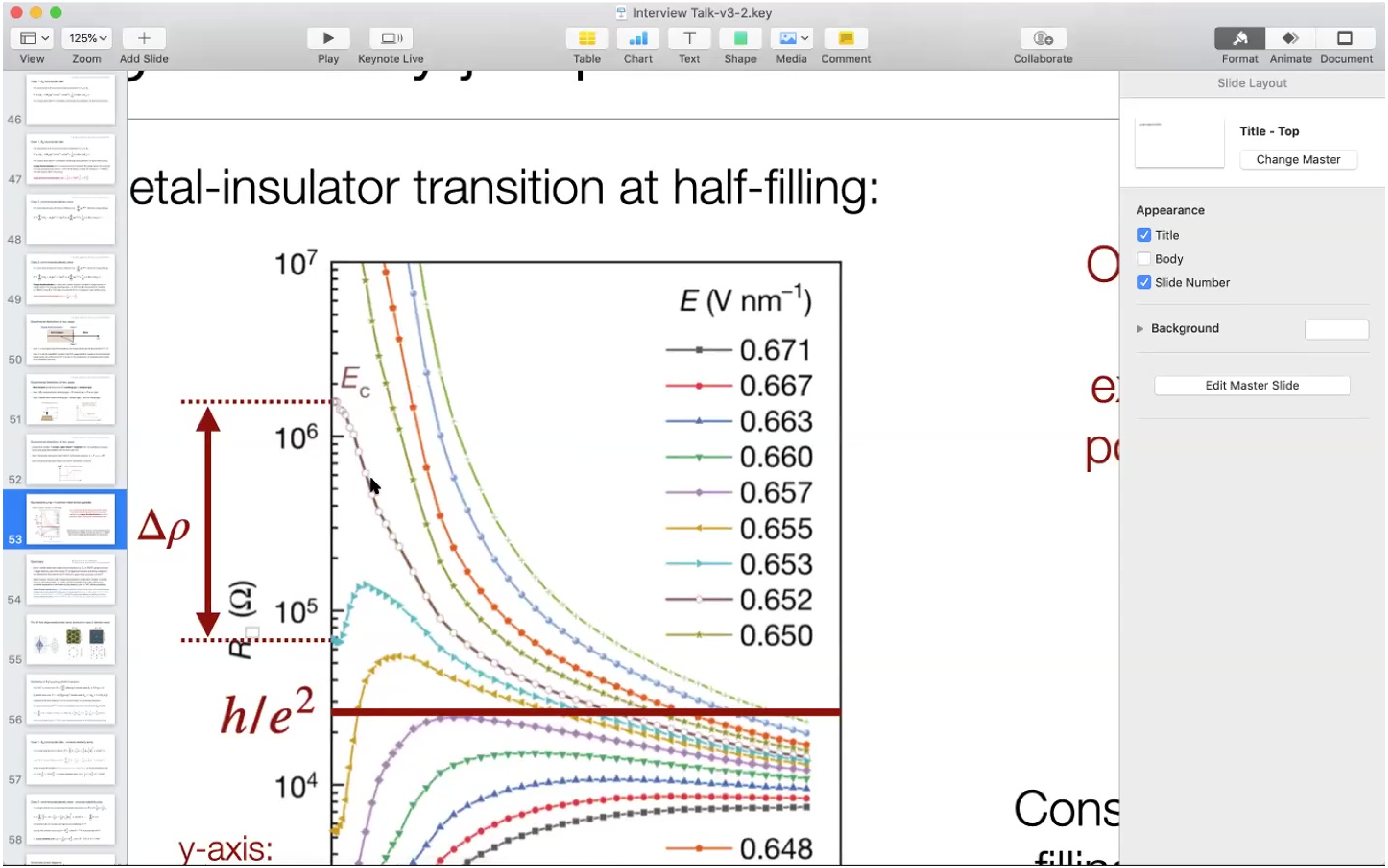
Metal-insulator transition at half-filling:



Shan, Mak et al. Nature (2021)

Our construction at half-filling (arXiv:2106.14910): the observed big resistivity jump is potentially explained by **charge fractionalization** at critical point (two cases: topological order/density wave).

Construction of metal to Wigner crystal transition at 1/6-filling by Musser-Senthil-Chowdhury arXiv:2111.09894 also involves charge fractionalization at critical point.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10^6
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10^4
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$\Delta\rho$
h/e^2
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y-axis:
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0.663
0.660
0.657
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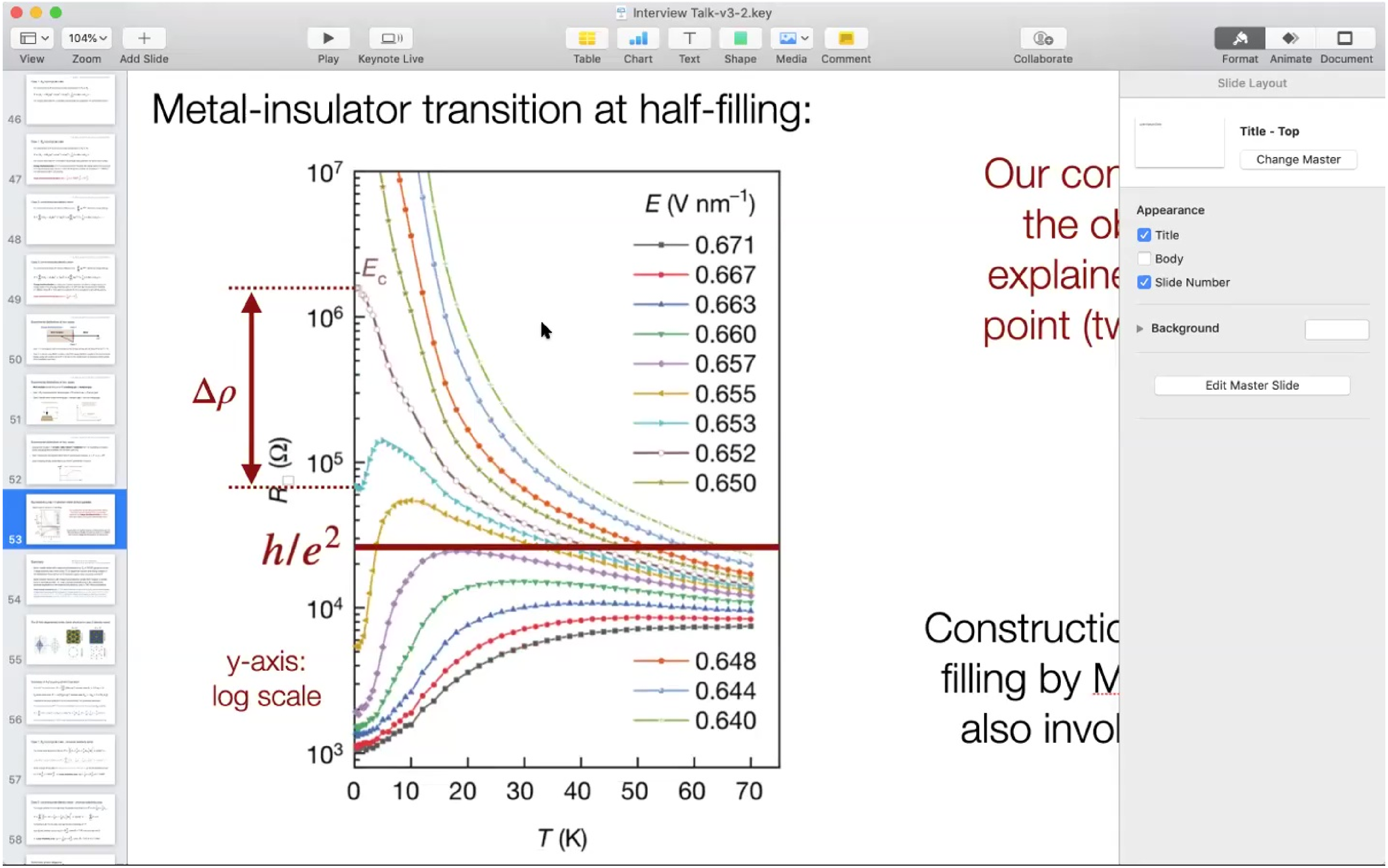
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Metal-insulator transition at half-filling:
E (V nm^{-1})
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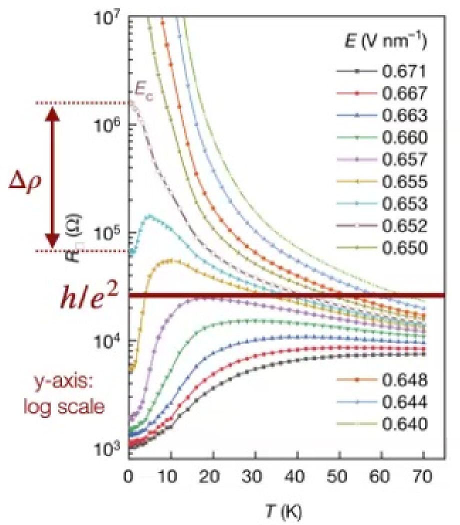
Big resistivity jump in transition metal dichalcogenides

Metal-insulator transition at half-filling:



Shan, Mak et al. Nature (2021)

Our construction at half-filling (arXiv:2106.14910): the observed big resistivity jump is potentially explained by **charge fractionalization** at critical point (two cases: topological order/density wave).

Construction of metal to Wigner crystal transition at 1/6-filling by Musser-Senthil-Chowdhury arXiv:2111.09894 also involves charge fractionalization at critical point.

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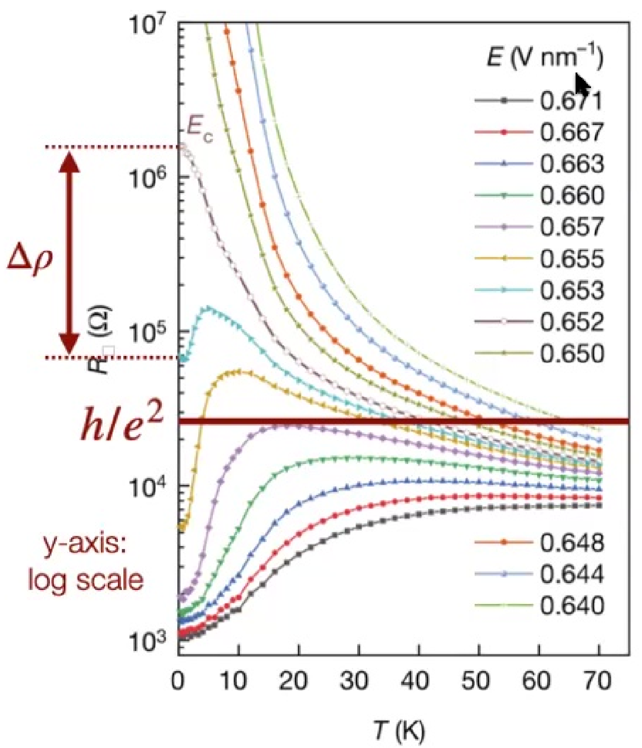
Big resistivity jump in transition metal dichalcogenides

Metal-insulator transition at half-filling:



Shan, Mak et al. Nature (2021)

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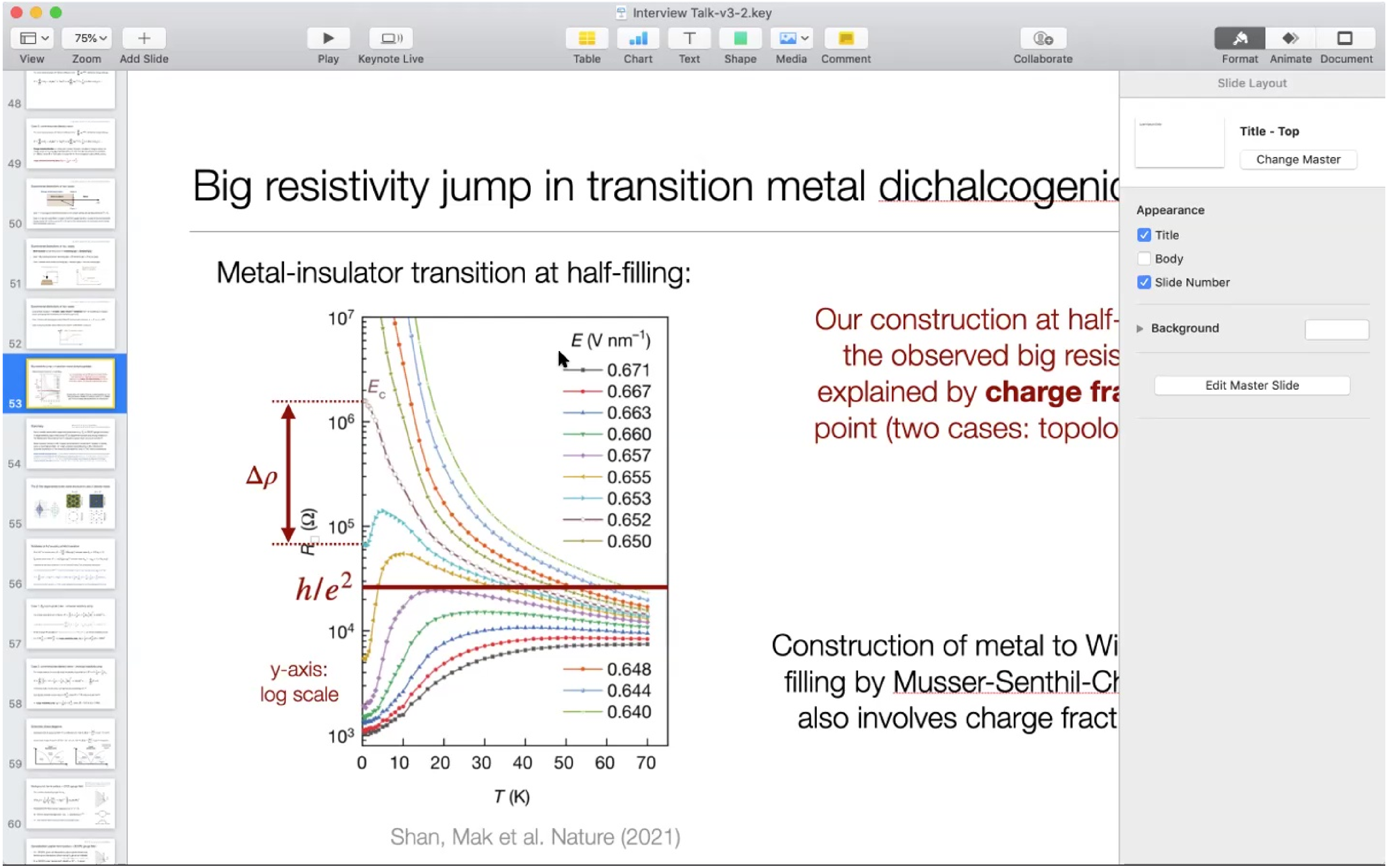
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