Title: Regulating Loops in dS - Akhil Premkumar

Speakers:

Series: Cosmology & Gravitation

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Abstract: Perturbative QFT calculations in de Sitter are plagued by a variety of divergences. One particular kind, the secular growth terms, cause the naive perturbation expansion to break down at late times. Such contributions often arise from loop integrals, which are notoriously hard to compute in dS. We discuss an approach to evaluate such loop integrals, for a scalar field theory in a fixed de Sitter background. Our method is based on the Mellin-Barnes representation of correlation functions, which enables us to regulate divergences for scalars of any mass while preserving the symmetries of dS. The resulting expressions have a similar structure as a standard dimreg answer in flat space QFT. These features of the regulator are illustrated with two examples. Along the way, we illuminate the physical origin of these divergences and their interpretation within the framework of the dynamical renormalization group. Our calculations naturally reveal additional infrared divergences for massless scalar fields in de Sitter, that are not present in the massive case. Such loop corrections can be incorporated as systematic improvements to the Stochastic Inflation framework, allowing for a more precise description of the IR dynamics of massless fields in de Sitter.

Regulating Loops in de Sitter

arXiv: 2110.12504

Akhil Premkumar
(UC San Diego)

Background: Inflation

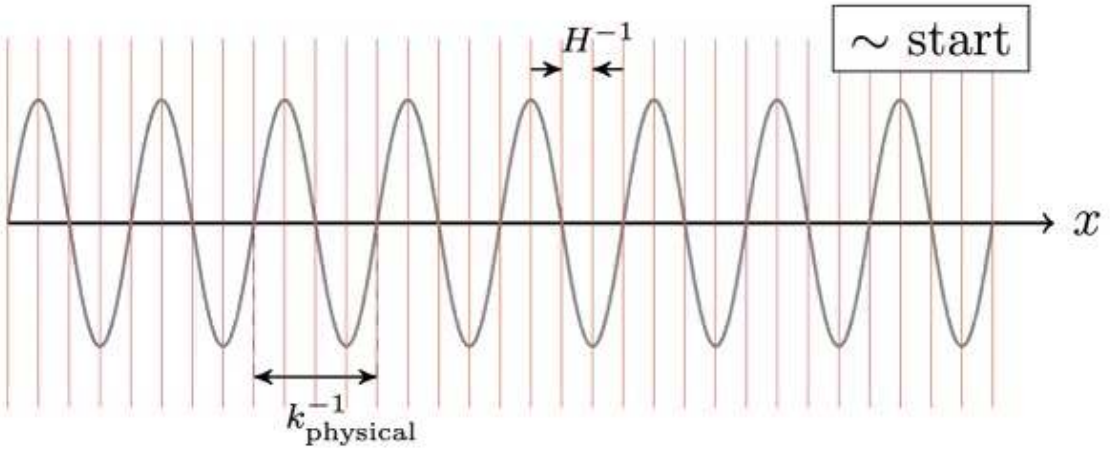
Rapid expansion

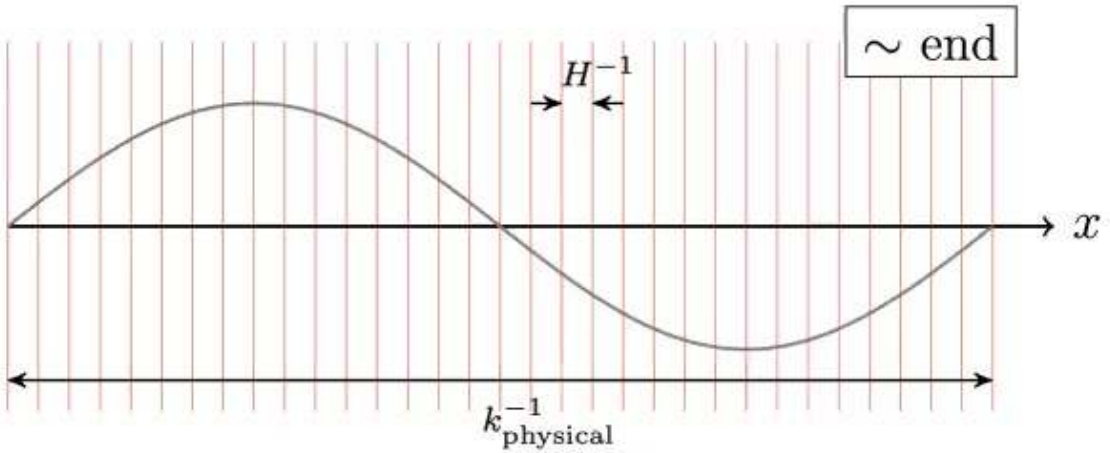
↓

"Freeze-out"

↓

Re-entry





Cosmology depends on $\phi(\vec{k}, t)$ with

$$\vec{k}_{\text{physical}} = \frac{k}{a(t)} \ll H$$

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

$$a(t) = e^{Ht}$$

Divergences in dS

- Our focus: Late-time/"secular" divergences in QFTs on a fixed dS background.
- (Weinberg 2005) This effect appears as at most $\log\left(\frac{k}{aH}\right)$.
- These are physical.
- Interpret with *Dynamical* Renormalization Group.
- Where does the DRG flow take us?

Secular Growth: A Classical Example

Rayleigh's equation:

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + y = \epsilon \left\{ \frac{\mathrm{d}y}{\mathrm{d}t} - \frac{1}{3}\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^3 \right\}, \quad \epsilon \ll 1$$

Chen et. al
arXiv:hep-th/9506161

Attempt a series soln.

$$y = y_0 + \epsilon y_1 + \epsilon^2 y_2 + \ldots$$

Breakdown at

$$\epsilon(t - t_0) \gtrsim 1$$

$$y(t) = R_0 \sin(t + \Theta_0) + \epsilon \left\{ -\frac{R_0^3}{96} \cos(t + \Theta_0) + \frac{R_0}{2}\left(1 - \frac{R_0^2}{4}\right)(t - t_0) \sin(t + \Theta_0) + \frac{R_0^3}{96} \cos 3(t + \Theta_0) \right\} + O\left(\epsilon^2\right)$$

Resolution: *Dynamical* Renormalization Group

Dynamical RG

Step 1: split $t - t_0 = t - T + T - t_0$ — New time scale

Step 2: $R_0(t_0) = Z_1(t_0, T) R(T), \quad \Theta_0(t_0) = \Theta(T) + O(\epsilon^2)$

$$Z_1 = 1 - \frac{1}{2}\left(1 - \frac{R^2}{4}\right)(T - t_0)\epsilon + O(\epsilon^2)$$

'Counterterm'

Renormalized soln.

$$y(t) = \left\{ R + \epsilon \frac{R}{2}\left(1 - \frac{R^2}{4}\right)(t - T) \right\} \sin(t + \Theta)$$
$$- \epsilon \frac{1}{96} R^3 \cos(t + \Theta) + \epsilon \frac{R^3}{96} \cos 3(t + \Theta) + O\left(\epsilon^2\right)$$

Step 3: $\left(\frac{\partial y}{\partial T}\right)_t = 0$ 'Callan-Symanzik equation'

Dynamical RG

$$\left(\frac{\partial y}{\partial T}\right)_t = 0 \quad \Longrightarrow \quad \frac{\mathrm{d}R}{\mathrm{d}T} = \epsilon\frac{1}{2}R\left(1-\frac{1}{4}R^2\right) + O\left(\epsilon^2\right), \quad \frac{d\Theta}{\mathrm{d}T} = O\left(\epsilon^2\right)$$

Solve…

$$R(t) = R(0)/\sqrt{e^{-\epsilon t} + \frac{1}{4}R(0)^2\left(1-e^{-\epsilon t}\right)}$$

$$\Theta(t) = \Theta(0) + O\left(\epsilon^2 t\right)$$

Initial conditions

$y(0) = 0, y'(0) = 2a$

Actual soln.

$$y(t) = R(t)\sin(t) + \frac{\epsilon}{96}R(t)^3\{\cos(3t) - \cos(t)\} + O\left(\epsilon^2\right)$$

$$\sim R(0)\sin(t) + \epsilon t(\cdots) + \ldots$$

Perturbative solution makes sense now!

Perturbative QFT in dS

$$S[\phi] = \frac{1}{2}\int \mathrm{d}^4x\sqrt{|g|}\left\{g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2 - \frac{\lambda}{4!}\phi^4\right\}$$

(fixed) dS metric in D+1 dimensions:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2\mathrm{d}\vec{x}^2 \overset{\mathrm{d}\tau = a(\tau)\mathrm{d}\tau}{\longrightarrow} a(\tau)^2(-\mathrm{d}\tau^2 + \mathrm{d}\vec{x}^2)$$

$$a(\tau) = -\frac{1}{H\tau}, \quad \tau \in (-\infty, 0)$$

'Propagator':

$$\langle\phi_{\vec{k}_1}(\tau_1)\phi_{\vec{k}_2}(\tau_2)\rangle'_{\mathrm{BD}} \sim (-\tau_1)^{\frac{D}{2}}(-\tau_2)^{\frac{D}{2}}\mathrm{H}_{i\nu}(-k_1\tau_1)\mathrm{H}^{\star}_{i\nu}(-k_2\tau_2)$$

$$\nu \overset{\mathrm{def.}}{=} i\sqrt{\frac{D^2}{4} - \frac{m^2}{H^2}}$$

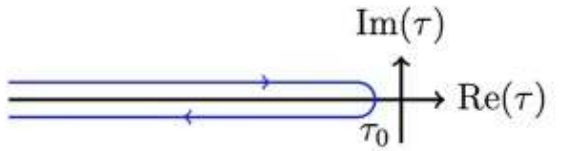
Conformal mass, $m^2 = 2H^2 \to \nu = \frac{i}{2}$

Massless, $m^2 = 0 \to \nu = i\frac{3}{2}$.

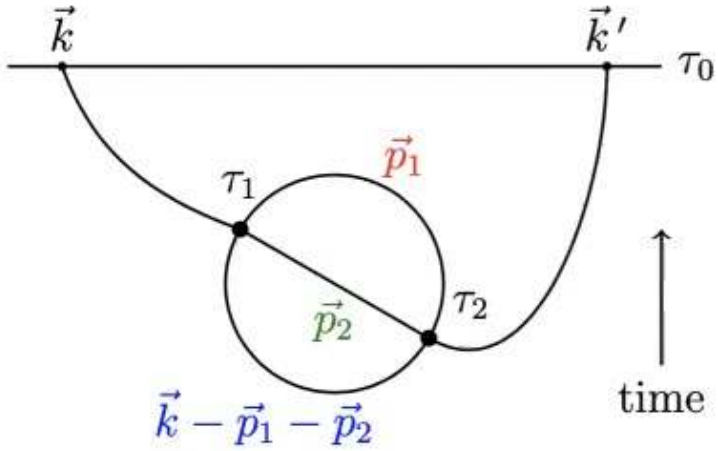
in-in formalism

$$\left\langle \phi_{\vec{k}_1}(\tau_0)\dots\phi_{\vec{k}_n}(\tau_0)\right\rangle_{\text{in-in}} = \int \mathrm{d}^3x a(\tau)^D \lambda\phi^4$$

$$\left\langle \bar{T}\exp\left[i\int_{-\infty(1+i\epsilon)}^{\tau_0} H_{\text{int}}(\tau)a(\tau)d\tau\right]\left(\phi_{\vec{k}_1}(\tau_0)\dots\phi_{\vec{k}_n}(\tau_0)\right) T\exp\left[-i\int_{-\infty(1-i\epsilon)}^{\tau_0} H_{\text{int}}(\tau)a(\tau)d\tau\right]\right\rangle$$



$$\langle\cdots\rangle + \lambda\langle\cdots\rangle + \frac{\lambda^2}{2}\langle\cdots\rangle + \dots$$



$$\left\langle \phi_{\vec{k}}(\tau_0)\phi_{\vec{k}}(\tau_0)\right\rangle_{\lambda^2} \supset$$

$$\sim \int_{-\infty}^{\tau_0}\mathrm{d}\tau_1 a(\tau_1)^{\#}\int_{-\infty}^{\tau_1}\mathrm{d}\tau_2 a(\tau_2)^{\#}\,\mathrm{H}_\nu(-k\tau_0)\mathrm{H}^\star_\nu(-k\tau_1)\mathrm{H}_\nu(-k'\tau_2)\mathrm{H}^\star_\nu(-k'\tau_0)$$

$$\times\int\frac{\mathrm{d}^Dp_1}{(2\pi)^D}\int\frac{\mathrm{d}^Dp_2}{(2\pi)^D}\mathrm{H}_\nu(-p_1\tau_1)\mathrm{H}^\star_\nu(-p_1\tau_2)\mathrm{H}_\nu(-p_2\tau_1)\mathrm{H}^\star_\nu(-p_2\tau_2)$$

$$\times\mathrm{H}_\nu(-|\vec{k}-\vec{p}_1-\vec{p}_2|\tau_1)\mathrm{H}^\star_\nu(-|\vec{k}-\vec{p}_1-\vec{p}_2|\tau_2)$$

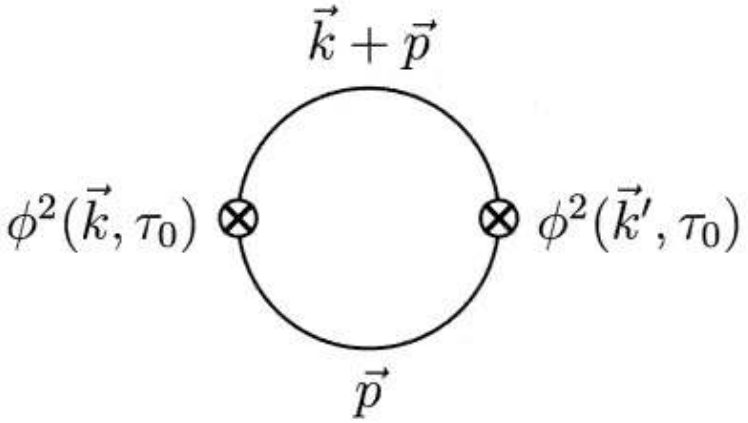
Secular growth hiding here

$$\nu \overset{\text{def.}}{=} i\sqrt{\frac{D^2}{4}-\frac{m^2}{H^2}}$$

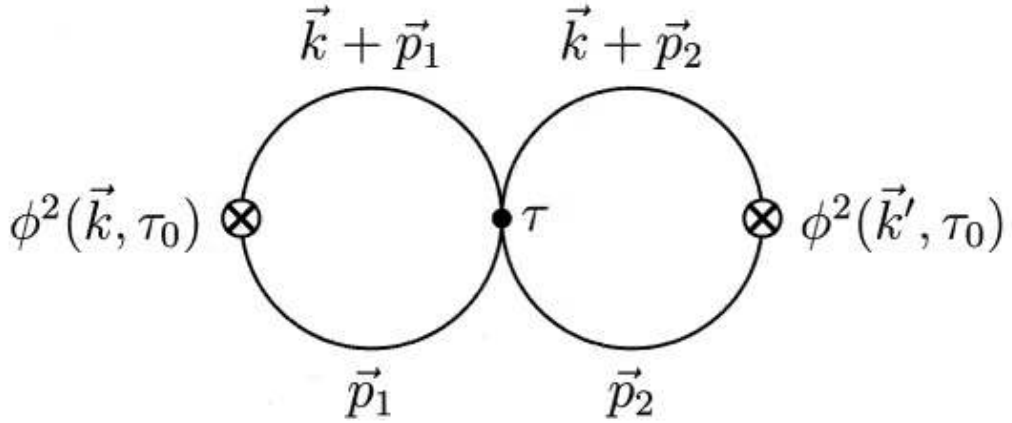
A 'doable' example

$$\nu \overset{\text{def.}}{=} i\sqrt{\frac{D^2}{4} - \frac{m^2}{H^2}}$$

$$\text{Conformal mass, } \nu = \frac{i}{2} \implies \mathrm{H}_{i\nu}(-k\tau) \sim -i\frac{e^{-ik\tau}}{\sqrt{k\tau}}$$



(a) $\langle\phi^2\phi^2\rangle$ at tree level.



(b) The first order correction to $\langle\phi^2\phi^2\rangle$.

$$\langle\phi^2\phi^2\rangle_\lambda \sim -\lambda\int_{-\infty}^{\tau_0} d\tau\; a^{D+1}(\tau)\left(\int \frac{d^Dp}{(2\pi)^D}\frac{1}{a(\tau)^{2\Delta_\phi}a(\tau_0)^{2\Delta_\phi}}\frac{e^{-i|\vec{k}+\vec{p}|\tau}}{2|\vec{k}+\vec{p}|}\frac{e^{-ip\tau}}{2\,p}\right)^2$$

$D = 3-\epsilon$

Time appears in the momentum integrals

D. Green, AP
arXiv: 2001.05974

A 'doable' example

$$\langle \phi^2(\vec{k},\tau_0)\phi^2(-\vec{k},\tau_0)\rangle = \frac{c}{2a(\tau_0)^{2\Delta_{\phi^2}}}\, k^{1-\epsilon}\left[1+\frac{\lambda}{32\pi^4 c}\left(\frac{1}{\epsilon}-\log\left(\frac{k}{a(\tau_0)H}\right)+\dots\right)\right]$$

$-k\tau_0$

$\log(-k\tau_0)$ blows up at late time $\tau_0 \to 0$ leading to a secular divergence.

Resummation of logs with DRG gives

$$\langle \phi^2(\vec{k},\tau_0)\phi^2(-\vec{k},\tau_0)\rangle : \underbrace{\frac{c}{2a(\tau_0)^{2\Delta_{\phi^2}}}\, k^{1-\epsilon}}_{\text{free}} \longrightarrow \underbrace{\frac{c\, H^{-2\gamma_{\phi^2}}}{2a(\tau_0)^{2\Delta_{\phi^2}+2\gamma_{\phi^2}}}\, k^{1-\epsilon+2\gamma_{\phi^2}}}_{\text{resummed}}$$

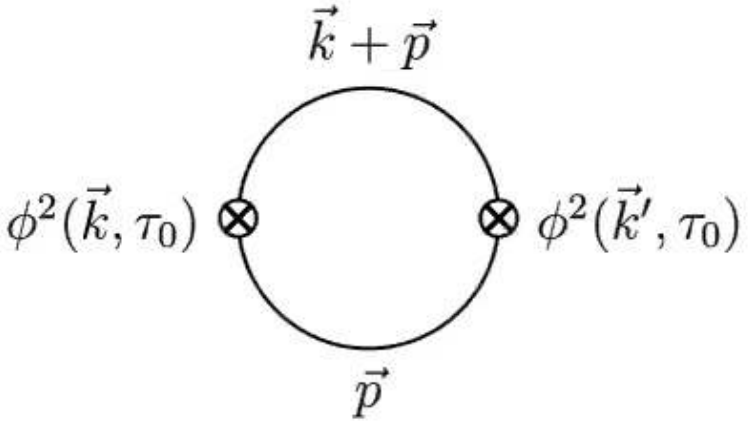
$$\Delta_{\phi^2} \to \Delta_{\phi^2}+\gamma_{\phi^2}$$

$$2-\epsilon \to 2-\epsilon+\frac{\lambda}{16\pi^2}$$

A 'doable' example

$$\nu \overset{\text{def.}}{=} i\sqrt{\frac{D^2}{4} - \frac{m^2}{H^2}}$$

$$\text{Conformal mass, } \nu = \frac{i}{2} \implies \mathrm{H}_{i\nu}(-k\tau) \sim -i\frac{e^{-ik\tau}}{\sqrt{k\tau}}$$



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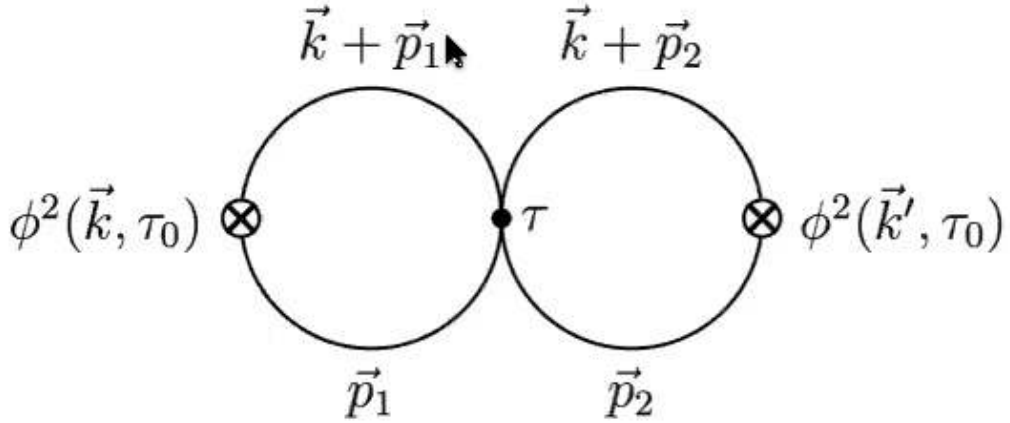

(b) The first order correction to $\langle\phi^2\phi^2\rangle$.

$$\langle\phi^2\phi^2\rangle_\lambda \sim -\lambda \int_{-\infty}^{\tau_0} d\tau\; a^{D+1}(\tau) \left(\int \frac{d^D p}{(2\pi)^D} \frac{1}{a(\tau)^{2\Delta_\phi} a(\tau_0)^{2\Delta_\phi}} \frac{e^{-i|\vec{k}+\vec{p}|\tau}}{2|\vec{k}+\vec{p}|} \frac{e^{-ip\tau}}{2\,p} \right)^2$$

$D = 3 - \epsilon$

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Loops of general mass

- Generic mass, ν

$$\nu \stackrel{\text{def.}}{=} i\sqrt{\frac{D^2}{4} - \frac{m^2}{H^2}}$$

- Momentum integrals are not scaleless
- Avoid hard cut-offs
- Repeatable procedure

PROBLEM: Bad basis *C. Sleight, M. Taronna*

Flat space: Time translations → Fourier transform

de Sitter: Dilatation invariance → Mellin transform

$$i\pi e^{\frac{\pi\nu}{2}} \mathrm{H}^{(1)}_{i\nu}(z) = \int_{c-i\infty}^{c+i\infty} \frac{\mathrm{d}s}{2\pi i} \Gamma\left(s + \frac{i\nu}{2}\right) \Gamma\left(s - \frac{i\nu}{2}\right) \left(-\frac{iz}{2}\right)^{-2s}$$

dS Loops in Mellin space

$$\nu \stackrel{\text{def.}}{=} i\sqrt{\frac{D^2}{4} - \frac{m^2}{H^2}}$$

$$\left\langle \phi^{\nu}_{\vec{k}_1}(\tau_0) \cdots \phi^{\nu}_{\vec{k}_n}(\tau_0) \right\rangle_{\text{in-in}}$$

$$i\pi e^{\frac{\pi\nu}{2}} \mathrm{H}^{(1)}_{i\nu}(z) = \int_{c-i\infty}^{c+i\infty} \frac{\mathrm{d}s}{2\pi i} \Gamma\left(s + \frac{i\nu}{2}\right) \Gamma\left(s - \frac{i\nu}{2}\right) \left(-\frac{iz}{2}\right)^{-2s}$$

$$\supset \prod_i \int^{\tau_0} \mathrm{d}\tau_i a(\tau_i)^{D+1} \prod_j \int \frac{\mathrm{d}^D p_j}{(2\pi)^D} \ldots \mathrm{H}_{i\nu}(a_m(\vec{k},\vec{p})\tau_i)\, \mathrm{H}_{i\nu}(a_{m+1}(\vec{k},\vec{p})\tau_{i+1}) \ldots$$

$\tau_0 \to 0$

$$\supset \prod_\ell \int \mathrm{d}s_\ell\, \Gamma\left(s_\ell + \tfrac{i\nu}{2}\right) \Gamma\left(s_\ell - \tfrac{i\nu}{2}\right) \prod_i \int^{\tau_0} \mathrm{d}\tau_i a(\tau_i)^{D+1} (-\tau_i)^{-2s_\ell}$$

$$\times \prod_j \int \frac{\mathrm{d}^D p_j}{(2\pi)^D} \ldots a_m(\vec{k},\vec{p})^{-2s_\ell}\, a_{m+1}(\vec{k},\vec{p})^{-2s_{\ell'}} \ldots .$$

$\delta\left(\frac{D}{2} - s_1 - \cdots - s_\ell - \cdots\right)$

s-conservation

$$\supset \prod_\ell{}' \int \mathrm{d}s_\ell\, \Gamma\left(s_\ell + \tfrac{i\nu}{2}\right) \Gamma\left(s_\ell - \tfrac{i\nu}{2}\right) Q(\boldsymbol{s}) \frac{\Gamma\left(D - s_{\ell'} - \ldots\right) \ldots}{\Gamma\left(s_\ell - \ldots\right) \ldots} b_\ell(\vec{k})^{s_\ell}$$

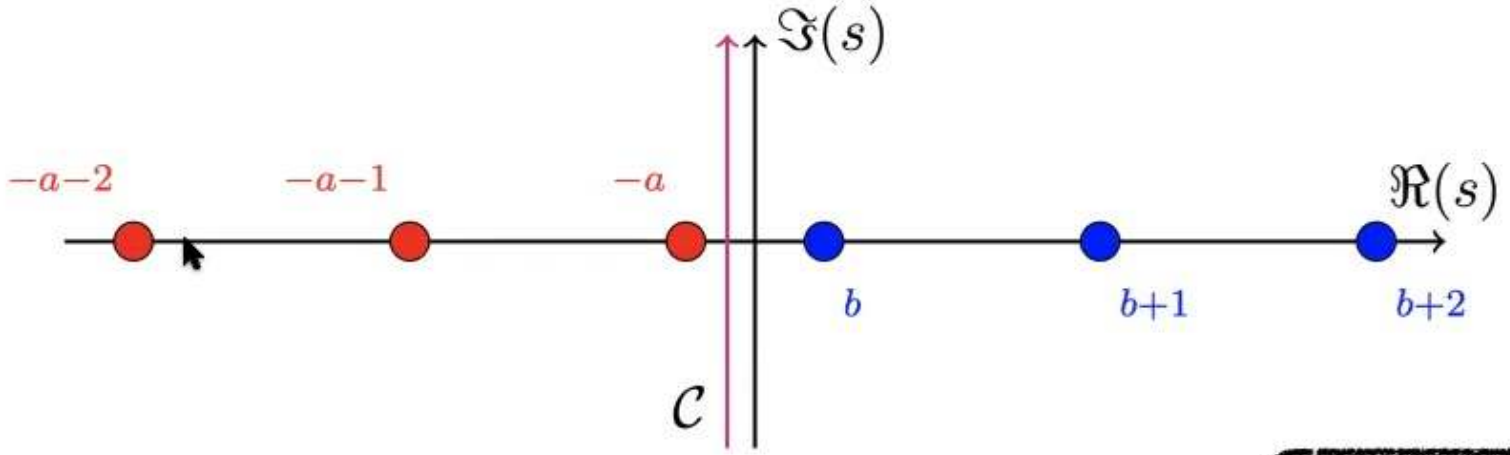
Divergences are encoded in this *Mellin-Barnes* integral

Mellin-Barnes (MB) Integrals

$$K(x) = \int_{-i\infty}^{+i\infty} \frac{\mathrm{d}s}{2\pi i} \Gamma(a+s)\Gamma(b-s)x^{-s}$$

$\Gamma(z)$ has poles at $z_\star = 0, -1, -2, \ldots$

$$\mathrm{Res}[\Gamma(z); -n] = \frac{(-1)^n}{n!}$$



$$\left.\begin{aligned} a + s^{\mathcal{C}} &> 0 \\ b - s^{\mathcal{C}} &> 0 \end{aligned}\right\} \implies -a < s^{\mathcal{C}} < b$$

Mellin contour prescription

Divergences in Mellin-Barnes Integrals

$$K(x) = \int_{-i\infty}^{+i\infty} \frac{\mathrm{d}s}{2\pi i} \Gamma(a+s)\Gamma(-s)x^{-s}$$

$\Gamma(z)$ poles: $z_\star = 0, -1, -2, \ldots$

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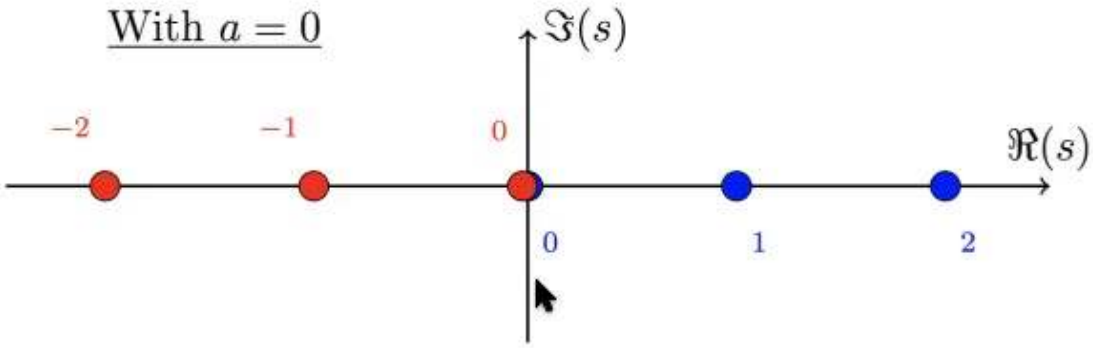
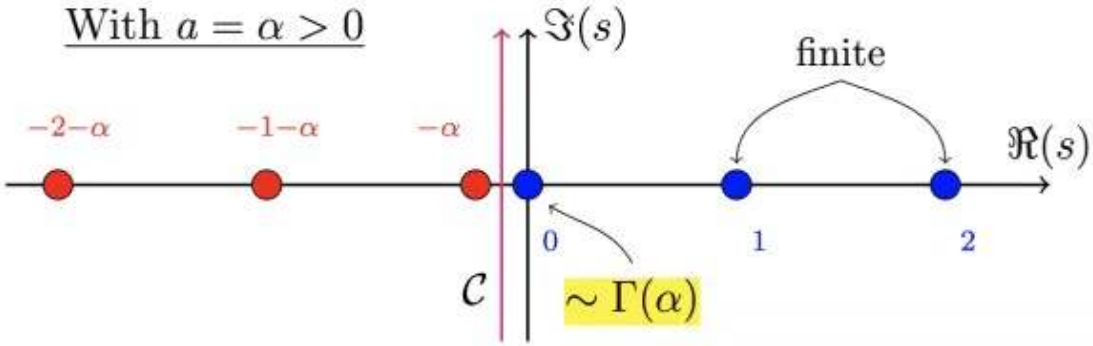


(a) Poles of $\Gamma(s)\Gamma(-s)$

$$\mathrm{Res}[\Gamma(s+\alpha)\Gamma(-s)x^{-s}; s=0] = \Gamma(\alpha)$$

Contour "pinching"



(b) Poles of $\Gamma(s+\alpha)\Gamma(-s)$

$$\int_{\mathcal{C}} \frac{\mathrm{d}s}{2\pi i}\Gamma(s+\alpha)\Gamma(-s)x^{-s} = -\mathrm{Res}[s=0] + \int_{\mathcal{C}'} \frac{\mathrm{d}s}{2\pi i}\Gamma(s+\alpha)\Gamma(-s)x^{-s}$$

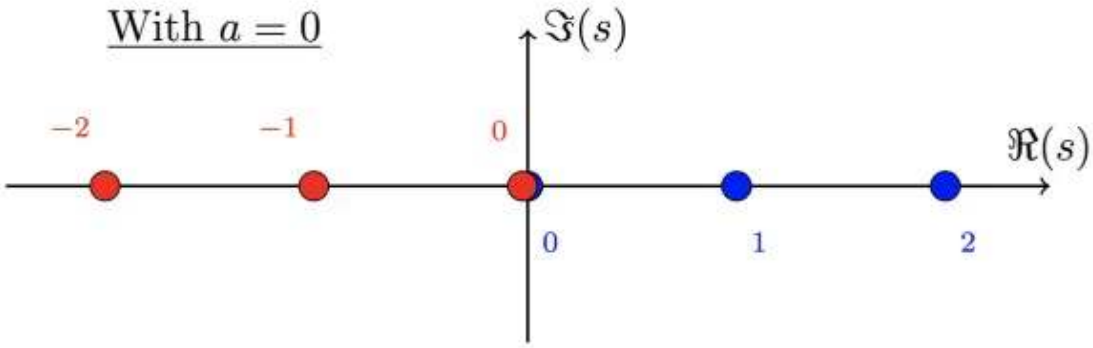
$O(\alpha^0)$

Divergences in Mellin-Barnes Integrals

$$K(x) = \int_{-i\infty}^{+i\infty} \frac{\mathrm{d}s}{2\pi i} \Gamma(a+s)\Gamma(-s)x^{-s}$$

$\Gamma(z)$ poles: $z_\star = 0, -1, -2, \ldots$

$$\mathrm{Res}[\Gamma(z); -n] = \frac{(-1)^n}{n!}$$



(a) Poles of $\Gamma(s)\Gamma(-s)$

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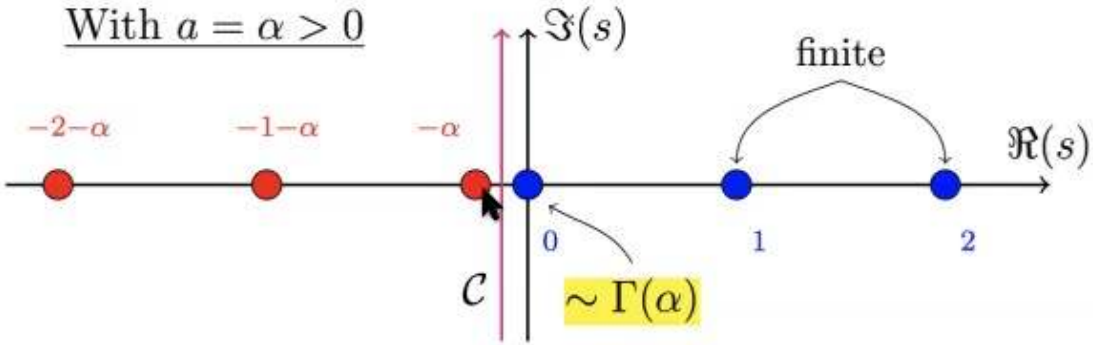


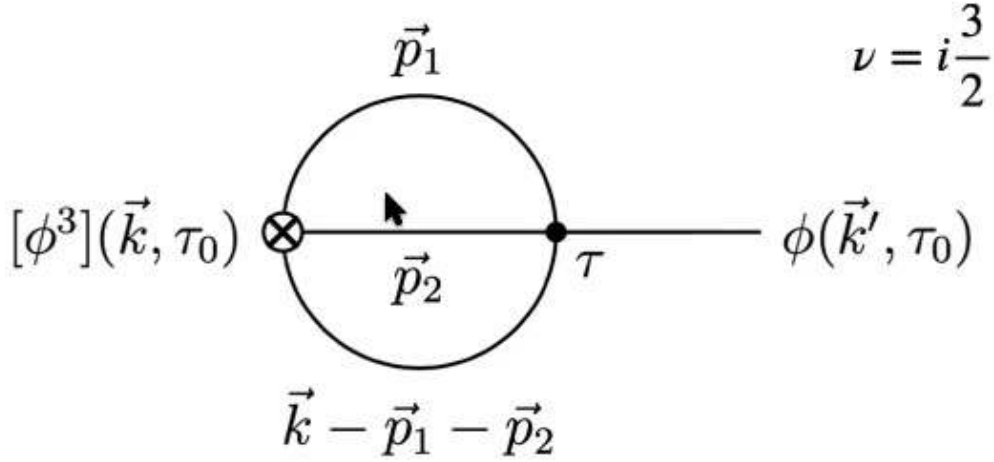
(b) Poles of $\Gamma(s+\alpha)\Gamma(-s)$

$$\int_{\mathcal{C}} \frac{\mathrm{d}s}{2\pi i}\Gamma(s+\alpha)\Gamma(-s)x^{-s} = -\mathrm{Res}[s=0] + \int_{\mathcal{C}'} \frac{\mathrm{d}s}{2\pi i}\Gamma(s+\alpha)\Gamma(-s)x^{-s}$$

$O(\alpha^0)$

Example: $\langle \phi^3 \phi \rangle_\lambda$ for a massless scalar

$$\nu \overset{\text{def.}}{=} i\sqrt{\frac{D^2}{4} - \frac{m^2}{H^2}}$$



$$\langle \phi^3 \phi \rangle'_\lambda = C \frac{8}{(4\pi)^3} \frac{1}{k^3} \int [\mathrm{d}s]_3 \, \Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4 \Gamma_5 \Gamma_6 \Gamma_7 \Gamma_8$$

$$\sim \frac{c_3}{\boldsymbol{\alpha}^3} + \frac{c_2}{\boldsymbol{\alpha}^2} + \frac{c_1}{\boldsymbol{\alpha}} + O\left(\boldsymbol{\alpha}^0\right)$$

α is the *regulator*

4 contour pinches, each $\sim \Gamma(\alpha)$,
up to 3 'active' simultaneously

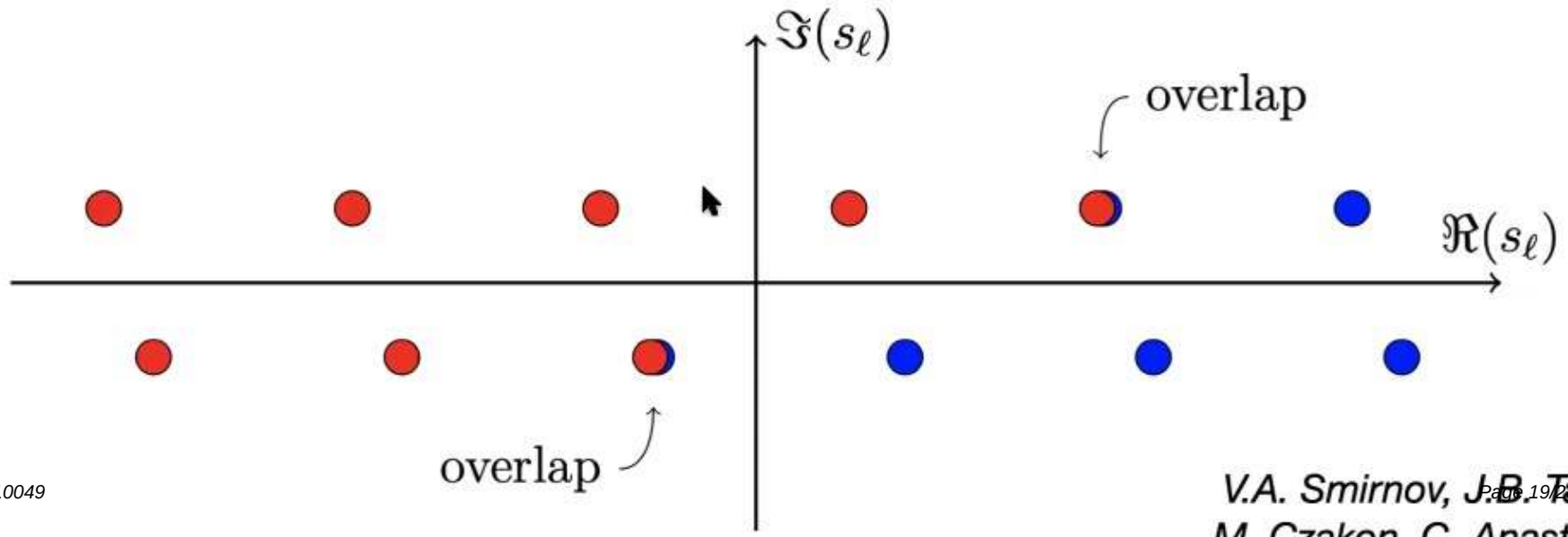
$$\Gamma_1 = \Gamma\left(\tfrac{3}{4} - s_1 - s_2 - s_3\right)$$
$$\Gamma_2 = \Gamma\left(-\tfrac{3}{4} + s_1 + s_2 + s_3\right)$$
$$\left.\begin{aligned} \Gamma_3 &= \Gamma\left(\tfrac{3}{4} - s_1\right) \\ \Gamma_4 &= \Gamma\left(-\tfrac{3}{4} + s_1\right) \end{aligned}\right\} \Gamma(z)\Gamma(-z) \text{ form}$$
$$\Gamma_5 = \Gamma\left(\tfrac{3}{4} - s_2\right)$$
$$\Gamma_6 = \Gamma\left(-\tfrac{3}{4} + s_2\right)$$
$$\Gamma_7 = \Gamma\left(\tfrac{3}{4} - s_3\right)$$
$$\Gamma_8 = \Gamma\left(-\tfrac{3}{4} + s_3\right)$$

N-dimensional MB integrals

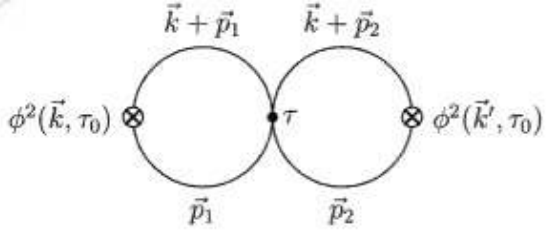
$$K(x_1, x_2, \ldots, x_N) = \int_{-i\infty}^{i\infty} \frac{\mathrm{d}s_1}{2\pi i} \cdots \int_{-i\infty}^{i\infty} \frac{\mathrm{d}s_N}{2\pi i} \frac{\prod_i \Gamma\left(U_i(\boldsymbol{s})\right)}{\prod_j \Gamma\left(V_j(\boldsymbol{s})\right)} x_1^{s_1} \cdots x_N^{s_N}$$

To identify and remove overlaps

$$U_i(\boldsymbol{s}) \stackrel{\text{def.}}{=} a_i + \sum_\ell b_{i\ell} s_\ell \longrightarrow a_i + \sum_\ell b_{i\ell} s_\ell + \sum_k c_{ik} \alpha_k$$



V.A. Smirnov, J.B. Tausk, M. Czakon, C. Anastasiou



Example: $\langle\phi^2\phi^2\rangle_\lambda$ again

$$\langle\phi^2\phi^2\rangle'_\lambda = C\frac{8}{(4\pi)^3}(-k\tau_0)^{-2\alpha}k\int[\mathrm{d}s]_3\,\frac{\Gamma_1\Gamma_2\Gamma_3\Gamma_4\Gamma_5\Gamma_6\Gamma_7\Gamma_8\Gamma_9\Gamma_{10}}{\Gamma_{11}\Gamma_{12}}$$

$$\begin{aligned}
\Gamma_1 &= \Gamma(-\tfrac{1}{4}+s_1)\\
\Gamma_2 &= \Gamma(\tfrac{5}{4}-s_1)\\
\Gamma_3 &= \Gamma(-\tfrac{1}{4}+s_2)\\
\Gamma_4 &= \Gamma(\tfrac{5}{4}-s_2)\\
\Gamma_5 &= \Gamma(-\tfrac{1}{4}+s_3)\\
\Gamma_6 &= \Gamma(\tfrac{5}{4}-s_3)\\
\Gamma_7 &= \Gamma(-\tfrac{1}{4}+s_1+s_2+s_3-\alpha)\\
\Gamma_8 &= \Gamma(\tfrac{5}{4}-s_1-s_2-s_3+\alpha)\\
\Gamma_9 &= \Gamma(\tfrac{1}{2}-s_1-s_2+\alpha)\\
\Gamma_{10} &= \Gamma(-1+s_1+s_2)\\
\Gamma_{11} &= \Gamma(1+s_1+s_2-\alpha)\\
\Gamma_{12} &= \Gamma(\tfrac{5}{2}-s_1-s_2)
\end{aligned}$$

Pinches not obvious, no apparent $\Gamma(z)\Gamma(-z)$

Mellin contour prescription $\Longrightarrow$

$$\frac{1}{2} < s_1^{\mathcal{C}} + s_2^{\mathcal{C}} < \frac{1}{2}+\alpha$$

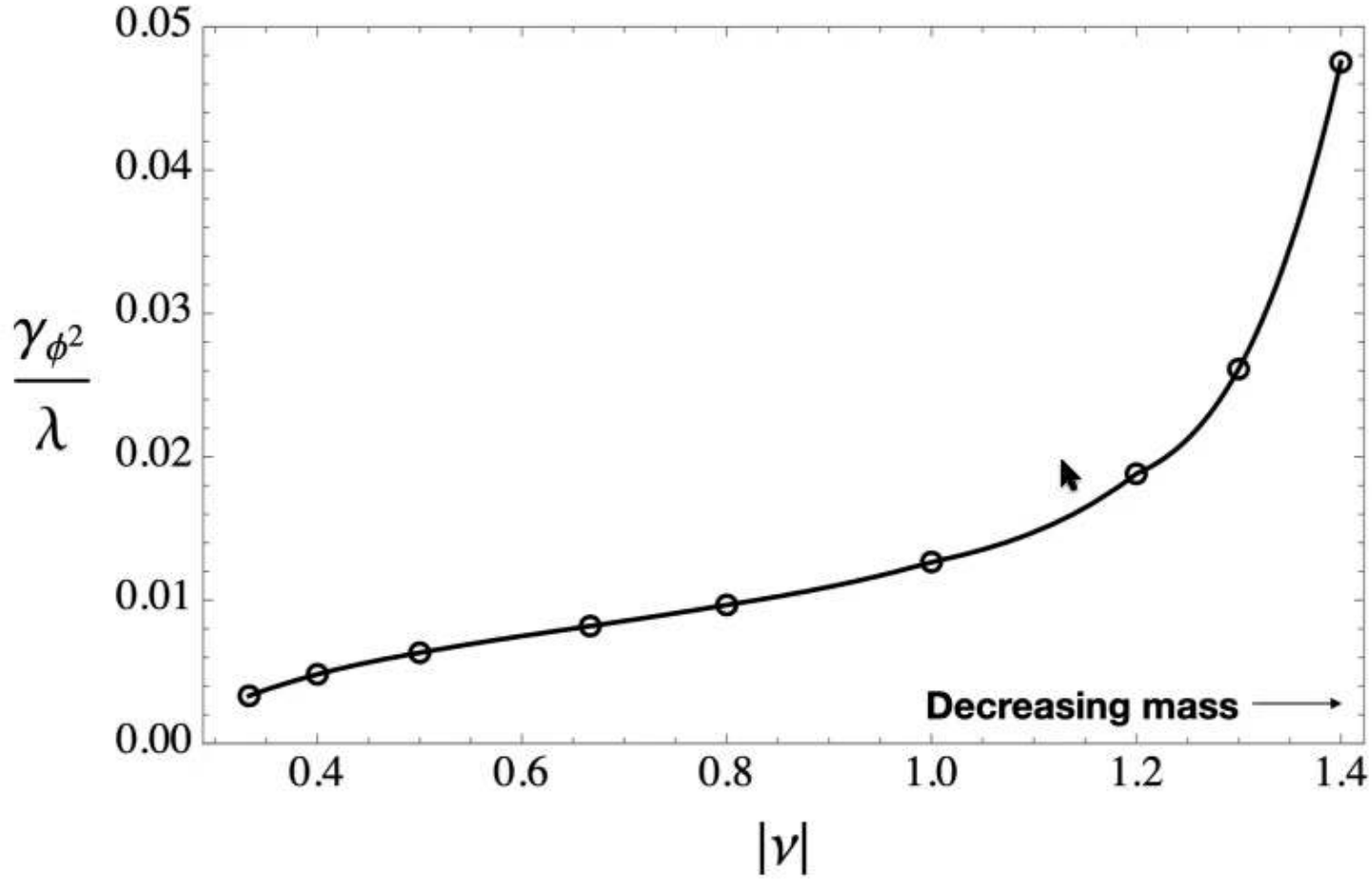
$$\frac{5}{4} < s_1^{\mathcal{C}} + s_2^{\mathcal{C}} + s_3^{\mathcal{C}} < \frac{5}{4}+\alpha$$

These are the pinches

$$\langle\phi^2\phi^2\rangle'_\lambda = -\frac{(-H\tau_0)^4}{512\pi^4}k\times -4\pi\Gamma(\alpha)(-k\tau_0)^{-2\alpha}$$

$$\overset{\alpha\to 0}{=} -\frac{(-H\tau_0)^4}{128\pi^4}k\left[\frac{1}{\alpha}-2\log(-k\tau_0)+\cdots\right].$$

γ_{ϕ^2} for different masses



$\lvert\nu\rvert$	γ_{ϕ^2}/λ
1/3	0.00333
2/5	0.00483
1/2	$\frac{1}{16\pi^2}$
2/3	0.00819
4/5	0.00967
1	$\frac{1}{8\pi^2}$
6/5	0.01882
13/10	0.02612
7/5	0.04753

Anomalous dimension of ϕ^2 for different masses. Larger values of $|\nu|$ correspond to smaller masses.

D. Green, AP
arXiv: 2001.05974

$$\nu \stackrel{\text{def.}}{=} i\sqrt{\frac{D^2}{4} - \frac{m^2}{H^2}}$$

Massless scalars

- No flat space analogue

A. Starobinsky, J. Yokoyama

- Framework: Stochastic Inflation

M. Baumgart, R. Sundrum

- Log resummation as a Fokker-Planck equation

- ϕ undergoes a random walk

$$\frac{\partial P(\phi,t)}{\partial t} = \frac{1}{3H}\frac{\partial}{\partial \phi}\left[V'(\phi)P(\phi,t)\right] + \frac{H^3}{8\pi^2}\frac{\partial^2}{\partial \phi^2}P(\phi,t)$$

$V(\phi) = \lambda\phi^4$

Drift

Diffusion

Stochastic Inflation at NNLO

T.Cohen, D. Green, AP, A. Ridgway

$$\frac{\partial}{\partial \mathtt{t}} P(\varphi_+, \mathtt{t}) = \frac{1}{3}\frac{\partial}{\partial \varphi_+}\left[V'_{\rm eff}(\varphi_+) P(\varphi_+, \mathtt{t})\right] + \frac{1}{8\pi^2}\frac{\partial^2}{\partial \varphi_+^2} P(\varphi_+, \mathtt{t}) + \frac{1}{1152\pi^2}\frac{\partial^3}{\partial \varphi_+^3}\left(\varphi_+ P(\varphi_+, \mathtt{t})\right)$$

$\phi \to H\varphi_+$

$$V'_{\rm eff} = \frac{\lambda}{3!}\left(\varphi_+^3 + \frac{\lambda}{18}\varphi_+^5 + \frac{\lambda^2}{162}\varphi_+^7 + \ldots\right).$$

- Soft de Sitter Effective Theory (SdSET)
- IR divergences of the 'full' theory → UV divergences of EFT.
- Phase transition to eternal inflation?

Conclusions

Loops in dS → 'Geometry' in s-space

- Generalize to higher loops
- Cosmological Bootstrap

A ‘doable’ example

$$\langle \phi^2(\vec{k},\tau_0)\phi^2(-\vec{k},\tau_0)\rangle = \frac{c}{2a(\tau_0)^{2\Delta_{\phi^2}}}\, k^{1-\epsilon}\left[1+\frac{\lambda}{32\pi^4 c}\left(\frac{1}{\epsilon}-\log\left(\frac{k}{a(\tau_0)H}\right)+\dots\right)\right]$$

$-k\tau_0$

$\log(-k\tau_0)$ blows up at late time $\tau_0 \to 0$ leading to a secular divergence.

Resummation of logs with DRG gives

$$\langle \phi^2(\vec{k},\tau_0)\phi^2(-\vec{k},\tau_0)\rangle : \underbrace{\frac{c}{2a(\tau_0)^{2\Delta_{\phi^2}}}\, k^{1-\epsilon}}_{\text{free}} \longrightarrow \underbrace{\frac{c\, H^{-2\gamma_{\phi^2}}}{2a(\tau_0)^{2\Delta_{\phi^2}+2\gamma_{\phi^2}}}\, k^{1-\epsilon+2\gamma_{\phi^2}}}_{\text{resummed}}$$

$$\Delta_{\phi^2} \to \Delta_{\phi^2}+\gamma_{\phi^2}$$

$$2-\epsilon \to 2-\epsilon+\frac{\lambda}{16\pi^2}$$

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T.Cohen, D. Green, AP, A. Ridgway

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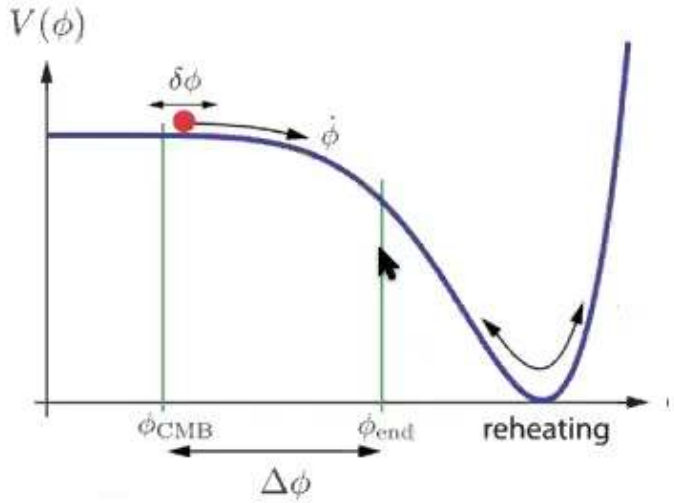
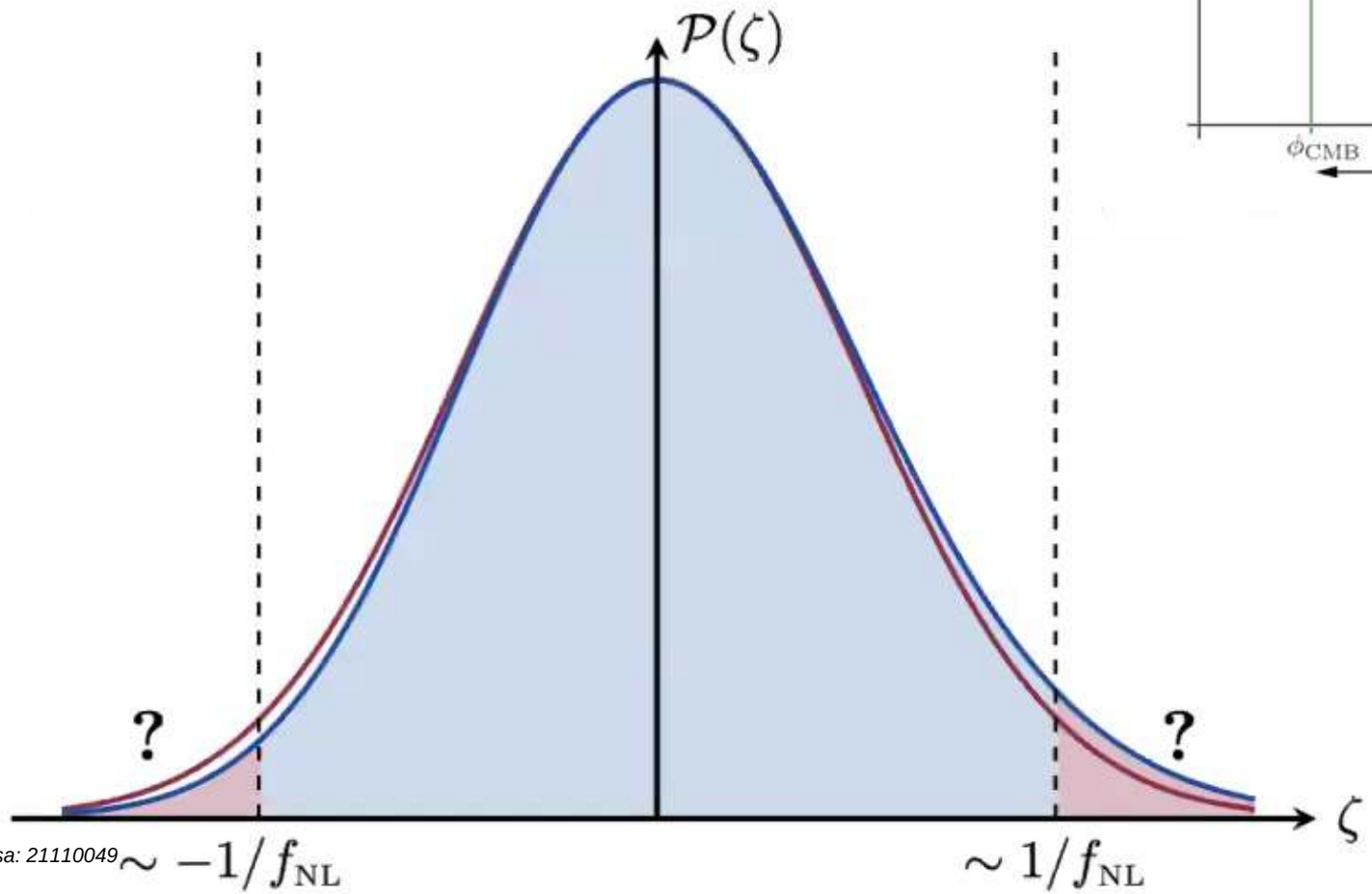
Eternal Inflation with non-Gaussianity





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