

Title: Gluon scattering in AdS from CFT

Speakers: Pietro Ferrero

Series: Quantum Fields and Strings

Date: November 30, 2021 - 2:00 PM

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Abstract: I will present a class of recently computed holographic correlators between half-BPS operators in a vast array of SCFTs with non-maximal superconformal symmetry in dimensions $d=3,4,5,6$. Via AdS/CFT, these four-point functions are dual to gluon scattering amplitudes in AdS. Exploiting the notion of MRV limit I will show that, at tree level, all such correlators are completely fixed by symmetries and consistency conditions. Our results encode a wealth of novel CFT data and exhibit various emergent structures, including Parisi-Sourlas supersymmetry, hidden conformal symmetry and color-kinematics duality. This talk will be based on <https://arxiv.org/pdf/2103.15830.pdf>.



Gluon scattering in AdS from CFT

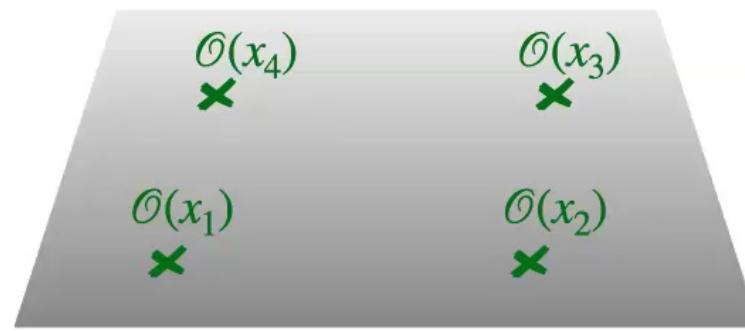
Based on work with

Fernando Alday, **Connor Behan** and **Xinan Zhou**:

arXiv: [2101.04114](https://arxiv.org/abs/2101.04114) [hep-th]
arXiv: [2103.15830](https://arxiv.org/abs/2103.15830) [hep-th]

AdS/CFT

Conformal field theory
on $\mathbb{R}^d = \partial \text{AdS}_{d+1}$

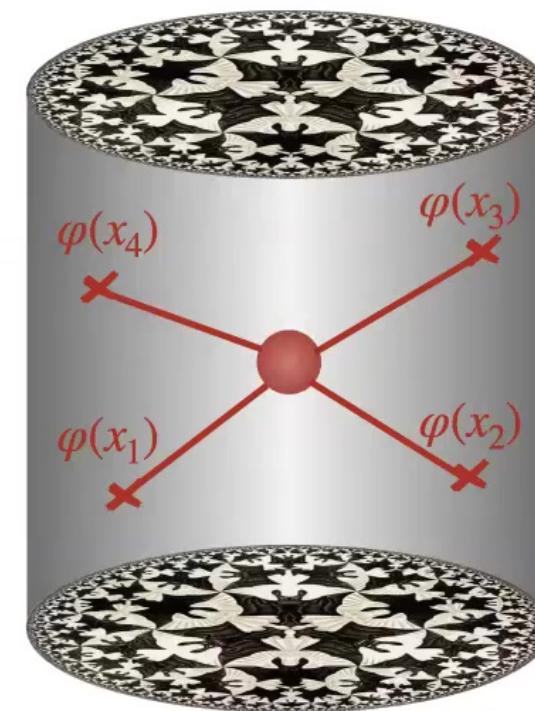


$$\langle \mathcal{O}(x_1) \mathcal{O}(x_2) \mathcal{O}(x_3) \mathcal{O}(x_4) \rangle$$

CFT correlator

$$\mathcal{O} \sim \varphi$$

 \iff



AdS scattering

Holographic 4-pt functions

$g \ll 1$: weakly coupled sugra in the bulk
e.g. $g = \frac{1}{N^2}$ in $\mathcal{N} = 4$ SYM

$$\langle 1234 \rangle = \text{Trivial} + g \text{ Here} + g^2 \text{ State of the art (only few cases)} + \dots$$

$\langle 1234 \rangle =$

+ g + g^2 + ...

Maximal SUSY

Background	SCFT	R symmetry
M-theory on $\text{AdS}_4 \times S^7$	$3d \mathcal{N} = 8$ ABJM	$\mathfrak{so}(8)$
IIB string theory on $\text{AdS}_5 \times S^5$	$4d \mathcal{N} = 4$ SYM	$\mathfrak{so}(6)$
M-theory on $\text{AdS}_7 \times S^4$	$6d \mathcal{N} = (2, 0)$	$\mathfrak{so}(5)$

Half-BPS operators: symmetric traceless of $\mathfrak{so}(n+1) \longrightarrow \mathcal{O}^{(i_1 \dots i_k)}$

$\mathbf{k = 2}$: CFT stress tensor multiplet \longleftrightarrow AdS graviton

$\mathbf{k > 2}$: Kaluza-Klein modes of the graviton on S^n

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k > 2 : Kaluza-Klein modes of the graviton on S^n

After 20 years of efforts, all $\langle k_1 k_2 k_3 k_4 \rangle$ correlators at tree level computed using a new idea: the **MRV limit**.

Alday, Zhou 2006.12505, 2006.06653 [hep-th]

Here: half-maximal SUSY

Here we apply the idea of

MRV limit: a special configuration of R-symmetry polarizations, that simplifies the correlators

to a large class of

SCFTs with half-maximal SUSY in $d = 3, 4, 5, 6$

and obtain all

all $\langle k_1 k_2 k_3 k_4 \rangle$ correlators, dual to
gluon scattering amplitudes in AdS

Motivation



Gravity from conformal field theory

AdS/CFT can be used as a definition of **quantum gravity in AdS**.

$$\mathcal{A}(\alpha', g_s; s, t, u) = \text{Diagram with 2 loops} + g_s^2 \text{Diagram with 3 loops} + g_s^4 \text{Diagram with 4 loops} + \dots$$
$$\mathcal{A}^{(g=0)}(\alpha'; s, t) = \underbrace{\frac{1}{s t u}}_{\text{2-derivative Supergravity}} + \underbrace{\alpha'^3}_{\text{Higher-derivative String corrections}} + \underbrace{\alpha'^5 (s^2 + t^2 + u^2)}_{\text{Higher-derivative String corrections}} + \dots$$

Loops

We can use **CFT techniques** to study

- The effect of **quantum gravity** in AdS
- **Effective actions** of string/M-theory perturbatively

Motivation



Scattering amplitudes program in AdS

For **scattering amplitudes in flat space**: rich story, plenty of interesting physical and mathematical **structures** hidden in Lagrangian description.

Can learn **lessons about QFTs** independently of Lagrangian!

A natural question: **what happens in curved space?**
Simplest case to look at is **AdS**.

- Generalizations of the **structures of flat space amplitudes?**
e.g. color-kinematics duality, double copy, MHV limit, CHY formulae, ...
- **New hidden features** for AdS amplitudes?
e.g. Parisi-Sourlas dimensional reduction, hidden conformal symmetry, ...

Motivation



Gravity from conformal field theory

AdS/CFT can be used as a definition of **quantum gravity in AdS**.

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$$\mathcal{A}^{(g=0)}(\alpha'; s, t) = \underbrace{\frac{1}{s t u}}_{\text{2-derivative Supergravity}} + \underbrace{\alpha'^3}_{\text{Higher-derivative String corrections}} + \underbrace{\alpha'^5 (s^2 + t^2 + u^2)}_{\text{Higher-derivative String corrections}} + \dots$$

Loops

We can use **CFT techniques** to study

- The effect of **quantum gravity** in AdS
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A brief history

The traditional method

In principle

Write effective **supergravity Lagrangian** for Kaluza-Klein modes up to quartic interactions



Extract “**Feynman rules**”



Add all the relevant **Witten diagrams** to obtain the result

In practice...

Really hard to make progress this way!

Still, great efforts led to the computation of:

- $AdS_5 \times S^5$: a bunch of results, no organizing principle
D'Hoker, Freedman, Mathur, Matusis, Rastelli, Arutyunov, Dolan, Frolov, Osborn, Sokatchev, Berdichevsky, Naaijkens, Urchurtu, Nirschl ...
- $AdS_7 \times S^4$: only $\langle 2222 \rangle$
Arutyunov, Sokatchev hep-th/0201145
- $AdS_4 \times S^7$: nothing, no closed form for exchange in position space

Mellin space

$$\langle \mathcal{O}_\Delta(x_1) \mathcal{O}_\Delta(x_2) \mathcal{O}_\Delta(x_3) \mathcal{O}_\Delta(x_4) \rangle = \frac{1}{x_{12}^{2\Delta} x_{34}^\Delta} \mathcal{G}(U, V) \quad U = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \quad V = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

$$\mathcal{G}(U, V) = \int_{-i\infty}^{i\infty} \frac{ds dt}{(4\pi i)^2} U^{\frac{s}{2}} V^{\frac{t}{2}} \Gamma\left[\Delta - \frac{s}{2}\right]^2 \Gamma\left[\Delta - \frac{t}{2}\right]^2 \Gamma\left[\Delta - \frac{u}{2}\right]^2 \mathcal{M}(s, t)$$

Mellin amplitude

s, t, u are the analogue of **Mandelstam variables** of flat space scattering:

$$s \sim -(p_1 + p_2)^2, \quad t \sim -(p_1 + p_3)^2, \quad u \sim -(p_1 + p_4)^2$$

$$s + t + u = 4\Delta$$

In fact:

$$\lim_{s \rightarrow \infty, t \rightarrow \infty} \mathcal{M}(s, t) = \mathcal{A}^{(\text{flat})}(s, t)$$

[Mack 0907.2407 \[hep-th\]](#)

[Penedones 1011.1485 \[hep-th\]](#)

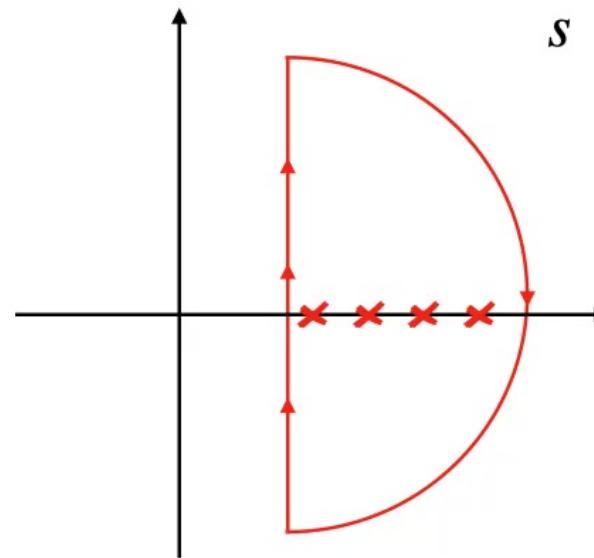
[Fitzpatrick, Kaplan 1111.6972 \[hep-th\]](#)

[Fitzpatrick, Kaplan, Penedones, Raju, van Rees 1107.1499 \[hep-th\]](#)

Mellin space

$$\mathcal{G}(U, V) = \int_{-i\infty}^{i\infty} \frac{ds dt}{(4\pi i)^2} U^{\frac{s}{2}} V^{\frac{t}{2}} \Gamma\left[\Delta - \frac{s}{2}\right]^2 \Gamma\left[\Delta - \frac{t}{2}\right]^2 \Gamma\left[\Delta - \frac{u}{2}\right]^2 \mathcal{M}(s, t)$$

Poles are what matters:



Mellin space

$$\mathcal{G}(U, V) = \int_{-i\infty}^{i\infty} \frac{ds dt}{(4\pi i)^2} U^{\frac{s}{2}} V^{\frac{t}{2}} \underbrace{\Gamma\left[\Delta - \frac{s}{2}\right]^2 \Gamma\left[\Delta - \frac{t}{2}\right]^2 \Gamma\left[\Delta - \frac{u}{2}\right]^2}_{\mathcal{M}(s, t)}$$

Double poles \leftrightarrow double trace operators

$$\mathcal{O} \times \mathcal{O} \sim 1 + ST + \mathcal{O} \square^n \partial_{\mu_1} \cdots \partial_{\mu_\ell} \mathcal{O}$$

At tree level

$$\mathcal{G}^{(1)}(U, V) \sim \sum_{n,l} U^{\Delta+n} \left(a_{n,\ell}^{(1)} + \frac{1}{2} a_{n,\ell}^{(0)} \gamma_{n,\ell}^{(1)} \left(\log U + \frac{\partial}{\partial n} \right) \right) g_{n,\ell}(U, V)$$

$$\text{Res}_{s=a} \left(U^s \frac{F(s)}{s-a} \right) = U^a F(a)$$

**Tree level
Anomalous dimensions**

$$\text{Res}_{s=a} \left(U^s \frac{F(s)}{(s-a)^2} \right) = U^a \left(F'(a) + F(a) \log U \right)$$

Mellin space

$$\mathcal{G}(U, V) = \int_{-i\infty}^{i\infty} \frac{ds dt}{(4\pi i)^2} U^{\frac{s}{2}} V^{\frac{t}{2}} \Gamma\left[\Delta - \frac{s}{2}\right]^2 \Gamma\left[\Delta - \frac{t}{2}\right]^2 \Gamma\left[\Delta - \frac{u}{2}\right]^2 \mathcal{M}(s, t)$$



Contact diagrams: $\mathcal{L}_{AdS} \supset \partial^{2L} \phi^4$

$\mathcal{M}(s, t) = \text{Poly}^{(L)}(s, t) \quad \Rightarrow \quad \text{only double trace operators in the OPE}$



Exchange diagrams: $\mathcal{L}_{AdS} \supset \phi^3$

$$\mathcal{M}_{\Delta, \ell}^{(s)}(s, t) = \sum_{m=0}^{\infty} \frac{Q_m^{(\ell)}(t, u)}{s - \Delta + \ell - 2m} + P^{(\ell-1)}(s, t)$$

Single pole: exchange of **single trace** operator (+ descendants)

Bootstrap methods

Forget the Lagrangian!

Superconformal invariance + crossing symmetry + consistency conditions

- $AdS_5 \times S^5$: full answer

Rastelli, Zhou [1608.06624](#), [1710.05923](#) [hep-th]

- $AdS_7 \times S^4$: partial results

Rastelli, Zhou [1712.02788](#) [hep-th]

- $AdS_4 \times S^7$: partial results

Rastelli, Zhou [1712.02800](#) [hep-th]

However: no organizing principle, especially for M-theory backgrounds...

The MRV limit

MRV = Maximally R-symmetry Violating

$$\begin{aligned} & (1032192 - 184320 s - 559104 t + 99840 s t + 96768 t^2 - 17280 s t^2 - 5376 t^3 + 960 s t^3 - 559104 u + 99840 s u + 302848 t u - 54080 s t u - 52416 t^2 u + \\ & 9360 s t^2 u + 2912 t^3 u - 520 s t^3 u + 96768 u^2 - 17280 s u^2 - 52416 t u^2 + 9360 s t u^2 + 9072 t^2 u^2 - 1620 s t^2 u^2 - 504 t^3 u^2 + 90 s t^3 u^2 - \\ & 5376 u^3 + 960 s u^3 + 2912 t u^3 - 520 s t u^3 - 504 t^2 u^3 + 90 s t^2 u^3 + 28 t^3 u^3 - 5 s t^3 u^3 - 4644864 \sigma + 1511424 s \sigma - 150528 s^2 \sigma + 5376 s^3 \sigma + \\ & 2774016 t \sigma - 895488 s t \sigma + 86912 s^2 t \sigma - 2912 s^3 t \sigma - 575232 t^2 \sigma + 183296 s t^2 \sigma - 17024 s^2 t^2 \sigma + 504 s^3 t^2 \sigma + 48384 t^3 \sigma - 15072 s t^3 \sigma + \\ & 1288 s^2 t^3 \sigma - 28 s^3 t^3 \sigma - 1344 t^4 \sigma + 400 s t^4 \sigma - 28 s^2 t^4 \sigma + 1511424 u \sigma - 391680 s u \sigma + 26880 s^2 u \sigma - 960 s^3 u \sigma - 895488 t u \sigma + \\ & 231360 s t u \sigma - 15520 s^2 t u \sigma + 520 s^3 t u \sigma + 183296 t^2 u \sigma - 47120 s t^2 u \sigma + 3040 s^2 t^2 u \sigma - 90 s^3 t^2 u \sigma - 15072 t^3 u \sigma + 3840 s t^3 u \sigma - \\ & 230 s^2 t^3 u \sigma + 5 s^3 t^3 u \sigma + 400 t^4 u \sigma - 100 s t^4 u \sigma + 5 s^2 t^4 u \sigma - 150528 u^2 \sigma + 26880 s u^2 \sigma + 86912 t u^2 \sigma - 15520 s t u^2 \sigma - 17024 t^2 u^2 \sigma + \\ & 3040 s t^2 u^2 \sigma + 1288 t^3 u^2 \sigma - 230 s t^3 u^2 \sigma - 28 t^4 u^2 \sigma + 5 s t^4 u^2 \sigma + 5376 u^3 \sigma - 960 s u^3 \sigma - 2912 t u^3 \sigma + 520 s t u^3 \sigma + 504 t^2 u^3 \sigma - 90 s t^2 u^3 \sigma - \\ & 28 t^3 u^3 \sigma + 5 s t^3 u^3 \sigma + 1032192 \sigma^2 - 559104 s \sigma^2 + 96768 s^2 \sigma^2 - 5376 s^3 \sigma^2 - 559104 t \sigma^2 + 302848 s t \sigma^2 - 52416 s^2 t \sigma^2 + 2912 s^3 t \sigma^2 + \\ & 96768 t^2 \sigma^2 - 52416 s t^2 \sigma^2 + 9072 s^2 t^2 \sigma^2 - 504 s^3 t^2 \sigma^2 - 5376 t^3 \sigma^2 + 2912 s t^3 \sigma^2 - 504 s^2 t^3 \sigma^2 + 28 s^3 t^3 \sigma^2 - 184320 u \sigma^2 + 99840 s u \sigma^2 - \\ & 17280 s^2 u \sigma^2 + 960 s^3 u \sigma^2 + 99840 t u \sigma^2 - 54080 s t u \sigma^2 + 9360 s^2 t u \sigma^2 - 520 s^3 t u \sigma^2 - 17280 t^2 u \sigma^2 + 9360 s t^2 u \sigma^2 - 1620 s^2 t^2 u \sigma^2 + \\ & 90 s^3 t^2 u \sigma^2 + 960 t^3 u \sigma^2 - 520 s t^3 u \sigma^2 + 90 s^2 t^3 u \sigma^2 - 5 s^3 t^3 u \sigma^2 - 4644864 \tau + 1511424 s \tau - 150528 s^2 \tau + 5376 s^3 \tau + 1511424 t \tau - \\ & 391680 s t \tau + 26880 s^2 t \tau - 960 s^3 t \tau - 150528 t^2 \tau + 26880 s t^2 \tau + 5376 t^3 \tau - 960 s t^3 \tau + 2774016 u \tau - 895488 s u \tau + 86912 s^2 u \tau - \\ & 2912 s^3 u \tau - 895488 t u \tau + 231360 s t u \tau - 15520 s^2 t u \tau + 520 s^3 t u \tau + 86912 t^2 u \tau - 15520 s t^2 u \tau - 2912 t^3 u \tau + 520 s t^3 u \tau - 575232 u^2 \tau + \\ & 183296 s u^2 \tau - 17024 s^2 u^2 \tau + 504 s^3 u^2 \tau + 183296 t u^2 \tau - 47120 s t u^2 \tau + 3040 s^2 t u^2 \tau - 90 s^3 t u^2 \tau - 17024 t^2 u^2 \tau + 3040 s t^2 u^2 \tau + \\ & 504 t^3 u^2 \tau - 90 s t^3 u^2 \tau + 48384 u^3 \tau - 15072 s u^3 \tau + 1288 s^2 u^3 \tau - 28 s^3 u^3 \tau - 15072 t u^3 \tau + 3840 s t u^3 \tau - 230 s^2 t u^3 \tau + 5 s^3 t u^3 \tau + \\ & 1288 t^2 u^3 \tau - 230 s t^2 u^3 \tau - 28 t^3 u^3 \tau + 5 s t^3 u^3 \tau - 1344 u^4 \tau + 400 s u^4 \tau - 28 s^2 u^4 \tau + 400 t u^4 \tau - 100 s t u^4 \tau + 5 s^2 t u^4 \tau - 28 t^2 u^4 \tau + \\ & 5 s t^2 u^4 \tau - 4644864 s \tau + 2774016 s t \tau - 575232 s^2 \sigma \tau + 48384 s^3 \sigma \tau - 1344 s^4 \sigma \tau + 1511424 t \sigma \tau - 895488 s t \sigma \tau + 183296 s^2 t \sigma \tau - \\ & 15072 s^3 t \sigma \tau + 400 s^4 t \sigma \tau - 150528 t^2 \sigma \tau + 86912 s t^2 \sigma \tau - 17024 s^2 t^2 \sigma \tau + 1288 s^3 t^2 \sigma \tau - 28 s^4 t^2 \sigma \tau + 5376 t^3 \sigma \tau - 2912 s t^3 \sigma \tau + \\ & 504 s^2 t^3 \sigma \tau - 28 s^3 t^3 \sigma \tau + 1511424 u \sigma \tau - 895488 s u \sigma \tau + 183296 s^2 u \sigma \tau - 15072 s^3 u \sigma \tau + 400 s^4 u \sigma \tau - 391680 t u \sigma \tau + 231360 s t u \sigma \tau - \\ & 47120 s^2 t u \sigma \tau + 3840 s^3 t u \sigma \tau - 100 s^4 t u \sigma \tau + 26880 t^2 u \sigma \tau - 15520 s t^2 u \sigma \tau + 3040 s^2 t^2 u \sigma \tau - 230 s^3 t^2 u \sigma \tau + 5 s^4 t^2 u \sigma \tau - 960 t^3 u \sigma \tau + \\ & 520 s t^3 u \sigma \tau - 90 s^2 t^3 u \sigma \tau + 5 s^3 t^3 u \sigma \tau - 150528 u^2 \sigma \tau + 86912 s u^2 \sigma \tau - 17024 s^2 u^2 \sigma \tau + 1288 s^3 u^2 \sigma \tau - 28 s^4 u^2 \sigma \tau + 26880 t u^2 \sigma \tau - \\ & 15520 s t u^2 \sigma \tau + 3040 s^2 t u^2 \sigma \tau - 230 s^3 t u^2 \sigma \tau + 5 s^4 t u^2 \sigma \tau + 5376 u^3 \sigma \tau - 2912 s u^3 \sigma \tau + 504 s^2 u^3 \sigma \tau - 28 s^3 u^3 \sigma \tau - 960 t u^3 \sigma \tau + \\ & 520 s t u^3 \sigma \tau - 90 s^2 t u^3 \sigma \tau + 5 s^3 t u^3 \sigma \tau + 1032192 \tau^2 - 559104 s \tau^2 + 96768 s^2 \tau^2 - 5376 s^3 \tau^2 - 184320 t \tau^2 + 99840 s t \tau^2 - 17280 s^2 t \tau^2 + \\ & 960 s^3 t \tau^2 - 559104 u \tau^2 + 302848 s u \tau^2 - 52416 s^2 t u \tau^2 + 2912 s^3 t u \tau^2 + 99840 t u \tau^2 - 54080 s t u \tau^2 + 9360 s^2 t u \tau^2 - 520 s^3 t u \tau^2 + \\ & 96768 u^2 \tau^2 - 52416 s u^2 \tau^2 + 9072 s^2 u^2 \tau^2 - 504 s^3 u^2 \tau^2 - 17280 t u^2 \tau^2 + 9360 s t u^2 \tau^2 - 1620 s^2 t u^2 \tau^2 + 90 s^3 t u^2 \tau^2 - 5376 u^3 \tau^2 + 2912 s u^3 \tau^2 - \\ & 504 s^2 u^3 \tau^2 + 28 s^3 u^3 \tau^2 + 960 t u^3 \tau^2 - 520 s t u^3 \tau^2 + 90 s^2 t u^3 \tau^2 - 5 s^3 t u^3 \tau^2) / (4 (-6 + s) (-4 + s) (-6 + t) (-4 + t) (-6 + u) (-4 + u) n^3) \end{aligned}$$

The MRV limit

MRV = Maximally R-symmetry Violating

$$\mathcal{O}_k(x; t) = \mathcal{O}(x)^{(i_1 \dots i_k)} t_{i_1} \dots t_{i_k}$$

$$\sigma = \frac{t_{13} t_{24}}{t_{12} t_{34}}, \quad \tau = \frac{t_{14} t_{23}}{t_{12} t_{34}}$$

$$\begin{aligned}
& (1832192 - 184329 s - 559104 t + 99440 s t - 95768 t^2 - 17280 s t^3 - 968 s t^4 - 559104 u + 99440 s u - 382448 t u - 544880 s t u - 52416 t^2 u + \\
& 9369 s^2 t^3 u - 2912 t^2 u - 528 s t^4 u - 96768 u^2 - 17280 s u^3 - 52416 s u^4 - 9369 s t u^2 - 9872 t^2 u^3 - 1620 s t^3 u^4 - 504 s^2 t^4 u^5 - 98 s^3 t^5 u^6 + \\
& 5376 u^7 - 968 s^2 t^6 u - 2912 t^2 u^7 - 528 s t^4 u^8 - 504 s^2 t^6 u^9 - 98 s^3 t^8 u^10 - 28 s^4 t^10 u^11 - 5 s^5 t^12 u^13 - 464 s^6 t^14 u^15 - 1331424 s^8 u^17 - 159328 s^9 u^19 - 33764 s^10 u^21 + \\
& 2774816 t u - 895488 s t u + 86912 s^2 t u - 2912 s^2 t^2 u - 575223 t^3 u - 183296 s t^4 u - 17024 s^2 t^5 u - 504 s^3 t^6 u - 48384 t^7 u - 15872 s^4 t^8 u + \\
& 1288 s^5 t^9 u - 28 s^6 t^10 u - 1344 s^7 t^11 u - 400 s^8 t^12 u - 26 s^9 t^13 u - 1304 s^10 t^14 u - 3946 s^11 t^15 u - 960 s^12 t^16 u - 895488 t u + \\
& 2313692 s t u - 19520 s^2 t u - 528 s^3 t^2 u - 183296 s^4 t^3 u - 47128 s^5 t^4 u - 98 s^6 t^5 u - 15072 s^7 t^6 u - 3840 s^8 t^7 u - \\
& 238 s^9 t^8 u - 5 s^10 t^9 u - 400 s^11 t^10 u - 26 s^12 t^11 u - 88912 s t^12 u - 15329 s^2 t^13 u - 124 s^3 t^14 u - \\
& 3080 s^4 t^15 u - 12388 s^5 t^16 u - 5376 s^6 t^17 u - 988 s^7 t^18 u - 2912 s^8 t^19 u - 504 s^9 t^20 u - 98 s^10 t^21 u - \\
& 28 s^11 t^22 u - 5 s^12 t^23 u - 1344 s^13 t^24 u - 558184 s^14 t^25 u - 96768 s^15 t^26 u - 5376 s^16 t^27 u - 504 s^17 t^28 u - 48384 s^18 t^29 u - 2912 s^19 t^30 u - \\
& 96768 s^20 t^31 u - 124 s^21 t^32 u - 504 s^22 t^33 u - 5376 s^23 t^34 u - 504 s^24 t^35 u - 48384 s^25 t^36 u - 2912 s^26 t^37 u - \\
& 96768 s^27 t^38 u - 124 s^28 t^39 u - 504 s^29 t^40 u - 5376 s^30 t^41 u - 504 s^31 t^42 u - 48384 s^32 t^43 u - 2912 s^33 t^44 u - \\
& 96768 s^34 t^45 u - 124 s^35 t^46 u - 504 s^36 t^47 u - 5376 s^37 t^48 u - 504 s^38 t^49 u - 48384 s^39 t^50 u - 2912 s^40 t^51 u - \\
& 96768 s^41 t^52 u - 124 s^42 t^53 u - 504 s^43 t^54 u - 5376 s^44 t^55 u - 504 s^45 t^56 u - 48384 s^46 t^57 u - 2912 s^47 t^58 u - \\
& 96768 s^48 t^59 u - 124 s^49 t^60 u - 504 s^50 t^61 u - 5376 s^51 t^62 u - 504 s^52 t^63 u - 48384 s^53 t^64 u - 2912 s^54 t^65 u - \\
& 96768 s^56 t^66 u - 124 s^57 t^67 u - 504 s^58 t^68 u - 5376 s^59 t^69 u - 504 s^60 t^70 u - 48384 s^61 t^71 u - 2912 s^62 t^72 u - \\
& 96768 s^64 t^73 u - 124 s^65 t^74 u - 504 s^66 t^75 u - 5376 s^67 t^76 u - 504 s^68 t^77 u - 48384 s^69 t^78 u - 2912 s^70 t^79 u - \\
& 96768 s^76 t^80 u - 124 s^77 t^81 u - 504 s^78 t^82 u - 5376 s^79 t^83 u - 504 s^80 t^84 u - 48384 s^81 t^85 u - 2912 s^82 t^86 u - \\
& 96768 s^88 t^87 u - 124 s^89 t^88 u - 504 s^90 t^89 u - 5376 s^91 t^90 u - 504 s^92 t^91 u - 48384 s^93 t^92 u - 2912 s^94 t^93 u - \\
& 96768 s^96 t^94 u - 124 s^97 t^95 u - 504 s^98 t^96 u - 5376 s^99 t^97 u - 504 s^100 t^98 u - 48384 s^101 t^99 u - 2912 s^102 t^100 u - \\
& 96768 s^104 t^101 u - 124 s^105 t^102 u - 504 s^106 t^103 u - 5376 s^107 t^104 u - 504 s^108 t^105 u - 48384 s^109 t^106 u - 2912 s^110 t^107 u - \\
& 96768 s^118 t^108 u - 124 s^119 t^109 u - 504 s^120 t^110 u - 5376 s^121 t^111 u - 504 s^122 t^112 u - 48384 s^123 t^113 u - 2912 s^124 t^114 u - \\
& 96768 s^126 t^115 u - 124 s^127 t^116 u - 504 s^128 t^117 u - 5376 s^129 t^118 u - 504 s^130 t^119 u - 48384 s^131 t^120 u - 2912 s^132 t^121 u - \\
& 96768 s^134 t^122 u - 124 s^135 t^123 u - 504 s^136 t^124 u - 5376 s^137 t^125 u - 504 s^138 t^126 u - 48384 s^139 t^127 u - 2912 s^140 t^128 u - \\
& 96768 s^142 t^129 u - 124 s^143 t^130 u - 504 s^144 t^131 u - 5376 s^145 t^132 u - 504 s^146 t^133 u - 48384 s^147 t^134 u - 2912 s^148 t^135 u - \\
& 96768 s^150 t^136 u - 124 s^151 t^137 u - 504 s^152 t^138 u - 5376 s^153 t^139 u - 504 s^154 t^140 u - 48384 s^155 t^141 u - 2912 s^156 t^142 u - \\
& 96768 s^158 t^143 u - 124 s^159 t^144 u - 504 s^160 t^145 u - 5376 s^161 t^146 u - 504 s^162 t^147 u - 48384 s^163 t^148 u - 2912 s^164 t^149 u - \\
& 96768 s^166 t^150 u - 124 s^167 t^151 u - 504 s^168 t^152 u - 5376 s^169 t^153 u - 504 s^170 t^154 u - 48384 s^171 t^155 u - 2912 s^172 t^156 u - \\
& 96768 s^174 t^157 u - 124 s^175 t^158 u - 504 s^176 t^159 u - 5376 s^177 t^160 u - 504 s^178 t^161 u - 48384 s^179 t^162 u - 2912 s^180 t^163 u - \\
& 96768 s^182 t^164 u - 124 s^183 t^165 u - 504 s^184 t^166 u - 5376 s^185 t^167 u - 504 s^186 t^168 u - 48384 s^187 t^169 u - 2912 s^188 t^170 u - \\
& 96768 s^190 t^171 u - 124 s^191 t^172 u - 504 s^192 t^173 u - 5376 s^193 t^174 u - 504 s^194 t^175 u - 48384 s^195 t^176 u - 2912 s^196 t^177 u - \\
& 96768 s^198 t^178 u - 124 s^199 t^179 u - 504 s^200 t^180 u - 5376 s^201 t^181 u - 504 s^202 t^182 u - 48384 s^203 t^183 u - 2912 s^204 t^184 u - \\
& 96768 s^206 t^185 u - 124 s^207 t^186 u - 504 s^208 t^187 u - 5376 s^209 t^188 u - 504 s^210 t^189 u - 48384 s^211 t^190 u - 2912 s^212 t^191 u - \\
& 96768 s^214 t^192 u - 124 s^215 t^193 u - 504 s^216 t^194 u - 5376 s^217 t^195 u - 504 s^218 t^196 u - 48384 s^219 t^197 u - 2912 s^220 t^198 u - \\
& 96768 s^222 t^199 u - 124 s^223 t^200 u - 504 s^224 t^201 u - 5376 s^225 t^202 u - 504 s^226 t^203 u - 48384 s^227 t^204 u - 2912 s^228 t^205 u - \\
& 96768 s^230 t^206 u - 124 s^231 t^207 u - 504 s^232 t^208 u - 5376 s^233 t^209 u - 504 s^234 t^210 u - 48384 s^235 t^211 u - 2912 s^236 t^212 u - \\
& 96768 s^238 t^213 u - 124 s^239 t^214 u - 504 s^240 t^215 u - 5376 s^241 t^216 u - 504 s^242 t^217 u - 48384 s^243 t^218 u - 2912 s^244 t^219 u - \\
& 96768 s^246 t^220 u - 124 s^247 t^221 u - 504 s^248 t^222 u - 5376 s^249 t^223 u - 504 s^250 t^224 u - 48384 s^251 t^225 u - 2912 s^252 t^226 u - \\
& 96768 s^254 t^227 u - 124 s^255 t^228 u - 504 s^256 t^229 u - 5376 s^257 t^230 u - 504 s^258 t^231 u - 48384 s^259 t^232 u - 2912 s^260 t^233 u - \\
& 96768 s^262 t^234 u - 124 s^263 t^235 u - 504 s^264 t^236 u - 5376 s^265 t^237 u - 504 s^266 t^238 u - 48384 s^267 t^239 u - 2912 s^268 t^240 u - \\
& 96768 s^270 t^241 u - 124 s^271 t^242 u - 504 s^272 t^243 u - 5376 s^273 t^244 u - 504 s^274 t^245 u - 48384 s^275 t^246 u - 2912 s^276 t^247 u - \\
& 96768 s^278 t^248 u - 124 s^279 t^249 u - 504 s^280 t^250 u - 5376 s^281 t^251 u - 504 s^282 t^252 u - 48384 s^283 t^253 u - 2912 s^284 t^254 u - \\
& 96768 s^286 t^255 u - 124 s^287 t^256 u - 504 s^288 t^257 u - 5376 s^289 t^258 u - 504 s^290 t^259 u - 48384 s^291 t^260 u - 2912 s^292 t^261 u - \\
& 96768 s^294 t^262 u - 124 s^295 t^263 u - 504 s^296 t^264 u - 5376 s^297 t^265 u - 504 s^298 t^266 u - 48384 s^299 t^267 u - 2912 s^300 t^268 u - \\
& 96768 s^302 t^269 u - 124 s^303 t^270 u - 504 s^304 t^271 u - 5376 s^305 t^272 u - 504 s^306 t^273 u - 48384 s^307 t^274 u - 2912 s^308 t^275 u - \\
& 96768 s^310 t^276 u - 124 s^311 t^277 u - 504 s^312 t^278 u - 5376 s^313 t^279 u - 504 s^314 t^280 u - 48384 s^315 t^281 u - 2912 s^316 t^282 u - \\
& 96768 s^318 t^283 u - 124 s^319 t^284 u - 504 s^320 t^285 u - 5376 s^321 t^286 u - 504 s^322 t^287 u - 48384 s^323 t^288 u - 2912 s^324 t^289 u - \\
& 96768 s^326 t^290 u - 124 s^327 t^291 u - 504 s^328 t^292 u - 5376 s^329 t^293 u - 504 s^330 t^294 u - 48384 s^331 t^295 u - 2912 s^332 t^296 u - \\
& 96768 s^334 t^297 u - 124 s^335 t^298 u - 504 s^336 t^299 u - 5376 s^337 t^300 u - 504 s^338 t^301 u - 48384 s^339 t^302 u - 2912 s^340 t^303 u - \\
& 96768 s^342 t^304 u - 124 s^343 t^305 u - 504 s^344 t^306 u - 5376 s^345 t^307 u - 504 s^346 t^308 u - 48384 s^347 t^309 u - 2912 s^348 t^310 u - \\
& 96768 s^350 t^311 u - 124 s^351 t^312 u - 504 s^352 t^313 u - 5376 s^353 t^314 u - 504 s^354 t^315 u - 48384 s^355 t^316 u - 2912 s^356 t^317 u - \\
& 96768 s^358 t^318 u - 124 s^359 t^319 u - 504 s^360 t^320 u - 5376 s^361 t^321 u - 504 s^362 t^322 u - 48384 s^363 t^323 u - 2912 s^364 t^324 u - \\
& 96768 s^366 t^325 u - 124 s^367 t^326 u - 504 s^368 t^327 u - 5376 s^369 t^328 u - 504 s^370 t^329 u - 48384 s^371 t^330 u - 2912 s^372 t^331 u - \\
& 96768 s^374 t^332 u - 124 s^375 t^333 u - 504 s^376 t^334 u - 5376 s^377 t^335 u - 504 s^378 t^336 u - 48384 s^379 t^337 u - 2912 s^380 t^338 u - \\
& 96768 s^382 t^339 u - 124 s^383 t^340 u - 504 s^384 t^341 u - 5376 s^385 t^342 u - 504 s^386 t^343 u - 48384 s^387 t^344 u - 2912 s^388 t^345 u - \\
& 96768 s^390 t^346 u - 124 s^391 t^347 u - 504 s^392 t^348 u - 5376 s^393 t^349 u - 504 s^394 t^350 u - 48384 s^395 t^351 u - 2912 s^396 t^352 u - \\
& 96768 s^398 t^353 u - 124 s^399 t^354 u - 504 s^400 t^355 u - 5376 s^401 t^356 u - 504 s^402 t^357 u - 48384 s^403 t^358 u - 2912 s^404 t^359 u - \\
& 96768 s^406 t^360 u - 124 s^407 t^361 u - 504 s^408 t^362 u - 5376 s^409 t^363 u - 504 s^410 t^364 u - 48384 s^411 t^365 u - 2912 s^412 t^366 u - \\
& 96768 s^414 t^367 u - 124 s^415 t^368 u - 504 s^416 t^369 u - 5376 s^417 t^370 u - 504 s^418 t^371 u - 48384 s^419 t^372 u - 2912 s^420 t^373 u - \\
& 96768 s^422 t^374 u - 124 s^423 t^375 u - 504 s^424 t^376 u - 5376 s^425 t^377 u - 504 s^426 t^378 u - 48384 s^427 t^379 u - 2912 s^428 t^380 u - \\
& 96768 s^430 t^381 u - 124 s^431 t^382 u - 504 s^432 t^383 u - 5376 s^433 t^384 u - 504 s^434 t^385 u - 48384 s^435 t^386 u - 2912 s^436 t^387 u - \\
& 96768 s^438 t^388 u - 124 s^439 t^389 u - 504 s^440 t^390 u - 5376 s^441 t^391 u - 504 s^442 t^392 u - 48384 s^443 t^393 u - 2912 s^444 t^394 u - \\
& 96768 s^446 t^395 u - 124 s^447 t^396 u - 504 s^448 t^397 u - 5376 s^449 t^398 u - 504 s^450 t^399 u - 48384 s^451 t^400 u - 2912 s^452 t^401 u - \\
& 96768 s^454 t^402 u - 124 s^455 t^403 u - 504 s^456 t^404 u - 5376 s^457 t^405 u - 504 s^458 t^406 u - 48384 s^459 t^407 u - 2912 s^460 t^408 u - \\
& 96768 s^462 t^409 u - 124 s^463 t^410 u - 504 s^464 t^411 u - 5376 s^465 t^412 u - 504 s^466 t^413 u - 48384 s^467 t^414 u - 2912 s^468 t^415 u - \\
& 96768 s^470 t^416 u - 124 s^471 t^417 u - 504 s^472 t^418 u - 5376 s^473 t^419 u - 504 s^474 t^420 u - 48384 s^475 t^421 u - 2912 s^476 t^422 u - \\
& 96768 s^478 t^423 u - 124 s^479 t^424 u - 504 s^480 t^425 u - 5376 s^481 t^426 u - 504 s^482 t^427 u - 48384 s^483 t^428 u - 2912 s^484 t^429 u - \\
& 96768 s^486 t^430 u - 124 s^487 t^431 u - 504 s^488 t^432 u - 5376 s^489 t^433 u - 504 s^490 t^434 u - 48384 s^491 t^435 u - 2912 s^492 t^436 u - \\
& 96768 s^494 t^437 u - 124 s^495 t^438 u - 504 s^496 t^439 u - 5376 s^497 t^440 u - 504 s^498 t^441 u - 48384 s^499 t^442 u - 2912 s^500 t^443 u - \\
& 96768 s^502 t^444 u - 124 s^503 t^445 u - 504 s^504 t^446 u - 5376 s^505 t^447 u - 504 s^506 t^448 u - 48384 s^507 t^449 u - 2912 s^508 t^450 u - \\
& 96768 s^510 t^451 u - 124 s^511 t^452 u - 504 s^512 t^453 u - 5376 s^513 t^454 u - 504 s^514 t^455 u - 48384 s^515 t^456 u - 2912 s^516 t^457 u - \\
& 96768 s^518 t^458 u - 124 s^519 t^459 u - 504 s^520 t^460 u - 5376 s^521 t^461 u - 504 s^522 t^462 u - 48384 s^523 t^463 u - 2912 s^524 t^464 u - \\
& 96768 s^526 t^465 u - 124 s^527 t^466 u - 504 s^528 t^467 u - 5376 s^529 t^468 u - 504 s^530 t^469 u - 48384 s^531 t^470 u - 2912 s^532 t^471 u - \\
& 96768 s^534 t^472 u - 124 s^535 t^473 u - 504 s^536 t^474 u - 5376 s^537 t^475 u - 504 s^538 t^476 u - 48384 s^539 t^477 u - 2912 s^540 t^478 u - \\
& 96768 s^542 t^479 u - 124 s^543 t^480 u - 504 s^544 t^481 u - 5376 s^545 t^482 u - 504 s^546 t^483 u - 48384 s^547 t^484 u - 2912 s^548 t^485 u - \\
& 96768 s^550 t^486 u - 124 s^551 t^487 u - 504 s^552 t^488 u - 5376 s^553 t^489 u - 504 s^554 t^490 u - 48384 s^555 t^491 u - 2912 s^556 t^492 u - \\
& 96768 s^558 t^493 u - 124 s^559 t^494 u - 504 s^560 t^495 u - 5376 s^561 t^496 u - 504 s^562 t^497 u - 48384 s^563 t^498 u - 2912 s^564 t^499 u - \\
& 96768 s^566 t^500 u - 124 s^567 t^501 u - 504 s^568 t^502 u - 5376 s^569 t^503 u - 504 s^570 t^504 u - 48384 s^571 t^505 u - 2912 s^572 t^506 u - \\
& 96768 s^574 t^507 u - 124 s^575 t^508 u - 504 s^576 t^509 u - 5376 s^577 t^510 u - 504 s^578 t^511 u - 48384 s^579 t^512 u - 2912 s^580 t^513 u - \\
& 96768 s^582 t^514 u - 124 s^583 t^515 u - 504 s^584 t^516 u - 5376 s^585 t^517 u - 504 s^586 t^518 u - 48384 s^587 t^519 u - 2912 s^588 t^520 u - \\
& 96768 s^590 t^521 u - 124 s^591 t^522 u - 504 s^592 t^523 u - 5376 s^593 t^524 u - 504 s^594 t^525 u - 48384 s^595 t^526 u - 2912 s^596 t^527 u - \\
& 96768 s^598 t^528 u - 124 s^599 t^529 u - 504 s^600 t^530 u - 5376 s^601 t^531 u - 504 s^602 t^532 u - 48384 s^603 t^533 u - 2912 s^604 t^534 u - \\
& 96768 s^606 t^535 u - 124 s^607 t^536 u - 504 s^608 t^537 u - 5376 s^609 t^538 u - 504 s^610 t^539 u - 48384 s^611 t^540 u - 2912 s^612 t^541 u - \\
& 96768 s^614 t^542 u - 124 s^615 t^543 u - 504 s^616 t^544 u - 5376 s^617 t^545 u - 504 s^618 t^546 u - 48384 s^619 t^547 u - 2912 s^620 t^548 u - \\
& 96768 s^622 t^549 u - 124 s^623 t^550 u - 504 s^624 t^551 u - 5376 s^625 t^552 u - 504 s^626 t^553 u - 48384 s^627 t^554 u - 2912 s^628 t^555 u - \\
& 96768 s^630 t^556 u - 124 s^631 t^557 u - 504 s^632 t^558 u - 5376 s^633 t^559 u - 504 s^634 t^560 u - 48384 s^635 t^561 u - 2912 s^636 t^562 u - \\
& 96768 s^638 t^563 u - 124 s^639 t^564 u - 504 s^640 t^565 u - 5376 s^641 t^566 u - 504 s^642 t^567 u - 48384 s^643 t^568 u - 2912 s^644 t^569 u - \\
& 96768 s^646 t^570 u - 124 s^647 t^571 u - 504 s^648 t^572 u - 5376 s^649 t^573 u - 504 s^650 t^574 u - 48384 s^651 t^575 u - 2912 s^652 t^576 u - \\
& 96768 s^654 t^577 u - 124 s^655 t^578 u - 504 s^656 t^579 u - 5376 s^657 t^580 u - 504 s^658 t^581 u - 48384 s^659 t^582 u - 2912 s^660 t^583 u - \\
& 96768 s^662 t^584 u - 124 s^663 t^585 u - 504 s^664 t^586 u - 5376 s^665 t^587 u - 504 s^666 t^588 u - 48384 s^667 t^589 u - 2912 s^668 t^590 u - \\
& 96768 s^670 t^591 u - 124 s^671 t^592 u - 504 s^672 t^593 u - 5376 s^673 t^594 u - 504 s^674 t^595 u - 48384 s^675 t^596 u - 2912 s^676 t^597 u - \\
& 96768 s^678 t^598 u - 124 s^679 t^599 u - 504 s^680 t^600 u - 5376 s^681 t^601 u - 504 s^682 t^602 u - 48384 s^683 t^603 u - 2912 s^684 t^604 u - \\
& 96768 s^686 t^605 u - 124 s^687 t^606 u - 504 s^688 t^607 u - 5376 s^689 t^608 u - 504 s^690 t^609 u - 48384 s^691 t^610 u - 2912 s^692 t^611 u - \\
& 96768 s^694 t^612 u - 124 s^695 t^613 u - 504 s^696 t^614 u - 5376 s^697 t^615 u - 504 s^698 t^$$

MHV amplitudes

MHV = Maximally Helicity Violating



$$\mathcal{A}(1^+ 2^+ 3^+ \dots n^+) = 0$$

$$\mathcal{A}(1^- 2^+ 3^+ \dots n^+) = 0$$

$$\mathcal{A}(1^- 2^- 3^+ \dots n^+) = i \frac{\langle 1 2 \rangle^4}{\langle 1 2 \rangle \langle 2 3 \rangle \dots \langle n 1 \rangle}$$

Parke, Taylor PRL 56 (23) 2459

Holographic theories with half-maximal SUSY

$4d \mathcal{N} = 4$ SYM with flavors

N D3-branes in flat space $\leftrightarrow AdS_5 \times S^5$ near horizon $\leftrightarrow 4d \mathcal{N} = 4$ SYM

Add N_F D7-branes. If $N_F \ll N$ ignore back-reaction: still conformal.

	AdS_5					S^5				
	1	2	3	4	5	6	7	8	9	10
N D3	x	x	x	x						
N_F D7	x	x	x	x	x	x	x	x		

$S^3 \subset S^5$

$\mathcal{N} = 4$ SYM with flavors

- **Break 1/2 SUSY:** the theory on the D3 is now $4d \ \mathcal{N} = 2$
- **Break the $SO(6)$ isometry of S^5 :** the breaking pattern is
$$SO(6) = SO(4) \times SO(2) = \boxed{SU(2)_L} \times \boxed{SU(2)_R \times U(1)_r}$$
$$S^5 = S^3 \star S^1 \quad \text{Global R symmetry}$$
- **Add gluons:** the D7 branes carry a $8d$ super Yang-Mills with gauge group $G_F = SU(N_F) \rightarrow$ flavor group of the $4d \ \mathcal{N} = 2$ CFT
- **1/2-BPS operators:** in the $4d \ \mathcal{N} = 2$ living on the D3-branes, dual to the Kaluza-Klein modes of the D7 gluons on the S^3

Branes probing singularities

SCFT	Holographic description	Global symmetry
E-string: $6d \mathcal{N} = 1$	M5 on end-of-the-world M9	$SU(2)_L \times SU(2)_R \times G_F$
Seiberg: $5d \mathcal{N} = 1$	D4-D8 system	$SU(2)_L \times SU(2)_R \times G_F$
$4d \mathcal{N} = 2$	D3 near F-theory singularities	$SU(2)_L \times SU(2)_R \times U(1)_r \times G_F$

↓ ↓ ↓
SUSY
half-maximal **Singular locus**
 $AdS_{d+1} \times S^3$ **Symmetry**
 $SU(2)_L \times \text{R-symm} \times G_F$

Flavor branes

SCFT	Holographic description
$4d \mathcal{N} = 4$ SYM + flavors: $4d \mathcal{N} = 2$	D7 wrapping $AdS_5 \times S^3 \subset AdS_5 \times S^5$
$3d \mathcal{N} = 6$ ABJM + flavors: $3d \mathcal{N} = 3$	D6 wrapping $AdS_4 \times \mathbb{RP}^3 \subset AdS_4 \times \mathbb{CP}^3$

↓

Note: only **6 supercharges**,
but same features and techniques

Always add $N_F \ll N$ flavor branes: ignore back-reaction.

Same generic features highlighted for the theories in the previous slide.

The 1/2-BPS sector: super-gluons

Δ	ℓ	j_R	j_L	G_F
ϵk	0	$\frac{k}{2}$	$\frac{k-2}{2}$	adj

$$\epsilon = \frac{d-2}{2}$$

$k = 2$: conserved **current** of $G_F \leftrightarrow$ AdS gluons with gauge group G_F

$k > 2$: Kaluza-Klein modes of gluons on S^3

$$\mathcal{O}_k^I(x; v, \bar{v}) \equiv \mathcal{O}(x)^{I; \alpha_1 \dots \alpha_k; \bar{\alpha}_1 \dots \bar{\alpha}_{k-2}} v_{\alpha_1} \dots v_{\alpha_k} \bar{v}_{\alpha_1} \dots \bar{v}_{\alpha_{k-2}}$$

The 1/2-BPS sector: super-gluons

Δ	ℓ	j_R	j_L	G_F
ϵk	0	$\frac{k}{2}$	$\frac{k-2}{2}$	adj

$$\epsilon = \frac{d-2}{2}$$

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$k > 2$: Kaluza-Klein modes of gluons on S^3

$$\mathcal{O}_k^I(x; v, \bar{v}) \equiv \mathcal{O}(x)^I; \alpha_1 \dots \alpha_k; \bar{\alpha}_1 \dots \bar{\alpha}_{k-2} \underbrace{v_{\alpha_1} \dots v_{\alpha_k}}_{\substack{SU(2)_R \\ \text{polarizations}}} \underbrace{\bar{v}_{\alpha_1} \dots \bar{v}_{\alpha_{k-2}}}_{\substack{SU(2)_L \\ \text{polarizations}}}$$

↓
Adj of G_F

Four-point kinematics

For simplicity focus on $\langle k k k k \rangle$, but full control on $\langle k_1 k_2 k_3 k_4 \rangle$

$$\langle \mathcal{O}(1) \dots \mathcal{O}(4) \rangle = \left(\frac{\nu_{12} \nu_{34}}{x_{12}^{2\epsilon} x_{34}^{2\epsilon}} \right)^k (\bar{\nu}_{12} \bar{\nu}_{34})^{k-2} \mathcal{G}^{I_1 I_2 I_3 I_4}(U, V; \alpha, \beta)$$

$$U = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z \bar{z}, \quad V = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z}), \quad \alpha = \frac{\nu_{13} \nu_{24}}{\nu_{12} \nu_{34}}, \quad \beta = \frac{\bar{\nu}_{13} \bar{\nu}_{24}}{\bar{\nu}_{12} \bar{\nu}_{34}}$$

N.B. \mathcal{G} is a **polynomial** in α, β of $\deg = k, k-2$ resp.

Superconformal invariance also enforces the
Superconformal Ward Identity (SCWI):

$$(z \partial_z - \epsilon \alpha \partial_\alpha) \mathcal{G}^{I_1 I_2 I_3 I_4}(z, \bar{z}; \alpha, \beta) \Big|_{\alpha=1/z} = 0$$

Dolan, Gallot, Sokatchev hep-th/0405180

All gluon four-point functions

Gluon scattering in AdS

Key fact: at lowest order in $1/N$ we have

$$\frac{1}{C_J} \sim \text{diagram with two gluons} \gg \text{diagram with three gluons} \sim \frac{1}{C_T} \Rightarrow \text{Pure gluon scattering in AdS}$$

$$\mathcal{M} = c_s \mathcal{M}_s + c_t \mathcal{M}_t + c_u \mathcal{M}_u$$

$$c_s = f^{I_1 I_2 J} f^{J I_3 I_4}$$

$$c_t = f^{I_1 I_4 J} f^{J I_2 I_3}$$

$$c_u = f^{I_1 I_3 J} f^{J I_4 I_2}$$

$$\mathcal{M}_s \sim \text{diagram with two gluons and a loop labeled } \mathcal{S}_p + \text{diagram with two gluons}$$

$$\mathcal{O}_{k_1} \times \mathcal{O}_{k_2} = \sum_p c_{k_1 k_2 p} \mathcal{O}_p + \text{double trace}$$

A two-steps procedure

For $\langle kkkk \rangle$ we set

$$\mathcal{M}_s = \sum_{p=2}^{2k-2} c_{kkp}^2 \mathcal{Y}_{p-2}(\beta) \mathcal{S}_p + \text{contact}$$

Exchange of the full half-BPS multiplet
N.B. not just the super-primary!

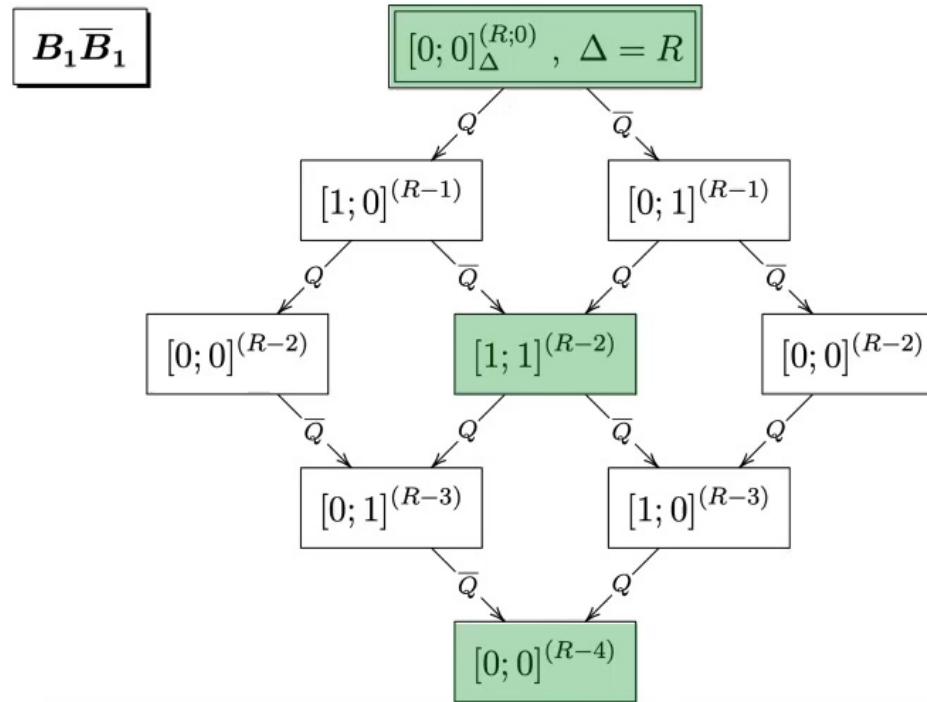
- We fix the individual \mathcal{S}_p using the MRV limit
- We apply the SCWI to fix the OPE coefficients $c_{k_1 k_2 k_3}$ and the contact terms

Super-gluon exchange

However, one has to determine the full **supermultiplet exchange!**

For example, look at the $1/2$ -BPS multiplets of $4d \mathcal{N} = 2$.

Only three **representations allowed by symmetry:**



Cordova, Dumitrescu, Intriligator 1612.00809 [hep-th]

Super-gluon exchange

For all short multiplets of all super algebras of interest, the **representations that can be exchanged by two super-primaries** are

component field	Δ	ℓ	j_R	j_L	G_F
s_p^I	ϵp	0	$\frac{p}{2}$	$\frac{p-2}{2}$	adj
$A_{p,\mu}^I$	$\epsilon p + 1$	1	$\frac{p}{2} - 1$	$\frac{p-2}{2}$	adj
r_p^I	$\epsilon p + 2$	0	$\frac{p}{2} - 2$	$\frac{p-2}{2}$	adj

The **exchange of super-gluons \mathcal{S}_p** must take into account **all of these contributions!**

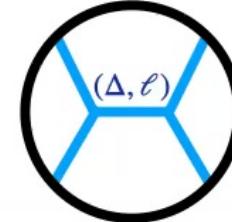
Super-gluon exchange

R-symmetry polynomials

→ for $SU(2)$: Jacobi polynomials

$$\mathcal{S}_p = \mathcal{Y}_p(\alpha) M_{\epsilon p,0}(s, t, u) + \lambda_{A_p} \mathcal{Y}_{p-2}(\alpha) M_{\epsilon p+1,1}(s, t, u) + \lambda_{r_p} \mathcal{Y}_{p-4}(\alpha) M_{\epsilon p+2,0}(s, t, u)$$

$M_{\Delta,\ell} \leftrightarrow$ exchange of $\mathcal{O}_{\Delta,\ell}$
(known for all Δ, ℓ)



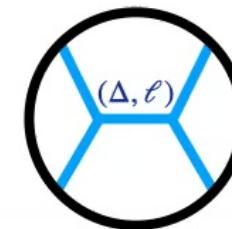
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$M_{\Delta,\ell} \leftrightarrow \text{exchange of } \mathcal{O}_{\Delta,\ell}$
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How do we fix the relative coefficients?

The MRV limit

Choose a configuration of R-symmetry polarizations

$$\text{u-channel MRV : } v_1 = v_3 \Rightarrow \alpha = 0$$

Two main consequences:

1

$\mathcal{M}(s, t)$ has no poles in u

2

$\mathcal{M}(s, t)$ has a single zero at $u = 2\epsilon k$

The MRV limit

$$\mathcal{S}_p = \mathcal{Y}_p(\alpha) M_{\epsilon p,0}(s,t,u) + \boxed{\lambda_{A_p}} \mathcal{Y}_{p-2}(\alpha) M_{\epsilon p+1,1}(s,t,u) + \boxed{\lambda_{r_p}} \mathcal{Y}_{p-4}(\alpha) M_{\epsilon p+2,0}(s,t,u)$$

Set $\alpha = 0$ and require a zero at $u = 2\epsilon k$



Fixes the λ 's completely!

$$\mathcal{S}_p \sim \underset{\rightarrow}{(u - 2\epsilon k)} \sum_i \frac{\mu_i}{s - \nu_i}$$

- **Improved Regge behavior:** $\mathcal{S}_p \sim s^{-1}$ in u-channel Regge limit
($s, t \rightarrow \infty$, u fixed)

The MRV limit

1

$\mathcal{M}(s, t)$ has **no poles in u**

If we set $v_1 = v_3$, then **only $SU(2)_R$ irreps with** $j_R = \frac{k_1 + k_3}{2}$ can be exchanged in the u-channel

However, the **vanishing of extremal 3-pt functions** dictates that

$$\langle \mathcal{O}_{k_1} \mathcal{O}_{k_3} \mathcal{O}_{k_1+k_3} \rangle = 0$$

Hence, **no exchange of short operators** in the u-channel is allowed

→ **No poles in u**

Or: all **R-symmetry polynomials** in the u-channel are proportional to $\alpha^\#$, so they **vanish when $\alpha = 0$** . Hence, cannot have u-channel exchanges.

The MRV limit

2

$\mathcal{M}(s, t)$ has a single **zero** at $u = 2\epsilon k$

Consider for simplicity $\langle k k k k \rangle$.

In a large N theory all **long operators** are double-trace operators, and the ones with **lowest twist** have

$$\tau = \Delta - \ell = 2\epsilon k$$

For a **long super-multiplet** to be exchanged in the u-channel, and to be **visible in the MRV limit**, the super-primary must have

$$j_R = k - 2$$

But then **all operators** in the super-multiplet with twist $\tau = 2\epsilon k$ decouple in the MRV limit. Hence, they **cannot develop an anomalous dimension**. To achieve this, we must cancel the double pole in the Gamma functions.

Operator	Twist
$(\Delta)_\ell^{(k-2)}$	$\tau_0 \equiv \Delta - \ell$
$(\Delta + 1)_{\ell+1}^{(k-2)\pm 1; (k-2)}$	τ_0
$(\Delta + 1)_{\ell-1}^{(k-2)\pm 1; (k-2)}$	$\tau_0 + 2$
$(\Delta + 2)_{\ell+2}^{(k-2)}$	τ_0
$(\Delta + 2)_\ell^{(k-2)\pm 2; (k-2)\pm 1; (k-2)}$	$\tau_0 + 2$
$(\Delta + 2)_{\ell-2}^{(k-2)}$	$\tau_0 + 4$
$(\Delta + 3)_{\ell+1}^{(k-2)\pm 1; (k-2)}$	$\tau_0 + 2$
$(\Delta + 3)_{\ell-1}^{(k-2)\pm 1; (k-2)}$	$\tau_0 + 4$
$(\Delta + 4)_\ell^{(k-2)}$	$\tau_0 + 4$

Operator	Twist
$(\Delta)_\ell^{(k-2)}$	$\tau_0 \equiv \Delta - \ell$
$(\Delta + 1)_{\ell+1}^{(k-2)\pm 1; (k-2)}$	τ_0
$(\Delta + 1)_{\ell-1}^{(k-2)\pm 1; (k-2)}$	$\tau_0 + 2$
$(\Delta + 2)_{\ell+2}^{(k-2)}$	τ_0
$(\Delta + 2)_\ell^{(k-2)\pm 2; (k-2)\pm 1; (k-2)}$	$\tau_0 + 2$
$(\Delta + 2)_{\ell-2}^{(k-2)}$	$\tau_0 + 4$
$(\Delta + 3)_{\ell+1}^{(k-2)\pm 1; (k-2)}$	$\tau_0 + 2$
$(\Delta + 3)_{\ell-1}^{(k-2)\pm 1; (k-2)}$	$\tau_0 + 4$
$(\Delta + 4)_\ell^{(k-2)}$	$\tau_0 + 4$

Only one operator, with $j_R = k$, survives the MRV limit

Hence, the operators with twist $\tau_0 \equiv \Delta - \ell$ decouple in the limit

If we take $\tau_0 = 2\epsilon\hat{k}$, the long multiplet with minimal twist, this means that when taking the MRV limit no long operator with $\tau_0 = 2\epsilon k$ is present

The ansatz

This procedure fixes completely the super-exchanges:

$$\mathcal{S}_p = \sum_{m=0}^{\infty} \sum_{i=0}^{p/2} \frac{R_{p,m}^i(t, u)}{s - \epsilon p - 2m} (1 - \alpha)^i$$

Residues fixed
by MRV limit

In terms of these, we write for the s-channel contribution

$$\mathcal{M}_s = \sum_{p=2}^{2k-2} c_{12p} c_{34p} \mathcal{Y}_{p-2}(\beta) \mathcal{S}_p + \text{contact}$$

Degree 0 in s, t, u :

In the flat space limit it must match
the contact term of Yang-Mills theory

Apply SCWI

$$\mathcal{M} = c_s \mathcal{M}_s + c_t \mathcal{M}_t + c_u \mathcal{M}_u$$
$$[c_s + c_t + c_u = 0]$$

Now we apply the SCWI: $(z \partial_z - \epsilon \alpha \partial_\alpha) \mathcal{G}^{I_1 I_2 I_3 I_4}(z, \bar{z}; \alpha, \beta) \Big|_{\alpha=1/z} = 0$

Fixes all **OPE coeff.** c_{pqr}
in terms of c_{222}

NEW!!!

Fixes **contact = 0**
in our ansatz
[only 3-pt couplings matter]

The main result

Now, we only **need** c_{222} . We have

$$(c_{222})^2 = \frac{2(2\epsilon + 1)}{\epsilon} \frac{1}{C_J} \quad \langle JJ \rangle \sim C_J$$

In **all theories** that we consider

$$C_J \sim N^\epsilon \quad \ll \quad C_T \sim N^{1+\epsilon}$$

The **explicit value** of C_J gives

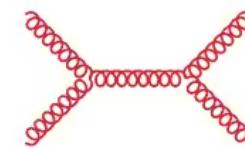
All $\langle k_1 k_2 k_3 k_4 \rangle$ in $d = 3, 4, 5, 6$

Only known results before us: 2222 in d=5,6 from Zhou 1804.02397 [hep-th]

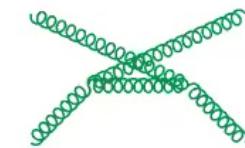
$\langle 22kk \rangle$ in $d = 4$

$$\mathcal{M} = c_s \mathcal{M}_s + c_t \mathcal{M}_t + c_u \mathcal{M}_u$$

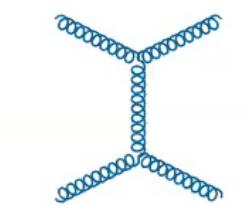
$$\mathcal{M}_s = \frac{-2\alpha(k+2) + k + \alpha t + \alpha u - u + 2}{(k-2)!(s-2)} \frac{6}{C_J}$$



$$\mathcal{M}_t = \frac{(1-\alpha)(-2\alpha k - k + \alpha s + u - 2)}{(k-2)!(t-k)} \frac{6}{C_J}$$



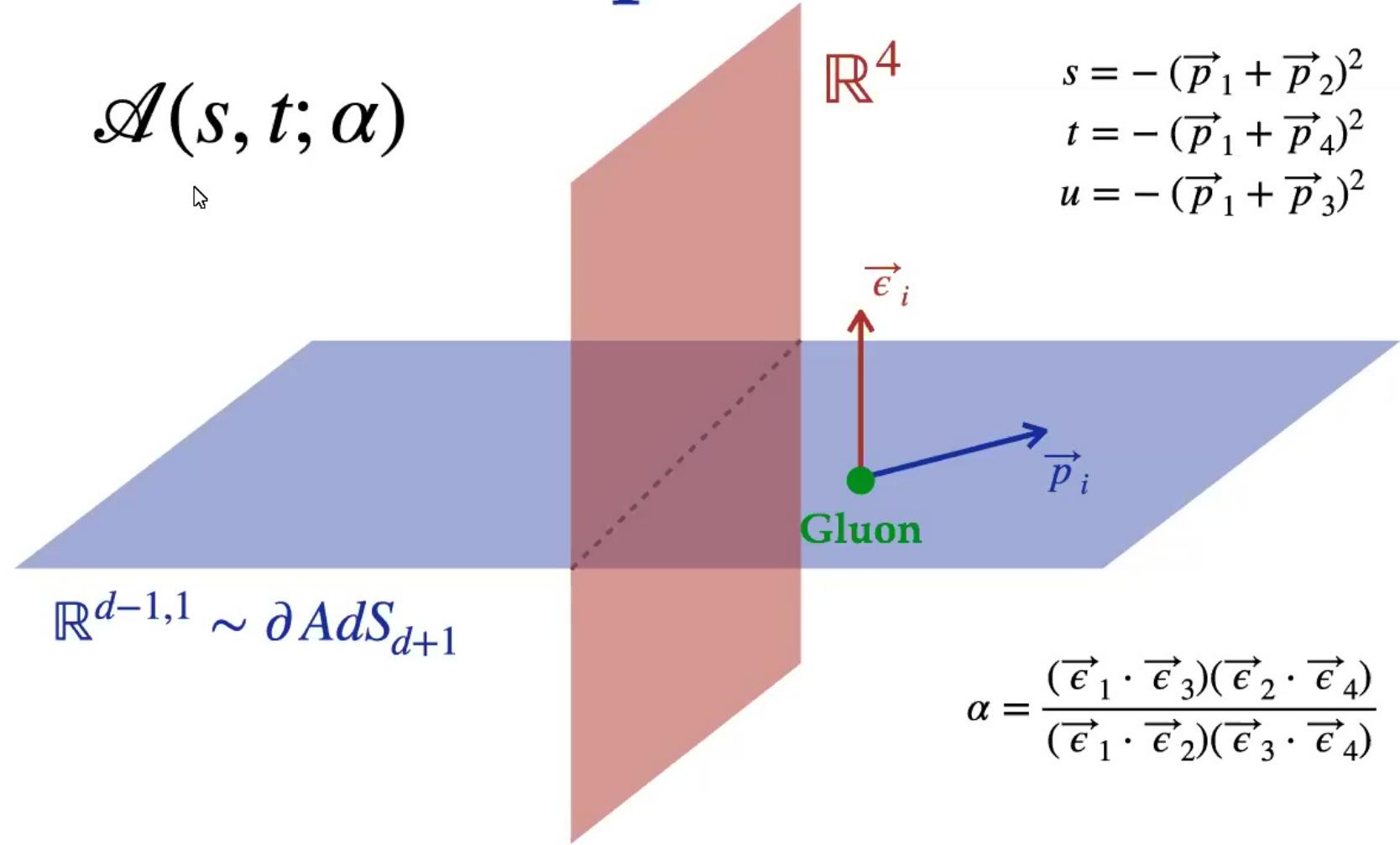
$$\mathcal{M}_u = \frac{-\alpha(2\alpha k - 3k - \alpha s + s + t - 2)}{(k-2)!(u-k)} \frac{6}{C_J}$$



$$\frac{1}{C_J} \sim \frac{1}{N} \quad \gg \quad \frac{1}{C_T} \sim \frac{1}{N^2}$$

Hidden structures

Flat space limit



Color-kinematics duality

$$\mathcal{M}_{2222} = \textcolor{red}{c}_s \textcolor{red}{n}_s \left(\frac{1}{s - 2\epsilon} + \dots \right) + \textcolor{green}{c}_t \textcolor{green}{n}_t \left(\frac{1}{t - 2\epsilon} + \dots \right) + \textcolor{blue}{c}_u \textcolor{blue}{n}_u \left(\frac{1}{u - 2\epsilon} + \dots \right)$$

$$\textcolor{red}{n}_s = u - 4\epsilon + \alpha s, \quad \textcolor{green}{n}_t = (1 - \alpha)((\alpha - 1)(u - 4\epsilon) + \alpha t), \quad \textcolor{blue}{n}_u = -\alpha(\alpha(s - 4\epsilon) + u)$$

$$\textcolor{red}{c}_s + \textcolor{green}{c}_t + \textcolor{blue}{c}_u = 0$$

Jacobi identity
(purely algebraic)

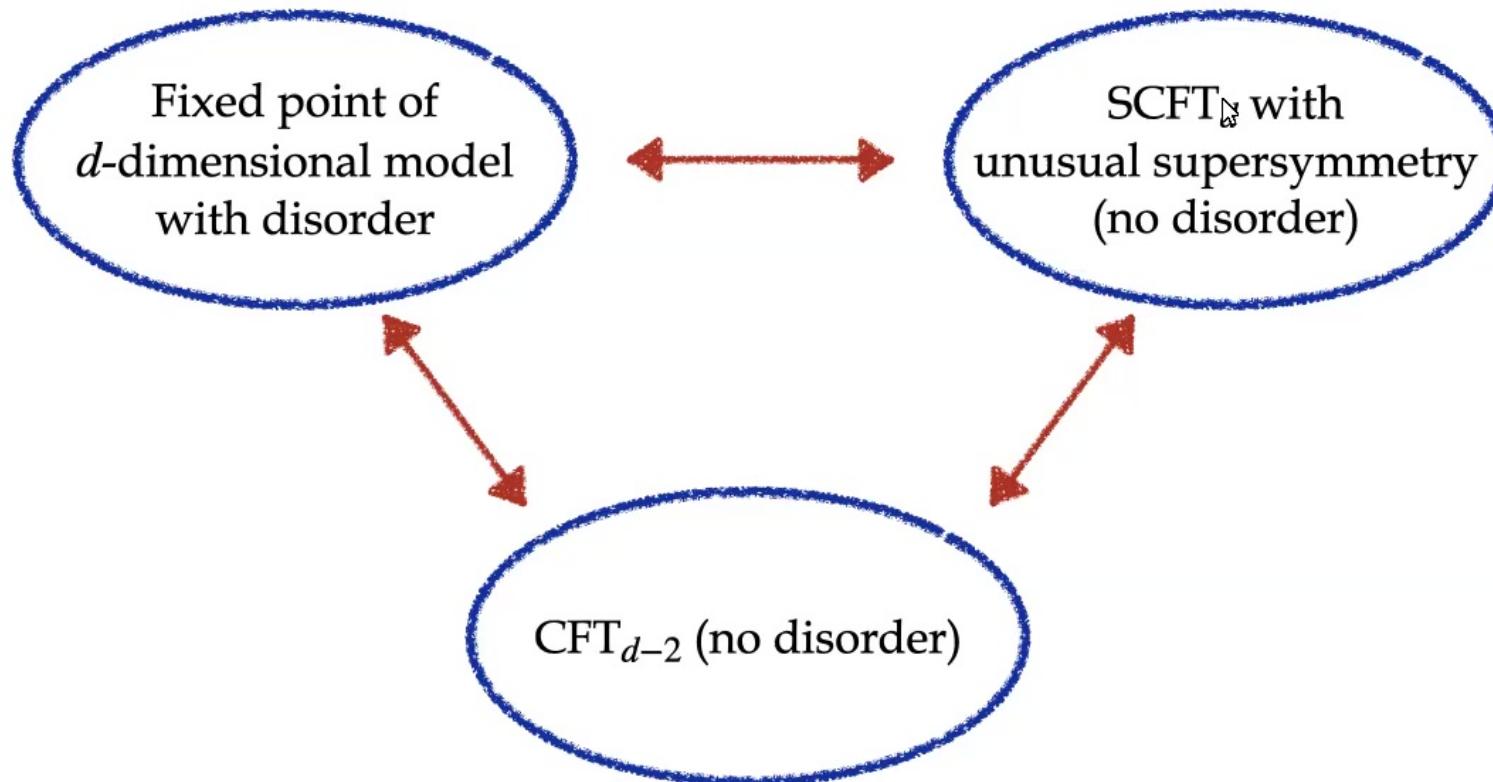
$$\textcolor{red}{n}_s + \textcolor{green}{n}_t + \textcolor{blue}{n}_u = 0$$

Kinematic analogue,
known in flat space

See also Zhou 2106.07651 [hep-th]: CK duality and double copy in d=4

Parisi-Sourlas supersymmetry

Parisi, Sourlas Phys. Rev. Lett. 43 (1979) 744



Parisi-Sourlas supersymmetry

Recall that we write the s-channel exchange multiplet as

$$\mathcal{M}_s = \sum_p c_{12p} c_{34p} \mathcal{S}_p$$

It turns out that

$$\mathcal{S}_p = \mathbb{K}_p \circ \underbrace{\mathbb{Y}_{p,2}(\alpha) \mathbb{Y}_{p-2,0}(\beta)}_{\text{Polynomials}} M_{\epsilon p,0}^{(d-1)}$$

Differential
Operator

Scalar exchange
in AdS_{d-1}

Hidden conformal symmetry

In the **flat space limit**

$$\widetilde{\mathcal{M}}_{2222} \rightarrow \left(\frac{c_s}{su} - \frac{c_t}{tu} \right)$$

Which is annihilated by the **SCT generator** in momentum space

$$K_\mu = \sum_{i=1}^4 \left(\frac{p_{i\mu}}{2} \frac{\partial}{\partial p_i^\nu} \frac{\partial}{\partial p_{i,\nu}} - p_i^\nu \frac{\partial}{\partial p_i^\nu} \frac{\partial}{\partial p_i^\mu} - \frac{d-2}{2} \frac{\partial}{\partial p_i^\mu} \right)$$

Only when the spacetime dimension is $d = 8$:
worldvolume of the D7-branes!

Similar to $AdS_5 \times S^5$ for $d = 10$ and $AdS_3 \times S^3 \times K3$ for $d = 6$

Caron-Huot, Trinh 1809.09173 [hep-th] Rastelli, Roumpedakis, Zhou 1905.11983 [hep-th]

Outlook



Color-kinematics duality & double copy

CK duality found here for 2222 in all dimensions.
Generalized to arbitrary KK modes, with related DC relations, for $d=4$.
What about other d ? Non supersymmetric theories?



Higher-point functions

Is CK duality present in higher-point gluon amplitudes in AdS?
↳ Double copy? Properties of color-ordered amplitudes?



Subleading corrections

These would include: loops, graviton exchanges, higher-derivative corrections (partially addressed in our work), mixed graviton-gluon.



CFT data

From our correlators: new protected and unprotected CFT data. Can extract them, explore structures, feed to and compare with numerical bootstrap.