Title: Gluon scattering in AdS from CFT

Speakers: Pietro Ferrero

Series: Quantum Fields and Strings

Date: November 30, 2021 - 2:00 PM

URL: https://pirsa.org/21110048

Abstract: I will present a class of recently computed holographic correlators between half-BPS operators in a vast array of SCFTs with non-maximal superconformal symmetry in dimensions d=3,4,5,6. Via AdS/CFT, these four-point functions are dual to gluon scattering amplitudes in AdS. Exploiting the notion of MRV limit I will show that, at tree level, all such correlators are completely fixed by symmetries and consistency conditions. Our results encode a wealth of novel CFT data and exhibit various emergent structures, including Parisi-Sourlas supersymmetry, hidden conformal symmetry and color-kinematics duality. This talk will be based on https://arxiv.org/pdf/2103.15830.pdf.

Pietro Ferrero
University of Oxford

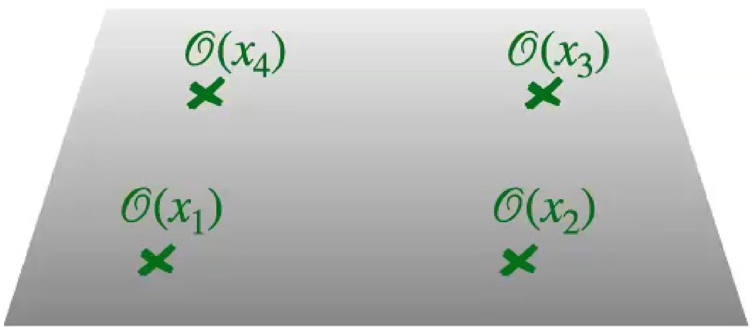
Gluon scattering in AdS from CFT

Based on work with
Fernando Alday, **Connor Behan** and **Xinan Zhou**:

arXiv: **2101.04114** [hep-th]
arXiv: **2103.15830** [hep-th]

AdS/CFT

Conformal field theory
on $\mathbb{R}^d = \partial \mathrm{AdS}_{d+1}$



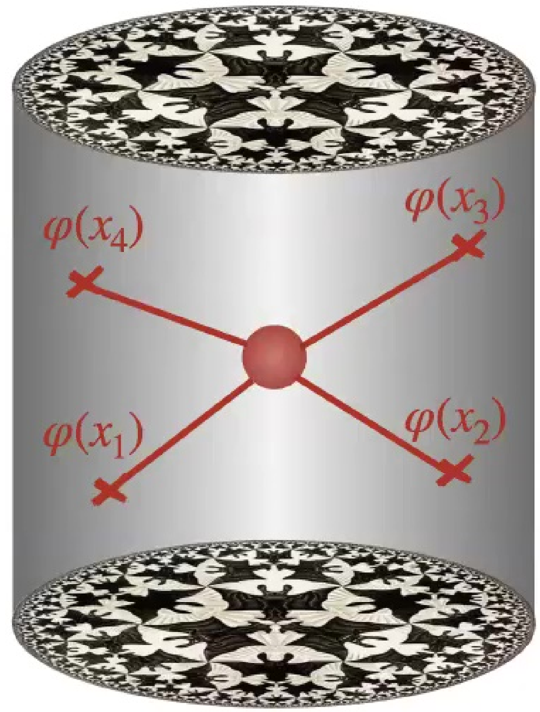
$\langle\, \mathcal{O}(x_1)\, \mathcal{O}(x_2)\, \mathcal{O}(x_3)\, \mathcal{O}(x_4)\, \rangle$

CFT correlator

$\mathcal{O} \sim \varphi$

$\Longleftrightarrow$

String / M-theory
on $\mathrm{AdS}_{d+1} \times M$

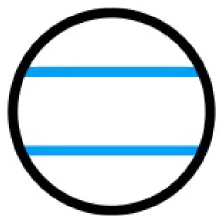
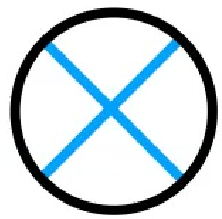
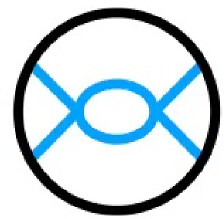


AdS scattering

Holographic 4-pt functions

$g \ll 1$: weakly coupled sugra in the bulk

e.g. $g = \frac{1}{N^2}$ in $\mathcal{N} = 4$ SYM

$\langle 1234 \rangle =$  $+ g$  $+ g^2$  $+ \ldots$

Trivial

Here

State of the art
(only few cases)

Maximal SUSY

Background	SCFT	R symmetry
M-theory on $\mathrm{AdS}_4 \times S^7$	$3d\ \mathcal{N}=8$ ABJM	$\mathfrak{so}(8)$
IIB string theory on $\mathrm{AdS}_5 \times S^5$	$4d\ \mathcal{N}=4$ SYM	$\mathfrak{so}(6)$
M-theory on $\mathrm{AdS}_7 \times S^4$	$6d\ \mathcal{N}=(2,0)$	$\mathfrak{so}(5)$

Half-BPS operators: symmetric traceless of $\mathfrak{so}(n+1) \longrightarrow \mathcal{O}^{(i_1 \cdots i_k)}$

k = 2 : CFT **stress tensor** multiplet $\longleftrightarrow$ AdS **graviton**

k > 2 : Kaluza-Klein modes of the graviton on S^n

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After 20 years of efforts, all $\langle k_1\, k_2\, k_3\, k_4 \rangle$ correlators at tree level computed using a new idea: the **MRV limit**.

Alday, Zhou 2006.12505, 2006.06653 [hep-th]

Here: half-maximal SUSY

Here we apply the idea of

MRV limit: a special configuration of R-symmetry polarizations, that simplifies the correlators

to a large class of

SCFTs with half-maximal SUSY in $d = 3,4,5,6$

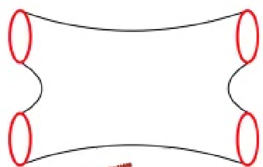
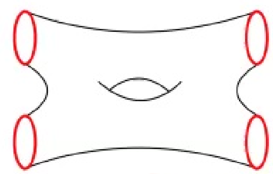
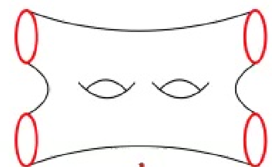
and obtain all

all $\langle k_1\, k_2\, k_3\, k_4 \rangle$ correlators, dual to
gluon scattering amplitudes in AdS

Motivation

Gravity from conformal field theory

AdS/CFT can be used as a definition of **quantum gravity in AdS**.

$\mathscr{A}(\alpha', g_s; s, t, u) =$  $+ g_s^2$  $+ g_s^4$  $+ \ldots$

$$\mathscr{A}^{(g=0)}(\alpha'; s, t) = \frac{1}{s\,t\,u} + \alpha'^3 + \alpha'^5(s^2+t^2+u^2) + \ldots$$

2-derivative Supergravity

Higher-derivative String corrections

Loops

We can use **CFT techniques** to study

- The effect of **quantum gravity** in AdS
- **Effective actions** of string/M-theory perturbatively

Motivation

Scattering amplitudes program in AdS

For **scattering amplitudes in flat space**: rich story, plenty of interesting physical and mathematical **structures** hidden in Lagrangian description.

Can learn **lessons about QFTs** independently of Lagrangian!

A natural question: **what happens in curved space?**
Simplest case to look at is **AdS**.

- Generalizations of the **structures of flat space amplitudes**?

  e.g. color-kinematics duality, double copy, MHV limit, CHY formulae, …

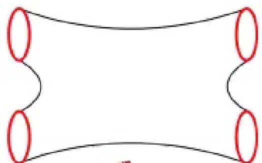
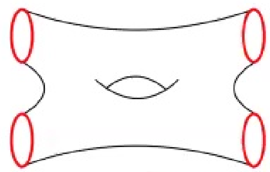
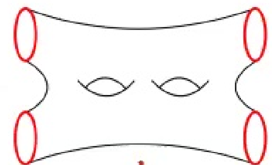
- **New hidden features** for AdS amplitudes?

  e.g. Parisi-Sourlas dimensional reduction, hidden conformal symmetry, …

Motivation

Gravity from conformal field theory

AdS/CFT can be used as a definition of **quantum gravity in AdS**.

$\mathscr{A}(\alpha', g_s; s, t, u) =$  $+ g_s^2$  $+ g_s^4$  $+ \dots$

$$\mathscr{A}^{(g=0)}(\alpha'; s, t) = \frac{1}{s\,t\,u} + \alpha'^3 + \alpha'^5 (s^2 + t^2 + u^2) + \dots$$

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A brief history

The traditional method

In principle

Write effective **supergravity Lagrangian** for Kaluza-Klein modes up to quartic interactions

↓

Extract “**Feynman rules**”

↓

Add all the relevant **Witten diagrams** to obtain the result

In practice…

Really hard to make progress this way!

Still, great efforts led to the computation of:

- $AdS_5 \times S^5$: a bunch of results, no organizing principle

  D'Hoker, Freedman, Mathur, Matusis, Rastelli, Arutyunov, Dolan, Frolov, Osborn, Sokatchev, Berdichevsky, Naaijkens, Urchurtu, Nirschl ...

- $AdS_7 \times S^4$: only $\langle 2222 \rangle$

  Arutyunov, Sokatchev hep-th/0201145

- $AdS_4 \times S^7$: nothing, no closed form for exchange in position space

Mellin space

$$\langle \mathcal{O}_\Delta(x_1)\,\mathcal{O}_\Delta(x_2)\,\mathcal{O}_\Delta(x_3)\,\mathcal{O}_\Delta(x_4)\rangle = \frac{1}{x_{12}^{2\Delta}x_{34}^{\Delta}}\,\mathcal{G}(U,V) \qquad U = \frac{x_{12}^2\,x_{34}^2}{x_{13}^2\,x_{24}^2}, \quad V = \frac{x_{14}^2\,x_{23}^2}{x_{13}^2\,x_{24}^2}$$

$$\mathcal{G}(U,V) = \int_{-i\infty}^{i\infty}\frac{dsdt}{(4\pi i)^2}U^{\frac{s}{2}}V^{\frac{t}{2}}\,\Gamma\left[\Delta-\frac{s}{2}\right]^2\Gamma\left[\Delta-\frac{t}{2}\right]^2\Gamma\left[\Delta-\frac{u}{2}\right]^2\boxed{\mathcal{M}(s,t)}$$

Mellin amplitude

s, t, u are the analogue of **Mandelstam variables** of flat space scattering:

$$s \sim -(p_1+p_2)^2\,, \quad t \sim -(p_1+p_3)^2\,, \quad u \sim -(p_1+p_4)^2$$

$$s+t+u = 4\,\Delta$$

In fact:

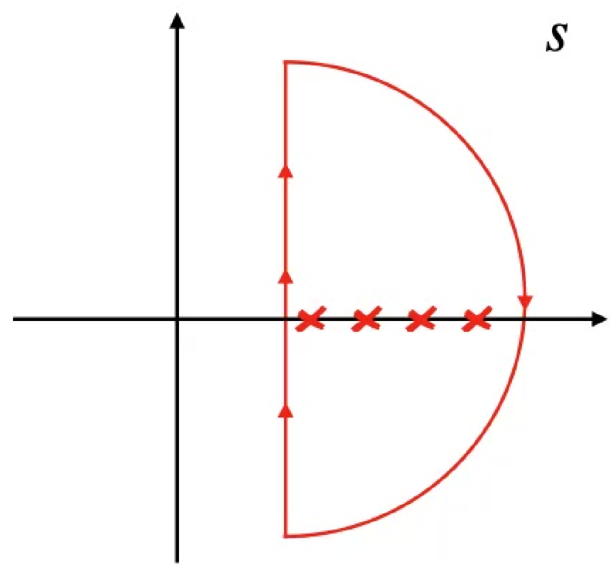
$$\lim_{s\to\infty,\,t\to\infty}\mathcal{M}(s,t) = \mathcal{A}^{(\text{flat})}(s,t)$$

Mack 0907.2407 [hep-th] **Penedones 1011.1485 [hep-th]**
Fitzpatrick, Kaplan 1111.6972 [hep-th]
Fitzpatrick, Kaplan, Penedones, Raju, van Rees 1107.1499 [hep-th]

Mellin space

$$\mathcal{G}(U,V) = \int_{-i\infty}^{i\infty} \frac{dsdt}{(4\pi i)^2} U^{\frac{s}{2}} V^{\frac{t}{2}}\, \Gamma\left[\Delta - \frac{s}{2}\right]^2 \Gamma\left[\Delta - \frac{t}{2}\right]^2 \Gamma\left[\Delta - \frac{u}{2}\right]^2 \mathcal{M}(s,t)$$

Poles are what matters:



Mellin space

$$\mathcal{G}(U,V) = \int_{-i\infty}^{i\infty} \frac{dsdt}{(4\pi i)^2} U^{\frac{s}{2}} V^{\frac{t}{2}}\, \Gamma\left[\Delta - \frac{s}{2}\right]^2 \Gamma\left[\Delta - \frac{t}{2}\right]^2 \Gamma\left[\Delta - \frac{u}{2}\right]^2 \mathcal{M}(s,t)$$

Double poles ↔ double trace operators

$$\mathcal{O} \times \mathcal{O} \sim 1 + \mathrm{ST} + \mathcal{O} \,\square^n \partial_{\mu_1} \cdots \partial_{\mu_\ell} \mathcal{O}$$

At **tree level**

$$\mathcal{G}^{(1)}(U,V) \sim \sum_{n,l} U^{\Delta+n} \left(a^{(1)}_{n,\ell} + \frac{1}{2} a^{(0)}_{n,\ell} \gamma^{(1)}_{n,\ell} \left(\log U + \frac{\partial}{\partial n} \right) \right) g_{n,\ell}(U,V)$$

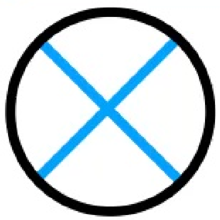
$$\mathrm{Res}_{s=a} \left(U^s \frac{F(s)}{s-a} \right) = U^a F(a)$$

Tree level
Anomalous dimensions

$$\mathrm{Res}_{s=a} \left(U^s \frac{F(s)}{(s-a)^2} \right) = U^a \left(F'(a) + F(a) \log U \right)$$

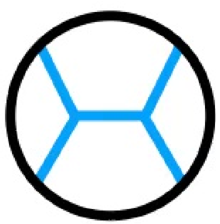
Mellin space

$$\mathcal{G}(U,V) = \int_{-i\infty}^{i\infty} \frac{ds dt}{(4\pi i)^2} U^{\frac{s}{2}} V^{\frac{t}{2}} \, \Gamma\left[\Delta - \frac{s}{2}\right]^2 \Gamma\left[\Delta - \frac{t}{2}\right]^2 \Gamma\left[\Delta - \frac{u}{2}\right]^2 \mathcal{M}(s,t)$$



Contact diagrams: $\mathcal{L}_{AdS} \supset \partial^{2L}\phi^4$

$\mathcal{M}(s,t) = \mathrm{Poly}^{(L)}(s,t) \quad \Rightarrow \quad$ only **double trace** operators in the OPE



Exchange diagrams: $\mathcal{L}_{AdS} \supset \phi^3$

$$\mathcal{M}^{(s)}_{\Delta,\ell}(s,t) = \sum_{m=0}^{\infty} \frac{Q_m^{(\ell)}(t,u)}{s - \Delta + \ell - 2m} + P^{(\ell-1)}(s,t)$$

Single pole: exchange of **single trace** operator (+ descendants)

Bootstrap methods

Forget the Lagrangian!

Superconformal invariance + crossing symmetry + consistency conditions

- $AdS_5 \times S^5$: full answer

  Rastelli, Zhou 1608.06624, 1710.05923 [hep-th]

- $AdS_7 \times S^4$: partial results

  Rastelli, Zhou 1712.02788 [hep-th]

- $AdS_4 \times S^7$: partial results

  Rastelli, Zhou 1712.02800 [hep-th]

However: no organizing principle, especially for M-theory backgrounds…

The MRV limit

MRV = Maximally R-symmetry Violating

$$\big(1032192 - 184320 s - 559104 t + 99840 s t + 96768 t^2 - 17280 s t^2 - 5376 t^3 + 960 s t^3 - 559104 u + 99840 s u + 302848 t u - 54080 s t u - 52416 t^2 u +$$
$$9360 s t^2 u + 2912 t^3 u - 520 s t^3 u + 96768 u^2 - 17280 s u^2 - 52416 t u^2 + 9360 s t u^2 + 9072 t^2 u^2 - 1620 s t^2 u^2 - 504 t^3 u^2 + 90 s t^3 u^2 -$$
$$5376 u^3 + 960 s u^3 + 2912 t u^3 - 520 s t u^3 - 504 t^2 u^3 + 90 s t^2 u^3 + 28 t^3 u^3 - 5 s t^3 u^3 - 4644864 \sigma + 1511424 s \sigma - 150528 s^2 \sigma + 5376 s^3 \sigma +$$
$$2774016 t \sigma - 895488 s t \sigma + 86912 s^2 t \sigma - 2912 s^3 t \sigma - 575232 t^2 \sigma + 183296 s t^2 \sigma - 17024 s^2 t^2 \sigma + 504 s^3 t^2 \sigma + 48384 t^3 \sigma - 15072 s t^3 \sigma +$$
$$1288 s^2 t^3 \sigma - 28 s^3 t^3 \sigma - 1344 t^4 \sigma + 400 s t^4 \sigma - 28 s^2 t^4 \sigma + 1511424 u \sigma - 391680 s u \sigma + 26880 s^2 u \sigma - 960 s^3 u \sigma - 895488 t u \sigma +$$
$$231360 s t u \sigma - 15520 s^2 t u \sigma + 520 s^3 t u \sigma + 183296 t^2 u \sigma - 47120 s t^2 u \sigma + 3040 s^2 t^2 u \sigma - 90 s^3 t^2 u \sigma - 15072 t^3 u \sigma + 3840 s t^3 u \sigma -$$
$$230 s^2 t^3 u \sigma + 5 s^3 t^3 u \sigma + 400 t^4 u \sigma - 100 s t^4 u \sigma + 5 s^2 t^4 u \sigma - 150528 u^2 \sigma + 26880 s u^2 \sigma + 86912 t u^2 \sigma - 15520 s t u^2 \sigma - 17024 t^2 u^2 \sigma +$$
$$3040 s t^2 u^2 \sigma + 1288 t^3 u^2 \sigma - 230 s t^3 u^2 \sigma - 28 t^4 u^2 \sigma + 5 s t^4 u^2 \sigma + 5376 u^3 \sigma - 960 s u^3 \sigma - 2912 t u^3 \sigma + 520 s t u^3 \sigma + 504 t^2 u^3 \sigma - 90 s t^2 u^3 \sigma -$$
$$28 t^3 u^3 \sigma + 5 s t^3 u^3 \sigma + 1032192 \sigma^2 - 559104 s \sigma^2 + 96768 s^2 \sigma^2 - 5376 s^3 \sigma^2 - 559104 t \sigma^2 + 302848 s t \sigma^2 - 52416 s^2 t \sigma^2 + 2912 s^3 t \sigma^2 +$$
$$96768 t^2 \sigma^2 - 52416 s t^2 \sigma^2 + 9072 s^2 t^2 \sigma^2 - 504 s^3 t^2 \sigma^2 - 5376 t^3 \sigma^2 + 2912 s t^3 \sigma^2 - 504 s^2 t^3 \sigma^2 + 28 s^3 t^3 \sigma^2 - 184320 u \sigma^2 + 99840 s u \sigma^2 -$$
$$17280 s^2 u \sigma^2 + 960 s^3 u \sigma^2 + 99840 t u \sigma^2 - 54080 s t u \sigma^2 + 9360 s^2 t u \sigma^2 - 520 s^3 t u \sigma^2 - 17280 t^2 u \sigma^2 + 9360 s t^2 u \sigma^2 - 1620 s^2 t^2 u \sigma^2 +$$
$$90 s^3 t^2 u \sigma^2 + 960 t^3 u \sigma^2 - 520 s t^3 u \sigma^2 + 90 s^2 t^3 u \sigma^2 - 5 s^3 t^3 u \sigma^2 - 4644864 \tau + 1511424 s \tau - 150528 s^2 \tau + 5376 s^3 \tau + 1511424 t \tau -$$
$$391680 s t \tau + 26880 s^2 t \tau - 960 s^3 t \tau - 150528 t^2 \tau + 26880 s t^2 \tau + 5376 t^3 \tau - 960 s t^3 \tau + 2774016 u \tau - 895488 s u \tau + 86912 s^2 u \tau -$$
$$2912 s^3 u \tau - 895488 t u \tau + 231360 s t u \tau - 15520 s^2 t u \tau + 520 s^3 t u \tau + 86912 t^2 u \tau - 15520 s t^2 u \tau - 2912 t^3 u \tau + 520 s t^3 u \tau - 575232 u^2 \tau +$$
$$183296 s u^2 \tau - 17024 s^2 u^2 \tau + 504 s^3 u^2 \tau + 183296 t u^2 \tau - 47120 s t u^2 \tau + 3040 s^2 t u^2 \tau - 90 s^3 t u^2 \tau - 17024 t^2 u^2 \tau + 3040 s t^2 u^2 \tau +$$
$$504 t^3 u^2 \tau - 90 s t^3 u^2 \tau + 48384 u^3 \tau - 15072 s u^3 \tau + 1288 s^2 u^3 \tau - 28 s^3 u^3 \tau - 15072 t u^3 \tau + 3840 s t u^3 \tau - 230 s^2 t u^3 \tau + 5 s^3 t u^3 \tau +$$
$$1288 t^2 u^3 \tau - 230 s t^2 u^3 \tau - 28 t^3 u^3 \tau + 5 s t^3 u^3 \tau - 1344 u^4 \tau + 400 s u^4 \tau - 28 s^2 u^4 \tau + 400 t u^4 \tau - 100 s t u^4 \tau + 5 s^2 t u^4 \tau - 28 t^2 u^4 \tau +$$
$$5 s t^2 u^4 \tau - 4644864 \sigma \tau + 2774016 s \sigma \tau - 575232 s^2 \sigma \tau + 48384 s^3 \sigma \tau - 1344 s^4 \sigma \tau + 1511424 t \sigma \tau - 895488 s t \sigma \tau + 183296 s^2 t \sigma \tau -$$
$$15072 s^3 t \sigma \tau + 400 s^4 t \sigma \tau - 150528 t^2 \sigma \tau + 86912 s t^2 \sigma \tau - 17024 s^2 t^2 \sigma \tau + 1288 s^3 t^2 \sigma \tau - 28 s^4 t^2 \sigma \tau + 5376 t^3 \sigma \tau - 2912 s t^3 \sigma \tau +$$
$$504 s^2 t^3 \sigma \tau - 28 s^3 t^3 \sigma \tau + 1511424 u \sigma \tau - 895488 s u \sigma \tau + 183296 s^2 u \sigma \tau - 15072 s^3 u \sigma \tau + 400 s^4 u \sigma \tau - 391680 t u \sigma \tau + 231360 s t u \sigma \tau -$$
$$47120 s^2 t u \sigma \tau + 3840 s^3 t u \sigma \tau - 100 s^4 t u \sigma \tau + 26880 t^2 u \sigma \tau - 15520 s t^2 u \sigma \tau + 3040 s^2 t^2 u \sigma \tau - 230 s^3 t^2 u \sigma \tau + 5 s^4 t^2 u \sigma \tau - 960 t^3 u \sigma \tau +$$
$$520 s t^3 u \sigma \tau - 90 s^2 t^3 u \sigma \tau + 5 s^3 t^3 u \sigma \tau - 150528 u^2 \sigma \tau + 86912 s u^2 \sigma \tau - 17024 s^2 u^2 \sigma \tau + 1288 s^3 u^2 \sigma \tau - 28 s^4 u^2 \sigma \tau + 26880 t u^2 \sigma \tau -$$
$$15520 s t u^2 \sigma \tau + 3040 s^2 t u^2 \sigma \tau - 230 s^3 t u^2 \sigma \tau + 5 s^4 t u^2 \sigma \tau + 5376 u^3 \sigma \tau - 2912 s u^3 \sigma \tau + 504 s^2 u^3 \sigma \tau - 28 s^3 u^3 \sigma \tau - 960 t u^3 \sigma \tau +$$
$$520 s t u^3 \sigma \tau - 90 s^2 t u^3 \sigma \tau + 5 s^3 t u^3 \sigma \tau + 1032192 \tau^2 - 559104 s \tau^2 + 96768 s^2 \tau^2 - 5376 s^3 \tau^2 - 184320 t \tau^2 + 99840 s t \tau^2 - 17280 s^2 t \tau^2 +$$
$$960 s^3 t \tau^2 - 559104 u \tau^2 + 302848 s u \tau^2 - 52416 s^2 u \tau^2 + 2912 s^3 u \tau^2 + 99840 t u \tau^2 - 54080 s t u \tau^2 + 9360 s^2 t u \tau^2 - 520 s^3 t u \tau^2 +$$
$$96768 u^2 \tau^2 - 52416 s u^2 \tau^2 + 9072 s^2 u^2 \tau^2 - 504 s^3 u^2 \tau^2 - 17280 t u^2 \tau^2 + 9360 s t u^2 \tau^2 - 1620 s^2 t u^2 \tau^2 + 90 s^3 t u^2 \tau^2 - 5376 u^3 \tau^2 + 2912 s u^3 \tau^2 -$$
$$504 s^2 u^3 \tau^2 + 28 s^3 u^3 \tau^2 + 960 t u^3 \tau^2 - 520 s t u^3 \tau^2 + 90 s^2 t u^3 \tau^2 - 5 s^3 t u^3 \tau^2\big) \big/ \big(4 (-6 + s) (-4 + s) (-6 + t) (-4 + t) (-6 + u) (-4 + u) n^3\big)$$

The MRV limit

MRV = Maximally R-symmetry Violating

$$\mathcal{O}_k(x;t) = \mathcal{O}(x)^{(i_1 \dots, i_k)}\, t_{i_1} \dots t_{i_k} \qquad \sigma = \frac{t_{13}\, t_{24}}{t_{12}\, t_{34}}, \quad \tau = \frac{t_{14}\, t_{23}}{t_{12}\, t_{34}}$$

$\langle 2222 \rangle$ in $AdS_7 \times S^4$: $\mathcal{M}(s,t;\sigma,\tau) =$ [illegible]

If you set $t_1 = t_3 \;\Rightarrow\; \sigma = 0,\; \tau = 1$ (**u-channel MRV limit**):

$$\mathcal{M}(s,t;0,1) = \frac{(u-8)(u-10)}{n^3}\left(\frac{1}{s-4} + \frac{1}{4(s-6)} + \frac{1}{t-4} + \frac{1}{4(t-6)}\right)$$

Alday, Zhou 2006.12505, 2006.06653 [hep-th]

MHV amplitudes

MHV = Maximally Helicity Violating

$$\mathscr{A}(1^+ 2^+ 3^+ \cdots n^+) = 0$$

$$\mathscr{A}(1^- 2^+ 3^+ \cdots n^+) = 0$$

$$\mathscr{A}(1^- 2^- 3^+ \cdots n^+) = i \frac{\langle 1\,2 \rangle^4}{\langle 1\,2 \rangle \langle 2\,3 \rangle \cdots \langle n\,1 \rangle}$$

Parke, Taylor PRL 56 (23) 2459

Holographic theories with half-maximal SUSY

$4d\ \mathcal{N} = 4$ SYM with flavors

N **D3-branes in flat space** $\leftrightarrow AdS_5 \times S^5$ near horizon $\leftrightarrow 4d\ \mathcal{N} = 4$ SYM

Add N_F **D7-branes**. If $N_F \ll N$ ignore back-reaction: still conformal.

AdS_5 (columns 1–5) S^5 (columns 6–10)

	1	2	3	4	5	6	7	8	9	10
N D3	×	×	×	×						
N_F D7	×	×	×	×	×	×	×	×		

$S^3 \subset S^5$ (columns 6–8)

$\mathcal{N} = 4$ SYM with flavors

- **Break 1/2 SUSY:** the theory on the D3 is now $4d$ $\mathcal{N} = 2$

- **Break the $SO(6)$ isometry of S^5:** the breaking pattern is

$$SO(6) = SO(4) \times SO(2) = \boxed{SU(2)_L} \times \boxed{SU(2)_R \times U(1)_r}$$

$$S^5 = S^3 \star S^1 \qquad \textbf{Global} \qquad \textbf{R symmetry}$$

- **Add gluons:** the D7 branes carry a $8d$ **super Yang-Mills** with gauge group $G_F = SU(N_F) \to$ flavor group of the $4d$ $\mathcal{N} = 2$ CFT

- **1/2-BPS operators:** in the $4d$ $\mathcal{N} = 2$ living on the D3-branes, dual to the **Kaluza-Klein modes of the D7 gluons on the S^3**

Branes probing singularities

SCFT	Holographic description	Global symmetry
E-string: $6d$ $\mathcal{N}=1$	M5 on end-of-the-world M9	$SU(2)_L \times SU(2)_R \times G_F$
Seiberg: $5d$ $\mathcal{N}=1$	D4-D8 system	$SU(2)_L \times SU(2)_R \times G_F$
$4d$ $\mathcal{N}=2$	D3 near F-theory singularities	$SU(2)_L \times SU(2)_R \times U(1)_r \times G_F$

↓ ↓ ↓

SUSY
half-maximal

Singular locus
$AdS_{d+1} \times S^3$

Symmetry
$SU(2)_L \times$ R-symm $\times G_F$

Flavor branes

SCFT	Holographic description
$4d\ \mathcal{N}=4$ SYM + flavors: $4d\ \mathcal{N}=2$	D7 wrapping $AdS_5 \times S^3 \subset AdS_5 \times S^5$
$3d\ \mathcal{N}=6$ ABJM + flavors: $3d\ \mathcal{N}=3$	D6 wrapping $AdS_4 \times \mathbb{RP}^3 \subset AdS_4 \times \mathbb{CP}^3$

Note: only **6 supercharges**,
but same features and techniques

Always add $N_F \ll N$ **flavor branes**: ignore back-reaction.

Same generic features highlighted for the theories in the previous slide.

The 1/2-BPS sector: super-gluons

Δ	ℓ	j_R	j_L	G_F
$\epsilon\, k$	0	$\frac{k}{2}$	$\frac{k-2}{2}$	adj

$$\epsilon = \frac{d-2}{2}$$

k = 2 : conserved **current** of G_F $\leftrightarrow$ AdS **gluons** with gauge group G_F

k > 2 : Kaluza-Klein modes of gluons on S^3

$$\mathcal{O}_k^I(x; v, \bar{v}) \equiv \mathcal{O}(x)^{I;\,\alpha_1 \ldots \alpha_k; \bar{\alpha}_1 \ldots \bar{\alpha}_{k-2}}\, v_{\alpha_1} \ldots v_{\alpha_k} \bar{v}_{\alpha_1} \ldots \bar{v}_{\alpha_{k-2}}$$

The 1/2-BPS sector: super-gluons

Δ	ℓ	j_R	j_L	G_F
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k > 2 : Kaluza-Klein modes of gluons on S^3

$$\mathcal{O}^I_k(x; v, \bar{v}) \equiv \mathcal{O}(x)^{I;\, \alpha_1 \ldots \alpha_k; \bar{\alpha}_1 \ldots \bar{\alpha}_{k-2}} \underbrace{v_{\alpha_1} \cdots v_{\alpha_k}}_{\substack{SU(2)_R \\ \text{polarizations}}} \underbrace{\bar{v}_{\alpha_1} \cdots \bar{v}_{\alpha_{k-2}}}_{\substack{SU(2)_L \\ \text{polarizations}}}$$

I → Adj of G_F

Four-point kinematics

For simplicity focus on $\langle k\,k\,k\,k\rangle$, but full control on $\langle k_1\,k_2\,k_3\,k_4\rangle$

$$\langle \mathcal{O}(1)\ldots\mathcal{O}(4)\rangle = \left(\frac{v_{12}\,v_{34}}{x_{12}^{2\epsilon}\,x_{34}^{2\epsilon}}\right)^{k} \left(\bar{v}_{12}\,\bar{v}_{34}\right)^{k-2} \mathcal{G}^{I_1I_2I_3I_4}(U,V;\alpha,\beta)$$

$$U = \frac{x_{12}^2x_{34}^2}{x_{13}^2x_{24}^2} = z\,\bar{z}\,, \quad V = \frac{x_{14}^2x_{23}^2}{x_{13}^2x_{24}^2} = (1-z)\,(1-\bar{z})\,, \quad \alpha = \frac{v_{13}v_{24}}{v_{12}v_{34}}\,, \quad \beta = \frac{\bar{v}_{13}\bar{v}_{24}}{\bar{v}_{12}\bar{v}_{34}}$$

N.B. $\mathcal{G}$ is a **polynomial in** α, β of deg $= k, k-2$ resp.

Superconformal invariance also enforces the
Superconformal Ward Identity (SCWI):

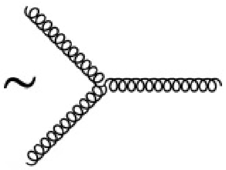
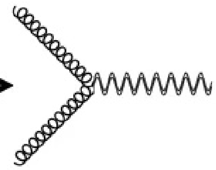
$$\left(z\,\partial_z - \epsilon\,\alpha\,\partial_\alpha\right)\, \mathcal{G}^{I_1I_2I_3I_4}(z,\bar{z};\alpha,\beta)\Big|_{\alpha=1/z} = 0$$

Dolan, Gallot, Sokatchev hep-th/0405180

All gluon four-point functions

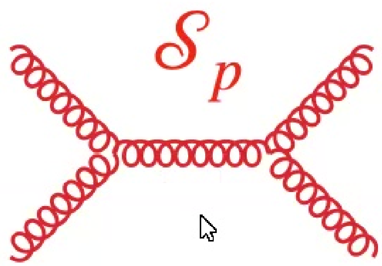
Gluon scattering in AdS

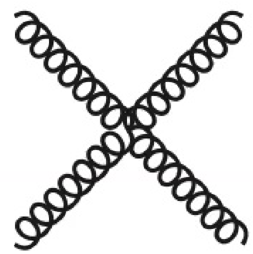
Key fact: at lowest order in $1/N$ we have

$\frac{1}{C_J} \sim$  $\gg$  $\sim \frac{1}{C_T} \Rightarrow$ **Pure gluon scattering in AdS**

$$\mathcal{M} = \mathsf{c}_s \mathcal{M}_s + \mathsf{c}_t \mathcal{M}_t + \mathsf{c}_u \mathcal{M}_u$$

$$\mathsf{c}_s = f^{I_1 I_2 J} f^{J I_3 I_4}$$
$$\mathsf{c}_t = f^{I_1 I_4 J} f^{J I_2 I_3}$$
$$\mathsf{c}_u = f^{I_1 I_3 J} f^{J I_4 I_2}$$

$\mathcal{M}_s \sim$ 

$+$ 

$$\mathcal{O}_{k_1} \times \mathcal{O}_{k_2} = \sum_p c_{k_1 k_2 p} \mathcal{O}_p + \text{double trace}$$

A two-steps procedure

For $\langle kkkk \rangle$ we set

$$\mathcal{M}_s = \sum_{p=2}^{2k-2} c_{kkp}^2 \, \mathcal{Y}_{p-2}(\beta) \boxed{\mathcal{S}_p} + \text{contact}$$

Exchange of the full half-BPS multiplet

N.B. not just the super-primary!

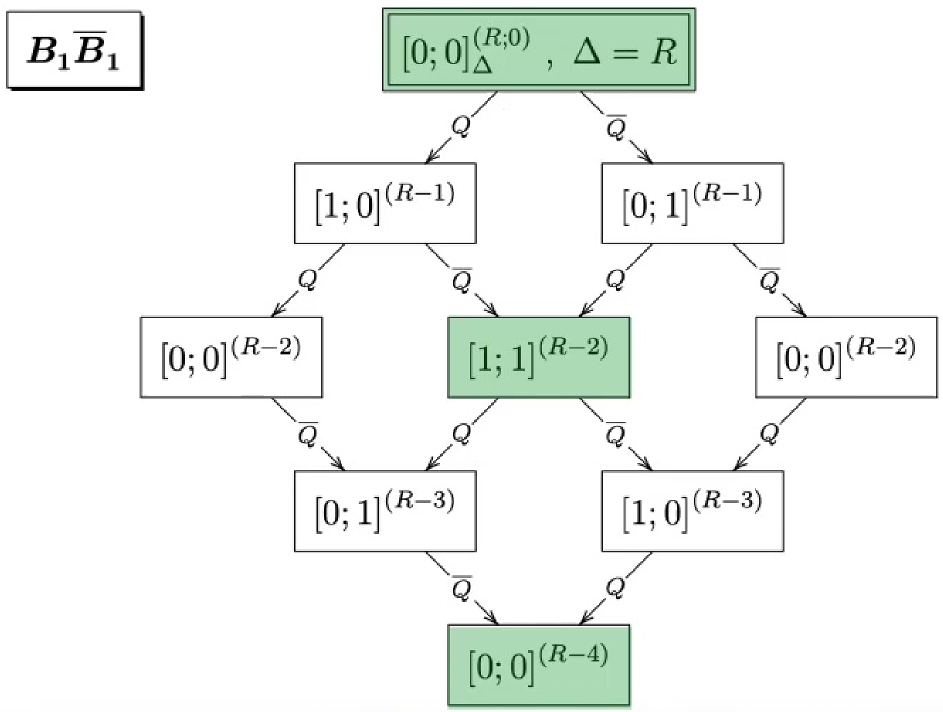
- We fix the individual $\mathcal{S}_p$ using the MRV limit
- We apply the SCWI to fix the OPE coefficients $c_{k_1 k_2 k_3}$ and the contact terms

Super-gluon exchange

However, one has to determine the full **supermultiplet exchange!**

For example, look at the 1/2-BPS multiplets of $4d$ $\mathcal{N} = 2$.

Only three representations allowed by symmetry:



Cordova, Dumitrescu, Intriligator 1612.00809 [hep-th]

Super-gluon exchange

For all short multiplets of all super algebras of interest, the **representations that can be exchanged by two super-primaries** are

component field	Δ	ℓ	j_R	j_L	G_F
s_p^I	$\epsilon\, p$	0	$\frac{p}{2}$	$\frac{p-2}{2}$	adj
$A_{p,\mu}^I$	$\epsilon\, p+1$	1	$\frac{p}{2}-1$	$\frac{p-2}{2}$	adj
r_p^I	$\epsilon\, p+2$	0	$\frac{p}{2}-2$	$\frac{p-2}{2}$	adj

The **exchange of super-gluons $\mathcal{S}_p$** must take into account **all of these contributions**!

Super-gluon exchange

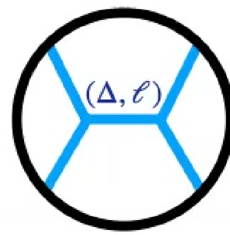
R-symmetry polynomials

→ for $SU(2)$: Jacobi polynomials

$$\mathcal{S}_p = \mathcal{Y}_p(\alpha)\, M_{\epsilon p,0}(s,t,u) + \lambda_{A_p}\, \mathcal{Y}_{p-2}(\alpha)\, M_{\epsilon p+1,1}(s,t,u) + \lambda_{r_p}\, \mathcal{Y}_{p-4}(\alpha)\, M_{\epsilon p+2,0}(s,t,u)$$

$M_{\Delta,\ell}$ ↔ **exchange of** $\mathcal{O}_{\Delta,\ell}$

(known for all Δ, ℓ)



Super-gluon exchange

R-symmetry polynomials

$\rightarrow$ for $SU(2)$: Jacobi polynomials

$$\mathcal{S}_p = \mathcal{Y}_p(\alpha)\, M_{\epsilon p,0}(s,t,u) + \lambda_{A_p} \mathcal{Y}_{p-2}(\alpha)\, M_{\epsilon p+1,1}(s,t,u) + \lambda_{r_p} \mathcal{Y}_{p-4}(\alpha)\, M_{\epsilon p+2,0}(s,t,u)$$

$M_{\Delta,\ell} \leftrightarrow$ **exchange of** $\mathcal{O}_{\Delta,\ell}$

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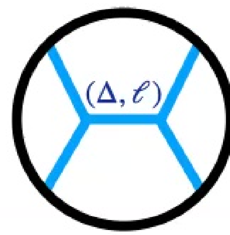


How do we fix the relative coefficients?

The MRV limit

Choose a configuration of R-symmetry polarizations

u-channel MRV : $v_1 = v_3 \quad \Rightarrow \quad \alpha = 0$

Two main consequences:

1. $\mathscr{M}(s,t)$ has **no poles in** u
2. $\mathscr{M}(s,t)$ has a single **zero at** $u = 2\epsilon k$

The MRV limit

$$\mathcal{S}_p = \mathcal{Y}_p(\alpha)\, M_{\epsilon p,0}(s,t,u) + \lambda_{A_p} \mathcal{Y}_{p-2}(\alpha)\, M_{\epsilon p+1,1}(s,t,u) + \lambda_{r_p} \mathcal{Y}_{p-4}(\alpha)\, M_{\epsilon p+2,0}(s,t,u)$$

Set $\alpha = 0$ and require a zero at $u = 2\epsilon k$

Fixes the λ's completely!

$$\mathcal{S}_p \sim (u - 2\epsilon k) \sum_i \frac{\mu_i}{s - \nu_i}$$

- **Improved Regge behavior:** $\mathcal{S}_p \sim s^{-1}$ in u-channel Regge limit ($s, t \to \infty, \quad u$ fixed)

The MRV limit

1 $\mathscr{M}(s,t)$ has **no poles in u**

If we set $v_1 = v_3$, then **only** $SU(2)_R$ **irreps with** $j_R = \frac{k_1 + k_3}{2}$

can be exchanged in the u-channel

However, the **vanishing of extremal 3-pt functions** dictates that

$$\langle \mathcal{O}_{k_1} \mathcal{O}_{k_3} \mathcal{O}_{k_1+k_3} \rangle = 0$$

Hence, **no exchange of short operators in the u-channel** is allowed

→ **No poles in u**

Or: all **R-symmetry polynomials** in the **u-channel** are proportional to $\alpha^{\#}$, so they **vanish when** $\alpha = 0$. Hence, cannot have u-channel exchanges.

The MRV limit

2 $\mathcal{M}(s,t)$ has a single **zero at** $u = 2\epsilon k$

Consider for simplicity $\langle k\,k\,k\,k \rangle$.
In a large N theory all **long operators** are double-trace operators, and the ones with **lowest twist** have

$$\tau = \Delta - \ell = 2\epsilon k$$

For a **long super-multiplet** to be exchanged in the u-channel, and to be **visible in the MRV limit**, the super-primary must have

$$j_R = k - 2$$

But then **all operators** in the super-multiplet **with twist** $\tau = 2\epsilon k$ **decouple** in the MRV limit. Hence, they **cannot develop an anomalous dimension**. To achieve this, we must cancel the double pole in the Gamma functions.

Operator	Twist
$(\Delta)_{\ell}^{(k-2)}$	$\tau_0 \equiv \Delta - \ell$
$(\Delta+1)_{\ell+1}^{(k-2)\pm1;(k-2)}$	τ_0
$(\Delta+1)_{\ell-1}^{(k-2)\pm1;(k-2)}$	$\tau_0 + 2$
$(\Delta+2)_{\ell+2}^{(k-2)}$	τ_0
$(\Delta+2)_{\ell}^{(k-2)\pm2;(k-2)\pm1;(k-2)}$	$\tau_0 + 2$
$(\Delta+2)_{\ell-2}^{(k-2)}$	$\tau_0 + 4$
$(\Delta+3)_{\ell+1}^{(k-2)\pm1;(k-2)}$	$\tau_0 + 2$
$(\Delta+3)_{\ell-1}^{(k-2)\pm1;(k-2)}$	$\tau_0 + 4$
$(\Delta+4)_{\ell}^{(k-2)}$	$\tau_0 + 4$

Operator	Twist
$(\Delta)_{\ell}^{(k-2)}$	$\tau_0 \equiv \Delta - \ell$
$(\Delta+1)_{\ell+1}^{(k-2)\pm1;(k-2)}$	τ_0
$(\Delta+1)_{\ell-1}^{(k-2)\pm1;(k-2)}$	$\tau_0 + 2$
$(\Delta+2)_{\ell+2}^{(k-2)}$	τ_0
$(\Delta+2)_{\ell}^{(k-2)\pm2;(k-2)\pm1;(k-2)}$	$\tau_0 + 2$
$(\Delta+2)_{\ell-2}^{(k-2)}$	$\tau_0 + 4$
$(\Delta+3)_{\ell+1}^{(k-2)\pm1;(k-2)}$	$\tau_0 + 2$
$(\Delta+3)_{\ell-1}^{(k-2)\pm1;(k-2)}$	$\tau_0 + 4$
$(\Delta+4)_{\ell}^{(k-2)}$	$\tau_0 + 4$

Only one operator, with $j_R = k$, survives the MRV limit

Hence, the operators with twist $\tau_0 \equiv \Delta - \ell$ decouple in the limit

If we take $\tau_0 = 2\epsilon k$, the long multiplet with minimal twist, this means that when taking the MRV limit no long operator with $\tau_0 = 2\epsilon k$ is present

The ansatz

This procedure **fixes completely the super-exchanges:**

$$\mathcal{S}_p = \sum_{m=0}^{\infty} \sum_{i=0}^{p/2} \frac{R^i_{p,m}(t,u)}{s - \epsilon p - 2m} (1-\alpha)^i$$

Residues fixed by MRV limit

In terms of these, we write for the s-channel contribution

$$\mathcal{M}_s = \sum_{p=2}^{2k-2} c_{12p} c_{34p} \mathcal{Y}_{p-2}(\beta)\, \mathcal{S}_p + \text{contact}$$

Degree 0 in s,t,u:

In the flat space limit it must match the contact term of Yang-Mills theory

Apply SCWI

$$\mathscr{M} = c_s\,\mathscr{M}_s + c_t\,\mathscr{M}_t + c_u\,\mathscr{M}_u$$

$$[c_s + c_t + c_u = 0]$$

Now we apply the **SCWI:** $\left(z\,\partial_z - \epsilon\,\alpha\,\partial_\alpha\right)\ \mathscr{G}^{I_1 I_2 I_3 I_4}(z,\bar{z};\alpha,\beta)\Big|_{\alpha=1/z} = 0$

Fixes all **OPE coeff.** c_{pqr} in terms of c_{222}

NEW!!!

Fixes **contact** $= 0$ in our ansatz

[only 3-pt couplings matter]

The main result

Now, we only **need** c_{222}. We have

$$(c_{222})^2 = \frac{2(2\epsilon+1)}{\epsilon}\frac{1}{C_J} \qquad \langle J J \rangle \sim C_J$$

In **all theories** that we consider

$$C_J \sim N^{\epsilon} \quad \ll \quad C_T \sim N^{1+\epsilon}$$

The **explicit value** of C_J gives

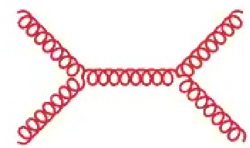
All $\langle k_1 \, k_2 \, k_3 \, k_4 \rangle$ in $d = 3,4,5,6$

Only known results before us: 2222 in d=5,6 from Zhou 1804.02397 [hep-th]

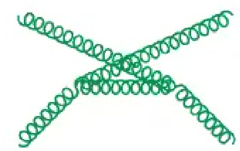
$\langle 2\,2\,k\,k \rangle$ in $d = 4$

$$\mathcal{M} = \mathsf{c}_s\,\mathcal{M}_s + \mathsf{c}_t\,\mathcal{M}_t + \mathsf{c}_u\,\mathcal{M}_u$$

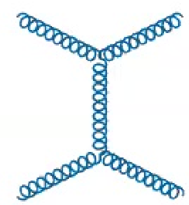
$$\mathcal{M}_s = \frac{-2\alpha(k+2) + k + \alpha\,t + \alpha\,u - u + 2}{(k-2)!(s-2)}\,\frac{6}{C_J}$$



$$\mathcal{M}_t = \frac{(1-\alpha)(-2\,\alpha\,k - k + \alpha\,s + u - 2)}{(k-2)!(t-k)}\,\frac{6}{C_J}$$



$$\mathcal{M}_u = \frac{-\alpha\,(2\,\alpha\,k - 3\,k - \alpha\,s + s + t - 2)}{(k-2)!(u-k)}\,\frac{6}{C_J}$$



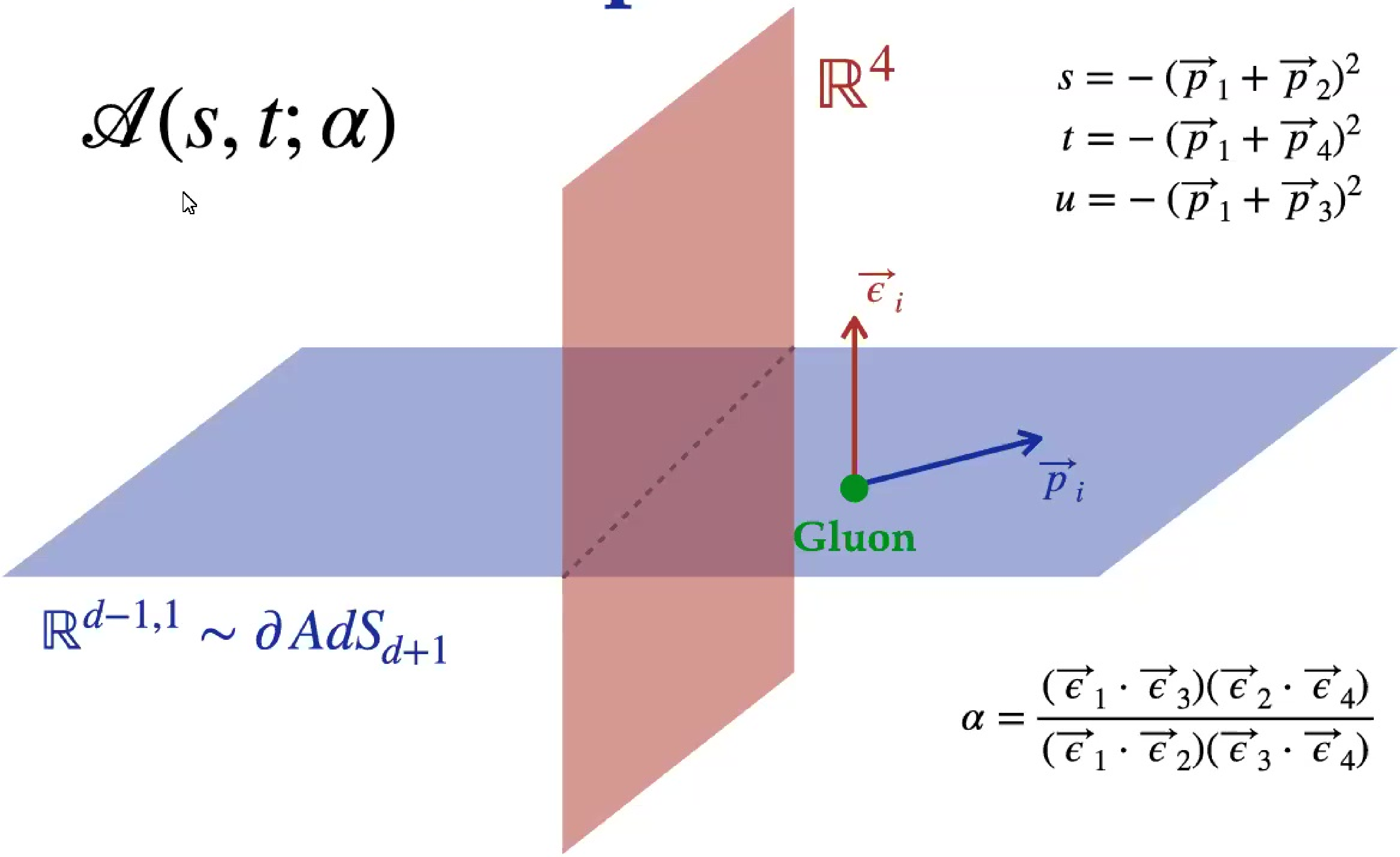
$$\frac{1}{C_J} \sim \frac{1}{N} \quad \gg \quad \frac{1}{C_T} \sim \frac{1}{N^2}$$

Hidden structures

Flat space limit

$$\mathscr{A}(s, t; \alpha)$$

$$s = -(\vec{p}_1 + \vec{p}_2)^2$$
$$t = -(\vec{p}_1 + \vec{p}_4)^2$$
$$u = -(\vec{p}_1 + \vec{p}_3)^2$$



$$\alpha = \frac{(\vec{\epsilon}_1 \cdot \vec{\epsilon}_3)(\vec{\epsilon}_2 \cdot \vec{\epsilon}_4)}{(\vec{\epsilon}_1 \cdot \vec{\epsilon}_2)(\vec{\epsilon}_3 \cdot \vec{\epsilon}_4)}$$

Color-kinematics duality

$$\mathcal{M}_{2222} = \mathtt{c}_s \mathtt{n}_s \left(\frac{1}{s-2\epsilon} + \ldots \right) + \mathtt{c}_t \mathtt{n}_t \left(\frac{1}{t-2\epsilon} + \ldots \right) + \mathtt{c}_u \mathtt{n}_u \left(\frac{1}{u-2\epsilon} + \ldots \right)$$

$$\mathtt{n}_s = u - 4\epsilon + \alpha s \,, \quad \mathtt{n}_t = (1-\alpha)((\alpha-1)(u-4\epsilon) + \alpha t) \,, \quad \mathtt{n}_u = -\alpha(\alpha(s-4\epsilon)+u)$$

$$\mathtt{c}_s + \mathtt{c}_t + \mathtt{c}_u = 0$$

Jacobi identity
(purely algebraic)

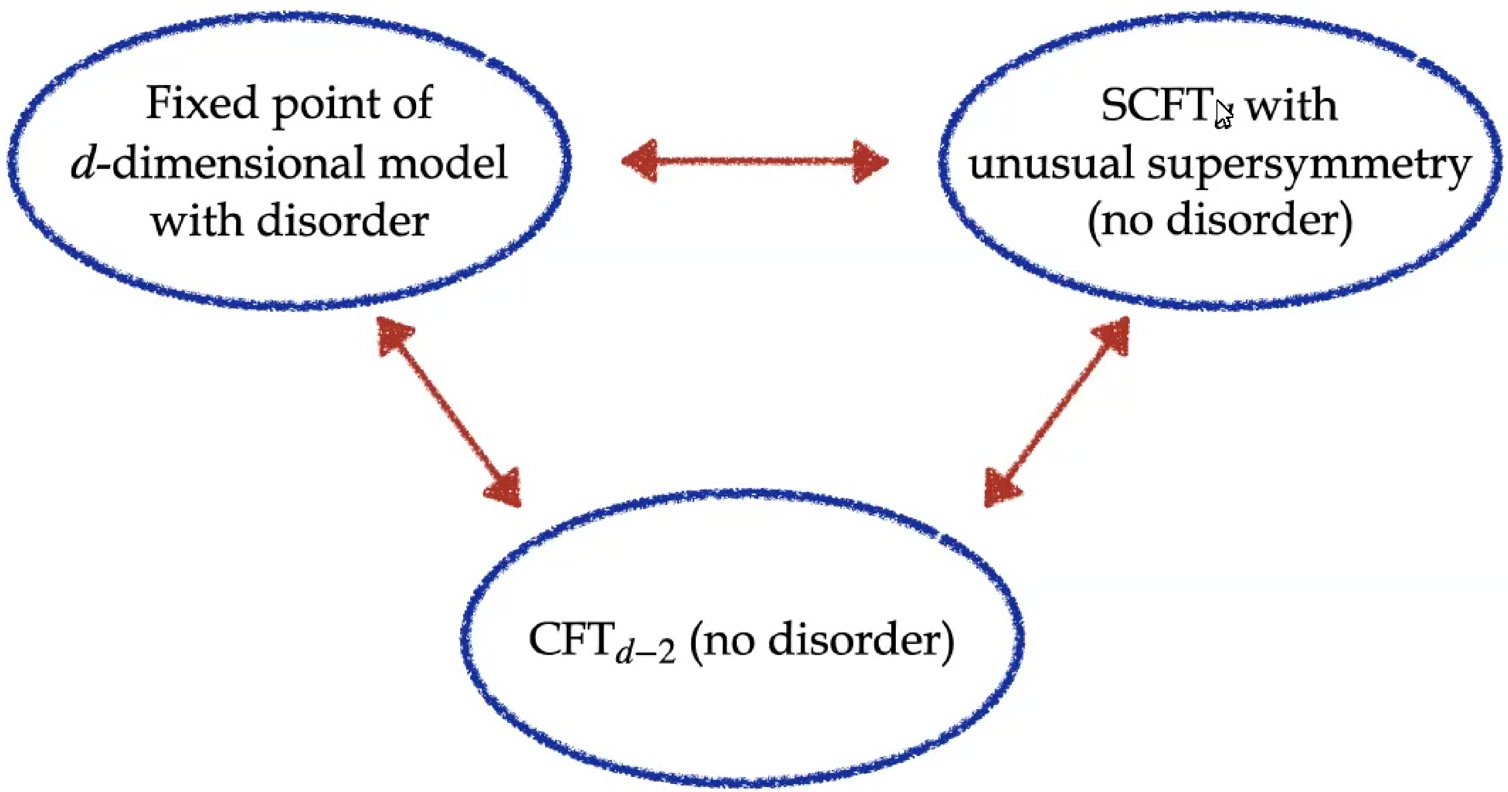
$$\mathtt{n}_s + \mathtt{n}_t + \mathtt{n}_u = 0$$

Kinematic analogue,
known in flat space

See also Zhou 2106.07651 [hep-th]: CK duality and double copy in d=4

Parisi-Sourlas supersymmetry

Parisi, Sourlas Phys. Rev. Lett. 43 (1979) 744



Parisi-Sourlas supersymmetry

Recall that we write the s-channel exchange multiplet as

$$\mathscr{M}_s = \sum_p c_{12p}\, c_{34p}\, \mathcal{S}_p$$

It turns out that

$$\mathcal{S}_p = \mathbb{K}_p \circ \underbrace{\mathbb{Y}_{p,2}(\alpha)\, \mathbb{Y}_{p-2,0}(\beta)}_{\text{Polynomials}}\, M^{(d-1)}_{\epsilon p,0}$$

Differential Operator ($\mathbb{K}_p$)

Polynomials

Scalar exchange in AdS_{d-1} ($M^{(d-1)}_{\epsilon p,0}$)

Hidden conformal symmetry

In the **flat space limit**

$$\widetilde{\mathcal{M}}_{2222} \longrightarrow \left(\frac{\mathsf{c}_s}{s\,u} - \frac{\mathsf{c}_t}{t\,u} \right)$$

Which is annihilated by the **SCT generator** in momentum space

$$K_\mu = \sum_{i=1}^{4} \left(\frac{p_{i\mu}}{2} \frac{\partial}{\partial p_i^\nu} \frac{\partial}{\partial p_{i,\nu}} - p_i^\nu \frac{\partial}{\partial p_i^\nu} \frac{\partial}{\partial p_i^\mu} - \frac{d-2}{2} \frac{\partial}{\partial p_i^\mu} \right)$$

Only when the spacetime dimension is $d = 8$:
worldvolume of the D7-branes!

Similar to $AdS_5 \times S^5$ for $d = 10$ and $AdS_3 \times S^3 \times K3$ for $d = 6$

Caron-Huot, Trinh 1809.09173 [hep-th] Rastelli, Roumpedakis, Zhou 1905.11983 [hep-th]

Outlook

Color-kinematics duality & double copy

CK duality found here for 2222 in all dimensions.
Generalized to arbitrary KK modes, with related DC relations, for d=4.
What about other d? Non supersymmetric theories?

Higher-point functions

Is CK duality present in higher-point gluon amplitudes in AdS?
Double copy? Properties of color-ordered amplitudes?

Subleading corrections

These would include: loops, graviton exchanges, higher-derivative corrections (partially addressed in our work), mixed graviton-gluon.

CFT data

From our correlators: new protected and unprotected CFT data. Can extract them, explore structures, feed to and compare with numerical bootstrap.