Title: The Entropic Dynamics approach to Quantum Mechanics

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Abstract: Entropic Dynamics (ED) is a framework in which Quantum Mechanics is derived as an application of entropic methods of inference. In ED the dynamics of the probability distribution is driven by entropy subject to constraints that are codified into a quantity later identified as the phase of the wave function. The challenge is to specify how those constraints are themselves updated.

The important ingredients are two: the cotangent bundle associated to the probability simplex inherits (1) a natural symplectic structure from ED, and (2) a natural metric structure from information geometry.

The requirement that the dynamics preserves both the symplectic structure (a Hamilton flow) and the metric structure (a Killing flow) leads to a Hamiltonian dynamics of probabilities in which the linearity of the Schrödinger equation, the emergence of a complex structure, Hilbert spaces, and the Born rule, are derived rather than postulated.

The Entropic Dynamics approach to Quantum Mechanics

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Thank you!

Where do we stand on Quantum Mechanics?

anti-realist		
realist		

Where do we stand on Quantum Mechanics?

	epistemic Ψ	ontic Ψ
anti-realist	Copenhagen QBism	X
realist		early Schrödinger Bohm many worlds stochastic mech.

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Where do we stand on Quantum Mechanics?

	epistemic Ψ	ontic Ψ
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	entropic dynamics	
realist	Einstein Ballentine	early Schrödinger Bohm many worlds stochastic mech.

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The subject: **Quantum mechanics**.

The goal: **To derive the mathematical formalism**.

In the traditional approach the Hilbert space comes first.

Why probabilities? “Quantum” probabilities? Born rule?

Linear unitary evolution vs. wave function collapse?

What is real? Ontic vs. epistemic?

An alternative approach: probability comes first.

Why wave functions? Why complex numbers?

Why a linear unitary evolution? Why Hilbert spaces?

1. Kinematics

Ontological clarity: What is real, ontic?
What is epistemic?

Discrete ontic microstates: $j = 1, \ldots, n$

e.g., an n-sided "quantum" die

Epistemic probabilities: $\rho(j) = \rho^j$ Bayesian,…

but not personalistic,

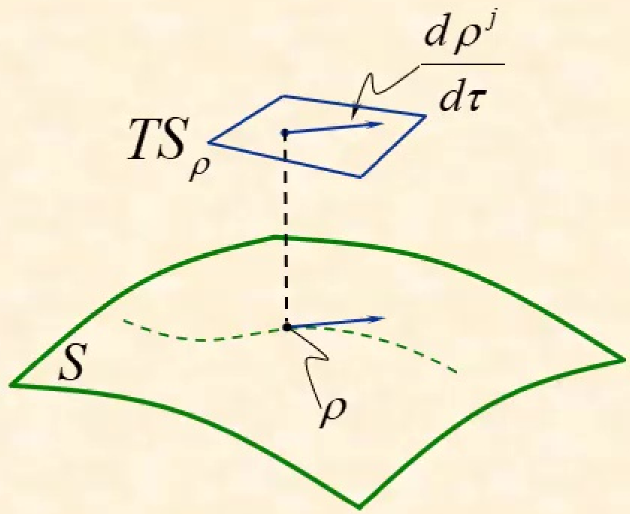
and not "quantum" probabilities.

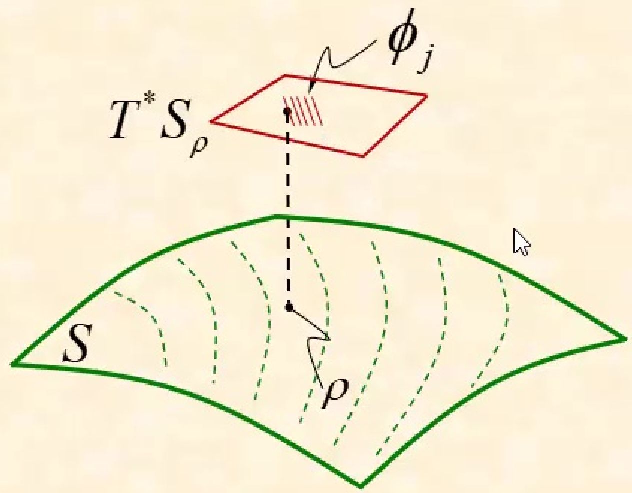
Our goal: to study curves on the $(n-1)$-dimensional simplex,

$$S = \{\rho \mid \rho^j \geq 0 \,;\, \sum_{j=1}^{n} \rho^j = 1\}$$

… but this is only a kinematical prelude to dynamics.

Some geometry





S = **e-configuration space**

TS = Tangent bundle

T^*S = **e-phase space**

= Cotangent bundle

Some notation

Point: $X = (\rho, \phi)$ $\quad X^{\alpha j} = (X^{1j}, X^{2j}) = (\rho^j, \phi_j)$

Vector: $\bar{V} = V^{\alpha j} \dfrac{\partial}{\partial X^{\alpha j}}$ $\quad V^{\alpha j} = \dfrac{dX^{\alpha j}}{d\tau} = \begin{pmatrix} d\rho^j / d\tau \\ d\phi_j / d\tau \end{pmatrix}$

Gradient: $\tilde{\nabla} F(X) = \dfrac{\partial F}{\partial \rho^j} \tilde{\nabla} \rho^j + \dfrac{\partial F}{\partial \phi^j} \tilde{\nabla} \phi^j \quad = \dfrac{\partial F}{\partial X^{\alpha j}} \tilde{\nabla} X^{\alpha j}$

An important technicality: Normalization

Embed S into the space S^+ of unnormalized probabilities,

$$S^+ = \{ \rho \mid \rho^j \geq 0 \}$$

Symplectic geometry of T^*S^+

Symplectic form: $\Omega = \tilde{\nabla}\rho^j \otimes \tilde{\nabla}\phi_j - \tilde{\nabla}\phi_j \otimes \tilde{\nabla}\rho^j$

$$\Omega(\bar{V},\bar{U}) = \Omega_{\alpha j,\beta k} V^{\alpha j} U^{\beta k} \qquad \Omega_{\alpha j,\beta k} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \delta_{jk}$$

Vector fields $\bar{H}(X)$ such that $\mathcal{L}_{\bar{H}}\Omega = 0$ are called Hamiltonian flows.

Hamiltonian flows: $\mathcal{L}_{\bar{H}}\Omega = 0$

Poincare's lemma: there exists a scalar function $\tilde{H}(X)$ such that

$$\Omega(\bar{H},\bullet) = \tilde{\nabla}\tilde{H}(\bullet)$$

$$\frac{d\rho^j}{d\tau} = \frac{\partial \tilde{H}}{\partial \phi_j} \quad \text{and} \quad \frac{d\phi_j}{d\tau} = -\frac{\partial \tilde{H}}{\partial \rho^j}$$

Hamilton's equations !

Furthermore… $\Omega(\bar{V},\bar{U}) = \{\tilde{V},\tilde{U}\}$ Poisson brackets !

and even more… $\dfrac{dF(X)}{d\tau} = \{F,\tilde{H}\}$

The normalization constraint

$$|\rho| = \sum_{j=1}^{n} \rho^j \qquad \tilde{N} = 1 - |\rho| \qquad \tilde{N} = 0$$

$\tilde{N}(X)$ generates a Hamiltonian flow $\bar{N}(X)$

$$\rho^j(\nu) = \rho^j(0) \qquad \phi_j(\nu) = \phi_j(0) + \nu$$

Rays !!

We want $\dfrac{d\tilde{N}}{d\tau} = \{\tilde{N}, \tilde{H}\} = 0$ but then $\{\tilde{H}, \tilde{N}\} = 0 = \dfrac{d\tilde{H}}{d\nu}$

$\Rightarrow$ $\tilde{N}$ generates a "gauge" symmetry !

$\Rightarrow$ $\tilde{H}$ must be "gauge" invariant.

The information geometry of T^*S^+

S^+ is an n-dim statistical manifold:

$$\delta\ell^2 = g_{jk}\delta\rho^j\delta\rho^k \quad \text{with} \quad g_{jk} = A(|\rho|) + \frac{B(|\rho|)}{2\rho^j}\delta_{jk}$$

For the $2n$-dim T^*S^+:

$$\delta\tilde{\ell}^2 = g_{jk}\delta\rho^j\delta\rho^k + g^{jk}\delta\phi_j\delta\phi_k$$

Flow-reversal symmetry: $\beta = 0$

Information geometry of e-phase space T^*S

Consider two neighboring points on the simplex, $|\rho| = 1$

$$(\rho^j, \phi_j) \quad \text{and} \quad (\rho^j + \delta\rho^j\,,\ \phi_j + \delta\phi_j)$$

then $$\delta\tilde{\ell}^2(\nu) = g_{jk}\delta\rho^j\delta\rho^k + g^{jk}(\delta\phi_j + \nu)(\delta\phi_k + \nu)$$

The distance between two neighboring rays is

$$\delta\tilde{s}^2 = \min_{\nu} \delta\tilde{\ell}^2(\nu) = \sum_{j=1}^{n}\left[\frac{B(1)}{2\rho^j}(\delta\rho^j)^2 + \frac{2\rho^j}{B(1)}(\delta\pi_j - \langle\delta\pi\rangle)^2\right]$$

which is the Fubini-Study metric !!

Since the particular embedding space T^*S^+ does not matter, choose

$$A(|\rho|) = 0 \qquad \text{and} \qquad B(|\rho|) = 1$$

which makes T^*S^+ flat.

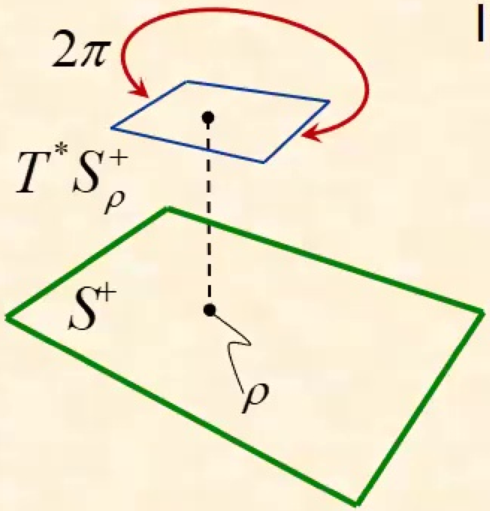
$$\delta\tilde{\ell}^2 = \sum_{j=1}^{n}\left[\frac{1}{2\rho^j}(\delta\rho^j)^2 + 2\rho^j(\delta\phi_j)^2\right] = G_{\alpha j,\beta k}\,\delta X^{\alpha j}\,\delta X^{\beta k}$$

Furthermore…

$$-G^{\alpha j,\gamma\ell}\,\Omega_{\gamma\ell,\beta k} = J^{\alpha j}{}_{\beta k} = \begin{pmatrix} 0 & -2\rho^j \\ 1/2\rho^i & 0 \end{pmatrix}\delta_{jk}$$

$J\,J = -\hat{1}$ … and we have a complex structure !!

For QM we must refine the choice of cotangent space



Introduce complex coordinates

$$\psi_j = \rho_j^{1/2} e^{i\phi_j} \quad \text{and} \quad i\psi_j^* = i\rho_j^{1/2} e^{-i\phi_j}$$

$$\psi^{\mu j} = \begin{pmatrix} \psi_j \\ i\psi_j^* \end{pmatrix} \qquad J^{\mu j}{}_{\nu k} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \delta_{jk}$$

ϕ_j is equivalent to $\phi_j + 2\pi$

The cotangent spaces are "hypercubes" of edge 2π with opposite faces identified.

Hamilton-Killing flows

We want flows $\bar{H}$ or $\tilde{H}$ such that

$$\mathcal{L}_{\bar{H}}\Omega = 0 \quad \text{and} \quad \mathcal{L}_{\bar{H}}G = 0$$

The conditions on $\tilde{H}(\psi,\psi^*)$ are

$$\frac{\partial^2 \tilde{H}}{\partial \psi_j \partial \psi_k} = 0 \quad \text{and} \quad \frac{\partial^2 \tilde{H}}{\partial \psi_j^* \partial \psi_k^*} = 0$$

Therefore,

$$\tilde{H}(\psi,\psi^*) = \sum_{j,k=1}^{n} \psi_j^* \hat{H}_{jk} \psi_k$$

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The HK flow is given by Hamilton's equations

$$\frac{d\psi_j}{d\tau} = \frac{\partial \tilde{H}}{\partial i\psi_j^*} \qquad \text{or} \qquad \frac{d\psi_j}{d\tau} = \{\psi_j, \tilde{H}\}$$

$$\tilde{H}(\psi,\psi^*) = \sum_{jk=1}^{n} \psi_j^* \hat{H}_{jk}\psi_k \quad \Rightarrow \quad i\frac{d\psi_j}{d\tau} = \sum_{k=1}^{n} \hat{H}_{jk}\psi_k$$

which is the linear Schrödinger equation.

Bohm 1952
Kibble 1979
Heslot 1985
Ashtekar Schilling 1998

Summary

- Ontological clarity:
  - Ontic microstates
  - Epistemic probabilities

- E-phase space is a cotangent bundle:

  Simplex plus "hypercubes" $\Longrightarrow$ Symplectic structure

  Information geometry $\Longrightarrow$ Metric structure

  $\Longrightarrow$ Complex structure

- HK-flows $\Longrightarrow$ Linear Schrödinger equation

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2. Entropic Dynamics

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We seek ontological clarity:

The goal is to predict the positions of particles, x.

Particles have definite but unknown positions $\Rightarrow \rho(x) = \rho^x$

to be inferred on the basis of relevant information

expressed in the form of constraints.

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Entropic Dynamics: Maximize an entropy

$$S[P,Q] = -\int dx' P(x'|x) \log \frac{P(x'|x)}{Q(x'|x)}$$

a point in
config. space

The Prior: Motion is continuous.

Impose short steps:

$$Q(x'|x) \propto \exp -\frac{1}{2}\sum_n \alpha_n \delta_{ab} \Delta x_n^a \Delta x_n^b$$

The main constraint:

Introduce a "phase field", $\phi(\vec{x}_1, \vec{x}_2, \ldots \vec{x}_N) = \phi_x$

Impose $\langle \Delta\phi \rangle = \kappa'$ or $\sum_n \frac{\partial \phi}{\partial x_n^a} \langle \Delta x_n^a \rangle = \kappa'$

Important: (a) Directionality, correlations, etc.

(b) Local in x but nonlocal in 3d space.

(c) The phase field is an "angle".

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Additional constraints: EM interactions, Spin ½, etc.

$$\langle \Delta x_n^a \rangle A_a(\vec{x}_n) = \kappa'' \qquad (n = 1 \ldots N)$$

vector potential

The transition probability :

$$P(x' \mid x) \propto \exp \sum_n \left(-\frac{1}{2} \alpha_n \delta_{ab} \Delta x_n^a \Delta x_n^b + \alpha' \left[\partial_{na} \phi - \beta_n A_a(\vec{x}_n) \right] \Delta x_n^a \right)$$

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The result:

$$P(x' \mid x) = \frac{1}{\zeta} \exp - \sum_n \left(\frac{1}{2} \alpha_n \delta_{ab} \Delta x_n^a \Delta x_n^b + \alpha' \partial_{na} \phi \, \Delta x_n^a \right)$$

"Entropic" Time

is introduced to keep track of the accumulation of many small changes.

(1) Introduce the notion of an instant

$$P(x', x) = P(x' \mid x)P(x) \;\Rightarrow\; P(x') = \int dx\, P(x' \mid x) P(x)$$

$$\rho_{t'}(x') = \int dx\, P(x' \mid x)\rho_t(x)$$

(2) Instants are ordered

... and there is an Arrow of Entropic Time

(3) Duration: the interval between instants

Define duration so that motion looks simple:

$$\Delta \bar{x}_n^a = \frac{\alpha'}{\alpha_n} \delta^{ab} \frac{\partial \phi}{\partial x_n^b} \qquad \langle \Delta w_n^a \Delta w_n^b \rangle = \frac{1}{\alpha_n} \delta^{ab}$$

$$\frac{\alpha'}{\alpha_n} = \frac{\hbar}{m_n} \Delta t$$

$$\frac{\Delta \bar{x}_n^a}{\Delta t} = \frac{\hbar}{m_n} \delta^{ab} \frac{\partial \phi}{\partial x_n^b}$$

The result for $\gamma = 2$ (Bohmian paths)

$$P(x' \mid x) = \frac{1}{\zeta} \exp - \sum_n \left[\frac{1}{2\eta \Delta t} m_{AB} \left(\frac{\Delta x^A}{\Delta t} - v^A \right) \left(\frac{\Delta x^B}{\Delta t} - v^B \right) \right]$$

mass tensor: $A = (n, a) \qquad m_{AB} = m_n \delta_{AB}$

expected particle velocity: $v^A = \dfrac{\langle \Delta x^A \rangle}{\Delta t}$

$$m_{AB} v^B = \partial_A \Phi \qquad \Phi = \hbar \phi$$

$$\left\langle \left(\frac{\Delta x^A}{\Delta t} - v^A \right) \left(\frac{\Delta x^B}{\Delta t} - v^B \right) \right\rangle = \eta m^{AB} \Delta t$$

Entropic dynamics:

Integral form: (1) $\rho_{t+\Delta t}(x') = \int dx\, P(x' \mid x)\rho_t(x)$

Differential form: (2) $\partial_t \rho = -\partial_A(\rho v^A)$

(3) $\partial_t \rho_x = \dfrac{\delta \tilde{H}}{\delta \Phi_x}$

$$\tilde{H}[\rho,\Phi] = \int dx \frac{1}{2} \rho m^{AB} \partial_A \Phi \partial_B \Phi + F[\rho]$$

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Symplectic form: $$\Omega = \int dx\, \mathrm{d}\rho^x \wedge \mathrm{d}\Phi_x$$

$$\Omega_{\alpha x, \beta x'} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \delta_{xx'}$$

Metric structure: $$\delta\tilde{\ell}^2 = \int dx \left(\frac{\hbar}{2\rho_x} \delta\rho_x^2 + \frac{2}{\hbar} \rho_x \delta\Phi_x^2 \right)$$

Complex structure: $$\Psi = \rho^{1/2} e^{i\Phi/\hbar}$$

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Time evolution? Hamiltonian?

Hamiltonian flow: $\mathcal{L}_{\bar{H}} \Omega = 0$

Killing flow: $\mathcal{L}_{\bar{H}} G = 0$

$$\tilde{H}[\Psi, \Psi^*] = \int dx\, dx'\, \Psi^*_x \hat{H}_{xx'} \Psi_{x'}$$

$$\tilde{H} = \int dx \left(\frac{\hbar^2}{2} m^{AB} \partial_A \Psi \partial_B \Psi^* + V \Psi \Psi^* \right)$$

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The equations of motion:

$$\partial_t \rho = \left\{\tilde{H}, \rho\right\} = \frac{\delta \tilde{H}}{\delta \Phi} \qquad \partial_t \Phi = \left\{\tilde{H}, \Phi\right\} = -\frac{\delta \tilde{H}}{\delta \rho}$$

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Summary:

- Entropic Dynamics $\Longrightarrow$ Quantum Mechanics
- Quantum Mechanics is a *Hamiltonian* dynamics (in the "classical" sense) with coordinates (ρ, Φ).

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Summary:

- Entropic Dynamics $\Longrightarrow$ Quantum Mechanics
- Quantum Mechanics is a *Hamiltonian* dynamics (in the "classical" sense) with coordinates (ρ, Φ).
- There is no need for *quantum* probabilities.
- Position is "ontic"; t is entropic time; m is mass.
- The Schrödinger equation is linear, time reversible, gauge invariant...
- No Hilbert spaces were postulated.

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