Title: Pivot Hamiltonians: a tale of symmetry, entanglement, and quantum criticality

Speakers: Nathanan Tantivasadakarn

Series: Quantum Matter

Date: November 29, 2021 - 3:30 PM

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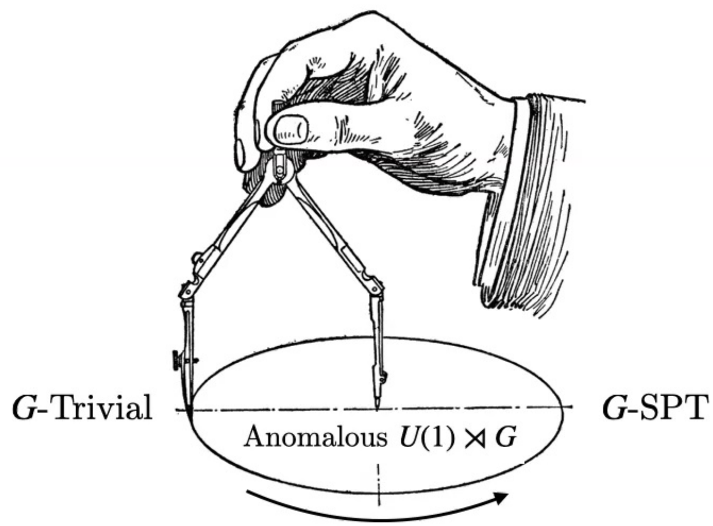
Abstract: I will introduce the notion of Pivot Hamiltonians, a special class of Hamiltonians that can be used to "generate" both entanglement and symmetry. On the entanglement side, pivot Hamiltonians can be used to generate unitary operators that prepare symmetry-protected topological (SPT) phases by "rotating" the trivial phase into the SPT phase. This process can be iterated: the SPT can itself be used as a pivot to generate more SPTs, giving a rich web of dualities. Furthermore, a full rotation can have a trivial action in the bulk, but pump lower dimensional SPTs to the boundary, allowing the practical application of scalably preparing cluster states as SPT phases for measurement-based quantum computation. On the symmetry side, pivot Hamiltonians can naturally generate U(1) symmetries at the transition between the aforementioned trivial and SPT phases. The sign-problem free nature of the construction gives a systematic approach to realize quantum critical points between SPT phases in higher dimensions that can be numerically studied. As an example, I will discuss a quantum Monte Carlo study of a 2D lattice model where we find evidence of a direct transition consistent with a deconfined quantum critical point with emergent SO(5) symmetry.

This talk is based on arXiv:2107.04019, 2110.07599, 2110.09512

Pivot Hamiltonians:
a tale of symmetry, entanglement, and quantum criticality



Nat Tantivasadakarn

Nov 30th, 2021

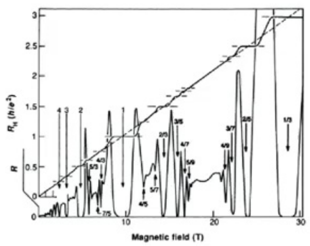
Ryan Thorngren

Ashvin Vishwanath

Ruben Verresen

arXiv:2107.04019 *NT, Vishwanath*
arXiv:2110.07599 *NT, Thorngren, Vishwanath, Verresen*
arXiv:2110.09512 *NT, Thorngren, Vishwanath, Verresen*

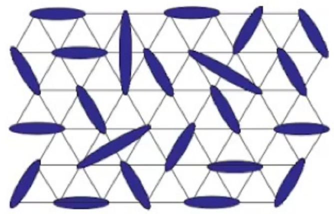
Topological Phases of matter



Fractional Quantum Hall

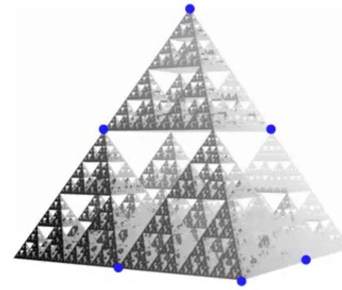
Laughlin, Störmer, Tsui

...



Quantum spin liquids

see Savary, Balents: Rep. Prog. Phys. 80, 016502 (2017)



Fractons (Haah's code)

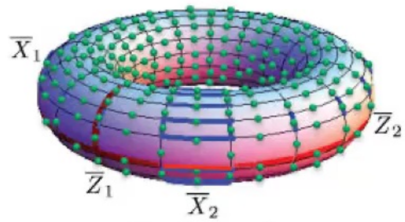
Chamon, Haah

...

Majorana chain

Kitaev

...

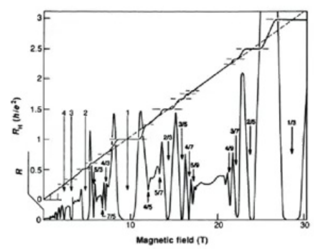


Toric code

Quantum error correcting codes that can perform topological quantum computation

see Nayak, Simon, Stern, Freedman, Das Sarma: RMP 80, 1083 (2008)

Topological Phases of matter



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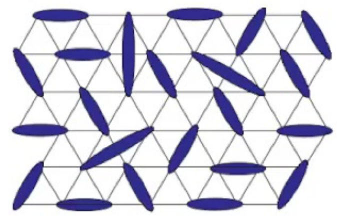

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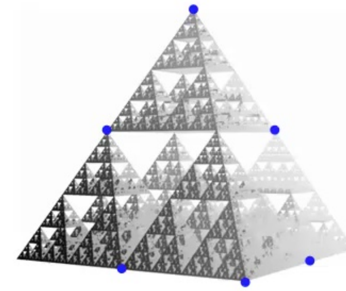

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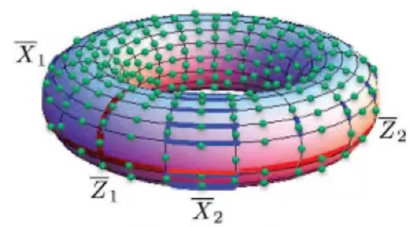


Toric code

Trivial phase

Quantum error correcting codes that can perform topological quantum computation

see Nayak, Simon, Stern, Freedman, Das Sarma: RMP 80, 1083 (2008)

Phases of matter with symmetry

Symmetry group G

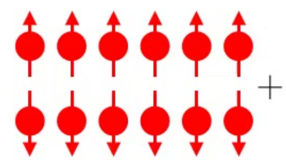
Spontaneous symmetry broken (SSB) phases

Trivial phase

SSB I

$H_1 \subset G$

preserved

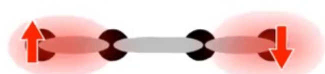


SSB II

$H_2 \subset G$

preserved

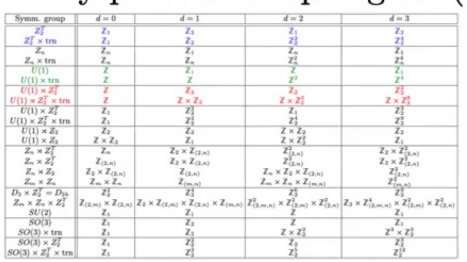
SPT phase I



SPT phase II

SPT phase III

Symmetry protected topological (SPT) phases



Symm. group	$d=0$	$d=1$	$d=2$	$d=3$
Z_2^T	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z}_1$	$\mathbb{Z}_2$
$Z_2^T \times trn$	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^4$
Z_n	$\mathbb{Z}_n$	$\mathbb{Z}_1$	$\mathbb{Z}_n$	$\mathbb{Z}_1$
$Z_n \times trn$	$\mathbb{Z}_n$	$\mathbb{Z}_n$	$\mathbb{Z}_n^2$	$\mathbb{Z}_n^4$
$U(1)$	$\mathbb{Z}$	$\mathbb{Z}_1$	$\mathbb{Z}$	$\mathbb{Z}_1$
$U(1) \times trn$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}^2$	$\mathbb{Z}^4$
$U(1) \rtimes Z_2^T$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2^2$
$U(1) \rtimes Z_2^T \times trn$	$\mathbb{Z}$	$\mathbb{Z} \times \mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}_2^3$	$\mathbb{Z} \times \mathbb{Z}_2^8$
$U(1) \times Z_2^T$	$\mathbb{Z}_1$	$\mathbb{Z}_2^2$	$\mathbb{Z}_1$	$\mathbb{Z}_2^3$
$U(1) \times Z_2^T \times trn$	$\mathbb{Z}_1$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^4$	$\mathbb{Z}_2^9$
$U(1) \rtimes Z_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}_2$	$\mathbb{Z}_2$
$U(1) \times Z_2$	$\mathbb{Z} \times \mathbb{Z}_2$	$\mathbb{Z}_1$	$\mathbb{Z} \times \mathbb{Z}_2^2$	$\mathbb{Z}_1$
$Z_n \rtimes Z_2^T$	$\mathbb{Z}_n$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}$	$\mathbb{Z}_{(2,n)}^2$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}^2$
$Z_n \times Z_2^T$	$\mathbb{Z}_{(2,n)}$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}$	$\mathbb{Z}_{(2,n)}^2$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}^2$
$Z_n \rtimes Z_2$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}$	$\mathbb{Z}_{(2,n)}$	$\mathbb{Z}_n \times \mathbb{Z}_2 \times \mathbb{Z}_{(2,n)}$	$\mathbb{Z}_{(2,n)}^2$
$Z_m \times Z_n$	$\mathbb{Z}_m \times \mathbb{Z}_n$	$\mathbb{Z}_{(m,n)}$	$\mathbb{Z}_m \times \mathbb{Z}_n \times \mathbb{Z}_{(m,n)}$	$\mathbb{Z}_{(m,n)}^2$
$D_2 \times Z_2^T = D_{2h}$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^4$	$\mathbb{Z}_2^6$	$\mathbb{Z}_2^3$
$Z_m \times Z_n \times Z_2^T$	$\mathbb{Z}_{(2,m)} \times \mathbb{Z}_{(2,n)}$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,m)} \times \mathbb{Z}_{(2,n)} \times \mathbb{Z}_{(m,n)}$	$\mathbb{Z}_{(2,m,n)}^2 \times \mathbb{Z}_{(2,m)}^2 \times \mathbb{Z}_{(2,n)}^2$	$\mathbb{Z}_2 \times \mathbb{Z}_{(2,m,n)}^4 \times \mathbb{Z}_{(2,m)}^2 \times \mathbb{Z}_{(2,n)}^2$
$SU(2)$	$\mathbb{Z}_1$	$\mathbb{Z}_1$	$\mathbb{Z}$	$\mathbb{Z}_1$
$SO(3)$	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z}$	$\mathbb{Z}_1$
$SO(3) \times trn$	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}_2^2$	$\mathbb{Z}^3 \times \mathbb{Z}_2^3$
$SO(3) \times Z_2^T$	$\mathbb{Z}_1$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2$	$\mathbb{Z}_2^3$
$SO(3) \times Z_2^T \times trn$	$\mathbb{Z}_1$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^5$	$\mathbb{Z}_2^{12}$

We have a pretty good understanding of what phases can exist given a symmetry G

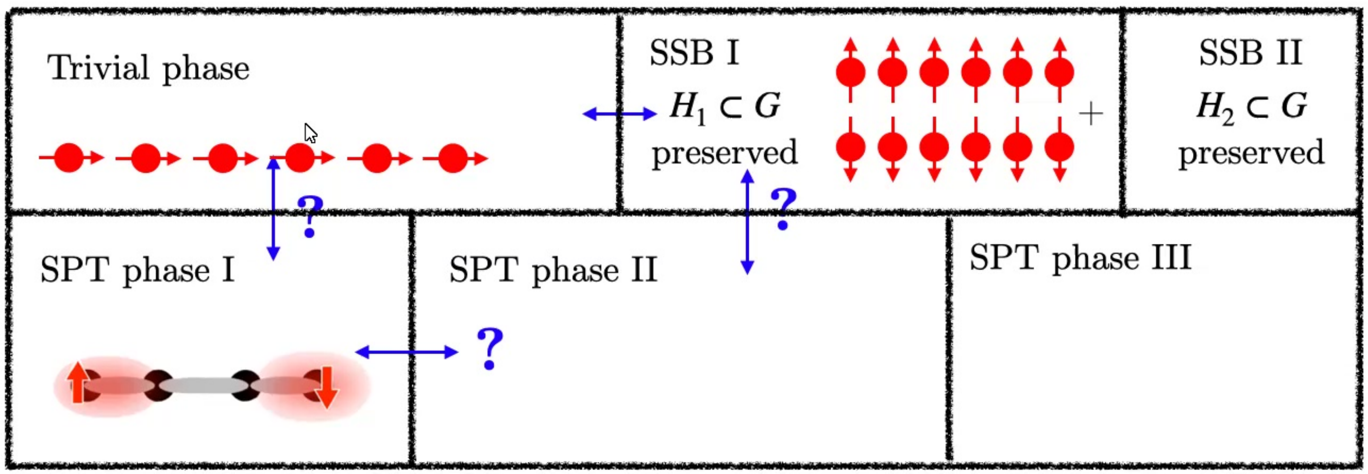
Q1. How are SSB and SPT phases related?

Pollmann et al 2010; Fidkowski, Kitaev 2011; Chen et al 2011, Kapustin 2012

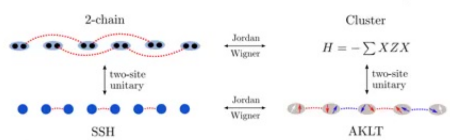
Phases of matter with symmetry

Symmetry group G

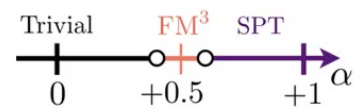
Spontaneous symmetry broken (SSB) phases



Symmetry protected topological (SPT) phases



Verresen, Moessner, Pollmann 2016



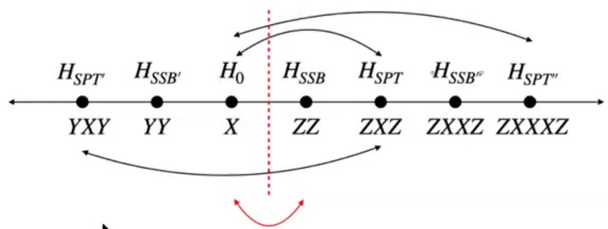
(Dupont, Gazit, Scaffidi 2008.11206)

Q2. How to systematically study strongly interacting SPT phase transitions?

Pivot Hamiltonians

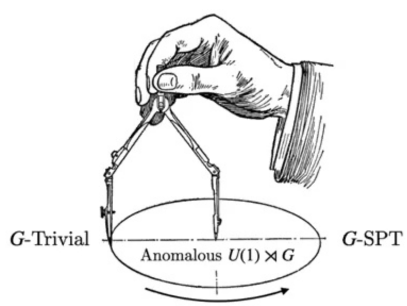
Q1. How are SSB and SPT phases related?

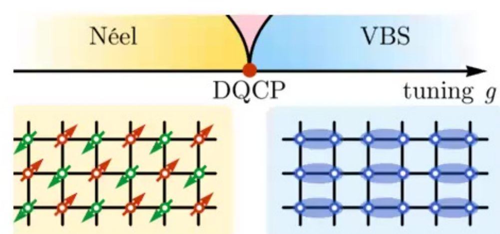
A1. Evolving with SSB/SPT phases can generate entanglement for other SSB/SPT phases



Q2. How to systematically study strongly interacting SPT phase transitions?

A2. Continuous symmetries generated from pivot Hamiltonians can help stabilize direct continuous transitions



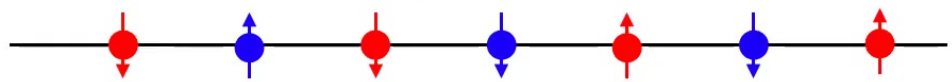


Connections to other unconventional transitions such as deconfined criticality!

Example: 1D Cluster state

$$G = \mathbb{Z}_2 \times \mathbb{Z}_2$$

$$P_1 = \prod_R X_n \quad P_2 = \prod_B X_n$$



$$H_0 = -\sum_n X_n$$

Trivial (paramagnet)

$$H_{SPT} = -\sum_n Z_{n-1} X_n Z_{n+1}$$

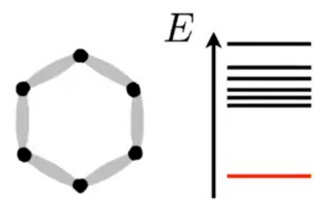
SPT (1D cluster state)

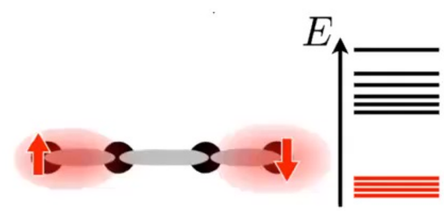
$$H_{Ising} = \frac{1}{4} \sum_n (-1)^n Z_n Z_{n+1}$$

$$H_{SPT} = U H_0 U^\dagger$$

$$U = e^{-\pi i H_{Ising}} = \prod_n CZ_{n,n+1}$$

Raussendorf, Briegel





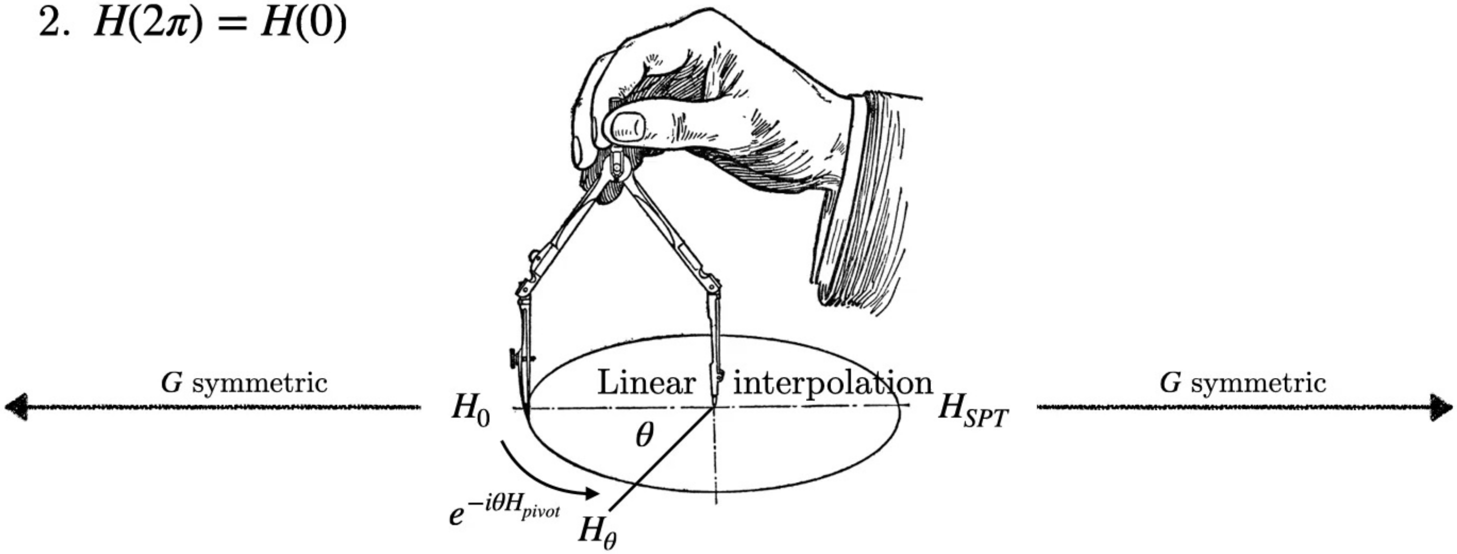
General Idea of Pivoting

Start with two Hamiltonians: H_0 (typically a G-paramagnet) and H_{pivot}

$$H(\theta) = e^{-i\theta H_{pivot}} H_0 e^{i\theta H_{pivot}}$$

H_{pivot} is chosen such that

1. $H_{SPT} = H(\pi)$ is a non-trivial SPT
2. $H(2\pi) = H(0)$



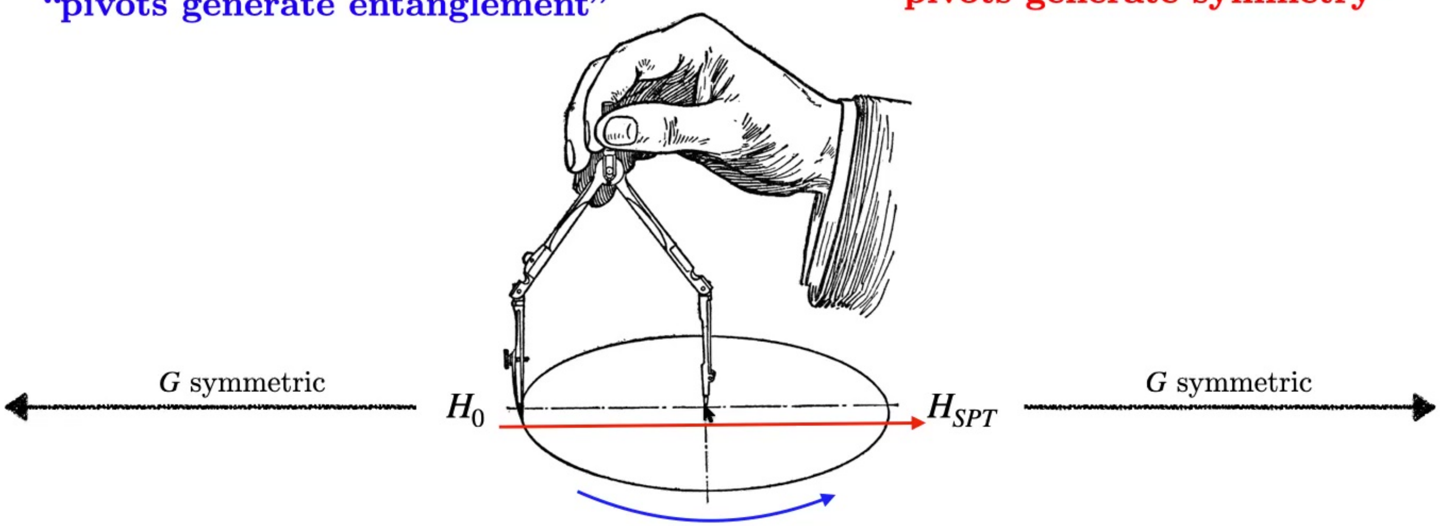
Outline

Part I: creating SPT models

"pivots generate entanglement"

Part II: SPT criticality

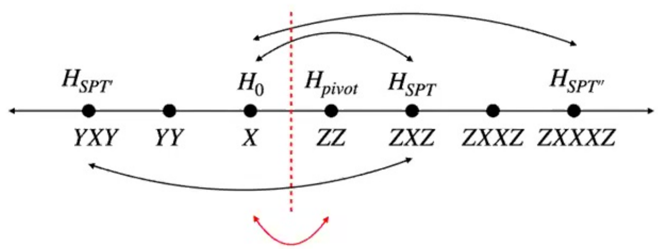
"pivots generate symmetry"



Part I: creating SPT models

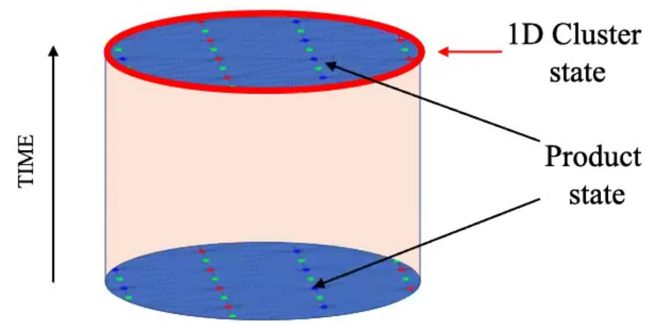
π rotation: SPT entanglers

A tool for relating SPT phases via a network of lattice dualities



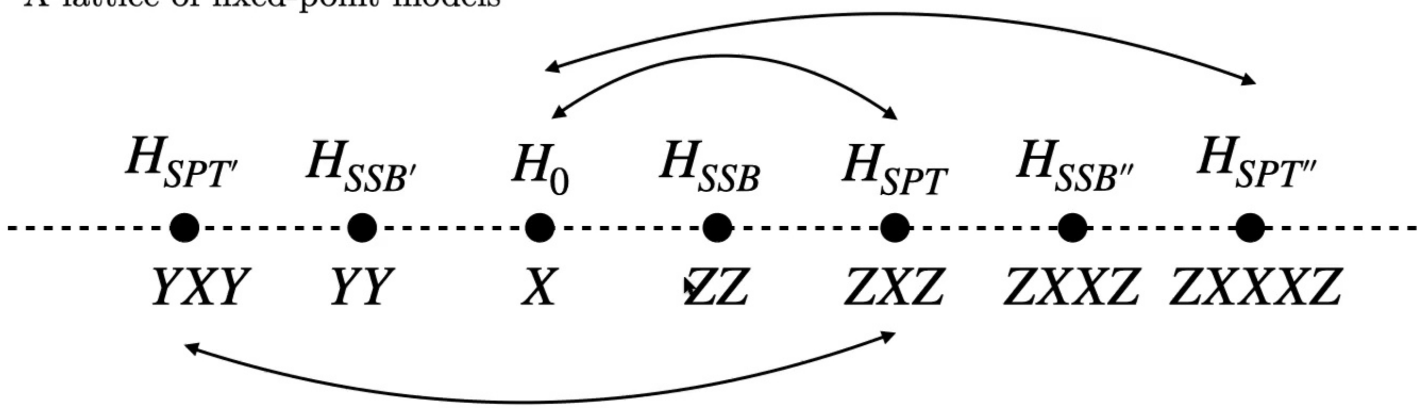
2π rotation: Quantum pumps

Preparation of cluster states for Measurement-based quantum computation



The pivoted becomes the pivot

A lattice of fixed-point models



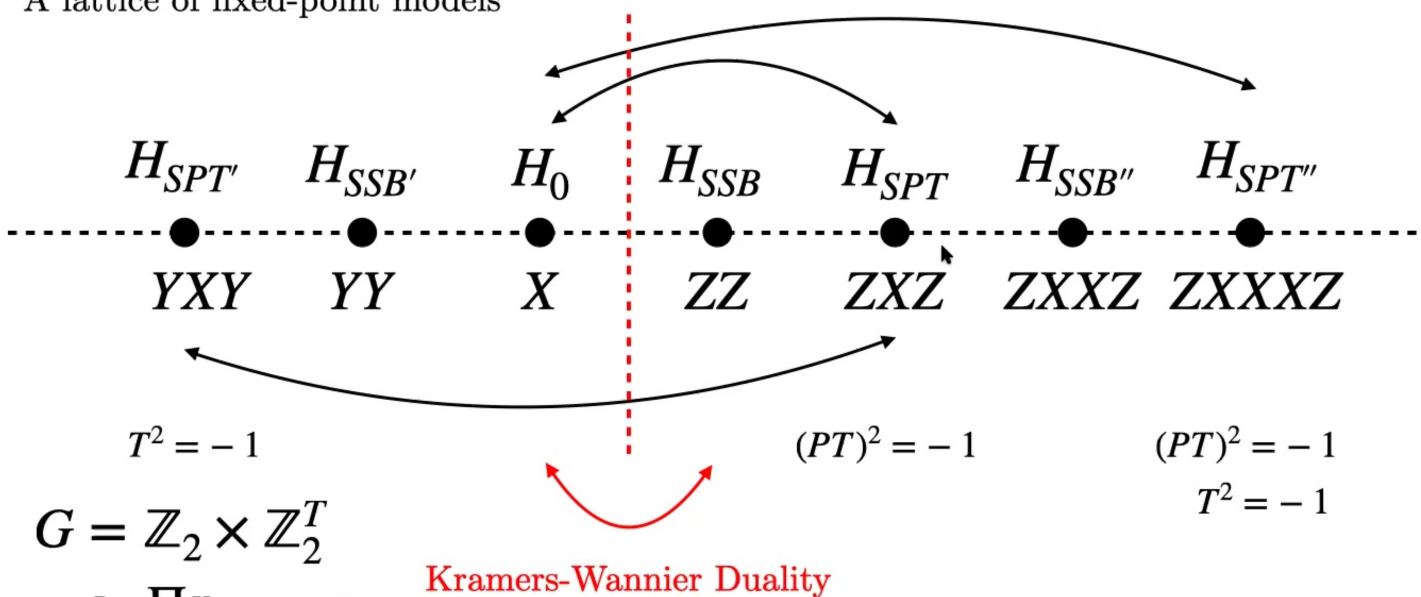
$T^2 = -1$ $\qquad$ $(PT)^2 = -1$ $\qquad$ $(PT)^2 = -1$, $T^2 = -1$

$$G = \mathbb{Z}_2 \times \mathbb{Z}_2^T$$

$$P = \prod_n X_n \qquad T = K$$

The pivoted becomes the pivot

A lattice of fixed-point models



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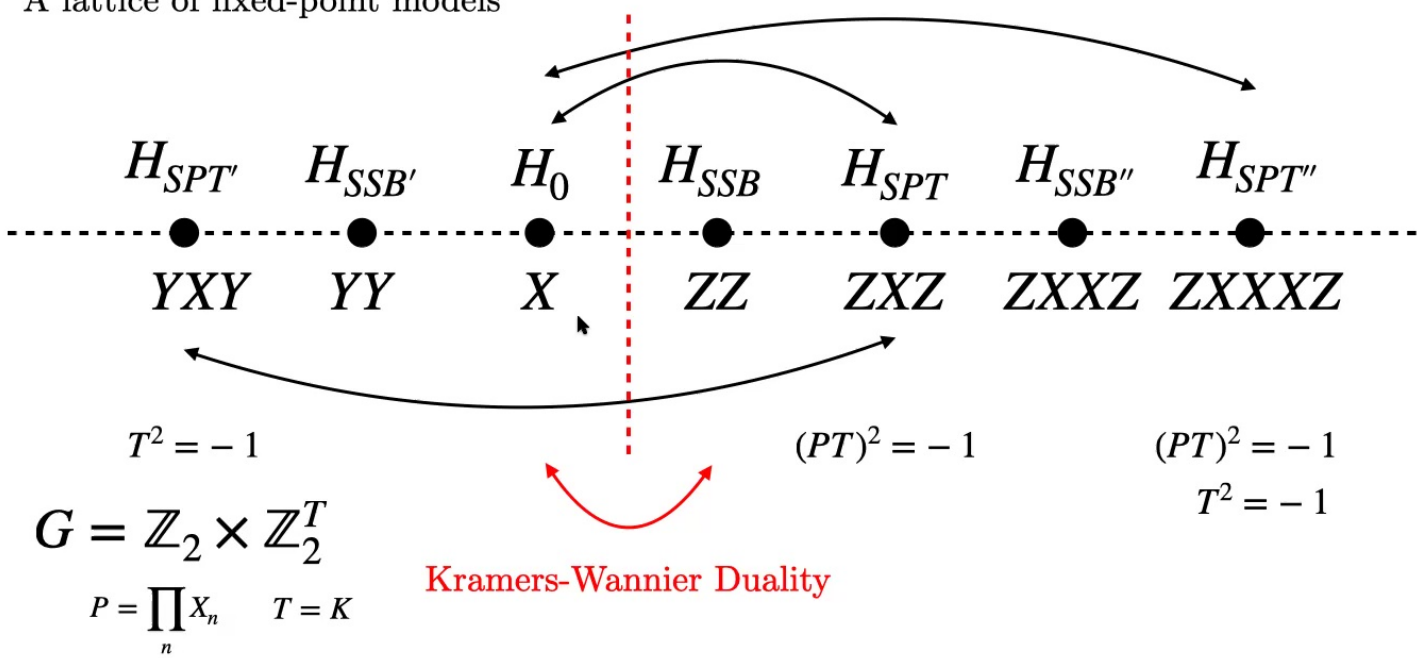
A lattice of fixed-point models



A network of dualities. Note that H_0, KW and pivoting can generate the whole sequence of models!

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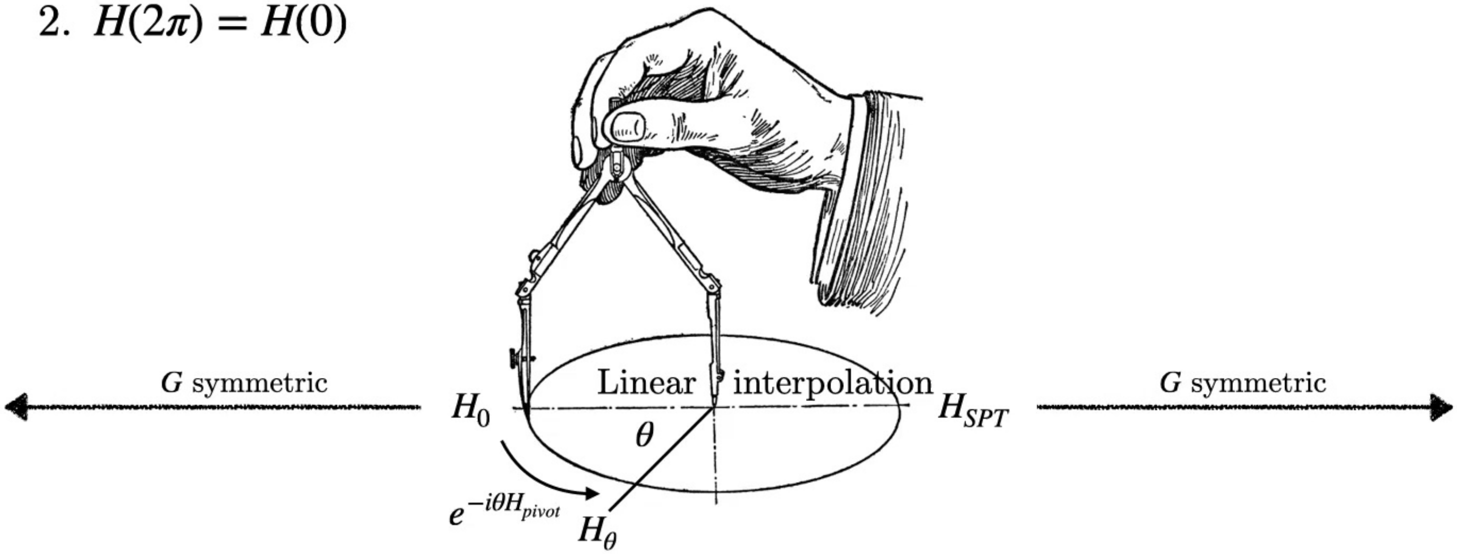
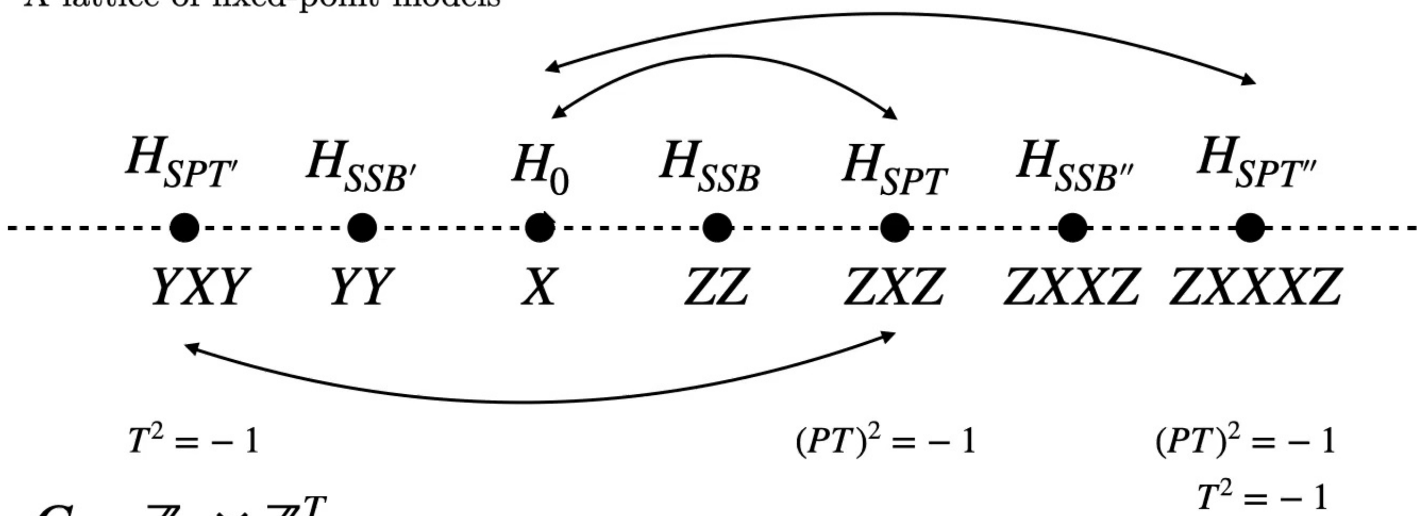


The pivoted becomes the pivot

A lattice of fixed-point models



$$G = \mathbb{Z}_2 \times \mathbb{Z}_2^T$$

$P = \prod_n X_n \qquad T = K$

Bootstrapping up in dimensions

Given a pivot that creates an 1D G-SPT of order two, we can construct a pivot that creates a 2D $\mathbb{Z}_2 \times G$ SPT

$$H^{2D}_{pivot} \sim \sum_{\Delta} \frac{Z}{2} H^{1D}_{pivot}$$

$$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$$

$$\prod_R X_v \quad \underbrace{\prod_G X_v \quad \prod_B X_v}_{\text{1D}}$$

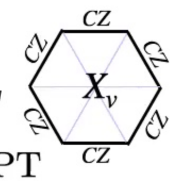
$$H_0 = -\sum_v X_v$$

Trivial

$$H_{pivot} = -\frac{1}{8}\sum_{\Delta} (-1)^{\Delta} Z_r Z_g Z_b$$

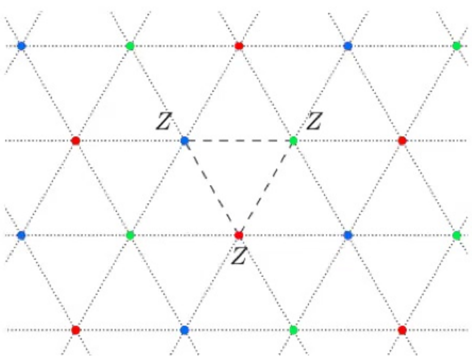
Pivot

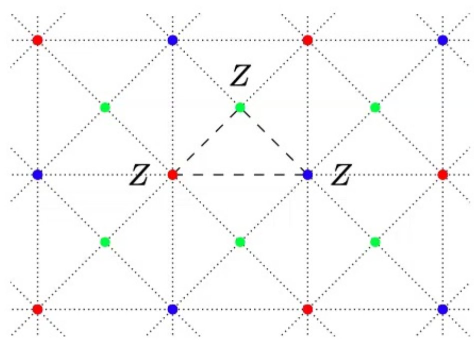
$$H_{SPT} = -\sum_v$$



SPT

Yoshida 1503.07208





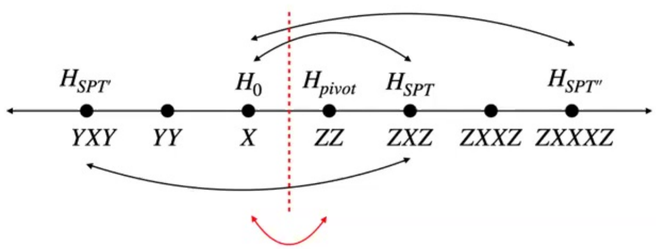
Cluster state decorated on $\mathbb{Z}_2$ domain walls

Chen, Lu, Vishwanath 1303.4301

Part I: creating SPT models

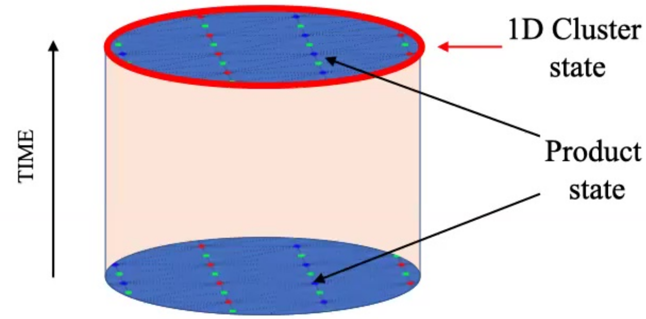
π rotation: SPT entanglers

A tool for constructing SPT phases via a network of lattice dualities



2π rotation: Quantum pumps

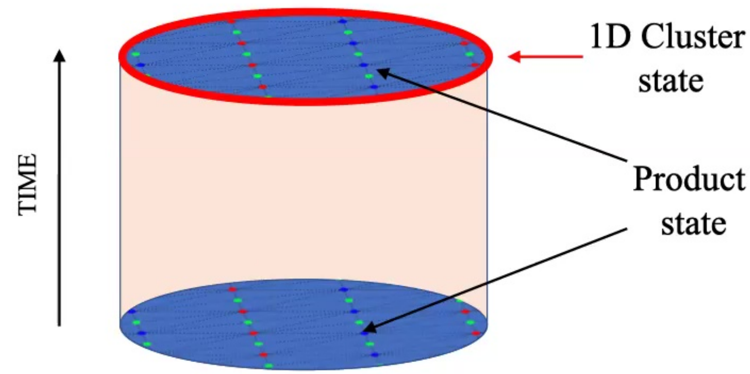
Preparation of cluster states for Measurement-based quantum computation



Pumps in higher dimensions

Pumps in higher dimensions pump SPT phases to the boundary.
This has been studied in the context of Floquet SPT phases

Else, Nayak; Potter, Morimoto, Vishwanath, von Keyserlingk, Sondhi; Roy, Harper...

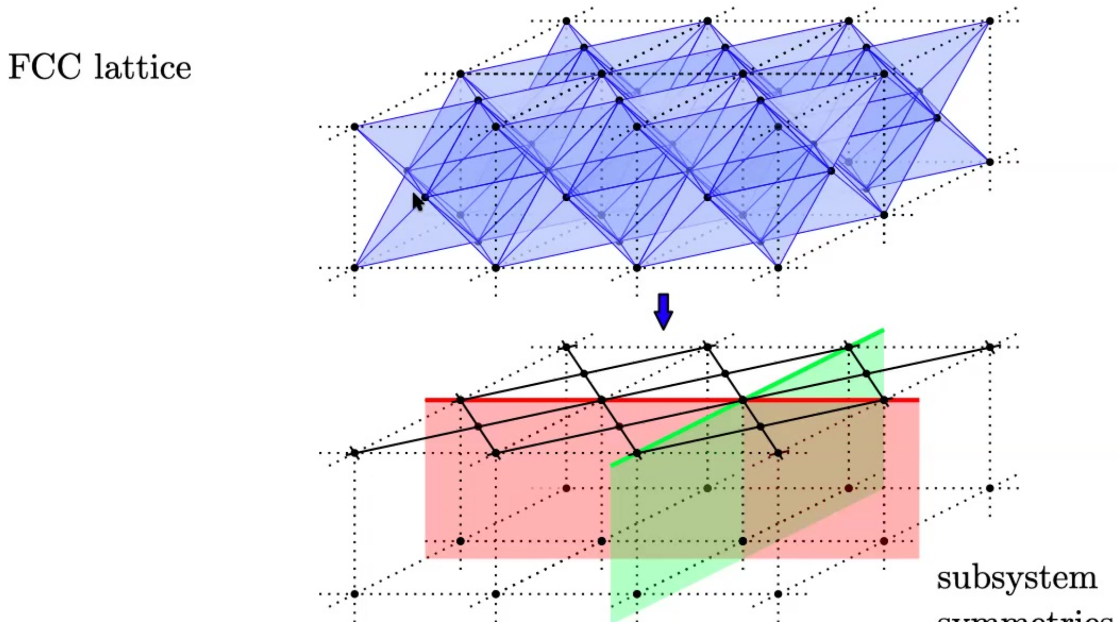


We can use this to prepare cluster states for measurement based quantum computation (MBQC)

Unlike the π evolution, this is robust against prepreration errors that respect the symmetry

2D Cluster state pump

$$H_{pivot} = -\frac{1}{8}\sum_{\text{tet}} ZZZZ$$



NT, Vishwanath 2107.04019

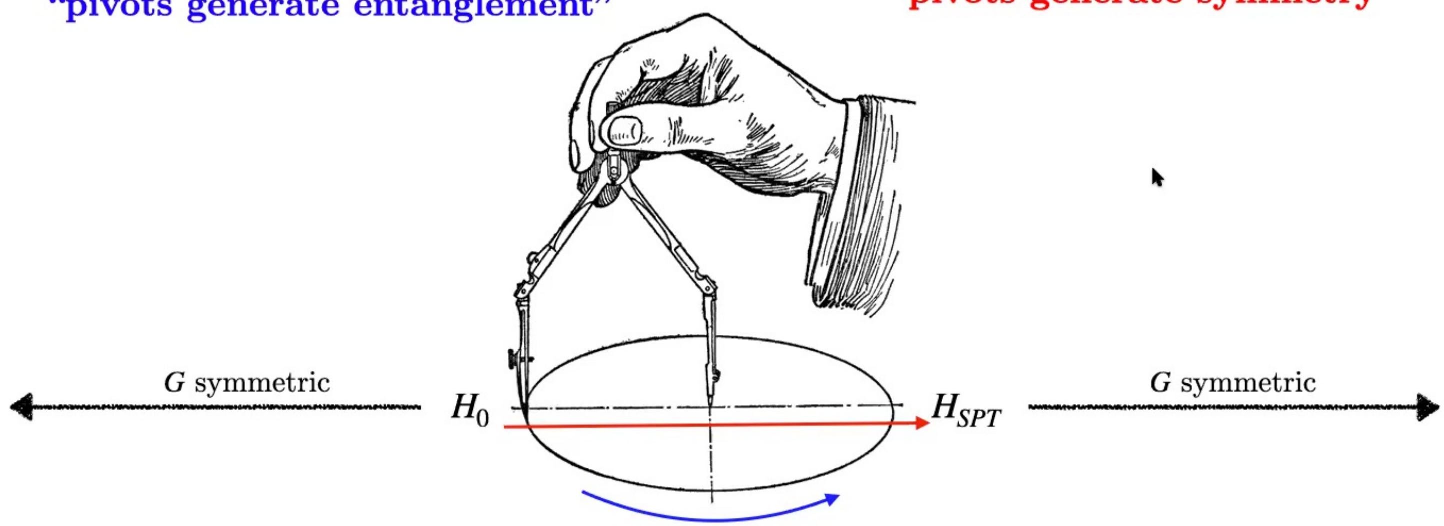
Questions?

Part I: creating SPT phases

"pivots generate entanglement"

Part II: SPT criticality

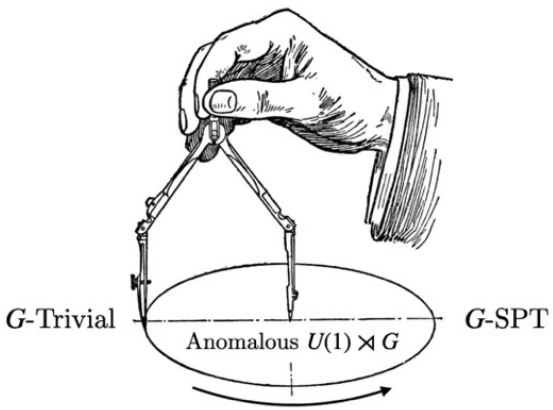
"pivots generate symmetry"



Part II: SPT criticality

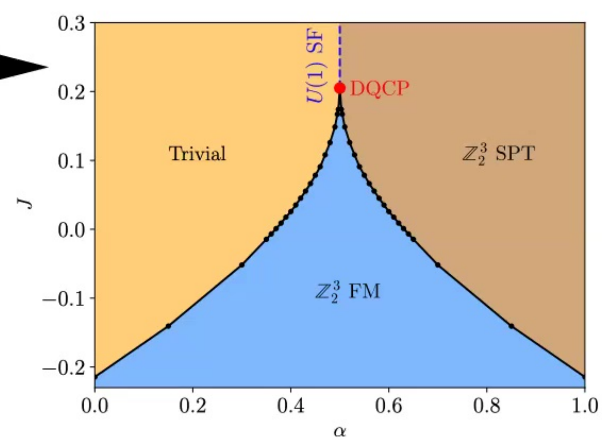
Symmetries

Anomalous continuous symmetries on the lattice

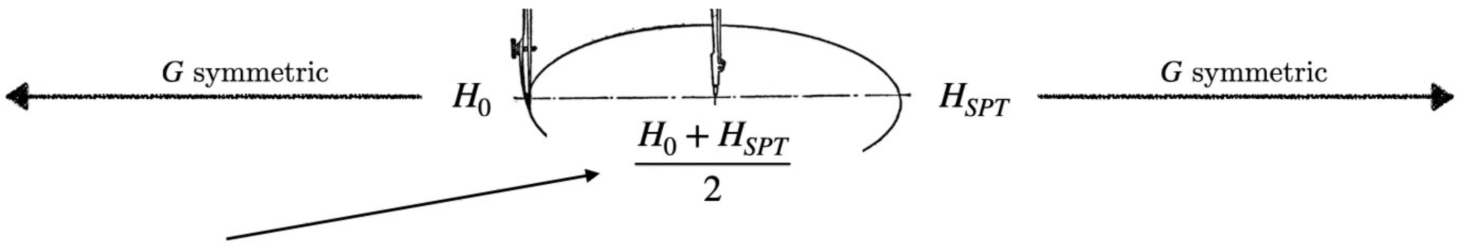


Criticality

Connecting SPT transitions and deconfined quantum critical points



$U(1)$ pivot symmetry



Midway point has additional $\mathbb{Z}_2$ symmetry $e^{-i\pi H_{pivot}}$ *Bultinck 1905.05790*

But in some cases, it is enlarged to a full $U(1)$ symmetry!
The pivot generates the symmetry at the midpoint

$$[H_{pivot}, H_0 + H_{SPT}] = 0$$

$U(1)$ is not onsite. Furthermore, there is no basis for which both $U(1)$ and G are onsite (if so, no entanglement would be created)

Mutual anomaly

$U(1)$ pivot symmetry in 1D

$H_0 = -\sum_n X_n$	$H_{pivot} = \frac{1}{4}\sum_n (-1)^n Z_n Z_{n+1}$	$H_{SPT} = -\sum_n Z_{n-1} X_n Z_{n+1}$
Trivial (paramagnet)	Pivot (Ising)	SPT (1D cluster state)

$$[H_{pivot}, H_0 + H_{SPT}] = 0$$

$H_0 + H_{SPT}$ has a conservation of domain wall number *Levin, Gu 1202.3120*

In fact, $H_0 + H_{SPT}$ is described by a compact boson CFT ($c = 1$)

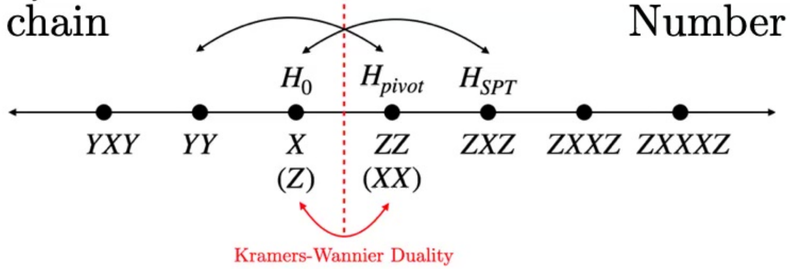
How to see? Use Kramers-Wannier duality (denote dual Ham. with tilde)

$$\tilde{H}_0 + \tilde{H}_{SPT} = \sum_i X_n X_{n+1} + Y_n Y_{n+1}$$

XY chain

$$\tilde{H}_{pivot} = \sum_n Z_n$$

Number conservation



2D Pivot on Δ lattice

$$G = \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$$

$$\prod_{R} X_v \quad \prod_{G} X_v \quad \prod_{B} X_v$$

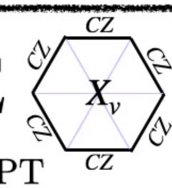
$$H_0 = -\sum_v X_v$$

Trivial

$$H_{pivot} = -\frac{1}{8}\sum_{\Delta} (-1)^{\Delta} Z_r Z_g Z_b$$

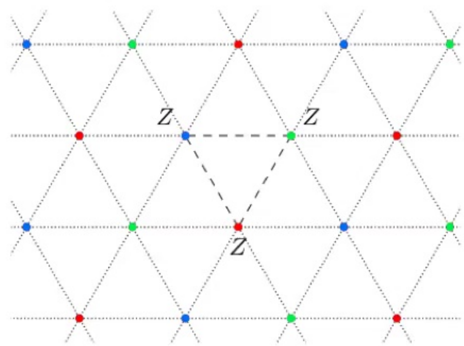
Pivot

$$H_{SPT} = -\sum_v$$



SPT

Yoshida 1503.07208



Cluster state decorated on $\mathbb{Z}_2$ domain walls

Chen, Lu, Vishwanath 1303.4301

Remarkably, we also find that $[H_{pivot}, H_0 + H_{SPT}] = 0$! How general is this?

Sufficient criterion for $U(1)$ pivot symmetry

Let $H_0 = -\sum_v X_v$ and consider pivots of the form $H_{pivot} = \frac{1}{2^N}\sum\left(\pm Z_{i_1}\cdots Z_{i_N}\right)$

Let k be the number of times that each vertex v is included in $Z_{i_1}\cdots Z_{i_N}$

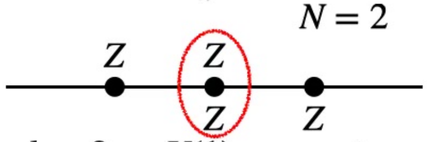
Theorem: if $k < 2^N$ then $[H_{pivot}, H_0 + H_{SPT}] = 0$

Examples:

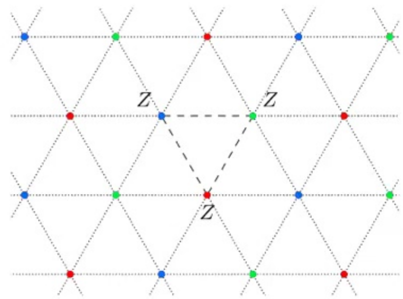
1D chain

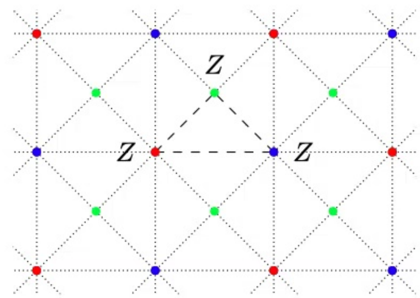
$$H_{pivot} = \frac{1}{4}\sum_n (-1)^n Z_n Z_{n+1}$$

$N = 2$



$k = 2 \rightarrow U(1)$ symmetry





$$H_{pivot} = -\frac{1}{8}\sum_\Delta (-1)^\Delta Z_r Z_g Z_b$$

$N = 3$

$k = 6 \rightarrow U(1)$

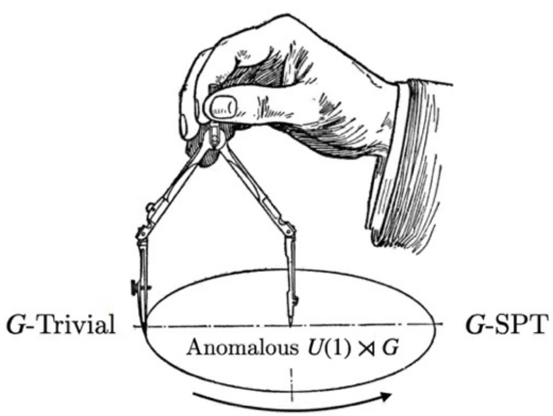
$k = 8 \rightarrow$ no $U(1)$

NT, Thorngren, Vishwanath, Verresen

Part II: SPT criticality

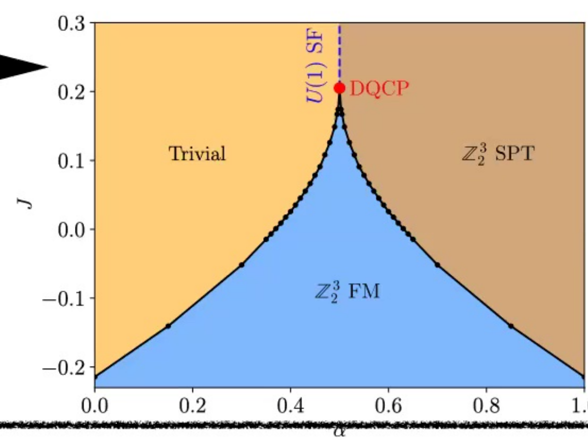
Symmetries

Anomalous continuous symmetries on the lattice



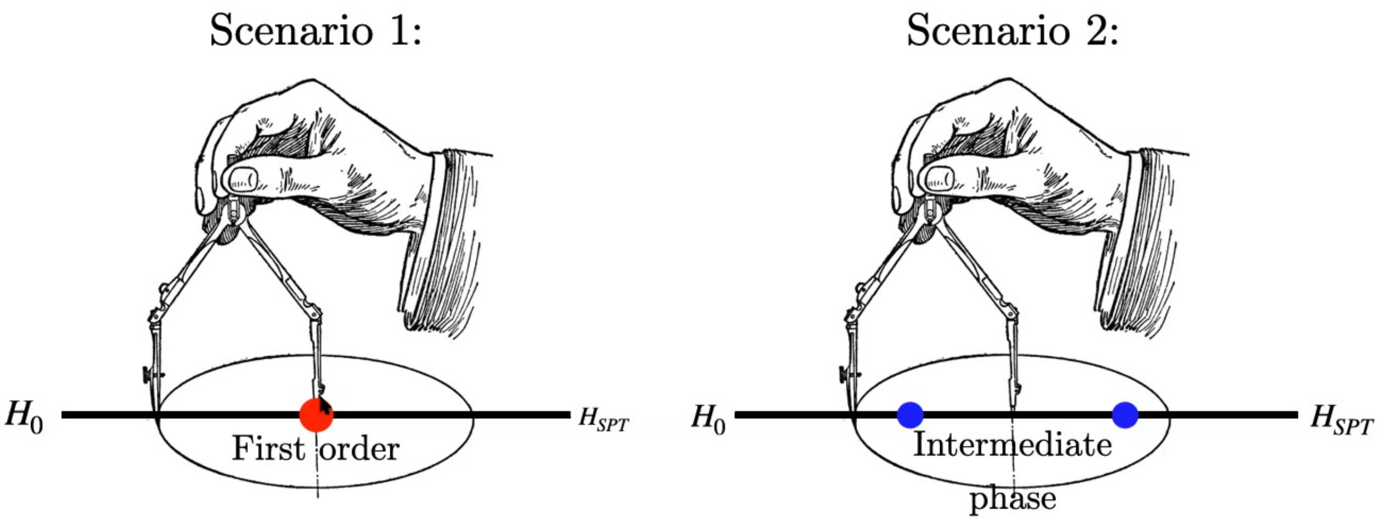
Criticality

Connecting SPT transitions and deconfined quantum critical points



SPT transitions in higher dimensions

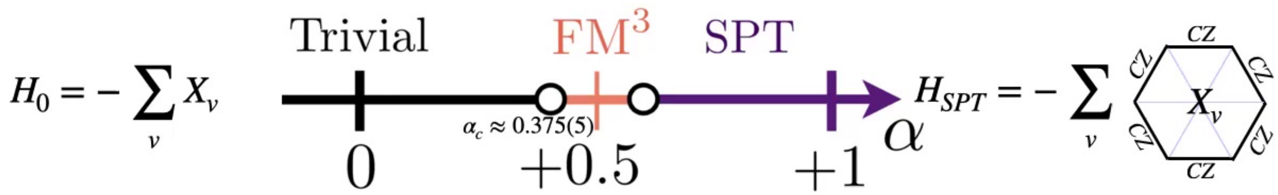
Although phase transitions in 1D are well studied, continuous direct transitions in 2D need fine tuning



The $U(1) \rtimes G$ anomaly constraints what can happen at the midpoint, helping us to fine-tune to multicriticality!

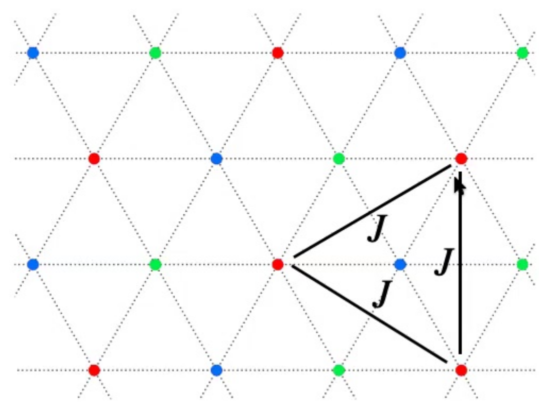
2D Phase Diagram

The direct interpolation was recently studied numerically *(Dupont, Gazit, Scaffidi 2008.11206)*



In search of a continuous direct transition, we further add antiferromagnetic Ising coupling $J > 0$ for each sub-lattice to suppress FM order (which commutes with the pivot)

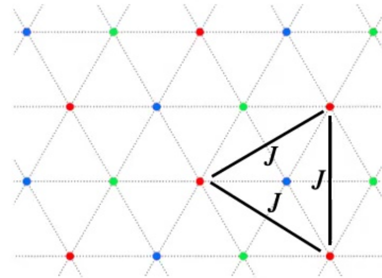
$$H = (1-\alpha)H_0 + \alpha H_{SPT} + J \sum_{\langle cc' \rangle \in R,G,B} Z_c Z_{c'}$$



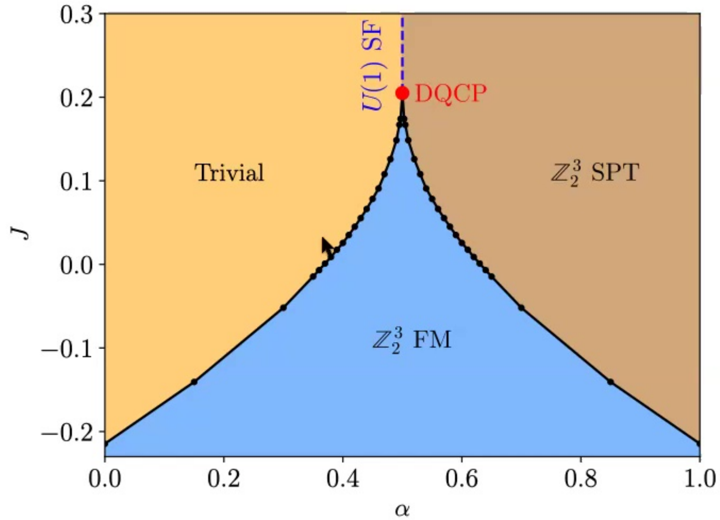
Ising coupling allows us to use efficient Quantum Monte Carlo algorithms, such as non-local cluster updates! *Sandvik PRE **68** 056701. Melko et. al.*

Numerical Study

$$H = (1-\alpha)H_0 + \alpha H_{SPT} + J \sum_{\langle cc' \rangle \in R,G,B} Z_c Z_{c'}$$



Sign problem free for $\alpha \leq 0.5$, but the pivoting by π rotation maps $\alpha \to 1-\alpha$

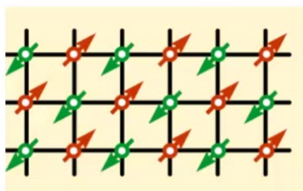


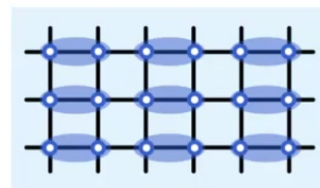
We argue in favor of a multicritical point described by the DQCP

NT, Thorngren, Vishwanath, Verresen 2110.09512

Deconfined criticality

Néel
$SO(3)$ SSB





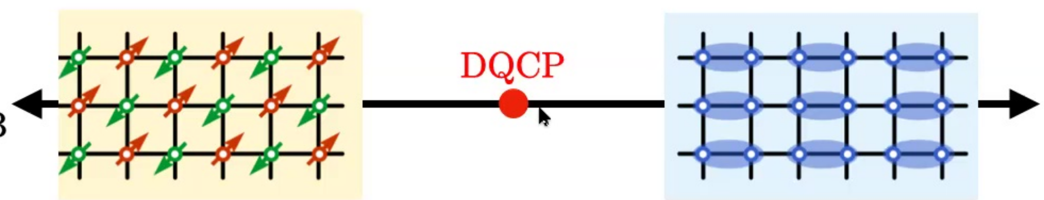
VBS
C_4 rotation SSB
($U(1)$ at DQCP)

Mutual anomaly between $SO(3)$ and C_4

Believed to be enhanced to $SO(5)$ at the DQCP

Senthil, Vishwanath, Balents, Sachdev, Fisher; Sandvik et. al.; Xu et al; Wang, He et. al.

Deconfined criticality



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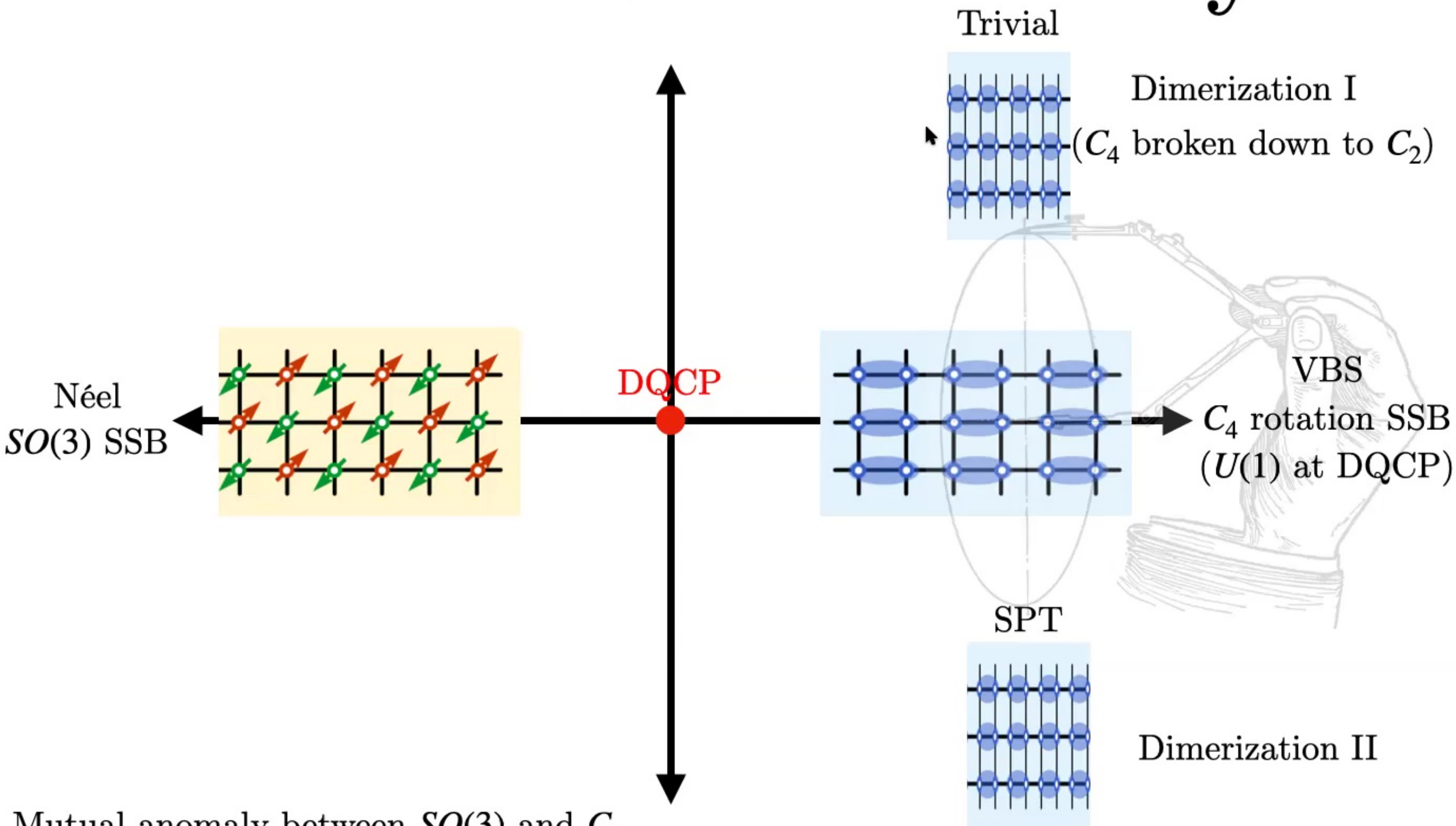
Deconfined criticality



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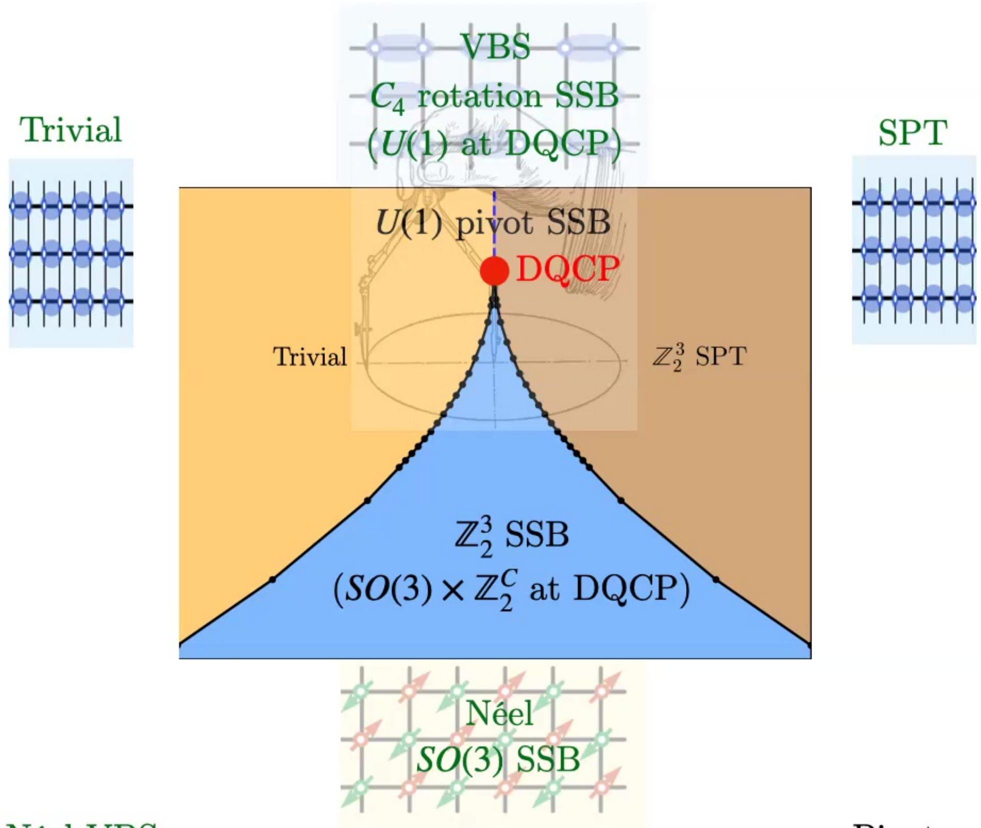
Believed to be enhanced to $SO(5)$ at the DQCP

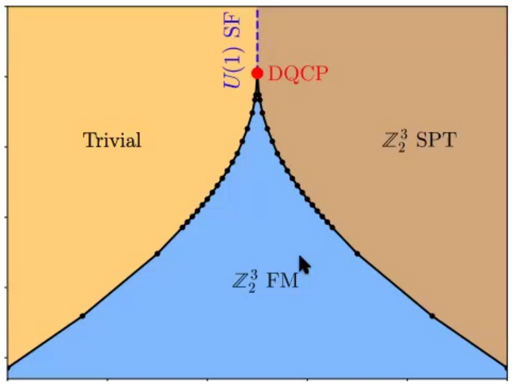
Senthil, Vishwanath, Balents, Sachdev, Fisher; Sandvik et. al.; Xu et al; Wang, He et. al.

Another incarnation of DQCP

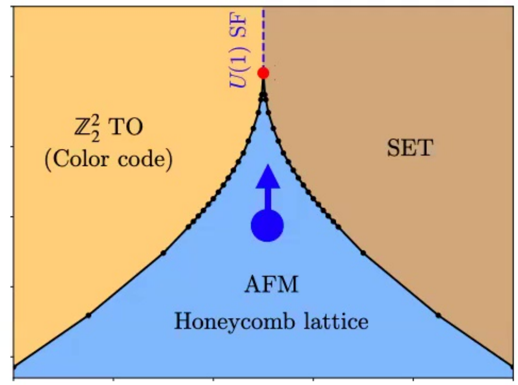


Néel-VBS
Mutual anomaly between $SO(3)$ and C_4 $\longleftrightarrow$ Pivot
Mutual anomaly between $\mathbb{Z}_2^3$ and $U(1)$

General construction of topological criticality



Gauge $\mathbb{Z}_2^2$ subgroup

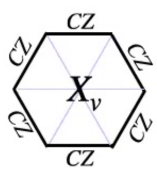


A honeycomb XY model

$$H_0 = -\sum_v X_v$$

$$\frac{1}{2}(\tilde{H}_0 + \tilde{H}_{\rm SPT}) = -\sum_{\hexagon} \sum_{v_i \in \hexagon} \sigma^+_{v_1}\sigma^+_{v_2}\sigma^+_{v_3}\sigma^-_{v_4}\sigma^-_{v_5}\sigma^-_{v_6}.$$

$$H_{SPT} = -\sum_v$$



$$\tilde{H}_{pivot} = \sum_i Z_i$$

$$\prod_{v \in \hexagon} Z_v = -1$$

Can drive to the multicritical point using NN Ising interactions

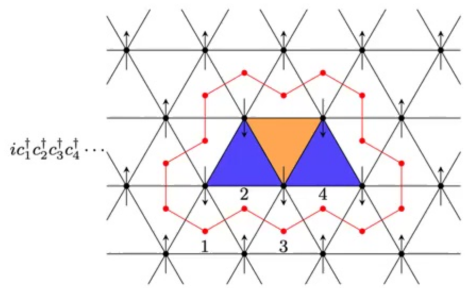
NT, Thorngren, Vishwanath, Verresen

Outlook

- Web of dualities in higher dimension: what's the total lattice of "space of models from pivoting"?
- Experimental platforms to realize the cluster state pumps.
- Pivot Hamiltonians beyond order two SPT phases
- Studying XY models to reach larger system size and extract critical exponents. Efficient QMC update schemes.
- Exploring higher dimensional versions of DQCP (both global and subsystem symmetry versions)

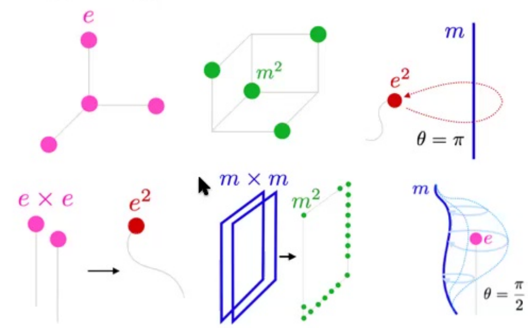
Other topics I am interested in

Interacting Fermionic SPT phases



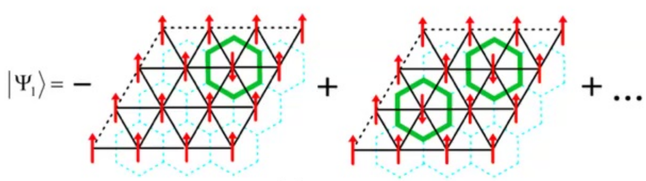
Meng Cheng, NT, Chenjie Wang 1705.08911
NT, Ashvin Vishwanath 1806.09709
Yu-An Chen, Tyler Ellison, NT 2008.05652

Topological & Fracton orders



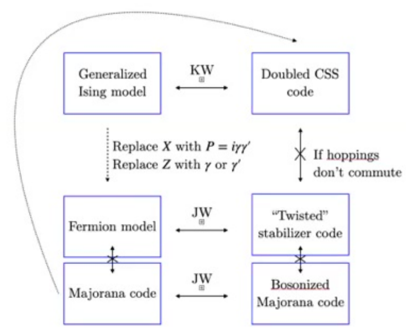
NT, Sagar Vijay 1912.02826
NT, Wenjie Ji, Sagar Vijay 2106.03842, 2107.04019s

Lattice Hamiltonian vs TQFT



Fidkowski, Haah, Hastings, NT 1906.04188

Bosonization & Dualities in lattice models



NT 2002.11345

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Examples:

1D chain

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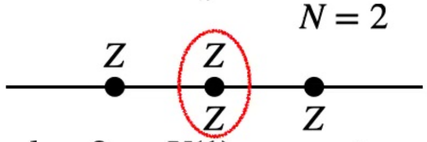
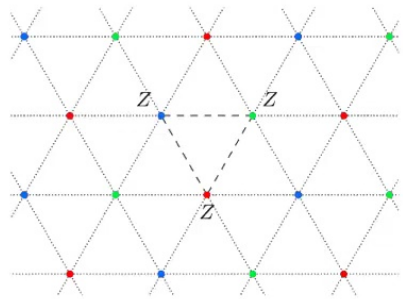
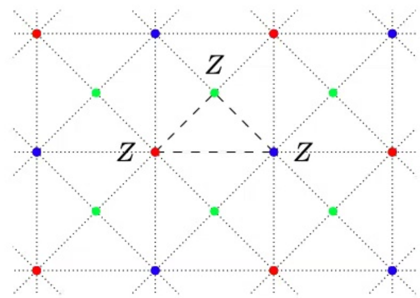
$N = 2$



$k = 2 \rightarrow U(1)$ symmetry

NT, Thorngren, Vishwanath, Verresen





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$N = 3$

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$k = 8 \rightarrow$ no $U(1)$