Title: Melonic large N limit of 5-index irreducible random tensors

Speakers: Sabine Harribey

Series: Quantum Gravity

Date: November 25, 2021 - 2:30 PM

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Abstract: The main feature of tensor models is their melonic large N limit, leading to applications ranging from random geometry and quantum gravity to many-body quantum mechanics and conformal field theories. However, this melonic limit is lacking for tensor models with ordinary representations of O(N) or Sp(N). We demonstrate that random tensors with sextic interaction transforming under rank-5 irreducible representations of O(N) have a melonic large N limit. This extends the recent proof obtained for rank-3 models with quartic interaction. After giving an introduction to random tensors, I will present the main ideas of our proof relying on recursive bounds derived from a detailed combinatorial analysis of the Feynman graphs.

Zoom Link: https://pitp.zoom.us/j/94691275506?pwd=RGFaN0NZR0FScFdOTXFzeFVXaXUvUT09

Melonic large N limit of 5-index irreducible random tensors

Sabine Harribey

Joint work with Sylvain Carrozza - arXiv:2104.03665

Perimeter Institute - November 25$^{\text{th}}$ 2021

CPHT Centre de Physique Théorique

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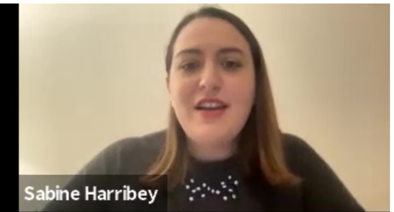
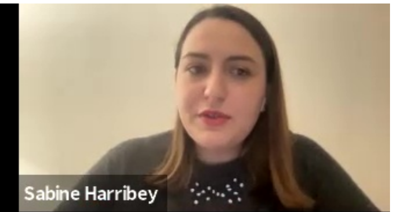


History of random tensors

- First introduced in zero dimension: random geometry and quantum gravity [Ambjorn Durhuus Jonsson '90, Boulatov '92, Ooguri '92, ...]
- Generalisation of matrix models in dimension higher than two
- Provide one possible definition of discretized Euclidean quantum gravity
- Gluing of tetrahedrons for rank 3.



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- Gluing of tetrahedrons for rank 3.

- No large N limit
- Very degenerate ways of gluing tetrahedra are allowed

$\Rightarrow$ Revived when a large N expansion was found [Gurau, Bonzom, Rivasseau, '10, ...]

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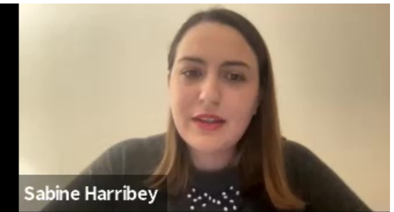
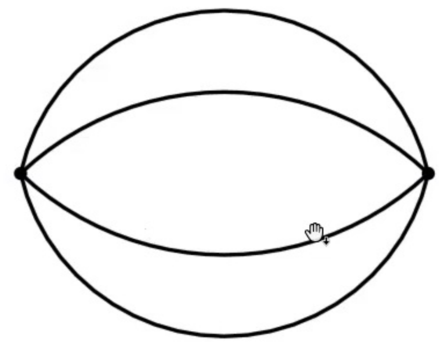
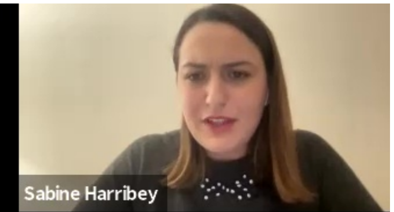


Melon diagrams





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Different types of melonic limit

- **Colored** tensor models: 4 tensor fields, $O(N)^6$ symmetry

$$\varphi^{i}_{abc}$$

- **Uncolored** tensor models: 1 tensor field, $O(N)^3$ symmetry

$$\varphi_{abc}$$

- SYK model

$$J_{i_1 i_2 i_3 i_4}\psi_{i_1}\psi_{i_2}\psi_{i_3}\psi_{i_4}$$

- Multi matrix models with a large number of matrices [Ferrari, Schaposnik Massolo, Valette ...], models in higher rank [Bonzom,Lionni, ...]

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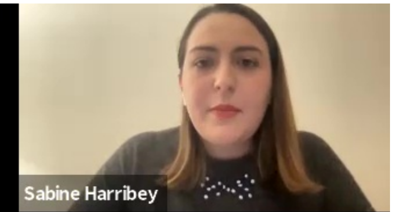
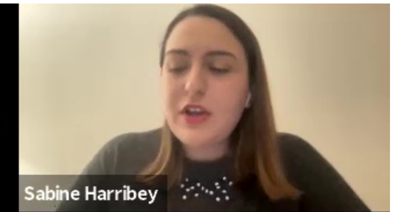
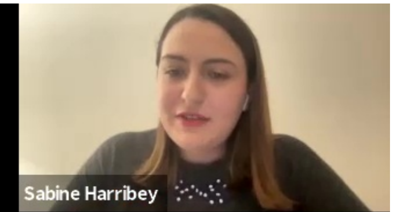


Melonic limit and quantum gravity



- Colored tensor models: colored triangulations
- Limitation: melons $\sim$ branched polymers [Gurau, Ryan, 2013]
- Open challenge [Bonzom, Oriti]

Melonic limit and quantum gravity



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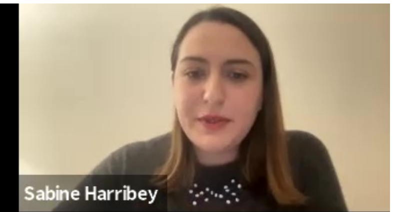
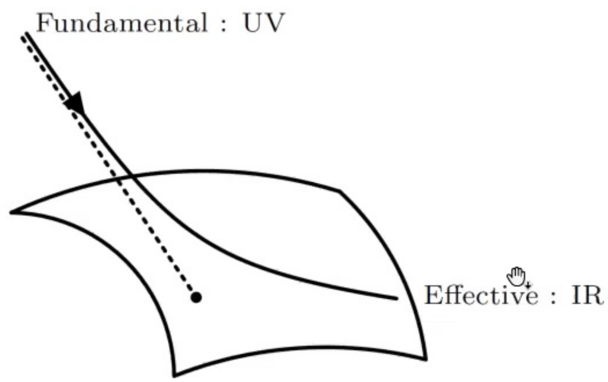
Can be used in another setting: large N field theory

General motivations: renormalization



Physics change with the energy scale: **Renormalization group**



Flow in the space of theories with respect to the energy scale [Wilson 1972, Polchinski 1984, . . .]:

$\rightarrow$ fixed points and trajectories

Melonic limit and quantum gravity

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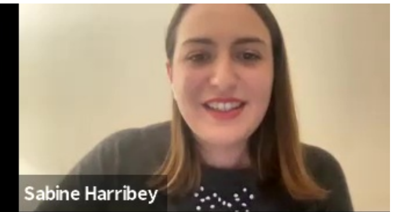
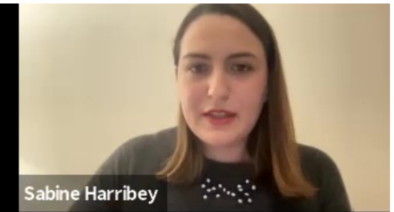


General motivations: renormalization



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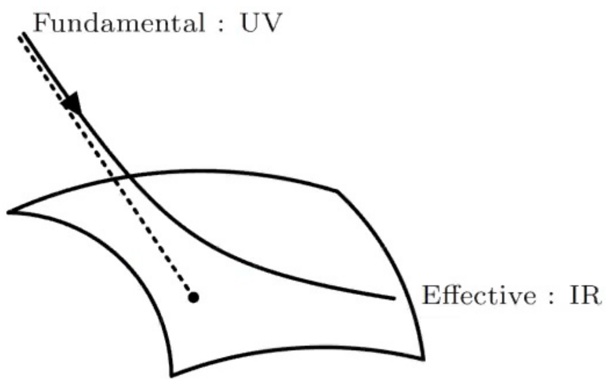


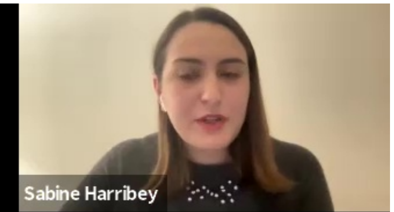
Flow in the space of theories with respect to the energy scale [Wilson 1972, Polchinski 1984, ...]:

$\rightarrow$ fixed points and trajectories

Weak versus strong coupling

Weak coupling	Strong coupling
Perturbation theory	?
Non perturbative ?	?

- Study non-trivial fixed points of the renormalization group [Wilson]
- Gain control over non-perturbative phenomena
- Spontaneous symmetry breaking [Coleman, Jackiw, Politzer]
- Dynamical mass generation [Gross, Neveu]



Weak versus strong coupling

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- Study non-trivial fixed points of the renormalization group [Wilson]
- Gain control over non-perturbative phenomena
- Spontaneous symmetry breaking [Coleman, Jackiw, Politzer]
- Dynamical mass generation [Gross, Neveu]
- Large N limit: helpful approximation scheme
- Vast array of applications: statistical mechanics, QCD, quantum gravity

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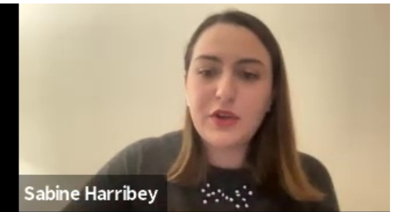
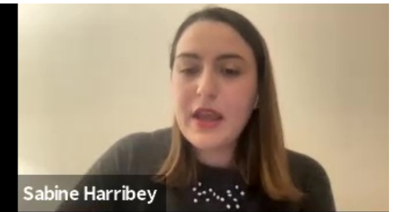


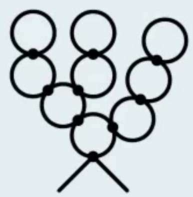
From vector to tensor models



Vector ϕ_a

$\frac{\lambda}{N}(\phi_a\phi_a)^2$

Cactus diagrams

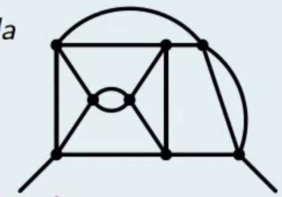


$\rightarrow$ Easy

Matrix M_{ab}

$\frac{\lambda}{N} M_{ab} M_{bc} M_{cd} M_{da}$

Planar diagrams

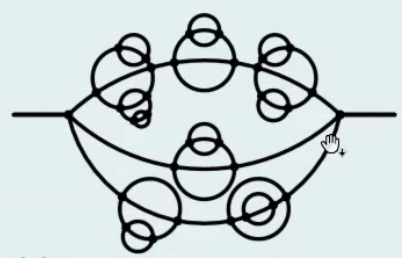


$\rightarrow$ Hard

Tensor T_{abc}

$\frac{\lambda}{N^{3/2}} T_{aeb} T_{cfb} T_{ced} T_{afd}$

Melon diagrams



$\rightarrow$ Tractable

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Melonic quantum mechanics

- Strongly coupled QFTs and holography ($d = 1$): SYK model without disorder [Witten, Klebanov, Tarnopolsky, ...]
- Tensor models in higher dimension: new class of conformal field theories

- Problem: divergences
- Renormalization group: computation of beta functions, stable IR fixed point? Unitarity ?

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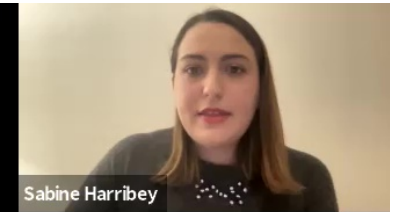
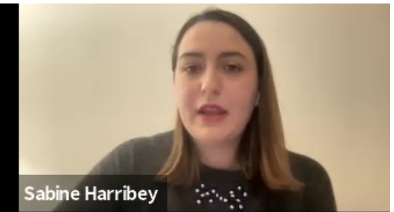


Main features of melonic CFTs

- Melonic dominance
  - Closed Schwinger-Dyson equation
  - Simplification of the four-point kernel



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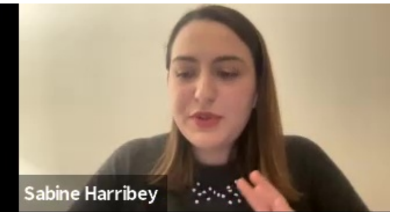
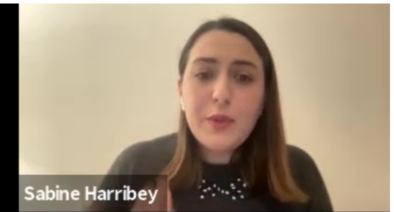


- Melonic dominance
  - Closed Schwinger-Dyson equation
  - Simplification of the four-point kernel
- Two fixed points mechanisms:
  - Usual $d = 4 - \epsilon$ regularization for short-range models
  - Exact marginality for long-range models
- Unitarity range: real scaling dimensions above unitarity bounds and real OPE coefficients

Irreducible tensor models

- Random geometry: colored tensor models
- Field theory: uncolored tensor models
- What about other tensor representations ?
- Completely symmetric tensors: no melonic large N limit
- Conjecture: melonic large N limit for irreducible representations of $O(N)$ or $Sp(N)$ [Klebanov, Tarnopolsky]
- Proof in rank 3 [Benedetti, Carrozza, Gurau, Kolanowski]
- Generalization in higher rank ?

$\Rightarrow$ Here for rank 5



Outline

1. The model
2. Perturbative expansion
3. Sketch of the proof

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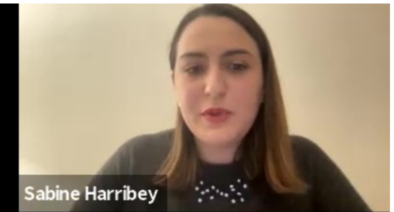


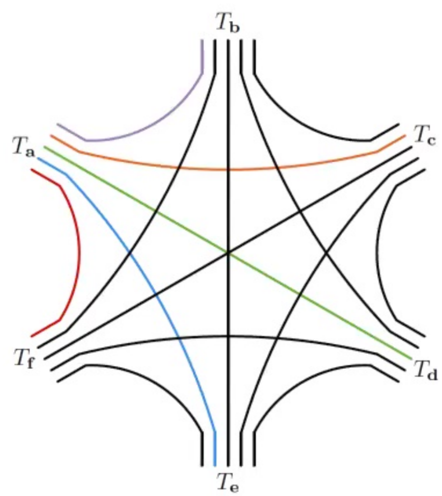
The model

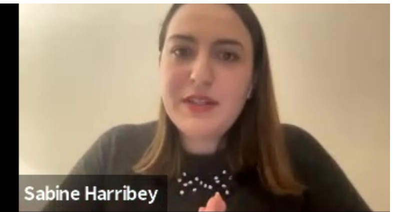
Free energy:

$$F_{\boldsymbol{P}}(\lambda) = \frac{6}{N^5}\lambda\partial_\lambda \ln\left\{\left[e^{\frac{1}{2}\partial_T \boldsymbol{P} \partial_T}\, e^{\frac{\lambda}{6N^5}\delta^h_{\boldsymbol{abcdef}} T_{\boldsymbol{a}} T_{\boldsymbol{b}} T_{\boldsymbol{c}} T_{\boldsymbol{d}} T_{\boldsymbol{e}} T_{\boldsymbol{f}}}\right]_{T=0}\right\}$$

- $\boldsymbol{P}$ one of seven orthogonal projectors on irreducible representations
- Interaction vertex

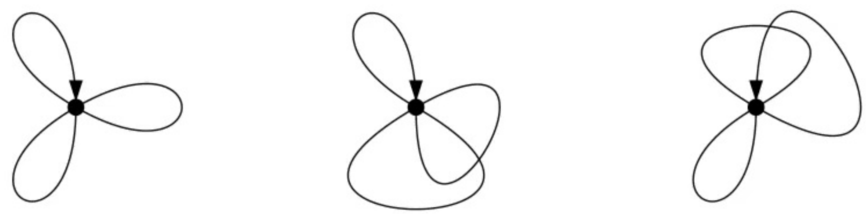
$$T_{a_1 a_2 a_3 a_4 a_5}\, T_{a_5 b_2 b_3 b_4 b_5}\, T_{b_5 a_4 c_3 c_4 c_5}\, T_{c_5 b_4 a_3 d_4 d_5}\, T_{d_5 c_4 b_3 a_2 e_5}\, T_{e_5 d_4 c_3 b_2 a_1}$$





Perturbative expansion

- $F_{\boldsymbol{P}}(\lambda)$: sum over rooted connected combinatorial maps $\mathcal{G}$



- Half-edge: represented with five strands
- 945 ways to connect two half-edges
- Projector: combination of those terms with different weights and signs
- Stranded map G: combinatorial map with a choice of one term per edge

$$F_{\boldsymbol{P}}(\lambda) = \sum_{G \text{ connected, rooted}} \lambda^{V(G)} \mathcal{A}(G)$$

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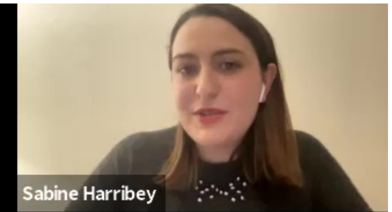
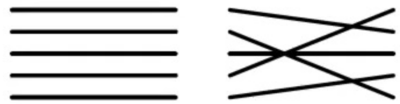
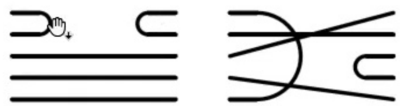


Types of edges

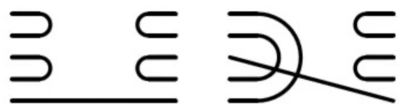
- Unbroken: all strands traverse

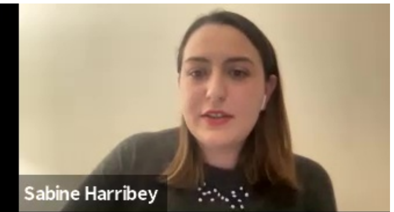


- Broken: a pair of corners is connected by a strand at each end of the edge. Rescaled by $1/N$



- Doubly broken: two pairs of corners are connected by a strand at each end of the edge. Rescaled by $1/N^2$





$1/N$ expansion

Amplitude of a stranded map:

$$\mathcal{A}(G) = K(G) N^{-\omega(G)} (1 + \mathcal{O}(1/N))$$

- $K(G)$ non-vanishing rational number independent of N
- One free sum = One factor of N per face
- Degree of a stranded map:

$$\omega(G) = 5 + 5V(G) + B(G) + 2B_2(G) - F(G)$$

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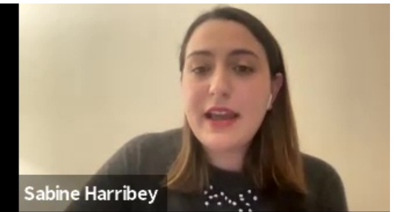

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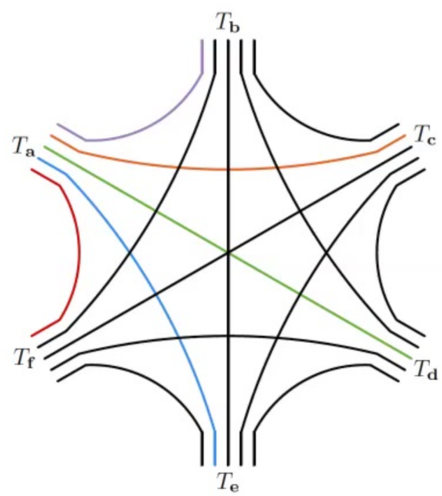

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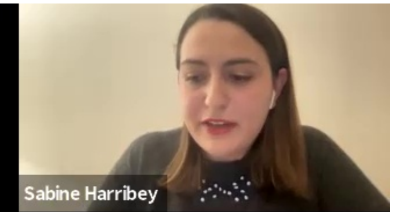

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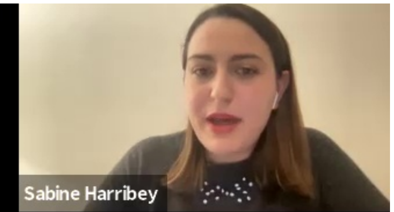
Goals:

$\rightarrow$ Non-negative degree

$\rightarrow$ Maps of zero degree are melonic



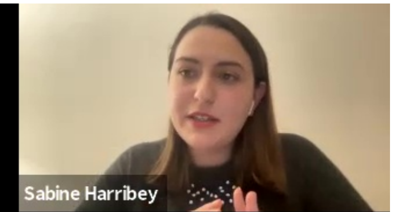
Problematic cases

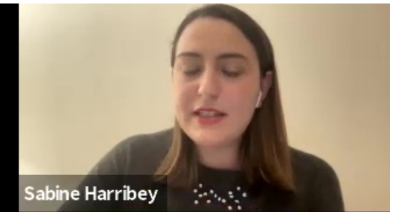
- Graphs with only long faces: positive degree
- Simplify the degree by considering number of faces of length p

$$\omega(G) = 5 + B(G) + 2B_2(G) + \sum_p F_p \left(\frac{p}{3} - 1\right)$$

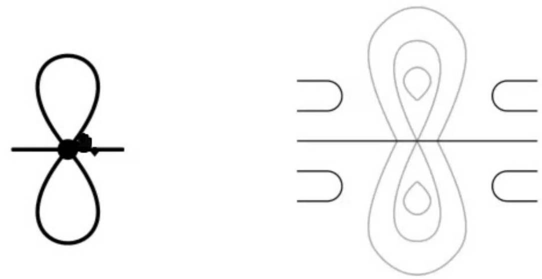
$\rightarrow$ Can be negative iff faces of length $p = 1$ or $p = 2$ (short-faces):

- $p = 1$: Tadpoles, double-tadpoles
- $p = 2$: Melons, dipoles





Bad double tadpoles



Chain of p double tadpoles:

- 4 faces per vertex
- Factor N^{-5} per vertex
- 2 faces when we glue two double-tadpoles

$$\left(\frac{1}{N}\right)^p N^{2p-1} = N^{p-1}$$

Unbounded from above

$\rightarrow$ non-trivial cancellations

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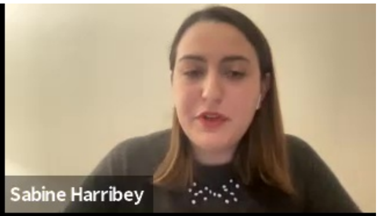
Bounds on combinatorial maps



- Stranded maps with negative degree
- Use irreducibility of the representation to bound the amplitude of the full combinatorial maps
- Double-tadpoles combinatorial maps well-behaved
- Melons: contribute to leading-order

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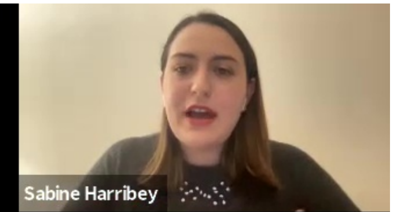
Bounds on combinatorial maps



- Stranded maps with negative degree
- Use irreducibility of the representation to bound the amplitude of the full combinatorial maps
- Double-tadpoles combinatorial maps well-behaved
- Melons: contribute to leading-order
- Problem: generalized double-tadpoles $\rightarrow$ arbitrarily negative degree
- Need to subtract both melons and double-tadpoles

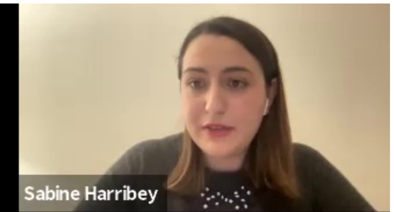
Main theorem

Theorem

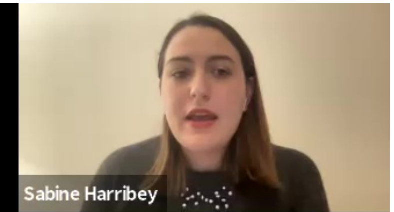
We have (in the sense of perturbation series):

$$F_{\boldsymbol{P}}(\lambda) = \sum_{\omega \in \mathbb{N}} N^{-\omega} F_{\boldsymbol{P}}^{(\omega)}(\lambda)\,. \tag{1}$$

- Subtract double-tadpoles and melons
- Restrict to unbroken edges
- Induction: remaining graphs have positive degree



Step 1: Subtraction of double-tadpoles and melons



- Consider a theory with modified covariance KP and subtracted interaction
- Show that we can choose K s.t. this is our original theory
- Partial resummation of the perturbative series: closed and algebraic SDE
- New perturbative expansion in terms of graphs with no double-tadpoles or melons

Step 2: Restriction to unbroken edges

- Cut and glue: changes the number of faces by -1, 0 or $+1$
- From doubly broken to unbroken propagator:

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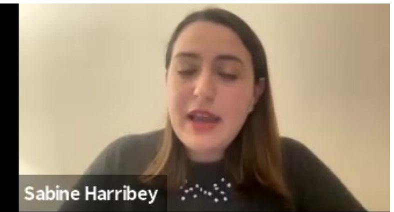
Step 2: Restriction to unbroken edges



- Cut and glue: changes the number of faces by -1, 0 or $+1$
- From doubly broken to unbroken propagator:

Remove broken edge:

- decrease the number of faces by at most one
- decrease the number of broken edges by one
- the degree can only decrease

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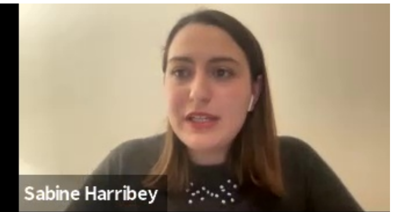
Step 3: Remaining graphs

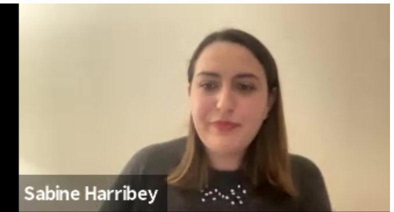


Let G be a stranded graph. If G has no double-tadpole and no melon, then $\omega(G) \geq 0$.

$\rightarrow$ proof by induction

- Look for a strict subgraph that can be deleted without increasing the degree and preserving the constraints
- Exhaustive graph-theoretic distinction of cases
- High number of particular two-point subgraphs to consider

Leading order



Stranded graphs with no melon can have vanishing degree

$\rightarrow$ Non-trivial cancellations

- Bounds on combinatorial maps
- Maps with no melons are subleading
- **Conclusion: A Feynman map is leading order iff it is melonic**

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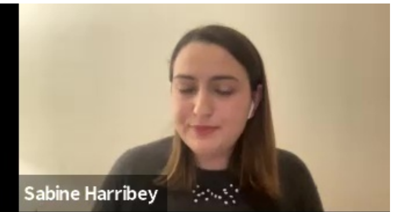
Conclusion and outlook

- Irreducible tensor models with 5-simplex interactions: melonic large-N expansion
- Recursive bounds from a detailed combinatorial analysis of the Feynman graphs.
- Estimated scaling of four and eight-point functions: could include other effective interactions
- Application to large N QFT
- Fermionic fields with $Sp(N)$ symmetry
- Generalization in arbitrary rank $r \geq 6$?



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Thank you !

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