Title: Quantum Lego: Building Quantum Error Correction Codes from Tensor Networks

Speakers: Charles Cao

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Abstract: We introduce a flexible and graphically intuitive framework that constructs complex quantum error correction codes from simple codes or states, generalizing code concatenation. More specifically, we represent the complex code constructions as tensor networks built from the tensors of simple codes or states in a modular fashion. Using a set of local moves known as operator pushing, one can derive properties of the more complex codes, such as transversal non-Clifford gates, by tracing the flow of operators in the network. The framework endows a network geometry to any code it builds and is valid for constructing stabilizer codes as well as non-stabilizer codes over qubits and qudits. For a contractible tensor network, the sequence of contractions also constructs a decoding/encoding circuit. To highlight the framework's range of capabilities and to provide a tutorial, we lay out some examples where we glue together simple stabilizer codes to construct non-trivial codes. These examples include the toric code and its variants, a holographic code with transversal non-Clifford operators, a 3d stabilizer code, and other stabilizer codes with interesting properties. Surprisingly, we find that the surface code is equivalent to the 2d Bacon-Shor code after "dualizing" its tensor network encoding map.

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# QUANTUM LEGO: BUILDING QECCS FROM TENSOR NETWORKS

CHUNJUN (CHARLES) CAO AND BRAD LACKEY

# OUTLINE

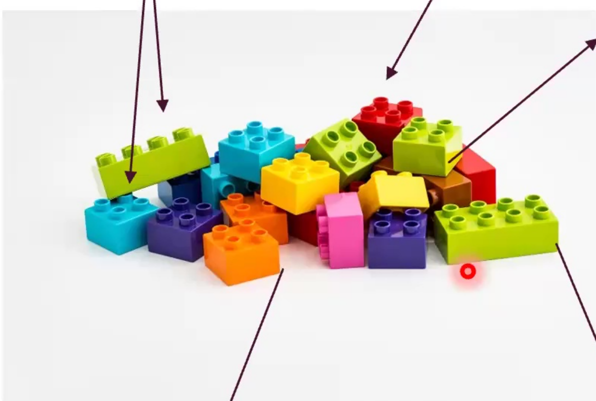
- General Idea
- Tensors
- Tensor Network, Operator Pushing and simple QECCs
  - Codes with transversal non-Clifford gate
- Examples
  - Toric code, surface code and variants
- Discussion and summary
  - Other codes (more general networks, 3d color code etc)
  - Decoding
  - Conclusion

# BUILDING CODES WITH TENSOR NETWORKS

simple non-additive codes

*And many more…*

4 qb Bacon-Shor code



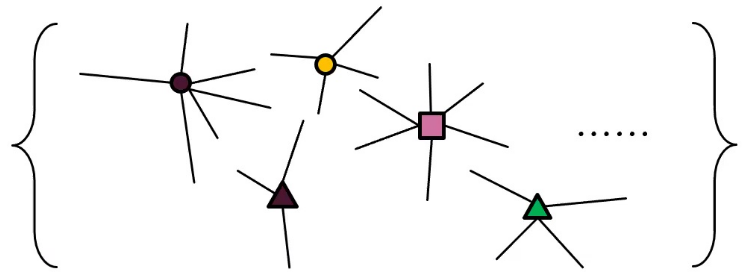
Steane code

5 qubit perfect code

QECC with desirable properties on complex quantum systems

# BUILDING CODES WITH TENSOR NETWORKS

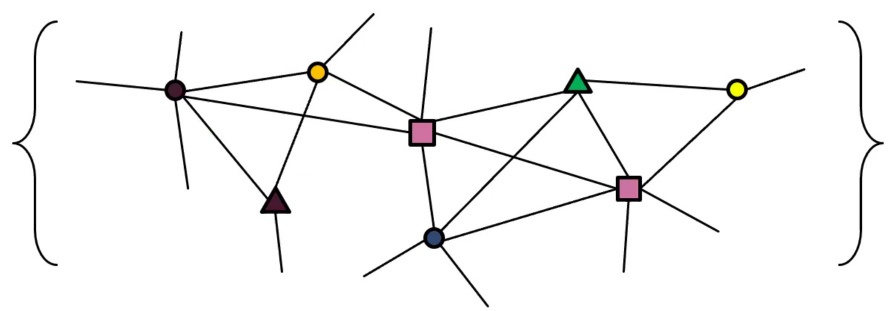
{"simple" quantum codes} ⟷



Tensors – "quantum lego blocks"

↓

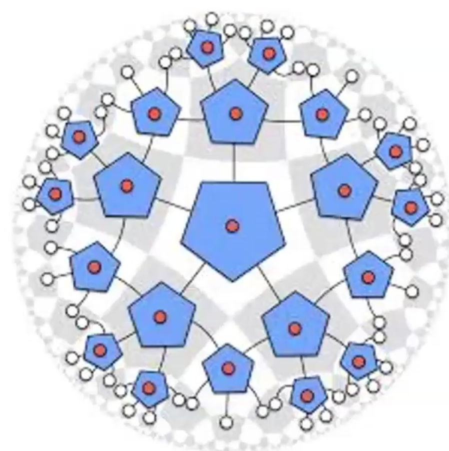
{"nice but complicated" quantum codes} ⟷



TN -- "quantum lego buildings"

# MOTIVATING EXAMPLES

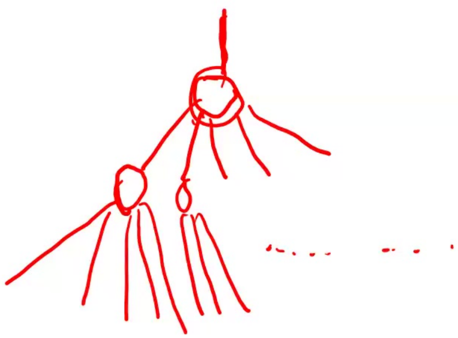
- TTN (code concatenation)
- HaPPY holographic code



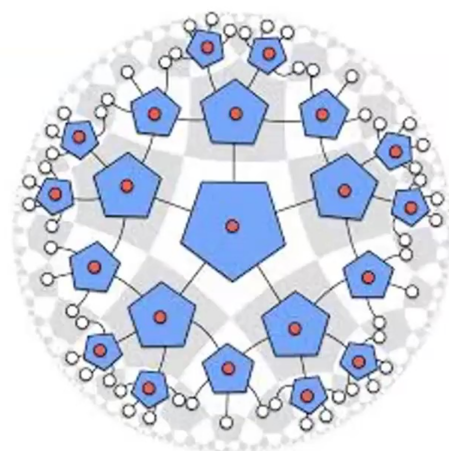
Pastawski, Yoshida, Harlow, Preskill (2015)

# MOTIVATING EXAMPLES

- TTN (code concatenation)



- HaPPY holographic code

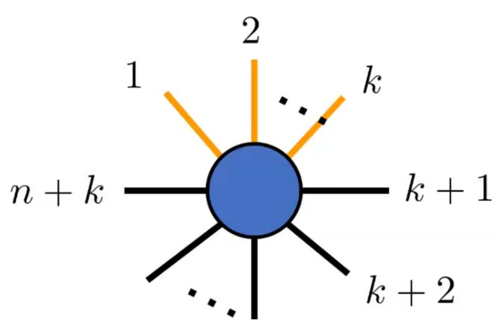


Pastawski, Yoshida, Harlow, Preskill (2015)

# QUANTUM LEGO BLOCKS:TENSORS

$$|V\rangle = \sum_{i_j} V_{i_1 \ldots i_{n+k}} |i_1, \ldots, i_{n+k}\rangle$$

$$V_{i_1,\ldots,i_{n+k}} =$$

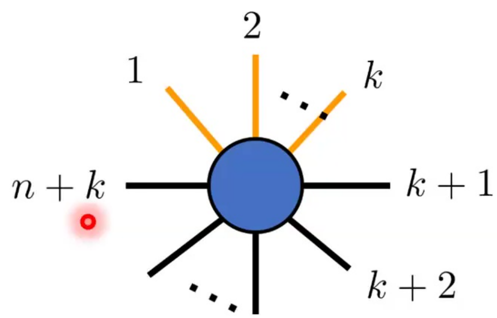


# QUANTUM LEGO BLOCKS:TENSORS

$$|V\rangle = \sum_{i_j} V_{i_1 \ldots i_{n+k}} |i_1, \ldots, i_{n+k}\rangle$$

$$V : \mathcal{H}_d^{\otimes k} \to \mathcal{H}_d^{\otimes n}, \quad \mathcal{H}_d = \mathbb{C}^d$$

$$V = \sum_{i_j} V_{i_1 \ldots i_{n+k}} |i_{k+1}, \ldots, i_{k+n}\rangle\langle i_1 \ldots, i_k|$$

$V_{i_1, \ldots, i_{n+k}} =$ 

# UNITARY (PRODUCT) STABILIZERS

- Tensors have a lot of components which we don't really need to track

- Characterize them by their unitary product stabilizers

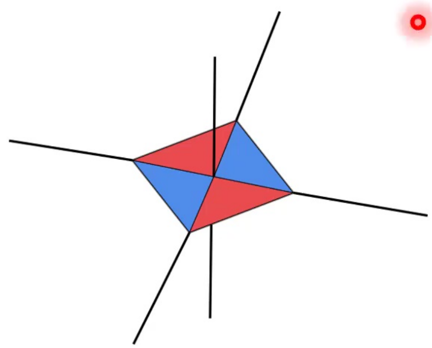
$$U_1 \otimes U_2 \otimes \cdots \otimes U_N |V\rangle = |V\rangle$$

- They capture the structure/"symmetries" of the tensor
- Directs operator flow and operator pushing (more on this later)

# EXAMPLE: [[4,2,2]]

Or a 6 qubit stabilizer state

$$|V_{[[6,0]]}\rangle \in \mathcal{H}_2^{\otimes 6}$$



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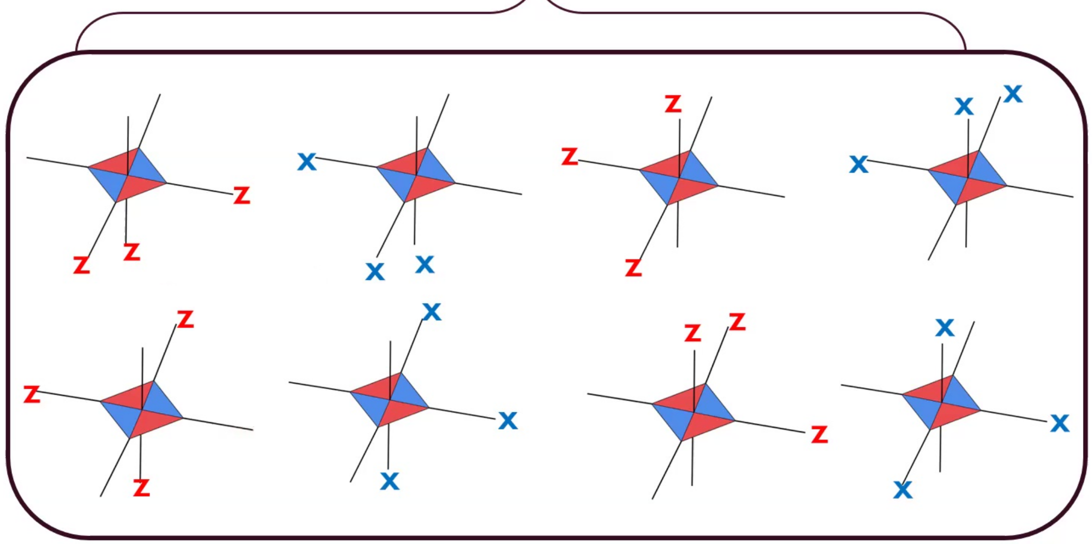
(Unitary product) stabilizers



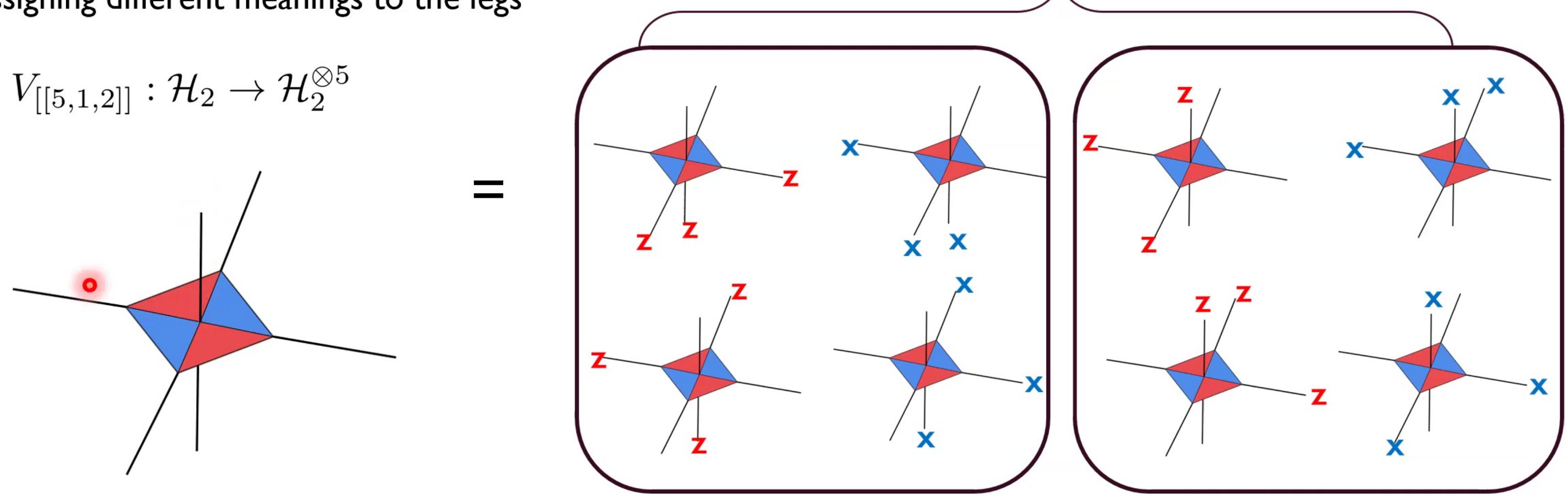
Stabilizers

# EXAMPLE: [[4,2,2]]

But I can also easily treat it as a [[5,1,2]] code by assigning different meanings to the legs

$$V_{[[5,1,2]]} : \mathcal{H}_2 \to \mathcal{H}_2^{\otimes 5}$$

(Unitary product) stabilizers

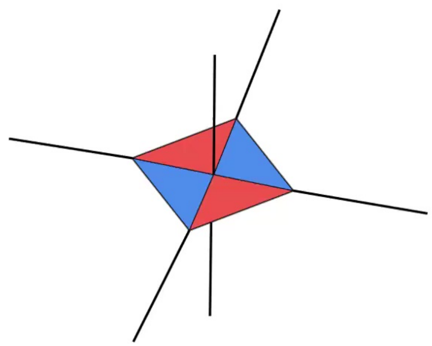


Stabilizers

Logical Operators

# EXAMPLE: [[4,2,2]]

$$V_{[[4,2,2]]} : \mathcal{H}_2^{\otimes 2} \to \mathcal{H}_2^{\otimes 4}$$



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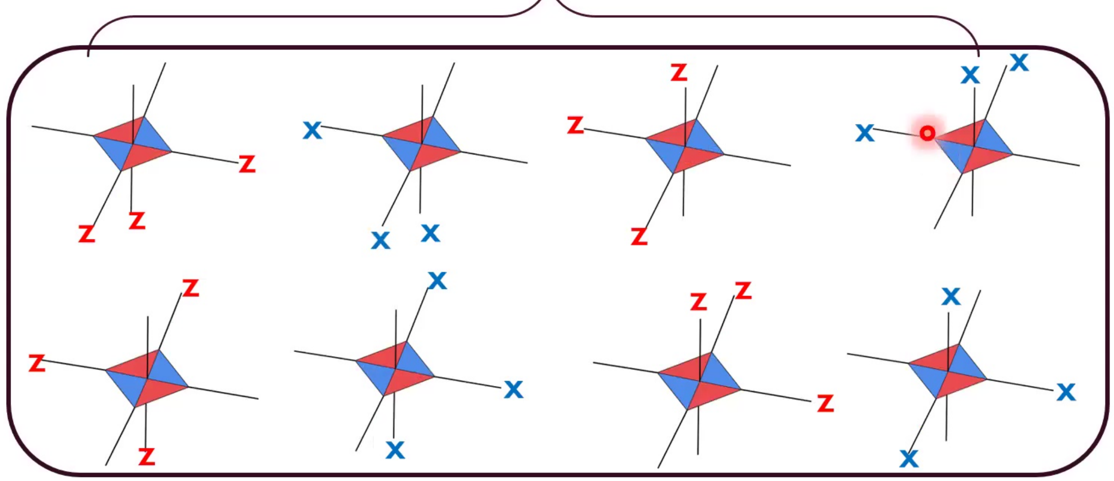
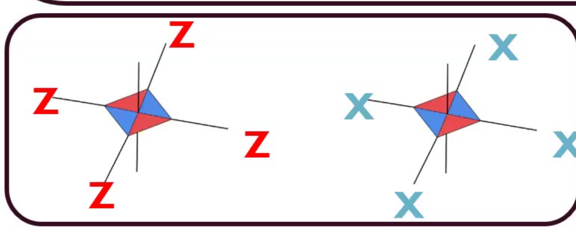
(Unitary product) stabilizers



Logical Operators

Stabilizers



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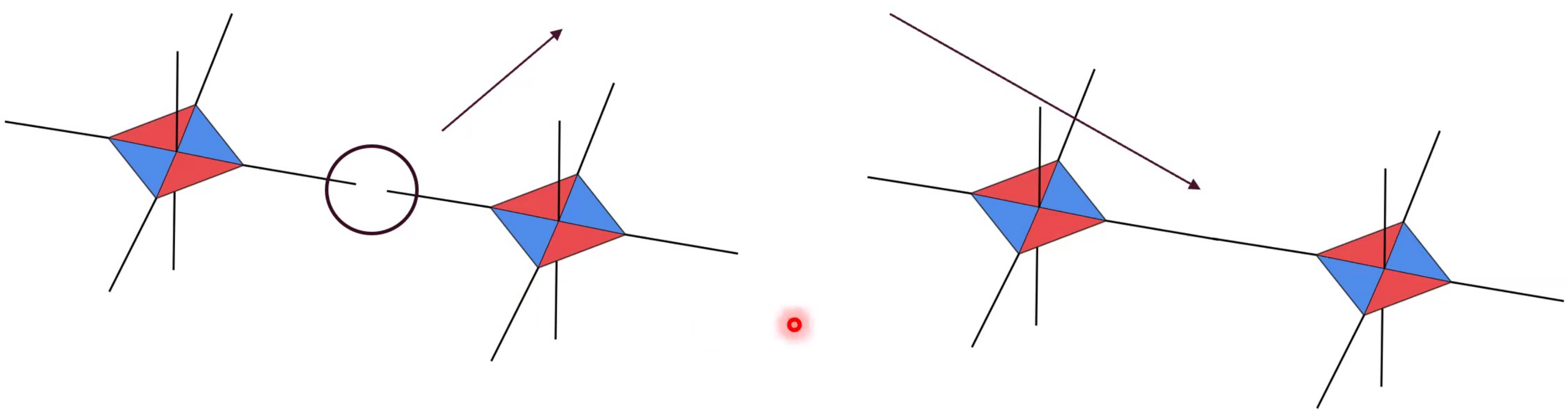
# JOINING THE "LEGO BLOCKS"

- This new language is powerful when we join the blocks together
- Generalizes code concatenation

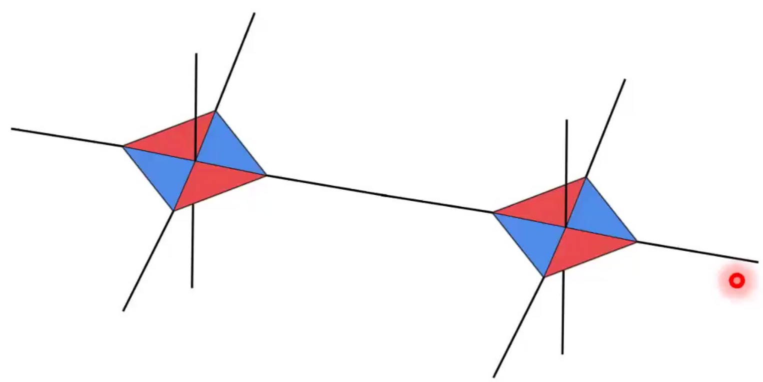
Project onto

$$|\Phi^+\rangle = |00\rangle + |11\rangle$$



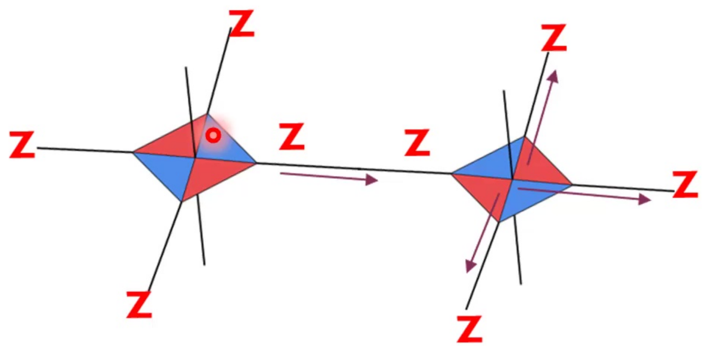
# JOINING THE "LEGO BLOCKS"

$$W_{[[6,4,2]]} : \mathcal{H}_2^{\otimes 4} \to \mathcal{H}_2^{\otimes 6}$$



# JOINING THE "LEGO BLOCKS"

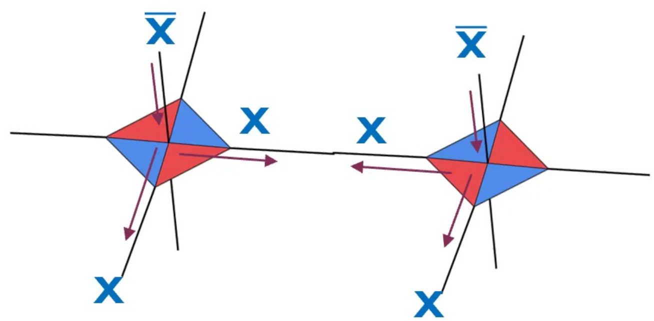
Stabilizer of the new code:



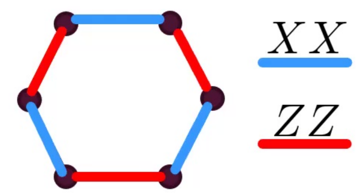

Matching condition:

$$O_A \otimes Q_B|\Phi^+\rangle = |\Phi^+\rangle$$

Logical operator of the new code:



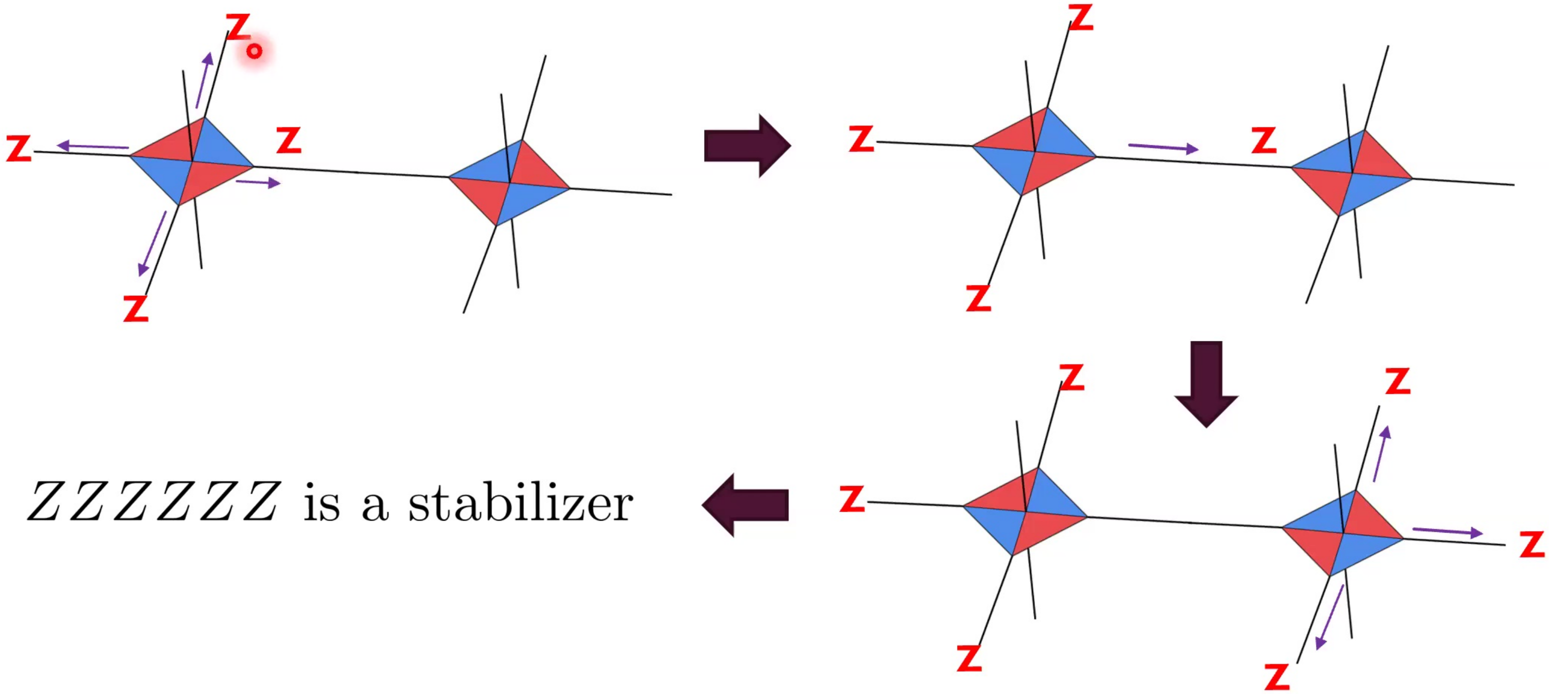

If demoting the bottom legs to gauge qubits




[[6,2,2]] generalized Bacon-Shor code

# JOINING THE "LEGO BLOCKS"

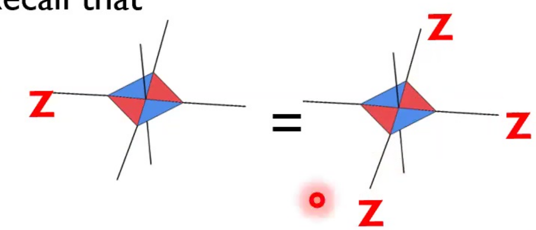
Generate stabilizers of the new code:



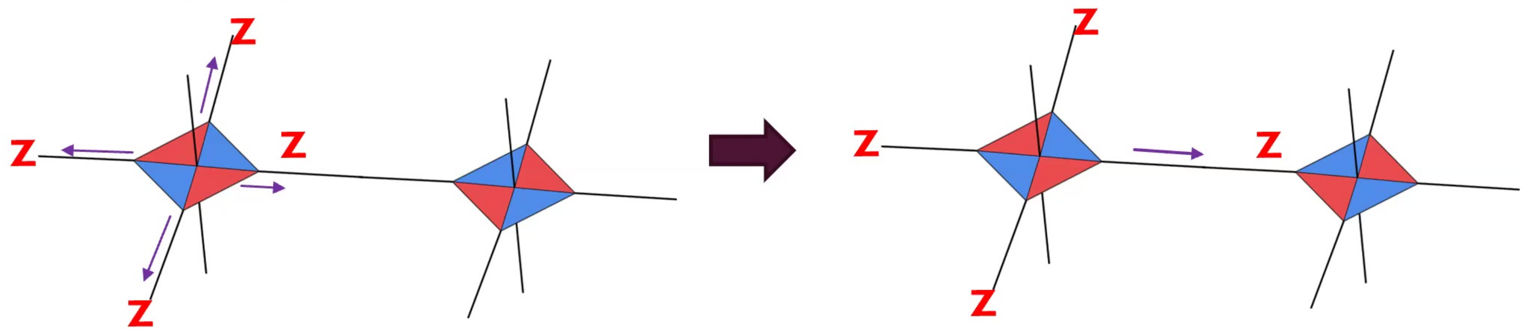

$ZZZZZZ$ is a stabilizer

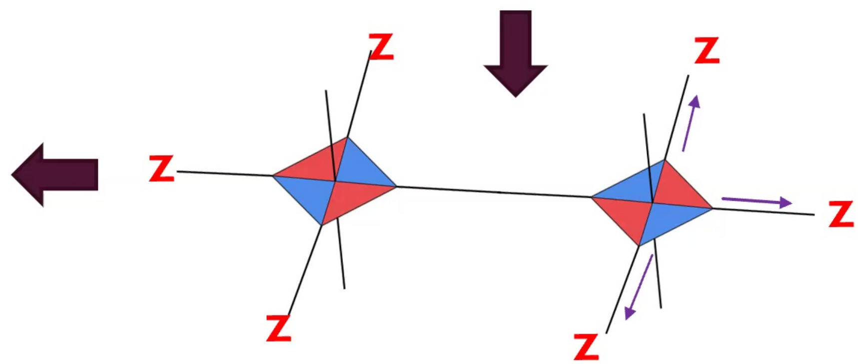
# JOINING THE "LEGO BLOCKS"

Recall that




Generate stabilizers of the new code:

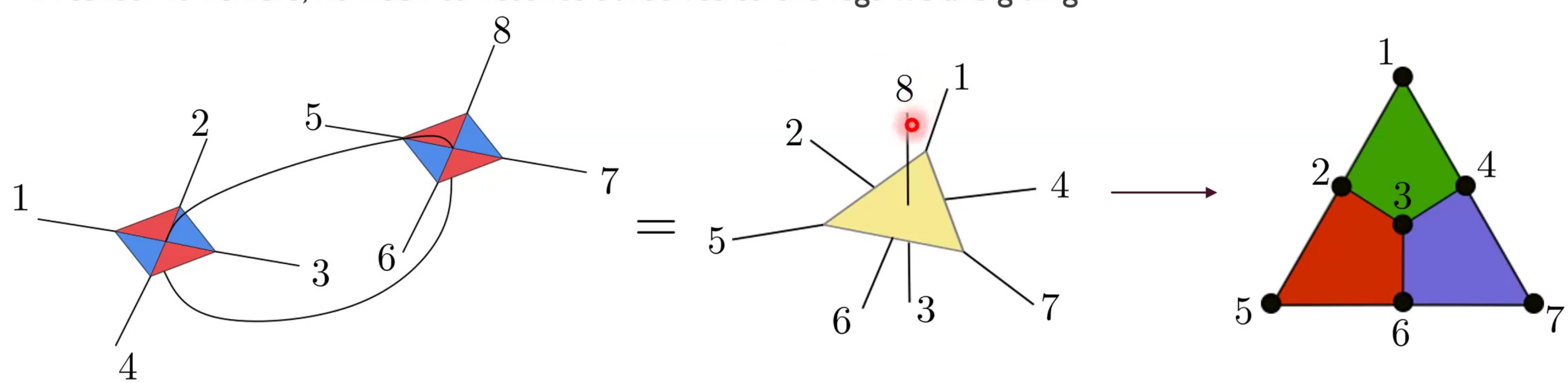



$ZZZZZZ$ is a stabilizer

# INCREASING THE CODE DISTANCE

- A tensor is flexible, no need to restrict ourselves to the legs we are gluing




Tracing the "logical dangling legs" of the [[4,2,2]] tensor yields the [[7,1,3]] Steane code

# INCREASING THE CODE DISTANCE

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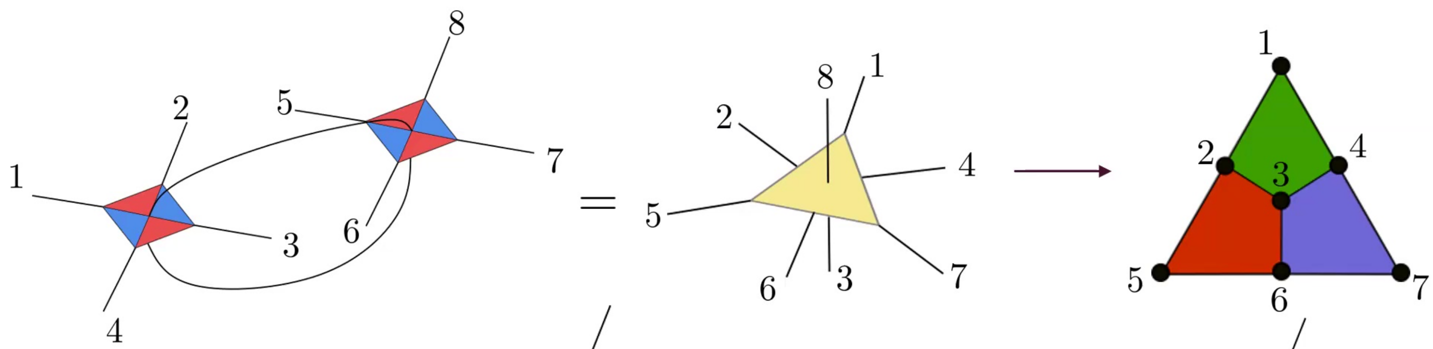
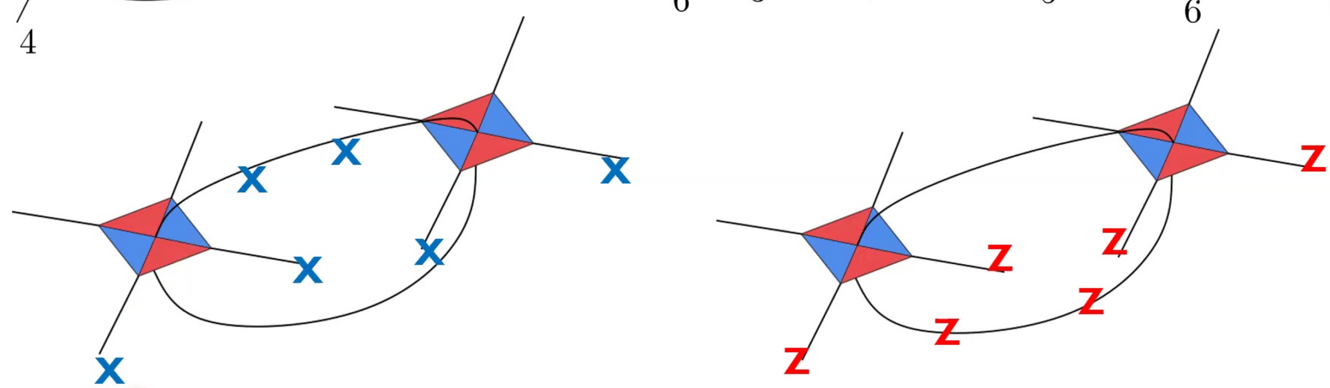
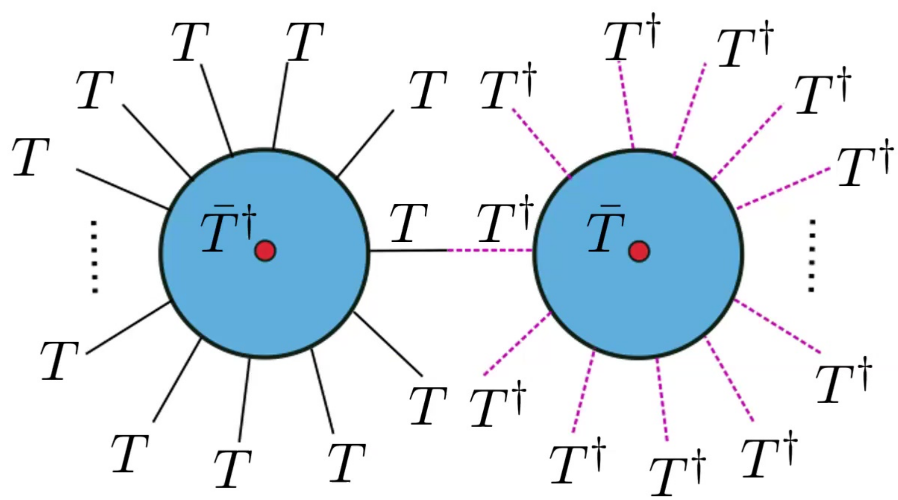



e.g.



# NON-CLIFFORD GATES

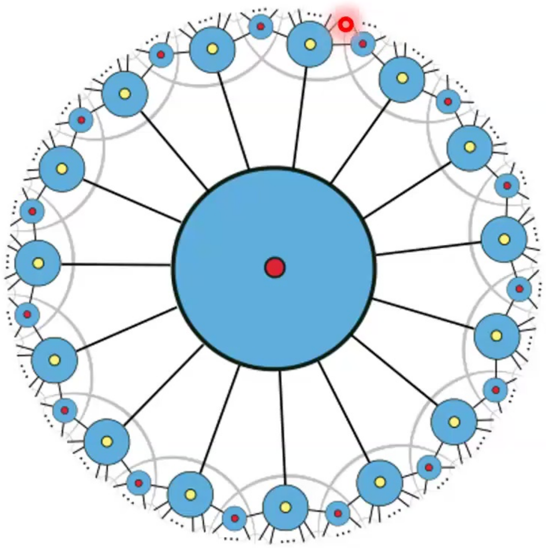
- For instance, $T$ and $T^{-1}$ are also matching: $T \otimes T^{-1}(|00\rangle + |11\rangle) = |00\rangle + |11\rangle$
- This implies the following TN has a transversal T-like operator




Where each blue tensor is the [[15,1,3]] Reed-Muller code with transversal T operator

# HOLOGRAPHIC CODE WITH NON-CLIFFORD GATE

- Therefore, one can make more complex codes by gluing together tensors where Ts can be pushed through
- As a simple example
- Need not restrict to a single tensor type



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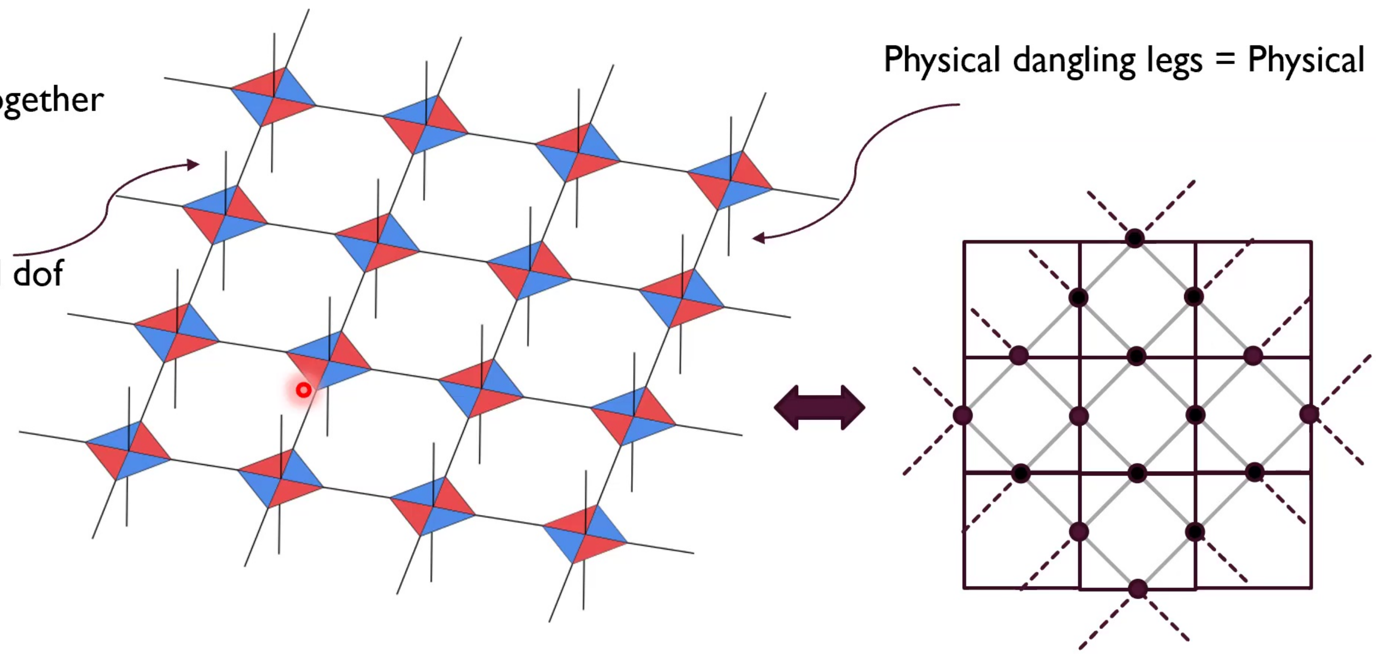
# TORIC CODE

- Having had some tutorials over simpler codes, let's look at something less trivial

"gluing [[5,1,2]] codes" together

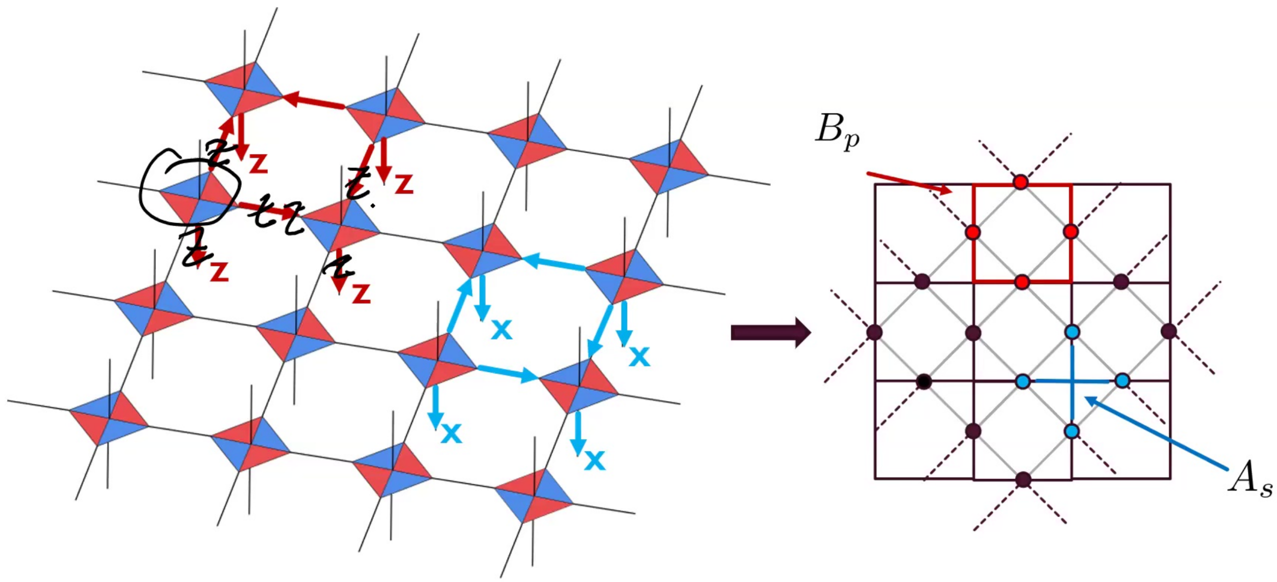
logical dangling legs=encoded dof

Physical dangling legs = Physical qubits



# TORIC CODE

- Making stabilizers via operator pushing



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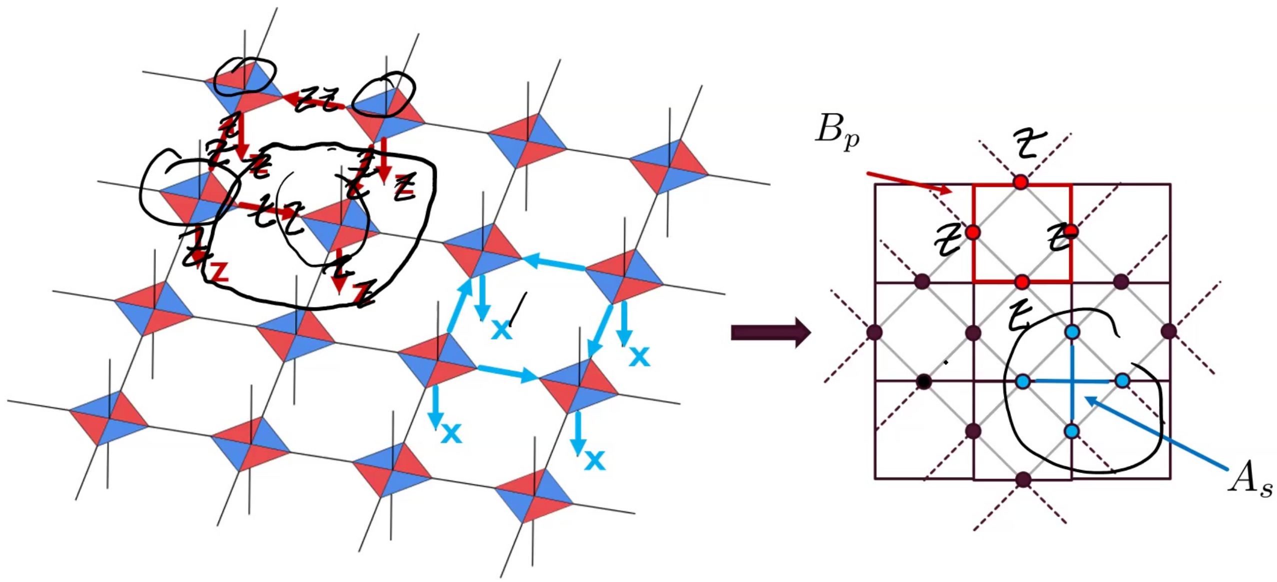
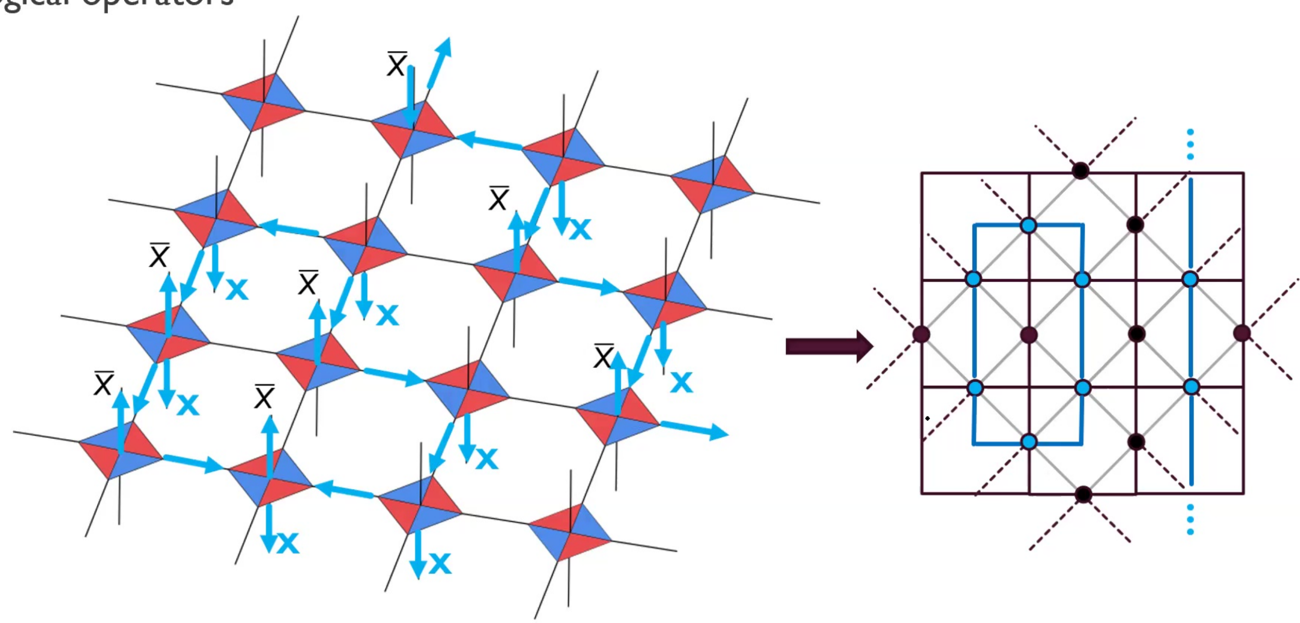


# TORIC CODE

- Similarly with logical operators



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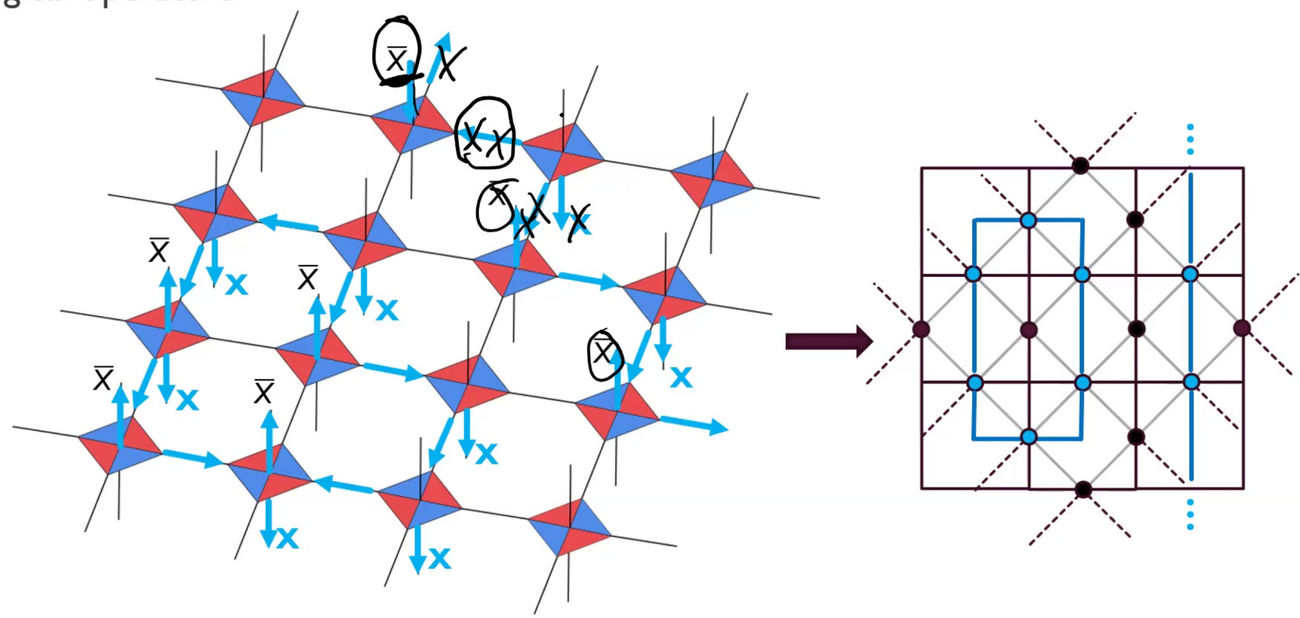

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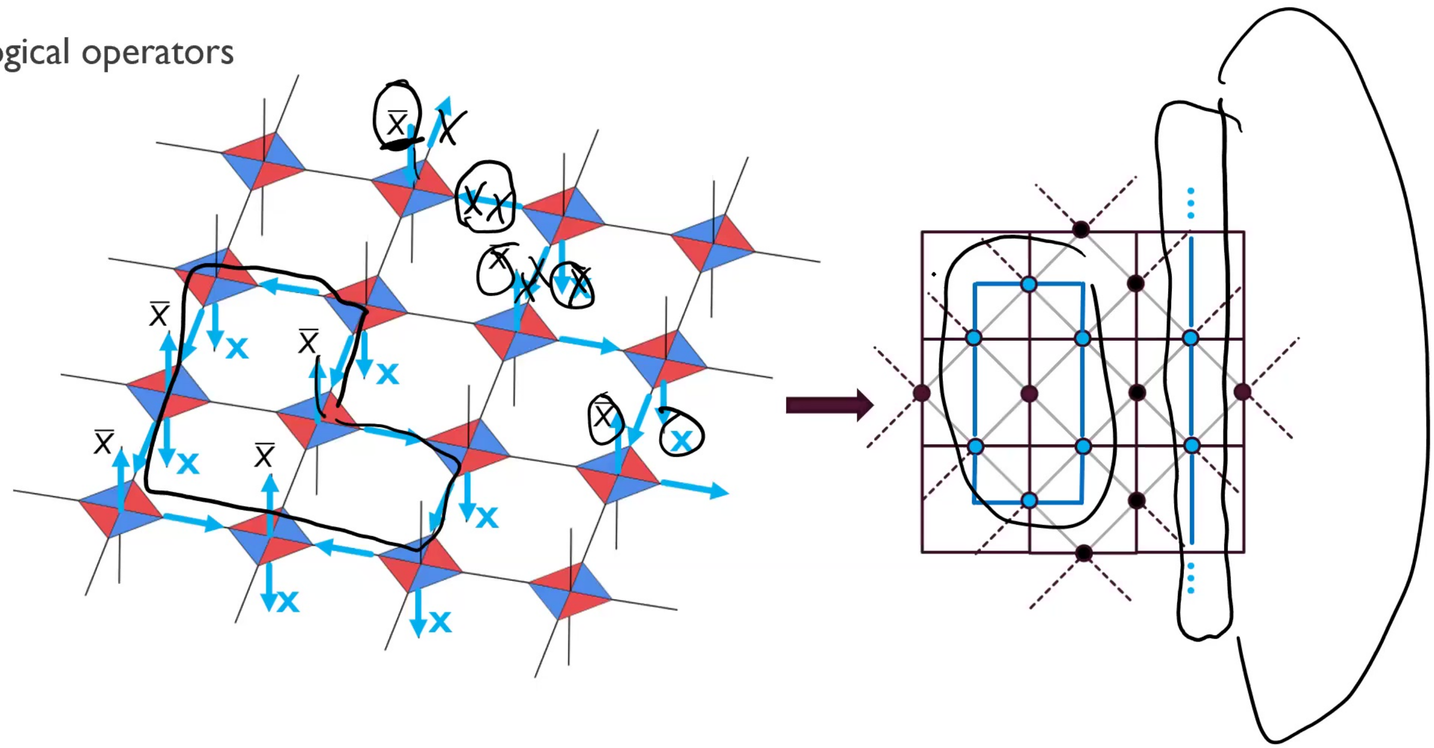

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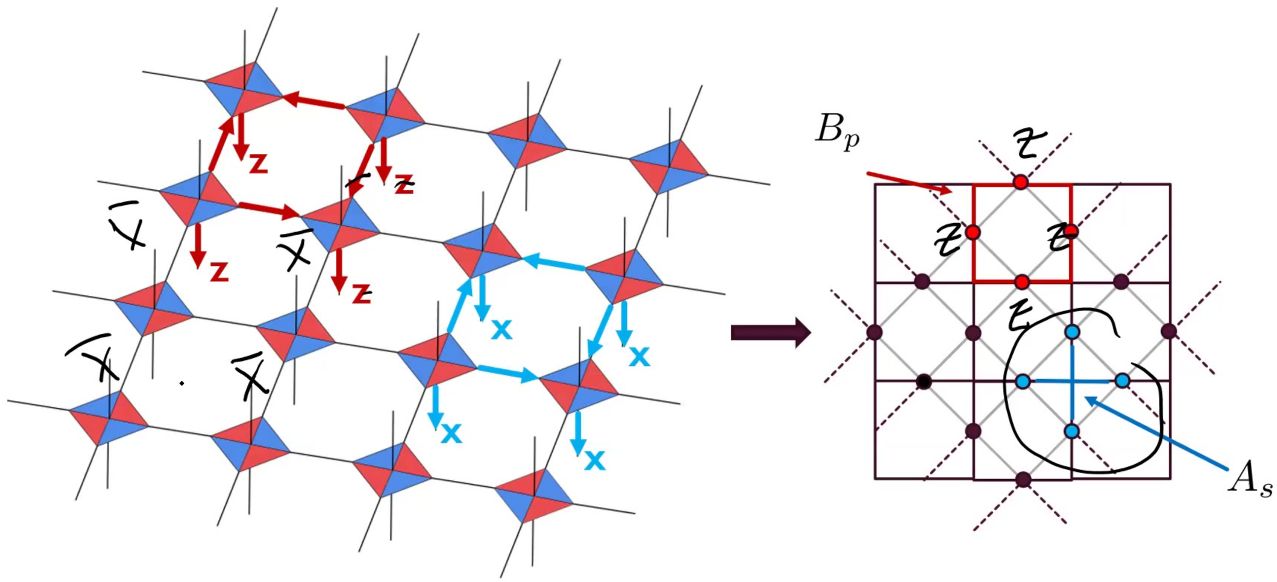

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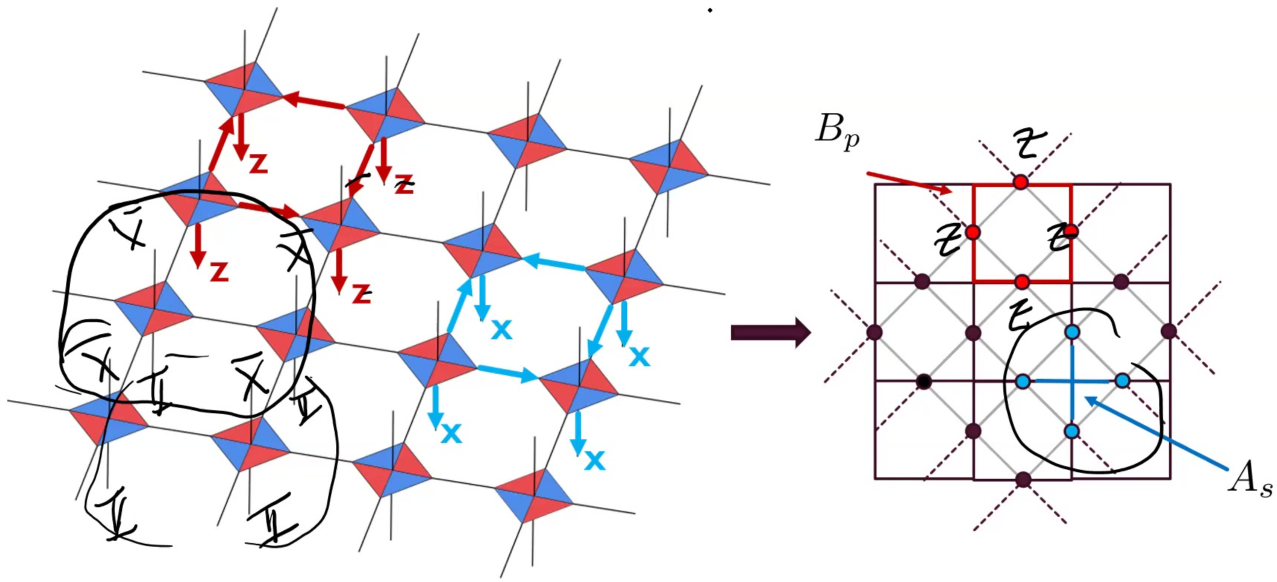

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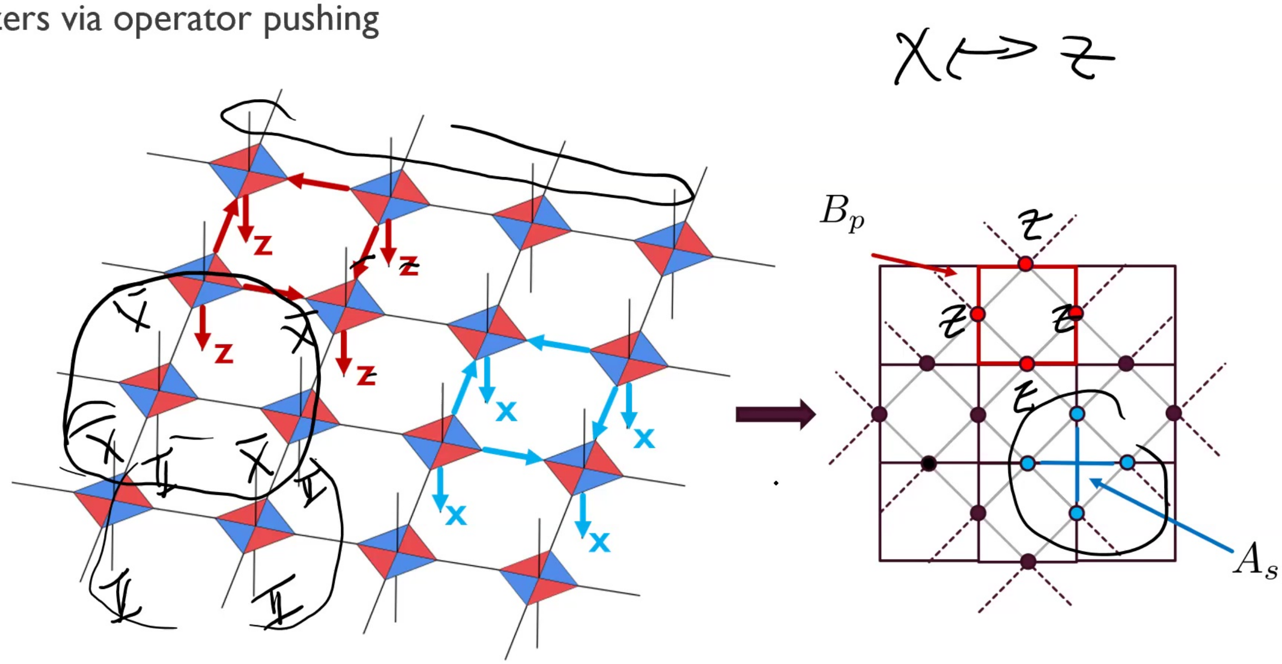

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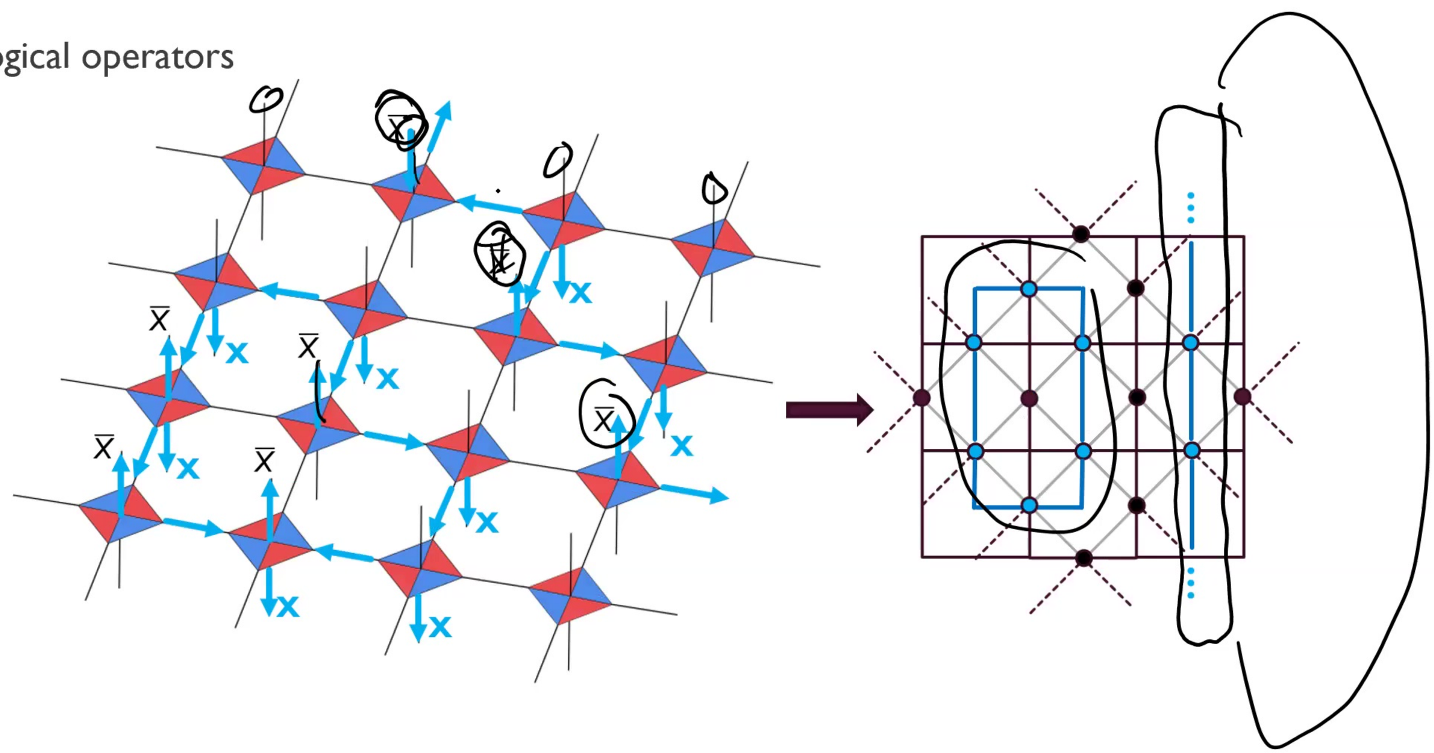


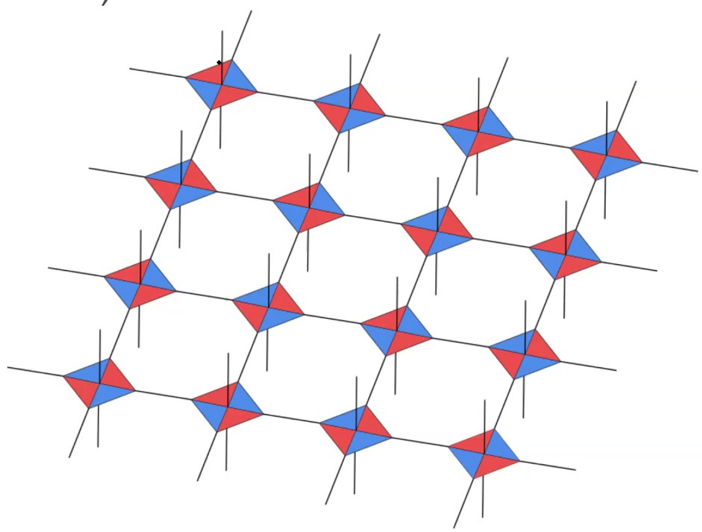
# TORIC CODE

- In some sense, we have stringed together essentially simple modules of [[4,2,2]] codes, the simplest qubit error detection code there is, to manufacture something interesting like the toric code

- What if we now modify some of the “modules” or lego blocks and see what we will get?

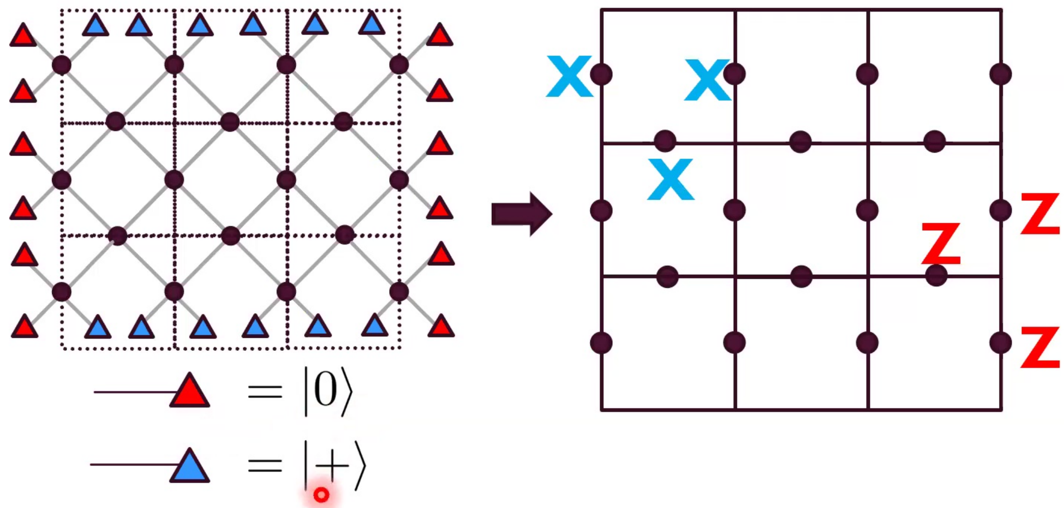
# CUSTOMIZATIONS: BOUNDARY

- Bare boundary condition: a subsystem code that looks like surface code in the bulk but like Bacon-Shor code near the boundary (weight 2 gauge operators)



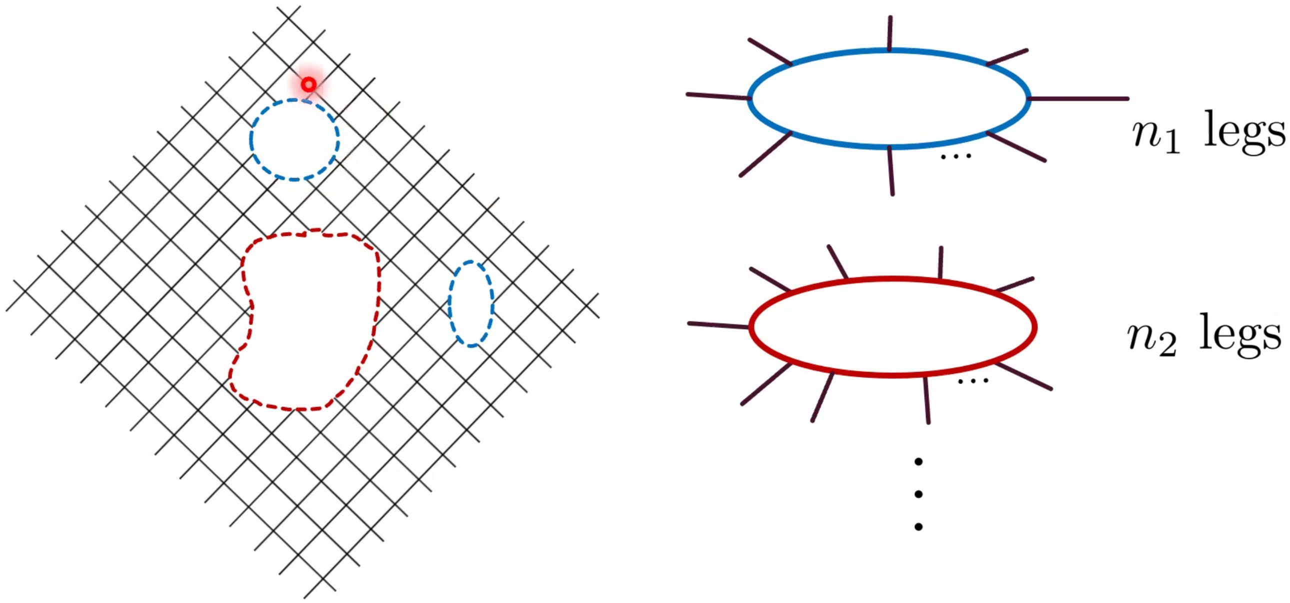
# CUSTOMIZATIONS: BOUNDARY

- Conventional surface code



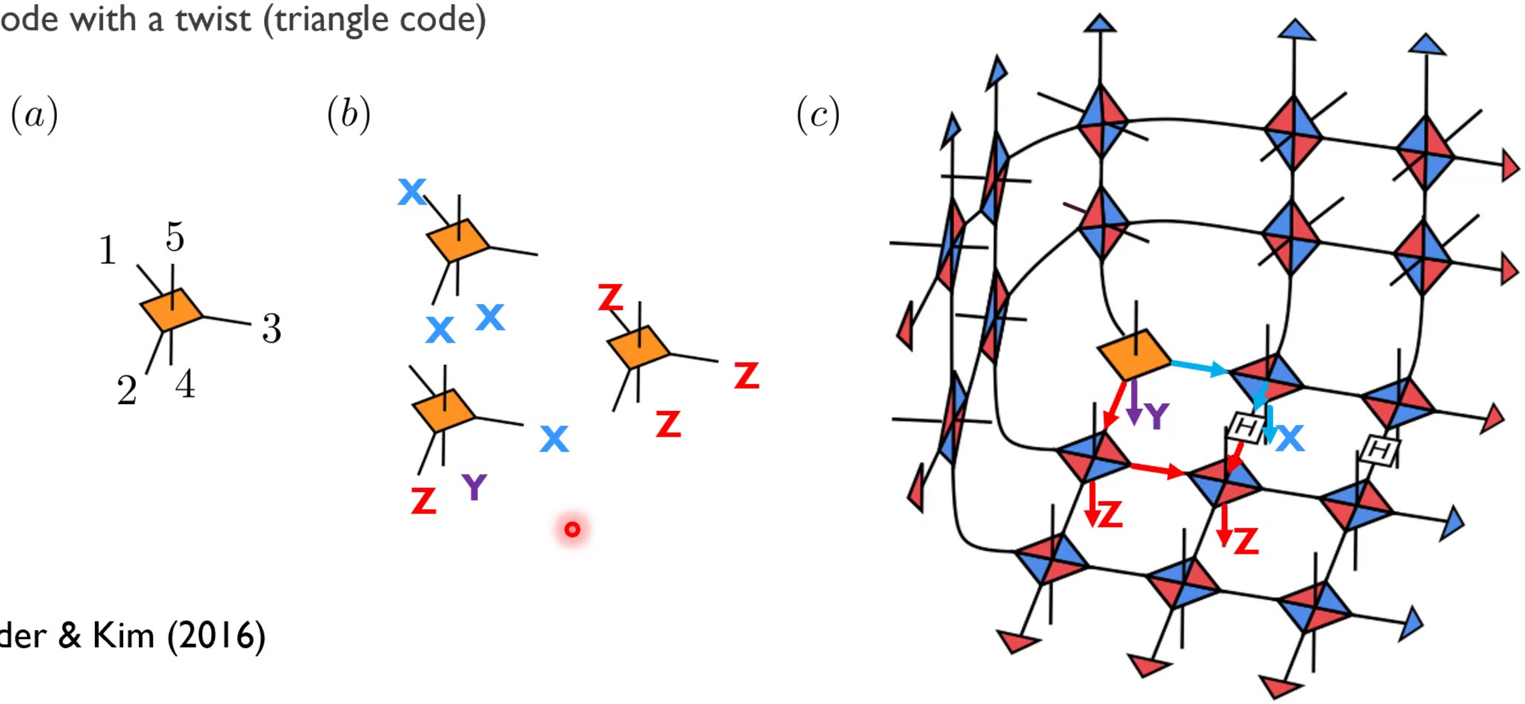
# CUSTOMIZATIONS: BOUNDARY

- Other boundary tensors: repetition codes



# CUSTOMIZATIONS: BULK

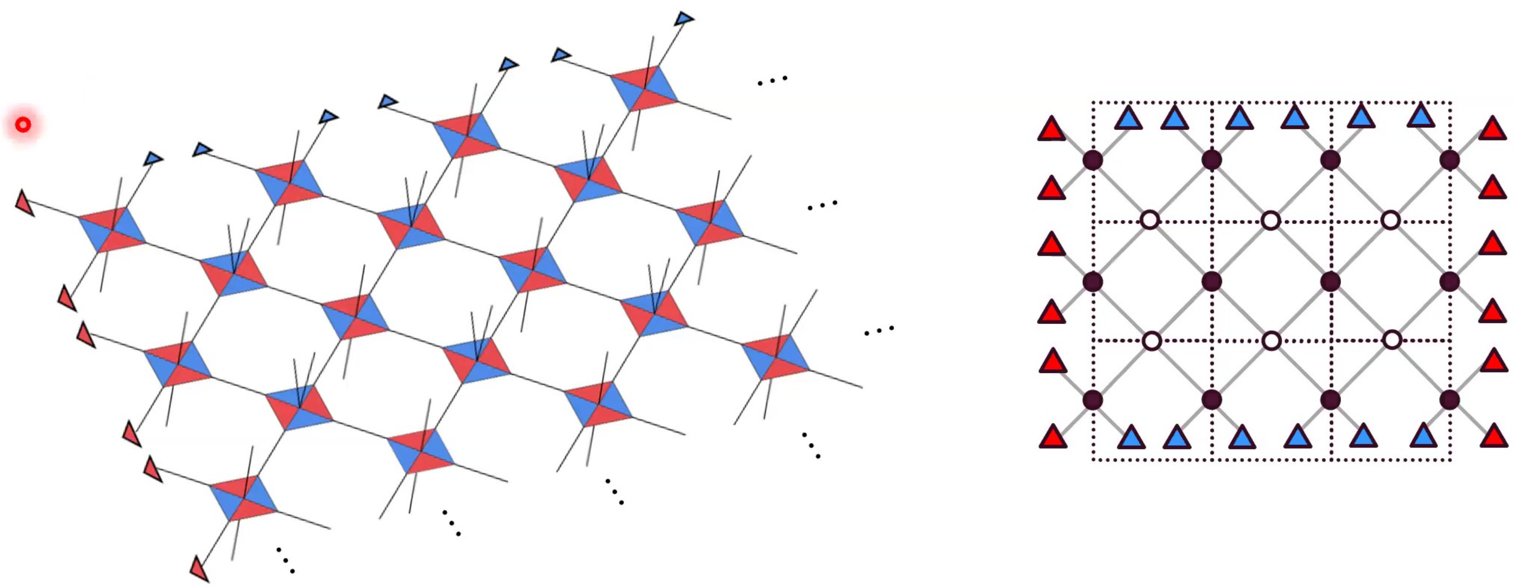
- Toric code with a twist (triangle code)




Yoder & Kim (2016)

# 2D BACON-SHOR CODE

- Reassign the dangling legs every other row to be purely logical (denoted as upward pointing)



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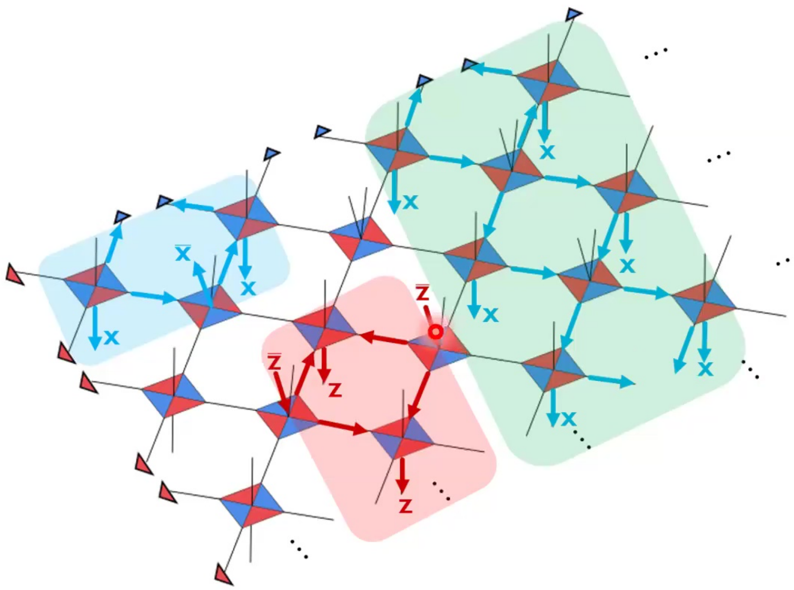
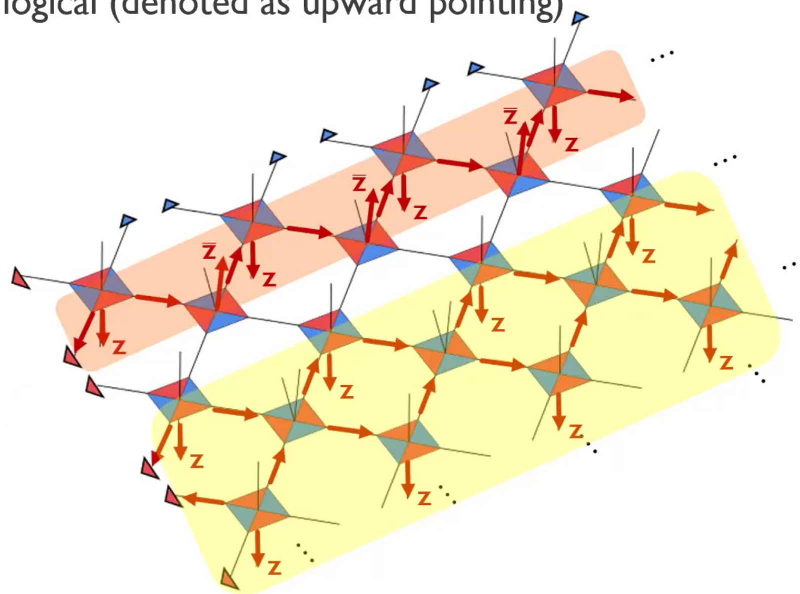
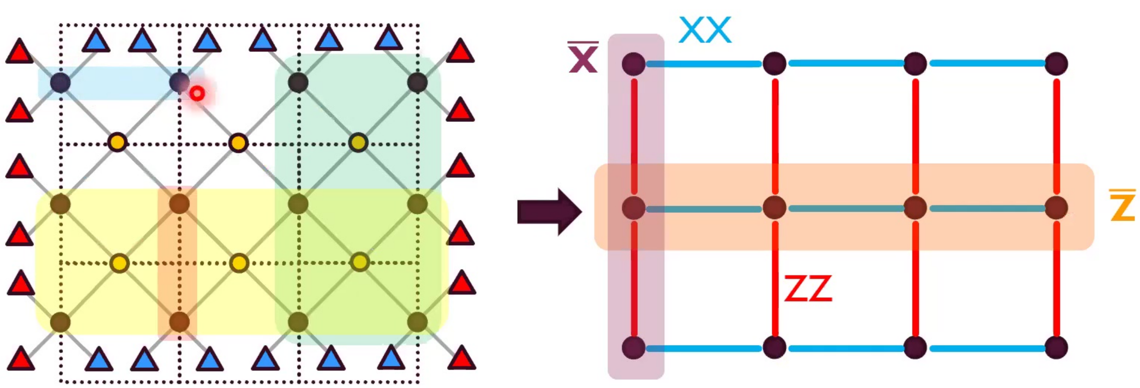




# 2D BACON-SHOR CODE

- We have turned the weight-4 plaquette and star operators into the gauge generators of the Bacon-Shor code



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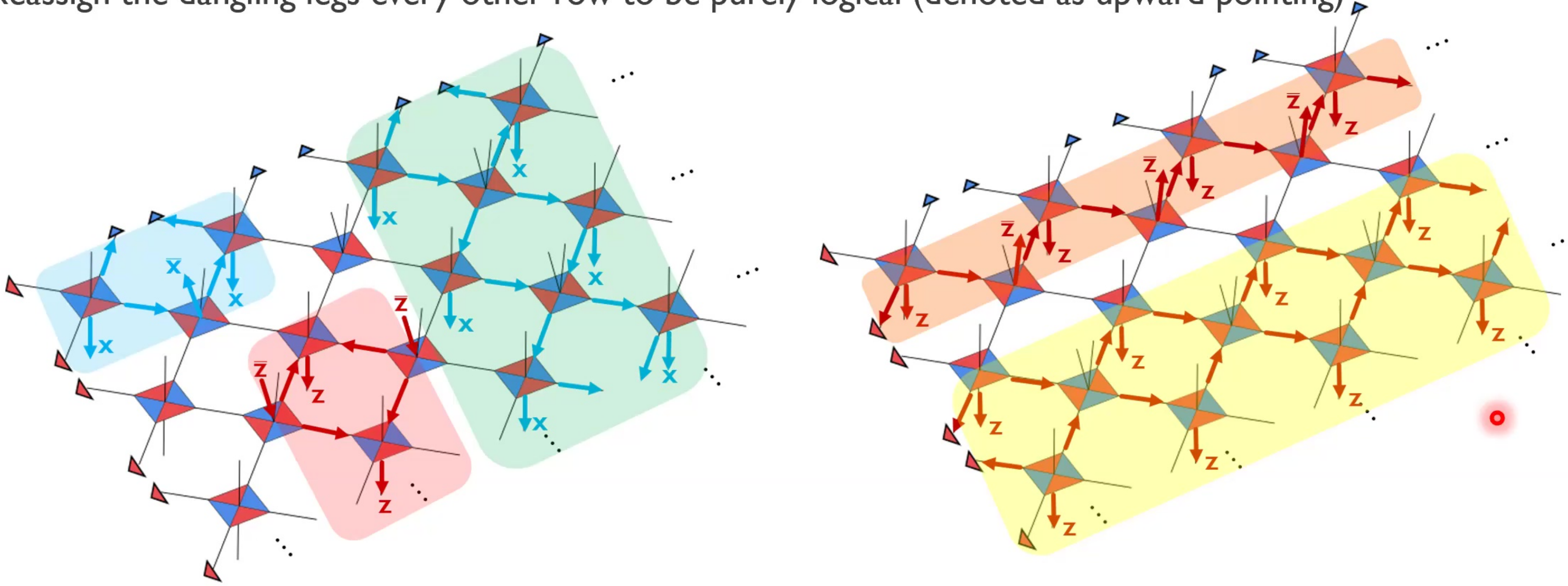

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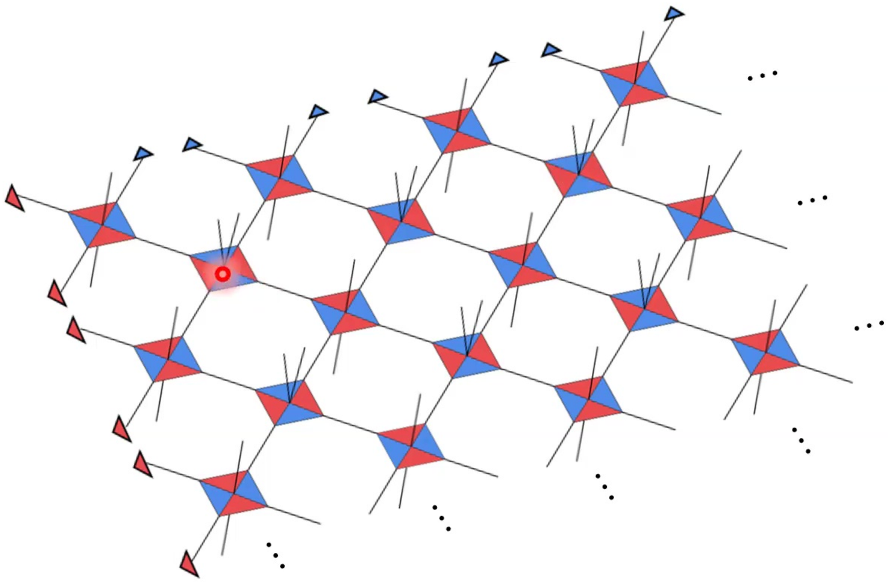
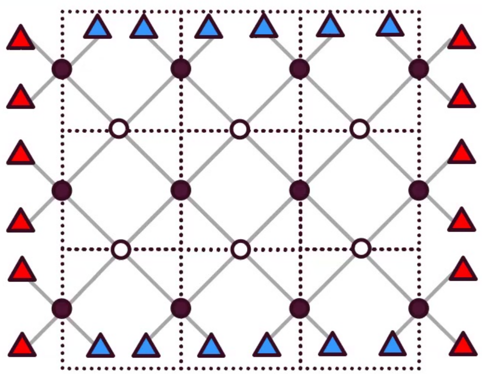
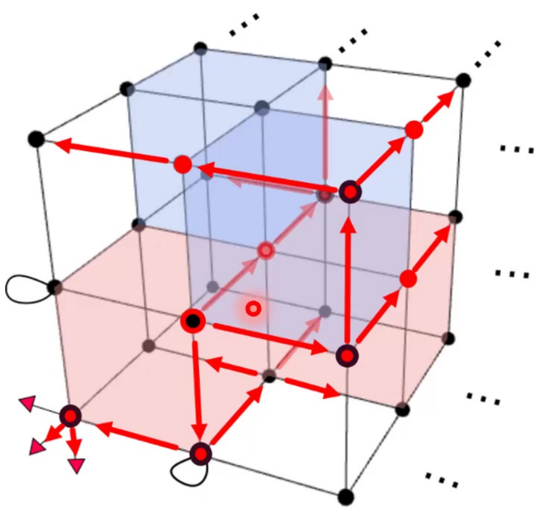
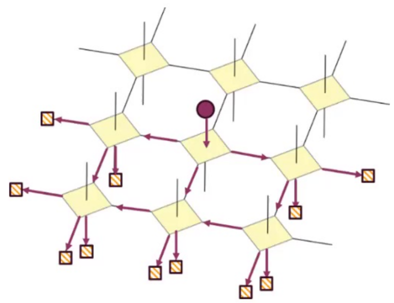


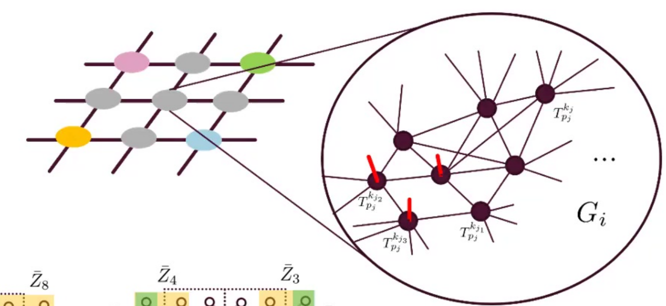


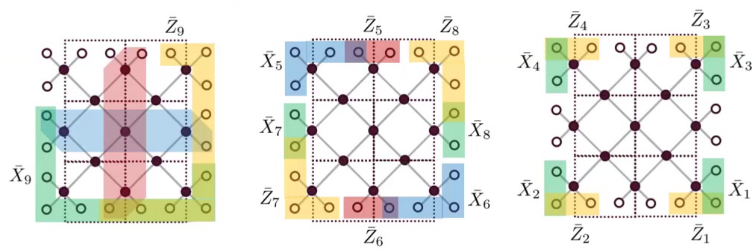
# MAKING NEW CODES

- These are some of the examples one can easily construct with predominantly 1 type of quantum lego block tied to the [[4,2,2]] code
- The framework is capable of producing a far wider variety of codes



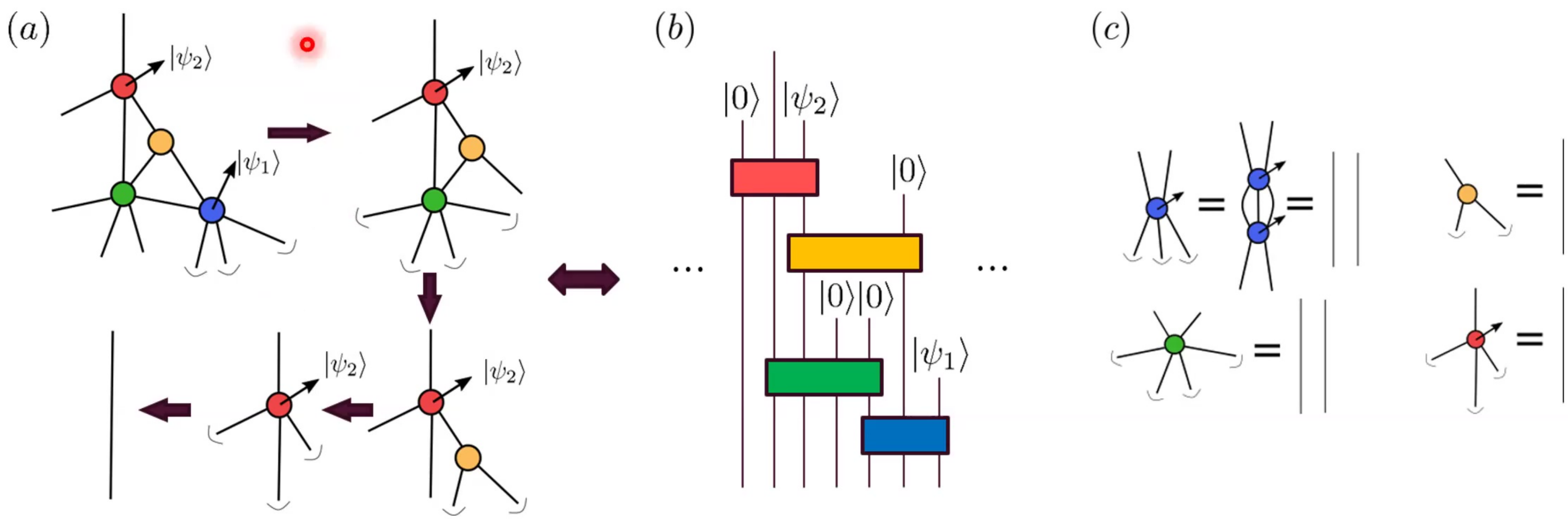




# DECODING

- Tensor network contraction maps to a decoding unitary circuit



# DECODING

- We do not provide a comprehensive decoding procedure as the quantum lego (QL) can describe a wide variety of QECCs

$$\mathrm{QL} \supset \mathrm{QbQLC} \supset \mathrm{TNC} \supset \mathrm{HSbC}$$

- Some of their subclasses do admit known decoders
  - E.g. certain Qubit Quantum Lego codes (QbQLC) can use the techniques by Ferris Poulin to compute Q(E|s) efficiently
  - Tensor network codes (TNC) are QLC with restricted forms of contractions, they admit a maximal likelihood decoder (Ferrally et. al. 2020)
  - Holographic stabilizer codes (HSbC) (Harris et. al.)

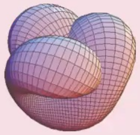
# SUMMARY AND REMARKS

- We constructed a graphically intuitive framework for building and studying QECCs
  - Valid for qubits and qudits
  - Applicable for non-stabilizer codes
  - Allow easily tractable derivation of code properties with operator flows
- Used TN to enhance the connection between existing codes
- Built new codes

# NICE FEATURES

**Flexible code designer toolbox**

Generalizes code concatentation
Tailor-made codes for different architectures/asymmetric errors
Allows easy modification and construction of code variants
Efficiently contractible TNs yields explicit decoding unitary circuit



**Geometrical Representation**

Re-interpret known codes geometrically
Recast into TN language for better connections with CMT, HEP, QG


**Accessibility**

Based on operator matching: anyone can contribute!
Can we teach a machine to play with quantum lego (QECC-ML)?
Efficiently generated - polynomial time algo (stabilizer codes)

# FUTURE WORK AND REMARK

- Take existing framework and play with it
  - Make more codes
  - Study existing codes with TN geometry
  - Variants of existing codes with nice properties
- Limitations and potential of quantum lego
  - Further development of the framework, tool sets
  - Code properties from TN
  - Decoding (circuit and algorithms)
- Reinforcement Learning and qLego
- Connections with CMT, QG, HEP

Thank you!