Title: Large-N solvable models of measurement-induced criticality

Speakers: Subhayan Sahu

Series: Quantum Matter

Date: November 23, 2021 - 3:30 PM

URL: https://pirsa.org/21110031

Abstract: Competition between unitary dynamics that scramble quantum information non-locally and local measurements that probe and collapse the quantum state can result in a measurement-induced entanglement phase transition. We introduce large-N Brownian hybrid circuits acting on clusters of qubits, which provide an analytically tractable model for measurement-induced criticality. The system is initially entangled with an equal sized reference, and the subsequent hybrid system dynamics either partially preserves or destroys this entanglement depending on the measurement rate. Our approach can access a variety of entropic observables, which are represented as a replica path integral with twisted boundary conditions. Saddle-point analysis reveals a second-order phase transition corresponding to replica permutation symmetry breaking below a critical measurement rate. The transition is mean-field-like and we characterize the critical properties near the transition in terms of a simple Ising field theory in 0+1 dimensions. By coupling the large-N clusters on a lattice, we also extend these solvable models to study the effects of power-law long-range couplings on measurement-induced phases. In one dimension, the long-range coupling is relevant for ?<3/2, with ? being the power-law exponent, leading to a nontrivial dynamical exponent at the measurement-induced phase transition. More interestingly, for ?<1 the entanglement pattern receives a sub-volume correction for both area-law and volume-law phases. The volume-law phase for ?<1 realizes a novel quantum error correcting code whose code distance scales as L^(2-2?).

References:
[1] Phys. Rev. B 104, 094304 (2021), ArXiv:2104.07688.
[2] ArXiv:2109.00013.

Large-N solvable models of measurement-induced criticality

Subhayan Sahu

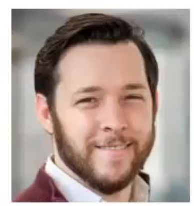
Quantum Matter Seminar, Perimeter Institute

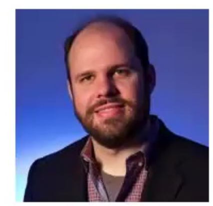
Nov 23, 2021

Work with Shao-Kai Jian, Gregory Bentsen and Brian Swingle

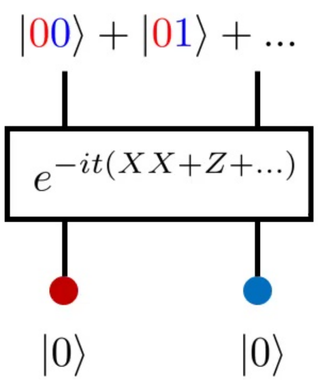
2104.07688 (PRB) - Bentsen, SS*, Swingle*

2109.00013 - SS, Jian*, Bentsen, Swingle*



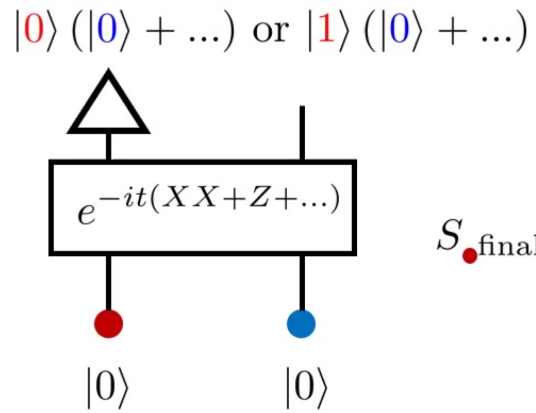


Killing entanglement one local measurement at a time



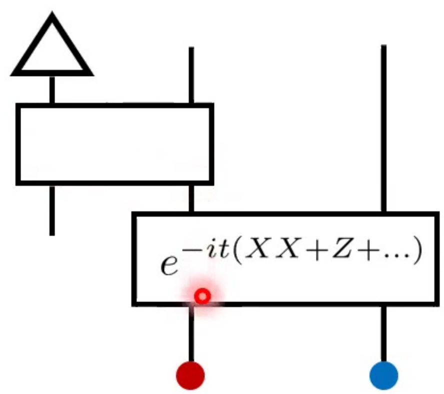
$S = -\mathrm{Tr}(\rho \ln \rho)$

$S_{\bullet \text{final}} > 0$



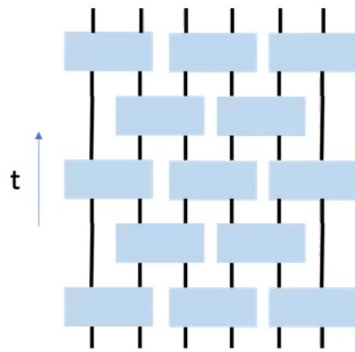
$S_{\bullet \text{final}} = 0$

Generic unitaries create entanglement, and local measurements (projective or weak) reduce entanglement.

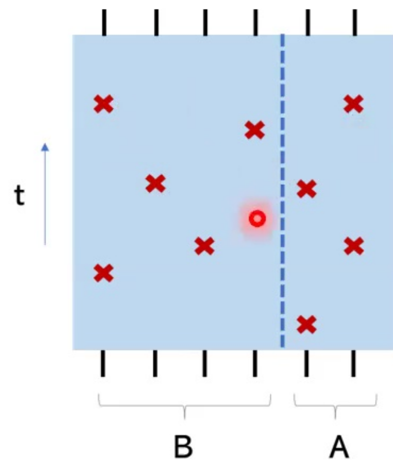


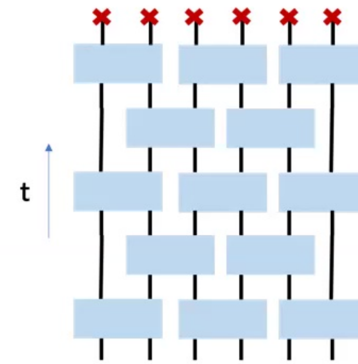
Measurement induced entanglement transition

✖ = Local Projective measurements   = Locality preserving scrambling unitary



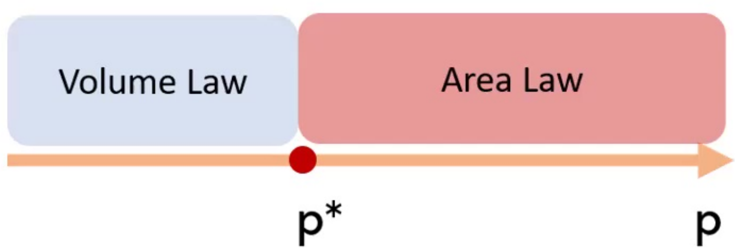
Any subregion has
Volume law entanglement
$S_A \sim A$





Any subregion has
Area law entanglement
$S_A \sim \partial A$

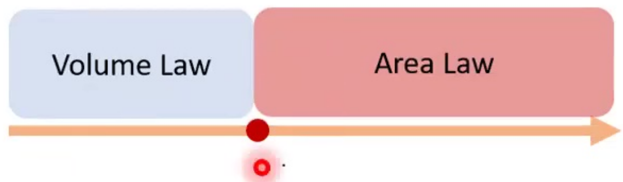
p = probability of measuring in one layer of the circuit



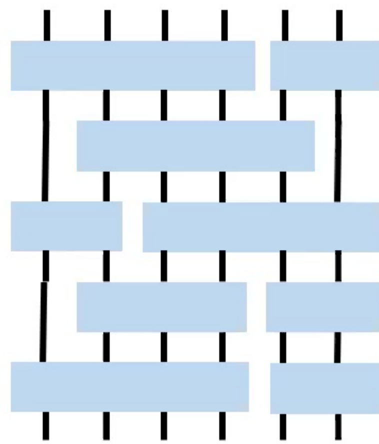
[Li, Chen, Fisher PRB 2018, 2019]
[Skinner, Ruhman, Nahum PRX 2019] …

Goals

- Can we understand the phases and the phase transitions analytically?

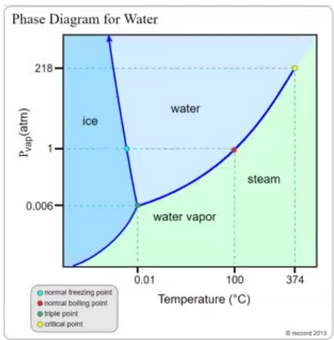


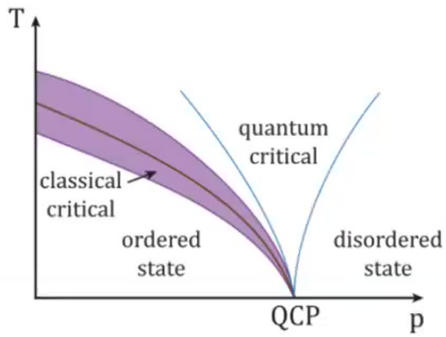
- Are there other dynamical phases possible if we add more ingredients, namely long-range interactions in the unitaries?



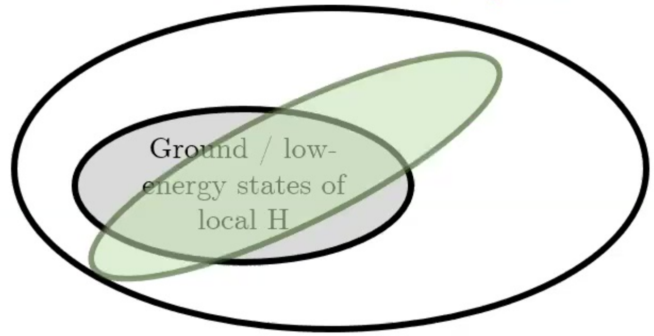
Phases and phase transitions

Condensed matter: study of phases of matter and phase transitions between them



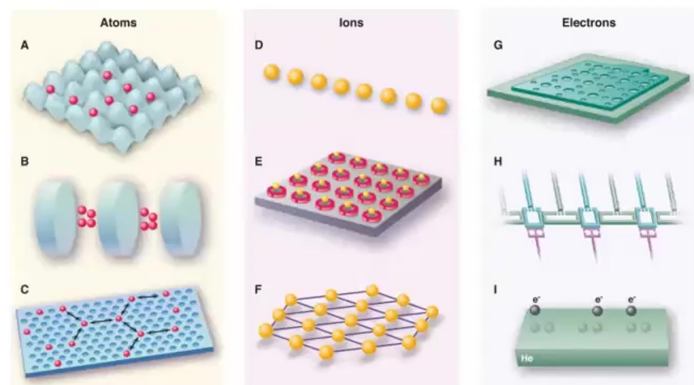


Equilibrium phases of local/simple Hamiltonians cover a small corner of the Hilbert space



Two relatively newer paradigms
(motivated by quantum information and quantum simulators)

- Dynamically generated quantum phases
- Entanglement phases and phase transitions



Outline

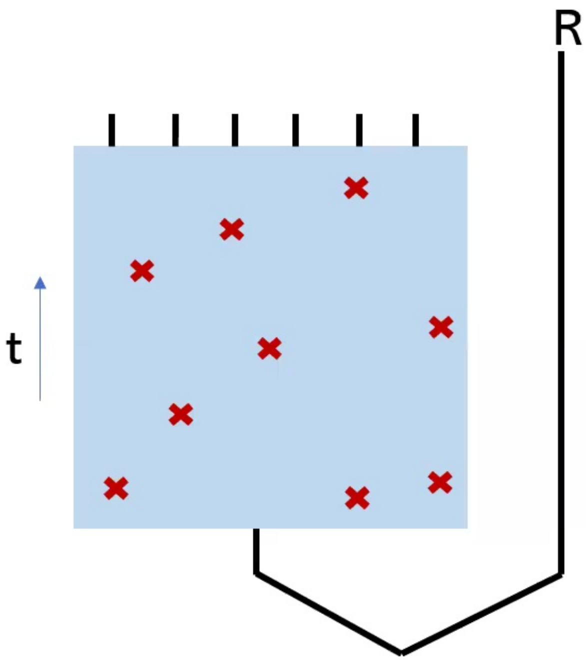
1. Setup
   a) Purification transition
   b) Brownian circuits on qubits
   c) Averaging of Renyi entropies over trajectories

2. Analytically accessible MIPT in the Brownian circuits
   a) Replica structure
   b) Replica symmetry breaking, MIPT, critical properties

3. Brownian circuits on a lattice with long-range couplings
   a) Effective statistical mechanical model
   b) Entanglement phase diagram
   c) Quantum error correcting properties

4. Future directions and Summary

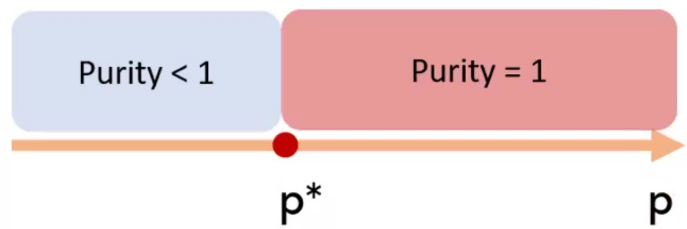
Measurement induced purification transition



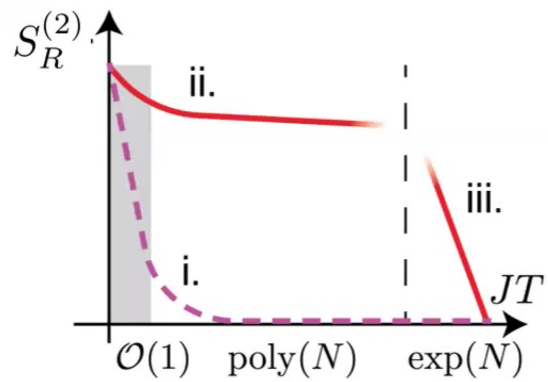
$$\text{Purity}_R(t) = \mathrm{Tr}\left(\rho_R^2(t)\right) = e^{-S_R^{(2)}}$$

$$\text{Rényi entropy: } S_R^{(n)} = \frac{1}{1-n}\ln \mathrm{Tr}\rho_R^n$$

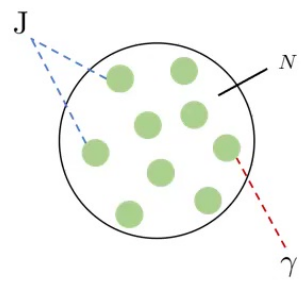
At times $T \sim poly(N)$



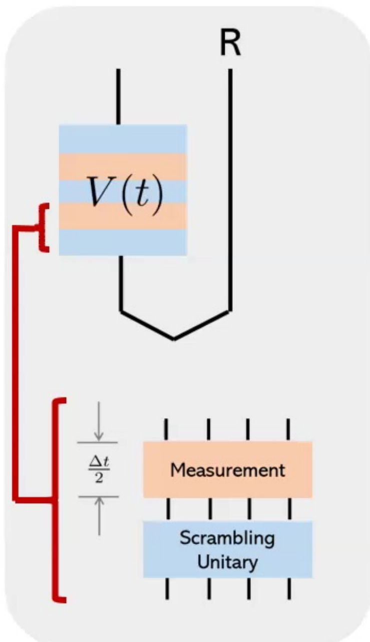
[Gullans, Huse PRX 2020] …



Spherical cow for MIPT: All to All Brownian circuit model

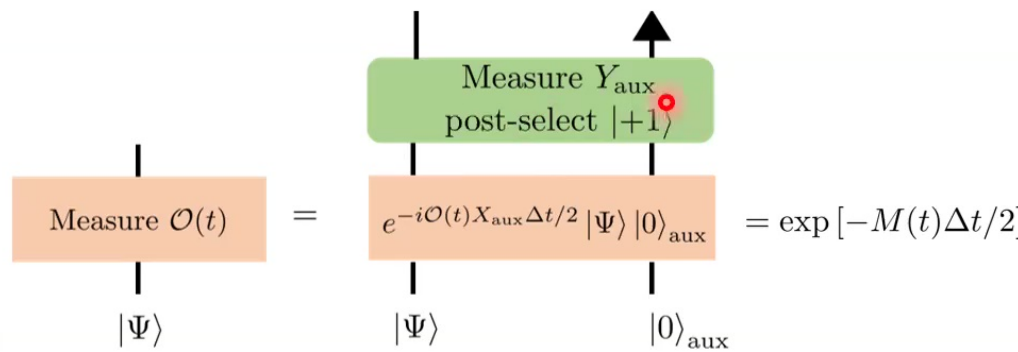


N qubits with all-to-all interaction



Scrambling Unitary $= \exp[-iH(t)\Delta t/2]$

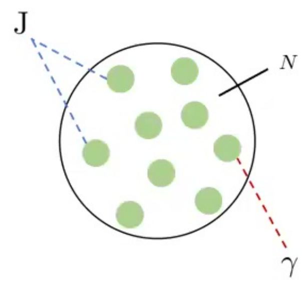
$$H(t) = \sum_{i\alpha} J^{\alpha_1\alpha_2}_{i_1 i_2}(t)\ S^{\alpha_1}_{i_1} S^{\alpha_2}_{i_2}$$



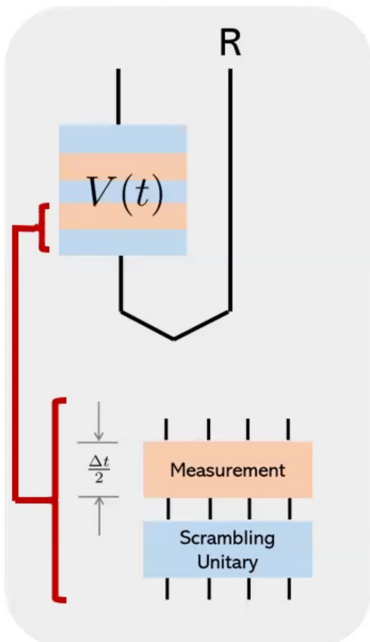
$$\mathcal{O}(t) = n_\alpha(t) S^\alpha$$

Forced measurement due to post-selected trajectories

Spherical cow for MIPT: All to All Brownian circuit model



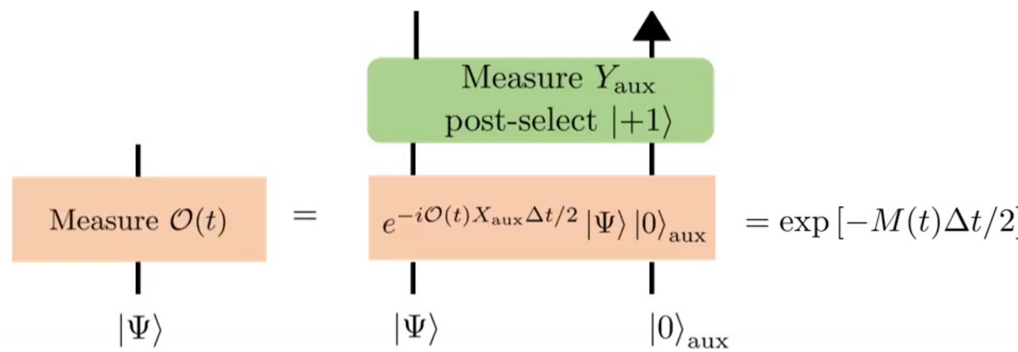
N qubits with all-to-all interaction



Scrambling Unitary $= \exp[-iH(t)\Delta t/2]$

$$H(t) = \sum_{i\alpha} J^{\alpha_1\alpha_2}_{i_1 i_2}(t)\; S^{\alpha_1}_{i_1} S^{\alpha_2}_{i_2}$$

$$\mathbb{E}\left[J(t)J(t')\right] \sim J\delta_{tt'}$$



$$\mathcal{O}(t) = n_\alpha(t) S^\alpha$$

$$\mathbb{E}\left[\mathcal{O}(t)\mathcal{O}(t')\right] \sim \gamma\delta_{tt'}$$

Forced measurement due to post-selected trajectories

Renyi entropy and circuit averaging

Measurements result in different trajectories / Kraus operators, $\rho_t = \sum_a K_a \rho_0 K_a^\dagger$

Post-selected circuit is a fixed non-unitary transformation, resulting in the un-normalized state,

$$\rho_t = V \rho_0 V^\dagger$$

We want to calculate the averaged Renyi entropy, which is hard to do analytically – have to rely on numerics [average of ratio]

$$S^{(2)} = -\mathbb{E}_{J,a} \ln \left[\frac{\mathrm{Tr}\rho_t^2}{\left(\mathrm{Tr}\rho_t\right)^2} \right] \underbrace{=}_{\text{post}-\text{selection}} -\mathbb{E}_J \ln \left[\frac{\mathrm{Tr}\rho_t^2}{\left(\mathrm{Tr}\rho_t\right)^2} \right]$$

We instead calculate a related entropic quantity, called quasi-Renyi entropy, which is easier to calculate analytically [ratio of averages]

$$\hat{S}^{(2)} = -\ln \left[\frac{\mathbb{E}_J \mathrm{Tr}\rho_t^2}{\mathbb{E}_J \left(\mathrm{Tr}\rho_t\right)^2} \right]$$

As experiments, both are hard to simulate [equal scaling of quantum resources]

Renyi entropy and circuit averaging

Measurements result in different trajectories / Kraus operators, $\rho_t = \sum_a K_a \rho_0 K_a^\dagger$

Post-selected circuit is a fixed non-unitary transformation, resulting in the un-normalized state,

$$\rho_t = V \rho_0 V^\dagger$$

We want to calculate the averaged Renyi entropy, which is hard to do analytically – have to rely on numerics [average of ratio]

$$S^{(2)} = -\mathbb{E}_{J,a} \ln \left[\frac{\mathrm{Tr}\rho_t^2}{\left(\mathrm{Tr}\rho_t\right)^2} \right] \underbrace{=}_{\text{post}-\text{selection}} -\mathbb{E}_J \ln \left[\frac{\mathrm{Tr}\rho_t^2}{\left(\mathrm{Tr}\rho_t\right)^2} \right]$$

We instead calculate a related entropic quantity, called quasi-Renyi entropy, which is easier to calculate analytically [ratio of averages]

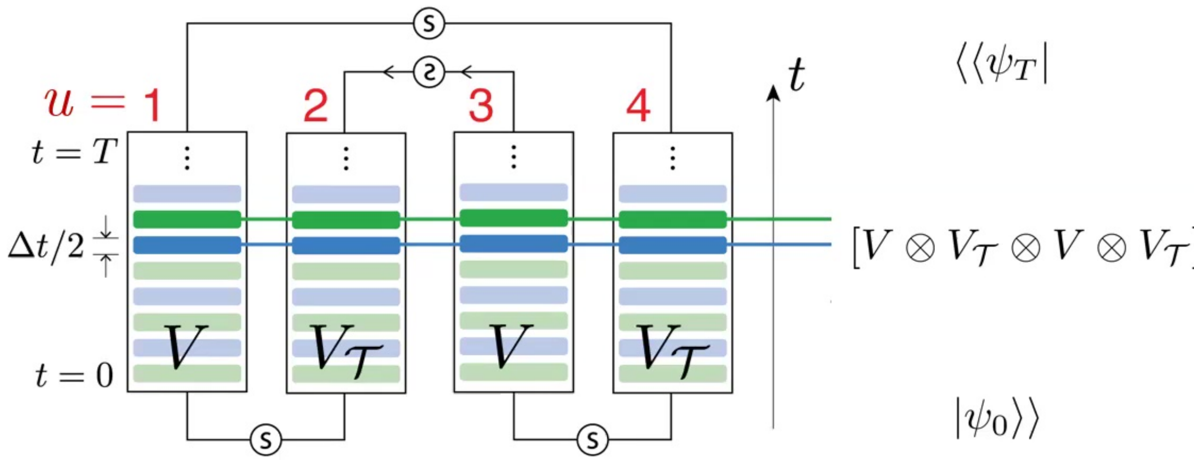
$$\hat{S}^{(2)} = -\ln \left[\frac{\mathbb{E}_J \mathrm{Tr}\rho_t^2}{\mathbb{E}_J \left(\mathrm{Tr}\rho_t\right)^2} \right]$$

As experiments, both are hard to simulate [equal scaling of quantum resources]

Replica structure of Renyi entropy

$$e^{-\hat{S}^{(2)}} = \frac{\mathbb{E}\mathrm{Tr}\rho_t^2}{\mathbb{E}\left(\mathrm{Tr}\rho_t\right)^2} = \frac{\mathbb{E}\mathrm{Tr}\ \mathrm{SWAP}_t\left(\rho_t \otimes \rho_t\right)}{\mathbb{E}\mathrm{Tr}\left(\rho_t \otimes \rho_t\right)} = \frac{\mathbb{E}\ \mathrm{Tr}V\rho_0 V^\dagger V \rho_0 V^\dagger}{\mathbb{E}\ \left(\mathrm{Tr}V\rho_0 V^\dagger\right)\left(TrV\rho_0 V^\dagger\right)}$$

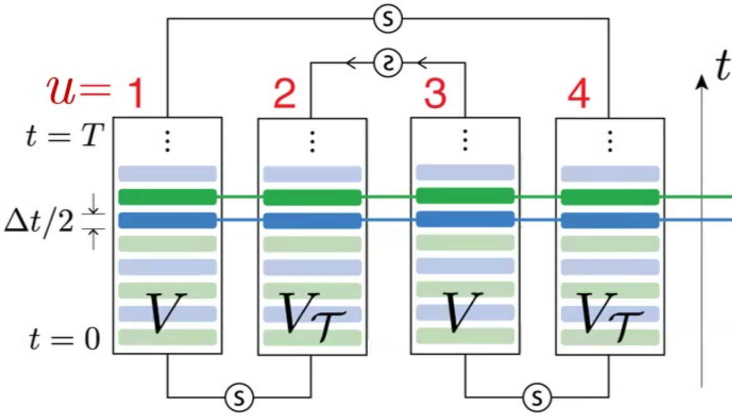
$$\mathbb{E}\ \mathrm{Tr}\underbrace{V}_{u=1}\rho_0\underbrace{V^\dagger}_{u=2}\underbrace{V}_{u=3}\rho_0\underbrace{V^\dagger}_{u=4} =$$



- Boundary conditions separate the numerator and denominator.
- There is an in-built replica permutation symmetry, for example $1 \leftrightarrow 3,\ 2 \leftrightarrow 4$
  Explicit replica permutation symmetry $(S_2 \times S_2) \rtimes Z_2$

Disorder averaging and path integral

Circuit averaging introduces inter-replica correlations



$$\langle U \otimes U_{\mathcal{T}} \otimes U \otimes U_{\mathcal{T}} \rangle_{\mathbf{J}} = e^{-N I_U \Delta t}, \quad I_U \sim J \left(\frac{1}{N} \sum_i S_i^u \cdot S_i^v \right)^2$$

$$\langle M \otimes M_{\mathcal{T}} \otimes M \otimes M_{\mathcal{T}} \rangle_{\mathbf{n}} = e^{-N I_M \Delta t}, \quad I_M \sim \gamma \underbrace{\left(\frac{1}{N} \sum_i S_i^u \cdot S_i^v \right)}_{\text{collective fields } G_{uv}}$$

intra-replica coupling $\rightarrow$ inter-replica Hubbard Stratonovich fields F_{uv}

$$\mathbb{E}\mathrm{Tr}\rho^2,\ \mathbb{E}\left(\mathrm{Tr}\rho\right)^2 \sim \int_{|\psi_0\rangle}^{|\psi_T\rangle} \left(\prod_{i,u} \mathcal{D} S_i^u \right) \exp\left(-N\left(I_U + I_M\right)\right) = \int \mathcal{D}G\mathcal{D}F \exp\left[-N f\left(F_{uv}, G_{uv}\right)\right]$$

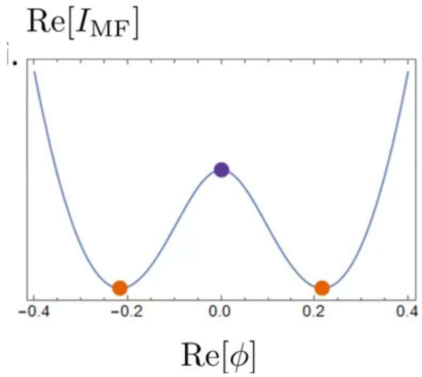
Large N $\rightarrow$ Saddle Point calculation

[SYK, Mean field spin glasses]
...

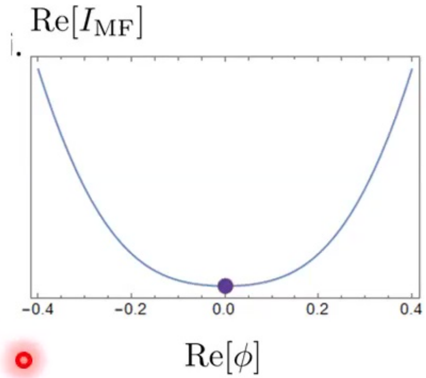
Replica Symmetry Breaking

$\mathbb{Z}_2$ subgroup of the replica symmetry is spontaneously broken for $\gamma < \gamma_c$

$\boxed{\gamma < \gamma_c}$



$\boxed{\gamma > \gamma_c}$



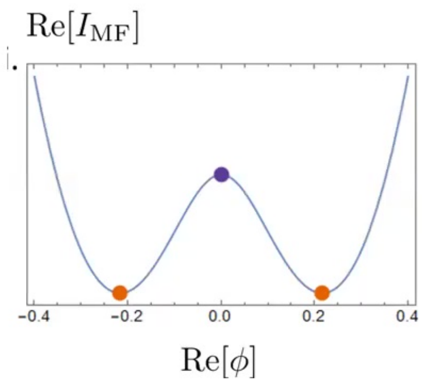
Replica-symmetry breaking field: $\phi \sim F_{12} + F_{34} - F_{14} - F_{23}$

[Parisi, 2021 Nobel]

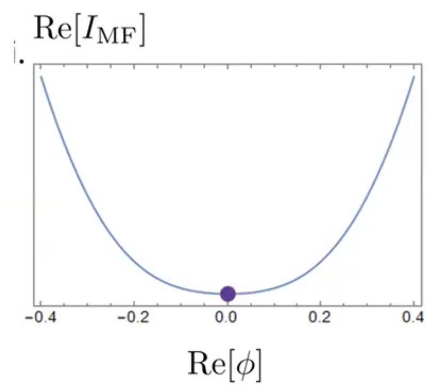
Replica Symmetry Breaking

$\mathbb{Z}_2$ subgroup of the replica symmetry is spontaneously broken for $\gamma < \gamma_c$

$\boxed{\gamma < \gamma_c}$



$\boxed{\gamma > \gamma_c}$



Replica-symmetry breaking field: $\phi \sim F_{12} + F_{34} - F_{14} - F_{23}$

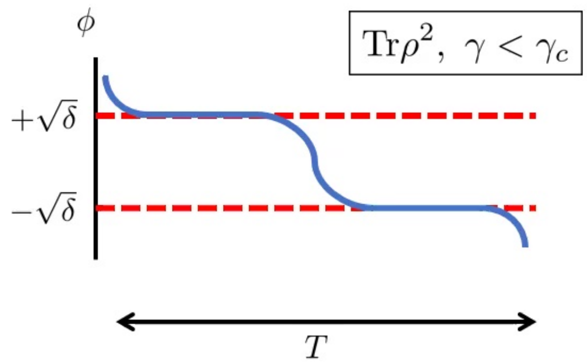
Close to the critical point, simple (0+1)D Ising field theory

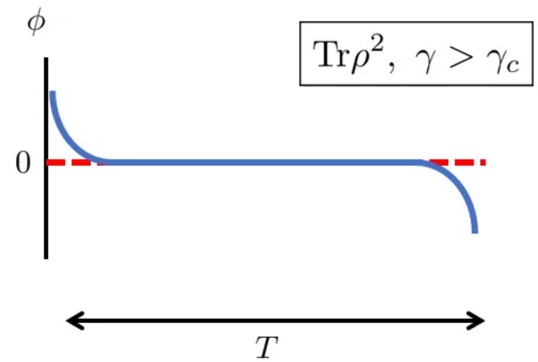
$$I = \int dt \left(\frac{1}{2}(\partial_t \phi)^2 - \delta \frac{\phi^2}{2} + \frac{\phi^4}{4} \right), \; \delta = \gamma_c - \gamma$$

[Parisi, 2021 Nobel]

Boundary effects and Instanton

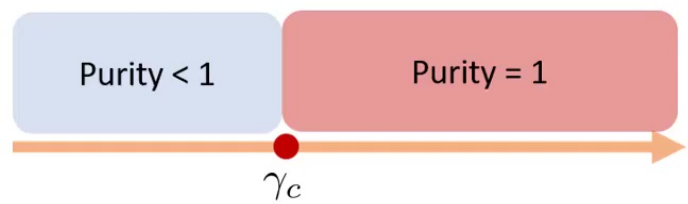
The boundary conditions in the 'spin' problem shows up in the ϕ fields





At $T \sim poly(N)$

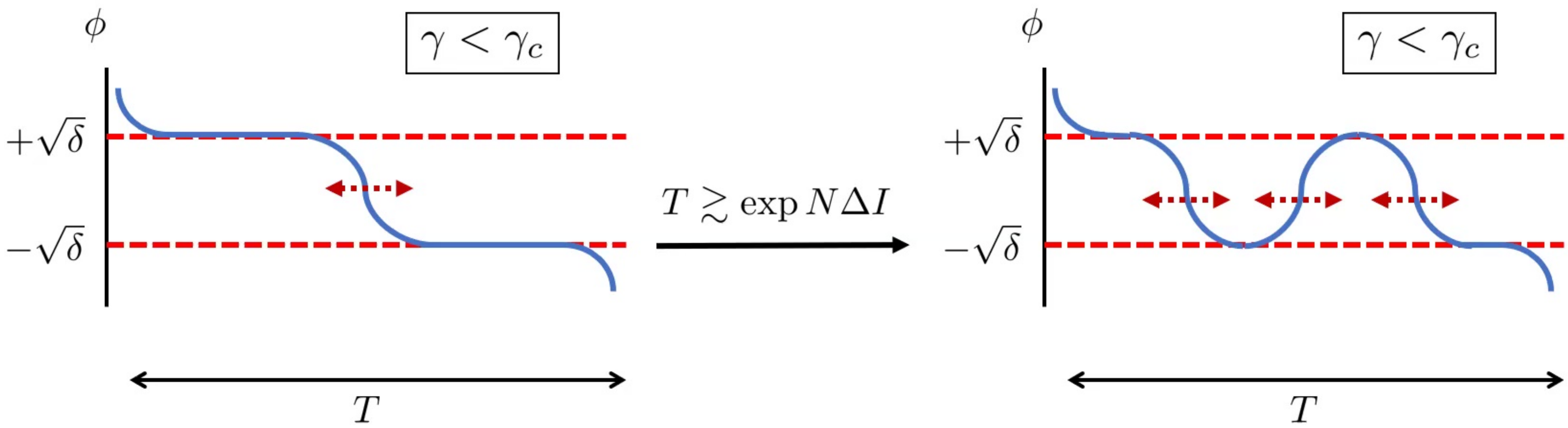
$$\mathbb{E}\ \text{Purity} \sim \begin{cases} T\exp\left(-N\Delta I_{\text{instanton}}\right) & \text{for } \gamma < \gamma_c \\ 1 & \text{otherwise.} \end{cases}$$



critical exponent

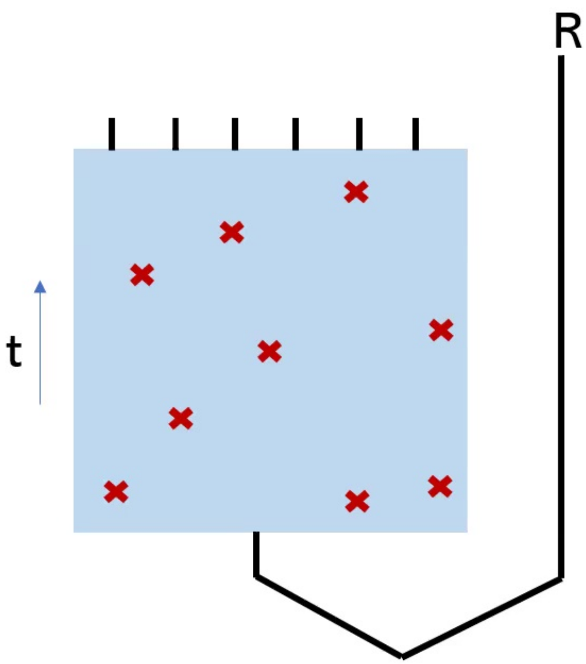
In the (0+1)D Ising field theory, $\Delta I_{\text{instanton}} \sim (\gamma_c - \gamma)^{3/2} \implies \hat{S}^{(2)} \sim (\gamma_c - \gamma)^{3/2}$

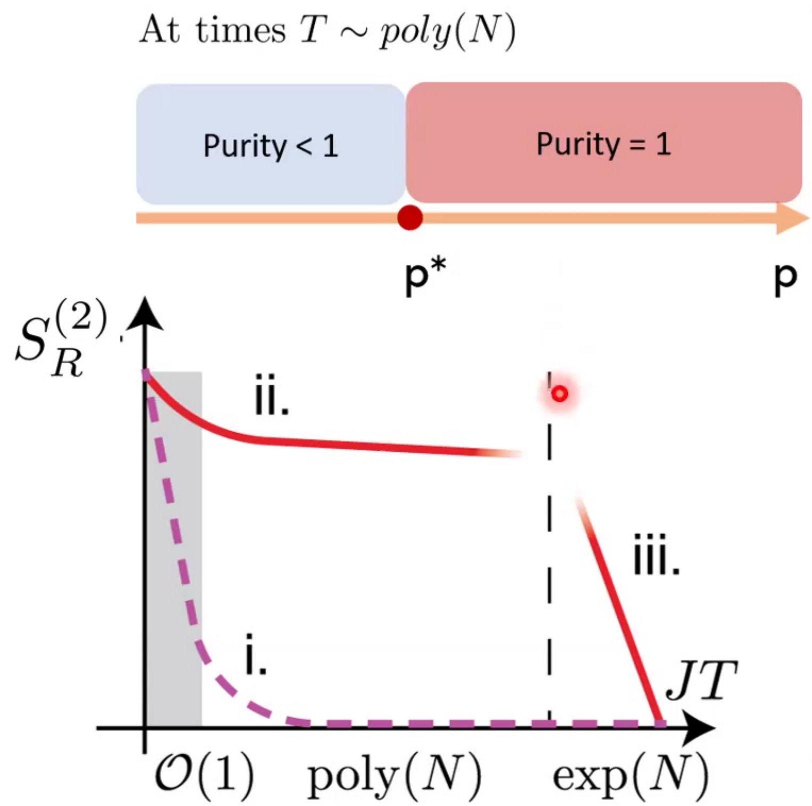
Late time purification



At $T \sim \mathcal{O}(\exp N)$, instantons proliferate, restores replica-symmetry and leads to Purity $= 1$.

Measurement induced purification transition





We obtained the MIPT phenomenology and critical properties in an analytically tractable model

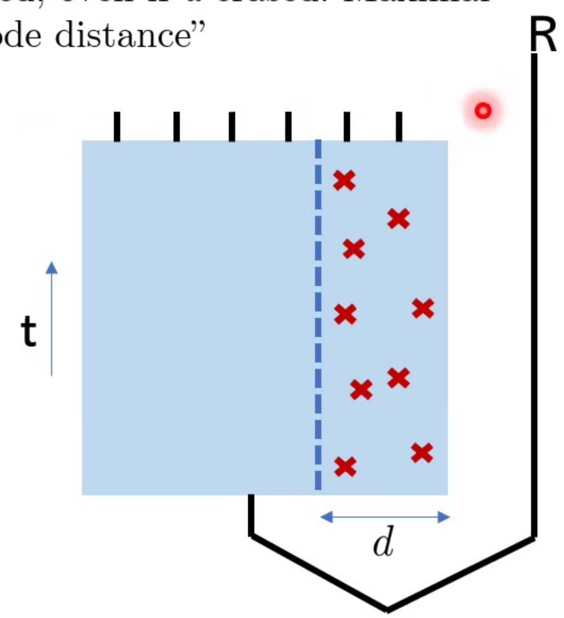
[Bentsen, SS*, Swingle PRB 2021]*

Mixed phase as quantum error correcting code

Entanglement survives in the mixed phase – can be thought of as a dynamically generated quantum error correcting code.

[Choi, Bao, Qi, Altman 2019, Gullans, Huse 2019, Li, Fisher 2020 …]

Information stored in R can be recovered, even if d erased. Maximal d = "code distance"

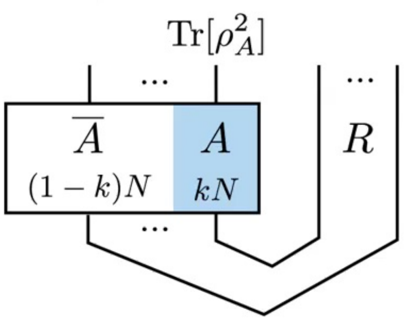


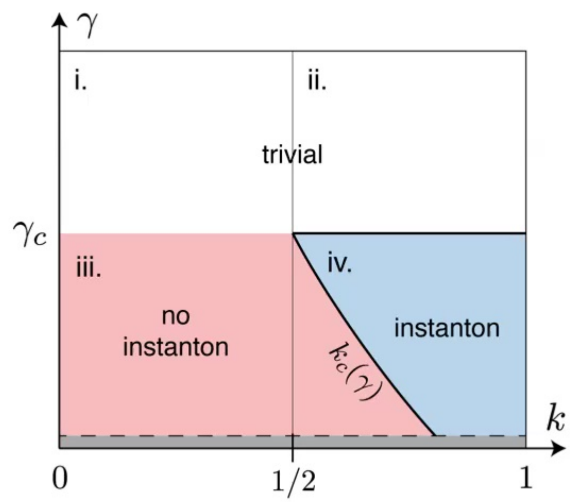
Mixed phase as quantum error correcting code

Entanglement survives in the mixed phase – can be thought of as a dynamically generated quantum error correcting code.

[Choi, Bao, Qi, Altman 2019, Gullans, Huse 2019, Li, Fisher 2020 …]

Subsystem entropy:





$$k_c - \frac{1}{2} \sim (\gamma_c - \gamma)^{\mu}, \text{ for } \gamma \text{ close to } \gamma_c \text{ with } \mu = 1.$$

For $\gamma < \gamma_c$ and $k > k_c$,

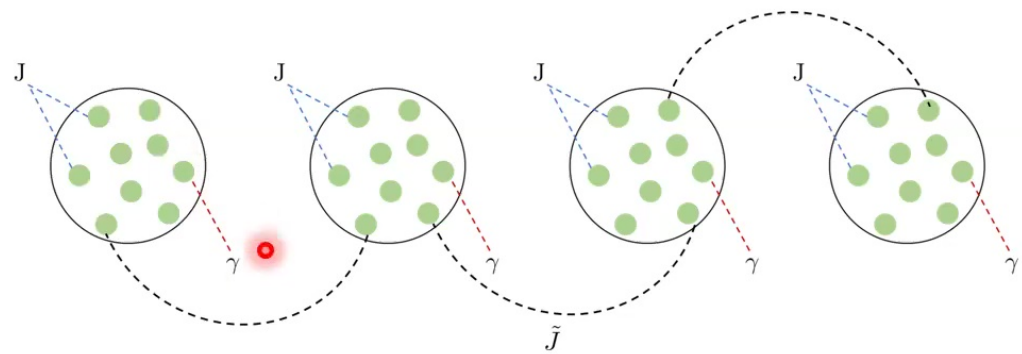
$$\hat{I}^{(2)}(\overline{A} : R) = \hat{S}^{(2)}_{1-k} + \hat{S}^{(2)}_{1} - \hat{S}^{(2)}_{k} \approx 0$$

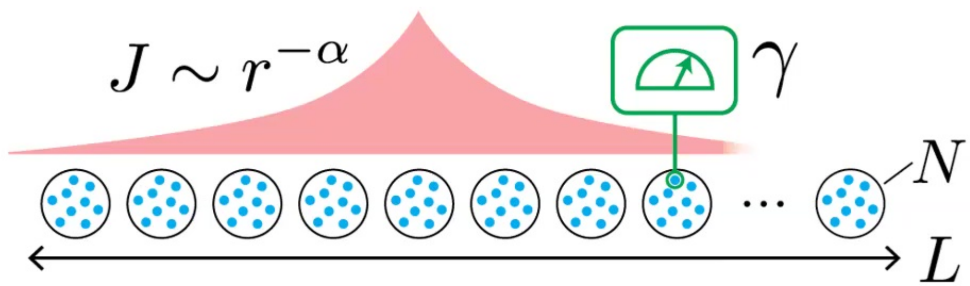
At $T \sim poly(N)$ for $\gamma < \gamma_c$

Code rate $\sim \hat{S}^{(2)}_R \approx N\Delta I$
Code distance $\sim N\,(1 - k_c(\gamma))$

[Bentsen, SS*, Swingle PRB 2021]*

Putting the clusters in a chain



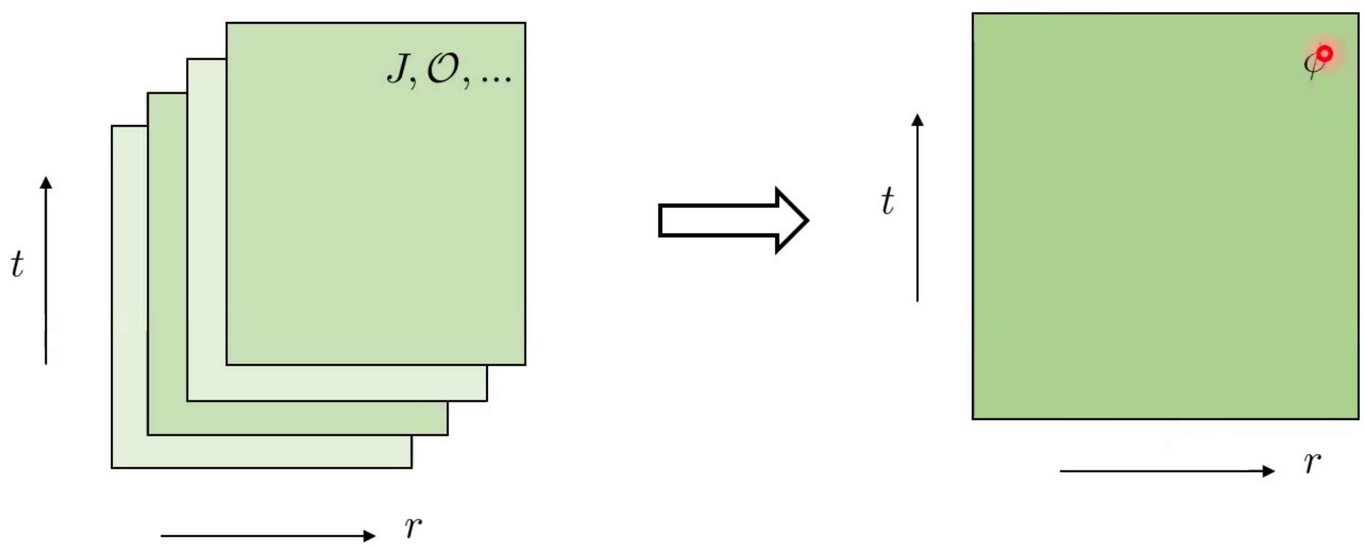


$$\mathbb{E}\, J_r(t) J_s(t') \sim |r-s|^{-2\alpha} \delta_{tt'}$$

Effective statistical mechanics model

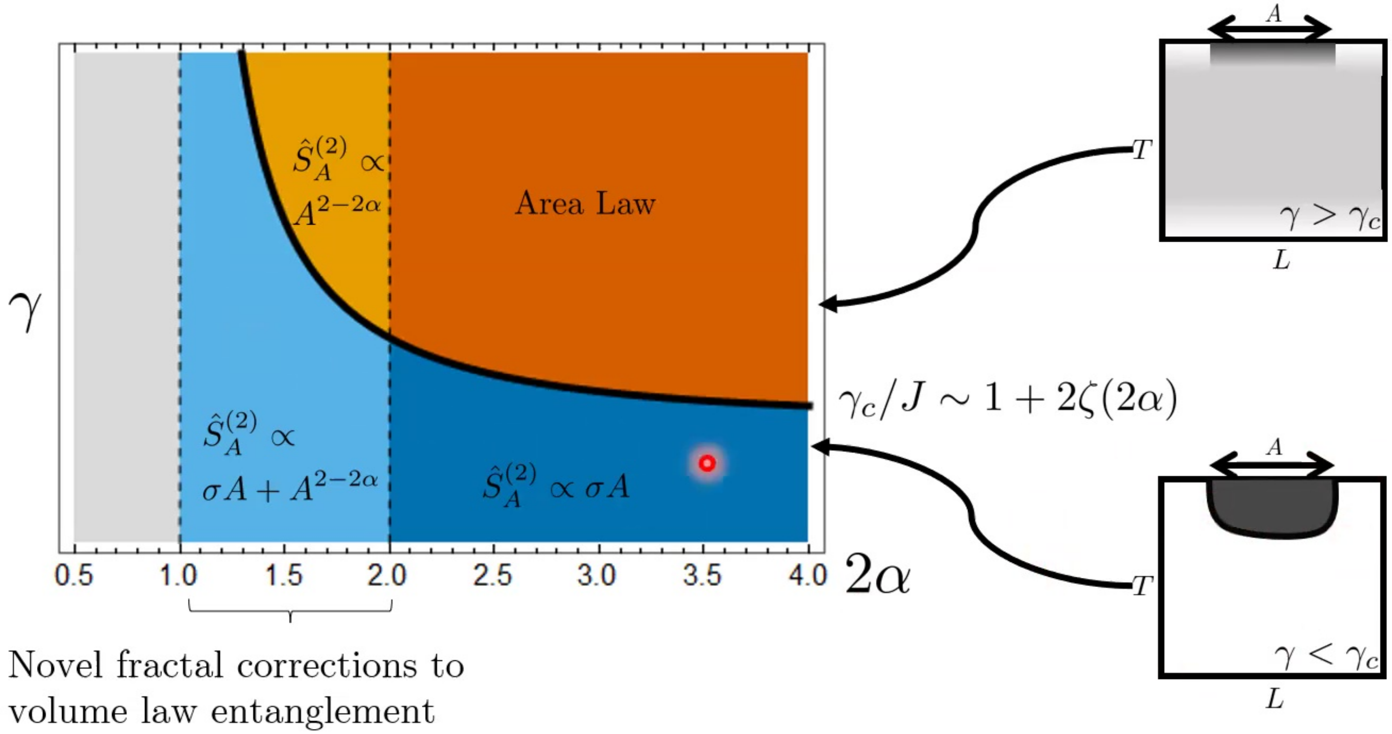
Now, the effective field theory is given by a (1+1)D anisotropic Ising field theory,

$$I \sim \int_{t,r} \left[-\phi \partial_t^2 \phi - \int_s \phi_r \phi_s |r-s|^{-2\alpha} - (\gamma_c - \gamma)\phi^2/2 + \phi^4/4 \right], \quad \phi \sim F_{12} + F_{34} - F_{14} - F_{23}$$



Entanglement phase diagram

$$I \sim \int_{t,r} \left[-\phi \partial_t^2 \phi - \int_s \phi_r \phi_s |r-s|^{-2\alpha} - (\gamma_c - \gamma)\phi^2/2 + \phi^4/4 \right]$$

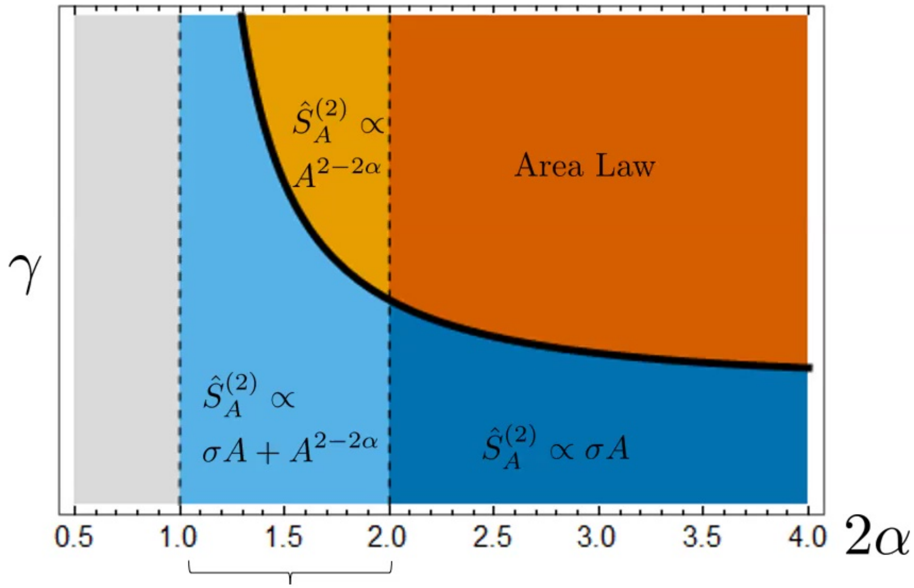


Novel fractal corrections to volume law entanglement

Fractal corrections to the entropy due to the long-range interaction on domain wall

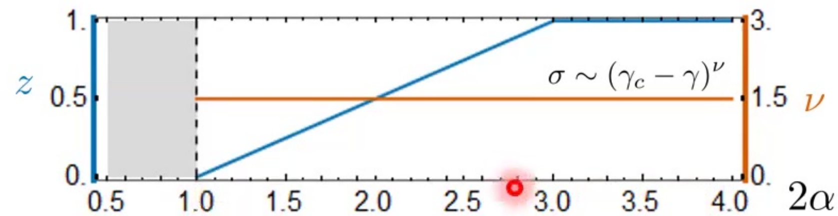
Entanglement phase diagram

$$I \sim \int_{t,r} \left[-\phi \partial_t^2 \phi - \int_s \phi_r \phi_s |r-s|^{-2\alpha} - (\gamma_c - \gamma)\phi^2/2 + \phi^4/4 \right]$$



Novel fractal corrections to volume law entanglement

Critical properties of the transition – non-CFT like for $2\alpha < 3$

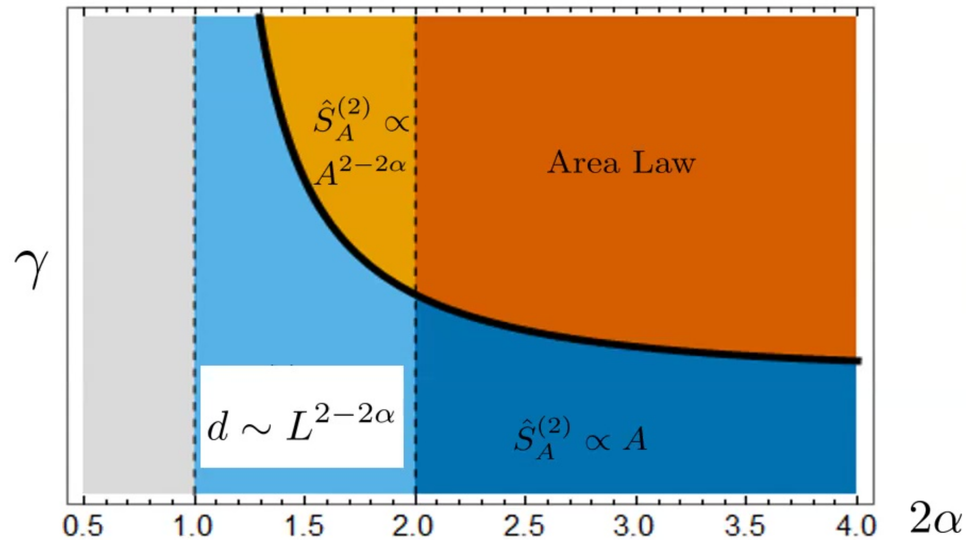


[SS, Jian*, Bentsen, Swingle ArXiv:2109.00013*
Related: Block, Bao, Choi, Altman, Yao 2021]

Quantum error correction in volume law phase

Entanglement survives in the volume law phase – can be thought of as a dynamically generated quantum error correcting code.

$\hat{I}(A_d : R) \approx 0$ for $d < L^{2-2\alpha}$ in the volume-law phase



At $T \sim poly(N)$ for $\gamma < \gamma_c$

Code rate $\sim \hat{S}_R^{(2)} \sim \sigma L$
Code distance $\sim L^{2-2\alpha}$

[SS*, Jian*, Bentsen, Swingle ArXiv:2109.00013]

Current directions

- In all analytical approaches to hybrid dynamics, focus has been on the 'annealed' entropy.

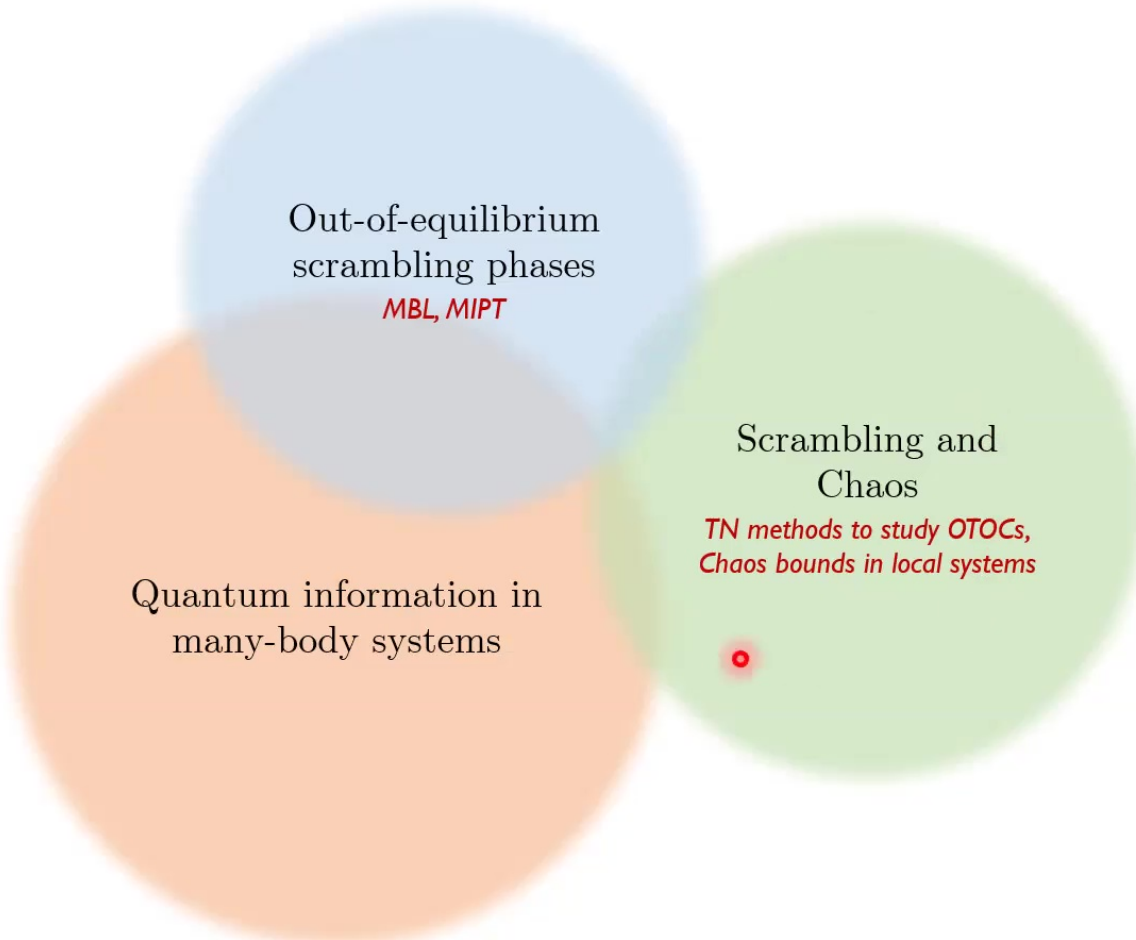
$$\hat{S}^{(2)} = -\ln\left[\frac{\mathbb{E}_J \mathrm{Tr}\rho_t^2}{\mathbb{E}_J \left(\mathrm{Tr}\rho_t\right)^2}\right]$$

Recently the limit $\hat{S}^{(n\to 1)}$ was taken [Jian, Swingle 2021] in a similar Brownian circuit for fermions. Can we take the $n \to 1$ limit for qubits? Can we calculate the 'quenched' entropy?

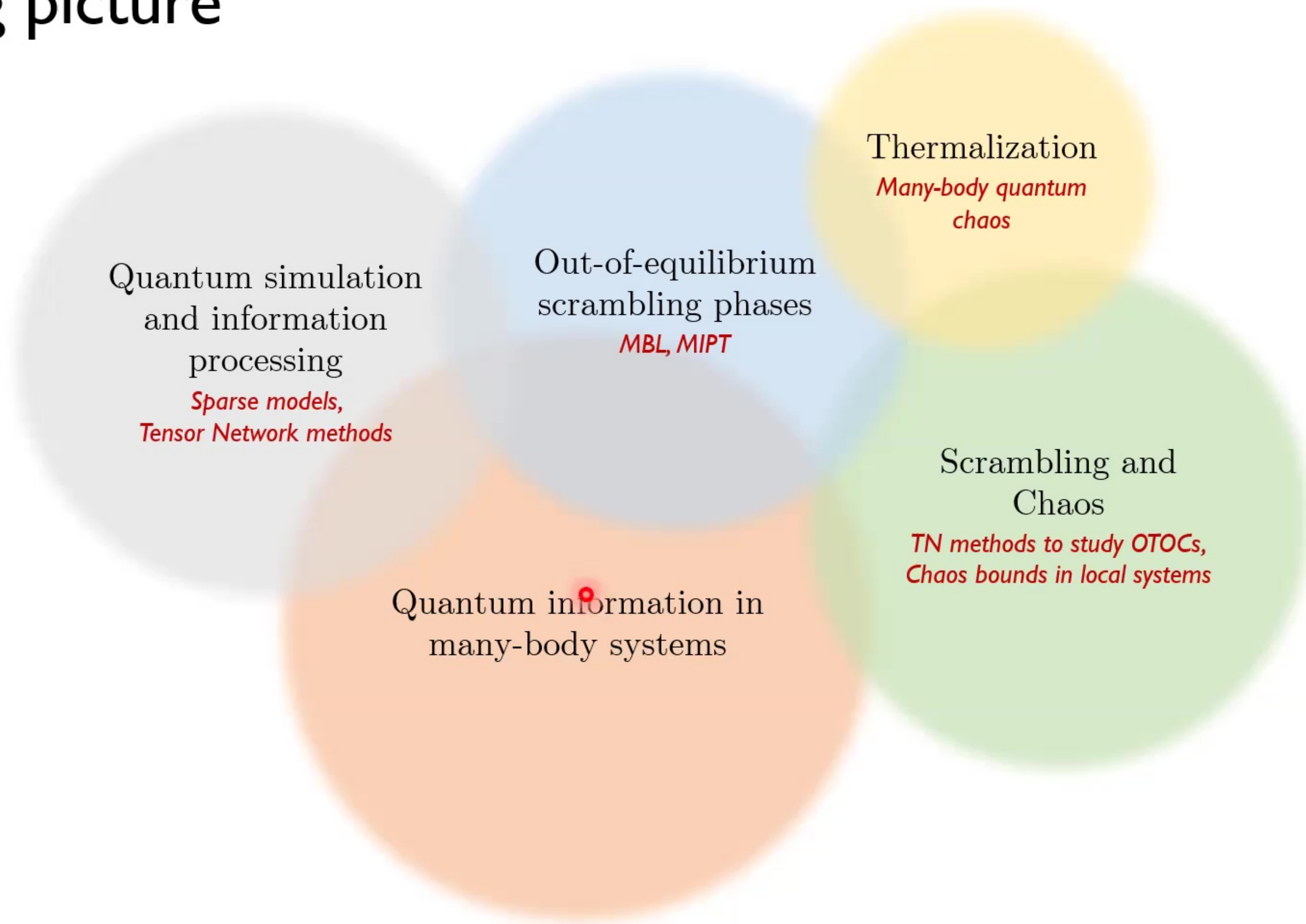
- Entangling (multi-particle) measurement protocols have been shown to harbor novel hybrid entanglement phases [Sang, Hsieh 2020, Lavasani, Alavirad, Barkeshli 2020 ...]. Can we realize them in solvable Brownian circuits?

- Brownian circuits to study other entanglement or spectral transitions, such as MBL? Can we retain solvability while considering a Floquet version?

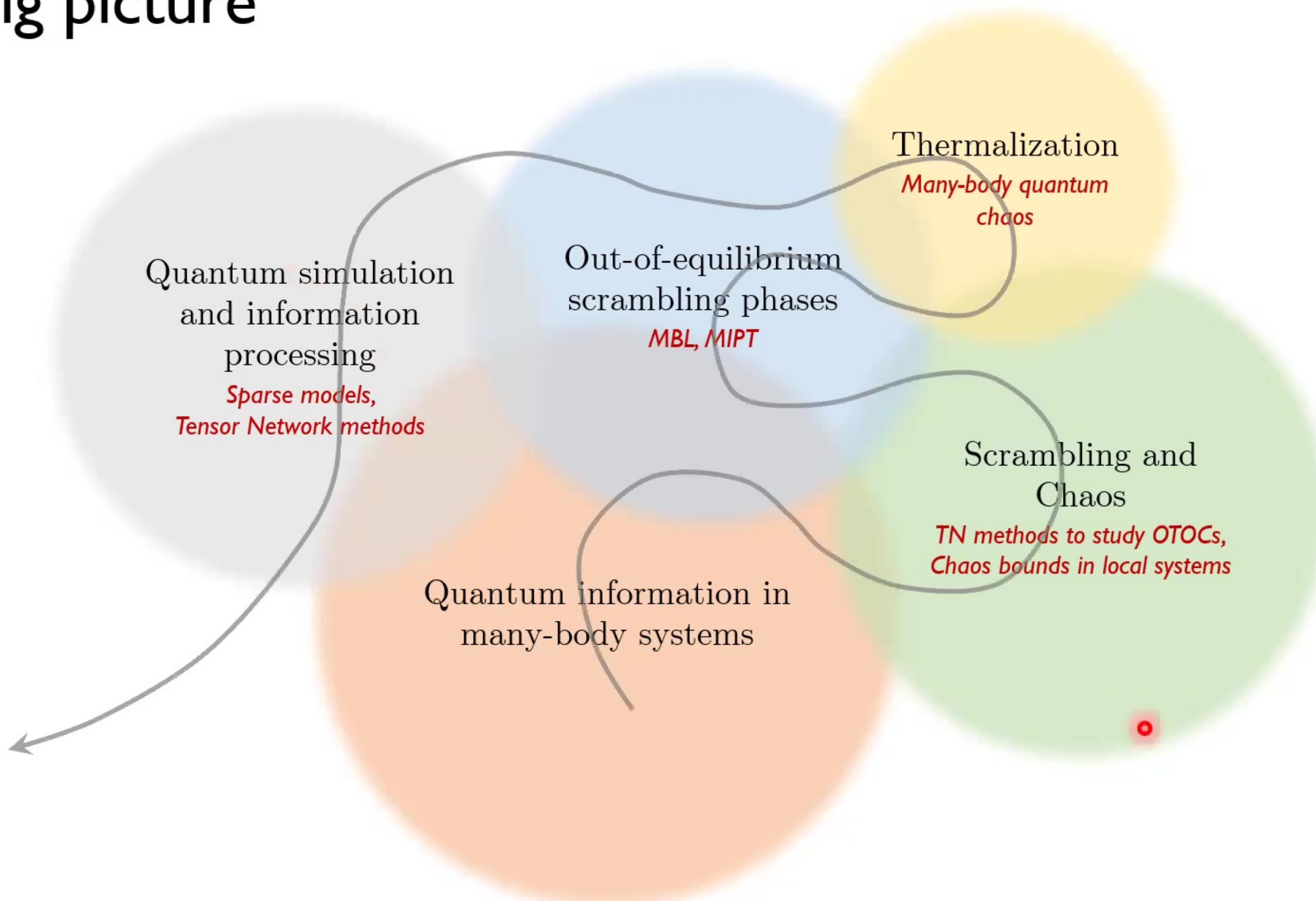
Big picture


Out-of-equilibrium
scrambling phases
MBL, MIPT
Scrambling and
Chaos
TN methods to study OTOCs,
Chaos bounds in local systems
Quantum information in
many-body systems

Big picture


Quantum simulation and information processing
Sparse models,
Tensor Network methods
Out-of-equilibrium scrambling phases
MBL, MIPT
Thermalization
Many-body quantum chaos
Scrambling and Chaos
TN methods to study OTOCs,
Chaos bounds in local systems
Quantum information in many-body systems

Big picture


Thermalization
Many-body quantum chaos
Quantum simulation and information processing
Sparse models,
Tensor Network methods
Out-of-equilibrium scrambling phases
MBL, MIPT
Scrambling and Chaos
TN methods to study OTOCs,
Chaos bounds in local systems
Quantum information in many-body systems

Summary and future questions

- We introduced analytically solvable models for measurement induced criticality.
- Identified replica symmetry breaking in the effective field theory as the mechanism responsible for MIPT. We derive a simple mean field description of the phase transition, with analytically obtained critical exponents.
- We generalize to one dimensional chain with power-law long range couplings. We identify novel entanglement phases, with quantum error correcting properties enhanced by the long-range interaction.
- Replica limit? Entanglement entropy?
- Other interesting, non-equilibrium, entanglement phases with measurement and scrambling? Can we probe MIPT with simpler, more experimentally feasible observables?

Summary and future questions

- We introduced analytically solvable models for measurement induced criticality.
- Identified replica symmetry breaking in the effective field theory as the mechanism responsible for MIPT. We derive a simple mean field description of the phase transition, with analytically obtained critical exponents.
- We generalize to one dimensional chain with power-law long range couplings. We identify novel entanglement phases, with quantum error correcting properties enhanced by the long-range interaction.
- Replica limit? Entanglement entropy?
- Other interesting, non-equilibrium, entanglement phases with measurement and scrambling? Can we probe MIPT with simpler, more experimentally feasible observables?

Thank you!

2104.07688 (PRB 2021) + 2109.00013