Title: Symmetry protected topological order in open quantum systems

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Abstract: I will discuss ongoing work on the robustness of symmetry protected topological (SPT) phases in open systems. By studying the evolution of non-local string order parameters, we find that one-dimensional SPT order is destabilized by couplings to the environment satisfying a weak symmetry condition that directly generalizes from closed systems. We introduce a stronger symmetry condition on channels that ensures SPT order is preserved. SPT phases of mixed states and their transformation under noisy channels is also discussed.

SPTO in Open Quantum Systems

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Topological phases in open systems

Questions

- Which topological phases of pure states are robust against dissipation?
- What are the topological phases of mixed states?

Defining phases

- In closed systems... phase equivalence is defined by

  low depth circuits of local gates (e.g. polylog(L) depth of constant size gates)

- In open systems... Coser & Pérez-García [1810.05092] propose using

  fast, local Lindbladian evolution $\rho \xrightarrow{\mathcal{L}} \rho'$

  $\implies$ defines a partial order on states... in particular, an equivalence

- C&PG motivate this definition. Classification is an open problem.

Symmetric noise destroys 1D SPT order

- 1D SPT phases are classified under this equivalence relation...
- C&PG: Any 1D state has a fast, local Lindbladian evolution to a product state.

$$\mathcal{L} = \sum_s \mathcal{L}_s \ , \qquad \mathcal{L}_s = \mathcal{T}_s - \mathbb{1}_s \ , \qquad \mathcal{T}_s(\rho) = \mathrm{Tr}_s[\rho] |\phi\rangle_s \langle\phi| \ .$$

$\Rightarrow$ Any two SPT orders $\omega_1, \omega_2 \in H^2(G; U(1))$ are connected by...

$$|\omega_1\rangle \sim |\omega_1\rangle \otimes |\omega_2^{-1}\rangle \otimes |\omega_2\rangle \xrightarrow{\mathcal{L}} |\mathrm{prod}\rangle \otimes |\omega_2\rangle \sim |\omega_2\rangle \ .$$

- Solution...? Require that the Lindbladian respect the symmetry.
- However... let impose $U_g|\phi\rangle = |\phi\rangle$. The $\mathcal{L}$ of C&PG satisfies

$$\mathcal{U}_g \circ \mathcal{L} \circ \mathcal{U}_g^\dagger = \mathcal{L} \ . \qquad \text{(weak symmetry)}$$

- Is there a stronger symmetry condition on $\mathcal{L}$ that protects SPT order?

Proposed SPT phase equivalence

- In this talk... I will propose, then motivate, a definition of SPT phase...

  ...based on the idea that order is fundamentally about order parameters.

SPT phases of mixed states

- Definition: Two mixed states belong to the same SPT phase if...

  they are related by a fast, local $\mathcal{L}$ satisfying a strong symmetry condition*

$$U_g H = H U_g \ , \qquad U_g L_i = L_i U_g \ , \qquad \forall \, i, g \ ,$$

where H and L_i are the system Hamiltonian and jump operators appearing in

$$\mathcal{L}(\rho) = -i[H, \rho] + \sum_{i>0} \left(L_i \rho L_i^\dagger - \frac{1}{2} L_i^\dagger L_i \rho - \frac{1}{2} \rho L_i^\dagger L_i \right) \ .$$

Motivating the definition

Main result

- For a class of mixed states called SPT ordered mixed states...

  ...two states belong to the same SPT phase if and only if SPT order parameters take the same values on them.

SPT ordered mixed states

- A mixed state is said to be SPT ordered if SPT order parameters take the same values on it as they do on some SPT pure state.
- The result implies that... pure state SPTO is robust in open systems if and only if the coupling to the environment is strongly symmetric.
- I will not address the classification of mixed states in general.

Motivating the definition

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Channels

- Work with channels $\mathcal{E}$. Have in mind Lindbladian evolutions $\mathcal{E}_t = e^{t\mathcal{L}}$.
- $\mathcal{E}$ is a trace-preserving, completely positive map $\rho \mapsto \mathcal{E}(\rho)$.
- Kraus representation

$$\mathcal{E}(\rho) = \sum_i K_i \rho K_i^\dagger \ , \qquad \sum_i K_i^\dagger K_i = \mathbb{1} \ .$$

- Example: a reversible channel has a single unitary Kraus operator:

$$\rho \mapsto W \rho W^\dagger \ .$$

- $\mathcal{L}$ is weakly symmetric precisely when $\mathcal{E}_t$ satisfies

$$\mathcal{U}_g \circ \mathcal{E} \circ \mathcal{U}_g^\dagger = \mathcal{E} \ . \qquad \text{(weak symmetry)}$$

at all times t.

Interference between trajectories

- Weak symmetry (WS) in terms of Kraus operators:

$$\sum (U_g K_i U_g^\dagger)\, \rho \, (U_g K_i U_g^\dagger)^\dagger = \sum K_i \rho K_i^\dagger \ , \quad \forall g \ .$$

$\Rightarrow$ For each g, there is a basis of Kraus operators K_i^g where

$$U_g K_i^g U_g^\dagger = e^{i\theta_i(g)} K_i^g \ , \quad \forall i \ .$$

- Different phases $\theta_i \neq \theta_j$ give rise to interference between trajectories.

- Strong symmetry (SS): trajectories transform with *the same* phase:

$$U_g K_i U_g^\dagger = e^{i\theta(g)} K_i \ , \quad \forall i, g \ .$$

- This condition is basis-independent, so the superscript g can be dropped.
- For this talk... neglect the weak invariant $\theta(g)$.

- Comment: WS and SS are the same for reversible channels $\rho \mapsto W \rho W^\dagger$.

Strong symmetry of Lindbladians

- Using the relation between the K_i and the jump operators L_i, we see that...

  ...families $\mathcal{E}_t = e^{t\mathcal{L}}$ of SS channels are generated by SS Lindbladians:

$$U_g H = H U_g \ , \qquad U_g L_i = L_i U_g \ , \qquad \forall\, i, g \ .$$

- The SS condition on $\mathcal{L}$ has appeared previously. [Buča , Prosen 12] [Albert , Jiang 13]

- The Lindbladian of C&PG, which destroys SPT order, is WS but not SS:

  Its jump operators are

$$L_{s,i} = |\phi\rangle_s\langle i| \qquad \Longrightarrow \qquad U_g L_{s,i} \neq L_{s,i} U_g \ .$$

Strong symmetry in the Heisenberg picture

- The Heisenberg picture is convenient for studying the evolution of observables:

$$\mathrm{Tr}[\mathcal{E}(\rho)\,\mathcal{O}] = \mathrm{Tr}\left[\rho\,\mathcal{E}^\dagger(\mathcal{O})\right] .$$

Here, $\mathcal{E}^\dagger$ is the *dual channel*, made with Kraus operators $K_i^\dagger$.

- Strong symmetry of $\mathcal{E}$ means that $\mathcal{E}^\dagger$ fixes the symmetry operators:

$$U_g K_i U_g^\dagger = K_i \ , \quad \forall\, i, g \qquad \Longleftrightarrow \qquad \mathcal{E}^\dagger(U_g) = U_g \ , \quad \forall\, g \ .$$

- $(\Longrightarrow)$

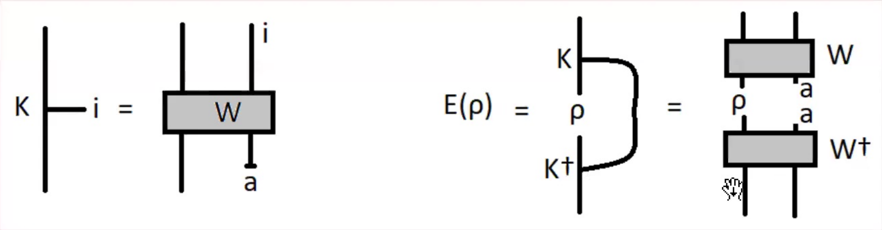
$$\mathcal{E}^\dagger(U_g) = \sum_i K_i^\dagger U_g K_i = \sum_i K_i^\dagger K_i U_g = U_g \ .$$

- $(\Longleftarrow)$

$$\sum_i [U_g, K_i]^\dagger [U_g, K_i] = \mathcal{E}^\dagger(\mathbb{1}) - \mathcal{E}^\dagger(U_g^\dagger)U_g - U_g^\dagger \mathcal{E}^\dagger(U_g) + U_g^\dagger \mathcal{E}^\dagger(\mathbb{1})U_g = 0 \ .$$

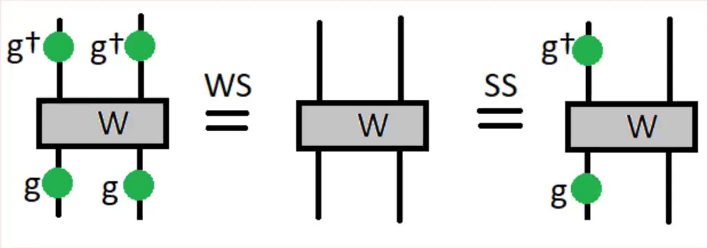
Strong symmetry in terms of purifications

- Purification: unitary W on $\mathcal{H} \otimes A$ such that $K_i = \langle e_i | W | a \rangle$.



- Claim: If $\exists$ a W symmetric with respect to some $U_g \otimes U_g^A$, the channel $\mathcal{E}$ is WS.

- Claim: The channel is SS if and only if there exists a W with $U_g^A = \mathbb{1}$.



- Strong symmetry means the system and bath couple by symmetric terms:

$$W = e^{-itH/\hbar}, \quad H = \sum_i H_i^S \otimes H_i^E \quad \overset{SS}{\Longrightarrow} \quad U_g H_i^S = H_i^S U_g \ , \forall\, i, g \ .$$

String operators

- For the remainder of the talk... take G be to finite abelian.

- String operators: long strings of symmetry operators U_g with end operators:

[den Nijs, Rommelse 89] [Perez-Garcia, Wolf, Sans, Verstraete, Cirac 08] [Pollmann, Turner 12]

- Let α label an irrep of G, then the end operators $\mathcal{O}_\alpha^{l,r}$ transform as

$$U_g^\dagger O_\alpha^l U_g = \chi_\alpha(g) O_\alpha^l \ , \qquad U_g^\dagger O_\alpha^r U_g = \chi_\alpha^*(g) O_\alpha^r \ .$$

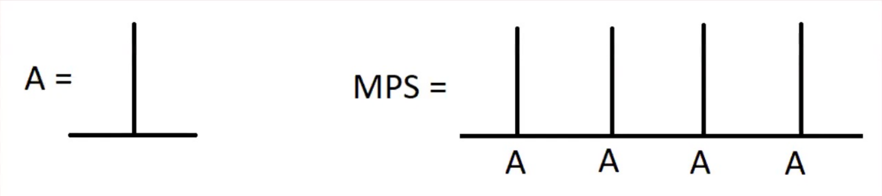
- Expectation values on gapped pure states display a "pattern of zeros"

$$\langle s(g, O_\alpha) \rangle = 0 \quad \text{unless} \quad \chi_\alpha(h) \frac{\omega(g,h)}{\omega(h,g)} = 1 \text{ for all } h \ ,$$

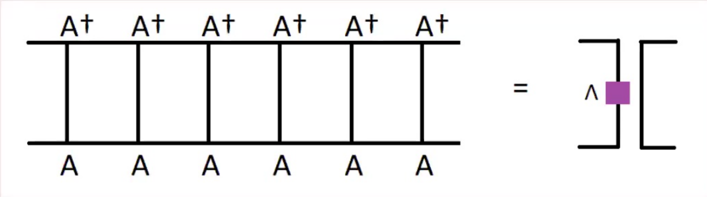
which captures the SPT invariant $[\omega]$ through the ratios ω/ω.

Patterns of zeros of MPS states

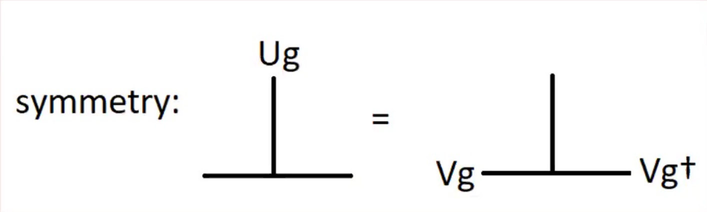
- Represent the state as a matrix product state state (MPS)



- Suppose A is injective and in canonical form, i.e. that its transfer matrix has unique left fixed point $\mathbb{1}$ (and right fixed point Λ):



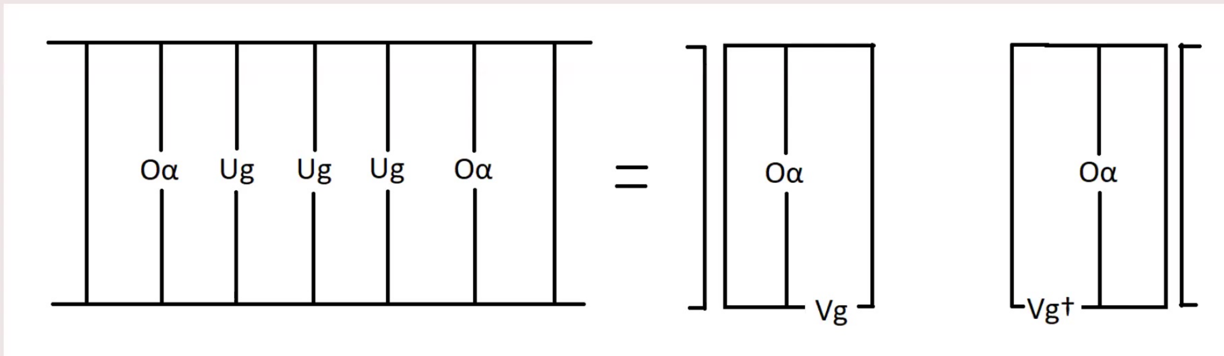
- Then if the MPS is invariant under the symmetry, the tensor A satisfies



- Projective representation $V_g V_h = \omega(g,h) V_{gh}$ encodes the phase invariant ω.

Patterns of zeros of MPS states (cont.)

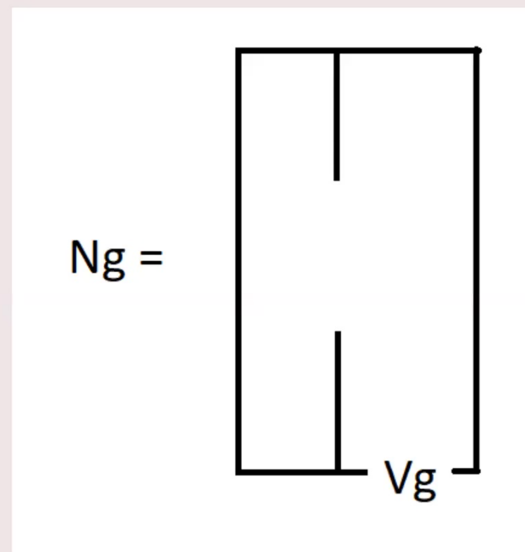
- Evaluate the string order parameter $\langle s \rangle$, using the relations of the MPS tensor:



- Left end evaluates to $\mathrm{Tr}[N_g O_\alpha]$. (Right end is similar.)
- O_α transforms as α, while N_g transforms as ω/ω.
- Vanishes unless these characters are equal:

$$\langle s(g, O_\alpha) \rangle = 0 \quad \text{unless} \quad \chi_\alpha(h)\frac{\omega(g,h)}{\omega(h,g)} = 1 \text{ for all } h \, .$$

- If they are equal, *generically* $\mathrm{Tr}[N_g O_\alpha] \neq 0$.



Reconstructing the SPT invariant

- Represent the pattern as an array with columns g, rows α.
- For each g, there is a unique α with $\langle s(g, O_\alpha)\rangle \neq 0$.

$$\langle s(g, O_\alpha)\rangle_{\text{trivial}} = \begin{pmatrix} \star & \star & \star & \star \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \langle s(g, O_\alpha)\rangle_{\text{Haldane}} = \begin{pmatrix} \star & 0 & 0 & 0 \\ 0 & \star & 0 & 0 \\ 0 & 0 & \star & 0 \\ 0 & 0 & 0 & \star \end{pmatrix}$$

- Claim: The ratios $\chi_\alpha = \omega/\omega$ completely determine the cohomology class $[\omega]$.
- Argument:
  - We show that the kernel of $\omega \mapsto \omega/\omega$ consists only of coboundaries.
  - Suppose $\omega/\omega = 1$. Then $V_g V_h = V_h V_g$, for all g, h.
  - By Schur's lemma, V_g is proportional to the identity: $V_g = \beta(g)\mathbb{1}$.
  - But then

$$\beta(g)\beta(h)\mathbb{1} = V_g V_h = \omega(g,h) V_{gh} = \beta(gh)\omega(g,h)\mathbb{1} \ ,$$

  so $\omega = \delta\beta$.

- If G is non-abelian, other non-local order parameters may be needed to fully reconstruct $[\omega]$. [Pollmann, Tur

Strongly symmetric uncorrelated noise on string operators

- Consider uncorrelated noise

$$\mathcal{E} = \mathcal{E}_1 \otimes \cdots \otimes \mathcal{E}_L \ .$$

- $\mathcal{E}$ is strongly symmetric if and only if the $\mathcal{E}_s$ are.

- The type (g, α) of the string operator is preserved

$$\mathcal{E}_s^\dagger(U_g) = U_g \ , \qquad U_g^\dagger \mathcal{E}_s^\dagger(O_\alpha) U_g = \chi_\alpha(g) \mathcal{E}_s^\dagger(O_\alpha)$$

$$\Longrightarrow \qquad \mathcal{E}^\dagger(s(g, O_\alpha)) = s(g, O'_\alpha) \ .$$

- Conversely, if a channel preserves all string operators, it must be SS.

Patterns of zeros

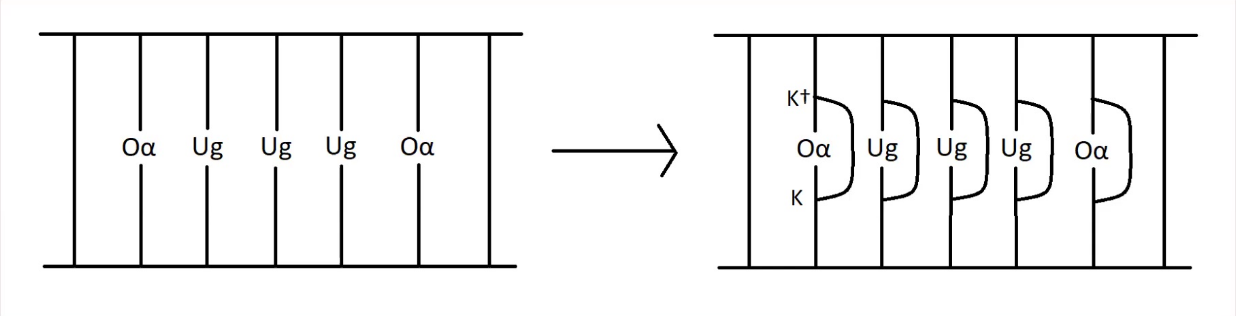
- Strong symmetry $\Longrightarrow$ order parameters $\langle s(g, O_\alpha)\rangle$ are preserved *generically*...

  ...i.e. if O_α is orthogonal to neither N_g nor $\mathcal{E}(N_g)$.
- Conversely...? Given a state, which channels preserve string order $\langle s(g, O_\alpha)\rangle$?

Strong symmetry of $\mathcal{L}$ is necessary and sufficient

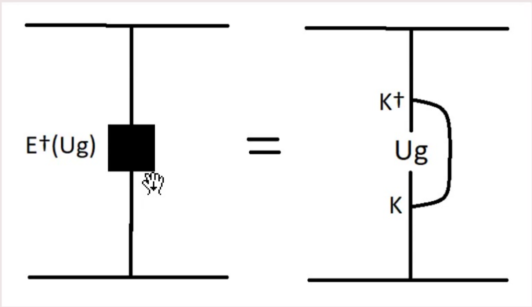
For a generic state with some pattern, a semigroup of channels of uncorrelated noise generically preserves the pattern at all times if and only if it is generated by a strongly symmetric Lindbladian.

- Generic states have injective MPS and exactly G symmetry (no more).
- Crucial to restrict to channels in a semigroup $\mathcal{E}_t = e^{t\mathcal{L}}$.



Transfer matrix argument

- String order vanishes unless the following transfer matrix has $\lambda_{\max} = 1$.



- For injective MPS, $\lambda_{\max} = 1$ implies the insertion is a symmetry. [Bridgeman, Chubb 17]
- Since G is the full symmetry group,

$$\mathcal{E}^{\dagger}(U_g) = U_{\sigma(g)} \ ,$$

for some endomorphism σ of G.

- The family $\mathcal{E}_t$ defines a continuous path σ_t from 1 to σ.
- For finite abelian G, this implies that $\sigma = 1$, which is the strong symmetry condition:

$$\mathcal{E}_t^{\dagger}(U_g) = U_g \ , \forall \, t, g \qquad \Rightarrow \qquad \mathcal{L} \text{ is SS}$$

Causal channels

- Local Lindbladians generate causal channels at finite time:

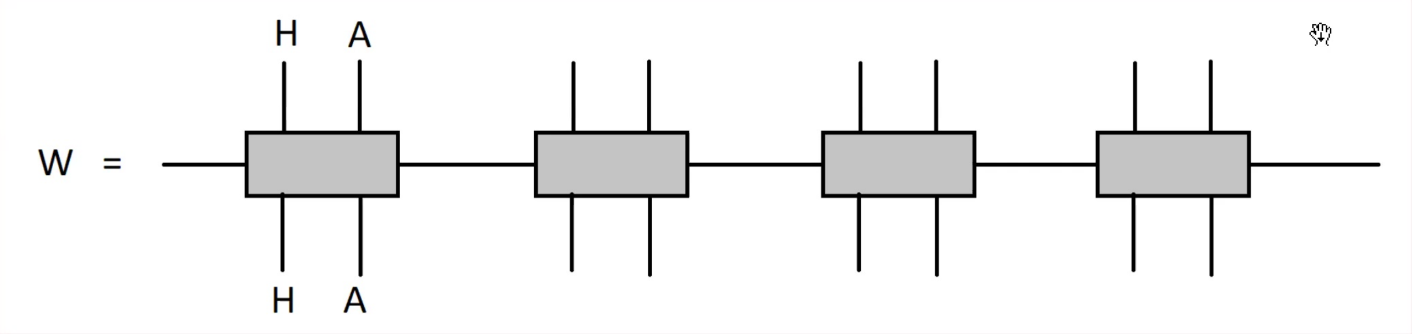
$$\mathcal{E}_t^\dagger : \mathcal{A}_{[a,b]} \to \mathcal{A}_{[a-k,b+k]}$$

- Fastness of $\mathcal{L}$ means the range k is small.

- Conjecturally, any causal channel is a convex combination of channels that purify to causal unitaries. [Piroli, Cirac 20]

- Causal unitaries have matrix product unitary (MPU) representations.

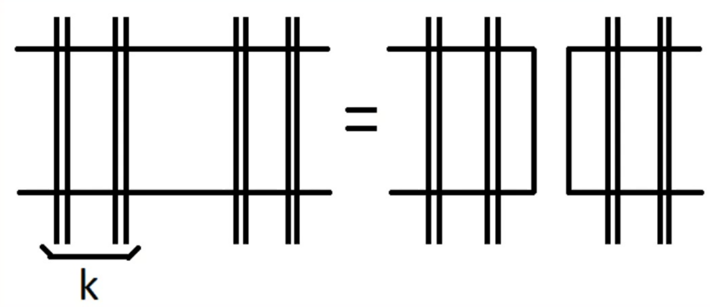
[Cirac, Pérez-Garcá, Schuch, Verstraete 17] , [Şahinğlu, Shukla, Bi, Chen 20]

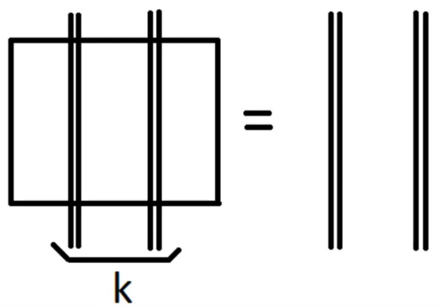


Causality

- An MPU tensor satisfies a simplicity condition on blocks of $k \leq D^4$ sites.

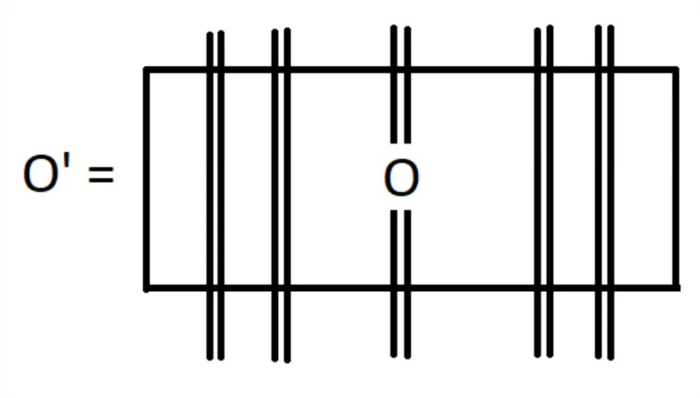
  [Cirac, Pérez-Garcá, Schuch, Verstraete 17] , [Şahinğlu, Shukla, Bi, Chen 20]





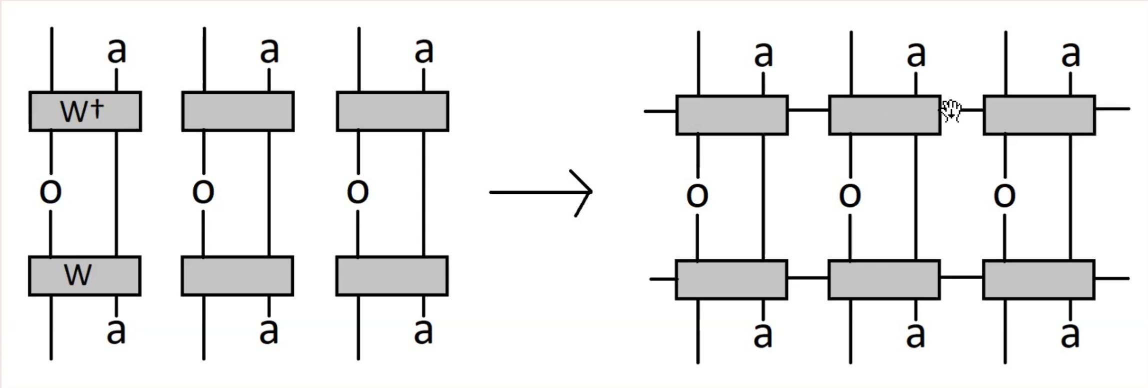
- $\implies$ Causality:

$$\mathcal{E}_t^\dagger : \mathcal{A}_{[a,b]} \to \mathcal{A}_{[a-k,b+k]}$$

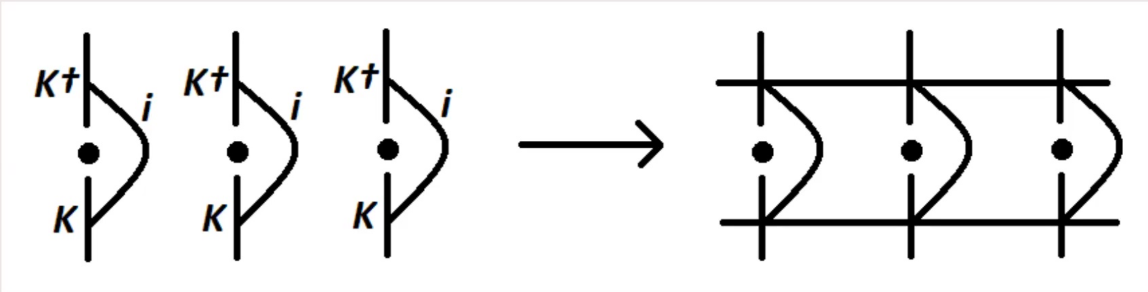


Beyond uncorrelated noise

- Generalize from uncorrelated noise to causal channels:

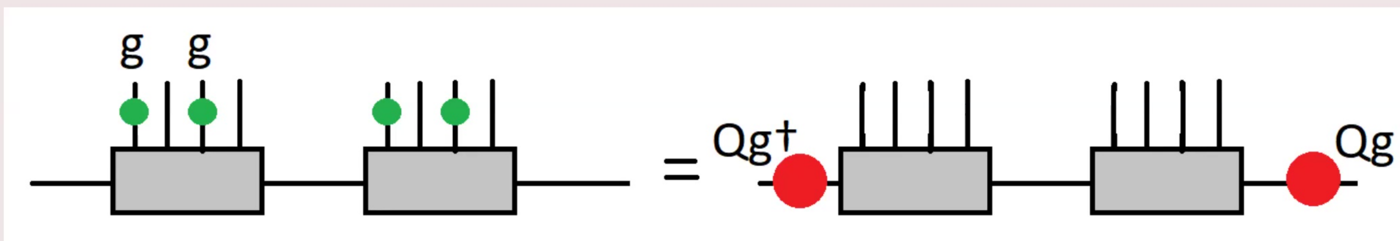


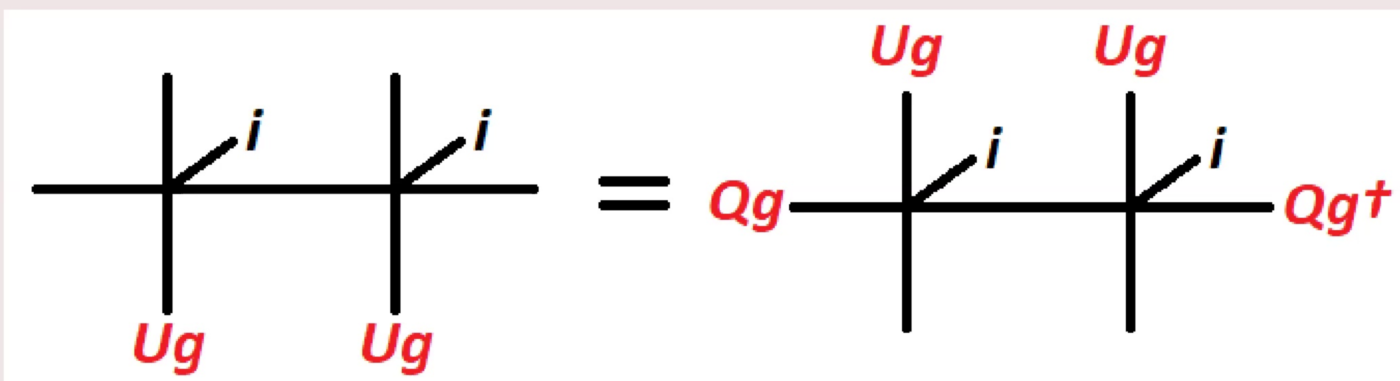
Kraus operators of $\mathcal{E}$ are matrix product operators (MPO):



Strong symmetry of MPO channels

- Folding the MPU yields an MPS that is injective on blocks of size k.
- Strong symmetry means this MPS has the symmetry $(U_g \otimes \mathbb{1}^A) \otimes (U_g \otimes \mathbb{1}^A)$.
- The symmetry fractionalizes over the blocks:





Local realization of strong symmetry

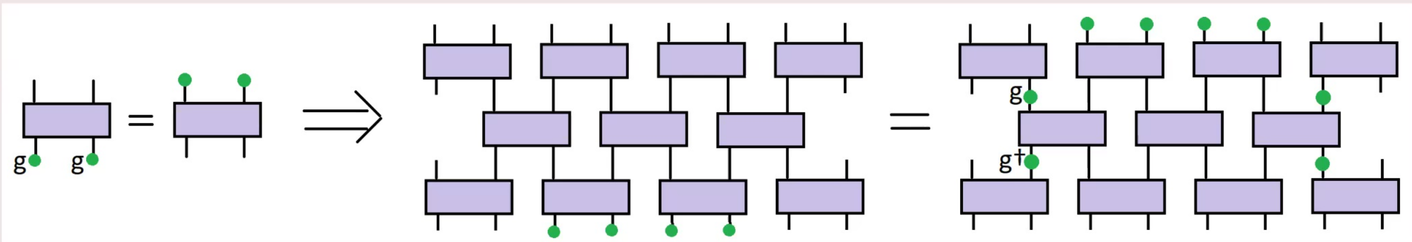
- Strong symmetry is not strong enough. Additionally, we must require that Q is a linear representation (up to phases):

$$Q_g Q_h = Q_{gh}$$

- This locally realized SS condition generalizes the condition on circuits:

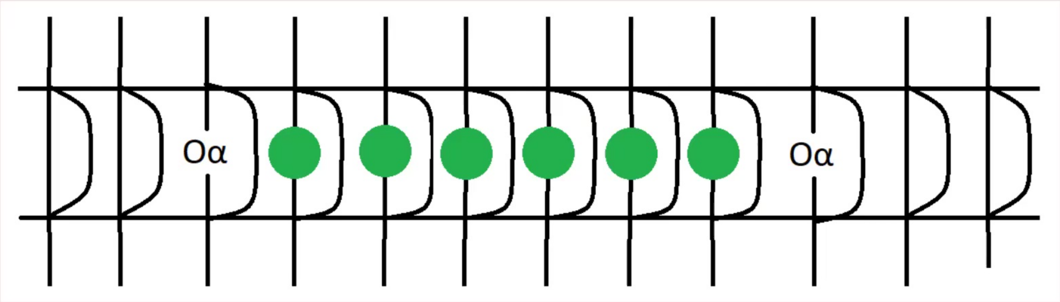
  – Closed system SPT phases are defined with circuits of symmetric gates, not merely symmetric circuits.

  – Symmetric gates mean Q is linear:

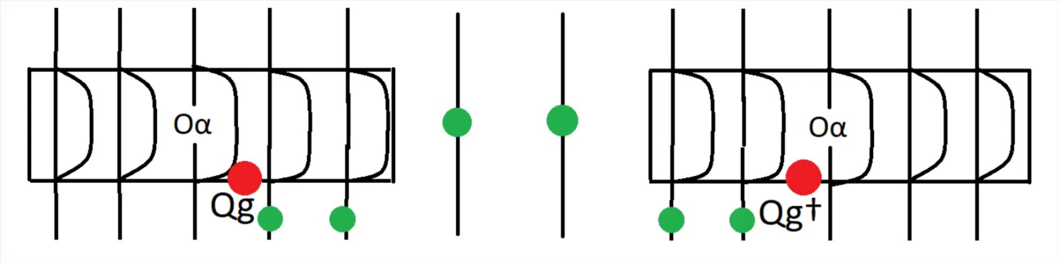


String operators under locally-SS MPO channels

- Under and MPO channel, the string operator becomes



- SS + simplicity conditions $\Longrightarrow$ another string operator



- Fattened end operators transform like O_α as long as Q is linear.
- Fastness / finite-range was key to keep the string bulk long.

Results and conjectures

String operators

- Claim (proof omitted from talk): the converse holds.

$\Rightarrow$ Theorem: Locally-SS causal channels are precisely the causal channels that preserve the types (g, α) of string operators.

Patterns of zeros

- Can we generalize our necessary-and-sufficient result from uncorrelated noise?
- By the above... Locally-SS $\mathcal{L}$ is sufficient to preserve the SPT invariant.
- Conjecture: For MPO channels on generic states, local-SS is necessary.
  - For convex combinations, the necessary condition may be weaker.

Twisted strong symmetry

- Recall from the transfer matrix argument...
  ...the evolved pattern of zeros is nonvanishing as long as

$$\mathcal{E}^\dagger(U_g) = U_{\sigma(g)} \ , \quad \forall\, g$$

  for some endomorphism σ of G.

- Equivalently,

$$U_g K_i = K_i U_{\sigma(g)} \ , \quad \forall\, i, g \ . \qquad \text{(} \sigma\text{-twisted SS condition)}$$

- If σ is nontrivial, $\mathcal{E}$ does not belong to a semigroup $\mathcal{E}_t = e^{t\mathcal{L}}$.

- Question: How do σ-SS channels act on SPT phases?

- Claim: σ-SS channels act within the space of SPT orders as

$$[\omega] \mapsto [\sigma^*\omega] \ , \qquad (\sigma^*\omega)(g, h) = \omega(\sigma(g), \sigma(h)) \ .$$

Endomorphisms

- Endomorphisms of $G = \mathbb{Z}_n \times \mathbb{Z}_n$

$$\mathrm{End}(\mathbb{Z}_n \times \mathbb{Z}_n) = M_2(\mathbb{Z}_n) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z}_n \right\} ,$$

$$g = \begin{pmatrix} w \\ x \end{pmatrix} , \qquad \sigma(g) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} w \\ x \end{pmatrix} = \begin{pmatrix} aw + bx \\ cw + dx \end{pmatrix} .$$

- Cocycles representing the SPT phases $H^2(G, U(1)) = \mathbb{Z}_n$:

$$\omega_k[(w, x), (y, z)] = k\, xy \ .$$

- Action on phases:

$$(\sigma^* \omega_k)[(w, x), (y, z)] = \omega_{k(\det \sigma)}[(w, x), (y, z)] + \text{coboundaries}$$

- The phase ω_k is preserved by σ with determinant d such that

$$kd \equiv k \mod n \ .$$

Action on patterns of zeros

- Invariant $[\omega]$ defines a pattern of zeros $\zeta_\omega : G \to G^*$

$$\zeta_\omega : g \mapsto \frac{\omega(\cdot, g)}{\omega(g, \cdot)} = \chi_\alpha \ .$$

$$\begin{pmatrix} \star & 0 & 0 & 0 \\ 0 & \star & 0 & 0 \\ 0 & 0 & \star & 0 \\ 0 & 0 & 0 & \star \end{pmatrix}$$

- If σ is invertible (an automorphism),

$$g \mapsto \sigma(g) \ , \qquad \alpha \mapsto \alpha \circ \sigma^{-1} \ .$$

- In general, the action on patterns of zeros is given by the rule...
  - Look up the β that appears in column $\sigma(g)$ of the old pattern.
  - Compose β with σ to obtain an α.
  - The new pattern has an entry at coordinates (g, α).

Twisted channels on string operators and patterns

- Bulk:

$$\mathcal{E}^\dagger(U_g) = U_{\sigma(g)}$$

- End operators (proof omitted):

$$\mathcal{E}^\dagger(O_\alpha) = \sum_i K_i^\dagger O_\alpha K_i = \sum_\beta O'_\beta \ ,$$

over β such that $\sigma^* \beta = \alpha$. Generically, none of the O'_β vanish.

Patterns of zeros

- Claim (proof omitted): The $\mathcal{E}$-evolved pattern is $\sigma \cdot \zeta_\omega$. Therefore

$$[\omega] \mapsto [\sigma^* \omega] \ .$$

SPT complexity

- The kernel of ζ is the "projective center"

$$K_\omega = \{ g : \omega(g,h) = \omega(h,g) \ \forall\, h \} \ .$$

  – K_ω consists of the entries in the top row of the pattern.

$$\langle s(g, O_\alpha)\rangle_{\text{trivial}} = \begin{pmatrix} \star & \star & \star & \star \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \ , \quad \langle s(g, O_\alpha)\rangle_{\text{Haldane}} = \begin{pmatrix} \star & 0 & 0 & 0 \\ 0 & \star & 0 & 0 \\ 0 & 0 & \star & 0 \\ 0 & 0 & 0 & \star \end{pmatrix}$$

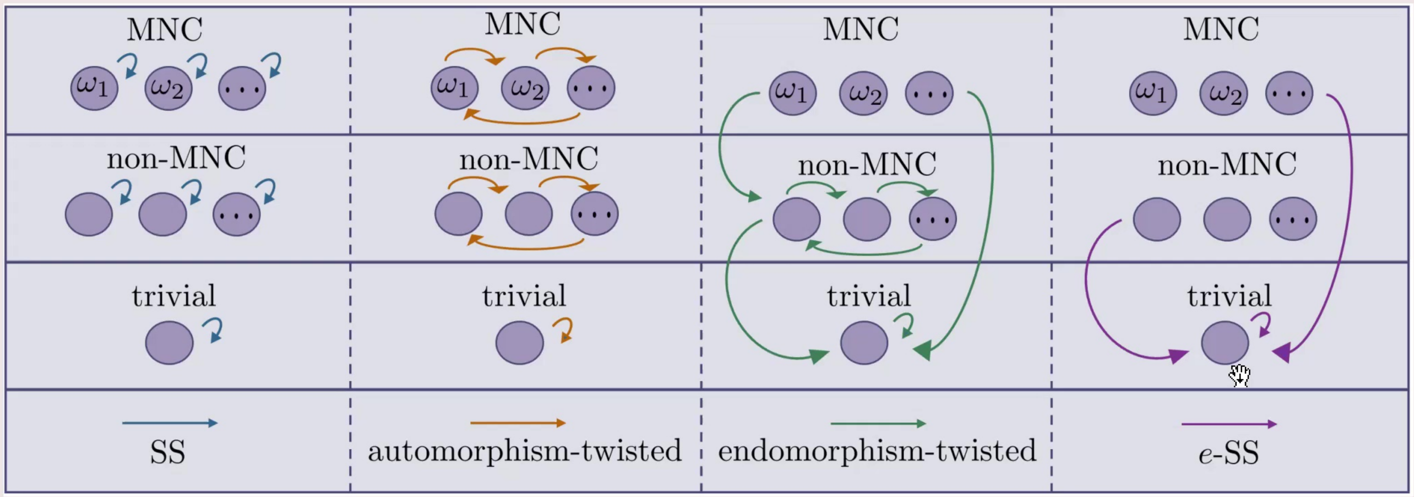
  – When $K_\omega = \{1\}$, ω is said to be maximally non-commutative (MNC).

- $D_\omega = \sqrt{|G|/|K_\omega|}$ is called the SPT complexity.

  – D_ω^2 is the symmetry-protected edge degeneracy.

Transforming between SPT phases

- σ-SS channels can never increase complexity:



- A σ-SS channel preserves the division between MNC and non-MNC phases if and only if σ is an automorphism.

Irreversibility

- Non-invertible endomorphisms are exclusive to irreversible channels!

$$U_g = W U_{\sigma(g)} W^\dagger \qquad \Longrightarrow \qquad \sigma \text{ invertible}$$

- Related: symmetric purifications do not exist for non-invertible σ.

- Example: $G = \mathbb{Z}_2 \times \mathbb{Z}_2$ with the constant endomorphism $\sigma : g \mapsto e$.

$$K_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad K_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad K_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad K_4 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

  is a well-defined channel: $K_1^\dagger K_1 + K_2^\dagger K_2 + K_3^\dagger K_3 + K_4^\dagger K_4 = \mathbb{1}$.
- No linear combination of these Kraus operators is unitary.
- This is the channel that C&PG's Lindbladian tends toward as $t \to \infty$:

$$L_{s,i} = |\phi\rangle\langle i| \,, \qquad \Longrightarrow \qquad U_g L_{s,i} \mathbb{1}^\dagger = L_{s,i} \,.$$

Conclusion

- SPTO is destroyed by weakly symmetric couplings to the environment.

- Strong symmetry of $\mathcal{E}$ $\longleftrightarrow$ string operator type is preserved.
  - True for all causal channels.
- Strong symmetry of $\mathcal{L}$ $\longleftrightarrow$ SPT order is preserved.
  - True for uncorrelated noise; conjectural for causal channels.

- σ-SS channels map within the space of SPT mixed states.