Title: Failure of the split property in gravity and the information paradox

Speakers: Suvrat Raju

Series: Colloquium

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Abstract: In an ordinary quantum field theory, the "split property" implies that the state of the system can be specified independently on a bounded subregion of a Cauchy slice and its complement. This property does not hold for theories of gravity, where observables near the boundary of the Cauchy slice uniquely fix the state on the entire slice. The original formulation of the information paradox explicitly assumed the split property and we follow this assumption to isolate the precise error in Hawking's argument. A similar assumption also underpins the monogamy paradox of Mathur and AMPS. Finally the same assumption is used to support the common idea that the entanglement entropy of the region outside a black hole should follow a Page curve. It is for this reason that computations of the Page curve have been performed only in nonstandard theories of gravity, which include a non-gravitational bath and massive gravitons. The fine-grained entropy at future null infinity does not obey a Page curve for an evaporating black hole in standard theories of gravity but we discuss possibilities for coarse graining that might lead to a Page curve in such cases.

Failure of the split property in gravity and the information paradox

Suvrat Raju

International Centre for Theoretical Sciences

Perimeter Institute Colloquium
17 November 2021

Collaborators and References

Alok Laddha

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- 2002.02448, Alok Laddha, Siddharth Prabhu, Pushkal Shrivastava, S.R.
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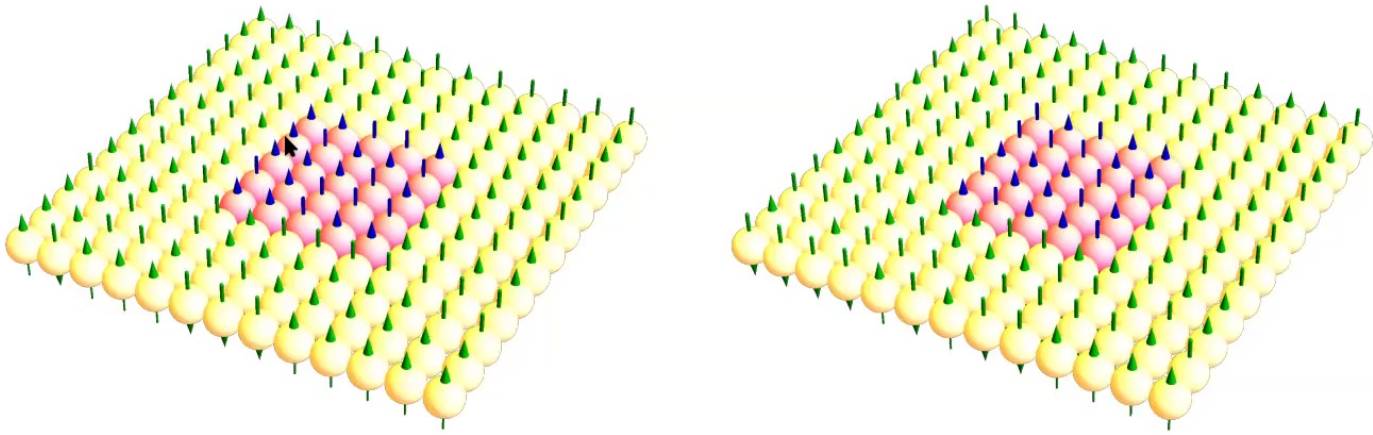
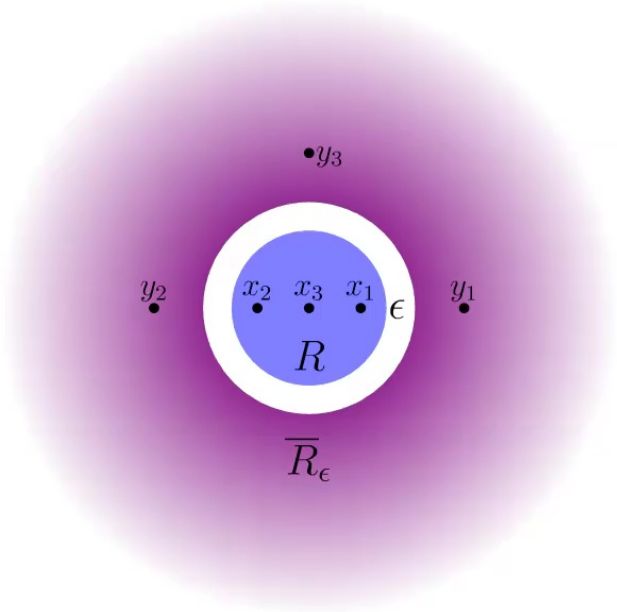
The split property in nongravitational theories



- In a nongravitational theory, the state inside and outside a bounded region can be specified independently.
- This is clear in a lattice regularization, where the Hilbert space factorizes.

The split property in nongravitational theories



In a nongravitational QFT,

- region R.
- collar ϵ
- complement $\overline{R}_\epsilon$

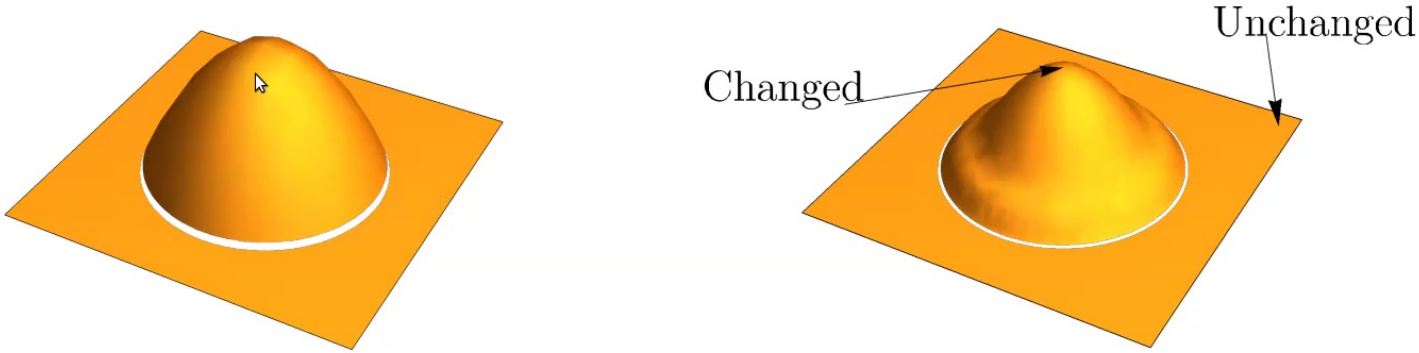
all on a single Cauchy slice

For *any states* $|\Psi_1\rangle$ and $|\Psi_2\rangle$ we can find a split state $|\Psi\rangle$

$$\langle\Psi|\phi(x_1)\dots\phi(x_n)\phi(y_1)\dots\phi(y_m)|\Psi\rangle$$
$$= \langle\Psi_1|\phi(x_1)\dots\phi(x_n)|\Psi_1\rangle \times \langle\Psi_2|\phi(y_1)\dots\phi(y_m)|\Psi_2\rangle$$

$x_i \in R, y_i \in \overline{R}_\epsilon$.

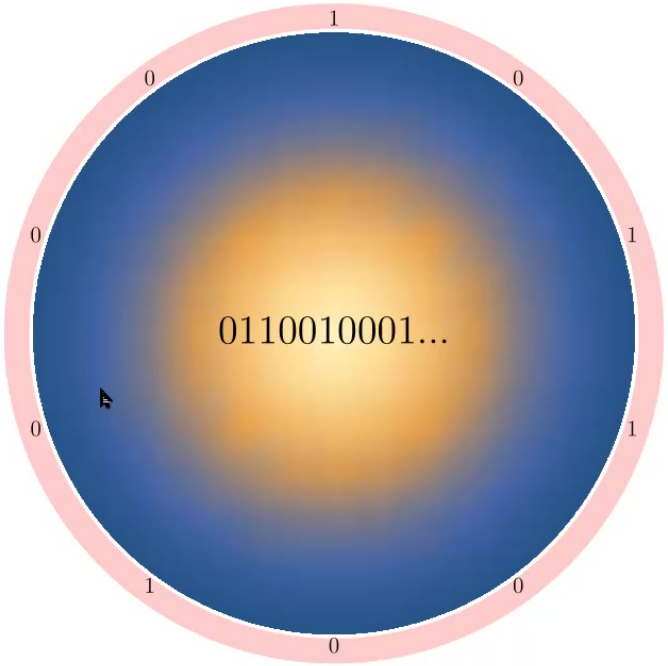
Ordinary localization of information in QFT



The split property is key to our usual idea that information is localized “inside” some region and is not available in its complement.

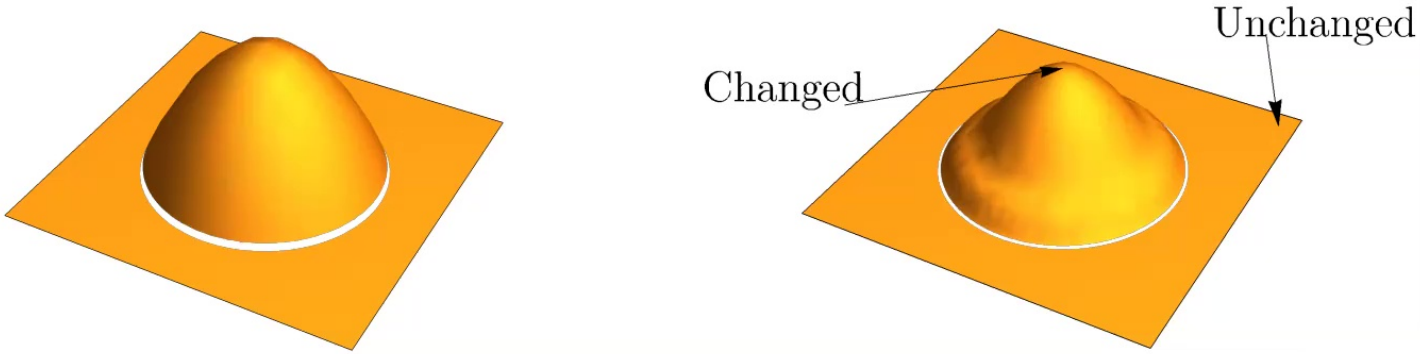
Principle of holography of information

In quantum gravity, information present in the bulk of a Cauchy slice is also available near its boundary.

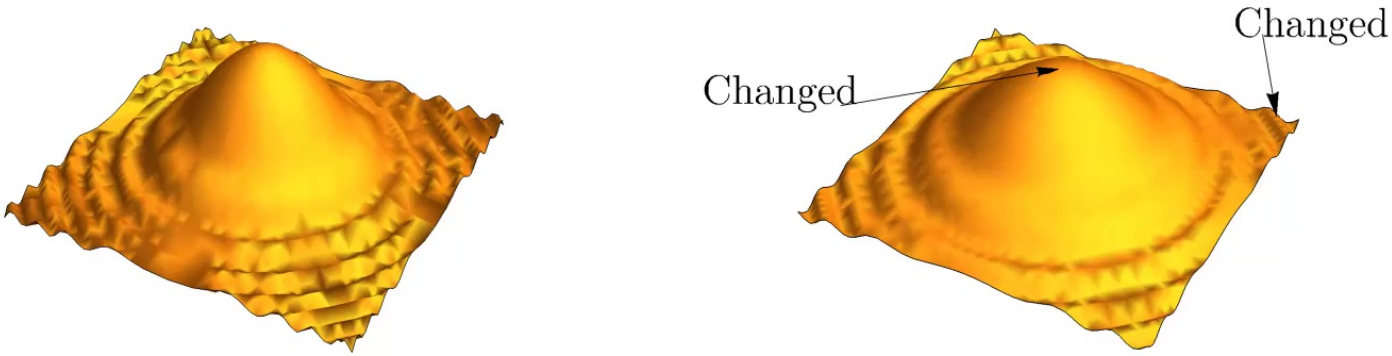


No split states in gravity

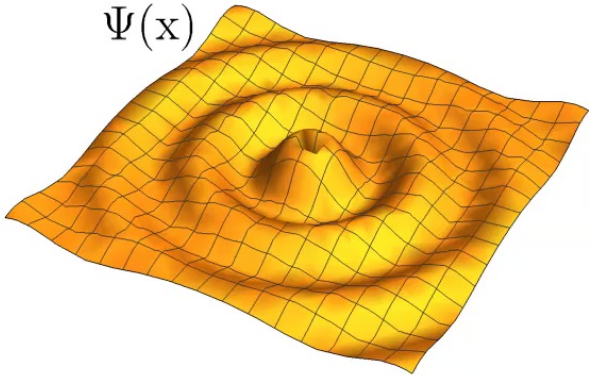
Nongravitational QFT:



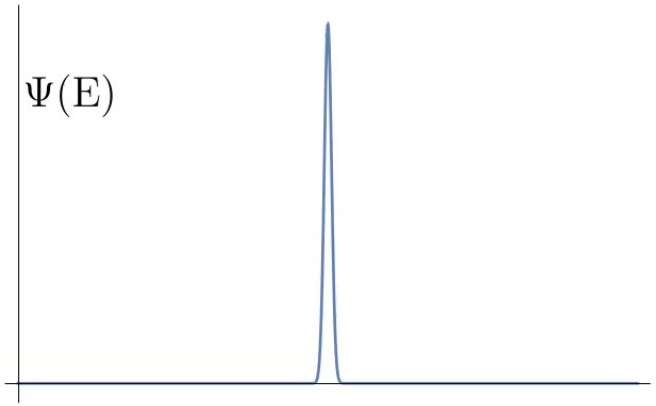
Gravity:



Uncertainty principle

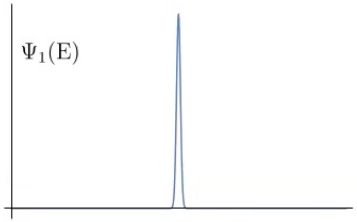


Position space



Energy spectrum

Localization via superposition



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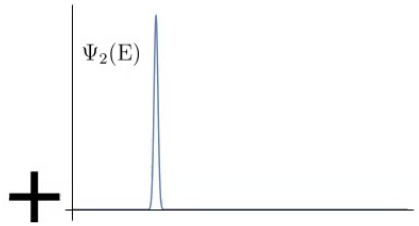

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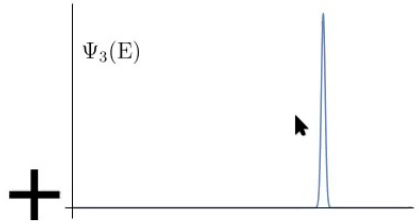

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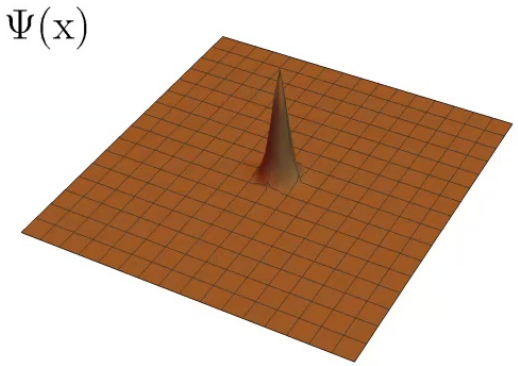
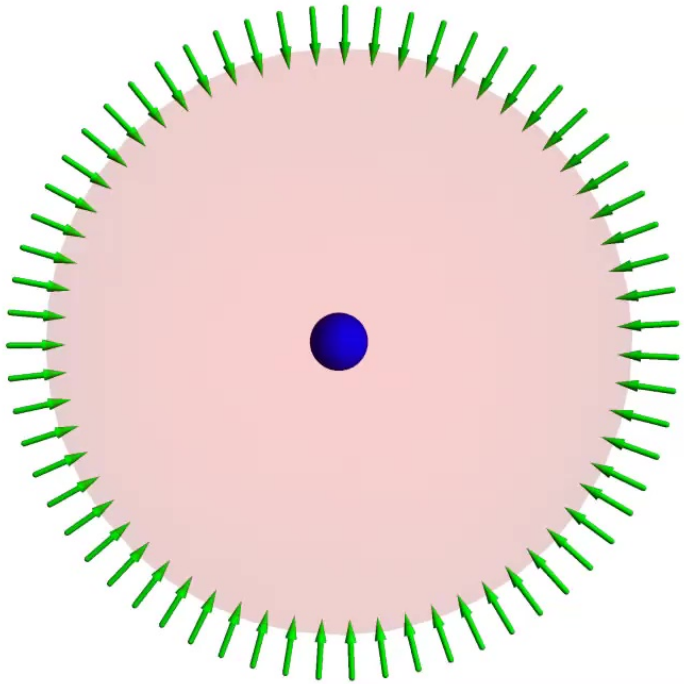
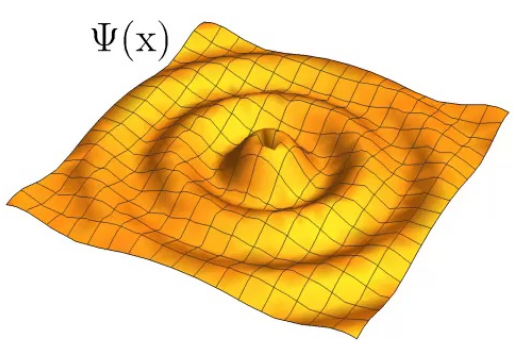


Gauss law



$$E = \frac{1}{16\pi G} \int \left(g_{jk,k} - g_{kk,j} \right) d^2\Omega_j$$

Gravitational tails



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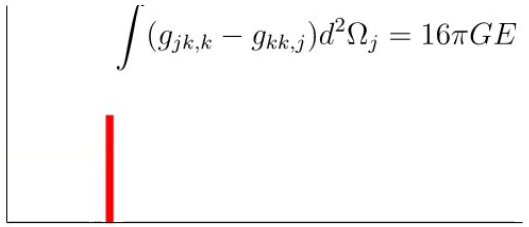
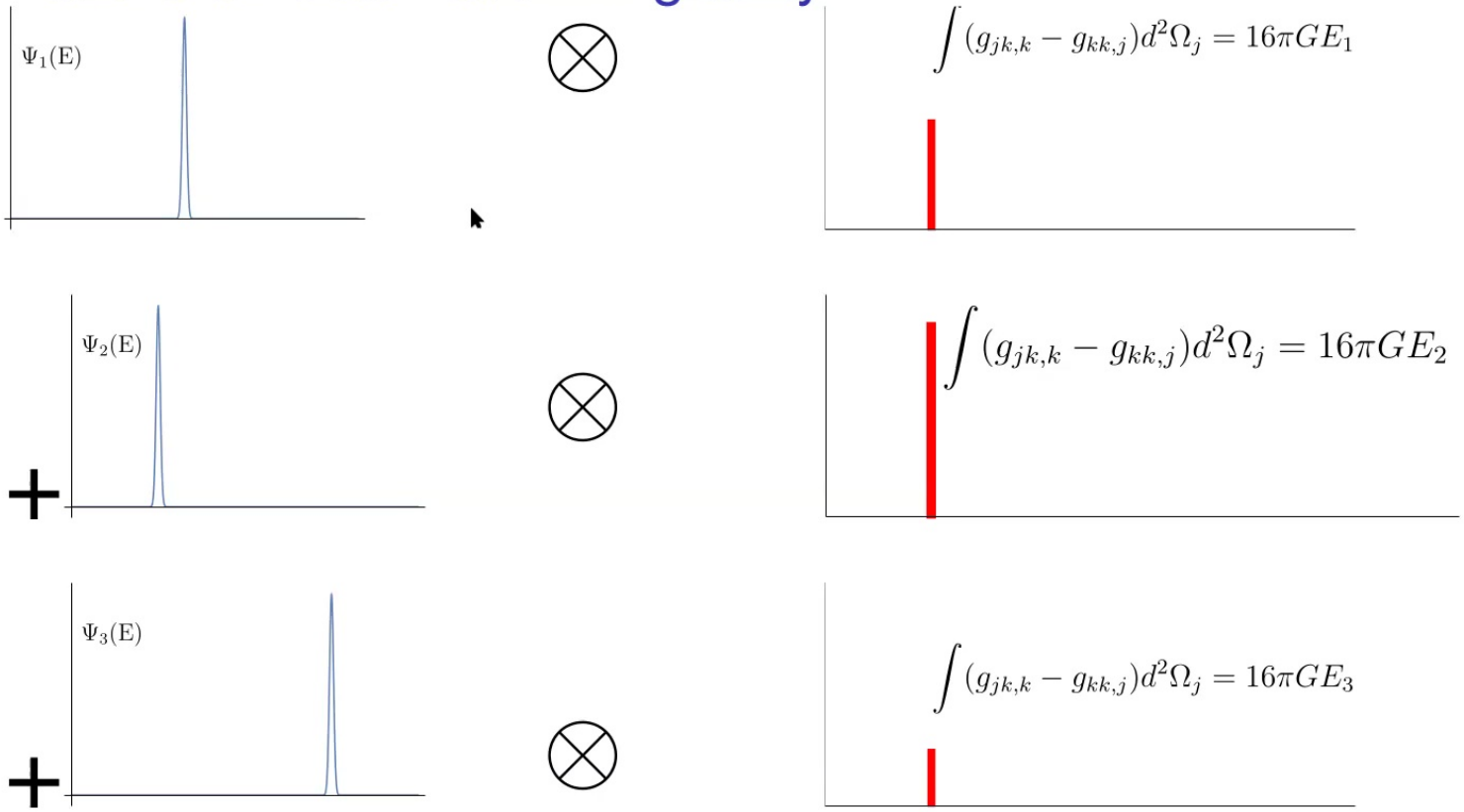


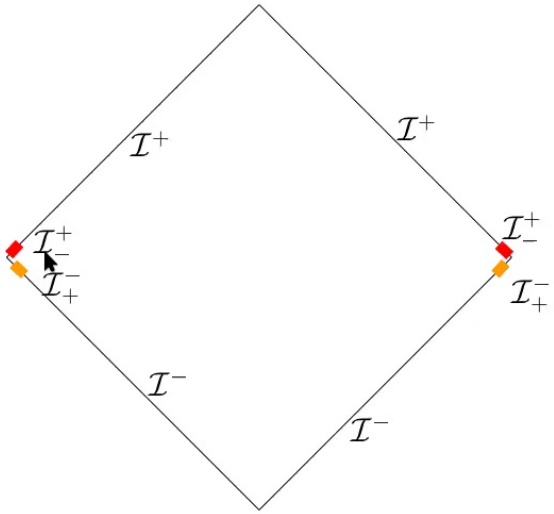
In the quantum theory, each excitation must be dressed with a gravitational tail.

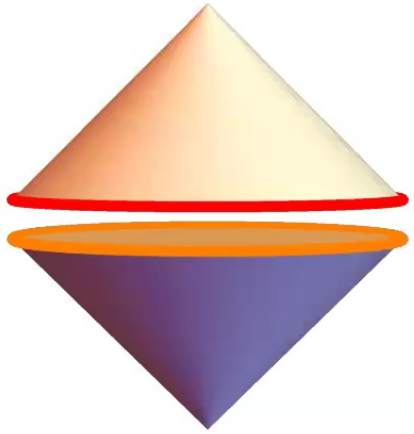
Failure of localization in gravity



Tails prevent destructive interference outside a bounded region.
Correlators of the energy and other observables carry information about the state.

Holography of information in flat space





Precise statement in 4d asymptotically flat spacetime: all information about massless particles is present near the past boundary of future null infinity.

[Laddha, Prabhu, S.R., Shrivastava, 2020]

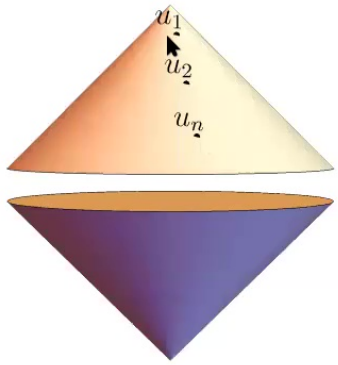
[Marolf, 2006–13]

[de Boer, Solodukhin, 2003]

[Bagchi, Grumiller, Pasterski, Shu-Heng Shao, Strominger, 2016–19]

Holography of information in flat space

Massless particles are conveniently described on $\mathcal{I}$ in asymptotically flat space.

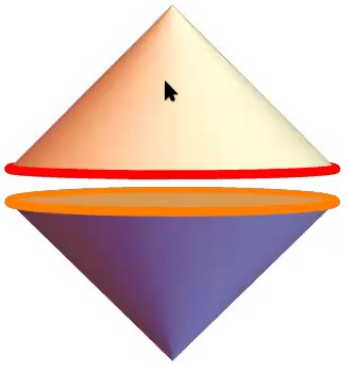


In a nongravitational theory, in 4D flat space, a complete specification of the state $|\Psi\rangle$ requires

$$\langle\Psi|\mathcal{Y}(u_1)\dots\mathcal{Y}(u_n)|\Psi\rangle, \qquad u_i \in [-\infty,\infty]$$

where $\mathcal{Y}(u_i)$ are observables on $\mathcal{I}^+$.

Result in 4d asymptotically flat space

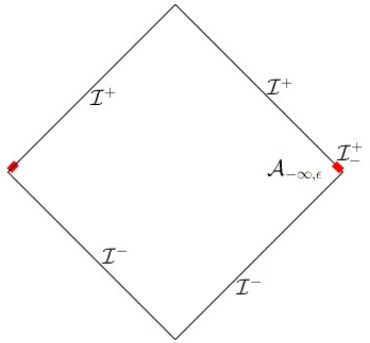


In gravity

$$\langle \Psi | \mathcal{Y}(u_1) \dots \mathcal{Y}(u_n) | \Psi \rangle, \qquad u_i \in [-\infty, -\frac{1}{\epsilon}]$$

is sufficient to specify the state $|\Psi\rangle$.

Comments on the result



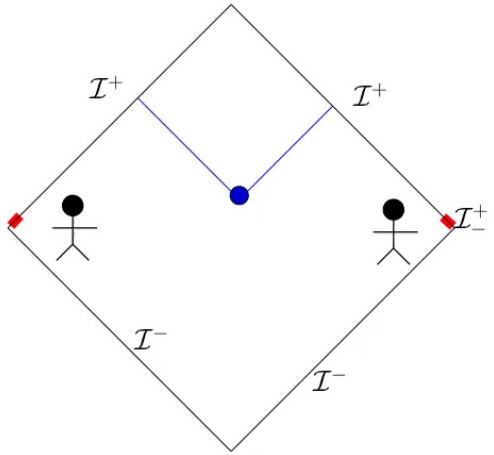
$$\langle\Psi|\mathcal{Y}(u_1)\dots\mathcal{Y}(u_n)|\Psi\rangle, \qquad u_i \in [-\infty, -\frac{1}{\epsilon}]$$

determine $|\Psi\rangle$.

- This result holds nonperturbatively.
- It relies on weak assumptions about the UV theory of gravity — (a) positivity of the Hamiltonian and (b) the vacuum can be identified by a boundary term in the full theory.

Perturbative evidence

The previous result is nonperturbative but it can be checked in perturbation theory for simple states.



- Take $f(x,\Omega)$ to have support in $x \in [0,1]$,

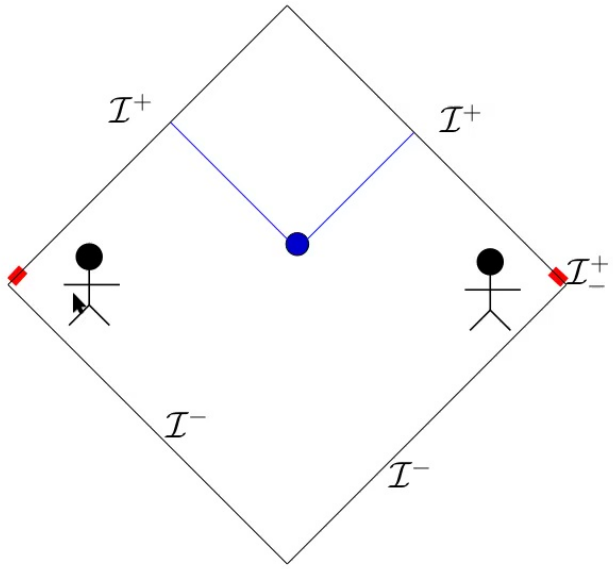
$$|f\rangle = e^{-i\lambda \int d^2\Omega \int_0^1 dx\, f(x,\Omega) O(x,\Omega)} |0\rangle.$$

- **Challenge:** Using perturbative quantum correlators near $\mathcal{I}^+_-$ determine $f(x,\Omega)$.

[Laddha, Prabhu, S.R., Shrivastava, 2020]

[Chowdhury, Papadoulaki, S.R., 2020]

Perturbative evidence



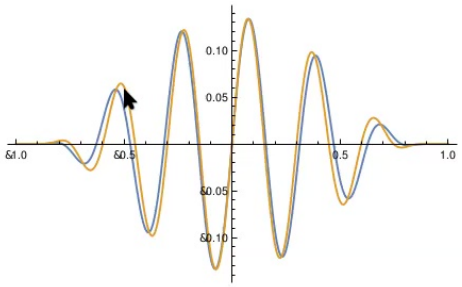
$$\langle f|M(-\infty)O(u,\Omega')|f\rangle = G\lambda \int_0^1 \frac{f(x,\Omega')}{(x-u-i\epsilon^+)}dx + \mathrm{O}\left(\lambda^2\right).$$

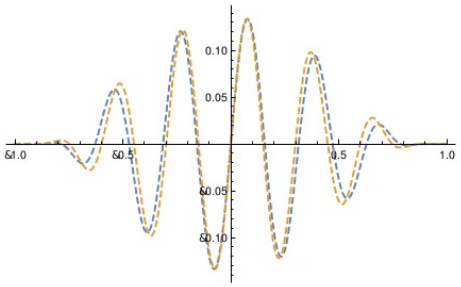
RHS is analytic when u is extended in the upper-half plane. So knowledge for $u \in (-\infty, -\frac{1}{\epsilon})$ allows reconstruction of f.

Perturbative evidence

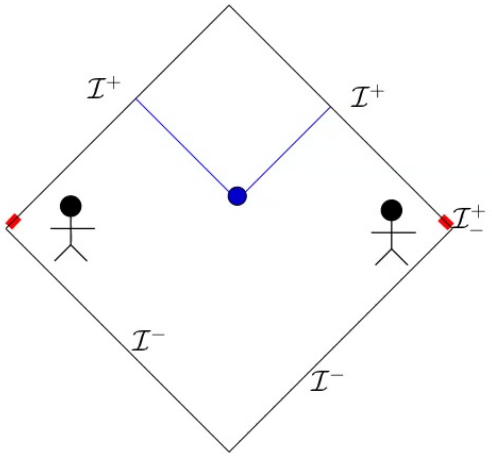
$$\int_0^1 \frac{f(x,\Omega')}{(x-u-i\epsilon^+)}dx = -\sum_{n=0}^{\infty} \frac{1}{u^{n+1}} \int_0^1 x^n f(x,\Omega')dx.$$

Gravitational effects allow reconstruction of moments of $f(x)$.





Significance of gravity



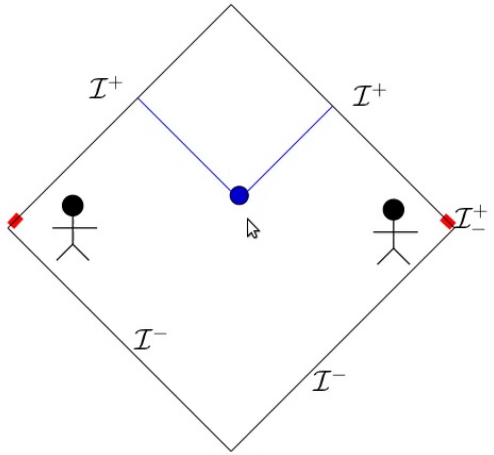
- This idea cannot work without gravity.
- Nongravitational gauge theories contain local gauge-invariant bulk operators.

$$|0\rangle \qquad \text{and} \qquad e^{i\mathrm{Tr}(F^2)(u=0)}|0\rangle$$

cannot be distinguished by any measurement near $\mathcal{I}^+_-$ without gravity.

Perturbative evidence

The previous result is nonperturbative but it can be checked in perturbation theory for simple states.



- Take $f(x,\Omega)$ to have support in $x \in [0,1]$,

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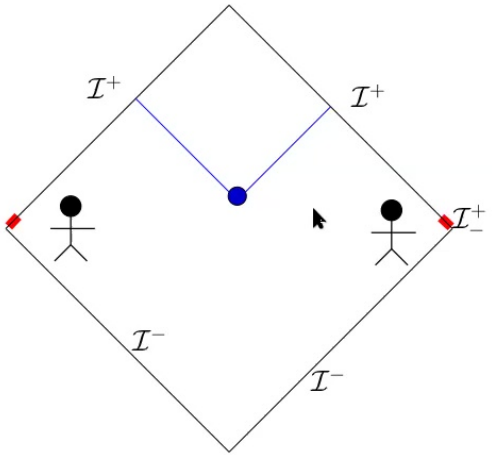
- **Challenge:** Using perturbative quantum correlators near $\mathcal{I}^+_-$ determine $f(x,\Omega)$.

[Laddha, Prabhu, S.R., Shrivastava, 2020]

[Chowdhury, Papadoulaki, S.R., 2020]

Significance of gravity



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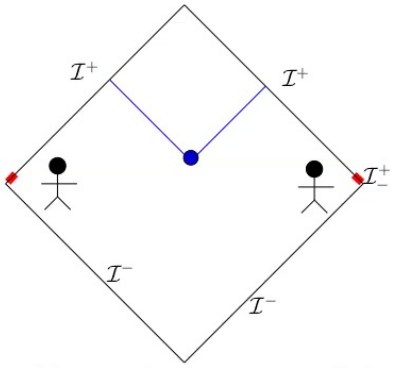
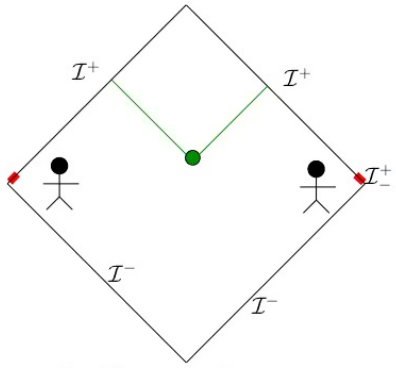
Global symmetry





What if we have two identical fields, O_1 and O_2 and

$$|f_1\rangle = e^{-i\lambda \int d^2\Omega \int_0^1 dx\, f(x,\Omega) O_1(x,\Omega)} |0\rangle.$$

$$|f_2\rangle = e^{-i\lambda \int d^2\Omega \int_0^1 dx\, f(x,\Omega) O_2(x,\Omega)} |0\rangle.$$

Then (up to O (λ))

$\langle f_1 \vert M(-\infty) O_1(u,\Omega') \vert f_1\rangle \neq 0$	$\langle f_1 \vert M(-\infty) O_2(u,\Omega') \vert f_1\rangle = 0$
$\langle f_2 \vert M(-\infty) O_1(u,\Omega') \vert f_2\rangle = 0$	$\langle f_2 \vert M(-\infty) O_2(u,\Omega') \vert f_2\rangle \neq 0.$

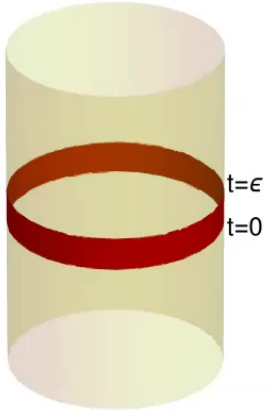
Holography of information in AdS

Similar arguments show that

Asymptotic correlators on an infinitesimal time band at the AdS boundary completely fix the bulk state.

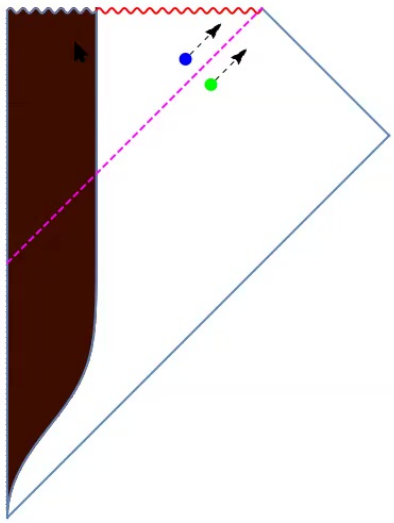
[Laddha, Prabhu, S.R., Shrivastava, 2020]

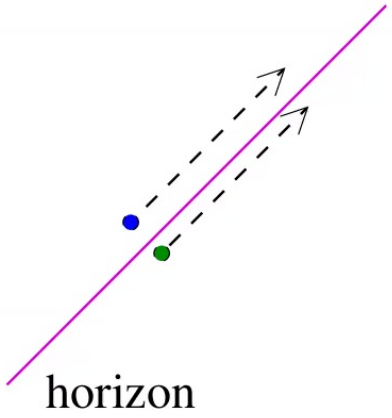
[Choudhury, Godet, Papadoulaki, S.R., 2107.14802]



This does not assume AdS/CFT. Rather, it shows gravitational theories must be holographic provided the UV theory obeys weak assumptions.

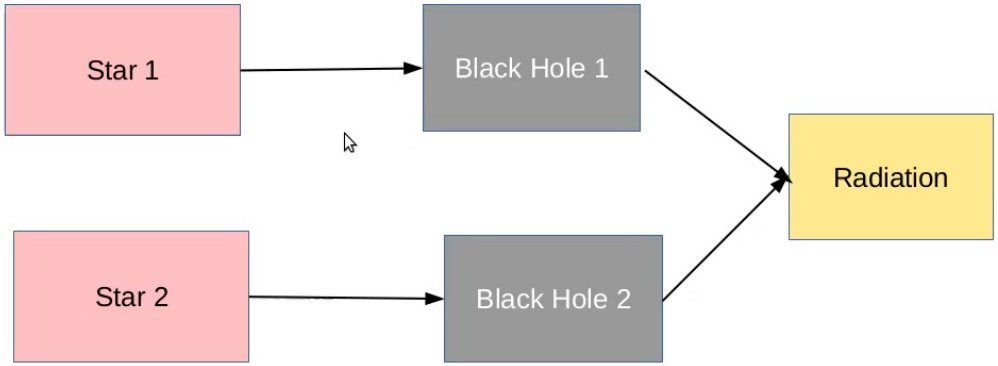
Pair creation





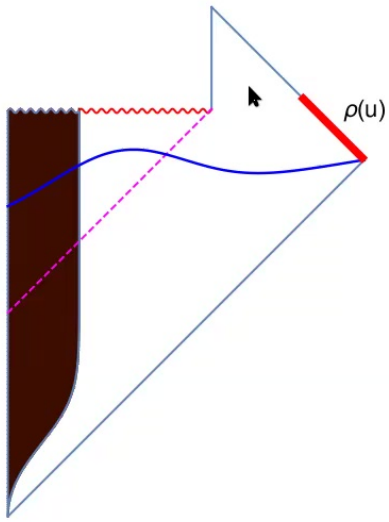
Hawking (1975) found that after a black hole forms, spontaneous pair creation at the horizon due to quantum effects leads to Hawking radiation that is seemingly uncorrelated with details of the initial state.

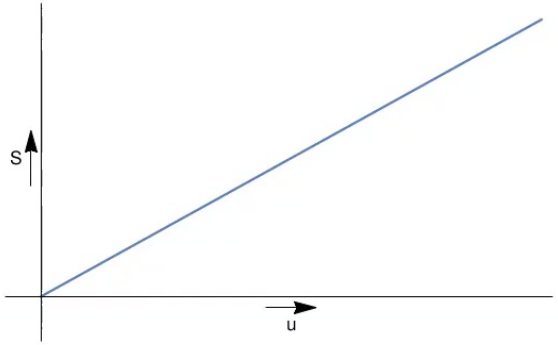
Information paradox



Hawking argued that since radiation is uncorrelated with the initial state, the time-evolution arrows cannot be reversed. This contradicts unitarity.

Entropy of Hawking radiation



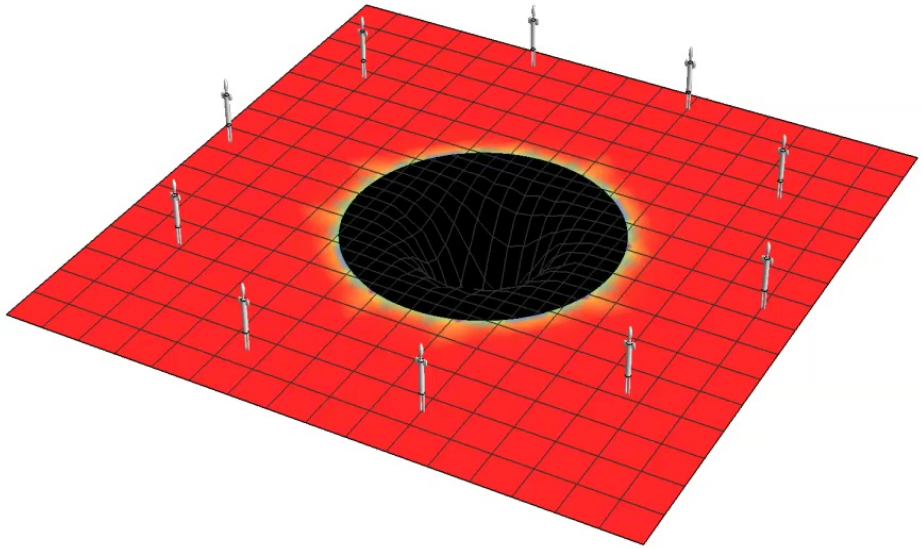


Hawking's arguments imply that $\frac{\partial S(u)}{\partial u} > 0$. This is a paradox since unitarity implies

$$S(\mathcal{I}^-) = S(\mathcal{I}^+)$$

and if $S(\mathcal{I}^-) = 0$ then $S(\mathcal{I}^+) = 0$.

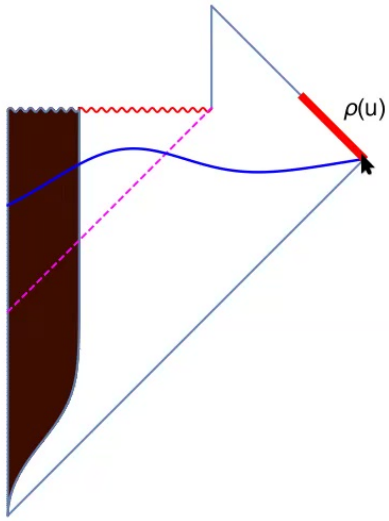
Black-hole information

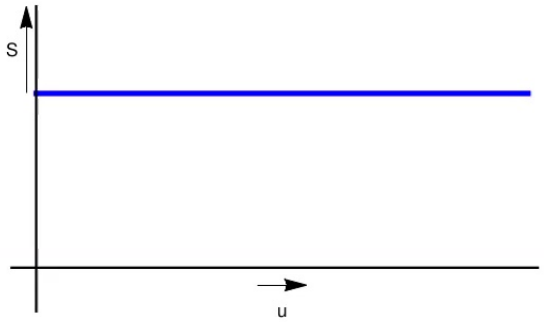


Holography of information implies that information about the black-hole microstate is always available outside with the right measurements.

[Related perspectives: Jacobson 2012, Jacobson, Nguyen 2019, Calmet, Casadio, Hsu, Kuipeers, 2021]

von Neumann entropy on $\mathcal{I}^+$

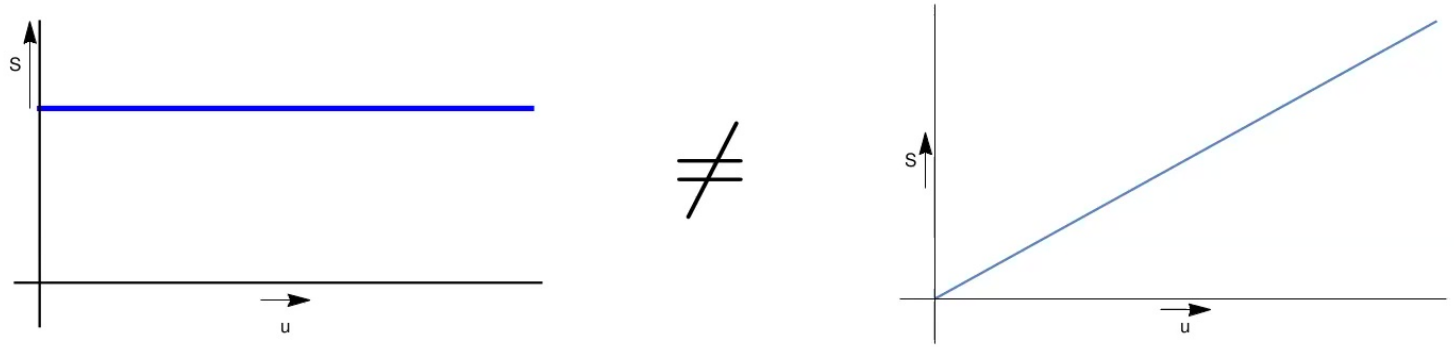




We can prove that the von Neumann entropy of a segment $(-\infty, u)$ of $\mathcal{I}^+$ is independent of u.

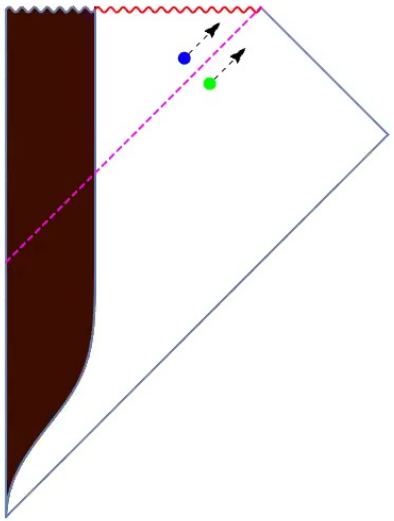
[Laddha, Prabhu, S.R., Shrivastava, 2020]

Resolving Hawking's paradox



- This contradicts Hawking's suggestion that the entropy should rise monotonically.

Error in Hawking's argument?



Hawking only computed low-point correlators.

$$\langle a_\omega a_\omega^\dagger \rangle = \frac{1}{1 - e^{-\beta\omega}}$$

But typical pure states are exponentially close to mixed states

$$\langle \Psi | A | \Psi \rangle = \mathrm{Tr}(\rho_{\mathrm{micro}} A) + \mathrm{O}\left(e^{-\frac{S}{2}}\right)$$

[Lloyd, 1988]

So low-point correlators cannot be used to conclude that the final state is mixed.

Where is the error in Hawking's argument?

Hawking provided an argument — "the principle of ignorance" — to suggest that his conclusion was robust against small corrections.

One therefore has to introduce a hidden surface around each of these holes and apply the principle of ignorance to say that all field configurations on these hidden surfaces are equally probable provided they are compatible with the conservation of mass, angular momentum, etc. which can be measured by surface integrals at a distance from the hole.

Let H_1 be the Hilbert space of all possible data on the initial surface, H_2 be the Hilbert space of all possible data on the hidden surface, and H_3 be the Hilbert space of all possible data on the final surface. The basic assumption of quantum theory is that there is some tensor S_{ABC} whose three indices refer to H_3, H_2, and H_1, respectively, such that if

$$\xi_C \in H_1, \quad \zeta_B \in H_2, \quad \chi_A \in H_3,$$

then

$$\sum\sum\sum S_{ABC}\chi_A\zeta_B\xi_C$$

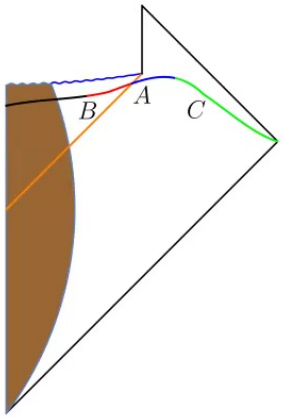
is the amplitude to have the initial state ξ_C, the final state χ_A, and the state ζ_B on the hidden surface. Given only the initial state ξ_C one cannot determine the final state but only the element $\sum S_{ABC}\xi_C$ of the tensor product $H_2 \otimes H_3$. Because one is ignorant of the state on the hidden surface one cannot find the amplitude for measurements on the final surface to give the answer χ_A but one can calculate the probability for this outcome to be $\sum\sum\sum \rho_{CD}\bar{\chi}_C\chi_D$, where

- Hawking assumed that the Hilbert space factorizes up to a few global constraints imposed by the no-hair theorem.
- Our previous discussion shows that this fails in quantum gravity.

Refinements of the paradox

More-recent refinements of the information paradox again assume the split property/factorization of the Hilbert space.

Monogamy paradox



Mathur (2009) and AMPS (2012) argued that

$$S_{AB} < S_B \qquad \text{if smooth horizon}$$

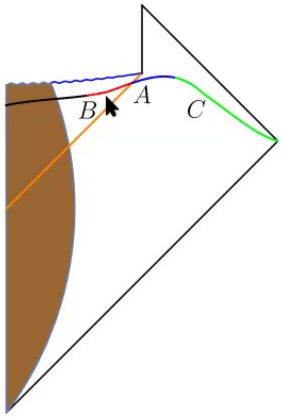
Further,

$$S_{AC} < S_C \qquad \text{if entropy declines at late times}$$

Seems to contradict strong subaddivity

$$S_{AC} + S_{AB} \geq S_C + S_B$$

Monogamy paradox and the split property



But strong subadditivity

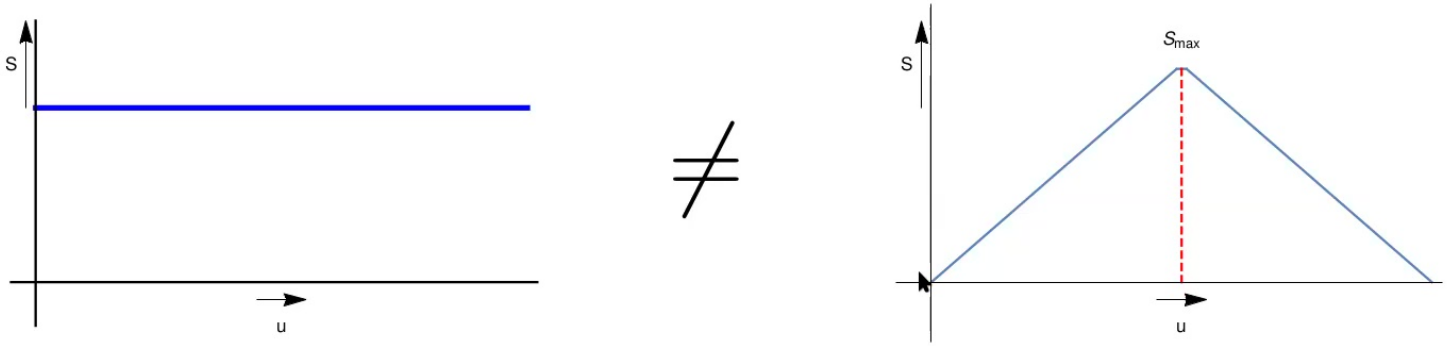
$$S_{AC} + S_{AB} \geq S_C + S_B$$

makes sense only if the Hilbert space factorizes.

But the Hilbert space does not factorize in gravity

$$H \neq H_A \otimes H_B \otimes H_C \otimes H_{\overline{ABC}}$$

Comparison with Page curve



Our result is also different from the conventional Page curve.

Inapplicability of Page's argument

Average Entropy of a Subsystem

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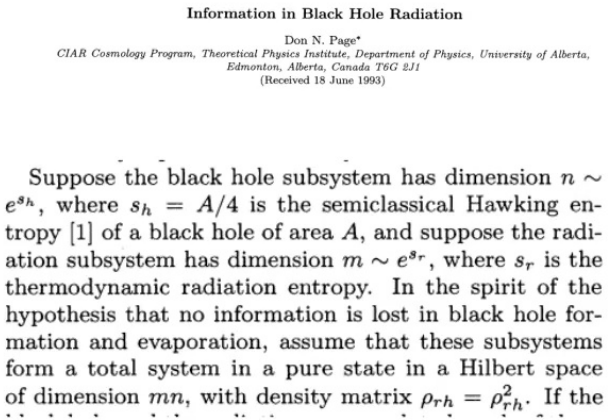
(Received 7 May 1993)

If a quantum system of Hilbert space dimension mn is in a random pure state, the average entropy of a subsystem of dimension $m \leq n$ is conjectured to be $S_{m,n} = \sum_{k=n+1}^{mn} \frac{1}{k} - \frac{m-1}{2n}$ and is shown to be $\simeq \ln m - \frac{m}{2n}$ for $1 \ll m \leq n$. Thus there is less than one-half unit of information, on average, in the smaller subsystem of a total system in a random pure state.

- Page's calculation also assumes factorization..
- Nongravitational systems must obey a Page curve.

Inapplicability of Page's argument



Information in Black Hole Radiation

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(Received 18 June 1993)

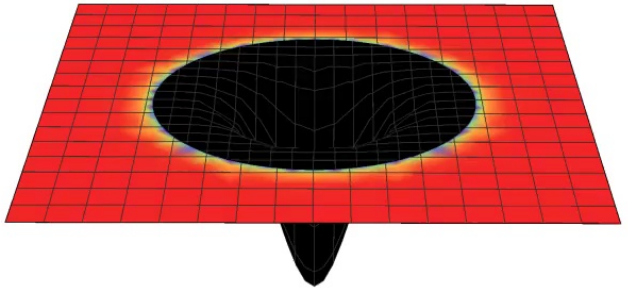
Suppose the black hole subsystem has dimension $n \sim e^{s_h}$, where $s_h = A/4$ is the semiclassical Hawking entropy [1] of a black hole of area A, and suppose the radiation subsystem has dimension $m \sim e^{s_r}$, where s_r is the thermodynamic radiation entropy. In the spirit of the hypothesis that no information is lost in black hole formation and evaporation, assume that these subsystems form a total system in a pure state in a Hilbert space of dimension mn, with density matrix $\rho_{rh} = \rho_{rh}^2$. If the

In applying this result to black holes, Page assumed

> *"the black hole subsystem has dimension n . . . the radiation subsystem has dimension m . . . [and] these subsystems form a total system in a pure state in a Hilbert space of dimension mn."*

- But since the Hilbert space for a black hole spacetime does not factorize in this manner, there is no reason to expect the entropy at $\mathcal{I}^+$ to follow a Page curve.

Black holes vs ordinary thermodynamic systems



We live in a world where gravity is presumably quantized.

So why do we still expect a Page curve for ordinary thermodynamic systems if not for black holes.

Coarse graining for ordinary systems

To see a Page curve we need to keep track of effects of size e^{-S}, where S is degeneracy of microstates.

- For ordinary systems, we can work in a limit where

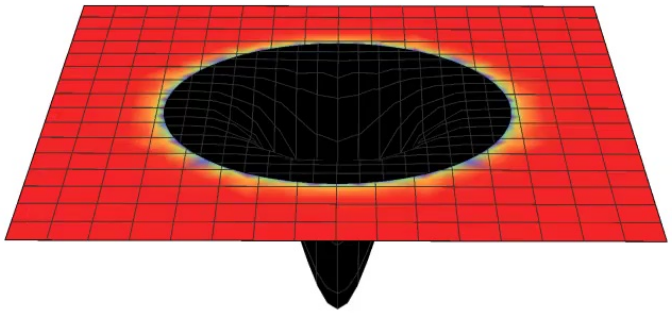
$$GE^{d-2} \to 0$$

but

S remains *finite*.

for ordinary systems, there is an obviously protocol for coarse-graining where we keep track of effects of size e^{-S} but ignore the unusual localization of information in gravity.

Coarse graining in gravity



For black holes

$$G E^{d-2} = \frac{\alpha}{S_{\rm BH}}$$

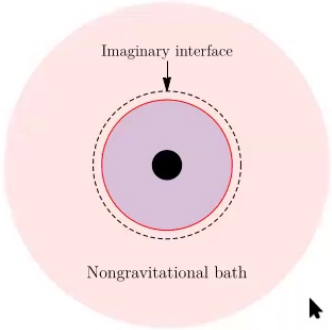
where $E =$ energy of Hawking quanta.

For a black hole, there is no obvious way to keep track of effects of size e^{-S} but forget about the unusual localization of information in gravity

Different levels of black-hole entropy

Coarse-grained observables (Few-point correlators of light fields)	Thermodynamic entropy
Fully fine-grained observables (All asymptotic correlators)	All information at all times.
Intermediate observables ?	Page curve?

Relationship to Page-curve computations



Recent computations of the Page curve do not contradict our results. These models involve non-standard gravity

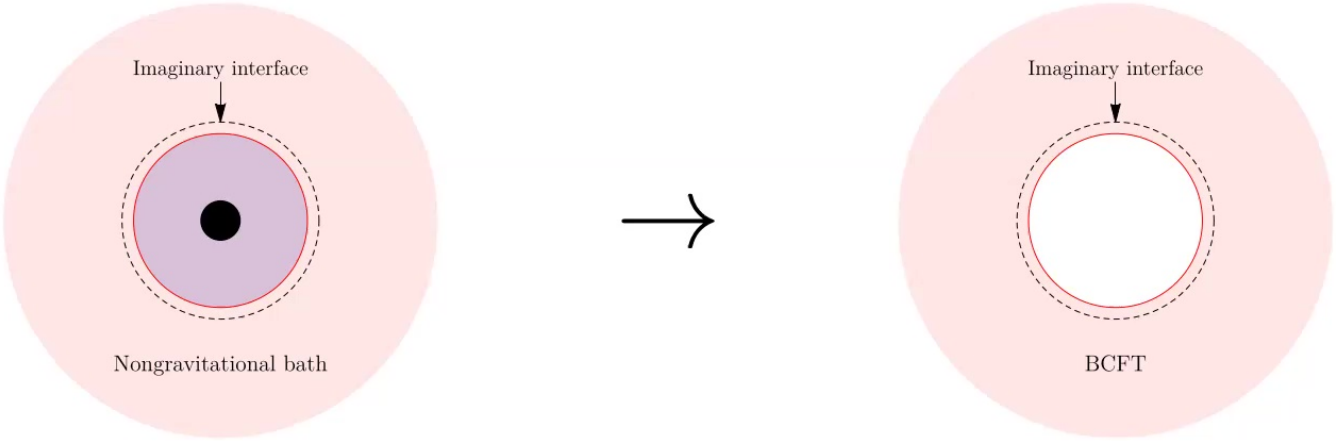
[Pennington, Almheiri, Engelhardt, Marolf, Mahajan,,]

[Maxfield, Maldacena, Hartman, Shaghoulian,]

[Tajdini, Stanford, Shenker, Yang..., 2019]

1. Gravity not dynamical throughout space. Models necessarily involve a nongravitational bath.
2. Graviton has a mass. No Gauss law in these models.

Nature of information transfer

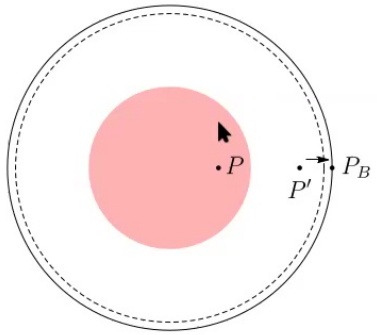


- Entire system has a nongravitational description. Page curve computes information transfer between nongravitating subregions.
- If the bath is made gravitating, the Page curve vanishes.

[Geng, Karch, Perez-Pardavila, S.R., Randall, Riojas, Shashi, 2020]

- So the calculations are genuinely nongravitational. Not computing Page curve of "Hawking radiation".

Puzzle with the Gauss law



Page curve computations involve the “island proposal” that suggests a factorization of the bulk Hilbert space.

But when Gauss law applies, islands suffer from inconsistency within perturbation theory without using the full power of the previous results.

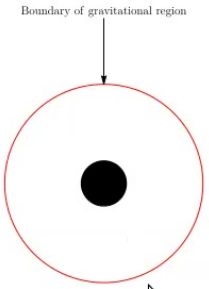
[Geng, Karch, Perez-Pardavila, S.R., Randall, Riojas, Shashi, 2021]

$$\frac{\partial}{\partial\lambda}\langle e^{-i\lambda\phi(P)} H \frac{\partial\phi(P')}{\partial t} e^{i\lambda\phi(P)}\rangle = \mathrm{O}\left(G_N\right)$$

Bath and graviton mass

But almost all computations of islands have been performed in massive theories of gravity.

[Geng, Karch, 2020]



- Before coupling to the bath,

$$\partial_\mu T^{\mu\nu} = 0.$$

After coupling to bath,

$$\partial_\mu T^{\mu\nu} \neq 0.$$

- So $T_{\mu\nu}$ picks up an anomalous dimension $\Rightarrow$ AdS graviton picks up a mass.

[Aharony, Clark, Karch, 2006]

Constraints in massive vs massless gravity

- In the linearized theory, in standard gravity

$$\frac{1}{16\pi G}\left(\partial_j\partial_i h_{ij} - \partial_j\partial_j h_{ii}\right) = \rho,$$

So

$$H = \int_V \rho = \frac{1}{16\pi G}\int_{S_\infty} n_j(\partial_i h_{ij} - \partial_j h_{ii}),$$

But, in massive gravity,

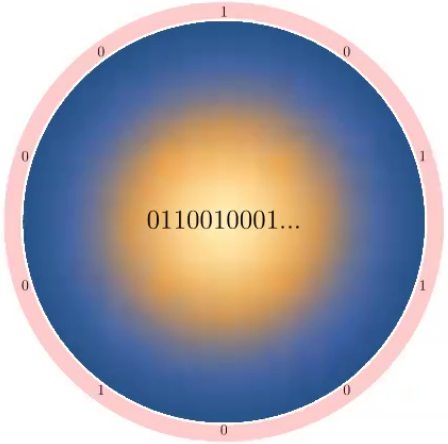
$$\frac{1}{16\pi G}\left(\partial_j\partial_i h_{ij} - \partial_j\partial_j h_{ii} + m^2 h_{ii}\right) = \rho.$$

So the Hamiltonian is not a boundary term in massive gravity $\Rightarrow$ Gauss law does not apply and neither does holography of information.

- Islands can be shown to be consistent in doubly holographic Karch-Randall models with massive gravity.

Summary

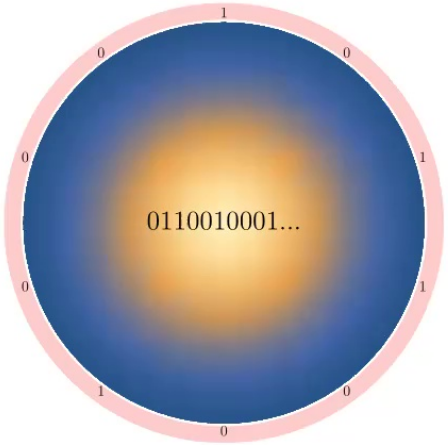
Standard theories of gravity store quantum information very differently from local quantum field theories.



- This is a nonperturbative result that relies on weak assumptions about the UV theory.
- But for simple states, it can be verified in perturbation theory.
- Sheds light on why gravitational theories are holographic.

Summary

Accounting for this unusual localization of information resolves several paradoxes about black holes.



- Forgetting about this property of gravity, and assuming factorization of the Hilbert space often leads to paradoxes!
- The idea that entropy of Hawking radiation obeys a “Page curve” also involves an assumption of factorization.
- Recent computations of the Page curve involve massive gravity and a nongravitational bath where the Hilbert space does factorize.