Title: Twisted eleven-dimensional supergravity and exceptional Lie algebras

Speakers: Ingmar Sabieri

Series: Mathematical Physics

Date: November 05, 2021 - 1:30 PM

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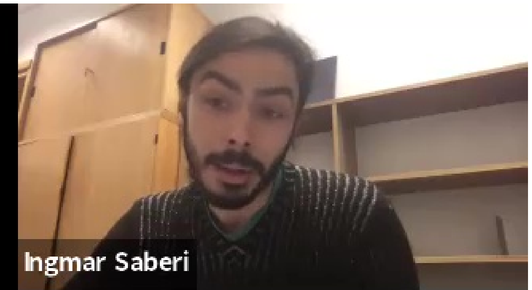
Abstract: In recent years, there has been a great deal of progress on ideas related to twisted supergravity, building on the definition given by Costello and Li. Much of what is explicitly known about these theories comes from the topological B-model, whose string field theory conjecturally produces the holomorphic twist of type IIB supergravity. Progress on eleven-dimensional supergravity has been hindered, in part, by the lack of such a worldsheet approach. I will discuss a rigorous computation of the twist of the free eleven-dimensional supergravity multiplet, as well as an interacting BV theory with this field content that passes a large number of consistency checks. Surprisingly, the resulting holomorphic theory on flat space is closely related to the infinite-dimensional exceptional simple Lie superalgebra E(5,10). This is joint work with Surya Raghavendran and Brian Williams.

Zoom Link: https://pitp.zoom.us/j/99622967785?pwd=YmlQWW1sNW1qS1FhQkV4NXFlY0Nsdz09

Twisted eleven-dim^{l} supergravity
and exceptional Lie superalgebras

Perimeter, Nov. 5, 2021

Based on: 2111.03049 (w. Raghavendran, Williams)
also: 2106.15639 w. Williams
(2111.01162 w. Eager/Hahner/Williams)



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An early example of an L_∞ alg. "controlling" a moduli problem:
Kodaira-Spencer thy of complex structure deformations.

As a field theory, this "wants" to be related to gravity.

(Example of a "gravity-like" moduli problem:
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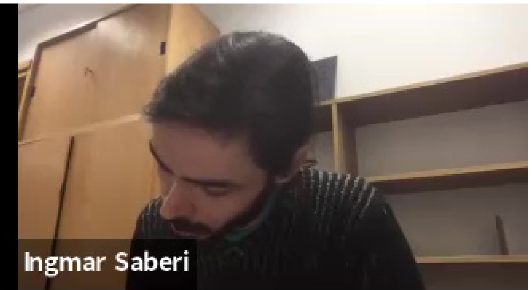

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Einstein metrics

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Work of Bershadsky-Cecotti-
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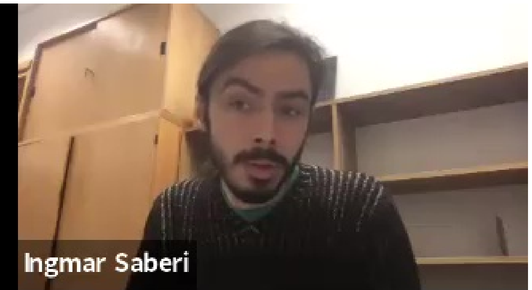

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- 1 -

Work of Bershadsky - Cecotti - Ooguri - Vafa interprets KS theory as "the string field theory of the B-model".

Pushed farther by Costello - Li

$\boxed{\text{Conj}}$: (A version of) the BCOV theory is holomorphically twisted IIB supergravity.

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the B-model".

Pushed farther by Costello-Li

[Conj]: (A version of) the BCOV theory is holomorphically twisted IIB supergravity.

Other supergravity theories (11d) were harder to work on: no worldsheet perspective.

① Twisting made easy

② An interacting twisted thy in 11d

③ Relation to the simple superalgebra $E(5,10)$

④ Evidence that this thy is actually 11d SUGRA

⑤ A strategy of proof for that fe

- 2 -

4 Seiten

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③ Relation to the simple superalgebra

④ Evidence that this thy is actually 11d SUGRA $E(5,10)$

⑤ A strategy of proof for that fact

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① Twisting via "pure spinors":

Y = space of square-zero elements in (any) supersymmetry alg

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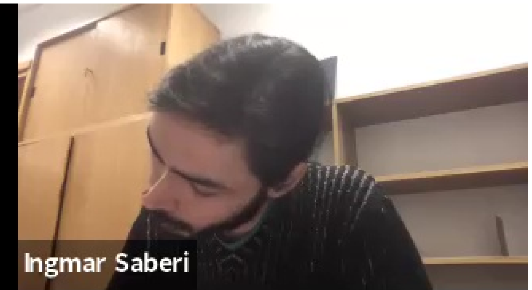

① Twisting via "pure spinors".

Y = space of square-zero elements in (any) supersymmetry alg.

$$\mathcal{O}_Y = H^0(\text{supertranslations})$$
$$\text{"} \quad \mathfrak{t}$$

There's a "most interesting" multiplet for $\mathfrak{t}$:

$$A^\bullet = C^\infty(\mathfrak{t})$$

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5 Seiten

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① Twisting via "pure spinors":

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$$\qquad\qquad\qquad\qquad \text{"} \; t$$

$$= \mathbb{C}[\lambda^\alpha]/I$$

There's a "most interesting" multiplet for t:

$$A^\bullet = \left(C^\infty(t) \otimes_{\mathbb{C}} \mathcal{O}_Y, \; D = \lambda^\alpha D_\alpha \right).$$

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5 Seiten

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$$A^\bullet = \left(C^\infty(t) \otimes_{\mathbb{C}} \mathcal{O}_Y, \ \mathcal{D} = \lambda^\alpha D_\alpha \right).$$

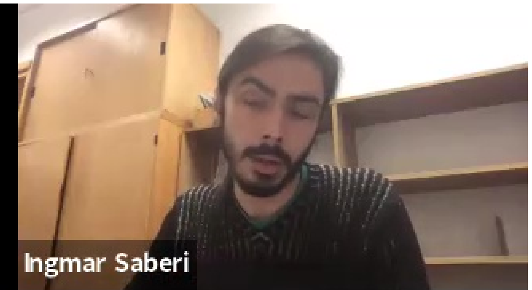
This algebra is filtered

-3-

$H^\bullet(\mathrm{Gr}\, A^\bullet)$ is sections of the bundle over spacetime, associated to the Koszul homology of $\mathcal{O}_Y$.

This is (a well-known) multiplet for the susy. alg.

<u>Fact</u>: Twisting commutes with forming the multiplet.



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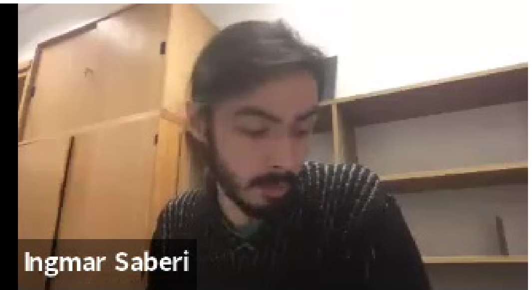

Fact: Twisting commutes with forming the multiplet.

Ex. 10d $N=1$ No odd elements in $H^\bullet(\mathrm{ad}_Q)$.

$\Longrightarrow \Omega^{0,*}(\mathbb{C}^5)$ = fields of twisted 10d SYM. (Baulieu).

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Ex. 10d IIB.

Odd elements = bivectors and one-forms.

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(Baulieu).

- 4 -

Ex. 10d IIB.

Odd elements = bivectors and one-forms.

Koszul homology:

$$\begin{array}{ccccc} 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & 5 & 1 & \cdot & \cdot \\ \cdot & \cdot & 11 & 10 & 1 \\ \cdot & \cdot & \cdot & \cdot & 1 \end{array} \cong \begin{array}{c} \Omega^0 \xrightarrow{\partial} \Omega^1 \xrightarrow{\partial} \Omega^2 \oplus \Omega^0 \\ \Omega^0 \xrightarrow{\partial} \Omega^1 \oplus PV^1 \xrightarrow{\partial} PV^0 \\ PV^0 \end{array}$$

These are the fields of (a particular
poten

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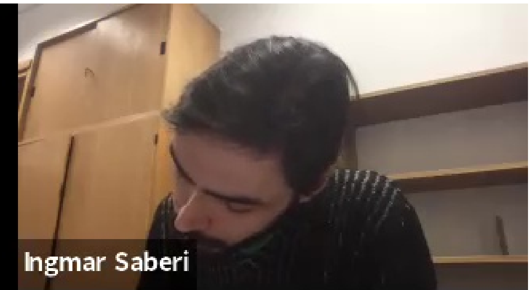

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\cdot & 5 & 1 & \cdot & \cdot \\
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\cdot & \cdot & \cdot & \cdot & 1
\end{array}
\cong
\begin{array}{ccc}
\Omega^0 & & \\
\downarrow \partial & & \\
\Omega^1 & \Omega^0 & PV^0 \\
\downarrow \partial & \downarrow \partial & \\
\Omega^2 \oplus \Omega^0 & \Omega^1 \oplus PV^1 & \\
 & \downarrow \partial & \\
 & PV^0 &
\end{array}
$$

These are the fields of (a particular "potential theory" for) minimal BCOV theory on $\mathbb{C}^5$.

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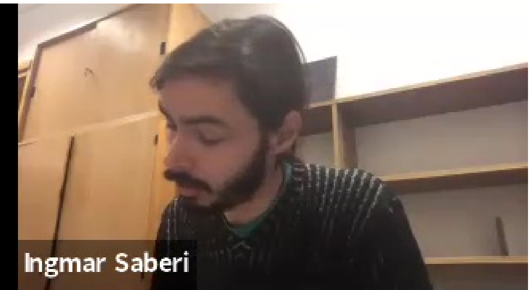

Koszul homology:

$$\begin{matrix}1&\cdot&\cdot&\cdot&\cdot\\ \cdot&5&1&\cdot&\cdot\\ \cdot&\cdot&11&10&1\\ \cdot&\cdot&\cdot&\cdot&1\end{matrix} \cong \begin{tikzcd}[column sep=tiny] \Omega^0 \arrow[dr, "\partial"] & & & & \\ & \Omega^1 \arrow[dr, "\partial"] & \Omega^0 \arrow[dr, "\partial"] & & PV^0 \\ & & \Omega^2 \oplus \Omega^0 & \Omega^1 \oplus PV^1 \arrow[dr, "\partial"] & \\ & & & & PV^0 \end{tikzcd}$$

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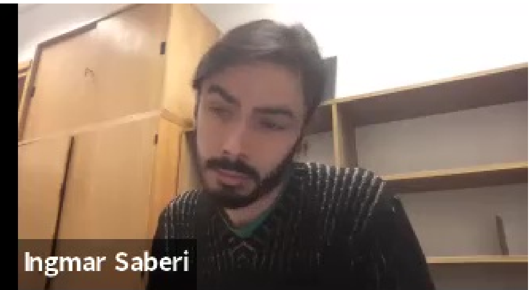


What about 11d?

odd elements = hol. 2-forms.

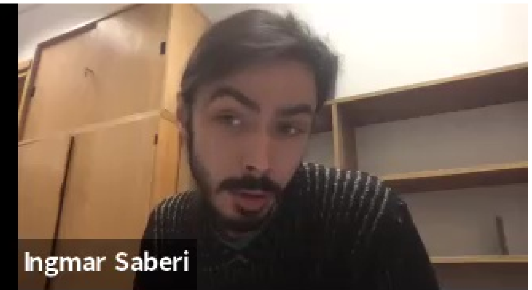
$$Y = Gr(2,5).$$

Koszul homology:

$$\begin{array}{cccc} 1 & \cdot & \cdot & \cdot \\ \cdot & 5 & 5 & \cdot \\ \cdot & \cdot & \cdot & 1 \end{array} \cong \quad \Omega^0 \xrightarrow{\partial} \Omega^1 \quad PV^1 \xrightarrow{\partial_\Omega} PV^0 .$$

Fields of the twisted 11d SUGRA multiplet.

$$\Gamma PV^1 = \left(PV^{1,\bullet}(\mathbb{C}^5) \otimes \Omega^\bullet(\mathbb{R}), \; \partial + d_{dR} \right).$$



6 Seiten

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$-$	$+$
PV^1	$\to PV^0$
Ω^0	$\to \Omega^1$.

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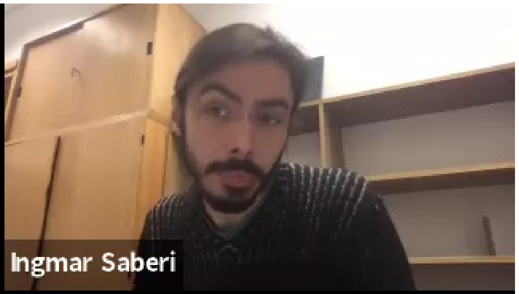

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$$PV^1 \to PV^0$$

$$\Omega^0 \to \Omega^1.$$

(Eager-Hahner did the maximal twist in components.)

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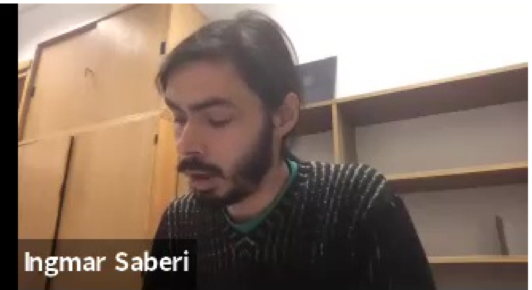

$$-\qquad\qquad +$$

$$PV^1 \longrightarrow PV^0 \quad \circledast$$

$$\Omega^0 \longrightarrow \Omega^1.$$

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§2 Interactions

There's an obvious way of putting interactions on this theory.

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- 6 -

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(Equip base w/ Schouten bracket.)

Form the BF theory.

$$I = \gamma[\mu,\mu] + \beta[\mu,\nu].$$

Another, equivalent,

7 Seiten

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Another, equivalent, way of writing this:

$$I = \frac{1}{1-\gamma}\,\partial\gamma(\mu \wedge \mu).$$

This action admits a defor

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9 Seiten

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Another, equivalent, way of writing this:

$$(\star)\quad I = \frac{1}{1-\gamma}\,\partial\gamma(\mu\wedge\mu).$$

This action admits a <u>deformation</u> by

$$J = \gamma\wedge\partial\gamma\wedge\partial\gamma.$$

Related to the CS term in 11d SUGRA.

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<u>Claim</u>: $(I+J)$ is

(Conjectu

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Related to the CS term in 11d SUGRA.

Claim: (I + J) is (should be)
(Conjecture) the action for
hol. twisted 11d SUGRA.

There are a number of consistency checks that can be made.

- Reduces to type IIA in its sl

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There are a number of consistency checks that can be made.

- Reduces to type IIA in its $\mathfrak{sl}(4)$ twist (conj. by Costello-Li).

- Further twist: equivalent to Poisson Chern-Simons thy.

- Residual SUSY maps into the gauge algebra.

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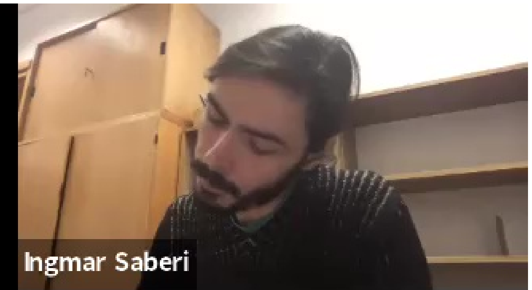
the gauge algebra.

In fact,
$$H^\bullet(\text{m2brane}, \mathrm{ad}_{\mathbb{Q}}) \longrightarrow \text{Fields}[-1].$$

What's the L_∞ alg. structure on fields? We can consider its minimal model on flat space

Coh:

+	−
$\mathrm{Vect}^{\mathrm{div}}(\mathbb{C}^5)$	$\cdot$
$\mathbb{C}$	$\Omega^2_{cl}(\mathbb{C}^5)$



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$$\cong \quad E(5,10) \oplus \mathbb{C}$$

Lie structure: as written.

There's a 3-ary higher operation:

$$(\mu, \mu', \alpha) \longmapsto \alpha(\mu \wedge \mu')\big|_{z=0} \in \mathbb{C}.$$

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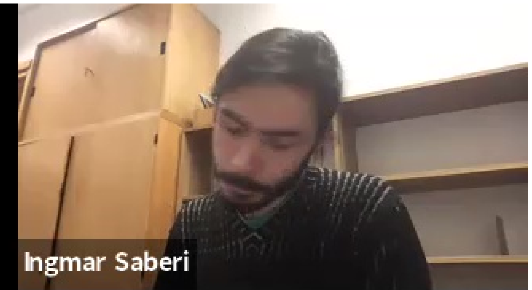

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$H^\bullet(\underbrace{\text{m2-brane}}, \mathrm{ad}_Q)$:

$+$	$-$	$+$
$sl(5) \oplus \mathbb{C}$	$\Omega^2_{\text{const.}}$	$\mathbb{C}^5$

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$z=0$

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$H^\bullet(\text{m2-brane}, \mathrm{ad}_Q)$:

$$\begin{array}{ccc} + & - & + \\ \mathfrak{sl}(5) \oplus \mathbb{C} & \Omega^2_{\text{const.}} & \mathbb{C}^5 \end{array}$$

$$\mu_3 : (v, v', \alpha) \longmapsto \alpha(v \wedge v').$$

Thm :

$$H^\bullet(\text{m2brane}, \mathrm{ad}_Q) \longrightarrow \widehat{E(5,10)}.$$

odd.

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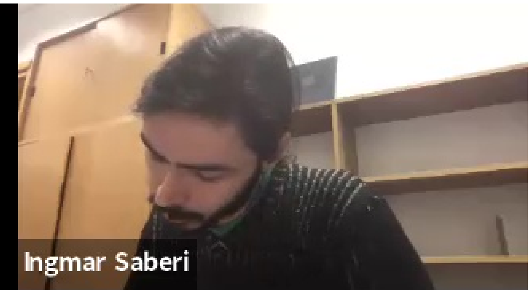

$$+ \qquad - \qquad +$$

$$\mathfrak{sl}(5) \qquad \Omega^2_{\text{const.}} \qquad \mathbb{C}^5$$

$$\oplus$$

$$\mathbb{C}$$

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Thm :

$$H^\bullet(\text{m2brane}, \mathrm{ad}_Q) \longrightarrow \widehat{E(5,10)}.$$

odd elements

$$\mapsto z_i\, dz_j - z_j\, dz_i \in \Omega^1_\gamma$$

$$\mathfrak{sl}(5) \mapsto z_i \partial_j \in PV^1_\mu$$

$$\mathbb{C}^5 \mapsto \partial_j \in PV^1_\mu.$$

$$\mathbb{C} \mapsto 1 \in \Omega^0_\beta.$$

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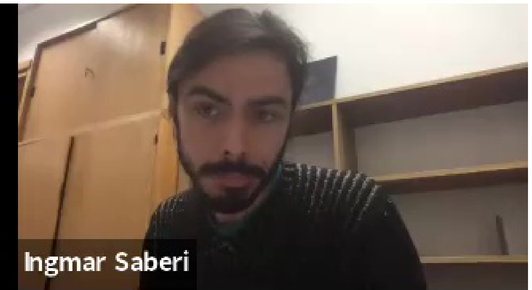

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Can also do this for twisted superconformal algebras.

M2: $osp(6|1)$

M5: $osp(6|1)$

There are two distinct maps

$$\begin{tikzcd} osp(6|1) \arrow[r, bend left, "m2"] \arrow[r, bend right, "m5"'] & 11d \text{ fields} \end{tikzcd}$$

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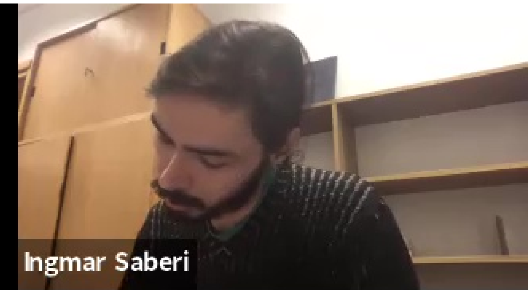

twisted superconformal algebras:

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There are two distinct maps

$$\begin{tikzcd} osp(6|1) \arrow[r, bend left, "m2"] \arrow[r, bend right, "m5"'] & \underbrace{\text{11d fields}}_{(\text{or } E(5,10))} \end{tikzcd}$$

These factor through two other exceptional simple algebras:

m5: $E(3,6)$

m2: CK_6

Further: Twist of abelian $(2,0)$ multiplet has a module structure